Study of Damage Detection and Quantification in Cantilevered Beams using Modal Strain Energy & Genetic Algorithm

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THESIS ACCEPTANCE CERTIFICATE

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Dedication

Dedicated to my exceptional parents and adored siblings whose tremendous support and cooperation led me to this wonderful achievement

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First and foremost, I am thankful to Allah Almighty for His countless blessings and for giving me the strength and courage to undertake this novel project and see to its outcomes. I am also thankful for the tremendous support, courage and prayers of my parents and family members, especially my father Zafar Iqbal. I wish to record my deepest obligation to my parents and siblings for their prayers, encouragement and financial as well as moral support during my studies.

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ABSTRACT

Structural health monitoring (SHM) is known to be a demanding area of research which has its applications in large scale load carrying structures like bridges, aircrafts, automobiles, offshore platforms, and submarines. Deteriorating and aging structures may require early detection and quantification of damage to ensure safety and service life. Various techniques based on modal parameters and optimization have been proposed in literature. The drawback of using conventional and deterministic optimization algorithms is that complete set of data and input parameters may be required to proceed. While GA can operate even with incomplete set of data and with minimum number of input modal parameters i-e mode shapes & natural frequencies. So, GA based methodology has the advantage in real life applications over deterministic and conventional optimization methods and therefore considered for damage detection in current study. Estimation of objective function is required for entire set of population of chromosomes at the end of each generation. Therefore, genetic algorithm (GA) is computationally expensive tool. The current study employs a multistage methodology to reduce the solution parameters of the GA. To compare the vibrational properties of the undamaged and damaged structures, modal analysis of the structure was conducted. The ratio of change in modal strain energy (MSECR) was used to localize damage and reduce the size of the solution space of GA. True damage percentage of identified damaged elements was calculated using a GA-based on mode shapes and natural frequencies. Numerical studies of cantilever beam have been carried out to validate the methodology. Numerical studies disclose multistage approach is found to be robust and rapidly convergent at a reduced computational cost.

Key Words: Damage, MAC, Natural frequency, Modal strain energy, Detection, Quantification, Genetic algorithm, Selection, Mutation, Crossover.

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LIST OF ABBREVIATIONS

SHM	Structural Health Monitoring
GA	Genetic Algorithm
MSECR	The Ratio of Change in Modal Strain Energy
VBDIT	Vibration-Based Techniques for Damage Identification
MSE	Modal Strain Energy
MAC	Modal Assurance Criteria
SRFs	Stiffness Reduction Factors
VIBDD	Vibration-Based Damage Detection
SDI	Stub’s Damage Index
DI	Damage Index
MSED	Modal Strain Energy Decomposition
MSEC	Change of Modal Strain Energy
CMSE	Cross Modal Strain Energy
CMCM	Cross-Model Cross Mode
MSEBI	Modal Strain Energy Based Indicator
PSO	Particle Swarm Optimization
IMSE	Improved Modal Strain Energy
BA	Bat Algorithm
MSF	Modal Scale Factor
FRF	Frequency Response Function
DOF	Degree Of Freedom
FE	Finite Element
TMM	Transfer matrix method

CHAPTER 1: INTRODUCTION

1.1 BACKGROUND

In order to ensure the safe operation of load-bearing structures like buildings, bridges, aircraft, and automobiles, damage detection and evaluation are essential. Vibration-based techniques for damage identification (VBDIT) are crucial to the damage identification process. VBDIT locates and measures damage by investigating changes in vibrational properties such as modal frequencies and mode shapes. The localization and estimation of the percentage of damage are made possible by modal parameters, which makes VBDIT an effective tool for tracking the health of structural components. In the current study, damage is located and quantified using a multistage methodology that incorporates an evolutionary optimization algorithm and modal strain energy (MSE). The drawback of using conventional and deterministic optimization algorithms is that complete set of data and input parameters may be required to proceed. While GA can operate even with incomplete set of data and with minimum number of input modal parameters i-e mode shapes & natural frequencies. So, GA based methodology has the advantage in real life applications over deterministic and conventional optimization methods and therefore considered for damage detection in current study. To reduce solution space for GA, damaged elements of beam are located using MSECR.

Srinivas et al. [1] proposed damage identification technique based on MSECR and GA based on changes in vibrational characteristics. Methodology was validated successfully for a simply supported beam and truss problem. As far as novelty of the study is concerned, this multistage methodology with multi-objective optimization and reduced solution space has not previously been validated for cantilever beams as per literature review conducted. Therefore, it is validated for cantilever beam in current research work.

Artar et al. [2] used GA using natural frequency based objective function. The technique was successfully validated for simply supported beam, cantilever beam and 8 bar space frame system. However, they never used MSECR for damage localization which would have made optimization process computationally expensive. Modal assurance criteria (MAC) based objective function was also not used hence, results would be unsatisfactory in case of localized damage.

