Quantum Walks of Two Entangled Photons

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Dedication

"This thesis is respectfully dedicated to my late parents. It is with deep gratitude that I also offer a special dedication to my uncle, without whose unwavering support and guidance this achievement would not have been possible. Additionally, I extend my heartfelt appreciation to my brothers for their steadfast support throughout this academic endeavor. While our parents may no longer be with us, their legacy lives on through our shared commitment to learning and achievement."

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Abstract

The quantum walk serves as the quantum counterpart to the classical random walk. This study delves into the exploration of quantum walks using two entangled photons and linear optical elements. Through analysis of various quantum states, including entangled and separable states, the study aims to comprehend the quantum random walk phenomenon. The research uncovers a substantial dependency of the probability of two-photon detection on the quantum nature of their states. Probabilities are calculated using the coined discrete-time quantum walk method, complemented by simulations. The findings hold significant implications for quantum computers, leveraging entangled bits to accelerate information processing. Additionally, applications such as super dense coding, quantum algorithms, teleportation and secure quantum key distribution stand to benefit from the insights gained.

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Chapter 1

Introduction

1.1 Preface

In the world of science, one of the most effective ways to investigate the nature of an occurrence or process is to use measurements. Physicists have focused their attention on precise measurements to gain a deeper comprehension of the workings of the natural world. As part of these efforts, in the year 1900, they formulated the theory of quantum mechanics with the goal of gaining previously unattainable insights into the workings of the natural world. The theory of quantum mechanics is a fundamental component of the study of physics because it explains how matter and energy operate on the smallest of scales, our understanding of the microscopic world underwent a revolutionary shift because of it upending conventional notions of determinism and presenting a probabilistic viewpoint on the natural world. At its core, quantum mechanics is a branch of physics that creates mathematical theories to describe how fundamental entities like electrons, photons, and atoms behave. In contrast to classical mechanics, which deals with macroscopic objects and adheres to deterministic laws, quantum mechanics presents a wave-particle duality in which particles exhibit both wave-like and particle-like characteristics. In addition, Heisenberg's discovery of the uncertainty principle is used to express the concept of uncertainty in the quantum mechanics theory. According to this theory, there is a basic upper bound on the level of accuracy that can be attained when understanding specific pairings of physical attributes simultaneously, such as location and momentum. The notion of superposition is one of the most intriguing ideas to emerge from the field of quantum physics. If a particle is not measured or observed, this principle maintains that it is possible for it to exist in several different states at the same time. These superposed states, which can interfere with one another, can give rise to certain strange phenomena, such as interference patterns and quantum entanglement, due to the fact that they can interact with one another. Quantum mechanics is also an essential component of the field of quantum information science, which investigates how quantum phenomena could be used into the development of innovative technologies such as quantum computing, quantum cryptography, and quantum sensing. Researchers are still working hard to comprehend the intricacies of quantum physics, which will, once accomplished, allow for remarkable leaps forward in both science and technology.

1.2 Motivation

Quantum walks, which are the quantum equivalents of random walks in general, could be seen as the most useful result or feature of quantum information theory. This is due to the fact that quantum walks provide as a foundation for other computational models, help to realize the computing power of algorithms, and drive the creation of a quantum computer that can effectively solve challenging issues. In 1994, Shors introduced the first quantum factorization technique, which offered exponential speed improvements over classical algorithms [29]. A continuous-time quantum walk technique for graph connectivity has been presented by Childs et al [30], which has an exponential speedup over any of the known classical approaches, whereas Shenvi et al [31]. Quantum technique for identifying a marked vertex on a hypercube has quadratically faster search times. Other quantum algorithms that offer speedups over classical algorithms have been developed, [32, 33, 34].

1.3 Preliminary introduction

Classical mechanics, which provides a framework to explain the motion and behavior of macroscopic objects, has long served as the foundation for our understanding of the physical universe.The classical random walk is one of the established walking phenomena. In a classical random walk, the walker makes discrete movements in a lattice of higher dimensions or along a one-dimensional line. The walker randomly selects a direction (such as left or right) and proceeds to the next spot with equal probability .Random walks have applications in many different fields of study, ranging from the simulation of molecular diffusion to the analysis of financial markets and search algorithms. Classical random walks served as inspiration for the idea of quantum walks. It is possible to think of quantum walks as generalization of classical random walks that incorporates quantum mechanics.In a quantum walk, the particle can be in many places at once, allowing for interference effects and intricate dynamics. Through different operations, such as unitary transformations, measurements, and manipulation of the particle's quantum states and evolution. Coined quantum walk is the basic example of quantum walk [1].The term "coined quantum walk" refers to a modification of the quantum walk in which the walker's position is additionally given a "coin" degree of freedom. This variation of the quantum walk is referred to as "coined quantum walk." The expansion of the quantum system contained within the coin, in conjunction with the movement of the walker, makes probabilistic exploration and quantum interference effects possible. Coined quantum walks, in comparison to classical algorithms, have applications in quantum algorithms and simulations, The resulting quantum factorization methods, like Shor's algorithm from 1994 [2]. Several different types of quantum algorithms, such as quantum teleportation and quantum state transfer are all founded on the concept of quantum walks. This increase in speed provided by quantum walks is a key benefit of quantum algorithms over classical algorithms. The study of quantum walks with photons, which are particles of light and are effective bearers of quantum information, has been the primary focus of research in more recent times.Our objectives include utilizing optical components, particularly polarizing beam splitters (PBS) and half-wave plates (HWP), to investigate the behavior of photons engaging in quantum walks. To find nth quantum state and probabilities of separable and entangled state.To evaluate the outcomes using separable state and entangled state then comparing these results.We are using optical components like half-wave plates (HWP) and polarizing beam splitters (PBS), regulate the behavior of photons that are involved in quantum walks. These optical components make it possible to establish superposition states and for distinct paths to interfere, which leads to a dynamic that is both rich and sophisticated in nature during a quantum walk. It is essential to the development of quantum technology to have a thorough understanding of the behavior of photons during quantum walks and to make use of the unique qualities that they possess. It makes possible the development of innovative algorithms for the processing of quantum information, the simulation of complex quantum systems, and the investigation of basic questions pertaining to quantum physics. To do quantum walks utilizing optical components, the polarization of the photons as well as their spatial degrees of freedom need to be altered. Using optical components such as polarizing beam splitters and half-wave plates, one can alter and exert control over the behavior of photons. To accurately describe the polarization of a photon, it is possible to employ a superposition of various states, such as horizontal (x) and vertical (y) polarization. When a photon travels through halfwave plate (HWP), the polarization state of the photon might shift from one orientation to another depending on how the plate is oriented. A polarizing beam splitter(PBS) can split a photon into many different paths depending on the polarization of the photon, it plays an important role in the interaction between a changing polarization state and a photon. This makes it possible to create superposition states and investigate the effects of quantum interference.The methodology we will use to study quantum walk of photns by using optical elements is discrete time quantum walk. The motivation behind studying quantum walks of photons using optical elements stems from several key factors.The fundamental building component for many quantum algorithms and quantum information processing tasks is the quantum walk. Researchers hope to further the study of quantum computation, quantum communication, and other quantum technologies by comprehending and modifying quantum walks of photons.We have done qaunutm walk of photons by using separable and entangeled state but in future we can also utilize chorent states and can see the probabilistic nature of photons by using chorent states.

1.4 Literature review

1.4.1 Literature review on quanutm walk

The study of quantum random walks is a relatively new topic; A great deal of progress has been made since the publication of the paper by Aharonov et al. in 1993 [3]. This was the first research to formally coin the phrase quantum random walk; however, it was not necessarily the first paper to establish the concept of a quantum walk. In the 1940s, Mr. Feynman was the one who accomplished it [4]. There is a graph that corresponds to each QW, and the vertices of that network represent the position of a hypothetical walker. The edges of that graph indicate the paths that a walker can take to move from one vertex to another. Recently, a novel approach to the investigation of random walks has been developed [5, 6, 7, 8, 9, 10, 11, 52, 13, 14, 15, 16, 17, 18, 19, 20].

1.4.2 literature review on quantum walk by using optical elements

A workable plan for implementing quantum random walks along a line using only linear optics components as the building blocks of the system. The existing technology for single-photon interference enables very large numbers of additional steps to be experimentally achieved. These additional steps can be extended to very huge number [21]. Using linear optical elements, Do et al [22] have realized the quantum random walk (QRW). In their experiments, they used a weak coherent field rather than a field that had a large quantum component. If we consider the findings of Knight et al [23], who showed that QRW of a single walker can be implemented making use of classical fields, then we can see that this is acceptable. Prior to this, other authors [24], [25] have also explored a similar arrangement. Jeong et al.'s [26] analysis of the cases of a walker in a coherent state and in a single photon state revealed that the final probability distribution was the same in both instances, despite differing from that of the classical random walk (i.e., a walker without the additional quantum degree of freedom). Two photons have the potential to form QRW in a variety of quantum states, including states in which they are entangled. They make the discovery that the final state is entangled even if the two photons started out in separate Fock states this is because the final state is determined by quantum mechanics . This is a really intriguing quantum characteristic that does not have a corresponding property in the classical world. They talk about the QRW of a single photon as it enters the setup through either of the two input ports. The entering photon has a condition that can be described as "pure." They compare the exact numerical results with an estimated analysis that they offer for the chance of locating the photon at a site after a large number of steps [27], [28]. In their study of QRW of two non-identical walkers with entangled initial states, Omar have demonstrated that the final state can be entangled depending on the initial entanglement.

1.5 Outline of Thesis

Outline for structuring the thesis report.

- Chapter 1: Provide an overview of the research topic and its significance. Present the objectives of the study in addition to the issues that will be investigated and research questions. Talk about the things that prompted the idea for the study in the first place.Discuss the motivation behind the study.Review relevant literature and theories related to the topic.

- Chapter 2: In this section, we will go over several of the essential ideas that form the foundation of quantum mechanics. Entanglement, composite systems, and an introduction to states on the most fundamental level are some of the primary issues that are investigated in this part.

- Chapter 3: This research begins by discussing the states and their corresponding operators. Subsequently, the experimental setup is detailed, providing a comprehensive overview of the equipment and procedures utilized in the investigation. Following the experimental setup, the methodology for calculating probabilities and determining key parameters is presented. This includes a step-by-step explanation of data collection techniques and analysis methods, combining both experimental and mathematical approaches to ensure a robust understanding of the research.

- Chapter 4: In this chapter the results will be carefully presented and interpreted. This will be done as thoroughly as possible. In order to acquire a more in-depth comprehension of the data, the study will investigate numerous probability distributions and conduct an analysis of their probability peaks. We will compare different graphs, Through conducting this comparative analysis, one will be able to determine the validity and accuracy of the analytical models in terms of how accurately they represent the experimental data.

- Chapter 5: We will provide a summary of the most important findings of the study, as well as a description of the objectives of the study and the approaches taken to achieve them and suggestions for potential study topics and further examination areas.

Chapter 2

Fundamental Principles of Quantum Walk

It's crucial to comprehend classical walks, the quantum counterpart, before delving into the preliminaries of quantum walks. Particles move at random in a discrete space, such as a lattice or graph, during classical walks. On the basis of predetermined probabilities, they switch between nearby places. The various varieties of classical walks include Markov chain walks, simple random walks. We may lay a strong foundation for comprehending the fundamental ideas and variations that underlie quantum walks by studying classical walks. This will make it possible for us to learn more about the special quantum characteristics and uses of quantum walks.

2.1 Classical Random Walk

The term "random walk" refers to a path that is created by taking a series of steps that are picked at random in space. It is usual practice to refer to a random process as a Markov process. The walker's current location is the only thing that matters for determining where it will go next; its prior locations have no bearing on the matter. It is impossible to predict the outcome of the process because the walker constantly moves between distinct or limited zones that it occupies. Pearson calculating the likelihood of coming across drunk person after 'n' units of time, first put out the concept of the random walk [35]. The origin was discovered to have the highest probability. One

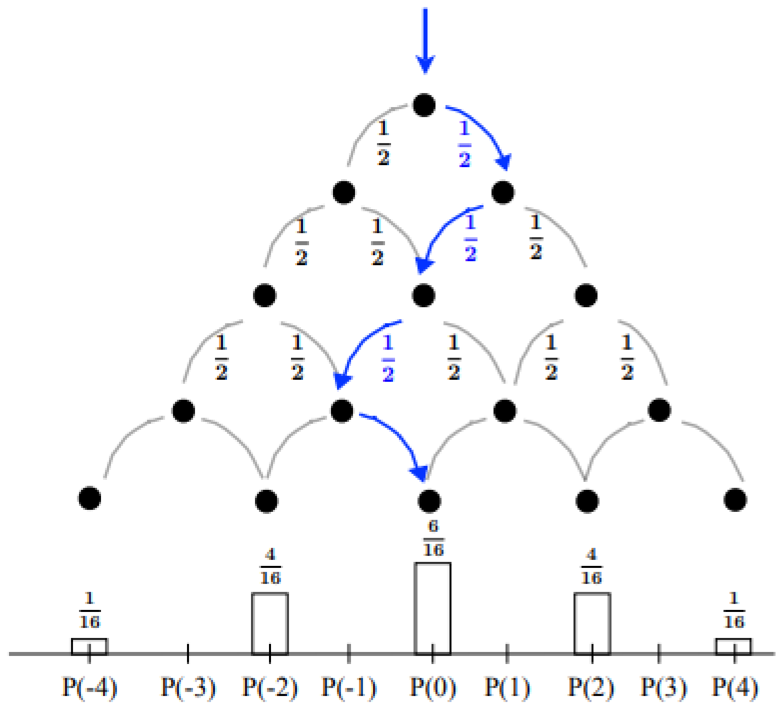


Figure 2.1: The Galton board [17]: A ball drops onto a board of nails and has equal probability to fall on either side of each nail it touches. Only either odd or even positions can be reached by the walker in each step. The walker's final probability distribution, constrained to the available positions, is binomial. One possible path is indicated explicitly by the blue arrows.

instance of a stochastic process that can be explained by the Classical Random Walk (CRW) model is the mobility of a particle suspended in liquid. The path that the particle takes is completely unpredictable, much like the path that a walker might take. Before each step, a coin is flipped, and the direction that the particle moves in is determined by which side of the coin lands face up. Either the left or the right. There are n distinct time steps included in each of the n repetitions of this process.

2.1.1 Experimental Implementation of Classical Random walk

The Galton Board [36] , which can be seen in Fig. (2.1), is one example of an experimental implementation of a classical random walk (CRW) that has been developed. As the ball descends from the top to the bottom of the system, it travels through n

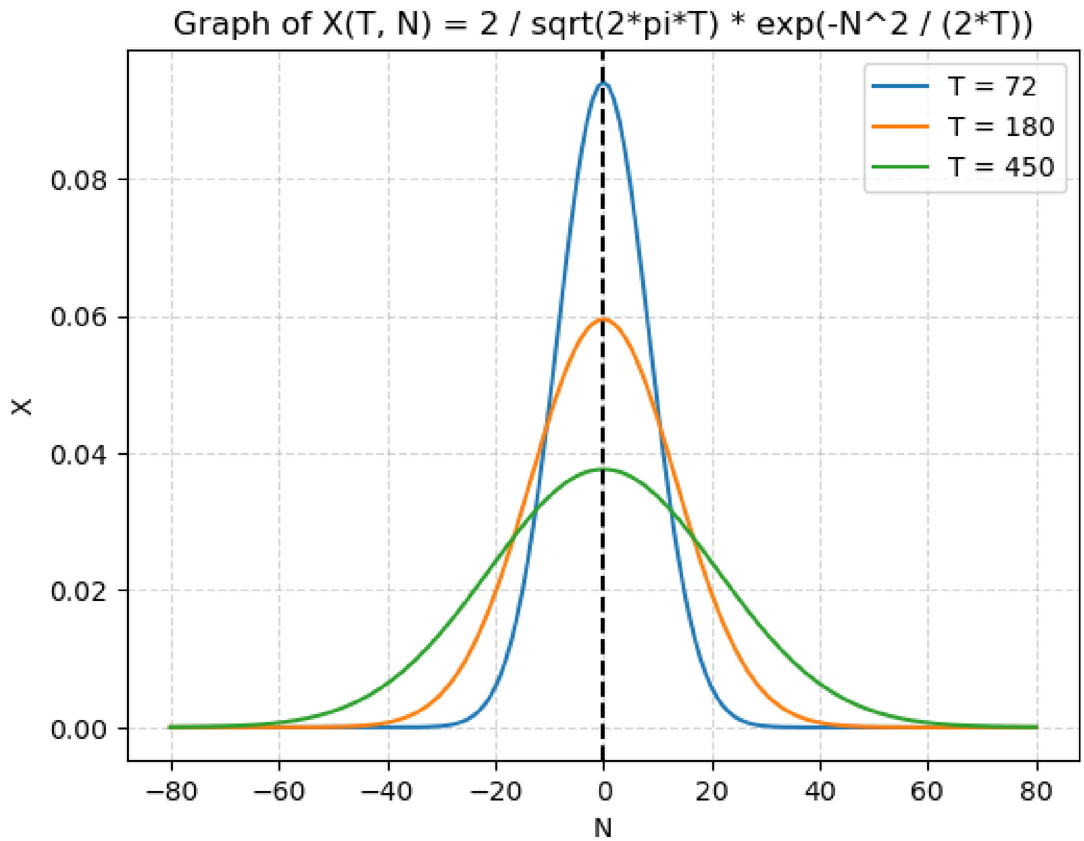


Figure 2.2: Probability distribution of a classical random walk on a line for t = 72, t = 180 and t = 450.

levels of nails that are arranged in such a way that it has an equal chance of landing on either side of each nail. This ensures that the system is fair. The CRW's development over time is essentially determined by random chance. For an unbiased CRW, the probability of a step to the left or right has a value of probability equal to half. The beginning and end points are connected by a route that is made up of n individual steps, just like the nails in Figure (2.1).

