STRENGTH AND FAILURE ANALYSIS OF COMPOSITE LAMINATES WITH HOLES USING FINITE ELEMENT METHOD

By:

ADEELA KHAN

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Thesis Supervisor:

Dr. Basharat –Ullah-Malik

College of Electrical & Mechanical Engineering

National University of Sciences & Technology

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CHAPTER 1

INTRODUCTION

1.1. Composite

The term "*composite material*" indicates a material which is formed by combining at least two different materials on a macroscopic scale. Infact, the term composite could mean almost anything if taken at face value, since all materials are composed of dissimilar subunits if examined at close enough detail. Some materials are found in nature, which possess all of the characteristics of composite materials; however, they are not categorized as composites. According to the above definition, composites are artificially manufactured to have predetermined properties.

In modern materials engineering, composites are composed of "fibers" of various forms, and a "matrix". The matrix groups the fibers together and puts them in fixed positions. Also, it protects the fibers from environmental attacks such as corrosion, oxidation, etc. mechanically, the matrix provides the load transferring mechanism to the fibers, so that a laminate can still carry a load after fibers break. Fibers play a significant role in composites. They provide the superior properties of a composite including the high strength and stiffness. For instance, the term "FRP" (for Fiber Reinforced Plastic) usually indicates a thermosetting polyester matrix containing glass fibers, and this particular composite has the lion's share of today's commercial market.

The main purpose of combining two or more constituent materials to form a composite is to generate a new material having better overall properties than its constituent and often some qualities that neither constituent possesses [44]. The properties that can be improved by forming a composite material include:

Composite materials offer "*high strength*" and "*low weight*", the properties that compel composite materials into new arenas. Apart from these, composite materials offer "*low coefficient of thermal expansion (CTE)*" and "*good vibrational damping*". Composites provide excellent "*fatigue resistance*", "*corrosion resistance*", *"temperature resistance"* and also "*wear resistance",* which are very helpful to reduce product life cycle cost. Composites provide "*excellent design/fabrication flexibility*" which significantly decreases the number of parts needed for a specific application. Also composites improve the overall appearance of the material making it more "*attractive*". These characteristics have propelled composites into wider use.

1.2. Classification of Composite Materials

There are two classification systems of composites materials. One of them is based on the matrix material (metal, ceramic, and polymer) and the second is based on the material structure:

1.2.1. Classification Based on Matrix Material

1) Metal Matrix Composites (MMC)
2) Ceramic Matrix Composites (CMC)
3) Polymer Matrix Composites (PMC)

1) Metal Matrix Composites (MMC)

MMC are composed of a metallic matrix (aluminium, magnesium, iron, cobalt, copper) and a dispersed ceramic (oxides, carbides) or metallic (lead, tungsten, molybdenum) phase.

2) Ceramic Matrix Composites (CMC)

CMC are composed of a ceramic matrix and imbedded fibers of other ceramic material (dispersed phase).

3) Polymer Matrix Composites (PMC)

PMC are composed of a matrix from "Thermoset" (Phenolics, Polyester, and Epoxy) or "Thermoplastic" (Acrylics, PEEK, and Carbonates)

Polymeric composites are primarily used for structural applications, whereas Metal-matrix composites and Ceramic composites are mainly used for components that are exposed to high temperature environments.

1.2.2. Classification Based on Material Structure

On the basis of material structure, composites can be classified as;

1) Fibrous Composites
2) Structural Composites
3) Particulate Composites

1) Fibrous Composites

Fibrous composites consist of fibers in a matrix. These are further divided into two categories;

- Continuous fiber-reinforced composites
- Discontinuous fiber-reinforced composites

The arrangement or orientation of the fibers relative to one another, the fiber concentration, and the distribution all have a significant influence on the strength and other properties of fiber-reinforced composites. Consideration of orientation and fiber length for a particular composite depends on the level and nature of the applied stress as well as fabrication cost.

Reinforcing fibers can be made up of Carbon, Kevlar, or Glass. Fibers increase the modulus of matrix material [44]. Also fibers are difficult to process into composites, which makes FRP relatively expensive. These are used in some of the most advanced, and expensive sports equipment, such as time-trial racing bicycle frame which is made up of carbon fibers in a thermoset polymer matrix. Also glass fibers (or fiberglass) in a thermoset matrix is used in body parts of race cars. Discontinuous fibers (often randomly oriented in the matrix material) are normally used in applications involving multidirectional applied stresses. Also intricate shapes can be formed with discontinuous short-fiber composites (both aligned and randomly oriented) having high production rates, which are not possible with continuous fiber reinforcement. Aircraft and aerospace industries mainly use continuous fiber-reinforced composites because of their superior mechanical properties.

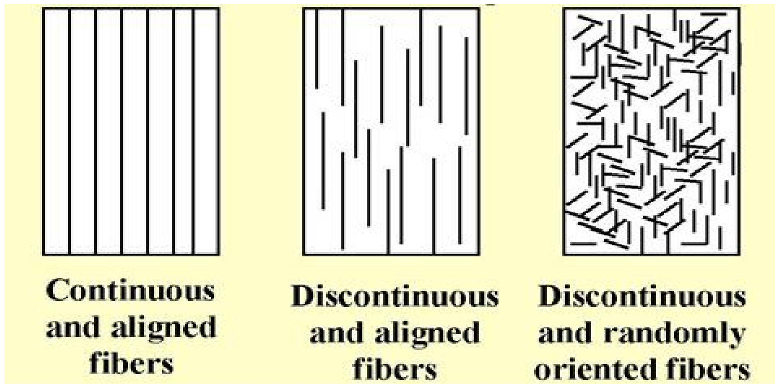

Fig. (1.1) Fiber Orientations in Fiber Reinforced Composites

2) Structural Composites

The properties of structural composites depend on “Constituents” and “Geometrical Design”.

These are further divided classified into two types;

- **Laminated Composites**

Laminated composites are constituted of successive layers (sometimes called plies) of reinforcements impregnated with resins. The layers differ by the constituents, the layer orientations, etc. In general, the reinforcement in each layer is of various kinds (strands, rovings, mats, cloths, glass fibers, carbon fibers, Kevlar fibers, etc.). Each layer is then designated by the nature of fibers (glass, carbon, Kevlar, etc.) and the types of reinforcement. Typical laminate construction is shown in figure (1.2)

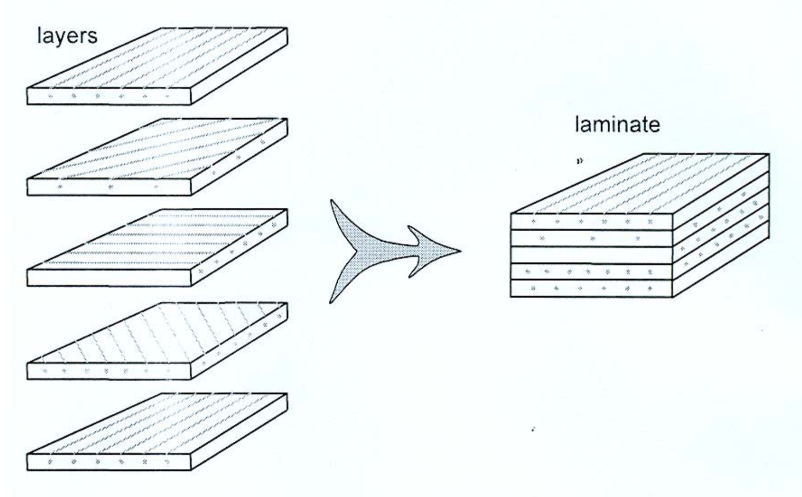

Fig. (1.2) Laminate Construction

The choice for the nature and the stacking sequence of layers depend upon the use of composite material, keeping in view the following points:

- Unidirectional layers have good mechanical properties in the direction of fibers.
- Mats have low resistance to tension and have a good resistance in compression.
- A cross-ply laminate is sensitive to interlaminar delamination.
- A lamination with atleast three fiber directions (0^o, 90^o, and 45^o) is necessary if quasi-isotropy is required in the plane of the laminate.
- Symmetric lamination usually avoids any warping of the laminate after demoulding.

Lamination is done to achieve the best aspects of the constituent layers such that the orientation of the high strength direction varies with each successive layer. Plywood is a common example of a laminated composite. The properties that can be emphasized by lamination are strength, stiffness, low weight, wear resistance, corrosion resistance, thermal insulation, acoustical insulation and attractiveness.

- **Sandwich Composites**

The principle of sandwich construction consists in coating two strong sheets, called *skins/ face sheets* on both sides of *core* with the help of adhesives.

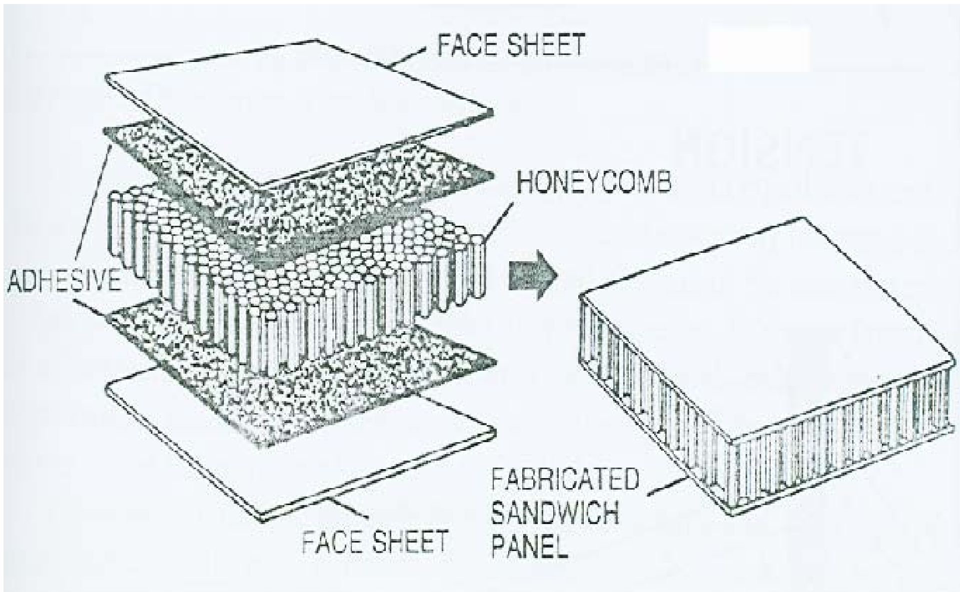

Fig. (1.3) Structural Sandwich Construction [43]

In composite sandwiches, the face sheets are most frequently constituted of composite panels (glass fibers, carbon fibers or Kevlar fibers). Light alloy sheets are also used. These are typically 0.01” to 0.5” thick and are chosen on the basis of weight, strength, and fabricability. They carry most of the bending load and stresses. The cores can be made up of many materials, but wood, rigid foam and honeycomb (thin sheets attached in such a way that cells are formed with a resulting appearance much like a bee honey comb) are the most common. Their densities are less than the face sheets. The cores carry shear/ compressive loads.

The objective of sandwich construction is to obtain a material having *lightness* and *high flexural stiffness.*

3) Particulate Composites

Particulate composites consist of particles of one or more materials suspended in a matrix of another material. Particles can be either metallic or non-metallic; similarly matrix can be either metallic or non-metallic. A common example of particle reinforced composites is concrete where the *aggregates* (sand and gravel) are the particles and *cement* is the matrix. Another example is *spheroidized steel* where cementite (Fe_3C) is the particle and ferrite (ά-iron) is the matrix, and cementite is transformed into a spherical shape which improves the machinability of the material.

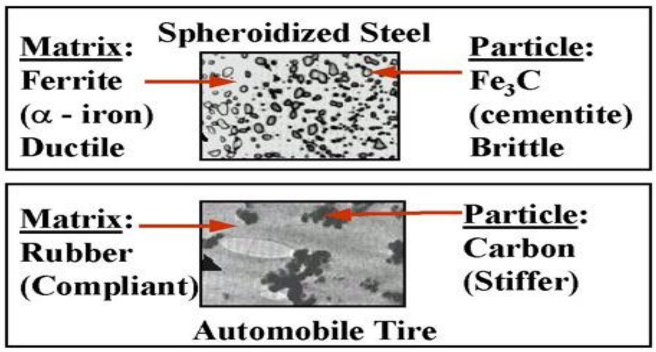

Fig. (1.4) Particle Reinforced Composites

Particles increase the modulus of the matrix. Besides that they also decrease the permeability and ductility of the matrix. Particles are also used to produce inexpensive composites. Particle reinforced composites support higher tensile, compressive and shear stresses.

Here is flowchart showing the classification of composite materials

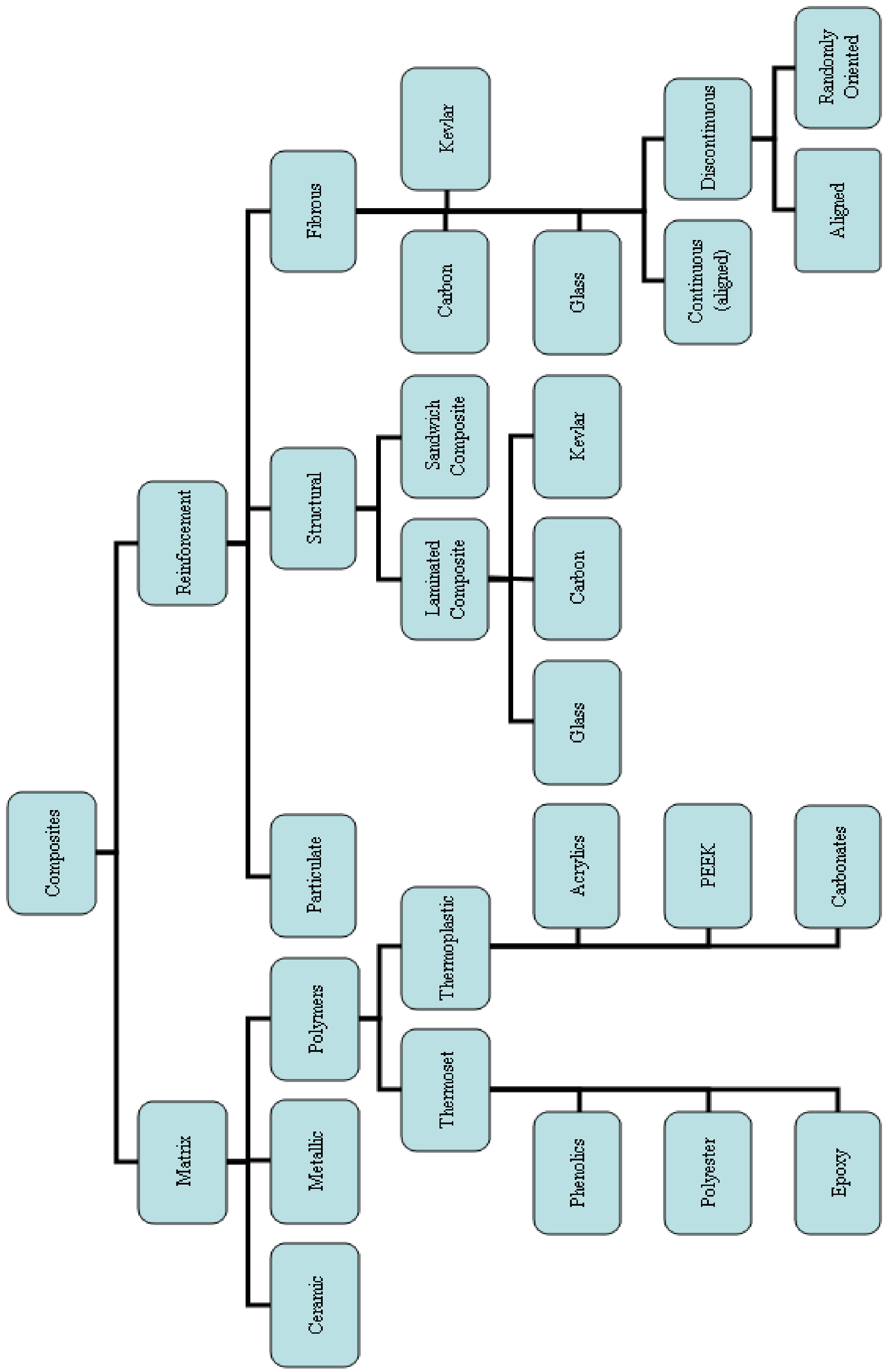

Fig. (1.5) Classification of Composite Materials

1.3. Applications of Composite Materials

Composite materials application can be traced back to the ancient history. For example, Israelites used straw to reinforce mud brick without which the bricks would have almost no strength [45]. Ancient Egyptians used plywood by rearranging wood to achieve superior strength, resistance to thermal expansion as well as swelling due to the presence of moisture. Also Medieval swords and armor were constructed with layers of different materials.

Today, composite materials are finding application in many industries.

- **Construction Applications**

Composite materials play a significant role in "*construction*", primarily in residential housing applications. Modular building panels made up of polyester and fiber glass are used for office partitions and for walls of portable or prefabricated dwellings. Fiberglass composites are finding niche applications in areas such as stay-in-place concrete forms, bridge decks, as well as entire bridges. Corrosion resistance, light weight (approximately one-fifth the weight of steel), high strength and ease of installation, are the properties which enable composite materials to be used as alternatives to the traditional materials to reduce dead load and extend structure life.

- **Marine Applications**

The most widespread use of composites in marine applications is for pressure hulls and buoyancy structures in submersibles. The most common material for these applications is fiberglass-reinforced epoxy and polyester. Also carbon fiber is making headway in a key deep sea well technology, the "*umbilical*", which is a bundled collection of steel or thermoplastic tubing and electric cabling used to transmit chemicals, hydraulic fluids, electric power and two-way communication and control between topside production vessels and subsea production equipment.

- **Sports Applications**

Composite materials have found application in sports goods also. For example, glass-reinforced composites (alone or in hybrids with other fibers) continue to replace wood and metal in fishing rods, tennis racquets, windsurfing masts, hockey sticks, kites, archery bows and arrows, skis and ski poles, water skis, tent poles and bicycle handlebars, as well as in niche applications such as fairings for recumbent bikes. Stiffness, strength, light weight, and aero-elastic tailoring of structural components to match the anticipated loads are the characteristics that enable the use of composite in place of traditional material. Sporting goods consume at least 11 million lb of carbon fiber annually, worldwide.

- **Electrical Applications**

Composites are also used in electrical applications, because of their nonconductive nature. The most common uses are printed circuit boards, insulators, and radomes. Most of the boards are made up of fabric reinforcement material, which gives some shear strength. Typical material used are paper / phenolic, cotton cloth / phenolic, nylon cloth / phenolic, glass cloth / phenolic, glass cloth / silicone, glass cloth / epoxy and paper / epoxy.

- **Automotive and Truck Applications**

Composite materials also found applications in automobiles and trucks, mostly in body panels, or in the case of the Corvette, the entire body. Besides that truck tractor bodies and sleeper units, as well as trailer bodies are made up of composite materials. The predominant materials used are fiberglass reinforced polyester, epoxies, or urethanes. Weight saving is the major incentive for using composites.

- **Space Structures Applications**

Composite materials are widely used in space structure applications also. Some areas in which composites have been used in apace are: trusses, platforms, pressure vessels and tanks, and shells. Weight saving, mechanical strength and stiffness are the main properties that compel composites to be used in such applications. Also composites have the ability to withstand the hostile environment of space (extreme temperature, high radiation, and high concentrations of molecular oxygen). Graphite/Epoxy, carbon reinforced epoxy composites are used in these applications

- **Rockets and Missiles**

The most important composite application in rocket and missiles is the rocket exit nozzle. Weight saving and thermal insulative capability of ablation-type composite materials are the main drivers for using them. The most common of these ablative composite materials is phenolic matrix reinforced with carbon fibers. Other applications include the nose cone and nozzle for a tactical missile, utilizing phenolic matrix reinforced with silica.

- **Aircraft Applications**

Composite materials have a wide range of application in both commercial and military aircraft. The main reason behind this is that composites satisfy mechanical strength requirements of various parts at a lower weight, which increases the range and maneuverability. Reduced manufacturing costs, improved corrosion, fatigue resistance, and improved flammability over traditional plastics materials are the main advantages. The use of composites is generally higher in military aircraft with regard to the percentage of total aircraft weight.

The first all- composite airplane was *Windecker Eagle* (1969) by the FAA. *Avtek 400* (1984) was fabricated entirely of composites (comprising mostly of Ke/Ep and Nomex honeycomb). In early 1980s, the *Beechcraft Starship 1* was fabricated for general aviation. It utilized nearly 100% composites for its body structures and was made up of AS4/3501-6 Graphite/Epoxy composite, making it the lightest among the airplanes of its size. Another aircraft made up of graphite/epoxy was *V-22* made by Bell and Boeing. The most important usage of composites is the *FY-22* and *B-2* aircraft, in which FY-22 uses more than 50% composites, while B-2 uses mostly composites comprising graphite/epoxy and high temperature polymeric composites. Large portions of *helicopter* structures are also made up composites.

(a) (b)

Fig. (1.6) All Composite Airplane (a) Avtek 400 (b) B-2

Unidirectional carbon reinforced epoxy is now being used in *Boeing 767* for door spring which were previously made up of titanium or steel, reducing its weight to about one-third as much as of steel spring and one-half as much as of the titanium spring while still giving equivalent mechanical performance. Aircraft *wing ribs* are also made by carbon reinforced thermoplastic composite.