The damage was located using MSE by Peng-hui et al. [3], who then quantified the damage using a GA with a natural frequency-based objective function. Methodology was applied to cantilever beam with different damage scenarios. However, MAC based objective function is not used, hence, results will be unsatisfactory in case of localized damage.

MSE was used by Ghasemi et al. [4] to locate the damage and quantify it using a modified GA with a natural frequency-based objective function. Methodology was tested using three different damage scenarios using ten bar planar trusses, thirty bar planar trusses, and seventy bar planar trusses. However, MAC based objective function is not used, hence, results will be unsatisfactory in case of localized damage.

Jeenkour et al. [5] used GA-based on combined natural frequency and MAC based objective functions for damage identification. The technique was validated for cantilever beam with different damage scenarios. However, damage localization was not achieved using MSECR hence, the process would be computationally expensive.

Cha et al. [6] presented hybrid GA for damage identification based on objective function comprising of modal strain energies. Methodology was successfully applied to 4-story irregular, 3-story asymmetric and 4-story prototype structures.

Hao et al. [7] used GA comprising of objective function based on mode shapes and modal frequencies used for damage identification. Framework was successfully applied to cantilever beam. However, MSECR was not used for damage localization, therefore, the process would be more computationally expensive.

The multistage methodology used in the current study explores the detection and measurement of damage in cantilever beams. In the beginning, the damage is located using MSECR. Variations in strain energy levels indicate locations of structural damage. Identified damaged elements along with their adjacent elements would be included in solution space for GA. Therefore, solution space of GA will be smaller and optimization process will become less computationally expensive.

The true damage percentage being calculated in the second step using a GA based on combined natural frequency and MAC based objective function. GA uses tournament selection criteria with a crossover probability of 0.9 and mutation rate of 0.01. Tournament selection operator means to select chromosome with highest value. Two-point crossover means to interchange strings of chromosomes at two points. Mutation involves replacing 1's to 0's and vice versa randomly. Stiffness reduction factors (SRFs) showing damage percentages are considered as solution parameters of GA. The multi-stage approach using both MSECR and the GA method is suitable for investigation of damage at large scale mechanical and civil structures [1].

1.2 PROBLEM STATEMENT

This thesis aimed to develop a multi-stage damage detection method based on MSE and GA, using the mode shapes and natural frequencies of damaged structures. Reduced computational and time cost with minimum modal input parameters from damaged structure provides the basis for a robust damage assessment for any structure.

1.3 MOTIVATION

The first and foremost motivation to conduct this study is to reduce time and computational cost for damage detection. Furthermore, this study aims to investigate the field of damage identification in a broader prospect. GA based methodology utilizes combined mode shapes and modal frequencies based objective function. This multi-objective optimization approach can estimate damage in localized as well as distributed damage cases. Moreover, the proposed methodology localizes the damage using MSE which considerably reduces solution space for GA. Therefore, this multi-stage methodology can be applied to large scale mechanical and civil structures where solution space would be very large.

1.4 CHALLENGES

Modal testing setup limitations and inaccuracy of measured modal parameters provide the basis for recent advances in the field of SHM. There is a set of certain limitations in any modal testing-based parameters which provides the motivation to propose this algorithm. Mode Shapes are contaminated due to noise and proposed algorithm can deal with noise. Impact hammer-based vibration testing does not provide higher modes for structure. Therefore, only first three modes (pure bending) can be considered for damage assessment. Damage detection is vital for structure life assessment and failure prediction as human life is at risk in many cases as shown in Figure 1.1.

Figure 1-1: Structural damage in a bridge

1.5 ORGANIZATION OF THESIS

The thesis follows the following format. The literature review in Chapter 2 discusses earlier research on vibration-based damage detection (VIBDD) techniques. The suggested methodology is explained in Chapter 3. The structures taken into consideration for damage detection are described in Chapter 4 along with the findings. The thesis is concluded in Chapter 5 and future research is mentioned. The study's findings turned out to be very satisfying and efficient.

CHAPTER 2: LITERATURE REVIEW

Buildings, bridges, aircraft structures, and automobiles all sustain damage over the course of their service lives. To forecast structural failure at early stages, damage detection is important [1]. VBDIT have an increasing importance in recent decades due to their better sensitivity to damage. The presence of damage is revealed by any change in vibrational properties. The percentage of the damage in that specific area of the structure is then calculated after the damage has been localized in the structure. The entire process helps to predict the current situation and failure in the future. MSE is used in most of the work for damage localization and quantification. This literature review will cover three major categories that describe vibration-based damage identification methods.