2.1.2 Classical Walk on line

One of the most elementary illustrations of a random walk is the motion of a particle along the integer points of a line, with the direction being determined by the flip of a coin [37]. If the coin lands on heads, the particle continues to the next vertex to the right; if it lands on tails, the particle continues to the next vertex to the left. This operation is carried out repeatedly. We are unable to make an accurate prediction regarding the location of a particle at a particular time. On the other hand, we can calculate the probability of being at a particular location (N) at a given time (T). Let's say the particle is at the starting point at time equal to zero. Therefore, X (T = 0, N = 0) = 1, as depicted in Figure (2.1) If time is equal to one, the position of the particle might be either N = -1 with a probability of 1/2 or N = 1 with the same probability. After n equals 0, there is no longer any chance of reaching that state. We can validate all the probabilities shown in Fig. (2.1) by repeatedly carrying out the steps in this approach. So, we can find probability by using binomial distribution,

$$\begin{pmatrix} A \\ B \end{pmatrix} = \frac{A!}{(A-B)!B!}, \tag{2.1}$$

Where A (forward step), B(backwadstep), A+B=T (T is total no of steps), A-B=N (gives the exact position of walker) and the probabilty is,

$$X(T,N) = \frac{1}{2^t} \begin{pmatrix} T \\ \frac{T+N}{2} \end{pmatrix}. \tag{2.2}$$

The probability distribution of the random walk can be estimated by applying Stirling's approximation for large t and binomial component of (2.1.2) in terms of factorials,

$$X(T,N) \simeq \frac{2}{\sqrt{2\pi T}} e^{-\frac{N^2}{2T}}. \tag{2.3}$$

Now, we can use this above probability expression to calculate the probability curves for different values of total no of steps(T). The three curves shown in Figure 2.2 correspond to the T = 72, T = 180, and T = 450 respectively. To be more precise, the curves represent the envelopes of the real probability distribution. This is because the probability of an odd n occurring while t is even equal to zero. Another approach to understand the curves is as the sum of the probability distributions X (T, N) and X (T + 1, N), which means that we have two distributions that overlap. It is important to note that, when T increases, the width of the curve widens and the height of the midpoint lowers.

The expected position is,

$$\langle N \rangle = \sum_{N=-\infty}^{\infty} N X(T, N). \tag{2.4}$$

Using the symmetry $X(T, N) = X(T, -N)$, we obtain,

$$\langle N \rangle = 0. \tag{2.5}$$

2.2 Quantum Walk

The quantum-mechanical equivalent of classical random walk is called quantum walk, where probability amplitudes take the place of probabilities. State vectors that contain all the information about the system in their probability amplitudes are used to represent these systems.The wave equation and their square give the likelihood of determining the particle's position in space-time. Superposition is the most remarkable characteristic of a quantum system, in which a particle can exist in more than one state at once unless a measurement is made.We can consider many examples to understand the quantum walk ,coined quantum walk/hadamard walk on line ,quantum walk on galton board , quantum walk on graphs.

2.2.1 Coined Quantum Walk

We have discovered in the past that, in a classical random walk, the walker will change positions just before a coin is flipped. In the same way, the transition between places in a quantum walk is determined by the outcome of the flip of a quantum coin. Any observable that is the representation of a quantum system having two discrete values, much like heads and tails, can be utilized productively as a coin in the realm of quantum mechanics. One of these observables is the internal degree of freedom of a particle, such as the spin of an electron, which can exist in one of two distinct states ($|\downarrow\rangle$,$|\uparrow\rangle$). A coin transformation operator is used to toss the state of the coin. In addition to giving the particles directional instructions, this operator transforms the coin state into a superposed state of its base. In addition to a variety of other unique unitary group matrices, the Hadamard gate can function as a quantum coin. The operation of hadamrd operator on spin and spin down state is,

$$\begin{aligned} \hat{G}|\uparrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle). \\ \hat{G}|\downarrow\rangle &= \frac{1}{\sqrt{2}}(|\uparrow\rangle - |\downarrow\rangle). \end{aligned} \tag{2.6}$$

Where the coin states are,

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \text{ and } |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \tag{2.7}$$

The conditional shift operator, which adjusts the system's position in accordance with the coin operator's instructions, comes after the Hadamard operation,

$$\hat{M} = \sum_i (|i-1\rangle\langle i|) \otimes |\downarrow\rangle\langle\downarrow| + \sum_i (|i+1\rangle\langle i|) \otimes |\uparrow\rangle\langle\uparrow|. \tag{2.8}$$

This suggests that the particle moves one step to the right $|i+1\rangle$, if the state is spin up, and one step to the left $|i-1\rangle$, if the state is spin down. A coin operation is performed onto the coin states of the system, followed by a conditional shift operation performed on the position states. This is the combined operation that is performed on the system. It is possible to express as,

$$\hat{O} = \hat{M}(I_{\text{position}} \otimes \hat{G}). \tag{2.9}$$

2.2.2 Symmetric Coin State

We can take different initial states like spin up coin state, sipn down coin state and symetric coin state.When we take single coin state can observe different probability distributions just by shifting the state of the coin and keeping the starting position of the walker same.We are considering symetric coin state(means superposition of spin up and sipn down state) with position of particle. Let the initial state be ,

$$|\chi_{ini}\rangle = |0\rangle_p \otimes \frac{1}{\sqrt{2}}(|\uparrow\rangle + i|\downarrow\rangle)_c \otimes |0\rangle_p. \tag{2.10}$$

The walker moves along the line by incrementally applying the hadamard and the conditional shift operator. State of the walker after two steps,

$$
\begin{aligned}
\hat{}|\chi_{ini}\rangle &= \frac{1}{2}[|,\uparrow,1\rangle + |\downarrow,-1\rangle + i|\uparrow,1\rangle - |\downarrow,-1\rangle] \\
{}^{2}\,|\chi_{imi}\rangle &= \frac{1}{2\sqrt{2}}[|\uparrow,2\rangle + |\downarrow,0\rangle + |\uparrow,0\rangle \\
, \qquad & - |\downarrow,-2\rangle + i|\uparrow,2\rangle + i|\downarrow,0\rangle \\
& -|\uparrow,0\rangle + i|^{\top}\downarrow,-2\Big\rangle\Big].
\end{aligned}
\tag{2.11}
$$

There is symmetry in the probability distribution for the case where the initial state of the walker is symmetric as shown in figure (2.3). The findings of the experiment make it abundantly clear that the CRW always adheres to the binomial probability distribution notwithstanding the initial state of the walker, whereas the quantum walk is extremely dependant on the walker's state at the beginning of the experiment.

2.3 Models For Quantum Walk

There are two models for quantum walk (1) Discrete time quantum walk, (2) Continuous time quantum walk .

2.3.1 Discrete Time Quantum Walk

The Discrete Time Quantum Walk (DTQW), proposed by meyer, consists of two quantum mechanical systems: a coin and a walker. In each discrete time step, an evolution operator is applied to the system. The evolution operator acts on the coin first, followed by its action on the walker. This operator is a unitary component that advances the system by one step along a line,

$$|x\rangle_2 = \hat{U}|x\rangle_1.$$

2.3.2 Continuous Time Quantum Walk

The continuous time quantum walk model was first presented by Farhi and Guttman. This model is made up of a walker, and the Hamiltonian of the system is used as the evolution operator. The walk is quite long, as the name implies continuous, without any limitations imposed by time, and capable of evolving in response to applications made by the operator at any time. The Schrodinger equation is the mathematical formula that determines how this model will develop over time,

$$|x\rangle_1 = exp(-i\|l)|x\rangle_1. \tag{2.12}$$

2.4 Difference Between Classical Walk And Quantum walk

Classical walks and quantum walks are two unique kinds of walks that represent the movement of particles or walkers through a series of steps.The main difference is classical walker will take only one step for walk and follows Gaussian probability distribution on the other hand in quantum walk particle will take two paths means it follows superposition principle of quantum mechanics and gives two superposition peaks.There are many other difference or reasons why we move from classical walks to quantum walks.

- Particle behavior: In classical walks, the particle moves along a predetermined path that is determined by probabilistic rules such as random selections or transition probabilities. The interference effects that result from the quantum superposition cause the particle to behave in a manner that is both deterministic and probabilistic when it is in a quantum walk.

- Interference: Interference phenomena are observed in quantum walks; specifically, the probability amplitudes associated with various pathways can interfere either constructively or destructively with one another. This interference can result in distinct patterns and dynamics, which are not seen in classical walks.

- State representation: A probability distribution across the various possible positions or states of the particle is used to represent the state of the system in classical walks. The state of the system is defined by a quantum superposition of various positions or states when carrying out quantum walks.

- Measurement: In classical walks, measurements are frequently taken to figure out the particle's final position or attributes in order to make calculations. The superposition can be collapsed, and a specific outcome revealed through the use of quantum walks. This can provide information about the position of the particle or other qualities that can be observed.

- Computational power: For some methods and problems, quantum walks have been shown to be computationally more advantageous than classical walks. For particular search, optimization, and graph-related activities, quantum walks can offer exponential speedups over classical random walks.

Overall, the behavior of quantum walks is governed by the laws of quantum mechanics, and they have special properties and computing advantages over classical walks. Quantum walking, such as quantum algorithms, quantum information processing, and quantum simulations, are attractive and promising areas of research because of these distinctions.

2.5 Preliminaries of Photonic Quantum Walk

Before talking about quantum walk of photons utilising optical elements, it's vital to set up some fundamental concepts and conditions. These fundamental overviews provide the necessary context for comprehending the essential concepts and procedures of quantum walks of photons employing optical components. We will define key terms like quantum state, superposition, entanglement, unitary operators, polarisation, and optical components utilised in quantum walks of photons in this section. Understanding these foundational concepts is necessary to completely appreciate the nature of quantum walks and their potential applications. These fundamental steps allow us to construct a robust framework for researching and exploring the fascinating realm of quantum walks of photons.

2.5.1 Qubit

Bits are the fundamental piece of data storage used in both computers and telephony. There are two options: 0 or 1. It is a logical state. In quantum information and computation, the quantum equivalent of a classical bit, also known as a qubit, is employed. A bit can only acquire one of the two values at a time in classical systems, however in quantum systems, the qubits remain in superposition of both values simultaneously until a measurement is made. Then, as a result of measurement, which destroys coherence (coherent superposition), the quantum system collapses into one of the two states. A two-dimensional vector is referred to as a qubit, which is a representation of a dual-level quantum system.A single photon's horizontal and vertical polarisation or an electron spin with up and down spin values are two examples of qubits. The orthonormal basis, which is a linear superposition of the basis vectors for the system, is the most versatile way to depict the quantum state of a qubit,

$$|\psi\rangle = d|0\rangle + d'|1\rangle. \tag{2.13}$$

The coefficients d and d_0 are the complex numbers termed as probability amplitudes. Their modulus squared reflect the probability of getting the states $|0\rangle$ and $|1\rangle$. The kets are the basis vectors $\{|0\rangle, |1\rangle\}$ which span the vector space. Since the total probability sums to 1, we have the following equation,

$$|d|^2 + |d_0|^2 = 1.$$

The kets can be denoted in their column representation as,

$$|\psi\rangle = d \begin{bmatrix} 1 \\ 0 \end{bmatrix} + d' \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{2.14}$$

This bipartite quantum system can also be expressed geometrically in a 2D polar coordinate system,

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{\iota\phi}\sin\frac{\theta}{2}|1\rangle. \tag{2.15}$$

A qubit may be in a multi-state superposition, when it is measured, it loses coherence and only receives one result.

2.5.2 Quantum State

It is possible to encode the information concerning these physical systems into a specific state that is referred to as a "quantum state." However, in quantum physics, it is hard to precisely extract all of the information due to the uncertainty principle. As a result, a probabilistic method is presented to collect information about the physical system by making use of the appropriate mathematics of quantum mechanics. This suggests that quantum states are important things in quantum physics,

$$|\alpha\rangle = \sum_{k\in\mathbb{N}} \mathrm{a}_k|k\rangle,$$

$$\sum_{k\in\mathbb{N}} |a_k|^2 = 1.$$

.

2.5.3 Superposition State

The superposition principle of quantum physics asserts that a quantum system can exist in several states at once. This shows that the system is not in a single state prior to a measurement but rather a superposition of several states.Imagine a particle, such as an electron, that has two distinct states, State A and State B. In classical mechanics the particle would be in either State A or State B, but in quantum mechanics, the particle might be in a superposition of both states. A wave function, also known as a state vector, is a mathematical concept that can be used to depict the superposition mathematically. The particle's wave function in this instance can be expressed as follows:

$$|\psi\rangle = \alpha|A\rangle + \beta|B\rangle.$$

Here, the complex numbers and are known as probability amplitudes, and |A| and |B| stand in for the states A and B, respectively. The absolute squares of the probability amplitudes $|\alpha|^2$ and $|\beta|^2$ represent the probabilities of finding the particle in State A or State B, respectively. We can perform the physical implementation of the superposition state just by taking a photon with x or y polarization state and when this photon passed through half-wave plate, it gives the superposition of polarization state like this,

$$|\psi\rangle = \alpha|x\rangle + \beta|y\rangle,$$

$$|\psi\rangle = \frac{i}{\sqrt{2}}(|X\rangle + \beta|Y\rangle).$$

Where, $\alpha = \beta = \frac{1}{\sqrt{2}}$ This indicates that the probability of finding the photon in either the v or y polarization condition is equal. We can use a tool called a half wave plate to comprehend the actual implementation. A partially reflecting mirror called a half wave plate can divide an incoming photon into two channels(polarization) depending on its polarization. Following interaction with a half wave plate , the state of the superposition state photon described above changes as follows,

$$\frac{1}{\sqrt{2}}(|X\rangle + |Y\rangle) \rightarrow \frac{1}{\sqrt{2}}(|X\rangle + |Y\rangle) + \frac{1}{\sqrt{2}}(|X\rangle - |Y\rangle).$$

.

2.5.4 Hilbert Space

This concept combines location and coin Hilbert spaces. The description of H_t for a discrete-time quantum walk is as follows,

$$H_t = H_p \otimes H_c, \tag{2.16}$$

The position space or walker sites are represented in Eq. (2.3.4) by H_p, the coin space is represented by H_c, and the Kronecker or tensor product is represented by $\otimes$. The whole Hilbert Space and the H_p position space, however, are the same in the case of a continuous quantum walk.

2.5.5 Quantum Optics

The study of the interaction between light and matter at the quantum level is the focus of the field of quantum optics. The quantum nature of light, including wave-particle duality and the quantization of energy in photons, is investigated in this study.

2.5.6 Photons

The building blocks of light and electromagnetic waves are called photons. They are the electromagnetic field's quanta, and they can simultaneously display characteristics of particles and waves.