Composites are mostly targeted to the fuselage, wing, wing boxes and empennage. UAVs are currently the fastest-growing segment of the aerospace sector. Composites are the material of choice for UAV airframes, which can range from a few inches in length to the size of commercial airliner. High strength-to-weight and limited radar signal transparency are the main drivers.

CHAPTER 2

LITERATURE REVIEW

Whenever a machine component changes the shape of its cross-section, the simple stress distribution no longer holds good and the neighborhood of the discontinuity is different. This irregularity in stress distribution caused by abrupt changes of form is called **"Stress Concentration"**. It occurs for all kinds of stresses in the presence of fillets, notches, holes, keyways, splines, surface roughness and scratches. Similarly, a hole in composite laminate greatly reduces the strength of the material because of the stress concentration around the hole. The stress distribution around the hole and the resulting damage leading to the failure of composite laminates is of obvious interest to the researchers.

Several researchers have employed various failure criterias for predicting the notched strength of composite laminate. The point stress and average stress criterias have been used by various researchers in the past [2, 3, 4, 5, 8, 9, 32]. These models have been used to predict the strength reduction of composite laminates due to the presence of hole of varying size. All of them assumed the characteristic length to be a material constant which is contrary to the experimental results. So Karlak R.F. [7] presented a criteria which shows the dependency of characteristic length on hole size.

Most recently, a two dimensional finite element analysis based on Yamada-Sun failure criteria was used by H.A. Whitworth, M. Othieno, and O. Barton [8] to evaluate joint failure of composite laminate. They applied Point stress failure criteria to find the characteristic length in tension and compression for a plate containing a circular inclusion. They compared the data with the available experimental data for graphite/epoxy laminate. The analysis showed that a power function relationship exists between the characteristic length in tension and hole size.

M. Yasar Kaltakei [15] used modified distortion energy and Tsai-Hill failure theories for finding the stress concentration in anisotropic plate with circular holes subjected to tension or compression. He found that the strength of the plates is highly dependent on the fiber orientation angle (α), the strength of the plates decreases with the inclusion of hole, and strength is minimum when $\alpha = 0$. Also the maximum stress concentration and its location are not dependent on the stress which causes the failure and its location.

Also Hashin and the maximum stress failure criteria were used by several authors [26] to determine the progressive ply failure. They predicted the bearing response of pin loaded composite plates with different stacking sequences and a comparison was made with the available experimental results. They concluded that when shear stress-stain relationship is linear, the use of maximum stress criterion for fiber failure leads to a more realistic and higher strength than Hashin criterion. Also if a nonlinear shear stress strain relationship is considered, both the criterias converge towards the same predictions.

The behavior of buckling of laminated composite circular plates having circular holes and subjected to uniform radial load using shear deformation theory has been investigated [11]. Finite element method was used for this. The effects of hole sizes, location of holes, thickness and boundary conditions on the buckling load were considered. It was concluded that when thickness is kept constant, circular plate is more resistive to buckling than the plate having variable thickness. Also chances of buckling increase as the distance between the hole and centre of plate decreases. Also increasing the hole size, increases the rate of buckling.

Moumita Roy [32] investigated the stress concentration around the hole with various ratios of hole size to the laminate width using finite element method. She concluded that the notched strength of a laminate decreases with increase in d/w ratio for a constant "θ". Also with a constant d/w ratio, increasing the value of "θ" cause a decrease in the value of notched strength.

Buket Okutan [33] investigated the stress and failure analysis of laminated composite pinned joints. He concluded that the net-tension strength of a single hole joint is strongly dependent on ply orientation and specimen width. $[90/\pm45]_s$ and $[\pm45]_s$ laminates are least effected with the change in w/d ratio, while $[90/0]_{2s}$ and $[0/90/0]_s$

laminates are most effected. The best performance is shown by $[0/\pm45]_s$ laminates. Also the shear strength of single hole joints is strongly dependent on the ply orientations. Shear stress at failure reduces to about 50% for all laminates except $[90/\pm45]_s$, while for $[90/\pm45]_s$ it reduces to about 66%.

Lay-up independent fracture model was proposed [31] for predicting the notched strength of composite laminates. Finite element analysis of notched composites was done to determine the behavior of stress concentration near the notch tip in finite width laminates. Effects of notch tip profile and notch width on stress field was investigated.

2.1. Failure Theories

In the past, various attempts have been made to predict the notched strength of composite laminates using different failure criterias. These include the extension of linear elastic fracture mechanics, to modified isotropic plate theory technique, to mechanics of materials analysis, to detailed finite element techniques. Each technique has its advantages and disadvantages and involves different assumptions, effort, and knowledge of material properties.

Various failure theories used by researchers for predicting notched strength of composite laminates are as follows.

2.1.1. Waddoups, Eisenmann, and Kaminski (WEK) Failure Theory

WEK model [6] is based on a plane strain Mode I crack for a laminate containing either a hole or a crack. They assume that there exists regions of intense energy of length 'a', developed at the edges of the hole in a direction transverse to the loading direction. Figure (2.1) represents a WEK fracture model.

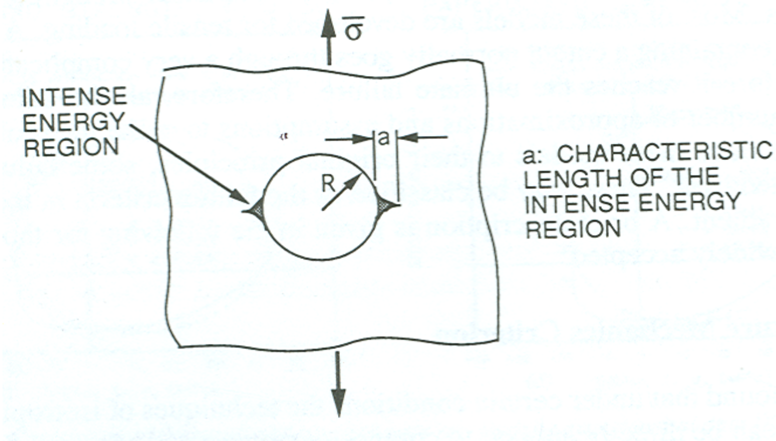

Fig. (2.1) Waddoups-Eisenmann-Kaminski Failure Criteria

These intense energy regions do exist because of stress concentrations around the hole or crack in the laminate. The model further assumes the characteristic length 'a', to be small, implying that failure strength of the laminate will occur at the vicinity of the crack.

A solution for the problem of symmetrical cracks emanating from a circular hole of radius R has been developed by Bowie and is given as;

$$K_I = \bar{\sigma}\sqrt{\pi a}\, f\left(a/R\right) \qquad \text{Eq. (2.1)}$$

The notched strength of the composite laminate at failure, can be obtained by substituting the applied stress by σ_N^{∞} in Eq (2.1) and is given as;

$$\sigma_N^{\infty} = \frac{K_{IC}}{\sqrt{\pi a}\, f\left(a/R\right)} \qquad \text{Eq. (2.2)}$$

Where

K_{IC} = Fracture Toughness

σ_N^{∞} = Notched Strength of a laminate of infinite width

a = Characteristic length of the intense energy region

R = Hole Radius

The strength of the laminate with no hole can be obtained from Eq. (2.2) by letting R equal to zero,

$$\sigma_o = \sigma_N^\infty \big|_{a/R \to \infty} = \frac{K_{IC}}{\sqrt{\pi a}(1.00)} \qquad \text{Eq. (2.3)}$$

The ratio of notched strength of a laminate of infinite width (σ_N^{∞}) and unnotched strength (σ_o) is obtained by combining Eq.(2.2) and Eq(2.3)

$$\frac{\sigma_N^\infty}{\sigma_o} = \frac{1}{f(a/R)} \qquad \text{Eq. (2.4)}$$

The stress intensity factor of an isotropic material containing a crack of length 2c is given by Griffith;

$$K_I = \bar{\sigma}\sqrt{\pi c} \qquad \text{Eq. (2.5)}$$

The critical stress intensity factor at failure is given as;

$$K_{IC} = \sigma_N^\infty \sqrt{\pi (c + a)} \qquad \text{Eq. (2.6)}$$

where a and (c+a) are the crack tip damage zone and the effective crack length, respectively.

For the case of unnotched laminates, the strengths can be obtained by letting c equal to zero.

$$K_{IC} = \sigma_o \sqrt{\pi a} \qquad \text{Eq. (2.7)}$$

Combining Eq. (2.6) and Eq. (2.7) results in

$$\frac{\sigma_N^{\infty}}{\sigma_o} = \sqrt{\frac{a}{c+a}} \quad \text{Eq. (2.8)}$$

This criterion involves two unknowns: the unnotched strength 'σ_o' and the characteristic length 'a' to be determined. It should be remembered that this criterion is valid for unidirectional laminates.

2.1.2. Whitney Nuismer (WN) Failure Theory [2]

An alternative approach to LEFM for predicting uniaxial notch strength was proposed by Whitney and Nuismer [2]. WN failure criterion uses the stress field to predict the notched strength without resorting to the classical concepts of linear elastic fracture mechanics. The development of these stress failure criterion is based on the observation of the stress fields around a hole at some characteristic distances which can be explained as the inelastic, nonlinear material behavior.

Originally Timoshenko showed the stress distribution of an infinite isotropic plate containing a circular hole as;

$$\frac{\sigma_y}{\overline{\sigma}} = 1 + \frac{1}{2}\left(\frac{R}{x}\right)^2 + \frac{3}{2}\left(\frac{R}{x}\right)^4 \quad \text{Eq. (2.9)}$$

where $\overline{\sigma}$ is the applied stress parallel to y-axis at infinity and R denotes hole radius.

The stress distribution around the hole is shown as;

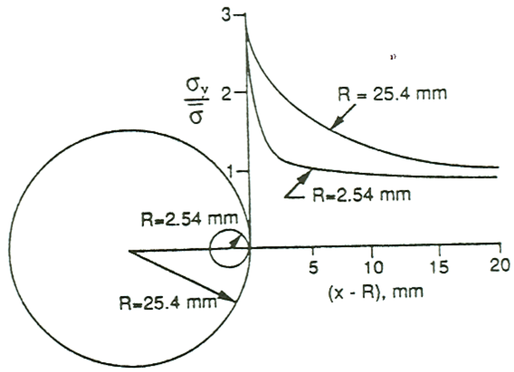

Fig. (2.2) Stress Distribution for an infinite isotropic plate containing a hole [44]

This approximation yields exact solution for quasi-isotropic laminates with a stress concentration factor, $K_T = 3$, but is inaccurate for orthotropic laminates where $K_T \neq 3$. For infinite orthotropic plates containing a circular hole, Konish and Whitney [1] extended Timoshenko's work to find the approximate solution of stress distribution along the axis perpendicular to the loading direction as;

$$\sigma_y(x,0) = \frac{\overline{\sigma}}{2}\left\{2 + \left(\frac{R}{x}\right)^2 + 3\left(\frac{R}{x}\right)^4 - \left(K_T^\infty - 3\right)\left[5\left(\frac{R}{x}\right)^6 - 7\left(\frac{R}{x}\right)^8\right]\right\}$$

x > R

Eq. (2.10)

where

$$K_T^\infty = 1 + \sqrt{\frac{2}{A_{66}}\left(\sqrt{A_{11}A_{22}} - A_{12} + \frac{A_{11}A_{22} - A_{12}^2}{2A_{66}}\right)}$$

Eq. (2.11)

Here K_T^∞ is stress concentration factor at the edge of the hole; $A_{ij,}$ i,j =1,2,6 are the components of the in-plane stiffness matrix as determined from the laminated plate theory.

Eq. (2.11) can be written in terms of engineering constants as;

$$K_T^{\infty} = 1 + \sqrt{2\left[\frac{E_y}{E_x} - \nu_{xy}\right] + \frac{E_y}{G_{xy}}} \qquad \text{Eq. (2.12)}$$

where E_y and E_x are the laminate stiffnesses in the y and x-direction respectively; υ_{xy} and G_{xy} are the Poisson's ratio and shear modulus, respectively.

The two alternative approaches to LEFM given by Whitney and Nuismer [2] are the point stress criteria and average stress criteria.

2.1.2.1. Point Stress Criteria

The Point Stress Criteria assumes that "Failure occurs when the stress, σ_y, over some distance, d_o, away from the opening is equal to or greater than the unnotched strength of the laminate".

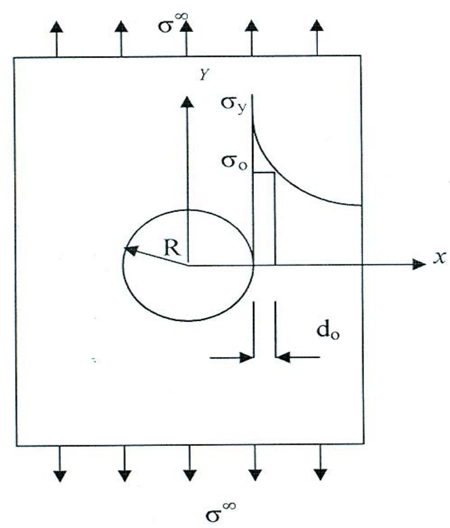

Fig. (2.3) Graphical representation of Point Stress Criteria [32]

From Fig. (2.3) it can be written as;

$$\sigma_y(x,0)\big|_{x = R + d_o} = \sigma_o \qquad \text{Eq. (2.13)}$$

where

d_o = Characteristic length

By substituting Eq. (2.10) into Eq. (2.13), the ratio of notched to unnotched strength can be written as;

$$\frac{\sigma_N^{\infty}}{\sigma_o} = \frac{2}{2 + \xi_1^{\,2} + 3\xi_1^{\,4} - \left(K_T^{\infty} - 3\right)\left(5\xi_1^{\,6} - 7\xi_1^{\,8}\right)} \qquad \text{Eq. (2.14)}$$

where

$$\xi_1 = \frac{R}{R + d_o} \qquad \text{Eq. (2.15)}$$

2.1.2.2. Average Stress Criteria

The Average Stress Criteria assumes that "Failure occurs when the average stress, σ_y, over some distance, a_o, away from the opening is equal to or greater than the unnotched strength of the laminate".

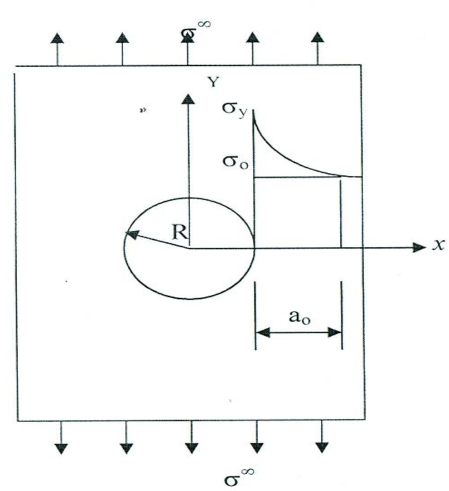

Fig. (2.4) Graphical representation of Average Stress Criteria [32]

From Fig. (2.4), it can be written as;

$$\frac{1}{a_o}\int_R^{R+a_o} \sigma_y(x,0)\,dx = \sigma_o \qquad \text{Eq. (2.16)}$$

where

a_o = Characteristic length

Using Eq. (2.10) with Eq. (2.16), the ratio of notched to unnotched strength can be written as;

$$\frac{\sigma_N^{\infty}}{\sigma_o} = \frac{2(1-\xi_2)}{2-\xi_2^{2}-\xi_2^{4}+\left(K_T^{\infty}-3\right)\left(\xi_2^{6}-\xi_2^{8}\right)} \qquad \text{Eq. (2.17)}$$

where

$$\xi_2 = \frac{R}{R+a_o} \qquad \text{Eq. (2.18)}$$

Like WEK model, the WN model contains two unknowns, i.e. , the unnotched strength, σ_o, and the characteristic length, "d_o" or "a_o", to predict the notched strength of the laminate. The characteristic length is determined experimentally. Firstly, the unnotched and notched strength are obtained from experiment. Then these values are substituted in either Eq. (2.14) or Eq. (2.17), and solved for d_o or a_o. Then, the notched strength of the composite laminate plate with a hole of any size can be predicted.

It should be noted that in WN failure criteria, the characteristic distance is believed to be a material constant, independent of lay-up and notch size, which is contrary to the experimental observation made by researchers. Therefore, the characteristic length obtained from tests on a particular laminate configuration may not be extrapolated to predict the failure of other laminates of the same material having different configuration.

One advantage of the WN failure criteria over the WEK failure criteria is the prediction of notched strength without the application of linear elastic fracture mechanics. Since LEFM is of questionable validity to composites, the Whitney-Nuismer Point Stress and Average Stress Criterion offer a significant improvement in the study of fracture in composites.

2.1.3. Karlak Failure Theory [7]

As mentioned above, WN failure criterion assumes the characteristic distance to be a material constant, irrespective of lay-up and notch size. Karlak [7] found that the notched strength of quasi-isotropic composite laminate depends upon the stacking sequence. Also the characteristic length, 'd_o', is not a material constant and is related to square root of the hole radius. So, he modified WNPS failure criterion as;

$$d_o \;\alpha\; \sqrt{R}$$

$$\Rightarrow d_o = k_o\sqrt{R} \qquad \text{Eq. (2.19)}$$

where

k_o = Curve fitting parameter determined experimentally for a material with particular stacking sequence.

The remaining analysis would be the same as in WN criterion by employing the value of "d_o" found above.

2.1.4. Pipes, Wetherhold and Gillespie (PWG)Failure Theory [5]

Pipes, Wetherhold, and Gillespie (PWG) [5] presented a three parameter model which is a further modification of WNPS failure criterion. Like Karlak, they also did not consider the characteristic distance, 'd_o' to be a material constant. The relationship between the hole radius and characteristic distance is given as;

$$d_o = \frac{(R/R_o)^m}{C} \qquad \text{Eq. (2.20)}$$

where

m = Exponential parameter

R_o = Reference radius

C = Notch sensitivity

It is possible to predict notched strength as the function of notch radius when parameters m, C, and σ_o are known. So, the modified WNPS failure criterion can be written as;

$$\frac{\sigma_N^\infty}{\sigma_o} = \frac{2}{2 + \lambda_1^2 + 3\lambda_1^4 - (K_T^\infty - 3)(5\lambda_1^6 - 7\lambda_1^8)} \qquad \text{Eq. (2.21)}$$

where

$$\lambda = \frac{1}{1 + R^{m-1} R_o^{-m} C^{-1}} \qquad \text{Eq. (2.22)}$$

For the notch radius of less than 1.0" and stress concentration range of 2 – 4, the notch sensitivity relations very nearly coincide for the intermediate value of C=10 and m = 0.5.

2.1.5. Mar-Lin (ML) Failure Theory [45]

As mentioned above, LEFM application is limited for composite materials. The basic LEFM equation for homogeneous material is given as;

$$\sigma_N^\infty = \frac{K_{IC}}{\sqrt{\pi c}} \qquad \text{Eq. (2.23)}$$

Mar and Lin [45] proposed LEFM fracture model, called Mar-Lin criterion for the notched strength of an orthotropic plate that has a form similar to Eq. (2.23) and given as;

$$\sigma_N^\infty = \frac{H_c}{(2c)^n} \qquad \text{Eq. (2.24)}$$

where

H_c = Composite fracture toughness

c = Either hole radius or half of the crack length.

n = Order of the singularity of a crack with its tip at the interface of two different materials (referred to as fiber and matrix)

The coefficients of 'H_c' and 'n' are determined by plotting the data on log-log scale. The value of 'n' is a function of the constituent material shear modulus and Poisson's ratio. Mar and Lin assumed that the fracture in the laminate must occur through the propagation of a crack lying in matrix material at the matrix/filament interface.

The model provides good correlation between the experimental data and the prediction and also very simple to apply. However, the fracture parameter, used in Mar-Lin criterion depends on the lay-up configuration. Therefore, experimental determination of 'H_c' for each laminate configuration is required for applying this criterion.

2.2. Objective of the Thesis

The purpose of this thesis is to further investigate the behavior of composite laminates with varying hole sizes when subjected to tensile loading. A detailed finite element analysis will be conducted by employing two criterias namely point stress and average stress. Two different materials AS4/3502 Graphite Epoxy and XAS/APC-1 Graphite PEEK having the same lay-up configuration $[0/\pm45/90]_{2s}$ will be considered. Effect of notch size on the reduction of strength of the material will be highlighted. The data will then be compared with the available experimental data [32] of notched strength.

CHAPTER 3

FINITE ELEMENT ANALYSIS

A thorough finite element analysis was done to determine the failure strength of notched laminated composites when subjected to tensile loading. Two materials were selected, namely ***AS4/3502 Graphite/Epoxy (Gr/Ep)*** and ***XAS/APC-1 Graphite/PEEK (Gr/PEEK)***, where Gr/Ep is a thermoset material and Gr/PEEK is a thermoplastic material. The stacking sequence for both the materials was **$[0/\pm45/90]_{2s}$**, so that both the materials were made up of 16 plies.