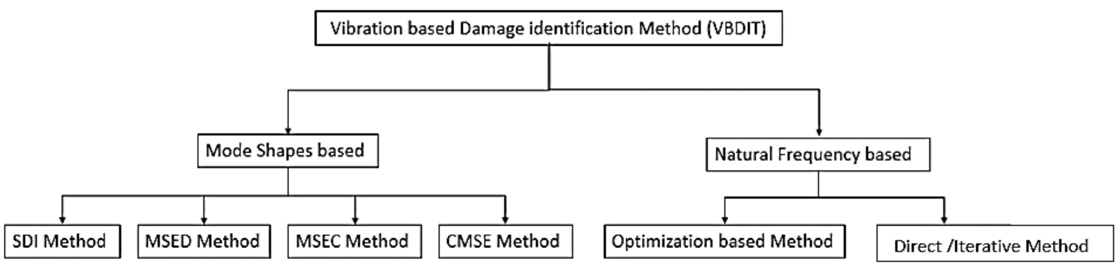

Figure 2-1: Categorization of VBDIT methods based on Modal Characteristics

A change in vibrational properties such as modal frequency and mode shape is one of the criteria for determining structural damage. MSE-based methods employ modal frequency and mode shapes to investigate damage in variety of issues. Techniques for identifying damage using MSE are based on the measured mode shapes and the stiffness of the structure. Any structure that has been damaged loses stiffness, which ultimately alters the MSE. Applications, benefits and limitations of these techniques are discussed in [8-14]. MSE based methods are typically noise-sensitive [15]. Mode shapes-based methods are considered as promising approach for damage detection [16]. In any experimental setup, modes shapes are subjected to contamination of noise and mostly measured mode shapes are incomplete due to limitation of sensors [17]. Frequency based damage identification requires less modal input as natural frequency is easy to measure and require even only one sensor for damage identification [18]. Optimization based damage detection methods couple mostly frequency with optimization algorithms for structural damage assessment [19]. The process of identifying damage is divided into four steps. 1. Identifying the damage 2. Damage localization 3. Calculating the damage percentage 4. Failure forecasting. A detailed discussion on proposed methodologies in the domain of mode shape based and frequency-based damage identification techniques will be provided and discussed.

2.1 METHODS BASED ON MODE SHAPES

These methods use changes in mode shapes as the criteria for damage assessment. Damage can be investigated by MSE derived from the mode shapes of the damaged structure. A higher damage Index value for an element indicates the presence of damage. These methods are characterized into five types as:

- Method of Stub's damage index (SDI)
- Decomposition of Modal strain energy (MSED) method
- Change of Modal Strain Energy (MSEC) method
- Method of Cross modal strain energy (CMSE)
- Methods based on Optimization techniques

2.1.1 Method of Stub's damage index (SDI)

MSE was first used by Stub for structural damage assessment [19]. Damage in all types of structures can be found using the Stub's index (SDI) method, which uses MSE. This technique for forecasting structural health was given the name damage index (DI) method after it was developed [13, 20, 21]. There are many improved forms of Stub's DI method but in this research thesis only the simple form of Stubs DI method is discussed [22]. DI method requires mode shapes from damage and intact structure to localize the damage. Numerous real-world applications, including an offshore platform have been used to numerically apply and experimentally verify this method. To validate this method, experimentally measured modal parameters that were subjected to contamination of noise and incomplete measurement were compared to numerically calculated modal parameters for intact structures [23]. SDI method was applied on I-40 bridge data and damage was localized based on the experimental data obtained from real structure [24-26]. Visual inspection supported the results of SDI method. Earlier it was believed that this method is limited to beam like structures but later this method successfully proved its application to metal & composite plates [27-30] and hollow cylinder [31]. SDI performed better than other damage detection algorithm in its earlier days as this performance was analyzed on an experimental data of I-40 bridge [20, 32]. SDI method is more stable under noisy conditions and results obtained from damage detection are more reliable [13, 33, 34]. SDI method is a better option for structural damage localization but for damage quantification an improved form of SDI method was introduced to localize and quantify the damage [35, 36]. Improved SDI method is a better approach for a regular damage localization and quantification [37]. As improved SDI promises improved damage quantification, but this method is not much successful for small damage under noisy conditions.

2.1.2 Decomposition of Modal strain energy (MSED) method

Considering the limitations of SDI method, another improved form of SDI method is developed which decomposes the stiffness of structure into its axial and transverse components [38, 39]. This method corresponds to axial and transverse stiffness of each nodal coordinate. Two damage indices are introduced for each element which indicates the damage in both DOF. This approach proves to be more elaborative for localizing the damage more accurately. MSED method is applied to offshore jacketed plate form and five story frame structure. Experimental validation with numerical studies proved the credibility of this approach for robust and accurate damage detection in structures [39-41]. Effective damage detection under temperature variation is another success of this technique [42]. Comparative studies of MSED method with SDI method make this method preferable over SDI for structural damage assessment [43]. Overall, MSED methods are found to be efficient and reliable for damage detection and quantification.