2.5.7 Wave Particle Duality

A key idea in quantum physics is the wave-particle duality, which states that depending on how they are observed or measured, objects like photons can behave both like waves and like particles. According to this theory, particles like photons can simultaneously display wave-like and particle-like characteristics.

2.5.8 Polarization

The term "polarization" describes the direction of the electric field vector connected to a photon or electromagnetic wave. It defines the course and alignment of the electric field's oscillations as the wave moves across space.

Types of p polarization

Three primary types of polarization.

- Linear polarization A photon is said to have linear polarization when the oscillation of its electric field takes place in a particular direction along a line. While the wave progression, the amplitude and direction of the electric field vector do not change. The orientation of linear polarization can be either vertical, horizontal or it can be at any angle in between.

- Circular Polarization

  When a wave travels, the electric field vector spins in a circular fashion, a phenomenon known as circular polarization occurs. This rotation can either be clockwise, which corresponds to right-handed circular polarization, when viewed in the direction of transmission, or anticlockwise, which corresponds to left-handed circular polarization.

- Elliptical Polarization

  Elliptical Polarization Light that is elliptically polarized possesses an electric field vector that traces out an ellipse as the wave moves from one location to another. The features of elliptical polarization are determined not only by the orientation of the ellipse but also by the eccentricity of the ellipse. It is possible to interpret it as a combination of linear and circular polarization.

Polarization in Quantum Walk of Photons

Two different polarization states are commonly referred to as the x-polarized state and the y-polarized state when discussing quantum walks. The y-polarized state is oriented along one axis, such as the vertical, while the x-polarized state is aligned along another, such as the horizontal. These show polarization orientations that are orthogonal. We refer to a photon as being x-polarized when its electric field vector oscillates along the x-axis. This is caused by the electric field vector's x-axis oscillation. Similar oscillations occur in the electric field vector of y-polarized photons along the y-axis. These two polarization states, which are orthogonal to one another, provide the basis of a description of photon polarization. It is possible to modify the polarization states of photons during quantum walks using optical devices like (HWP) and (PBS). This makes it possible to create quantum walks. By varying the orientation of these elements, we can either establish superposition states where a photon is in a combination of x and y polarization or change the polarization direction of photons.

2.5.9 Direction of Propagation

The term "direction of propagation" refers to the path that a wave, such as light waves or photons, travels along when it moves from one location to another. It reveals the spatial direction in which the wavefronts of the wave propagate as they move across space. Polarization is a way to characterize the orientation of the electric field vector that relates to the wave, and because of this, it has a tight relationship to the direction in which the wave is travelling. The vector of the electric field is oriented such that it is perpendicular to the direction in which the signal is propagating. Because polarization and direction of propagation are two interconnected features of the wave, it is imperative that both be taken into consideration at the same time.

Propagation in Quantum Walk of Photons

The direction of propagation is frequently considered along with polarization in the context of quantum walks and photon polarization because they are closely related. The path or trajectory that the photon is on is referred to as its direction of propagation, but the orientation of the photon's associated electric field is referred to as its polarization. If we consider the direction of propagation in addition to the polarization of the light, we can have a more thorough understanding of how photons behave in quantum walks and how they interact with a range of optical devices. Direction of propagation can be horizontal(h) and vertical(v). It enables us to explore the dynamic interaction between polarization and direction as well as the other factors that affect the path and characteristics of the photon.

2.5.10 Optical Elements

Devices that alter light in various ways, for as by manipulating photons, are referred to as optical elements. They were developed with the intention of regulating and altering the properties of light, including its wavelength, polarization, propagation, intensity, and phase. These components are necessary for optical systems and experiments because they make it possible to shape, regulate, and manipulate light—elements that are required for a wide range of varied functions.

Half Wave Plate

(HWP) Wave Plates, also known as Retarders, are birefringent optical elements that induce a regulated phase delay between the orthogonal polarization components of light. Wave plates are often referred to by their other name, retarders. They can change the polarization state, convert linear polarization to superposition polarization or vary the relative phase between the various polarization states. A half-wave plate uses the birefringence property of some materials as part of its operating system.Birefringence refers to a material's capacity to exhibit a range of refractive indices for light that is polarised in diverse directions. Half-wave plates are typically made from a uni-axial crystal with birefringence, such as quartz or calcite.

- Working of half wave plate in quantum walk of photon by using optical elements.

  The HWP serves the dual purpose of acting as a Hadamard gate when placed at a 45-degree angle to the polarization state, creating superposition, and preserving the polarization direction when aligned parallel or perpendicular to the polarization state.

- Half wave plate acts as a Hadamard gate

  When the HWP is placed at a 45-degree angle to the polarization state, it acts as a Hadamard gate. The Hadamard gate is a fundamental operation in quantum computing and quantum information processing, which can create superposition states. When a polarization state, such as an x-polarized or y-polarized photon, passes through the HWP at this angle, it can generate a superposition state where the photon is simultaneously in both x and y polarization states,

$$\begin{aligned} |X\rangle &\xrightarrow{HWP} \frac{1}{\sqrt{2}}(|X\rangle + |Y\rangle). \\ |Y\rangle &\xrightarrow{HWP} \frac{1}{\sqrt{2}}(|X\rangle - |Y\rangle). \end{aligned} \tag{2.17}$$

  In this quantum walk half wave plate acts as coin that we discussed in coined quantum walk . Another half wave plate formula we can use that can also gives superposition when we applied on polarized photon. (x or y) and that half wave plate operator represented by G,

$$\hat{G} = \frac{1}{\sqrt{2}}\big(|H,X\rangle\langle H,X| + |H,x\rangle\langle h,y| + |h,Y\rangle\langle V,X| - |H,Y\rangle\langle V,Y| \\ + |V,X\rangle\langle V,X| - |H,Y\rangle\langle V,Y| + |V,X\rangle\langle V,X| + |V,X\rangle\langle V,Y| \\ + |V,Y\rangle\langle H,X| - |V,Y\rangle|H,Y|\big). \tag{2.18}$$

Beam Splitter

An optical device known as a beam splitter is one that can separate a single incident light beam into two or more individual beams. It does this by dividing the incoming light energy and sending it off in a variety of directions, all of which are determined by certain optical qualities. In a wide variety of optical applications, such as imaging, interferometry, microscopy, and research involving quantum optics, beam splitters are a frequent and useful piece of equipment.

- Working of Beam Splitter.

  The operation of a beam splitter is based on the idea of partial transmission and partial reflection of the light beam that is incident on the device. It is made up of either a very thin optical coating or a collection of materials that each have their own unique optical properties. When light reaches the surface of the beam splitter, some of the light is reflected, while the remaining amount is transmitted through the splitter. The design and characteristics of the beam splitter will determine the ratio of transmission to reflection that it achieves.There are many types of beam splitter.

- Simple Beam Splitter.

  A simple beam splitter is a type of beam splitter that divides the incoming light beam into two different beams with the same intensity. This is made possible by transmitting half of the light that strikes it and reflecting the other half. Simple beam splitters are often constructed by applying a thin dielectric coating on a glass substrate.

- Polarizing Beam Splitter. A polarizing beam splitter (PBS), also known as a polarizing cube beam splitter, is a specialized kind of beam splitter that divides an incident beam based on its polarization state. It allows light with a specific polarization orientation to pass through, while reflecting light with an orthogonal polarization orientation. Polarizing beam splitters are constructed using birefringent substances like calcite or by coating a beam splitter with a thin film of a polarizing material. They are commonly used in applications that require polarization control, such as polarization microscopy, imaging, and optical communication systems.

- Working of Polarizing Beam Splitter in Quantum Walk of Photons by using Optical Elements. The experiment takes use of a polarizing beam splitter, or PBS, to divide the incoming photon beam into two distinct streams. Depending on the polarization of the photons and the direction in which they are travelling, the PBS can cause a positional shift in the photons. Since it allows for control over the spatial dispersion of the photons and is crucial to the quantum walk,

$$\begin{aligned}\hat{v} = {} & |H,X\rangle\langle H,X| \otimes |m+1\rangle\langle m| \\ & + |H,Y\rangle\langle H,Y| \otimes |m+1\rangle\langle m| \\ & + |V,X\rangle\langle V,X| \otimes |m-1\rangle\langle m| \\ & + |V,Y\rangle\langle V,Y| \otimes |m-1\rangle\langle m|.\end{aligned} \tag{2.19}$$

  A photon that is horizontally polarized and passes through the PBS observes a positional shift in the positive direction. On the other hand, a photon that has undergone vertical polarization experiences a shift in location that is opposite to the positive direction.

2.5.11 Unitary Operator

This kind of function only works in Hilbert space, maintains the inner product property, and complies with the following rules:

$$O^{\dagger}O = OO^{\dagger} = I. \tag{2.20}$$

$O^\dagger$ is the transpose complex conjugate of O, I is the identity matrix.The operator that will be utilised for the time evolution of quantum walks needs to be unitary. This means that the past position and the future position after applying the operator need to be distinct from one another. It is only possible if there is no loss of data as a result of the transition between the states of the system.The application of the unitary operator to the system wave function results in the production of amplitudes rather than probabilities[ref]. The equation that describes the wave function of the system after t iterations of the unitary operator is as follows:

$$|\psi\rangle_t = O^t|\psi\rangle_0. \tag{2.21}$$

ψ_t is the wave state vector after 't' steps, O^t is the unitary operator after 't' repetitions, and ψ_0 is the initial wave state vector defined as $|\psi\rangle_0 = |\text{position}\rangle_0 \otimes |\text{coin}\rangle_0$. A mathematical transformation that is utilised in quantum walks is referred to as the unitary operator.A "half-wave plate" (abbreviated as "HWP") and a "polarizing beam splitter" (abbreviated as "PBS") are combined to form a "unitary operator". Together, the HWP and PBS form an optical system that can alter both the polarization and the spatial characteristics of photons,

$$\hat{O} = \hat{V} \otimes \hat{G}. \tag{2.22}$$

The operator $\hat{D}$ in equation (2.6.11) is half wave plate operator for superposition and operator $\hat{T}$ is polarizing beam splitter operator used to shift the position of photon,

$$\begin{aligned}\hat{G} = \frac{1}{\sqrt{2}}\Big(&|H,X\rangle\langle H,X| + |H,x\rangle\langle h,y| + |h,Y\rangle\langle V,X| - |H,Y\rangle\langle V,Y| \\ &+ |V,X\rangle\langle V,X| - |H,Y\rangle\langle V,Y| + |V,X\rangle\langle V,X| + |V,X\rangle\langle V,Y| \\ &+ |V,Y\rangle\langle H,X| - |V,Y\rangle|H,Y|\Big),\end{aligned} \tag{2.23}$$

$$\begin{aligned}\hat{v} = &|H,X\rangle\langle H,X| \otimes |m+1\rangle\langle m| \\ &+ |H,Y\rangle\langle H,Y| \otimes |m+1\rangle\langle m| \\ &+ |V,X\rangle\langle V,X| \otimes |m-1\rangle\langle m| \\ &+ |V,Y\rangle\langle V,Y| \otimes |m-1\rangle\langle m|.\end{aligned} \tag{2.24}$$

When we apply this unitary operator to the initial state it will give next state and so on.

Chapter 3

Quantum Walk of Single and Two Photons

3.1 Introduction to the methodology

The main goal of the methods used in this research was to examine the quantum walk of photons utilizing birefringent plates and polarizing prisms. The experimental setup has been characterized, and the optical components as well as the orientation angles of the birefringent plate have been specified. The photon source is selected and regulated with great deliberation to achieve the desired qualities. These initial polarization states include x-polarization, y-polarization, separable state, entangled state. The quantum walk protocol, which involves sending the photons through the birefringent plate and polarizing prisms in a particular order, is carried out at this stage. Measurements are made to capture the photons' output states, and detectors or sensors are utilized to record the crucial data. The methods used to analyze the gathered data include computing probabilities and using statistical analysis or simulations and comparing the analytical calculation with numerical simulations results. The method we will follow to calculate the nth states and probabilities by using discrete time quantum walk method in which we will use unitary Operator. The conclusions collected from the data analysis are then utilized to assess the effectiveness of the quantum walk using HWP and PBS.This was one of the main findings of the study. The implementation of QRW takes use of a wide variety of classical sources as a direct result of this, including low-power lasers [38, 39], squeezed state and Electromagnet fields [40, 41]. In order to gain a deeper understanding of the quantum properties of random walks, we will analyse the situation in which two photons begin QRW from two separate beginning states.

3.2 Quantum Walk of Single Photon

We are simulating the behavior of a quantum walker represented by a photon in this quantum walk scenario by using optical components, specifically a birefringent plate /Half Wave Plate(HWP) and polarizing prism/polarizing beam splitter(PBS). The walker's state is defined by its polarization, specifically the X and Y polarization states, rather than a classical coin. A single photon that has been initiated in a particular polarization state—which might be either X or Y—begins the quantum walk. A superposition of the X and Y states may also be used to prepare the photon. We use a birefringent plate (HWP), an optical component, to produce a superposition by introducing a relative phase shift between the incoming light's X and Y polarization components. The polarization state is transformed into a superposition of X and Y polarization by the HWP, which performs a Hadamard-like operation on it. The polarizing prism (PBS) is used to simulate the walker's position changing after the superposition stage. The PBS transmits photons with a particular (X)polarization and reflects photons with an orthogonal (Y)polarization. We can successfully change the photon's position based on its polarization state by adding the PBS after the HWP.One photon prepared in a certain polarization state serves as our starting point. The polarization states in this situation are X and Y. The photon may be in a superposition of both polarization states or it may be in a single polarization state (either X or Y),

$$|\Psi_i\rangle = |Particle(photon)state\rangle \otimes |Polarizationstate\rangle. \tag{3.1}$$

Lets write initial single state of photon with X polarization and with Y polarization,

$$|\Psi_i\rangle = |m\rangle \otimes |H, X\rangle, \tag{3.2}$$

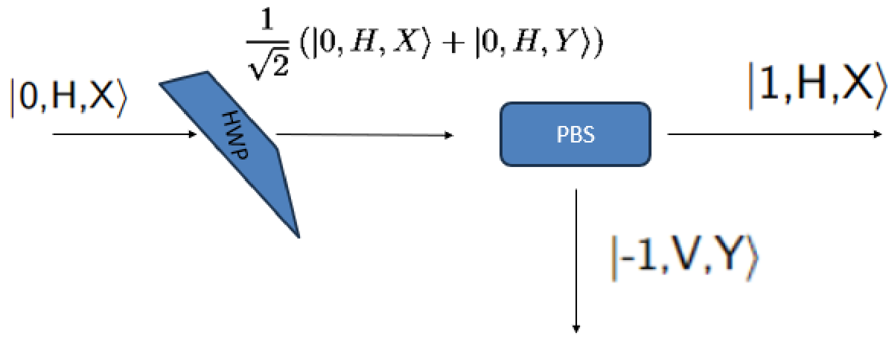


Figure 3.1: The schematic arrangements for realizing QRW of single photon walker. In the inset effect of polarization beam splitter is shown explicitly. The x-polarized photon is transmitted in the same direction but the y-polarized photon is reflected and changed the direction of propagation.

$$|\Psi_i\rangle = |m\rangle \otimes |V, Y\rangle, \tag{3.3}$$

In the above State m shows no of steps of photon , H and V shows direction of propagation and X and Y are polarization states .When polarized photon state let say state in equation (3.2) passes through HWP in figure (3.1) then it gives superposition state of the polarization states and when this superposition state passed through PBS, it will shift the position of photon only does not effect the direction of polarization.Unitary operator will be,

$$\hat{O} = \hat{V}(\hat{G} \otimes \hat{I}). \tag{3.4}$$

HWP operator acts as Hadamrd Operator,

$$\hat{G} = \frac{1}{\sqrt{2}}\big(|H, X\rangle\langle H, X| + |H, X\rangle\langle H, Y| + |H, Y\rangle\langle V, X| - |H, Y\rangle\langle V, Y| \tag{3.5}$$

$$+ |V, X\rangle\langle V, X| + |V, X\rangle\langle V, Y| + |V, Y\rangle\langle H, X| - |V, Y\rangle\langle H, Y|\big). \tag{3.6}$$