Material properties used for AS4/3502 Graphite epoxy are listed below; [45]

Parameter	***Graphite Epoxy (AS4-3502)***
Longitudinal Tensile Modulus, E_{11}	20.87 x 10^6(psi)
Transverse Tensile Modulus, E_{22}=E_{33}	1.72 x 10^6(psi)
Shear Modulus, G_{12}=G_{23}=G_{13}	0.97 x 10^6(psi)
Poisson's Ratio (major), v_{12}=v_{23}=v_{13}	0.326

Table (3.1) Typical Properties of Graphite Epoxy (AS4/3502) Composite Laminate

And material properties used for XAS/APC-1 Graphite Peek are listed as; [45]

Parameter	***Graphite Peek (XAS/APC-1)***
Longitudinal Tensile Modulus, E_{11}	17.56×10^6(psi)
Transverse Tensile Modulus, $E_{22}=E_{33}$	1.47×10^6(psi)
Shear Modulus, $G_{12}=G_{23}=G_{13}$	0.67×10^6(psi)
Poisson's Ratio (major), $v_{12}=v_{23}=v_{13}$	0.37

Table (3.2) Typical Properties of Graphite Peek (XAS/APC-1) Composite Laminate

The dimensions of the model [45] used for both the materials are as follows;

Length (in)	**Width (in)**	**Hole Dia (in)**	**Ply Thickness (in)**	**Thickness of Model (in)**
5	1.0	0.1	0.005	0.08
5	1.0	0.2	0.005	0.08
5	1.5	0.4	0.005	0.08
5	2.0	0.6	0.005	0.08

Table (3.3) Dimensions of the model

Ansys 10.0 software was used to perform finite element analysis (FEA). In this software, FEA is done by a numerical method of deconstructing a complex system into very small pieces called elements. Then Ansys implements equations that govern the behavior of these elements and solves all of them giving an explanation of how the system acts as a whole. The results are then generated in the form of table or graph.

3.1. Procedure for Finite Element Analysis

Steps followed for the finite element analysis of the current problem are as follows;

3.1.1. Choose Element Type

First the element type was defined. The eight-node quadrilateral shell element **SHELL99** was used, which is capable of modeling multiple plies in the laminate. Also shell element was chosen because these are used to model panel type structures where thickness is small compared to other dimensions of the part. The shell element is really a 2D element that is called 3D because it is not restricted to the XY plane like a 2D solid element

The quadrilateral element can be degenerated into triangular element if required. It has six degrees of freedom. Figures below shows the element geometry and output definition.

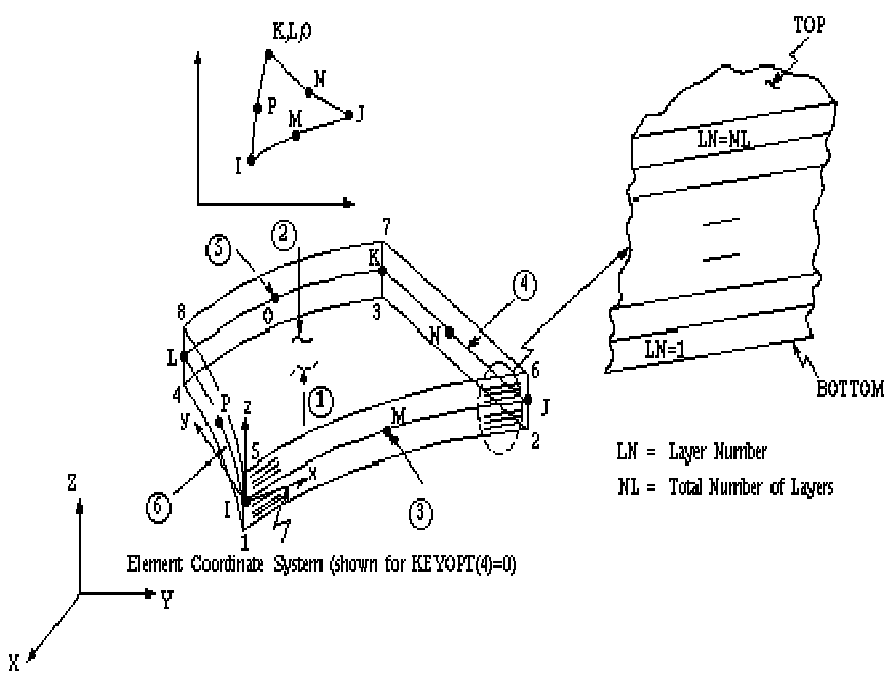

Fig. (3.1) SHELL99 Geometry

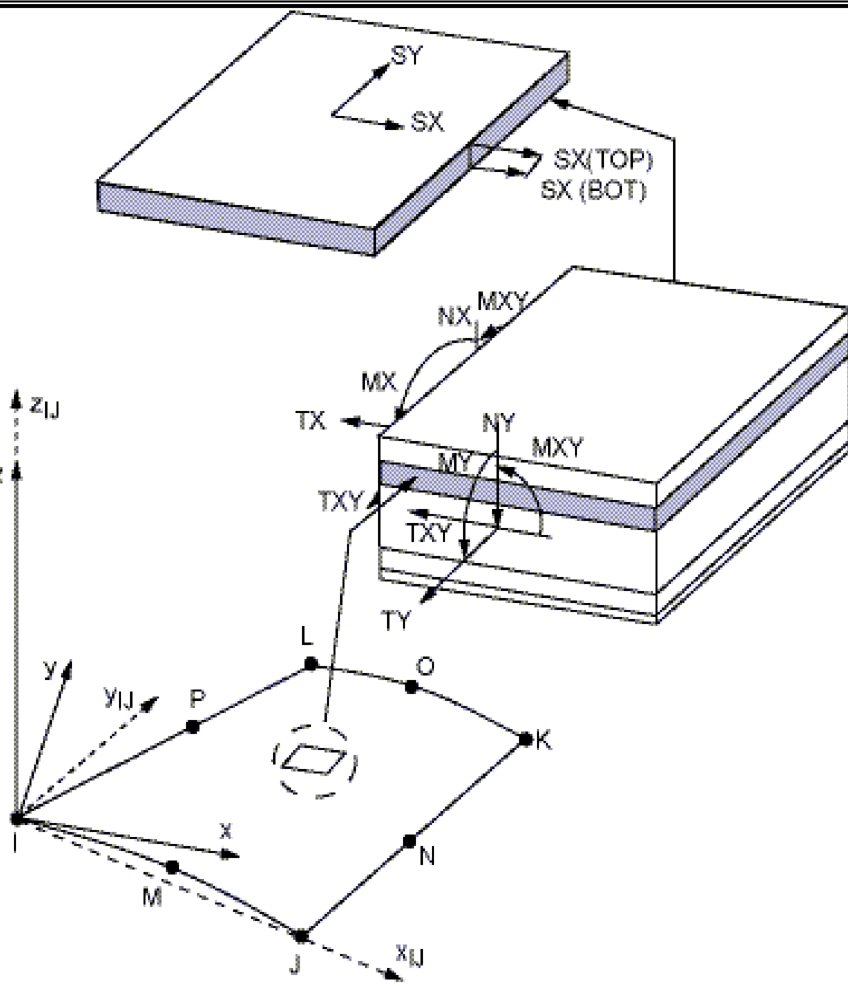

Fig. (3.2) SHELL99 Stress Output

3.1.2. Define Real Constants

Then model stacking sequence (orientation of angles) and each layer thickness were defined in real constant. The stacking sequence is as shown in figure.

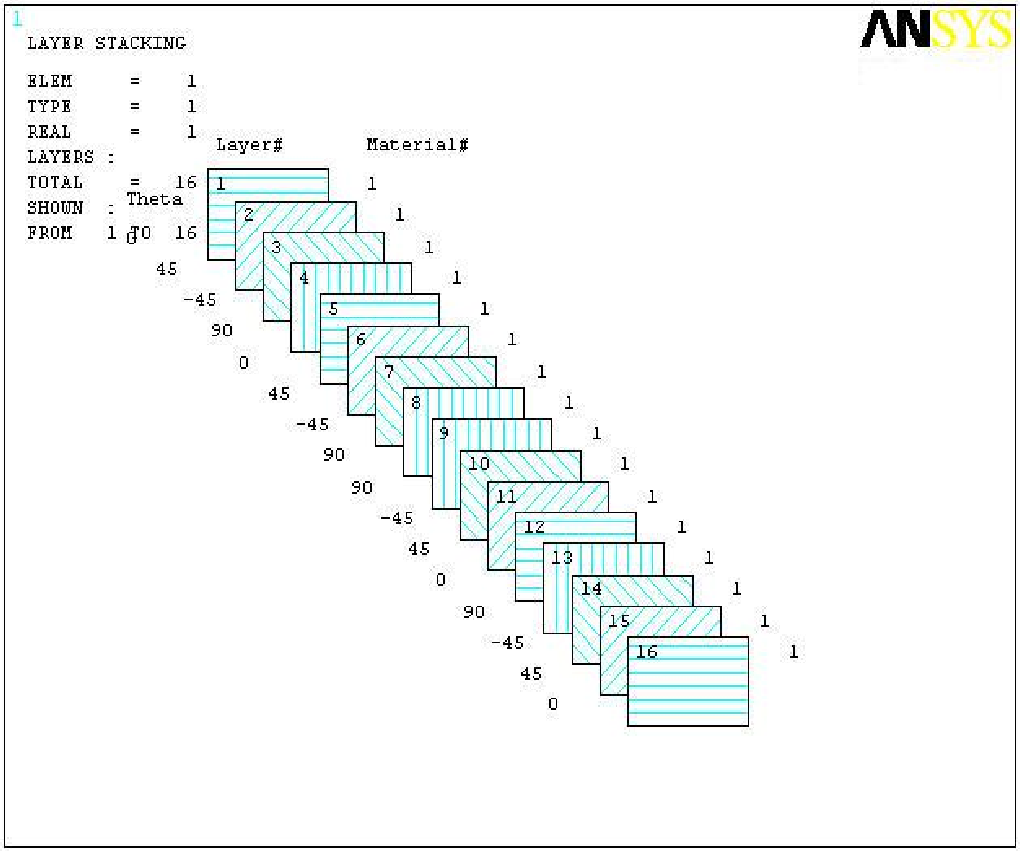

Fig. (3.3) Stacking Sequence of the Laminate

3.1.3. Define Material Properties

Now that the element had been defined, material properties indicated in table (3.1) or table (3.2) were defined for the orthotropic material in material models.

3.1.4. Build Geometry

A 2D model for the problem was created utilizing the geometry given in table (3.3). Because of the laminate symmetry about its mid-plane, only quarter of the model was considered. A rectangle was made first, and then a solid circle was made and subtracted from the rectangle to get a hole in the plate. The hole was located at the center of the laminate.

3.1.5. Generate Mesh

Meshing is a very important feature of finite element analysis, since the size of mesh largely affects the results in the model. To get a precise result, very fine meshing (SMRT,1) was done. Also, the number of elements and nodes in the model varied for different cases under consideration.

3.1.6. Apply Loads

First of all, the model was constrained on the surface of one end in x-direction. The bottom surface was constrained along y-direction. This is shown if fig. (3.4)

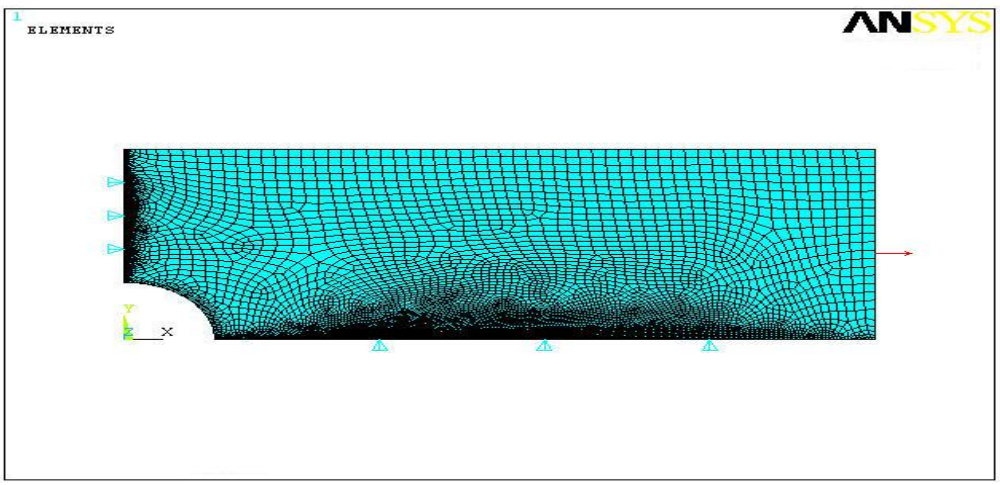

Fig. (3.4) Quarter model with meshing and applied boundary conditions.

Then the load was applied at the end keeping in view the WN model.

3.1.7. Obtain Solution

After applying loads, solution for the problem was done. For this, static analysis was chosen.

All these steps were repeated for each case of the model, taking into account both the materials.

CHAPTER 4

RESULTS AND DISCUSSION

Both thermoplastic composite (XAS/APC-1 Gr/PEEK) and thermoset composite (AS4/3502 Gr/Epoxy) were investigated for tensile loading through finite element analysis. The WN point stress criterion and average stress criterion were applied to predict the notched strength of the laminates. The results were then compared with the experimental results [32].

4.1. Characteristic Length in Tension

First of all, the value of stress concentration factor "K_T" around the hole was found by Eq. (2.11) for both the materials. The values obtained were $\boldsymbol{K_T = 2.6272}$ for Gr/PEEK, and $\boldsymbol{K_T = 2.633}$ for Gr/Ep. These values were then used to find the characteristic lengths **"d_o"** and **"a_o"**, as discussed in point stress and average stress criteria respectively. The values of "d_o" and "a_o" were determined by backward substitution of experimental notched strength data into Eq. (2.14)-(2.15) and Eq. (2.17)-(2.18) respectively. All of this was done in Matlab 7.0

Variation of characteristic length with varying hole sizes for both Gr/Epoxy and Gr/PEEK using point stress and average stress criteria is given as;

Hole Dia (in)	**AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$**		**XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$**	
	d_o (in)	**a_o (in)**	**d_o (in)**	**a_o (in)**
0.1	0.0225	0.09434	0.0344	0.1467
0.2	0.0359	0.1583	0.0365	0.1581
0.4	0.0654	0.2914	0.0397	0.218
0.6	0.0466	0.2964	0.017	0.2401

Table (4.1) Variation of Characteristic Length with varying Hole Sizes

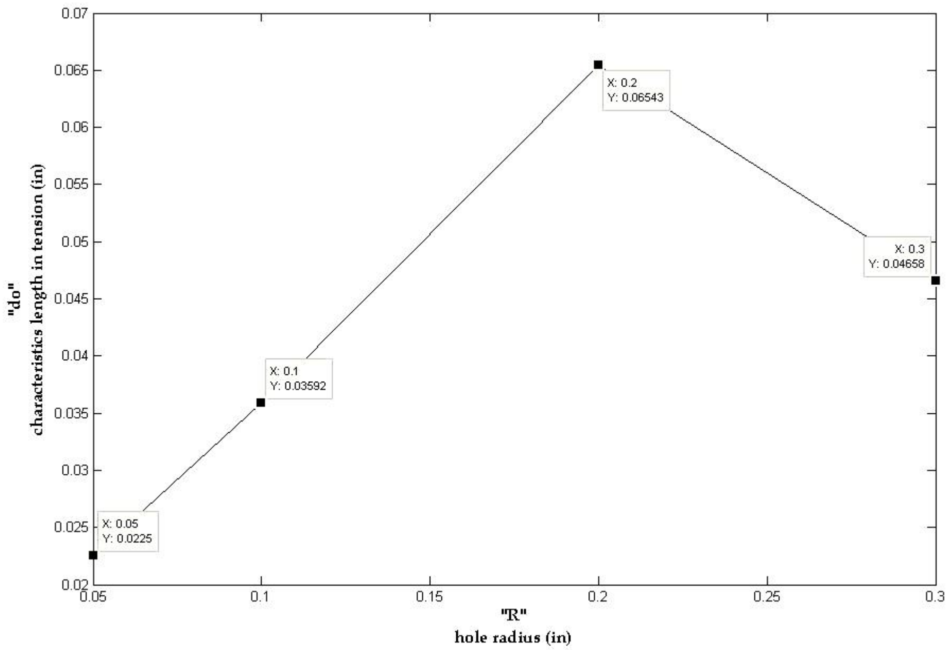

Fig. (4.1) Variation of "d_o" for Gr/ Ep with different hole sizes using Point Stress Criteria

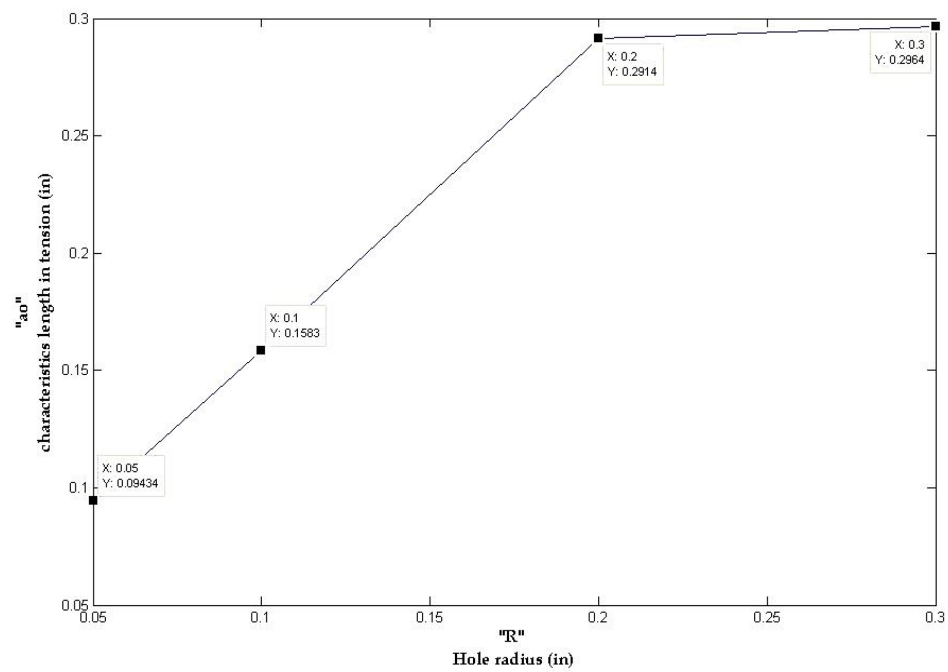

Fig. (4.2) Variation of "a_o" for Gr/Ep with different hole sizes using Average Stress Criteria

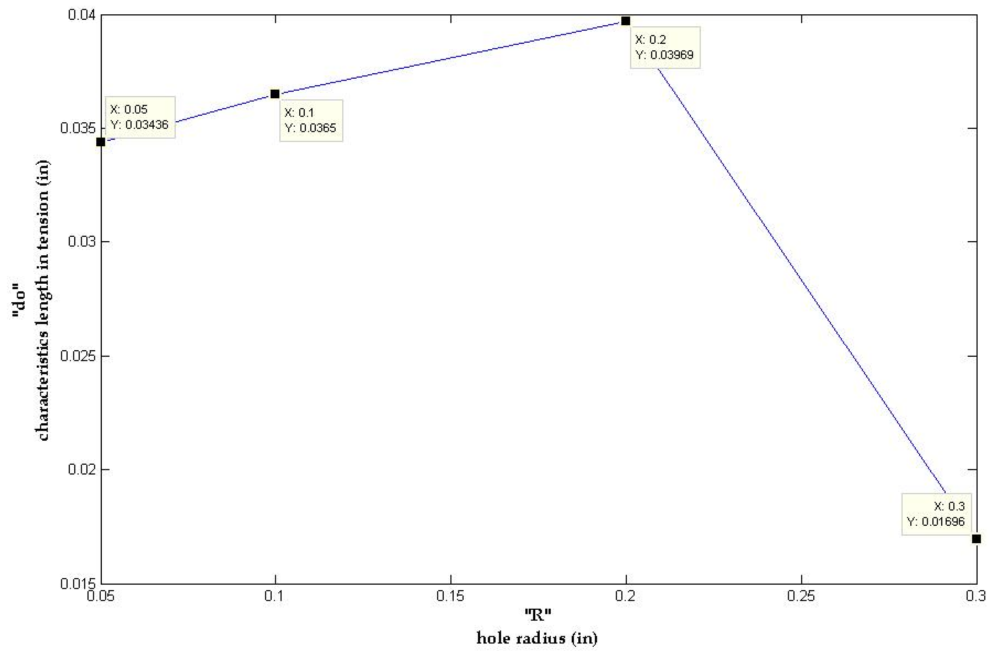

Fig. (4.3) Variation of "d_o" for Gr/PEEK with different hole sizes using Point Stress Criteria

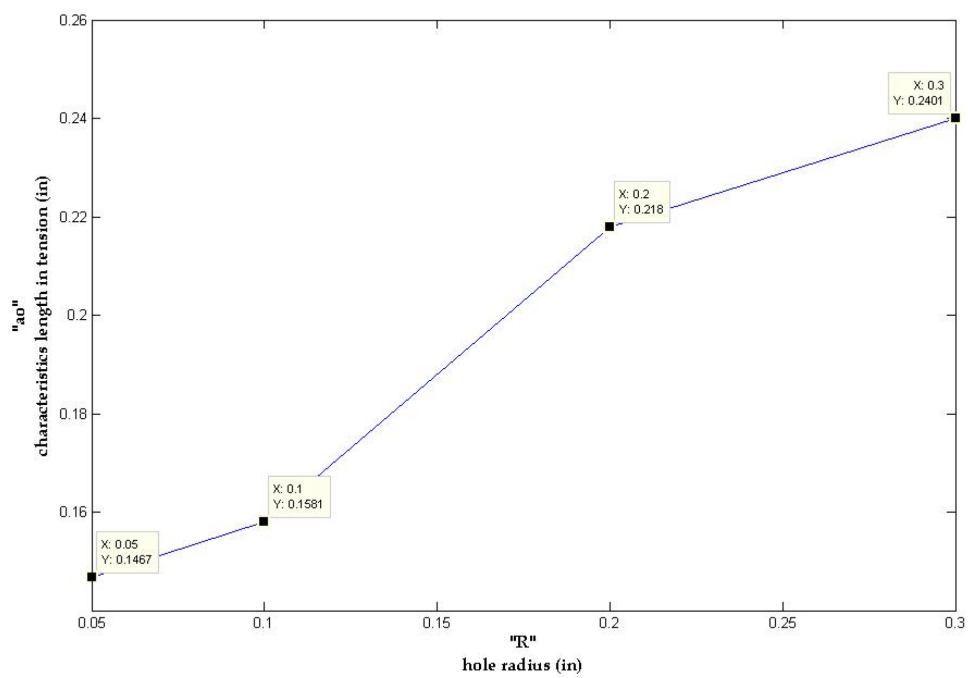

Fig. (4.4) Variation of "a_o" for Gr/PEEK with different hole sizes using Average Stress Criteria

From the above graphs, it can be seen that values of characteristic length "d_o" and "a_o" in point stress and average stress failure criteria respectively are dependent on the notch size. It is also dependent on laminate configuration. Therefore, the characteristic length obtained from tests on a particular laminate configuration may not be extrapolated to predict the failure of other laminates of the same material having different configuration. This is contrary to the previous work [32], which assumes the characteristic length to be a material constant, independent of stacking sequence and notch size.