Polarizing prism /Shift operator,

$$\begin{aligned}\hat{V} &= |H,X\rangle\langle H,X| \otimes |m+1\rangle\langle m| \\ &+ |H,Y\rangle\langle H,Y| \otimes |m+1\rangle\langle m| \\ &+ |V,X\rangle\langle V,X| \otimes |m-1\rangle\langle m| \\ &+ |V,Y\rangle\langle V,Y| \otimes |m-1\rangle\langle m|.\end{aligned} .$$

After applying these operators we can get next state.Lets take single input state , x polarized photon with horizontal direction of propagation .Input state will converted into superposition state when we apply HWP (G),

$$|m,H,X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m,H,X\rangle + |m,V,Y\rangle], \tag{3.7}$$

Apply shift operator on superposition state, it will give next state,

$$\frac{1}{\sqrt{2}}[|m,H,X\rangle + |m,V,Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|m+1,H,X\rangle + |m-1,V,Y\rangle]. \tag{3.8}$$

if m is 0 then means photon at origin on no line , we apply operators to get state next state (1),

$$|0,H,X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0,H,X\rangle + |0,V,Y\rangle], \tag{3.9}$$

$$\frac{1}{\sqrt{2}}[|0,H,X\rangle + |0,V,Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|1,H,X\rangle + |-1,V,Y\rangle]. \tag{3.10}$$

When we take superposition state of polarization(m=0) then its walk also determine by unitary operator,

$$|\Psi_i\rangle = |m\rangle \otimes \frac{1}{\sqrt{2}}\left(|H,X\rangle + |V,Y\rangle\right), \tag{3.11}$$

$$\frac{1}{\sqrt{2}}\left[|0,H,X\rangle + |0,V,Y\rangle\right] \xrightarrow{\hat{O}} \frac{1}{\sqrt{2}}\left[|1,H,X\rangle + |-1,V,Y\rangle - |1,H,Y\rangle + |-1,V,X\rangle\right]. \tag{3.12}$$

Evaluation of photon done by unitary operator and when we have to find nth state of the photon we have to apply nth times unitary operator on initial state,

$$|\text{nth state}\rangle = \hat{O}^n|\text{initial state}\rangle. \tag{3.13}$$

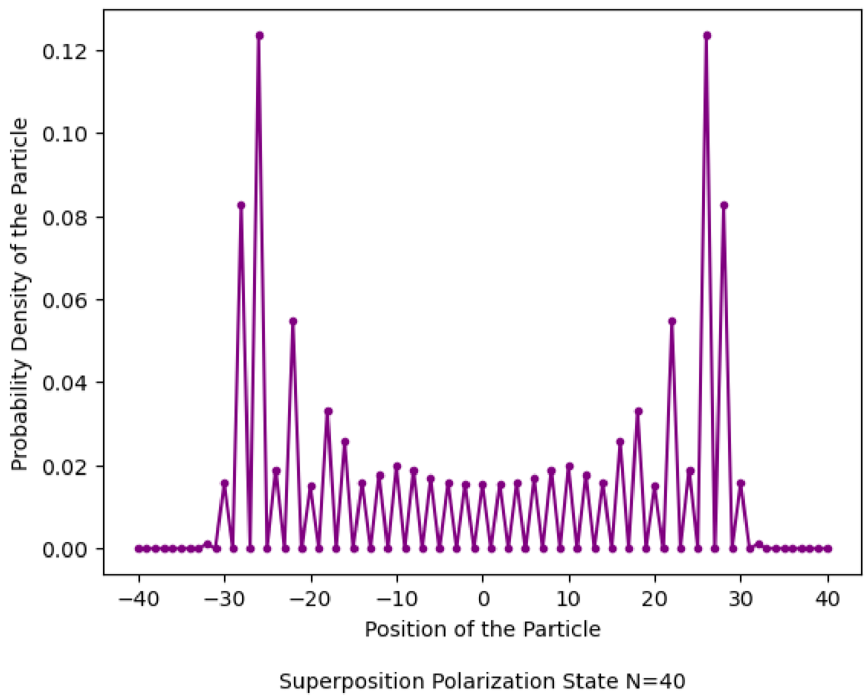


Figure 3.2: Probability distribution for a quantum random walk with N=40, superposition initial polarization state (3.11).

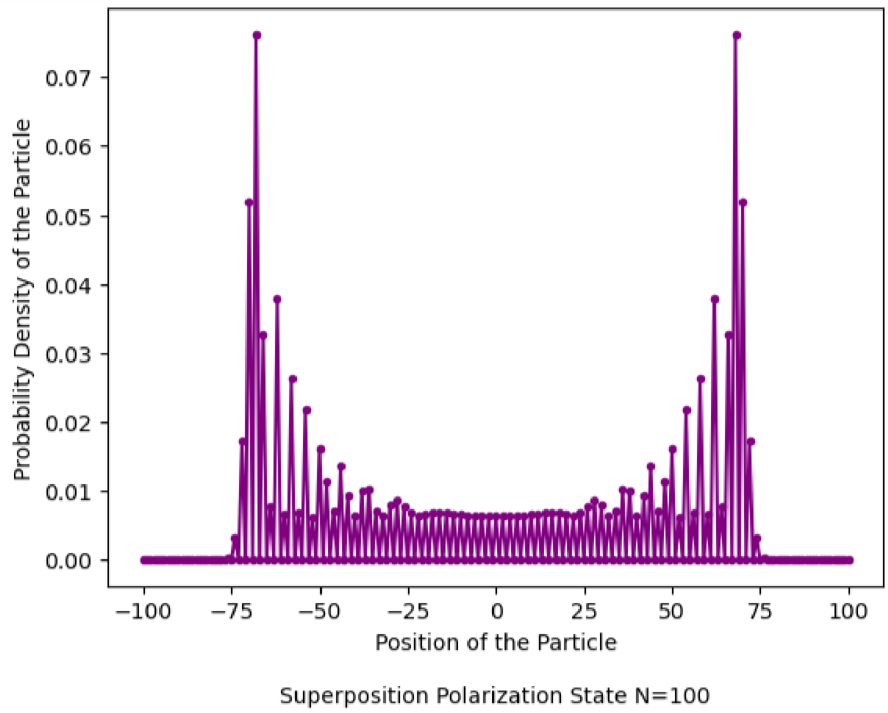


Figure 3.3: Probability distribution for a quantum random walk with N=100, superposition initial polarization state (3.11).

There is symmetry in the probability distribution for the walker's symmetric initial state. When we repeatedly applied the unitary operator to the starting state. The conditions of the walk that proceeds to the left do not supersede the conditions of the walk that moves to the right. The probability distributions are always added at the end.

3.3 Quantum Walks of Separable Photons

3.3.1 Experimental Implementation

We have one X-polarized photon that travels in a horizontal direction, and one Y-polarized photon that travels in a vertical direction. Both of these photons are distinct from one another.Given that the X-polarized photon's polarisation direction coincides with the direction of the birefringent wave plate (HWP), the passage of the X-polarized photon through the HWP has no effect on the photon. As a direct consequence of this, the original route taken by the X-polarized photon is not altered in any way. In a similar manner, the Y-polarized photon is able to go through the HWP without experiencing any change in the direction of its polarisation. This is because the HWP is aligned with the y polarisation. Therefore, the Y-polarized photon follows the exact same path as the unpolarized photon .After that, both photons arrive at the polarising prism, also known as the PBS. The PBS arranged to allow photons with horizontal polarisation to be transmitted while simultaneously allowing photons with vertical polarisation to be reflected. Due to the fact that the polarisation of the X-polarised photon coincides with the transmission property of the PBS, the photon is able to travel horizontally through the PBS without being affected in any way.On the other hand, the y-polarised photon that has vertical propagation is reflected by the PBS because the polarisation of the photon is similar to the polarisation that the PBS has while it is reflecting. As a consequence of this, the Y-polarised photon alters the path that it takes while travelling and becomes entangled with the X-polarised photon that is being transmitted. Therefore, after going through the HWP and PBS apparatus, we arrive at a state of entanglement in which the x-polarized photon with horizontal propagation is unaffected but the y-polarized photon with vertical propagation becomes entangled with the x-polarized photon that was sent.

3.3.2 Initial Separable States

Two photons play the part of two walkers in our scenario.Two photons enter the arrangement through two different input ports to initiate QRW from two different beginning states. One photon moves in a horizontal direction while the other moves in a vertical direction when they first come into contact,

$$|\Psi_i\rangle = |\Psi_1\rangle \otimes |\Psi_2\rangle, \tag{3.14}$$

Where $|\Psi_i\rangle$ is the state of two photons and

$$|\Psi_1\rangle \text{ and } |\Psi_2\rangle \text{ are states of single photons with different} \tag{3.15}$$

direction of propagation and polarization.

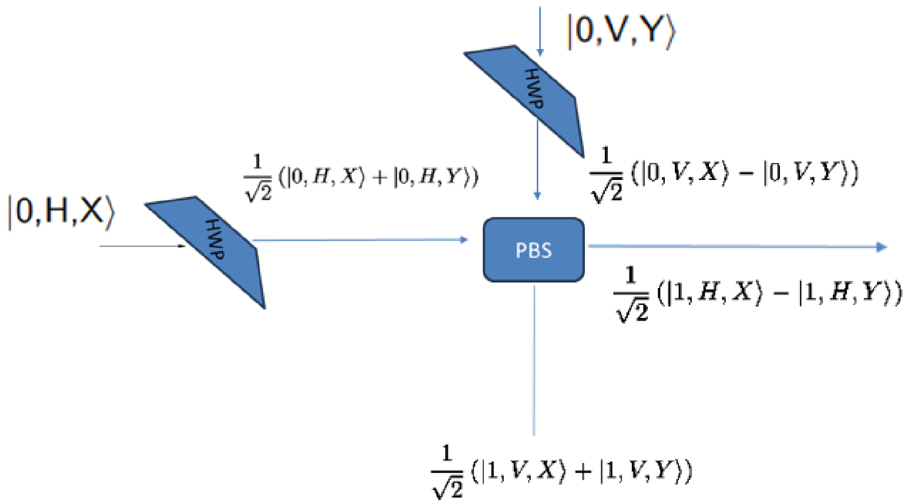


Figure 3.4: The conceptual structure for implementing two separable photons. The effect of the polarisation beam splitter is directly displayed in the inset. The y-polarized photon is reflected, changing its direction of transmission, whereas the x-polarized photon is transmitted in the same direction.

Four possible separable general state,

$$\begin{aligned} &|m,H,X\rangle \otimes |m,V,X\rangle \\ &|m,H,X\rangle \otimes |m,V,Y\rangle \\ &|m,H,Y\rangle \otimes |m,V,Y\rangle \\ &|m,H,Y\rangle \otimes |m,V,X\rangle. \end{aligned} \tag{3.16}$$

Photons starting quantum walk from initial position means origin.We can replace m with zeros,

$$|0,H,X\rangle \otimes |0,V,X\rangle, \tag{3.17}$$

$$|0,H,X\rangle \otimes |0,V,Y\rangle \tag{3.18}$$

$$|0,H,Y\rangle \otimes |0,V,Y\rangle \tag{3.19}$$

$$|0,H,Y\rangle \otimes |0,V,X\rangle. \tag{3.20}$$

In order to determine the separable next state, we need to begin by carrying out an evaluation of the single state computation before moving on to the separable state calculation. For calculating the next separable state, we will take initial states from (3.18) because initially we have x polarized and y polarized separable photons,

$$\begin{aligned} &|m, H, X\rangle \\ &|m, V, Y\rangle. \end{aligned} \tag{3.21}$$

The unitary operator consists of HWP operator (G) and PBS operator (V). As I indicated in the working of birefringent plate equations 2.6.6 and 2.6.7, we can apply both and get similar results, so I am utilizing formula technique equation 2.6.7. Half wave plate /Coin operator(G),

$$\hat{G} = \frac{1}{\sqrt{2}}\big(|h, x\rangle\langle H, X| + |H, X\rangle\langle H, Y| + |H, Y\rangle\langle V, X| - |H, Y\rangle\langle V, Y| \tag{3.22}$$

$$+ |V, X\rangle\langle V, X| + |V, X\rangle\langle V, Y| + |V, Y\rangle\langle H, X| - |V, Y\rangle\langle H, Y|\big). \tag{3.23}$$

Polarizing beam splitter /Shift operator (V),

$$\begin{aligned} \hat{V} = &|H, X\rangle\langle H, X| \otimes |m+1\rangle\langle m| \\ &+ |H, Y\rangle\langle H, Y| \otimes |m+1\rangle\langle m| \\ &+ |V, X\rangle\langle V, X| \otimes |m-1\rangle\langle m| \\ &+ |V, Y\rangle\langle V, Y| \otimes |m-1\rangle\langle m|. \end{aligned}$$

After applying these operators we can get next state.

3.3.3 Operation on the States

Evaluation of separable photons also done by operating unitary operator separately on each state,

$$|\text{separable state}\rangle = \hat{O}|m, H, X\rangle \otimes \hat{O}|m, V, Y\rangle. \tag{3.24}$$

Let's consider an input state consisting of an X-polarized photon with a horizontal direction of propagation. When we apply a half wave/birefringent wave plate operator (G) to the input state, it undergoes a conversion into a superposition state,

$$|m, H, X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m, H, X\rangle + |m, V, Y\rangle]. \tag{3.25}$$

Apply shift operator on superposition state, it will give next state,

$$\frac{1}{\sqrt{2}}[|m,H,X\rangle + |m,V,Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|m+1,H,X\rangle + |m+1,V,Y\rangle]. \tag{3.26}$$

If m = 0 then we apply operators to get state next state (1),

$$|0,H,X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0,H,X\rangle + |0,V,Y\rangle], \tag{3.27}$$

$$\frac{1}{\sqrt{2}}[|0,H,X\rangle + |0,V,Y\rangle] \xrightarrow{\hat{s}} \frac{1}{\sqrt{2}}[|1,H,X\rangle + |-1,V,Y\rangle]. \tag{3.28}$$

Similarly next state(1) will pass through apparatus and mathematically we can see the change by applying operators and get next state (2). lets take another state which is $|\ m,V,Y\rangle$, when this state pass through birefringent/half wave plate and polarizing beam splitter,

$$|m,V,Y\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m,VX\rangle - |m,H,Y\rangle]. \tag{3.29}$$

Apply shift operator on superposition state, it will give next state,

$$\frac{1}{\sqrt{2}}[|m,V,X\rangle - |m,H,Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|m-1,V,X\rangle - |m+1,H,Y\rangle], \tag{3.30}$$

We can apply unitary operator on previous state to find the next state. when we apply on initial state (o), we will get next state (1),

$$|0,V,Y\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0,V,X\rangle - |0,H,Y\rangle], \tag{3.31}$$

$$\frac{1}{\sqrt{2}}[|0,V,X\rangle - |0,H,Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[-1,V,X\rangle - |+1,H,Y\rangle], \tag{3.32}$$

We can see the behaviour of other states by applying unitary operator.

Taking state $|\ m,H,Y\rangle$,

$$|m,H,Y\rangle \xrightarrow{\hat{g}} \frac{1}{\sqrt{2}}[|m,H,X\rangle - |m,V,Y\rangle], \tag{3.33}$$

When we apply shift operator on superposition state, it will give next state,

$$\frac{1}{\sqrt{2}}[|m,H,X\rangle - |m,V,Y\rangle] \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m+1,H,X\rangle - |m-1,V,Y\rangle]. \tag{3.34}$$

When no of step (m) = 0,

$$|0, H, Y\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0, H, X\rangle - |0, V, Y\rangle], \tag{3.35}$$

Applying shift operator on superposition state , it will give next state,

$$\frac{1}{\sqrt{2}}[|0, H, X\rangle - |0, V, Y\rangle] \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|1, H, X\rangle - |-1, V, Y\rangle]. \tag{3.36}$$

Now state $|\ m, V, X\rangle$,

$$|m, V, X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m, V, X\rangle + |m, H, Y\rangle], \tag{3.37}$$

Apply shift operator on superposition state , it will give next state,

$$\frac{1}{\sqrt{2}}[|m, V, X\rangle + |m, H, Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|m-1, V, X\rangle + |m+1, H, Y\rangle]. \tag{3.38}$$

When photon at m=0,

$$|0, V, X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0, V, X\rangle + |0, H, Y\rangle], \tag{3.39}$$

Apply shift operator on superposition state, it will give next state,

$$\frac{1}{\sqrt{2}}[|0, V, X\rangle + |0, H, Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|-1, V, X\rangle + |+1, H, Y\rangle]. \tag{3.40}$$

For nth state the transformation acting on the initial state of the photon is equivalent to the one step of QRW. The iterative application of the transformation U for n times gives the QRW of n steps.The equation representing the nth state is given by,

$$|\text{nth state}\rangle = \hat{O}^n|\text{initial state}\rangle. \tag{3.41}$$

One of the states that can be chosen is the coin's initial state, which includes states such as $|H, X\rangle$, $|H, Y\rangle$, $|V, X\rangle$, and $|V, Y\rangle$, along with their possible super positions. Analytically this this is not possible to find nth state by operating unitary operator n times on the state ,so for calculating nth state we will do numerical simulations. We can choose different separable polarization states. If we take this separable state as initial input state,

$$(|m, H, X\rangle + |m, V, X\rangle) \otimes (|m, H, X\rangle + |m, V, X\rangle). \tag{3.42}$$

Initial state can be written as when m=0,

$$(|0, H, X\rangle + |0, V, X\rangle) \otimes (|0, H, X\rangle + |0, V, X\rangle). \tag{3.43}$$

Graph will be same shifted towards in one direction but with different central peak only.

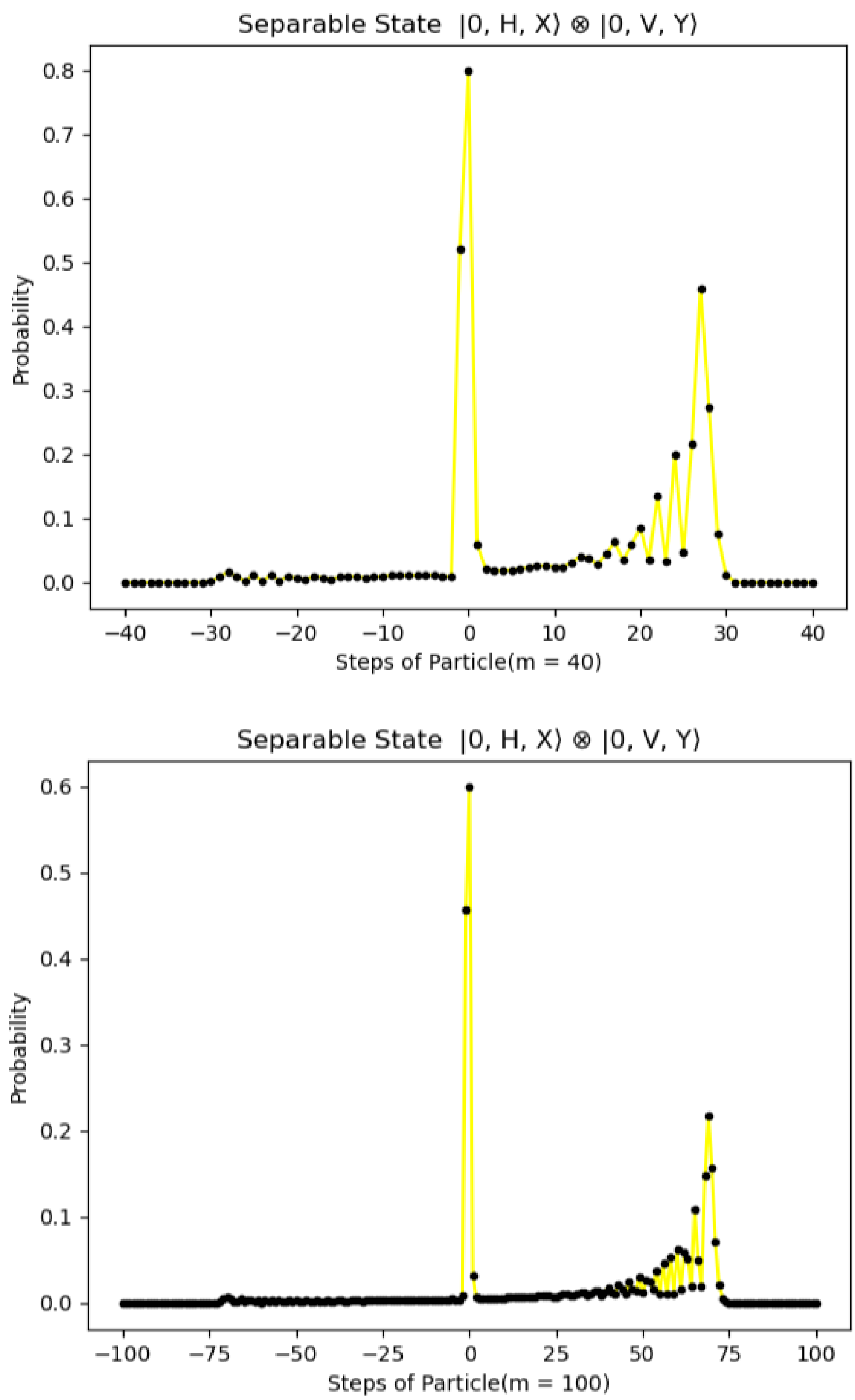


Figure 3.5: Probability distribution for detecting walkers after 40 and 100 steps, starting from an initial separable state, showing an asymmetric peak(right) with a central peak.

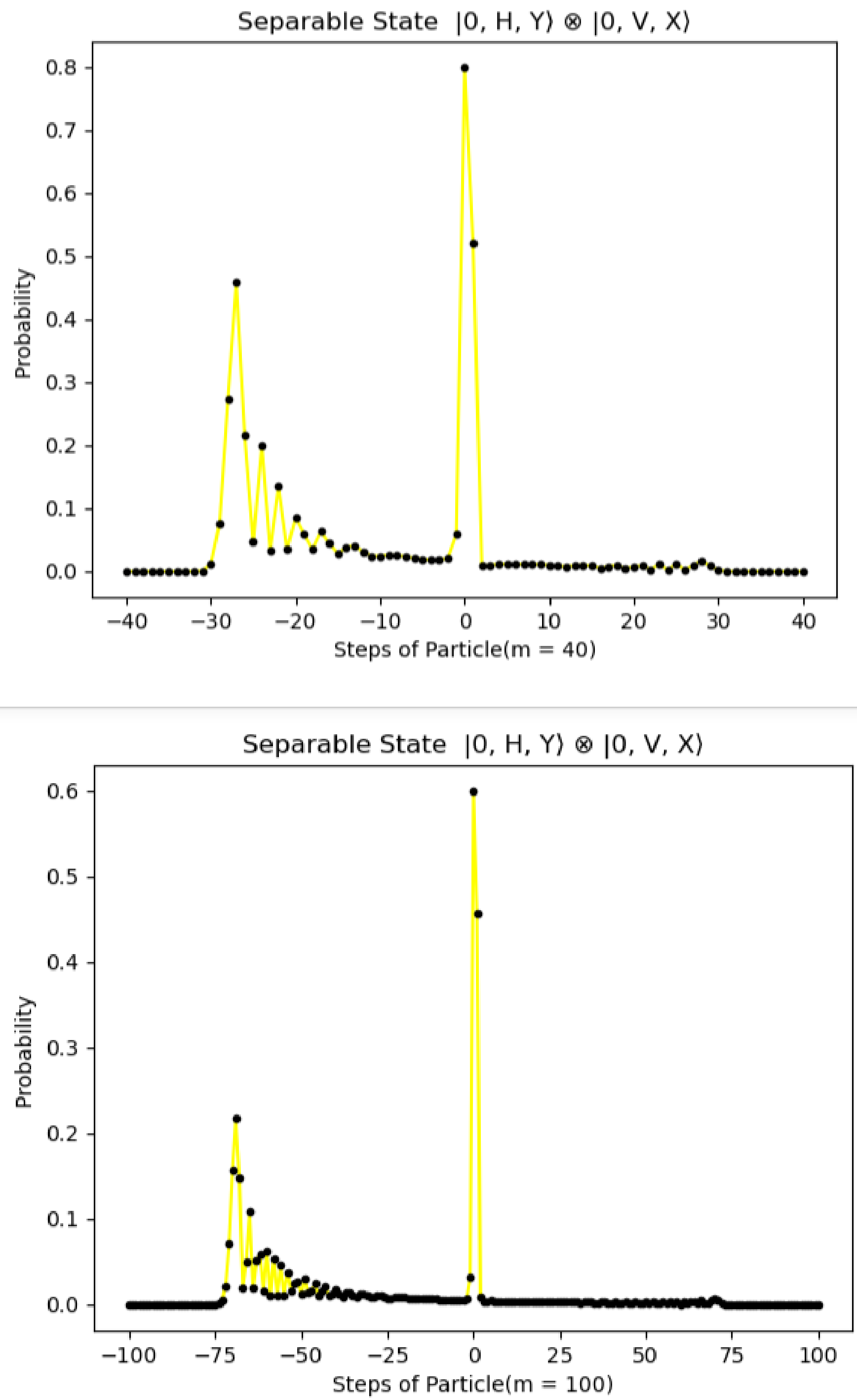


Figure 3.6: Probability distribution for detecting walkers after 40 and 100 steps, starting from an initial separable state, showing an asymmetric peak(left) with a central peak.

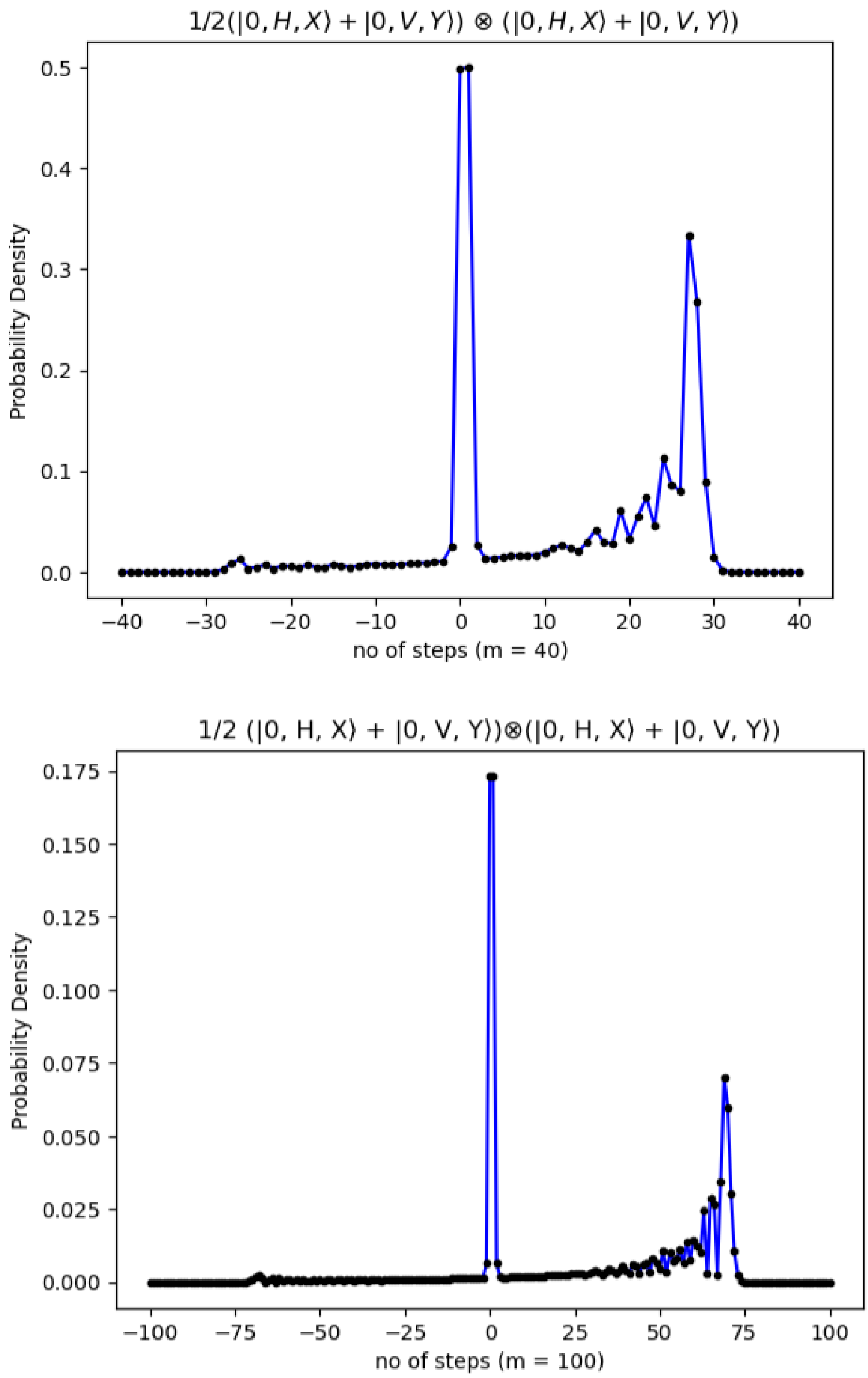


Figure 3.7: Probability distribution for detecting walkers after 40 and 100 steps, starting from an initial separable state, showing an asymmetric peak with a central peak.

3.4 Quantum Walks of Two Photons with Entangled State

3.4.1 Description

Omar et al [51] conducted research on the quantum random walk(QRW) of two different walkers who started out in an entangled state. They found that the final state of both walkers was also entangled, but the degree of entanglement varied depending on the degree of the beginning entanglement.In order to conclusively prove that the fields have a quantum origin, we compute photon entanglement, which has already been exploited to great advantage. [46, 47]. We provide evidence to demonstrate that the probability of detecting two photons and the State of photons has a quantum character. are significantly related. Below are detailed results for each of the four Bell states that the incoming photons held.The research that we have done so sheds light on the function that coherences and entanglement play in QRW.

3.4.2 Entanglement in Polarization

The entanglement in the polarisation refers to the correlation or dependence between the polarisation states of two or more photons. The polarisation of light is what defines the direction in which its electric field oscillates; this property of light is called polarisation.According to quantum mechanics, the polarisation of a photon is determined by the photon's quantum state, which can be a superposition of several different polarisation states.When two photons are entangled, which causes their polarisation states to become correlated, the polarisation of one photon is dependent on the polarisation of the other photon. Take into consideration a set of entangled photons that were produced by a method such as parametric down-conversion. It is possible for these photons to show entangled polarisation states. If this is the case, then when the polarisation of one photon is measured, the polarisation of the other photon may be instantly calculated, despite the fact that they are physically separated from one another. Understanding the entanglement of polarisation states can be achieved by using the superposition of quantum states. The (x) and (y) polarisation states, for example, can both be present in superposition with the photons. The combined state of the photons is identical to the states of the individual photons in a superposition known as a joint superposition, which is feasible when photons are entangled. In the event when the first photon has x polarisation and the second photon has y polarisation, or vice versa, the entangled state can be represented as (X1Y2 - Y1X2) in either case. The fact that the distinct polarisation states of the photons are connected to one another and dependent on one another suggests that it is impossible to determine each one of them separately as a result of entanglement. This entanglement of polarisation states is a fundamental component of quantum mechanics that has been extensively explored and utilised in a variety of applications. Some examples of these applications include quantum information processing, quantum communication, and quantum cryptography.