4.2. Notched Strength

First of all, the gross strength **"σ_N"** was calculated by the fracture load divided by the total cross-sectional area of the specimen. Then the Notched Strength **"σ_N^{∞}"** was calculated by multiplying the gross strength with finite-width correction factor (FWC). This was done because all the results obtained were for finite width specimen. Therefore, the ultimate strengths were corrected using finite-width correction (FWC) factors to obtain the notched strengths of the infinite-width plates.

Heywood's formula of FWC factor for isotropic plates containing a circular hole was used and given as;

$$\frac{K_T}{K_T^{\infty}} = \frac{2 + (1 - 2a/W)^3}{3(1 - 2a/W)} \qquad \text{Eq. (4.1)}$$

Where a = Hole Radius

Since the current investigation was based on Whitney Nuismer model, which contains two criterias, point stress and average stress. Considering first the point stress criteria which assume that failure occurs when the normal stress over some distance, "d_o", away from the opening is equal to or greater than the unnotched strength of the laminate. The value of unnotched strength "σ_o" was taken from the experimental results [32]. The effect of hole size was examined by considering circular holes of 0.1", 0.2", 0.4" and 0.6" diameter for both the materials. A great number of attempts were made to find out exactly at which load, the material will ultimately fail.

4.3. Point Stress Criteria

4.3.1. AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$

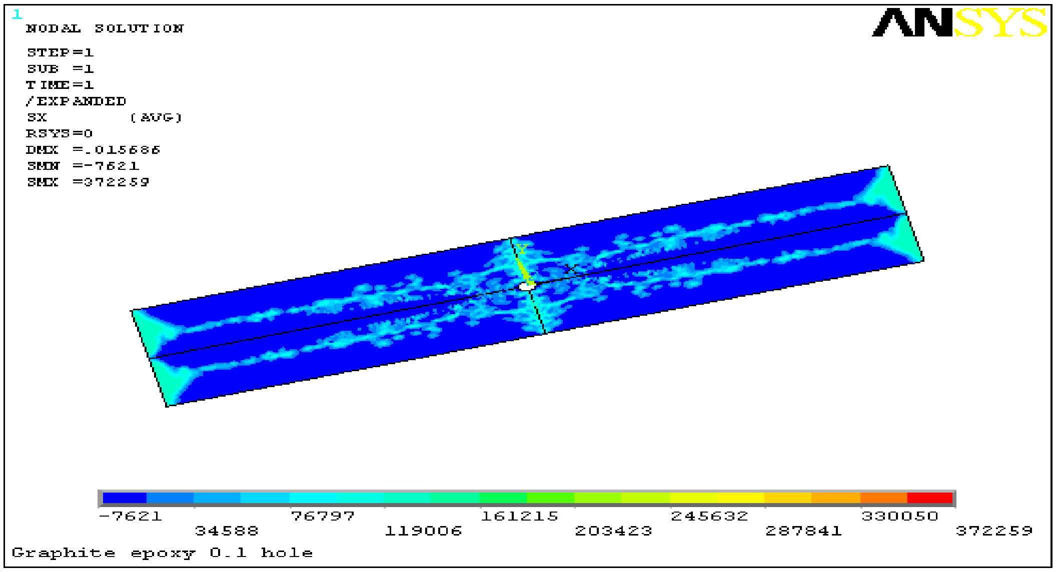

Fig. (4.5) Stress distribution for AS4/3502 Gr/Ep Composite Laminate containing 0.1" hole using Point Stress Criteria

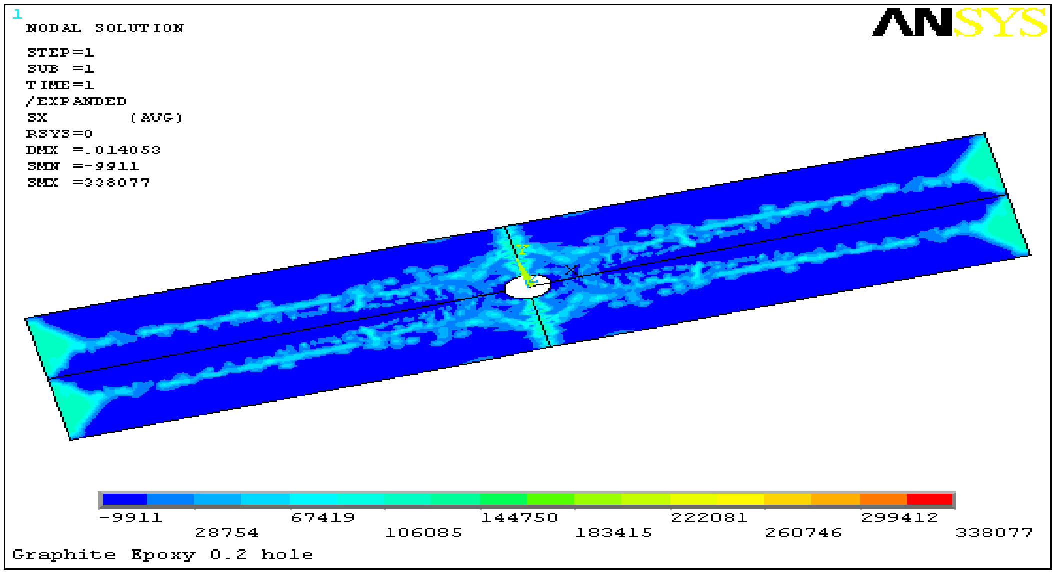

Fig. (4.6) Stress distribution for AS4/3502 Gr/Ep Composite Laminate containing 0.2" hole using Point Stress Criteria

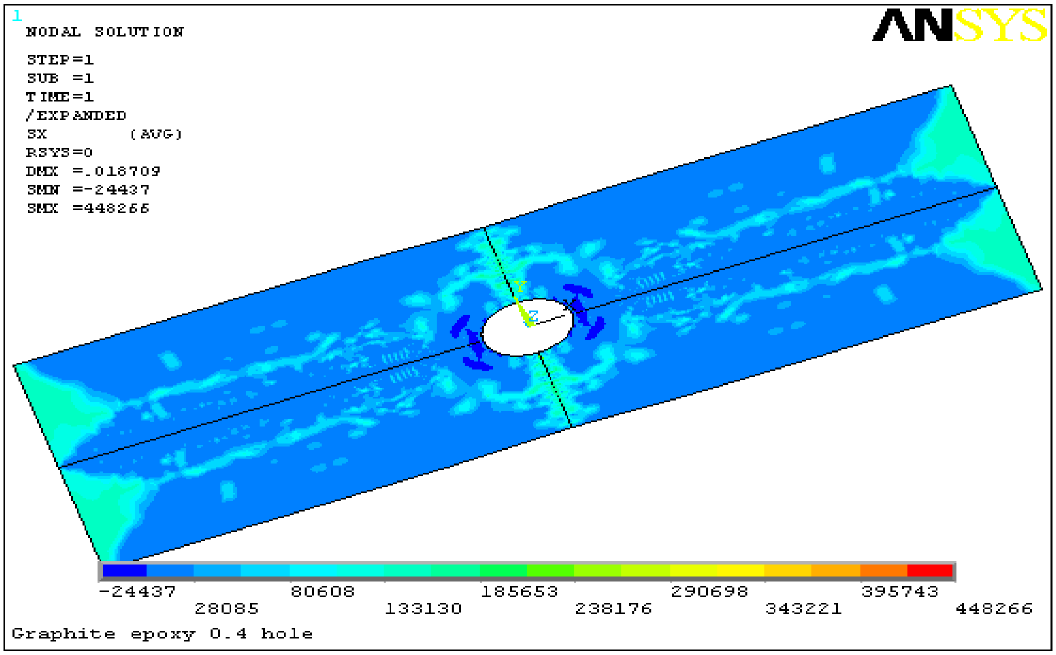

Fig. (4.7) Stress distribution for AS4/3502 Gr/Ep Composite Laminate containing 0.4" hole using Point Stress Criteria

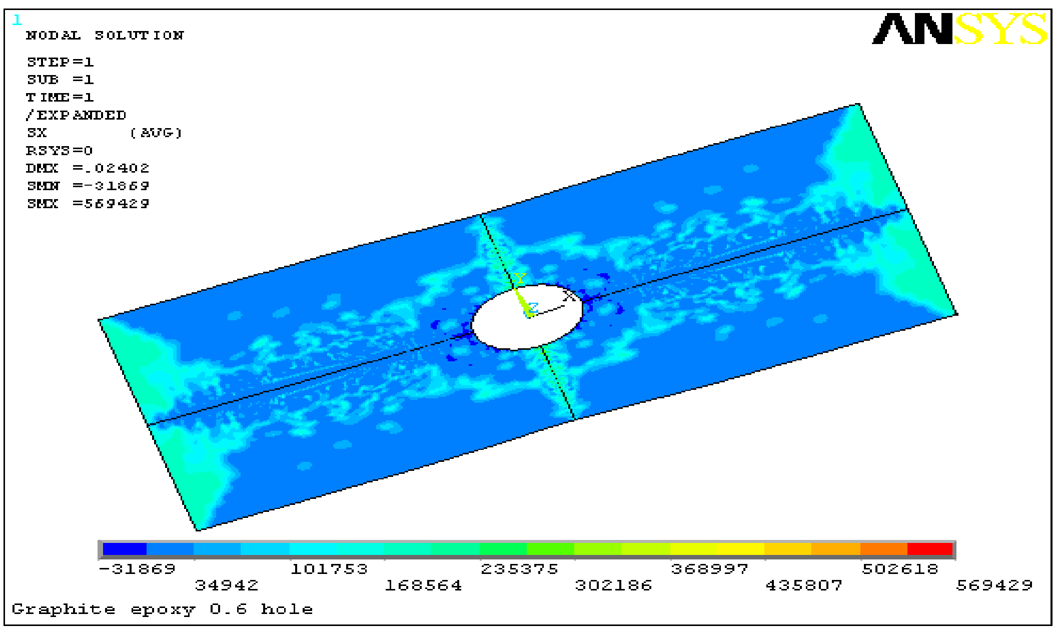

Fig. (4.8) Stress distribution for AS4/3502 Gr/Ep Composite Laminate containing 0.6" hole using Point Stress Criteria

The results obtained from above analysis are tabulated as follows;

Hole Dia "D" (in)	**Plate Width "W" (in)**	**X-Sec Area "A" (in)**	**Applied Load "P" (lbs)**	**Gross Strength "σ_N" (psi)**	**FWC K_T/K_T^∞**	**Notched Strength "σ_N^∞" (psi)**	**NSR σ_N^∞/σ_o**
0.1	1	0.08	4151	51,890	1.01074	52,448	0.512
0.2	1	0.08	3665	45,810	1.04667	47,948	0.468
0.4	1.5	0.12	4706	39,216	1.08835	42,681	0.416
0.6	2.0	0.16	5808	36,303	1.11571	40,504	0.395

Table (4.2) Ultimate Strength Data for AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$ Laminates using Point Stress Criteria

Where

σ_o = Unnotched Strength = 102,510 psi [32]

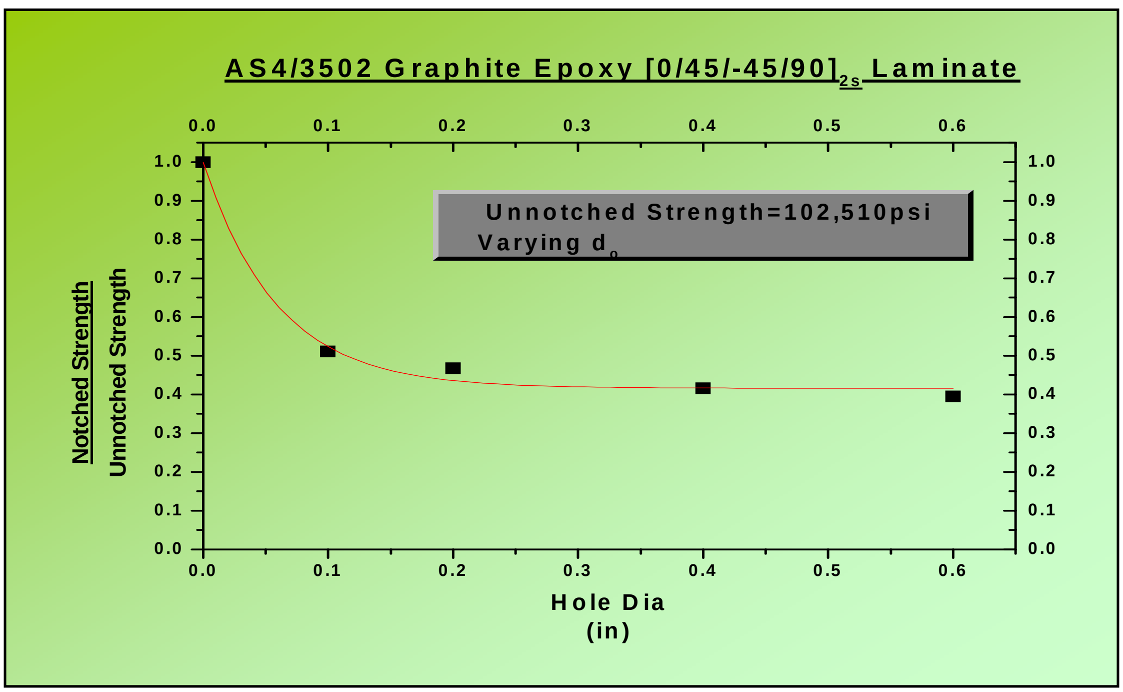

Fig. (4.9) Ultimate Strength Data for AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$ Laminates using Point Stress Criteria

It is evident from this table that the strength of the Graphite Epoxy greatly reduces due to the presence of notch when point stress criteria is applied. With 0.1" hole the strength decreases to about 50%. Increasing the notch size further, the strength continuous to decrease but at a slow rate. A graphical presentation of this table is shown in figure above.

4.3.2. XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$

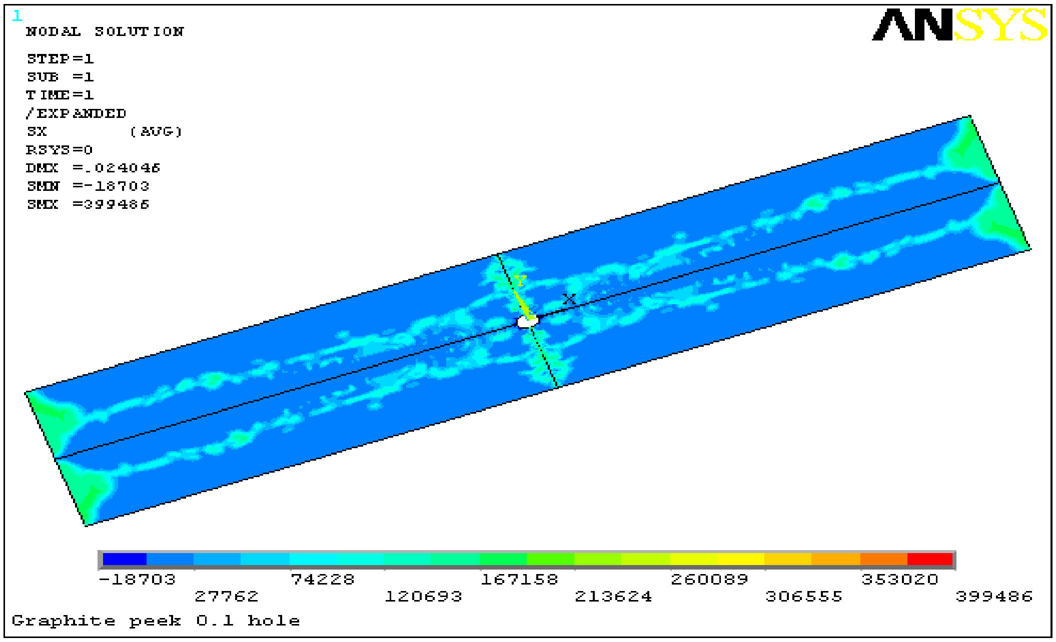

Fig. (4.10) Stress distribution for XAS/APC-1 Gr/PEEK Composite Laminate containing 0.1" hole using Point Stress Criteria

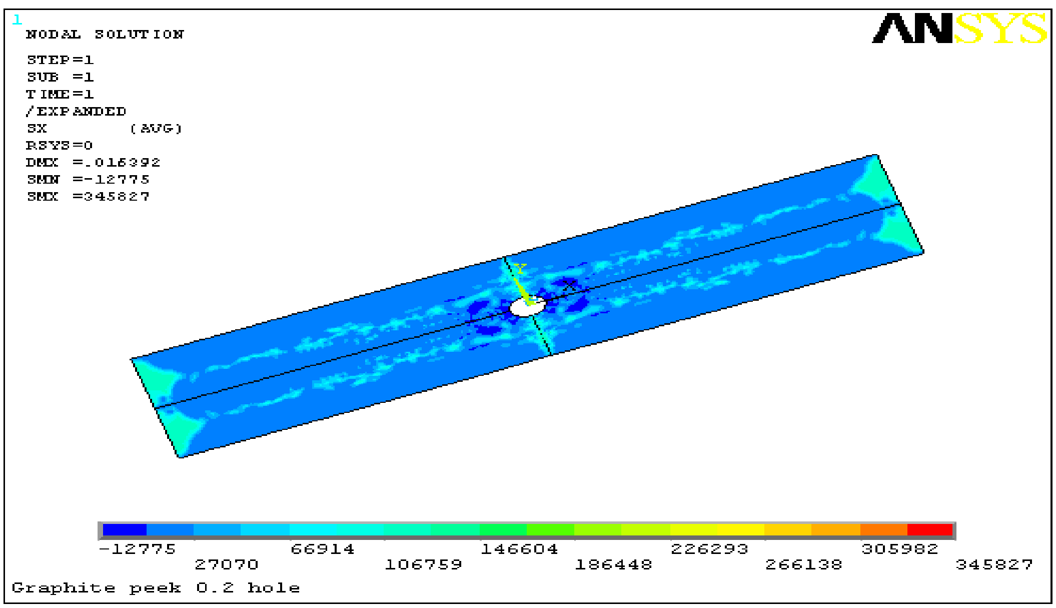

Fig. (4.11) Stress distribution for XAS/APC-1 Gr/PEEK Composite Laminate containing 0.2" hole using Point Stress Criteria

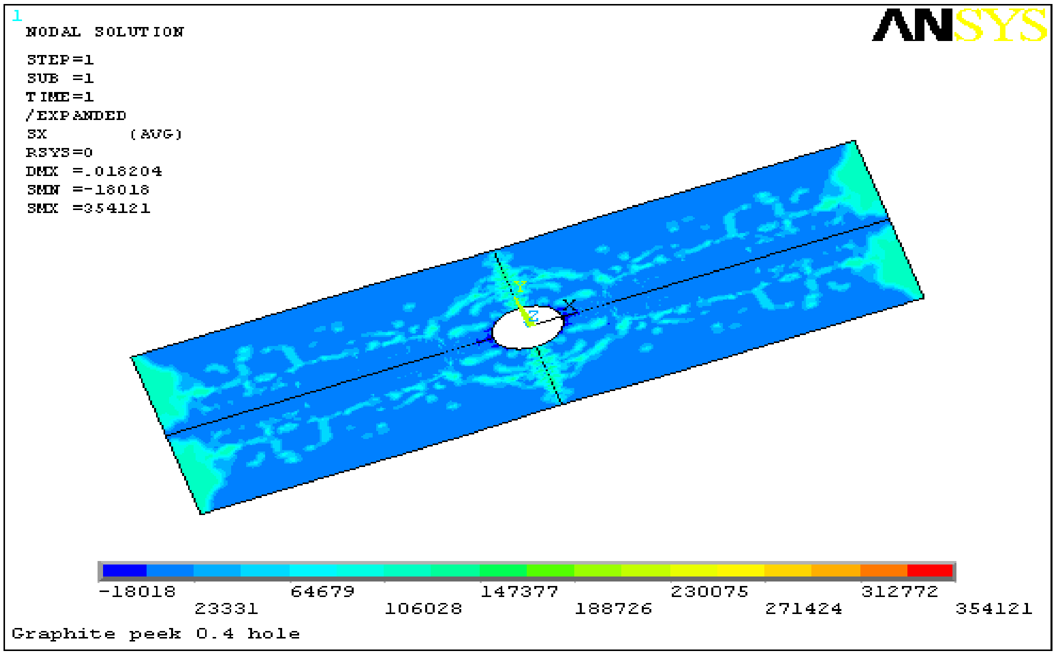

Fig. (4.12) Stress distribution for XAS/APC-1 Gr/PEEK Composite Laminate containing 0.4” hole using Point Stress Criteria