3.4.3 Experimental Implementation of Entangled Photons

In the scenario in which we have entangled photons, one of which has x polarisation and the horizontal direction of propagation, and the other of which has y polarisation and the vertical direction of propagation, let's look at what happens to these photons as they move through the various optical elements. After travelling through the HWP, the photon that is x-polarized and has horizontal propagation will continue to go in a straight line without any modification to either its polarisation or its direction of travel. In a similar manner, the y-polarized photon that is travelling vertically will not be altered in any way as it travels through the HWP because it will continue to travel straight. The polarising prism or PBS, is the next stop for both photons. It depends on the photons' polarisation whether the PBS will let them pass through or reflect them. When a photon is travelling horizontally and is x-polarized, the PBS will not block it because the polarisation state of the photon is aligned with the transmission axis of the PBS. On the other hand, because it is perpendicular to the transmission axis, the y-polarized photon with vertical propagation will be reflected by the PBS. After going through the HWP and the PBS, the photons will end up in an entangled state as a direct consequence of this process. The x-polarized photon, which is travelling

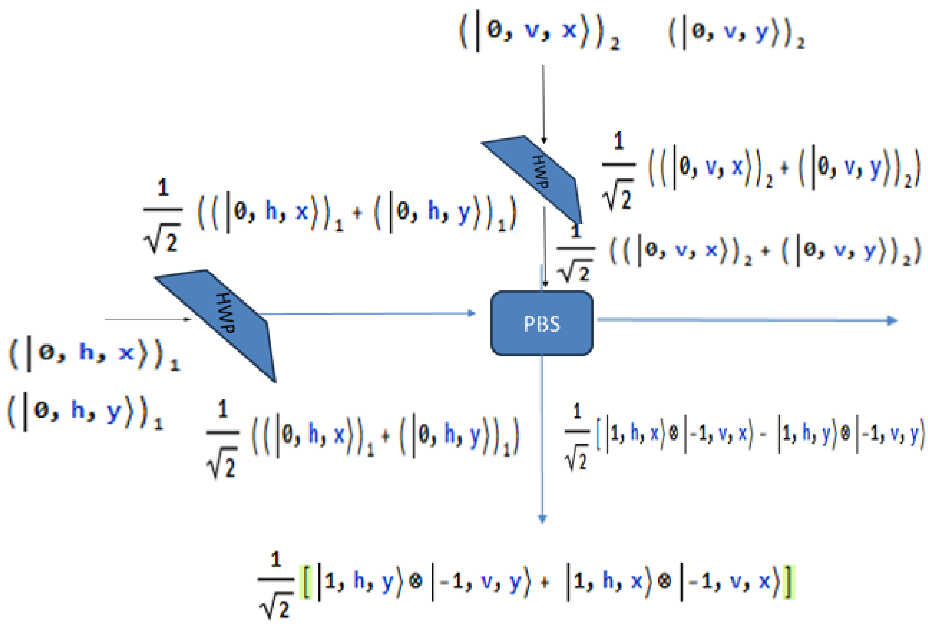


Figure 3.8: The conceptual structures for implementing two entangled photons quantum walk. The effect of the polarisation beam splitter is directly displayed in the inset. The y-polarized photon is reflected, changing its direction of transmission, whereas the x-polarized photon is transmitted in the same direction.

in a horizontal direction, it is transmitted , whereas the y-polarized photon, which is travelling in a vertical direction, is reflected. In this entangled condition, there is a correlation between the polarization of the two photons, despite the fact that they have travelled through the optical elements in separate orders [48].

3.4.4 Initial Entangled states

Bell's states of polarisation are a group of four entangled quantum states that can be produced by employing polarised photons as the source of information. These states, rather than being analogous to Bell's states in the context of spin, are analogous to polarisation.The four Bell's states of polarization are typically denoted as $|\Phi^+\rangle$, $|\Phi^-\rangle$, $|\Phi^+\rangle$, and $|\Phi^-\rangle$. Each state corresponds to a particular combination of polarization states for the two photons:

- This condition is the result of entanglement between two photons that have states of polarisation that are orthogonal to one another. For instance, one photon may have a polarisation state denoted by the letter x, whereas the other may have a polarisation state denoted by the letter y. It can be represented mathematically as;

$$\Phi^{\pm} = \frac{1}{\sqrt{2}} \left(|m, H, X\rangle \otimes |m, V, Y\rangle + |m, H, Y\rangle \otimes |m, V, X\rangle \right), \tag{3.44}$$

$$\Phi^{\pm} = \frac{1}{\sqrt{2}} \left(|0, H, X\rangle \otimes |0, V, Y\rangle + |0, H, Y\rangle \otimes |0, V, X\rangle \right). \tag{3.45}$$

- This state likewise reflects entanglement between orthogonal polarisation states; however, the signals of the entanglement are reversed. It can be represented mathematically as;

$$\Phi^{\pm} = \frac{1}{\sqrt{2}} \left(|0, H, X\rangle \otimes |0, V, Y\rangle - |0, H, Y\rangle \otimes |0, V, X\rangle \right). \tag{3.46}$$

- Entanglement between two photons with the identical polarisation states is represented by this state. For instance, the polarisation of both photons might be horizontal (H) or vertical (V), but not both at the same time. It can be represented mathematically as;

$$\phi^{\pm} = \frac{1}{\sqrt{2}} \left(|0, H, X\rangle \otimes |0, V, X\rangle + |0, H, Y\rangle \otimes |0, V, Y\rangle \right). \tag{3.47}$$

- Represents entanglement between the identical polarisation states but with opposite signs, hence it is analogous to the previous state. It can be represented mathematically;

$$\Phi^{\pm} = \frac{1}{\sqrt{2}} \left(|0, H, X\rangle \otimes |0, V, X\rangle - |0, H, Y\rangle \otimes |0, V, Y\rangle \right). \tag{3.48}$$

These are the four Bell's states of polarisation ,the outcomes of one photon's measurements are instantly correlated with the outcomes of the measurements of the other photon, regardless of the distance between them. They are useful for a variety of applications in quantum communication and quantum information processing, including quantum teleportation and quantum cryptography.

3.5 Operation on Entangled States

Evaluation of entangled photons states also done by operating unitary operator separately on each state and Unitary will be,

$$\hat{O} = \hat{V}(\hat{G} \otimes \hat{I}). \tag{3.49}$$

HWP operator acts as Hadamrd Operator,

$$\hat{G} = \frac{1}{\sqrt{2}}\big(|H,X\rangle\langle H,X| + |H,X\rangle\langle H,Y| + |H,Y\rangle\langle V,X| - |H,Y\rangle\langle V,Y| \tag{3.50}$$

$$+ |V,X\rangle\langle V,X| + |V,X\rangle\langle V,Y| + |V,Y\rangle\langle H,X| - |V,Y\rangle\langle H,Y|\big). \tag{3.51}$$

Polarizing prism /Shift operator,

$$\begin{aligned}\hat{V} = &|H,X\rangle\langle H,X| \otimes |m+1\rangle\langle m| \\ &+ |H,Y\rangle\langle H,Y| \otimes |m+1\rangle\langle m| \\ &+ |V,X\rangle\langle V,X| \otimes |m-1\rangle\langle m| \\ &+ |V,Y\rangle\langle V,Y| \otimes |m-1\rangle\langle m|.\end{aligned}$$

After applying these operators we can get next state,

$$|\text{Entangled Next State}\rangle = \frac{1}{\sqrt{2}}(\hat{O}|m,H,X\rangle \otimes \hat{O}|m,V,Y\rangle + \hat{O}|m,H,Y\rangle \otimes \hat{O}|m,V,X\rangle). \tag{3.52}$$

Let's consider an input state consisting of an X-polarized photon with a horizontal direction of propagation and second photon is Y polarized with vertical direction of propagation. When we apply a half wave/birefringent wave plate operator (G) to the input state, it undergoes a conversion into a superposition state.When we apply shift operator on superposition state this will give next state after one step,

$$|m,H,X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m,H,X\rangle + |m,V,Y\rangle]. \tag{3.53}$$

Apply shift operator on superposition state , it will give next state,

$$\frac{1}{\sqrt{2}}[|m, H, X\rangle + |m, V, Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|m+1, H, X\rangle + |m+1, V, Y\rangle]. \tag{3.54}$$

If m = o then we apply operators to get state next state (1),

$$|0, H, X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0, H, X\rangle + |0, V, Y\rangle], \tag{3.55}$$

$$\frac{1}{\sqrt{2}}[|0, H, X\rangle + |0, V, Y\rangle] \xrightarrow{\hat{s}} \frac{1}{\sqrt{2}}[|1, H, X\rangle + |-1, V, Y\rangle]. \tag{3.56}$$

Similarly next state(1) will pass through apparatus and mathematically we can see the change by applying operators and get next state (2).

lets take another state which is $| m, V, Y\rangle$, when this state pass through birefringent/half wave plate and polarizing beam splitter,

$$|m, V, Y\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m, V X\rangle - |m, H, Y\rangle], \tag{3.57}$$

Apply shift operator on superposition state , it will give next state,

$$\frac{1}{\sqrt{2}}[|m, V, X\rangle - |m, H, Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|m-1, V, X\rangle - |m+1, H, Y\rangle]. \tag{3.58}$$

We can apply unitary operator on previous state to find the next state.when we apply on initial state (o), we will get next state (1),

$$|0, V, Y\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0, V, X\rangle - |0, H, Y\rangle], \tag{3.59}$$

$$\frac{1}{\sqrt{2}}[|0, V, X\rangle - |0, H, Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[-1, V, X\rangle - |+1, H, Y\rangle]. \tag{3.60}$$

We can see the behaviour of other states by applying unitary operator.like this Taking state $| m, H, Y\rangle$,

$$|m, H, Y\rangle \xrightarrow{\hat{g}} \frac{1}{\sqrt{2}}[|m, H, X\rangle - |m, V, Y\rangle]. \tag{3.61}$$

When we apply shift operator on superposition state, it will give next state,

$$\frac{1}{\sqrt{2}}[|m,H,X\rangle - |m,V,Y\rangle] \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m+1,H,X\rangle - |m-1,V,Y\rangle]. \tag{3.62}$$

When no of step(m) = 0,

$$|0,H,Y\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0,H,X\rangle - |0,V,Y\rangle]. \tag{3.63}$$

Applying shift operator on superposition state , it will give next state,

$$\frac{1}{\sqrt{2}}[|0,H,X\rangle - |0,V,Y\rangle] \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|1,H,X\rangle - |-1,V,Y\rangle]. \tag{3.64}$$

Now state $|\ m,V,X\rangle$,

$$|m,V,X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|m,V,X\rangle + |m,H,Y\rangle], \tag{3.65}$$

Apply shift operator on superposition state , it will give next state,

$$\frac{1}{\sqrt{2}}[|m,V,X\rangle + |m,H,Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|m-1,V,X\rangle + |m+1,H,Y\rangle]. \tag{3.66}$$

When photon at m=0,

$$|0,V,X\rangle \xrightarrow{\hat{G}} \frac{1}{\sqrt{2}}[|0,V,X\rangle + |0,H,Y\rangle], \tag{3.67}$$

Apply shift operator on superposition state , it will give next state,

$$\frac{1}{\sqrt{2}}[|0,V,X\rangle + |0,H,Y\rangle] \xrightarrow{\hat{V}} \frac{1}{\sqrt{2}}[|-1,V,X\rangle + |+1,H,Y\rangle]. \tag{3.68}$$

For next state the transformation acting on the initial state of the photon is equivalent to the one step of QRW. The iterative application of the transformation U for n times gives the QRW of n steps. The equation representing the nth state is given by:

$$|\text{nth state}\rangle = \hat{O}^n|\text{initial state}\rangle. \tag{3.69}$$

Analytically this is not possible to find nth state by operating unitary operator n times on the state ,so for calculating nth state we will do numerical simulations.

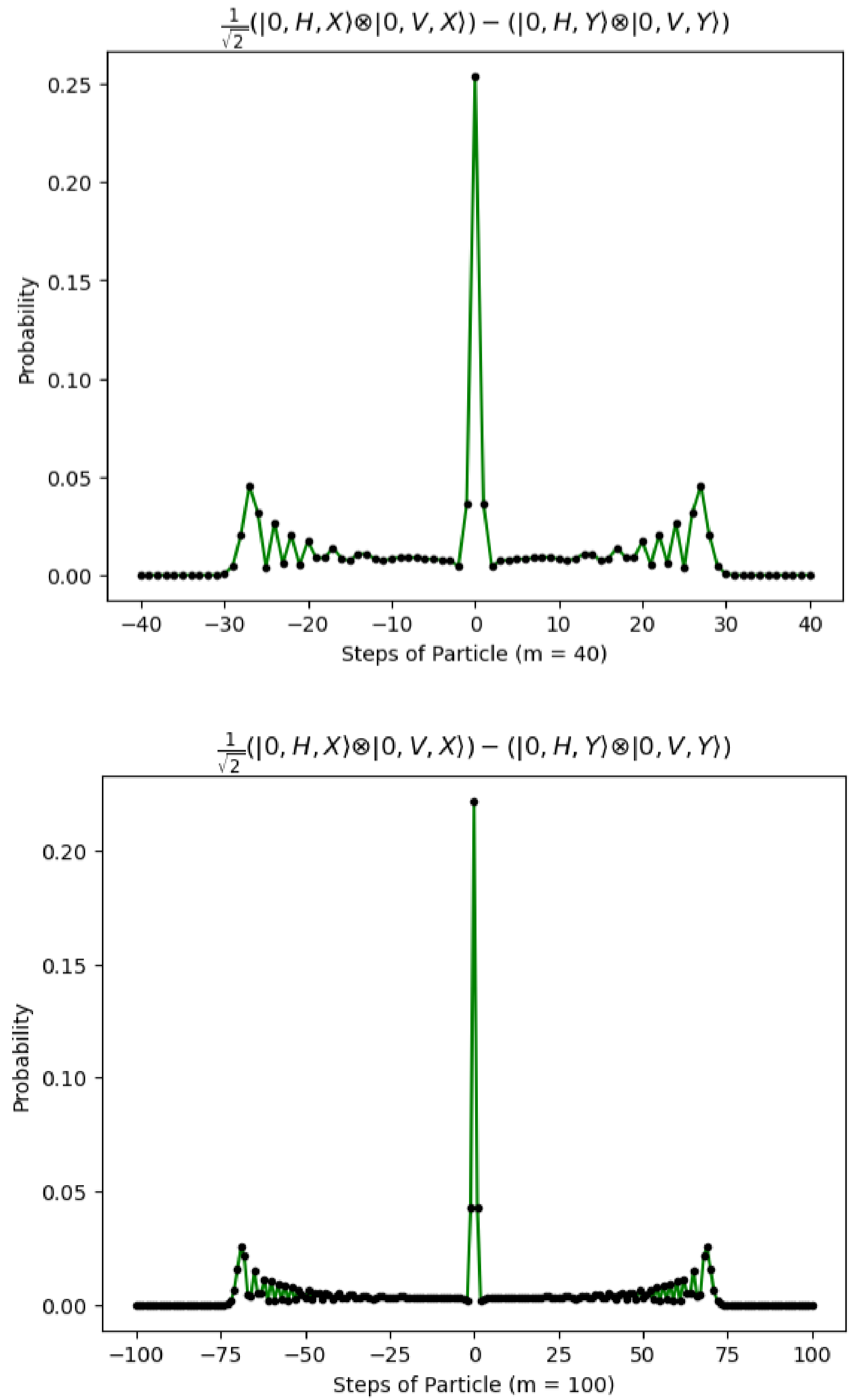


Figure 3.9: Probability distribution for detecting walkers after 40 and 100 steps, starting from an initial entangled state, showing symmetric peaks with a central peak.