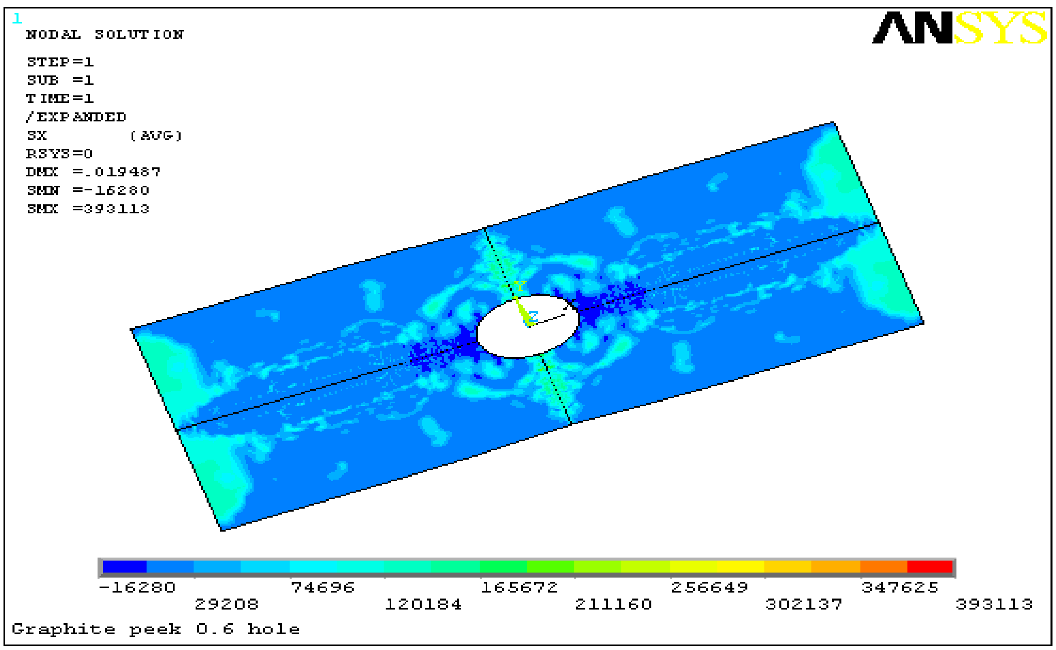

Fig. (4.13) Stress distribution for XAS/APC-1 Gr/PEEK Composite Laminate containing 0.6” hole using Point Stress Criteria

The results obtained from above analysis are listed below;

Hole Dia "D" (in)	Plate Width "W" (in)	X-Sec Area "A" (in)	Applied Load "P" (lbs)	Gross Strength "σ_N" (psi)	FWC K_T/K_T^{∞}	Notched Strength "σ_N^{∞}" (psi)	NSR $\sigma_N^{\infty}/\sigma_o$
0.1	1	0.08	5265	65,815	1.01074	66,521	0.692
0.2	1	0.08	3536	44,206	1.04667	46,269	0.481
0.4	1.5	0.12	3787	31,562	1.08835	34,350	0.357
0.6	2.0	0.16	3905	24,406	1.11571	27,230	0.283

Table (4.3) Ultimate Strength Data for XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$ Laminates using Point Stress Criteria

Where

σ_o = Unnotched Strength = 96,120 psi [32]

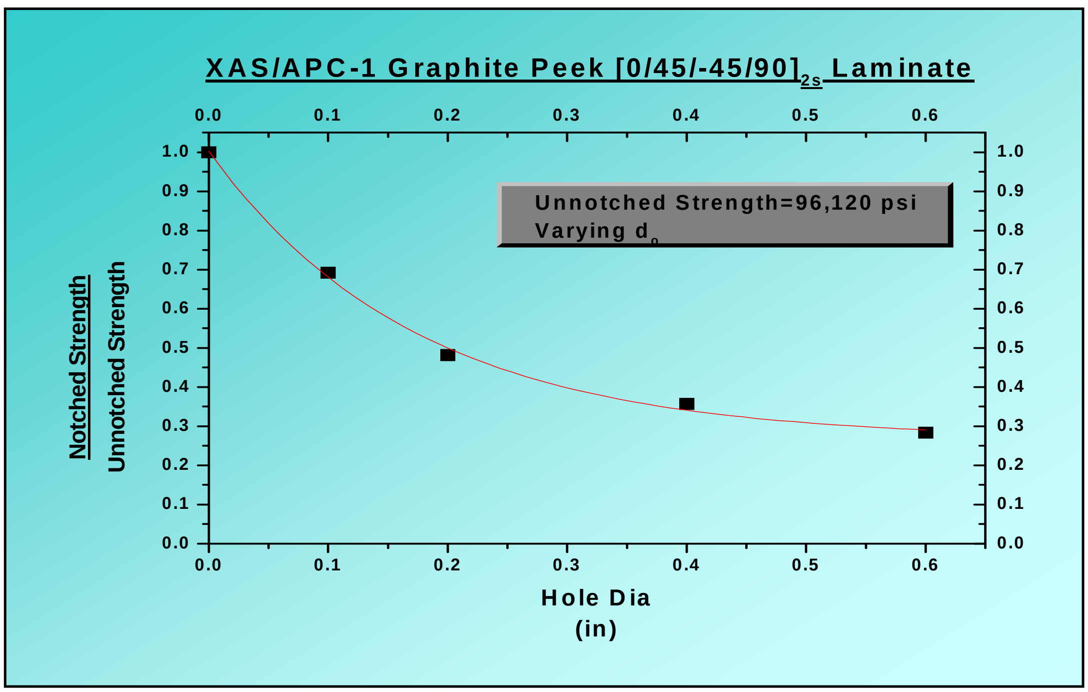

Fig. (4.14) Ultimate Strength Data for XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$ Laminate using Point Stress Criteria

In case of Graphite PEEK having the same lay-up configuration, we can see that notched strength of the material with 0.1 hole size is reduced to about 30%, whereas in case of Graphite Epoxy there was about 50% reduction when point stress criteria is applied. But as the hole size increases, the reduction in notched strength for Graphite PEEK takes place at a greater rate than in Graphite Epoxy.

4.4. Average Stress Criteria

4.4.1. AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$

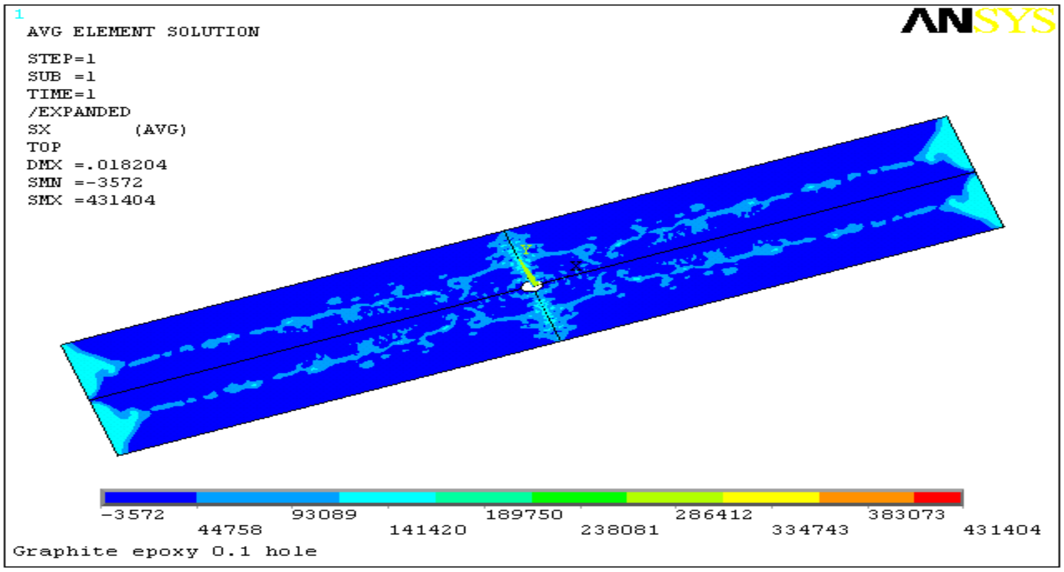

Fig. (4.15) Stress distribution for AS4/3502 Gr/Ep Composite Laminate containing 0.1" hole using Average Stress Criteria

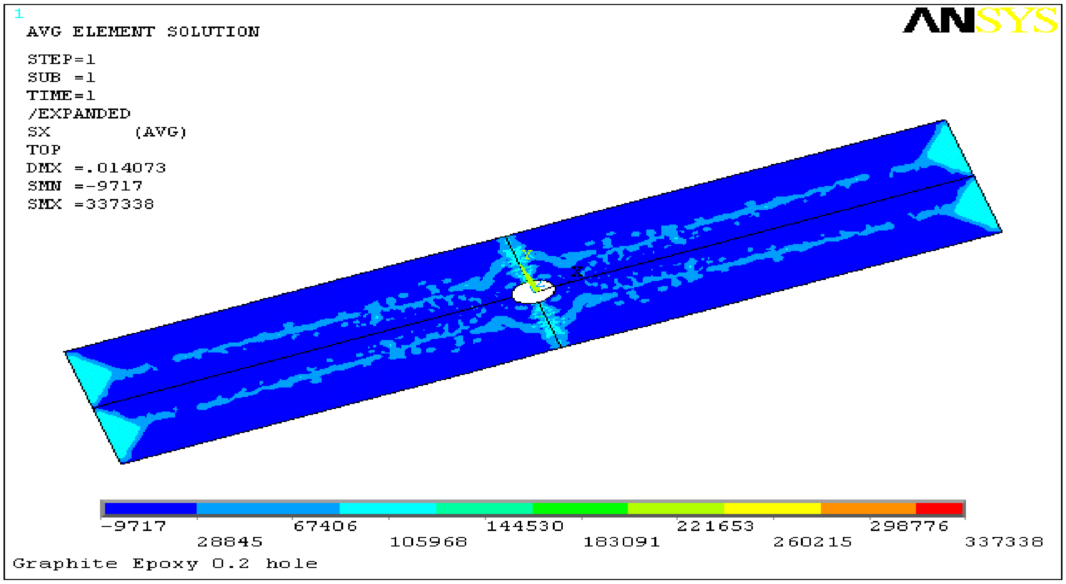

Fig. (4.16) Stress distribution for AS4/3502 Gr/Ep Composite Laminate containing 0.2" hole using Average Stress Criteria

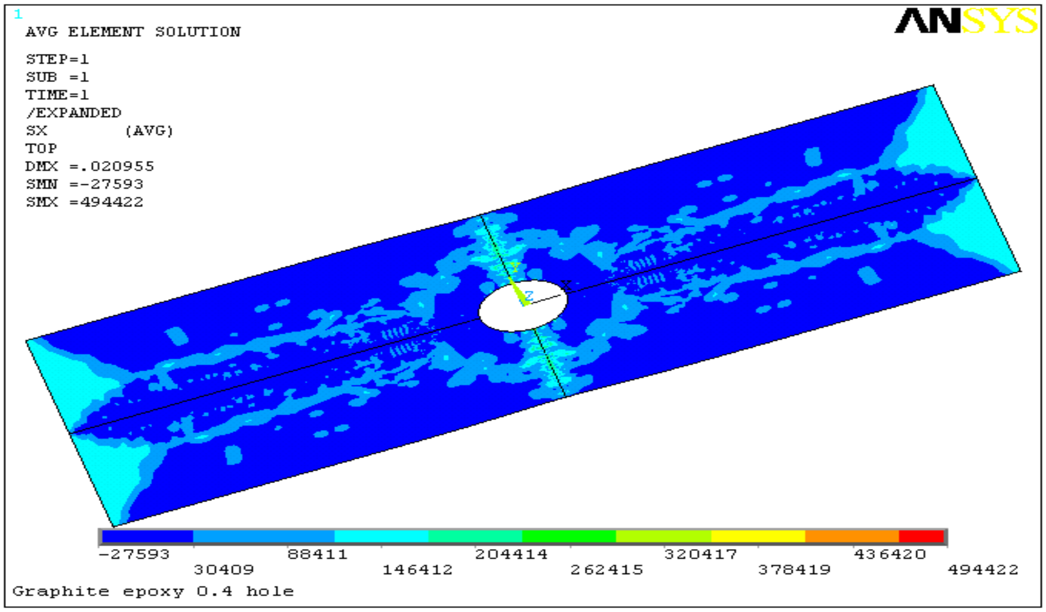

Fig. (4.17) Stress Distribution for AS4/3502 Gr/Ep Composite Laminate containing 0.4” hole using Average Stress Criteria

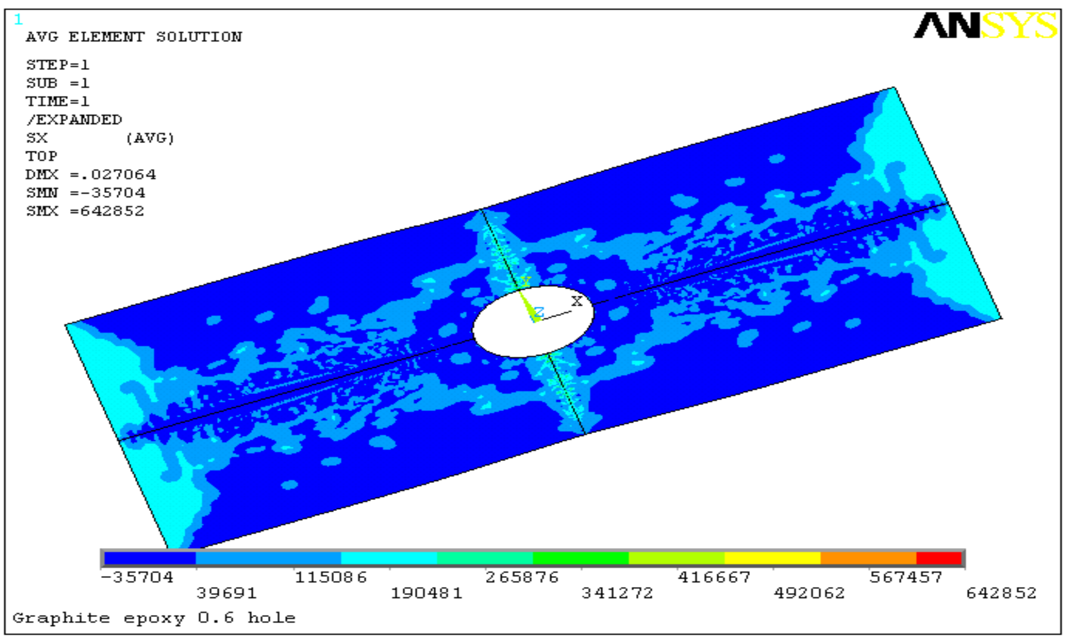

Fig. (4.18) Stress Distribution for AS4/3502 Gr/Ep Composite Laminate containing 0.6” hole using Average Stress Criteria

The results obtained from above analysis are tabulated as follows;

Hole Dia "D" (in)	**Plate Width "W" (in)**	**X-Sec Area "A" (in)**	**Applied Load "P" (lbs)**	**Gross Strength "σ_N" (psi)**	**FWC K_T/K_T^{∞}**	**Notched Strength "σ_N^{∞}" (psi)**	**NSR $\sigma_N^{\infty}/\sigma_o$**
0.1	1	0.08	4818	60,219	1.01074	60,866	0.594
0.2	1	0.08	3670	45,872	1.04667	48,013	0.468
0.4	1.5	0.12	5271	43,924	1.08835	47,804	0.466
0.6	2.0	0.16	6545	40,904	1.11571	45,637	0.445

Table (4.4) Ultimate Strength Data for AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$ Laminates using Average Stress Criteria

Where

σ_o = Unnotched Strength = 102,510 psi [32]

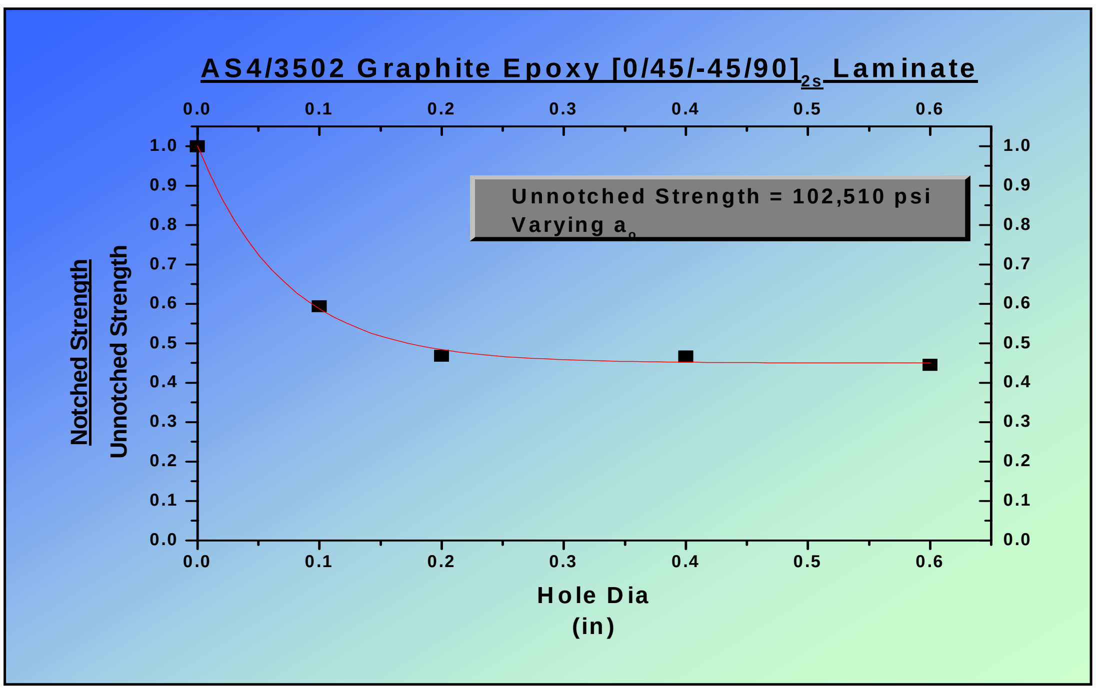

Fig. (4.19) Ultimate Strength for AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$ Laminate using Average Stress Criteria

When strength of Graphite Epoxy is evaluated using average strength criteria, it is found that the strength of the material decreases to about 40% with the inclusion of 0.1" hole in the specimen. But a very slow and steady rate in the reduction of notched strength is observed when hole size further increases. Therefore data scattering is found less. Also it gives a better result as compare to point stress criteria.

4.4.2. XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$

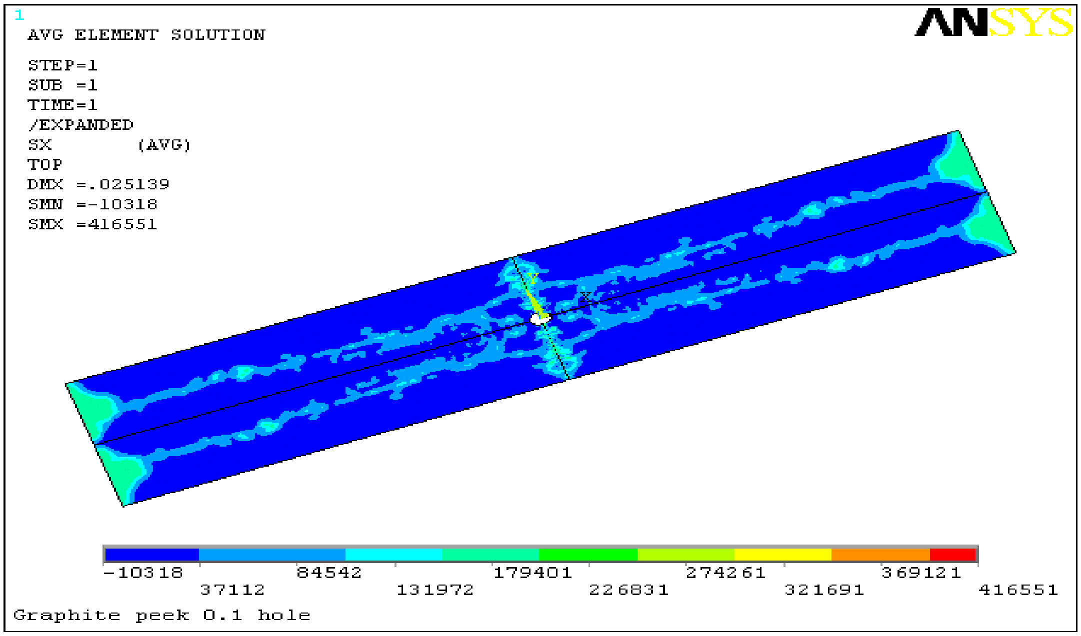

Fig. (4.20) Stress distribution for XAS/APC-1 Gr/PEEK Composite Laminate containing 0.1” hole using Average Stress Criteria

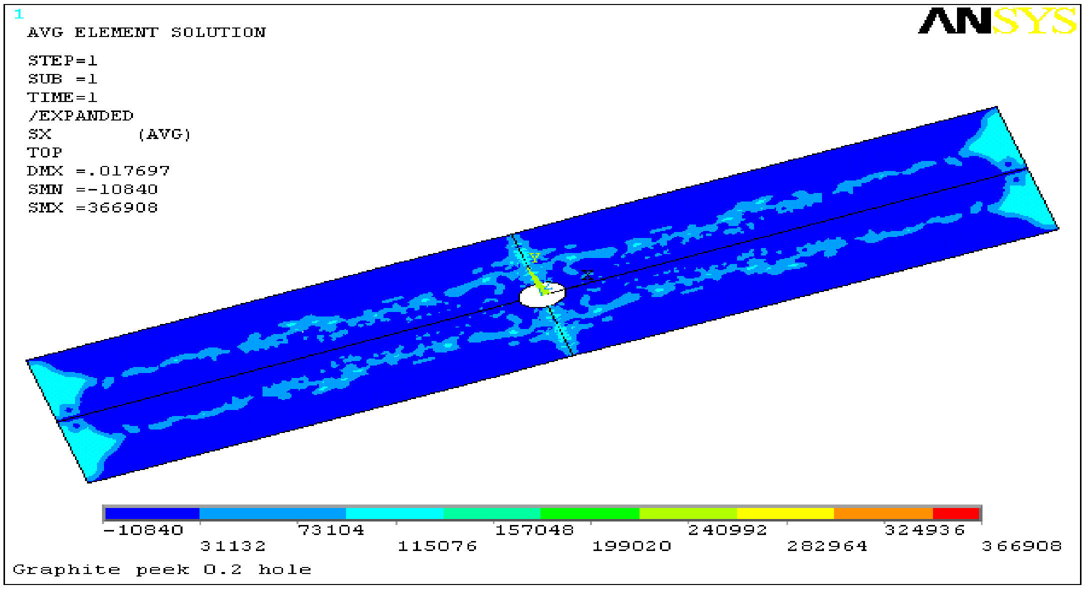

Fig. (4.21) Stress distribution for XAS/APC-1 Gr/PEEK Composite Laminate containing 0.2” hole using Average Stress Criteria

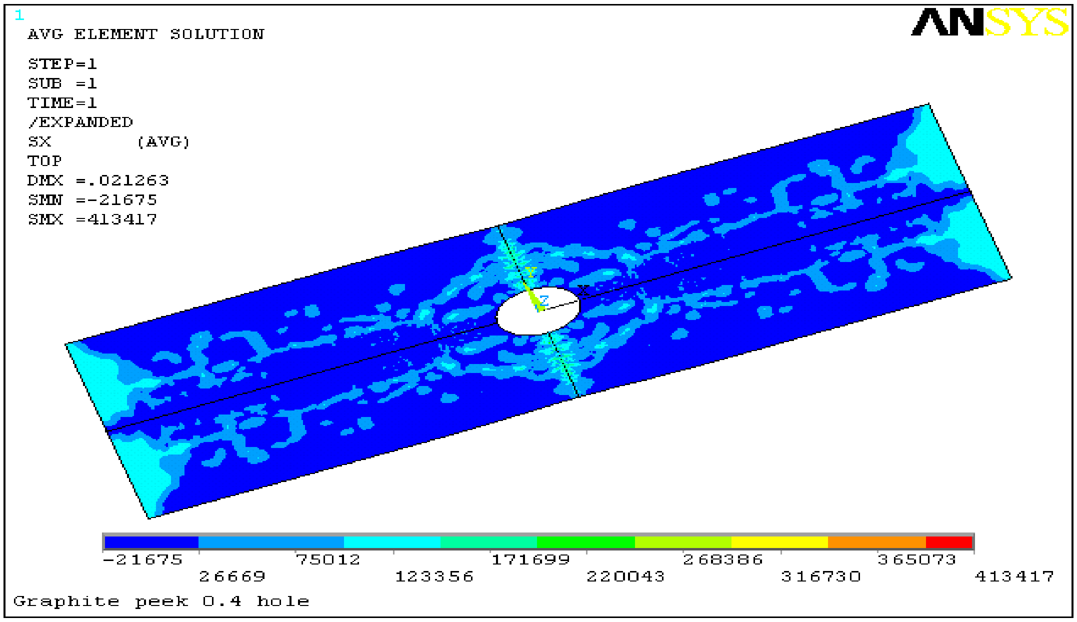

Fig. (4.22) Stress distribution for XAS/APC-1 Gr/PEEK Composite Laminate containing 0.4” hole using Average Stress Criteria

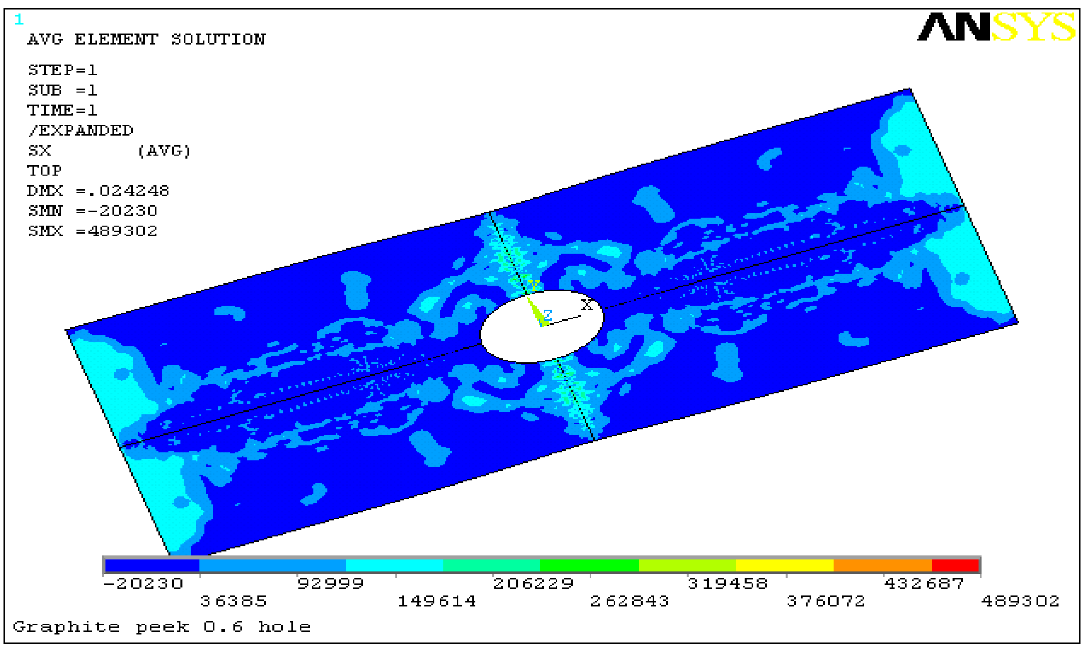

Fig. (4.23) Stress distribution for XAS/APC-1 Gr/PEEK Composite Laminate containing 0.6” hole using Average Stress Criteria

The results obtained from above analysis are listed below;

Hole Dia "D" (in)	Plate Width "W" (in)	X-Sec Area "A" (in)	Applied Load "P" (lbs)	Gross Strength "σ_N" (psi)	FWC K_T/K_T^∞	Notched Strength "σ_N^∞" (psi)	NSR σ_N^∞/σ_o
0.1	1	0.08	5505	68,807	1.01074	69,546	0.724
0.2	1	0.08	3818	47,725	1.04667	49,952	0.52
0.4	1.5	0.12	4424	36,865	1.08835	40,122	0.417
0.6	2.0	0.16	4859	30,369	1.11571	33,883	0.353

Table (4.5) Ultimate Strength Data for XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$ Laminates using Average Stress Criteria

Where

σ_o = Unnotched Strength = 96,120 psi [32]

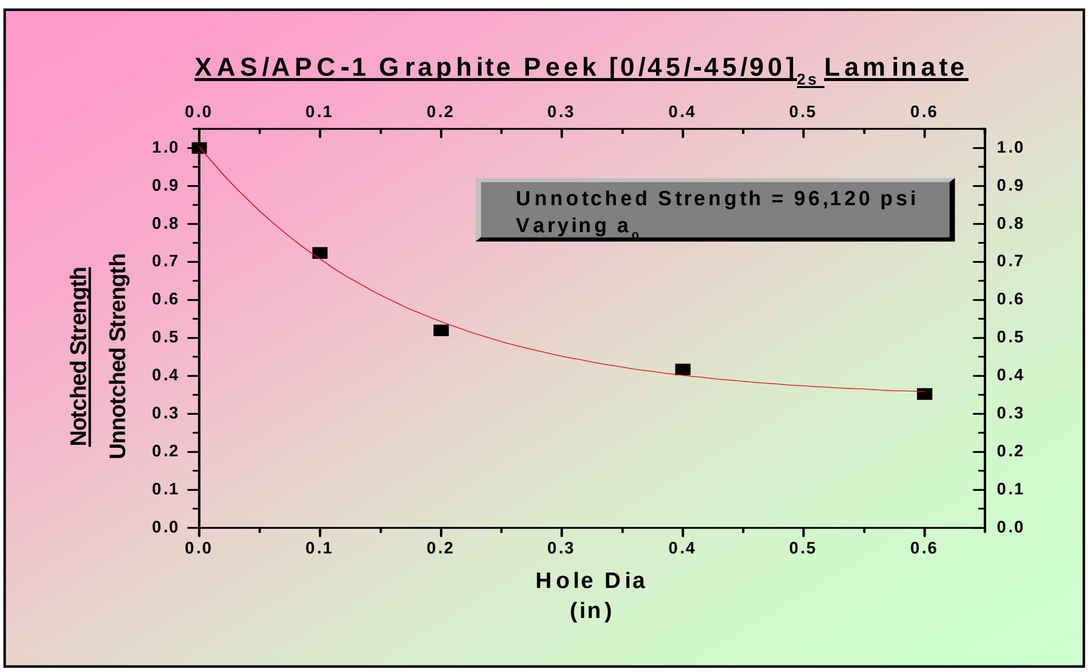

Fig. (4.24) Ultimate Strength for XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$ Laminate using Average Stress Criteria

For Graphite PEEK subjected to average stress criteria, it is found that with 0.1” hole size, the strength of the material reduces to about 28%. On increasing the hole size further, the notched strength decreases at a steady rate, but much faster than Graphite Epoxy.

CHAPTER 5

COMPARISON

A comparison was made between the available experimental data [32] for notched strength and the predicted strength found above through finite element analysis for each material.

5.1. Point Stress Criteria

5.1.1. AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$

Hole Dia (in)	Experimental		Finite Element Analysis	
	Notched Strength "σ_N^∞" (psi)	NSR "$\sigma_N^\infty / \sigma_o$"	Notched Strength "σ_N^∞" (psi)	NSR "$\sigma_N^\infty / \sigma_o$"
0.1	63,650	0.621	52,448	0.512
0.2	58,700	0.573	47,948	0.468
0.4	56,900	0.555	42,681	0.416
0.6	46,660	0.455	40,504	0.395

Table (5.1) Comparison of Experimental and Predicted Notched Strength for AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$ using Point Stress Criteria

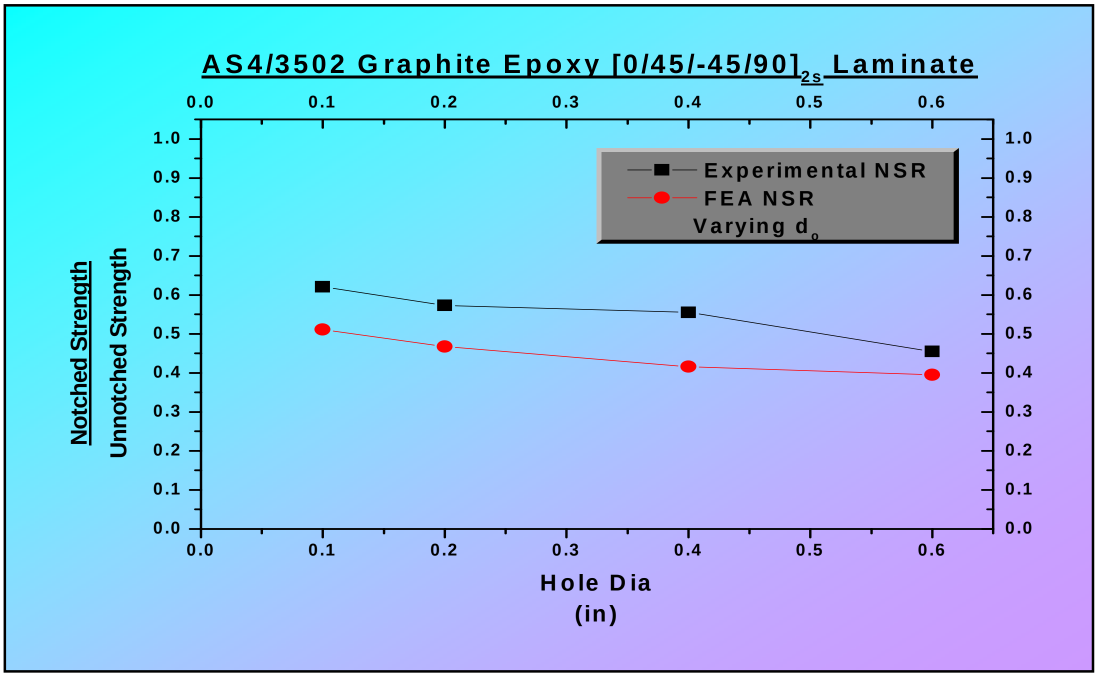

Fig. (5.1) Comparison of Experimental and Predicted Notched Strength for AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$ using Point Stress Criteria

As can be seen that a good correlation is observed between the experimental and the FEA model. The FEA model gives lower value of NSR as compared to the experimental model. But the result tend to converge at higher value of hole radius.

5.1.2. XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$

Hole Dia (in)	Experimental		Finite Element Analysis	
	Notched Strength "σ_N^∞" (psi)	NSR "$\sigma_N^\infty / \sigma_o$"	Notched Strength "σ_N^∞" (psi)	NSR "$\sigma_N^\infty / \sigma_o$"
0.1	69,560	0.724	66,521	0.692
0.2	55,400	0.576	46,269	0.481
0.4	46,168	0.480	34,350	0.357
0.6	38,710	0.403	27,230	0.283

Table (5.2) Comparison of Experimental & Predicted Notched Strength for XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$ using Point Stress Criteria

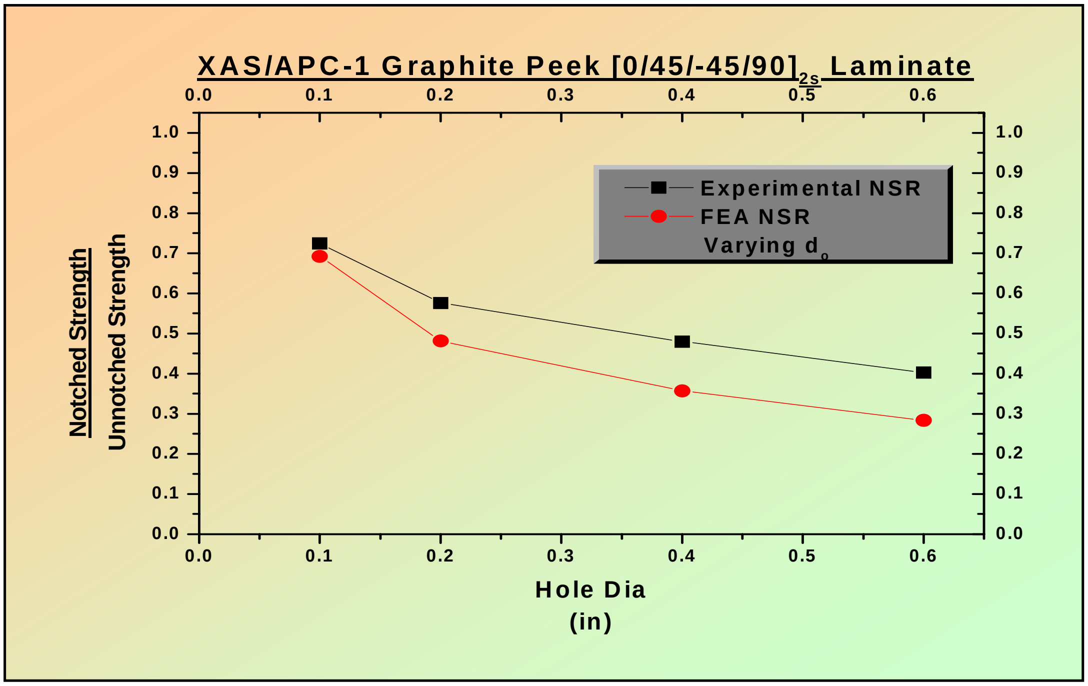

Fig. (5.2) Comparison of Experimental & Predicted Notched Strength for XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$ using Point Stress Criteria

Also for Graphite PEEK subjected to tensile loading using point stress criteria, a good correlation is observed. However, the value of NSR tend to converge initially with a 0.1" hole size, after that there is a little diversion with each increasing hole size.

5.2. Average Stress Criteria

5.2.1. AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$

Hole Dia (in)	Experimental		Finite Element Analysis	
	Notched Strength "σ_N^∞" (psi)	NSR "$\sigma_N^\infty / \sigma_o$"	Notched Strength "σ_N^∞" (psi)	NSR "$\sigma_N^\infty / \sigma_o$"
0.1	63,650	0.621	60,866	0.594
0.2	58,700	0.573	48,013	0.468
0.4	56,900	0.555	47,804	0.466
0.6	46,660	0.455	45,637	0.445

Table (5.3) Comparison of Experimental and Predicted Notched Strength for AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$ using Average Stress Criteria

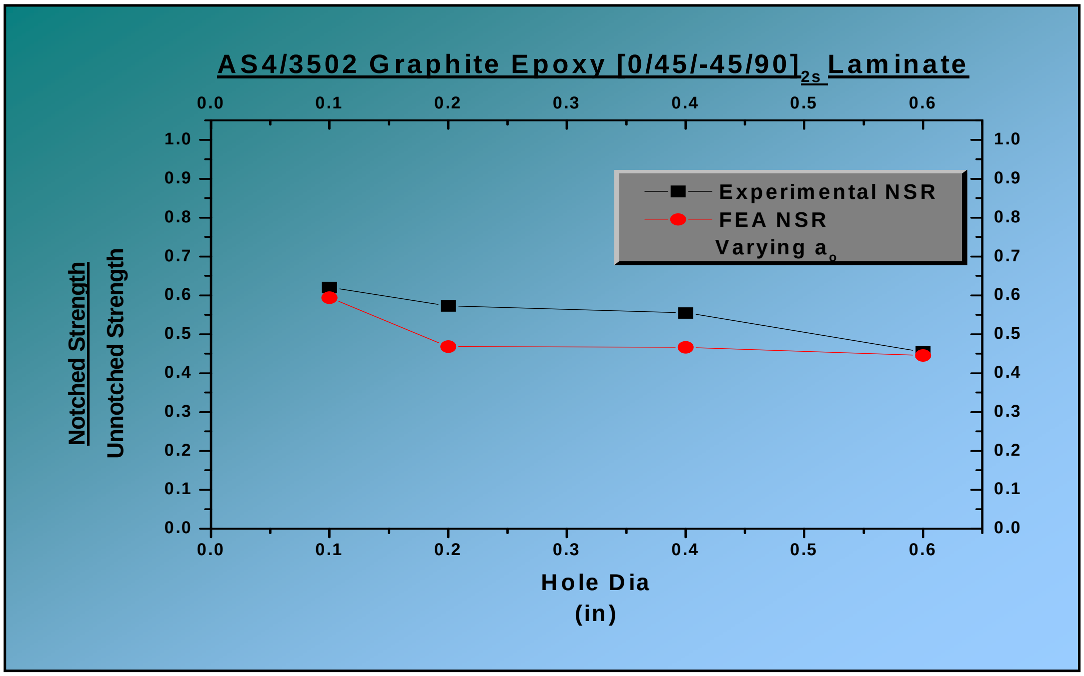

Fig. (5.3) Comparison of Experimental and Predicted Notched Strength for AS4/3502 Graphite Epoxy $[0/\pm45/90]_{2s}$ using Average Stress Criteria

When notched strength of Graphite Epoxy is compared with the experimental data using average stress criteria, a very good correlation is observed. The value of NSR tends to converge with the experimental data very closely at 0.1" and 0.6" diameter of hole. It deviates a little at 0.2" and 0.4" hole size, but still it is in very good relation with the experimental results.