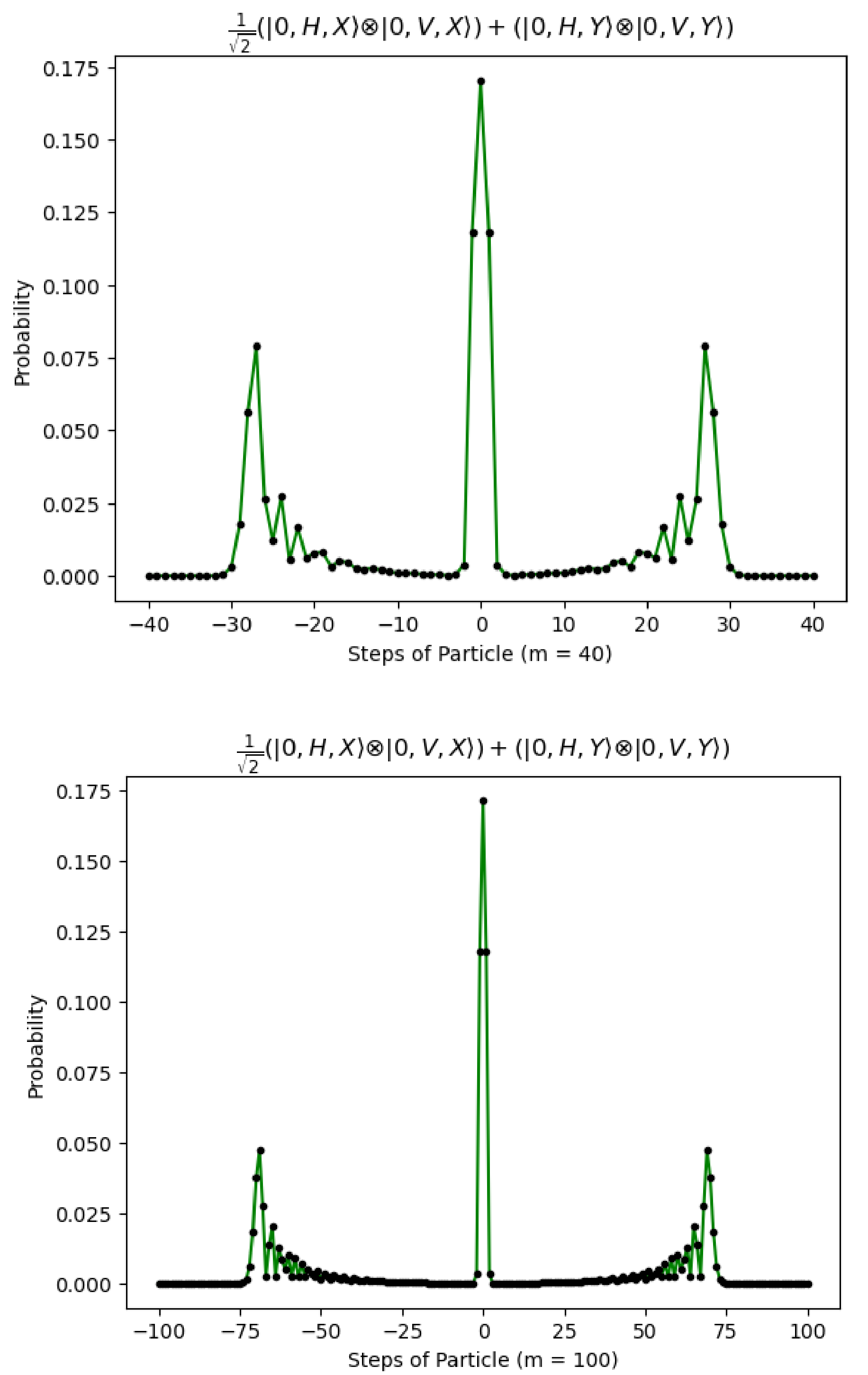


Figure 3.10: Probability distribution for detecting walkers after 40 and 100 steps, starting from an initial entangled state, showing symmetric peaks with a central peak.

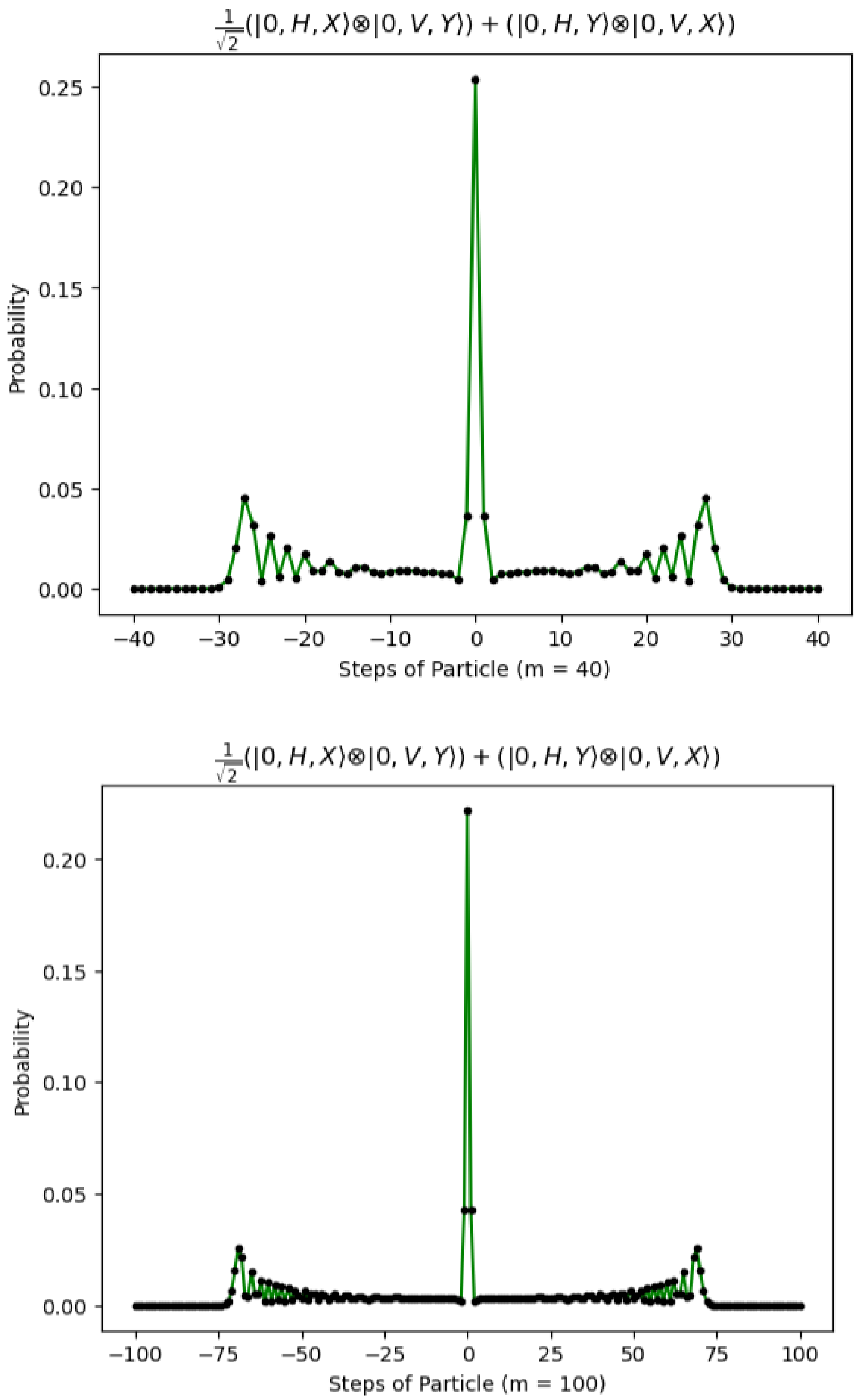


Figure 3.11: Probability distribution for detecting walkers after 40 and 100 steps, starting from an initial entangled state, showing symmetric peaks with a central peak.

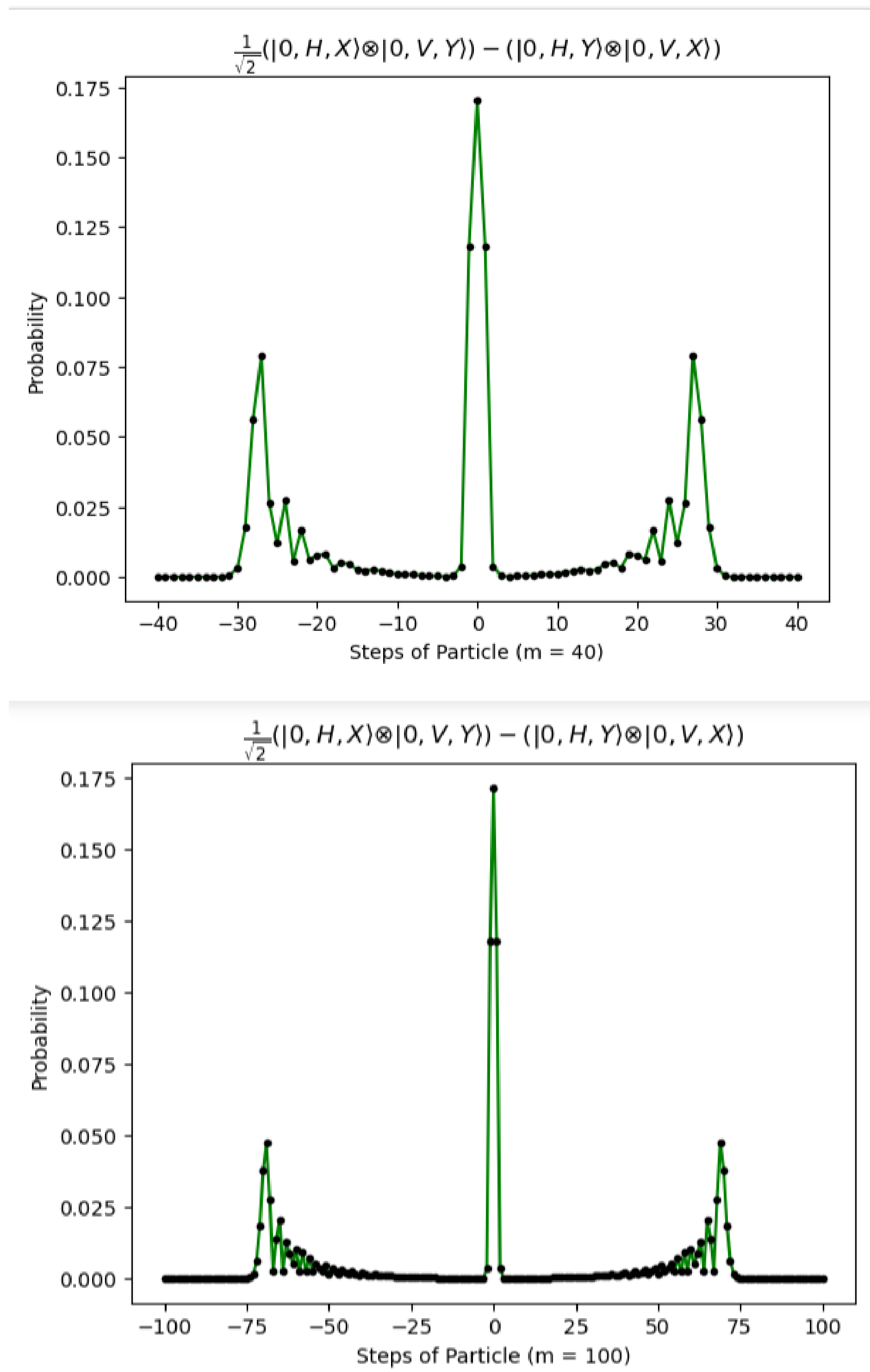


Figure 3.12: Probability distribution for detecting walkers after 40 and 100 steps, starting from an initial entangled state, showing symmetric peaks with a central peak.

Chapter 4

Results and Discussion

This section summarizes the findings acquired from our research into the behavior of photons that are participating in quantum walks. Specifically, we used optical components, such as polarizing prism (PBS) and birefringent -wave plates (HWP), in our examination of this phenomenon. The subsections that follow provide an overview of the outcomes associated with our research objectives:

- To Determine the Effects of PBS and HWP on the Behavior of Quantum Walks. In order to accomplish the first goal, we investigated how the use polarizing prism (PBS) and birefringent -wave plates (HWP), influences the way photons behave during quantum walks. We were able to observe significant influences on the dynamics of the quantum walk through the use of rigorous testing and data analysis. The presence of PBS and HWP resulted in superposition and next state of photons.These findings provide important new insights into the effect that optical components have on the behavior of photons when they are participating in quantum walks.

- The calculation of nth quantum states and probabilities.Finding the nth quantum state and the probabilities associated with separable and entangled states was a primary focus for us as we worked toward achieving the second goal of the thesis, to find the nth separable and entangled state we used a discrete time quantum walk method and find probabilities and compare the results of separable states with entangled states. In framework of quantum physics, these discoveries provide a deeper understanding of the quantum features of separable and entangled states.

4.1 Separable State

The probability distributions exhibited antisymmetry when considering the initial states. The distribution also showed a positive side peak for the starting state $(0, 0, h, x)(0, 0, v, y)$. This suggests a higher likelihood of discovering the photon at the origin and on one side of the $q1$-axis. Particularly, it indicates a higher likelihood in the positive direction. This behavior is depicted in Figure (4.1)a.

The initial state was represented by the notation $(0, 0, h, y)(0, 0, v, x)$, and the probability distribution displayed a side peak in the opposite direction. This indicates that there was a greater possibility of the photon being located on the opposite side of the $q1$-axis, particularly in the opposite (negative) direction in figure (4.2)b.

The probability distributions' antisymmetry denotes an uneven distribution of the photon's probability along the q1-axis. Depending on the structure of the starting state, the presence of side peaks implies that the photon is more likely to be discovered in particular locations away from the central axis. The asymmetric behavior and its implifications in the context of quantum walks are highlighted by these data, which offer insightful information about the impact of initial states on the photon distribution during the quantum walk.

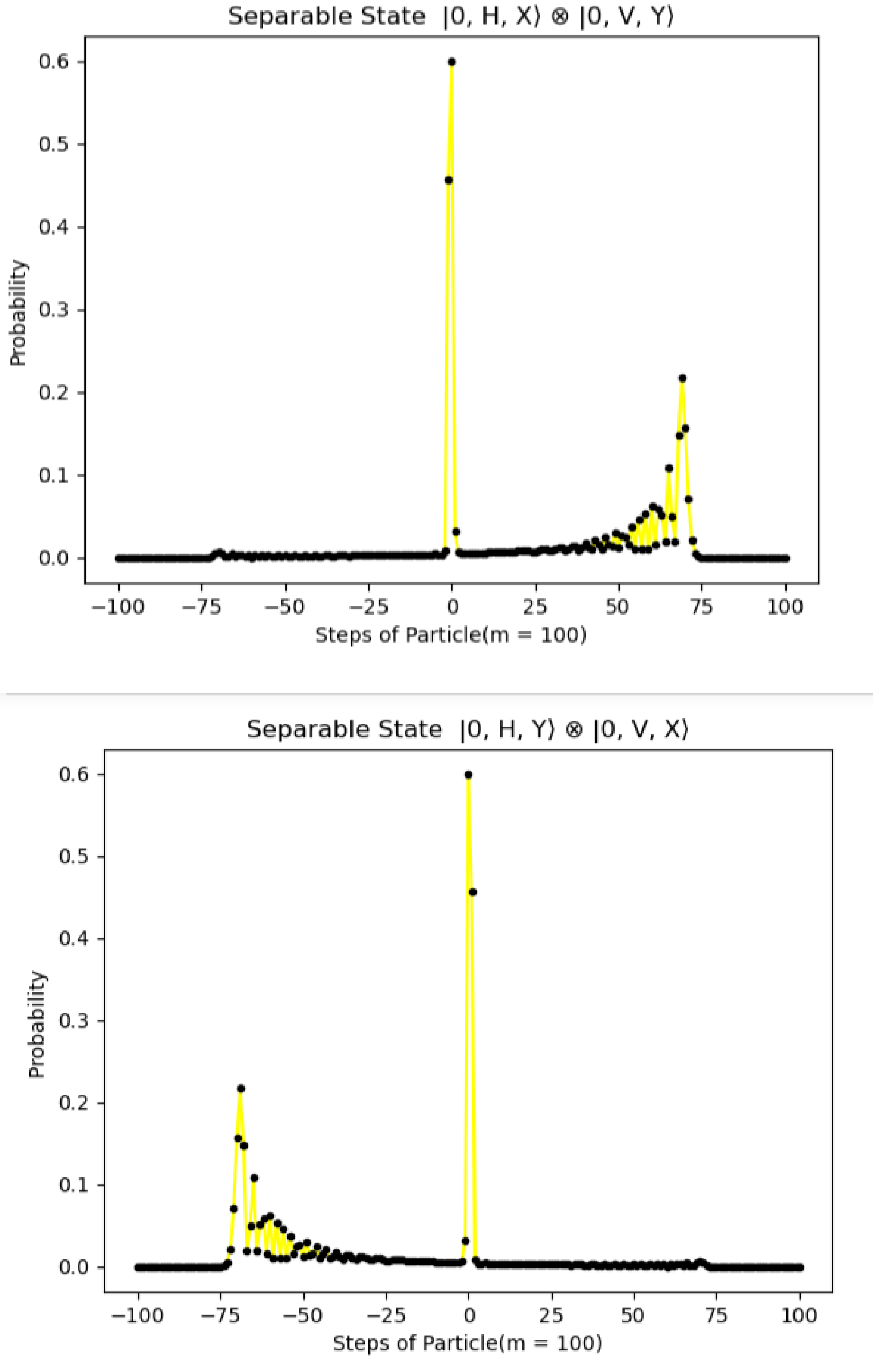


Figure 4.1: Probability distribution for detecting walkers after 100 o of steps, starting from an initial different separable states, showing asymmetric peaks with a central peak.