5.2.2. XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$

Hole Dia (in)	Experimental		Finite Element Analysis	
	Notched Strength "σ_N^∞" (psi)	NSR "$\sigma_N^\infty / \sigma_o$"	Notched Strength "σ_N^∞" (psi)	NSR "$\sigma_N^\infty / \sigma_o$"
0.1	69,560	0.724	66,546	0.724
0.2	55,400	0.576	49,952	0.52
0.4	46,168	0.480	34,350	0.417
0.6	38,710	0.403	27,230	0.353

Table (5.4) Comparison of Experimental & Predicted Notched Strength for XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$ using Average Stress Criteria

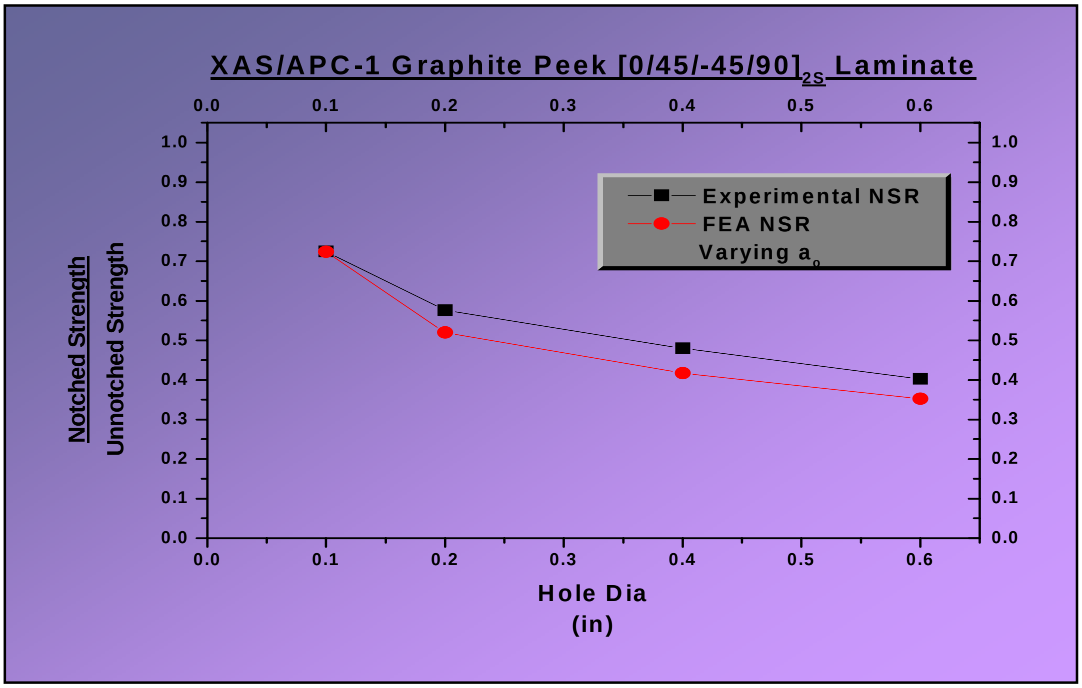

Fig. (5.4) Comparison of Experimental and Predicted Notched Strength for XAS/APC-1 Graphite PEEK $[0/\pm45/90]_{2s}$ using Average Stress Criteria

The best correlation between the experimental NSR and FEA model exists for Graphite PEEK when employing average stress criteria.

As can be seen from above results, the average stress criteria gives a better result when compared with the point stress criteria. This is probably due to the averaging process.

CONCLUSION

Following conclusions can be drawn from the above investigation.

- The value of characteristic length in tension does not found to be material constant. Rather, it shows dependency on notch size and lay-up configuration.

- Good correlation is observed when the results of notched strength from ANSYS are compared with the experimental data. Also values of notched strength obtained from Ansys are somewhat conservative than the experimental results.

- The strength of Graphite Epoxy decreases at a higher rate initially, but as the hole size further increases, the strength decreases at a slow and steady rate.

- Whereas the strength of Graphite PEEK decreases at a slow rate initially, but as the hole size further increases, the strength decreases at a faster rate.

- The average stress criteria gives a better result when compared with the experimental failure stresses than the point stress criteria. So data scattering is found much less using average stress criteria. The reason for this is because of the averaging process itself.

- The best correlation between the experimental failure stresses and FEA results exists for Graphite PEEK when employing average stress criteria.

- The good correlation of FEA model with the experimental data shows its ability to save time, efforts, and finances for carrying out expensive experiments for every application.

- The results generated here for the failure stresses are for a particular lay-up configuration of the material. Hence no general model can be presented which can be considered as completely material dependent.

- Finite element approach for the prediction of failure strength of composite laminates with holes is found to be valid and can be used as a replacement of costly and time consuming experimental work.

RECOMMENDATIONS

From the above analysis, it can be concluded that there are some unsolved issues. So there are some recommendations for the future work.

- It is recommended that further analysis should be done by refining the mesh size further to find the exact solution since the above analysis gives approximate result.

- Different failure criteria should be employed for predicting the failure strength of the same materials used and a comparison be made to find out which one is better.

- Since the result obtained through above analysis are for a particular lay-up configuration of the material, so different lay-up configurations of the same material should be tested.

- The above analysis is done for tensile loading, so it is recommended that the materials should be tested for compression loading also.

- Temperature effect should be considered in further analysis.

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APPENDIX – 1A

Point Stress Criterion

Graphite Epoxy

Calculation of Characteristic Length in Tension (d_o)

```
% "Strength and failure analysis of composite laminates using finite element
% method"
%
% Calculation for the value of characteristic length in tension "Rt/do" for
% AS4/3502 Graphite Epoxy [0/+45/-45/90]2s laminates using "Point Stress
% Criterion"
%
Q = ReducedStiffness(143.92,11.86,0.326,6.69);% Stiffness Matrix(E1,E2,v12,G)
% Qbar = [Qbar] matrix for layer k
Qbar1 = Qbar(Q,0);
Qbar2 = Qbar(Q,45);
Qbar3 = Qbar(Q,-45);
Qbar4 = Qbar(Q,90);
Qbar5 = Qbar(Q,0);
Qbar6 = Qbar(Q,45);
Qbar7 = Qbar(Q,-45);
Qbar8 = Qbar(Q,90);
Qbar9 = Qbar(Q,90);
Qbar10 = Qbar(Q,-45);
Qbar11 = Qbar(Q,45);
Qbar12 = Qbar(Q,0);
Qbar13 = Qbar(Q,90);
Qbar14 = Qbar(Q,-45);
Qbar15 = Qbar(Q,45);
Qbar16 = Qbar(Q,0);
% ----------------------------------------------------------------------
z1 = -0.04;
z2 = -0.035;
z3 = -0.03;
z4 = -0.025;
z5 = -0.02;
z6 = -0.015;
z7 = -0.01;
z8 = -0.005;
z9 = 0;
z10 = 0.005;
z11 = 0.01;
z12 = 0.015;
z13 = 0.02;
z14 = 0.025;
z15 = 0.03;
z16 = 0.035;
z17 = 0.04;
% ----------------------------------------------------------------------
% Amatrix  This function returns the [A] matrix after the layer k with
% stiffness [Qbar] is assembeled
```

```
% A = [A]matrix after layer k is assembeled
A = zeros(3,3);
A = Amatrix(A,Qbar1,z1,z2);
A = Amatrix(A,Qbar2,z2,z3);
A = Amatrix(A,Qbar3,z3,z4);
A = Amatrix(A,Qbar4,z4,z5);
A = Amatrix(A,Qbar5,z5,z6);
A = Amatrix(A,Qbar6,z6,z7);
A = Amatrix(A,Qbar7,z7,z8);
A = Amatrix(A,Qbar8,z8,z9);
A = Amatrix(A,Qbar9,z9,z10);
A = Amatrix(A,Qbar10,z10,z11);
A = Amatrix(A,Qbar11,z11,z12);
A = Amatrix(A,Qbar12,z12,z13);
A = Amatrix(A,Qbar13,z13,z14);
A = Amatrix(A,Qbar14,z14,z15);
A = Amatrix(A,Qbar15,z15,z16);
A = Amatrix(A,Qbar16,z16,z17); % This matrix will give the values of A11,
                                %A12, A22, and A66.
A11 = 4.9265;
A12 = 1.6718;
A22 = 4.7136;
A66 = 3.2547;
% Calculation of Stress Concentration Factor
Kt = 1+sqrt((2/A22).*(sqrt(A11.*A22)-A12+(((A11.*A22)-(A12.^2))./(2.*A66))))
%
% ----------------------------------------------------------------------
%
N1 = 0.621; % Ratio of notched to unnotched strength at 0.1(in) hole dia
E1 = [(7*N1*Kt-21*N1) 0 (15*N1-5*N1*Kt) 0 3*N1 0 N1 0 (2*N1-2)]; % Epsilon
e1 = roots(E1)
E1 = 0.6897;  % Consider only positive real value
R1 = 0.05;    % Radius of hole "0.05 in"
Rt1 = (R1./E1)-R1  % Characteristic length in tension
%
% -----------------------------------------------------------------------
%
N2 = 0.573; % Ratio of notched to unnotched strength at 0.2(in) hole dia
E2 = [(7*N2*Kt-21*N2) 0 (15*N2-5*N2*Kt) 0 3*N2 0 N2 0 (2*N2-2)]; % Epsilon
e2 = roots(E2)
E2 = 0.7357;   % Consider only positive real value
R2 = 0.1;      % Radius of hole "0.1 in"
Rt2 = (R2./E2)-R2   % Characteristic length in tension
%
% -----------------------------------------------------------------------
%
N3 = 0.555;  % Ratio of notched to unnotched strength at 0.4(in) hole dia
E3 = [(7*N3*Kt-21*N3) 0 (15*N3-5*N3*Kt) 0 3*N3 0 N3 0 (2*N3-2)]; % Epsilon
e3 = roots(E3)
E3 = 0.7535;    % Consider only positive real value
R3 = 0.2;       % Radius of hole "0.2 in"
Rt3 = (R3./E3)-R3    % Characteristic length in tension
%
% -----------------------------------------------------------------------
%
N4 = 0.455;  % Ratio of notched to unnotched strength at 0.6(in) hole dia
E4 = [(7*N4*Kt-21*N4) 0 (15*N4-5*N4*Kt) 0 3*N4 0 N4 0 (2*N4-2)]; % Epsilon
```

```
e4 = roots(E4)
E4 = 0.8656;    % Consider only positive real value
R4 = 0.3;       % Radius of hole "0.3 in"
Rt4 = (R4./E4)-R4   % Characteristic length in tension
%
% -----------------------------------------------------------------------
%
x = [0.05 0.1 0.2 0.3]
y1 = [Rt1(1,1) Rt2(1,1) Rt3(1,1) Rt4(1,1)]
plot(x,y1)
xlabel('hole radius (in)');
ylabel('characteristics length in tension (in)');
```

Calculation of Notched Strength

Graphite Epoxy with 0.1” hole (d_o = 0.0225”)

```
/TITLE,Graphite epoxy 0.1 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,20.87e6
MPDATA,EY,1,,1.72e6
MPDATA,EZ,1,,1.72e6
MPDATA,PRXY,1,,0.326
MPDATA,PRYZ,1,,0.326
MPDATA,PRXZ,1,,0.326
MPDATA,GXY,1,,0.97e6
MPDATA,GYZ,1,,0.97e6
MPDATA,GXZ,1,,0.97e6
RECTNG,0,2.5,0,0.5
CYL4,0,0,0.05
ASBA,1,2
LESIZE,9,,,300,0.1,,,,0
LESIZE,10,,,300,0.1,,,,0
LESIZE,5,,,14,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
```

```
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-4151.222,
FINISH
/SOL
SOLVE
FINISH
/POST1
SET,FIRST
NSORT,LOC,Y,0,0, ,SELECT
```

Graphite Epoxy with 0.2” hole (d_o = 0.0359”)

```
/TITLE,Graphite Epoxy 0.2 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,20.87e6
MPDATA,EY,1,,1.72e6
MPDATA,EZ,1,,1.72e6
MPDATA,PRXY,1,,0.326
MPDATA,PRYZ,1,,0.326
MPDATA,PRXZ,1,,0.326
MPDATA,GXY,1,,0.97e6
MPDATA,GYZ,1,,0.97e6
MPDATA,GXZ,1,,0.97e6
RECTNG,0,2.5,0,0.5
CYLl4,0,0,0.1
ASBA,1,2
LESIZE,9,,,300,0.1,,,,0
```

```
LESIZE,10,,,300,0.1,,,,0
LESIZE,5,,,25,,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-3664.7805,
FINISH
/SOL
SOLVE
FINISH
```

Graphite Epoxy with 0.4” hole (d_o = 0.0654”)

```
/TITLE,Graphite epoxy 0.4 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,20.87e6
MPDATA,EY,1,,1.72e6
MPDATA,EZ,1,,1.72e6
MPDATA,PRXY,1,,0.326
MPDATA,PRYZ,1,,0.326
MPDATA,PRXZ,1,,0.326
MPDATA,GXY,1,,0.97e6
MPDATA,GYZ,1,,0.97e6
```

```
MPDATA,GXZ,1,,0.97e6
RECTNG,0,2.5,0,0.75
CYL4,0,0,0.2
ASBA,1,2
LESIZE,9,,,450,0.1,,,,0
LESIZE,10,,,450,0.1,,,,0
LESIZE,5,,,52,,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-4705.911,
FINISH
/SOL
SOLVE
FINISH
/POST1
NSORT,LOC,Y,0,0, ,SELECT
```

Graphite Epoxy with 0.6” hole (d_o = 0.0466”)

```
/TITLE,Graphite epoxy 0.6 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,20.87e6
```

```
MPDATA,EY,1,,1.72e6
MPDATA,EZ,1,,1.72e6
MPDATA,PRXY,1,,0.326
MPDATA,PRYZ,1,,0.326
MPDATA,PRXZ,1,,0.326
MPDATA,GXY,1,,0.97e6
MPDATA,GYZ,1,,0.97e6
MPDATA,GXZ,1,,0.97e6
RECTNG,0,2.5,0,1
CYL4,0,0,0.3
ASBA,1,2
LESIZE,9,,,600,0.1,,,,0
LESIZE,10,,,600,0.1,,,,0
LESIZE,5,,,66,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-5808.4986,
FINISH
/SOL
SOLVE
FINISH
/POST1
NSORT,LOC,Y,0,0, ,SELECT
```

APPENDIX – 1B

Point Stress Criterion

Graphite Peek

Calculation of Characteristic Length in Tension (d_o)

```
% "Strength and failure analysis of composite laminates using finite element
% method"
%
% Calculation for the value of characteristic length in tension"Rt/do" for
% XAS/APC-1 Graphite Peek [0/+45/-45/90]2s laminates using "Point Stress
% Criterion"
%
Q = ReducedStiffness(121.1,10.14,0.37,4.65);  % Stiffness Matrix (E1,E2,v12,G)
% Qbar = [Qbar] matrix for layer k
Qbar1 = Qbar(Q,0);
Qbar2 = Qbar(Q,45);
Qbar3 = Qbar(Q,-45);
Qbar4 = Qbar(Q,90);
Qbar5 = Qbar(Q,0);
Qbar6 = Qbar(Q,45);
Qbar7 = Qbar(Q,-45);
Qbar8 = Qbar(Q,90);
Qbar9 = Qbar(Q,90);
Qbar10 = Qbar(Q,-45);
Qbar11 = Qbar(Q,45);
Qbar12 = Qbar(Q,0);
Qbar13 = Qbar(Q,90);
Qbar14 = Qbar(Q,-45);
Qbar15 = Qbar(Q,45);
Qbar16 = Qbar(Q,0);
% -----------------------------------------------------------------------
z1 = -0.04;
z2 = -0.035;
z3 = -0.03;
z4 = -0.025;
z5 = -0.02;
z6 = -0.015;
z7 = -0.01;
z8 = -0.005;
z9 = 0;
z10 = 0.005;
z11 = 0.01;
z12 = 0.015;
z13 = 0.02;
z14 = 0.025;
z15 = 0.03;
z16 = 0.035;
z17 = 0.04;
% -----------------------------------------------------------------------
% Amatrix  This function returns the [A] matrix after the layer k with
% stiffness [Qbar] is assembled
```

```
% A = [A]matrix after layer k is assembled
A = zeros(3,3);
A = Amatrix(A,Qbar1,z1,z2);
A = Amatrix(A,Qbar2,z2,z3);
A = Amatrix(A,Qbar3,z3,z4);
A = Amatrix(A,Qbar4,z4,z5);
A = Amatrix(A,Qbar5,z5,z6);
A = Amatrix(A,Qbar6,z6,z7);
A = Amatrix(A,Qbar7,z7,z8);
A = Amatrix(A,Qbar8,z8,z9);
A = Amatrix(A,Qbar9,z9,z10);
A = Amatrix(A,Qbar10,z10,z11);
A = Amatrix(A,Qbar11,z11,z12);
A = Amatrix(A,Qbar12,z12,z13);
A = Amatrix(A,Qbar13,z13,z14);
A = Amatrix(A,Qbar14,z14,z15);
A = Amatrix(A,Qbar15,z15,z16);
A = Amatrix(A,Qbar16,z16,z17); % This matrix will give the values of A11,
                                %A12, A22, and A66.
A11 = 4.1518;
A12 = 1.4623;
A22 = 4.1518;
A66 = 2.6894;
% Calculation of Stress Concentration Factor
Kt = 1+sqrt((2/A22).*(sqrt(A11.*A22)-A12+(((A11.*A22)-(A12.^2))./(2.*A66))))
%
% -----------------------------------------------------------------------
%
N1 = 0.724; % Ratio of notched to unnotched strength at 0.1(in) hole dia
E1 = [(7*N1*Kt-21*N1) 0 (15*N1-5*N1*Kt) 0 3*N1 0 N1 0 (2*N1-2)]; % Epsilon
e1 = roots(E1)
E1 = 0.5927;  % Consider only positive real value
R1 = 0.05;    % Radius of hole "0.05 in"
Rt1 = (R1./E1)-R1  % Characteristic length in tension
%
% ------------------------------------------------------------------------
%
N2 = 0.576; % Ratio of notched to unnotched strength at 0.2(in) hole dia
E2 = [(7*N2*Kt-21*N2) 0 (15*N2-5*N2*Kt) 0 3*N2 0 N2 0 (2*N2-2)]; % Epsilon
e2 = roots(E2)
E2 = 0.7326;   % Consider only positive real value
R2 = 0.1;      % Radius of hole "0.1 in"
Rt2 = (R2./E2)-R2   % Characteristic length in tension
%
% ------------------------------------------------------------------------
%
N3 = 0.480;  % Ratio of notched to unnotched strength at 0.4(in) hole dia
E3 = [(7*N3*Kt-21*N3) 0 (15*N3-5*N3*Kt) 0 3*N3 0 N3 0 (2*N3-2)]; % Epsilon
e3 = roots(E3)
E3 = 0.8344;    % Consider only positive real value
R3 = 0.2;       % Radius of hole "0.2 in"
Rt3 = (R3./E3)-R3    % Characteristic length in tension
%
% ------------------------------------------------------------------------
%
N4 = 0.403;  % Ratio of notched to unnotched strength at 0.6(in) hole dia
E4 = [(7*N4*Kt-21*N4) 0 (15*N4-5*N4*Kt) 0 3*N4 0 N4 0 (2*N4-2)]; % Epsilon
```

```
e4 = roots(E4)
E4 = 0.9465;    % Consider only positive real value
R4 = 0.3;       % Radius of hole "0.3 in"
Rt4 = (R4./E4)-R4   % Characteristic length in tension
%
% -------------------------------------------------------------------------
%
x = [0.05 0.1 0.2 0.3]
y1 = [Rt1(1,1) Rt2(1,1) Rt3(1,1) Rt4(1,1)]
plot(x,y1)
xlabel('hole radius (in)');
ylabel('characteristics length in tension (in)');
```

Calculation of Notched Strength

Graphite Peek with 0.1” hole (d_o = 0.033”)

```
/TITLE,Graphite peek 0.1 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,17.56e6
MPDATA,EY,1,,1.47e6
MPDATA,EZ,1,,1.47e6
MPDATA,PRXY,1,,0.37
MPDATA,PRYZ,1,,0.37
MPDATA,PRXZ,1,,0.37
MPDATA,GXY,1,,0.67e6
MPDATA,GYZ,1,,0.67e6
MPDATA,GXZ,1,,0.67e6
RECTNG,0,2.5,0,0.5
CYL4,0,0,0.05
ASBA,1,2
LESIZE,9,,,300,0.1,,,,0
LESIZE,10,,,300,0.1,,,,0
LESIZE,5,,,25,,,,,0
FINISH
/POST1
FINISH
/PREP7
SMRT,6
```

```
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-5265.164,
FINISH
/SOL
SOLVE
FINISH
```

Graphite Peek with 0.2” hole (d_o = 0.0365”)

```
/TITLE,Graphite peek 0.2 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,17.56e6
MPDATA,EY,1,,1.47e6
MPDATA,EZ,1,,1.47e6
MPDATA,PRXY,1,,0.37
MPDATA,PRYZ,1,,0.37
MPDATA,PRXZ,1,,0.37
MPDATA,GXY,1,,0.67e6
MPDATA,GYZ,1,,0.67e6
MPDATA,GXZ,1,,0.67e6
```

```
RECTNG,0,2.5,0,0.5
CYL4,0,0,0.1
ASBA,1,2
LESIZE,9,,,300,0.1,,,,0
LESIZE,10,,,300,0.1,,,,0
LESIZE,5,,,30,,,,,,0
FINISH
/SOL
SOLVE
FINISH
/PREP7
SMRT,6
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-3536.459,
FINISH
/SOL
SOLVE
FINISH
/POST1
NSORT,LOC,Y,0,0, ,SELECT
```