4.2 Entangled State

If the initial states of the photons are states that are maximally entangled with one another (3.45) and (3.48). We get distinct probability distributions depending on the entangled state that we are looking at. Differences in distributions between symmetric states and anti symmetric states are substantially greater than differences in distributions between anti symmetric states and anti symmetric states or between anti symmetric states and anti symmetric states.

When symmetric entangled state is used as the starting point for the calculation equation (3.4.8). Both negative and positive values of m1 can be accommodated by the distribution without disrupting its symmetry. It is important to note that there is a clear peak at the initial position, where $m1 = 0$. This peak reflects the place with the highest probability of detecting the walker. In addition, there is the appearance of side peaks, which highlights the quantum nature of the particle fig (4.2)a.

When an anti symmetric entangled state is used as the starting point for the calculation equation (3.45). Both negative and positive values of q1 can be accommodated by the distribution without disrupting its symmetry. It is important to note that there is a highest peak at the initial position, where $m1 = 0$. This peak reflects the place with the highest probability of detecting the photon but this highest peak at origin in asymmetric initial state case is less than highest peak at origin in symmetric probability distributions fig (4.2)b. In addition, there is the appearance of side peaks, which highlights the quantum nature of the particle and these peaks similar to figure(4.2)a.

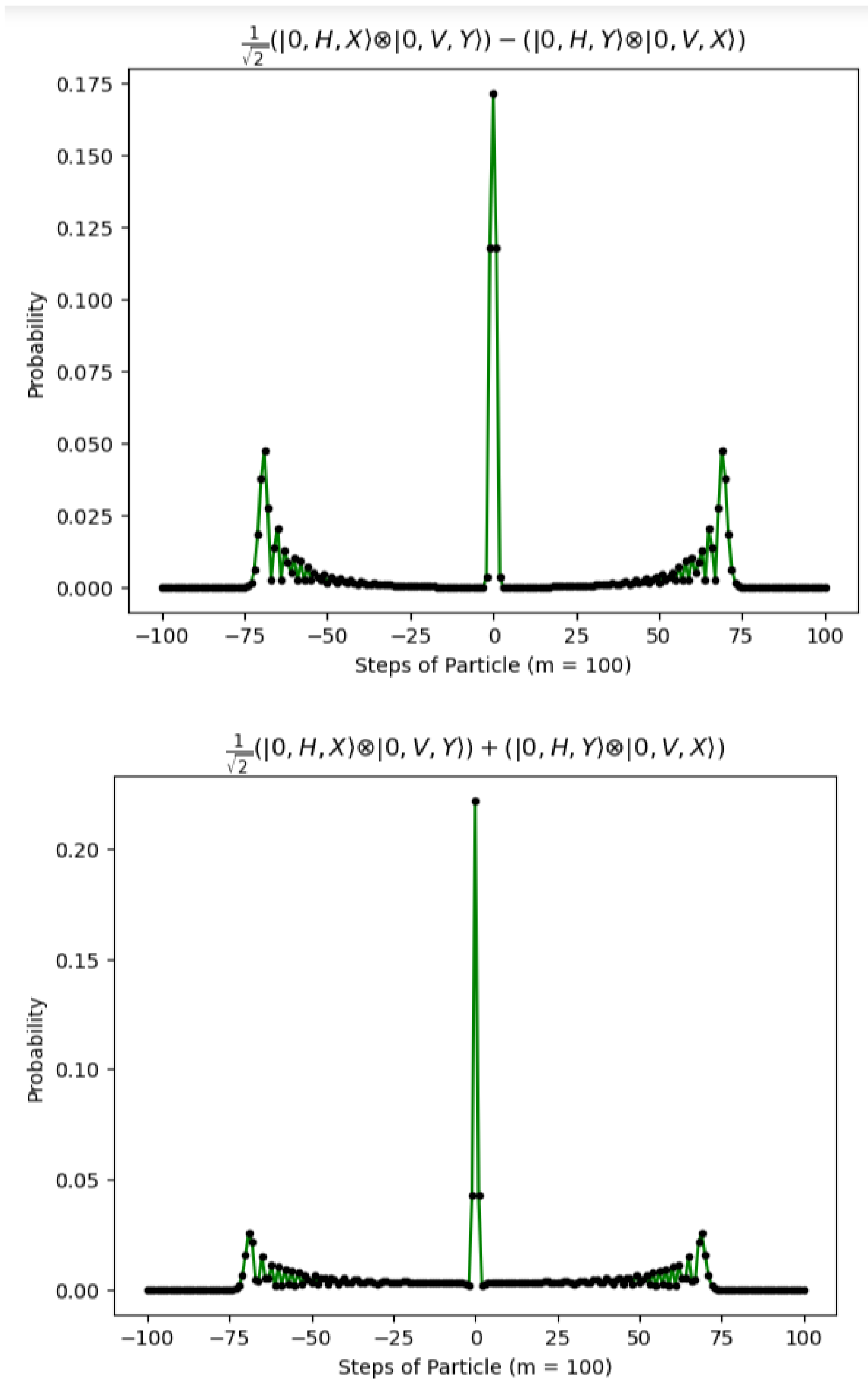


Figure 4.2: Probability distribution for detecting walkers after 100 number of steps, starting from an initial different entangled states, showing symmetric peaks with a central peak.

4.3 Comparison Between Separable State probabilities and Entangled State probabilities

Results from the separable state with anti symmetric polarisation showed an anti symmetric graph with a probability distribution that had a greater peak at the origin (representing the original state) and fewer peaks overall, indicating a more conventional behaviour in figure (4.3)a and (4.4)a. As opposed to the separable state, the results from the entangled state showed a symmetric probability graph with a lower probability at the origin. Separable and entangled graphs behave differently, highlighting their distinctive properties. The entangled state displayed a symmetrical distribution shown in figure (4.3)b and (4.4)b, suggesting a quantum nature where the probability was spread more evenly across positions away from the origin, in contrast to the separable state, which displayed an asymmetric probabilities peak with a prominent highest probability peak at the origin. Entangled quantum states are characterized by interdependent properties between constituent parts, leading to correlations that can defy classical expectations. While symmetric probabilities are one possible outcome of certain entangled systems, the behavior of probabilities is governed by complex mathematical equations. Entanglement's significance lies in applications like quantum computing, teleportation, cryptography, and sensors. This stands in contrast to separable states where properties of parts are not correlated, but the distinction between symmetric and asymmetric probabilities isn't solely determined by state separability. These results highlight how entanglement contributes to the generation of distinctive quantum effects and highlight the divergent behaviours of entangled states in quantum systems.

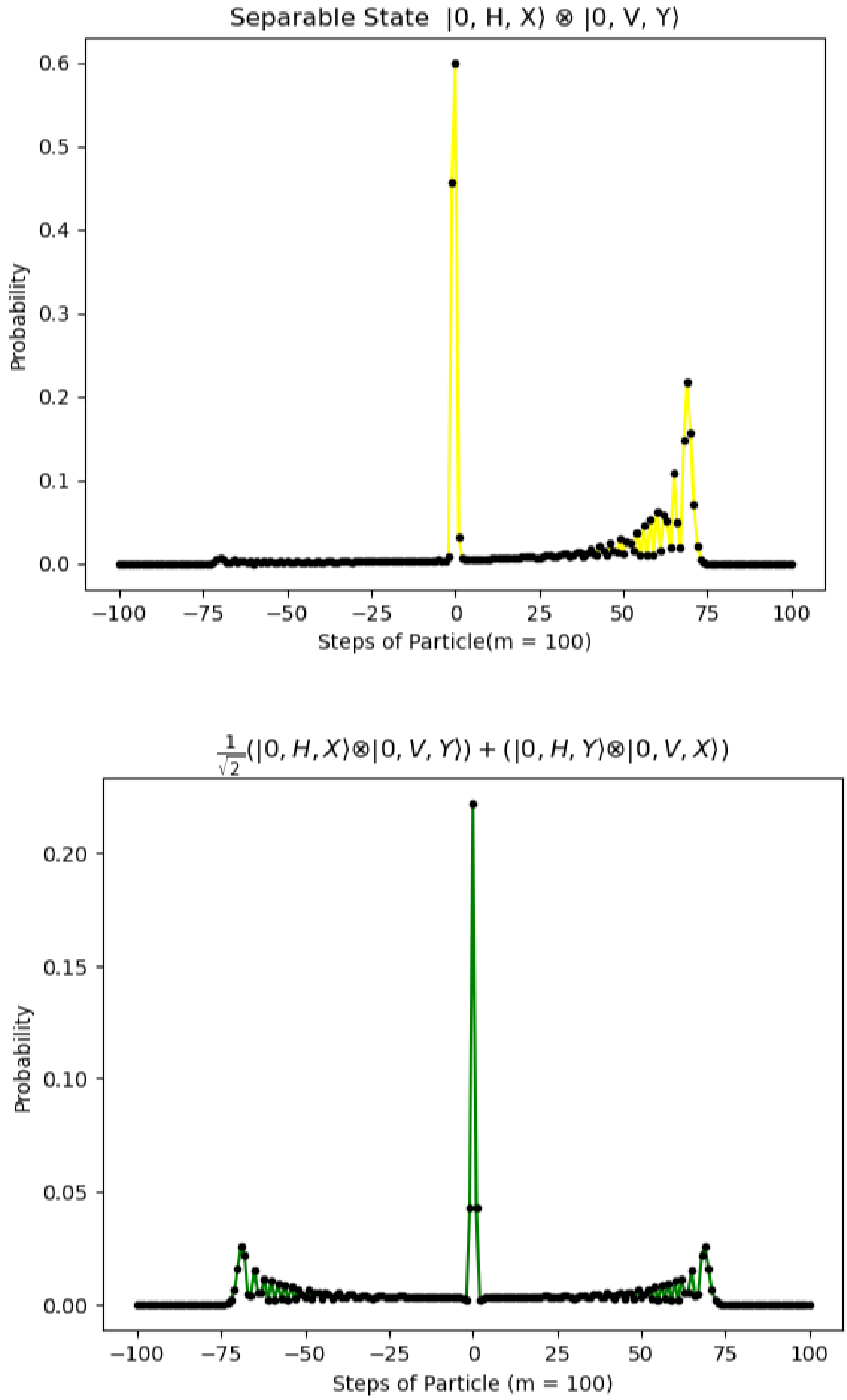
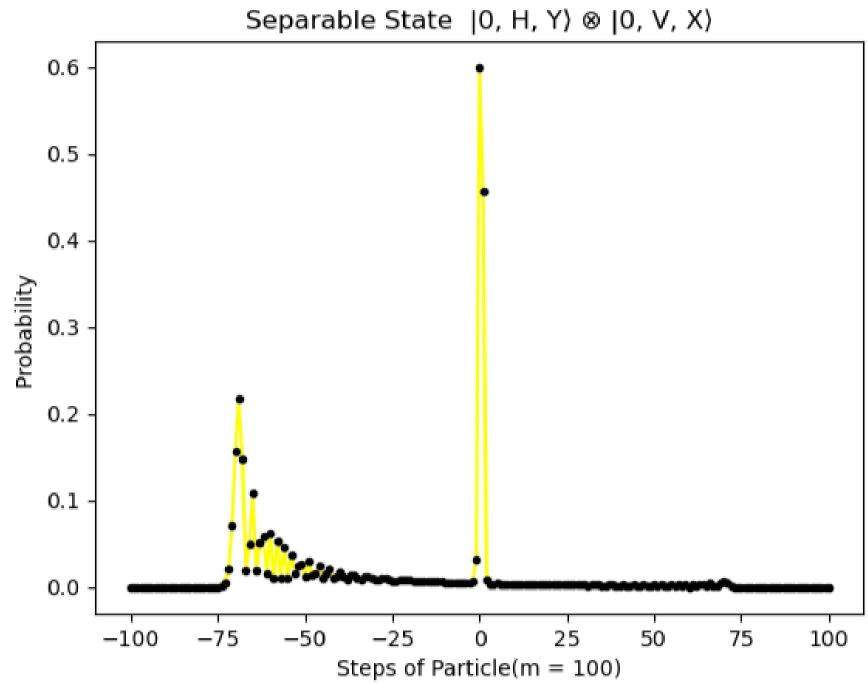


Figure 4.3: Comparison between probabilities for locating the walker at various positions after 100 steps of walking with separable and entangled states.



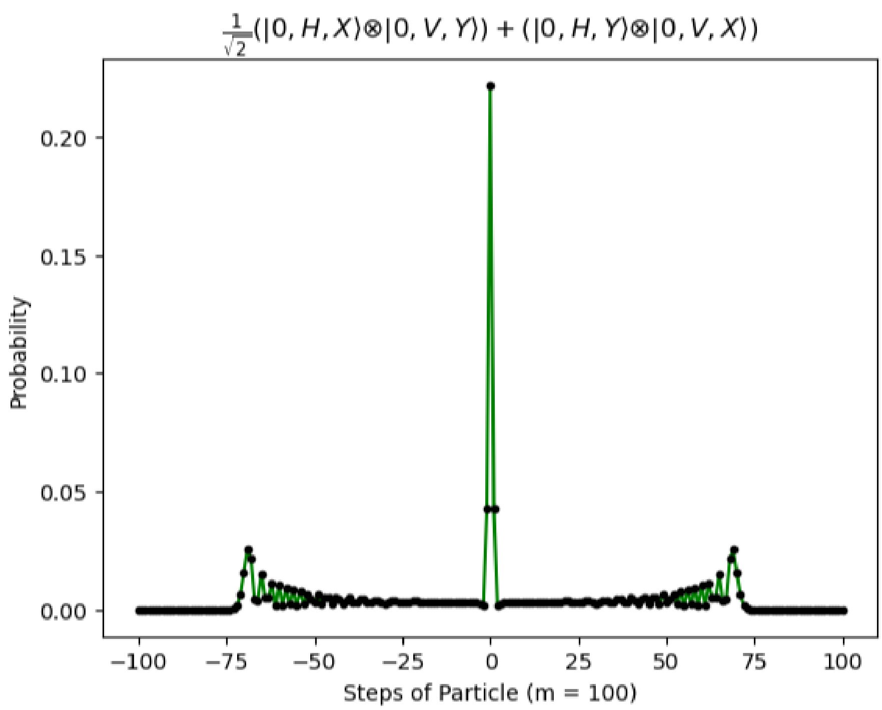


Figure 4.4: Comparison between probabilities for locating the walker at various positions after 100 steps of walking with separable and entangled states.

Chapter 5

Summary and Outlook

The purpose of this study was to examine the quantum walk of photons using optical elements, specifically half-wave plates and polarising beam splitters, with various states, including superposition state, separable state, partially entangled state and maximally entangled states, calculate nth states and their probabilities. The research encompassed both experimental and analytical approaches, complemented by simulations to validate the results.In analytical approach we used coined quantum walk model to find nth state and their probabilities.The behaviour of photons going through the optical components was studied during the experimentation phase and also seen the working of optical elements.When we measured superposition state probabilities, we got symmetric peaks both sides. The anti symmetric polarised separable state displayed an anti symmetric graph, this graph showed fewer peaks overall and a higher peak in the start stage.When we measured partially entangled state, we got different peaks ,first peak towards position side on number line with higher probability amplitude and second peak towards negative side with less probability amplitude than positive side peak's amplitude. In contrast to the separable state and partially entangled state, the maximally entangled state produced a symmetric probability graph with a lower probability near the origin. The probability was uniformly distributed across positions away from the origin, as evidenced by the symmetrical distribution, which implied a quantum nature.To further analyze the results, two approaches were employed. Firstly, an analytical approach based on discrete time quantum walks was utilized. The calculations made using this method were in line with the outcomes of the experiment, confirming its validity. In particular, the Coined (Hadamard) quantum walk and the beam splitter quantum walk are carefully evaluated for discrete time quantum walks on the line. To understand how entanglement affects quantum walks, the probability distribution of the walk is examined for separable states as well as entangled states, including various bell's states. It is thought that entanglement might enhance the probability distribution of the walker. To discover the likelihood outcomes, simulations were run.The findings hold significant implications for quantum computers, leveraging entangled bits to accelerate information processing. Additionally, applications such as super dense coding, teleportation, and secure quantum key distribution stand to benefit from the insights gained. Our analysis may be used to explain other systems, such as the quantum walk of many photons by using same optical setup and quantum walks of two or more photons with chorent state.

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