Graphite Peek with 0.4” hole (d_o = 0.0397”)

```
/TITLE,Graphite peek 0.4 hole /PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
```

```
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,17.56e6
MPDATA,EY,1,,1.47e6
MPDATA,EZ,1,,1.47e6
MPDATA,PRXY,1,,0.37
MPDATA,PRYZ,1,,0.37
MPDATA,PRXZ,1,,0.37
MPDATA,GXY,1,,0.67e6
MPDATA,GYZ,1,,0.67e6
MPDATA,GXZ,1,,0.67e6
RECTNG,0,2.5,0,0.75
CYL4,0,0,0.2
ASBA,1,2
LESIZE,9,,,450,0.1,,,,0
LESIZE,10,,,450,0.1,,,,0
LESIZE,5,,,54,,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-3787.389,
FINISH
/SOL
SOLVE
FINISH
/POST1
NSORT,LOC,Y,0,0, ,SELECT
```

Graphite Peek with 0.6" hole (d_o = 0.017")

```
/TITLE,Graphite peek 0.6 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,17.56e6
MPDATA,EY,1,,1.47e6
MPDATA,EZ,1,,1.47e6
MPDATA,PRXY,1,,0.37
MPDATA,PRYZ,1,,0.37
MPDATA,PRXZ,1,,0.37
MPDATA,GXY,1,,0.67e6
MPDATA,GYZ,1,,0.67e6
MPDATA,GXZ,1,,0.67e6
RECTNG,0,2.5,0,1
CYL4,0,0,0.3
ASBA,1,2
LESIZE,9,,,600,0.1,,,,0
LESIZE,10,,,600,0.1,,,,0
LESIZE,5,,,63,,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
```

```
FITEM,2,2
/GO
SFL,P51X,PRES,-3904.966,
FINISH
/SOL
SOLVE
FINISH
/POST1
NSORT,LOC,Y,0,0, ,SELECT
```

APPENDIX – 2A

Average Stress Criterion

Graphite Epoxy

Calculation of Characteristic Length in Tension (a_o)

```
% "Strength and failure analysis of composite laminates using finite element
% method"
%
% Calculation for the value of characteristic length in tension "Rt/ao" for
% AS4/3502 Graphite Epoxy [0/+45/-45/90]2s laminates using "Average Stress
% Criterion"
%
Q = ReducedStiffness(143.92,11.86,0.326,6.69);  % Stiffness Matrix
(E1,E2,v12,G)
% Qbar = [Qbar] matrix for layer k
Qbar1 = Qbar(Q,0);
Qbar2 = Qbar(Q,45);
Qbar3 = Qbar(Q,-45);
Qbar4 = Qbar(Q,90);
Qbar5 = Qbar(Q,0);
Qbar6 = Qbar(Q,45);
Qbar7 = Qbar(Q,-45);
Qbar8 = Qbar(Q,90);
Qbar9 = Qbar(Q,90);
Qbar10 = Qbar(Q,-45);
Qbar11 = Qbar(Q,45);
Qbar12 = Qbar(Q,0);
Qbar13 = Qbar(Q,90);
Qbar14 = Qbar(Q,-45);
Qbar15 = Qbar(Q,45);
Qbar16 = Qbar(Q,0);
% -----------------------------------------------------------------------
z1 = -0.04;
z2 = -0.035;
z3 = -0.03;
z4 = -0.025;
z5 = -0.02;
z6 = -0.015;
z7 = -0.01;
z8 = -0.005;
z9 = 0;
z10 = 0.005;
z11 = 0.01;
z12 = 0.015;
z13 = 0.02;
z14 = 0.025;
z15 = 0.03;
z16 = 0.035;
z17 = 0.04;
% -----------------------------------------------------------------------
% Amatrix  This function returns the [A] matrix after the layer k with
```

```
% stiffness [Qbar] is assembeled
% A = [A]matrix after layer k is assembeled
A = zeros(3,3);
A = Amatrix(A,Qbar1,z1,z2);
A = Amatrix(A,Qbar2,z2,z3);
A = Amatrix(A,Qbar3,z3,z4);
A = Amatrix(A,Qbar4,z4,z5);
A = Amatrix(A,Qbar5,z5,z6);
A = Amatrix(A,Qbar6,z6,z7);
A = Amatrix(A,Qbar7,z7,z8);
A = Amatrix(A,Qbar8,z8,z9);
A = Amatrix(A,Qbar9,z9,z10);
A = Amatrix(A,Qbar10,z10,z11);
A = Amatrix(A,Qbar11,z11,z12);
A = Amatrix(A,Qbar12,z12,z13);
A = Amatrix(A,Qbar13,z13,z14);
A = Amatrix(A,Qbar14,z14,z15);
A = Amatrix(A,Qbar15,z15,z16);
A = Amatrix(A,Qbar16,z16,z17); % This matrix will give the values of A11,
                                %A12, A22, and A66.
A11 = 4.9265;
A12 = 1.6718;
A22 = 4.7136;
A66 = 3.2547;
% Calculation of Stress Concentration Factor
Kt = 1+sqrt((2/A22).*(sqrt(A11.*A22)-A12+(((A11.*A22)-(A12.^2))./(2.*A66))))
%
% ---------------------------------------------------------------------
%
N1 = 0.621; % Ratio of notched to unnotched strength at 0.1(in) hole dia
E1 = [(3*N1-N1*Kt) 0 (N1*Kt-3*N1) 0 (-N1) 0 N1 2 (2*N1-2)]; % Epsilon
e1 = roots(E1)
E1 = 0.3464;  % Consider only positive real value
R1 = 0.05;    % Radius of hole "0.05 in"
Rt1 = (R1./E1)-R1  % Characteristic length in tension
%
% ----------------------------------------------------------------------
%
N2 = 0.573; % Ratio of notched to unnotched strength at 0.2(in) hole dia
E2 = [(3*N2-N2*Kt) 0 (N2*Kt-3*N2) 0 (-N2) 0 N1 2 (2*N2-2)]; % Epsilon
e2 = roots(E2)
E2 = 0.3872;   % Consider only positive real value
R2 = 0.1;      % Radius of hole "0.1 in"
Rt2 = (R2./E2)-R2   % Characteristic length in tension
%
% ----------------------------------------------------------------------
%
N3 = 0.555;  % Ratio of notched to unnotched strength at 0.4(in) hole dia
E3 = [(3*N3-N3*Kt) 0 (N3*Kt-3*N3) 0 (-N3) 0 N3 2 (2*N3-2)]; % Epsilon
e3 = roots(E3)
E3 = 0.4070;    % Consider only positive real value
R3 = 0.2;       % Radius of hole "0.2 in"
Rt3 = (R3./E3)-R3    % Characteristic length in tension
%
% ----------------------------------------------------------------------
%
N4 = 0.455;  % Ratio of notched to unnotched strength at 0.6(in) hole dia
```

```
E4 = [(3*N4-N4*Kt) 0 (N4*Kt-3*N4) 0 (-N4) 0 N4 2 (2*N4-2)]; % Epsilon
e4 = roots(E4)
E4 = 0.5030;    % Consider only positive real value
R4 = 0.3;       % Radius of hole "0.3 in"
Rt4 = (R4./E4)-R4   % Characteristic length in tension
%
% ------------------------------------------------------------------------
%
x = [0.05 0.1 0.2 0.3]
y1 = [Rt1(1,1) Rt2(1,1) Rt3(1,1) Rt4(1,1)]
plot(x,y1)
xlabel('hole radius (in)');
ylabel('characteristics length in tension (in)');
```

Calculation of Notched Strength

Graphite Epoxy with 0.1” hole (a_o = 0.0943”)

```
/TITLE,Graphite epoxy 0.1 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,20.87e6
MPDATA,EY,1,,1.72e6
MPDATA,EZ,1,,1.72e6
MPDATA,PRXY,1,,0.326
MPDATA,PRYZ,1,,0.326
MPDATA,PRXZ,1,,0.326
MPDATA,GXY,1,,0.97e6
MPDATA,GYZ,1,,0.97e6
MPDATA,GXZ,1,,0.97e6
RECTNG,0,2.5,0,0.5
CYL4,0,0,0.05
ASBA,1,2
LESIZE,9,,,300,0.1,,,,0
LESIZE,10,,,300,0.1,,,,0
LESIZE,5,,,14,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
```

```
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-4817.527,
FINISH
/SOL
SOLVE
FINISH
```

Graphite Epoxy with 0.2” hole (a_o = 0.1583”)

```
/TITLE,Graphite Epoxy 0.2 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,20.87e6
MPDATA,EY,1,,1.72e6
MPDATA,EZ,1,,1.72e6
MPDATA,PRXY,1,,0.326
MPDATA,PRYZ,1,,0.326
MPDATA,PRXZ,1,,0.326
MPDATA,GXY,1,,0.97e6
MPDATA,GYZ,1,,0.97e6
MPDATA,GXZ,1,,0.97e6
RECTNG,0,2.5,0,0.5
CYL4,0,0,0.1
ASBA,1,2
LESIZE,9,,,300,0.1,,,,0
LESIZE,10,,,300,0.1,,,,0
LESIZE,5,,,25,,,,,0
SMRT,1
```

```
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,        3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-3669.792,
FINISH
/SOL
SOLVE
FINISH
```

Graphite Epoxy with 0.4” hole (a_o = 0.2914”)

```
/TITLE,Graphite epoxy 0.4 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,20.87e6
MPDATA,EY,1,,1.72e6
MPDATA,EZ,1,,1.72e6
MPDATA,PRXY,1,,0.326
MPDATA,PRYZ,1,,0.326
MPDATA,PRXZ,1,,0.326
MPDATA,GXY,1,,0.97e6
MPDATA,GYZ,1,,0.97e6
MPDATA,GXZ,1,,0.97e6
RECTNG,0,2.5,0,0.75
CYL4,0,0,0.2
```

```
ASBA,1,2
LESIZE,9,,,450,0.1,,,,0
LESIZE,10,,,450,0.1,,,,0
LESIZE,5,,,52,,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-5270.838,
FINISH
/SOL
SOLVE
FINISH
```

Graphite Epoxy with 0.6" hole (a_o = 0.2964")

```
/TITLE,Graphite epoxy 0.6 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,20.87e6
MPDATA,EY,1,,1.72e6
MPDATA,EZ,1,,1.72e6
MPDATA,PRXY,1,,0.326
MPDATA,PRYZ,1,,0.326
MPDATA,PRXZ,1,,0.326
```

```
MPDATA,GXY,1,,0.97e6
MPDATA,GYZ,1,,0.97e6
MPDATA,GXZ,1,,0.97e6
RECTNG,0,2.5,0,1
CYL4,0,0,0.3
ASBA,1,2
LESIZE,9,,,600,0.1,,,,0
LESIZE,10,,,600,0.1,,,,0
LESIZE,5,,,66,,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-6544.616,
FINISH
/SOL
SOLVE
FINISH
```

APPENDIX – 2B

Average Stress Criterion

Graphite Peek

Calculation of Characteristic Length in Tension (a_o)

```
% "Strength and failure analysis of composite laminates using finite element
% method"
%
% Calculation for the value of characteristic length in tension"Rt" for
% XAS/APC-1 Graphite Peek [0/+45/-45/90]2s laminates using "Average Stress
% Criterion"
%
Q = ReducedStiffness(121.1,10.14,0.37,4.65);  % Stiffness Matrix (E1,E2,v12,G)
% Qbar = [Qbar] matrix for layer k
Qbar1 = Qbar(Q,0);
Qbar2 = Qbar(Q,45);
Qbar3 = Qbar(Q,-45);
Qbar4 = Qbar(Q,90);
Qbar5 = Qbar(Q,0);
Qbar6 = Qbar(Q,45);
Qbar7 = Qbar(Q,-45);
Qbar8 = Qbar(Q,90);
Qbar9 = Qbar(Q,90);
Qbar10 = Qbar(Q,-45);
Qbar11 = Qbar(Q,45);
Qbar12 = Qbar(Q,0);
Qbar13 = Qbar(Q,90);
Qbar14 = Qbar(Q,-45);
Qbar15 = Qbar(Q,45);
Qbar16 = Qbar(Q,0);
% -----------------------------------------------------------------------
z1 = -0.04;
z2 = -0.035;
z3 = -0.03;
z4 = -0.025;
z5 = -0.02;
z6 = -0.015;
z7 = -0.01;
z8 = -0.005;
z9 = 0;
z10 = 0.005;
z11 = 0.01;
z12 = 0.015;
z13 = 0.02;
z14 = 0.025;
z15 = 0.03;
z16 = 0.035;
z17 = 0.04;
% -----------------------------------------------------------------------
% Amatrix  This function returns the [A] matrix after the layer k with
% stiffness [Qbar] is assembeled
```

```
% A = [A]matrix after layer k is assembeled
A = zeros(3,3);
A = Amatrix(A,Qbar1,z1,z2);
A = Amatrix(A,Qbar2,z2,z3);
A = Amatrix(A,Qbar3,z3,z4);
A = Amatrix(A,Qbar4,z4,z5);
A = Amatrix(A,Qbar5,z5,z6);
A = Amatrix(A,Qbar6,z6,z7);
A = Amatrix(A,Qbar7,z7,z8);
A = Amatrix(A,Qbar8,z8,z9);
A = Amatrix(A,Qbar9,z9,z10);
A = Amatrix(A,Qbar10,z10,z11);
A = Amatrix(A,Qbar11,z11,z12);
A = Amatrix(A,Qbar12,z12,z13);
A = Amatrix(A,Qbar13,z13,z14);
A = Amatrix(A,Qbar14,z14,z15);
A = Amatrix(A,Qbar15,z15,z16);
A = Amatrix(A,Qbar16,z16,z17); % This matrix will give the values of A11,
                                %A12, A22, and A66.
A11 = 4.1518;
A12 = 1.4623;
A22 = 4.1518;
A66 = 2.6894;
% Calculation of Stress Concentration Factor
Kt = 1+sqrt((2/A22).*(sqrt(A11.*A22)-A12+(((A11.*A22)-(A12.^2))./(2.*A66))))
%
% -----------------------------------------------------------------------
%
N1 = 0.724; % Ratio of notched to unnotched strength at 0.1(in) hole dia
E1 = [(3*N1-N1*Kt) 0 (N1*Kt-3*N1) 0 (-N1) 0 N1 2 (2*N1-2)]; % Epsilon
e1 = roots(E1)
E1 = 0.2542;   % Consider only positive real value
R1 = 0.05;    % Radius of hole "0.05 in"
Rt1 = (R1./E1)-R1  % Characteristic length in tension
%
% ------------------------------------------------------------------------
%
N2 = 0.576; % Ratio of notched to unnotched strength at 0.2(in) hole dia
E2 = [(3*N2-N2*Kt) 0 (N2*Kt-3*N2) 0 (-N2) 0 N2 2 (2*N2-2)]; % Epsilon
e2 = roots(E2)
E2 = 0.3875;   % Consider only positive real value
R2 = 0.1;      % Radius of hole "0.1 in"
Rt2 = (R2./E2)-R2   % Characteristic length in tension
%
% ------------------------------------------------------------------------
%
N3 = 0.480;  % Ratio of notched to unnotched strength at 0.4(in) hole dia
E3 = [(3*N3-N3*Kt) 0 (N3*Kt-3*N3) 0 (-N3) 0 N3 2 (2*N3-2)]; % Epsilon
e3 = roots(E3)
E3 = 0.4785;    % Consider only positive real value
R3 = 0.2;       % Radius of hole "0.2 in"
Rt3 = (R3./E3)-R3    % Characteristic length in tension
%
% ------------------------------------------------------------------------
%
N4 = 0.403;  % Ratio of notched to unnotched strength at 0.6(in) hole dia
E4 = [(3*N4-N4*Kt) 0 (N4*Kt-3*N4) 0 (-N4) 0 N4 2 (2*N4-2)]; % Epsilon
```

```
e4 = roots(E4)
E4 = 0.5555;    % Consider only positive real value
R4 = 0.3;       % Radius of hole "0.3 in"
Rt4 = (R4./E4)-R4   % Characteristic length in tension
%
% ------------------------------------------------------------------------
%
x = [0.05 0.1 0.2 0.3]
y1 = [Rt1(1,1) Rt2(1,1) Rt3(1,1) Rt4(1,1)]
plot(x,y1)
xlabel('hole radius (in)');
ylabel('characteristics length in tension (in)');
```

Calculation of Notched Strength

Graphite Peek with 0.1” hole (a_o = 0.1467”)

```
/TITLE,Graphite peek 0.1 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,17.56e6
MPDATA,EY,1,,1.47e6
MPDATA,EZ,1,,1.47e6
MPDATA,PRXY,1,,0.37
MPDATA,PRYZ,1,,0.37
MPDATA,PRXZ,1,,0.37
MPDATA,GXY,1,,0.67e6
MPDATA,GYZ,1,,0.67e6
MPDATA,GXZ,1,,0.67e6
RECTNG,0,2.5,0,0.5
CYL4,0,0,0.05
ASBA,1,2
LESIZE,9,,,300,0.1,,,,0
LESIZE,10,,,300,0.1,,,,0
LESIZE,5,,,25,,,,,0
FINISH
/POST1
FINISH
/PREP7
SMRT,6
SMRT,1
MSHAPE,0,2D
```

```
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-5504.54
SFL ,
FINISH
/SOL
SOLVE
FINISH
```

Graphite Peek with 0.2" hole (a_o = 0.1581")

```
/TITLE,Graphite peek 0.2 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,17.56e6
MPDATA,EY,1,,1.47e6
MPDATA,EZ,1,,1.47e6
MPDATA,PRXY,1,,0.37
MPDATA,PRYZ,1,,0.37
MPDATA,PRXZ,1,,0.37
MPDATA,GXY,1,,0.67e6
MPDATA,GYZ,1,,0.67e6
MPDATA,GXZ,1,,0.67e6
RECTNG,0,2.5,0,0.5
CYL4,0,0,0.1
```

```
ASBA,1,2
LESIZE,9,,,300,0.1,,,,0
LESIZE,10,,,300,0.1,,,,0
LESIZE,5,,,30,,,,,,0
FINISH
/SOL
SOLVE
FINISH
/PREP7
SMRT,6
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-3818.003,
FINISH
/SOL
SOLVE
FINISH
```

Graphite Peek with 0.4” hole (a_o = 0.218”)

```
/TITLE,Graphite peek 0.4 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
```

```
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,17.56e6
MPDATA,EY,1,,1.47e6
MPDATA,EZ,1,,1.47e6
MPDATA,PRXY,1,,0.37
MPDATA,PRYZ,1,,0.37
MPDATA,PRXZ,1,,0.37
MPDATA,GXY,1,,0.67e6
MPDATA,GYZ,1,,0.67e6
MPDATA,GXZ,1,,0.67e6
RECTNG,0,2.5,0,0.75
CYL4,0,0,0.2
ASBA,1,2
LESIZE,9,,,450,0.1,,,,0
LESIZE,10,,,450,0.1,,,,0
LESIZE,5,,,54,,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-4423.752,
FINISH
/SOL
SOLVE
FINISH
```

Graphite Peek with 0.6” hole (a_o = 0.2401”)

```
/TITLE,Graphite peek 0.6 hole
/PREP7
ET,1,SHELL99
*SET,_RC_SET,1,
R,1
RMODIF,1,1,16,1,0,0,0,0
*SET,_RC_SET,1,
R,1
```

```
RMODIF,1,1,16,1,0,0,0,0
RMODIF,1,13,1,0,0.005,1,45,0.005,
RMODIF,1,19,1,-45,0.005,1,90,0.005,
RMODIF,1,25,1,0,0.005,1,45,0.005,
RMODIF,1,31,1,-45,0.005,1,90,0.005,
RMODIF,1,37,1,0,0,1,0,0,
RMODIF,1,43,1,0,0,1,0,0,
RMODIF,1,49,1,0,0,1,0,0,
RMODIF,1,55,1,0,0,1,0,0,
MPTEMP,,,,,,,,
MPTEMP,1,0
MPDATA,EX,1,,17.56e6
MPDATA,EY,1,,1.47e6
MPDATA,EZ,1,,1.47e6
MPDATA,PRXY,1,,0.37
MPDATA,PRYZ,1,,0.37
MPDATA,PRXZ,1,,0.37
MPDATA,GXY,1,,0.67e6
MPDATA,GYZ,1,,0.67e6
MPDATA,GXZ,1,,0.67e6
RECTNG,0,2.5,0,1
CYL4,0,0,0.3
ASBA,1,2
LESIZE,9,,,600,0.1,,,,0
LESIZE,10,,,600,0.1,,,,0
LESIZE,5,,,63,,,,,,0
SMRT,1
MSHAPE,0,2D
MSHKEY,0
CM,_Y,AREA
ASEL, , , ,       3
CM,_Y1,AREA
CHKMSH,'AREA'
CMSEL,S,_Y
AMESH,_Y1
CMDELE,_Y
CMDELE,_Y1
CMDELE,_Y2
FLST,2,1,4,ORDE,1
FITEM,2,9
/GO
DL,P51X, ,UY,0
FLST,2,1,4,ORDE,1
FITEM,2,10
/GO
DL,P51X, ,UX,0
FLST,2,1,4,ORDE,1
FITEM,2,2
/GO
SFL,P51X,PRES,-4858.965,
FINISH
/SOL
SOLVE
FINISH
```