2.1.3 Change of Modal Strain Energy (MSEC) method

The formulation used in this method is developed to localize the damage effectively and efficiently. The MSECR was proposed for damage detection in a 2D structure. Change in modal strain energy (MSEC) occurs as a result of damage in a structure, so it is used in MSEC-based methods for damage identification [44]. MSEC uses its sensitivity for damage size determination after its localization [45]. MSECR method is further improved by reducing the effect of truncation and modeling for higher modes for damage quantification [46]. MSEC method is an impressive method for damage detection as a few issues regarding the use of absolute value of MSE, convergence issues and sensitivity related problems are elaborated and discussed [45, 47]. Modified modal strain energy [48] based on an iterative process to detect damage was proposed to overcome the limitations as explained [45]. Further this method is applied to composite sandwich beams and bridge structures [49-51]. One of the MSECR method's drawbacks is its sensitivity to noise and inaccuracy in measuring mode shapes, which lowers the method's accuracy for quantifying the damage [44, 47]. Many damage detection techniques currently in use for damage detection in various applications were developed using MSECR. MSEC-based methodologies are divided into three categories. First order sensitivity formulae were made and used to solve the problem of damage investigation [52, 53]. To remove any differences between experimentally measured modal characteristics and analytically determined MSE, statistically closed form of MSE was also proposed [54]. Since it has been discovered that this method also provides evidence to examine the damage location identification, elemental MSE sensitivity-based methods have attracted the attention of researchers recently [55-57]. Elemental MSE for elements with larger modal displacements is more sensitive to structural stiffness than those with smaller displacements [52]. MSE methods are also useful to investigate the damage at specific locations with selected modes.

2.1.4 Method of Cross modal strain energy (CMSE)

The cross-modal strain energy (CMSE) method was developed to address the shortcomings of the DI and MSEC methods, which are unable to precisely identify the damage when subjected to certain practical limitations. The CMSE method uses measured MSE terms and cross-over analytical terms to quantify damage [37]. Damage localization techniques using MSEC and damage severity estimation techniques using CMSE are developed [58]. Similar work based on the combination of niche GA and CMSE was proposed for structural damage assessment [59]. The direct approach of

CMSE methods for damage quantification requires only a few measured modes of the damaged structure, and no such mode normalization is thought to be necessary for damage detection. This gives CMSE methods some advantages over other methods. In the early stages, this method was limited to only damage severity estimation practices later, a few improved forms of these methods were formulated for damage localization as well [60, 61]. This method was further explored by other researchers extending it for model updating Cross-model cross mode (CMCM) method was proposed for structural model updating [62-64] for damage detection and other domains [65-69].

2.1.5 Methods based on Optimization techniques

Another improved form of MSEC method is coupling MSEC with optimization algorithms. Damage is located by MSEC methods and further quantified by optimization technique comprising of an objective function [70]. With less computational resources, this method aids in localization, identification and quantification of damage in intricate and substantial structures [70]. Frequency and mode-shape criteria were developed using MSECR and GA for damage investigation in a simply supported beam and 2D structure [1, 71]. Another damage localization indicator known as modal strain energy based damage indicator (MSEBI) [72] was proposed for finding the locations of damage and then fused with particle swarm optimization (PSO) algorithm to quantify the damage [73, 74]. Data fusion techniques for multiple modes were developed and validated to increase the effectiveness of localization of damage because, in general, MSECR value increases for damaged elements and their neighbors [75, 76].

2.2 NATURAL FREQUENCY BASED METHODOLOGIES

Frequency based methods are further categorized into two types.

- Iterative and direct methods
- Methods based on optimization techniques

The recent developments on these methods will be discussed which will highlight the importance of these approaches.

2.2.1 Iterative and direct methods

It is simpler to find damage in any structure using frequency-based damage detection methods because frequency measurement in any experimental modal setup is more precise and requires little work [77]. Any damage detection algorithm which uses natural frequency for damage identification will be preferred over other methods. In the beginning, a method based on modal frequency for investigation of damage was proposed, giving a rough estimate of damage [78]. There are many advantages of frequency based damage identification methods over other methods as discussed [79]. An iterative method for damage quantification is formulated which uses natural frequency for this purpose [66, 78, 80], later on this method was extended to damage localization as well [55, 81]. These methods are validated on 1-D beam structures and 2-D planar frames. The extension of this technique to the 3-D offshore platform is another milestone achieved [80, 82]. The benefit of the improved modal strain energy (IMSE) approach is that it can deal with the restrictions of insufficiently measured modes and noise contamination.

2.2.2 Methods based on optimization techniques

A proposed damage detection technique based on Bat algorithm (BA) and PSO algorithm uses modal scale factor (MSF) and change in modal frequency [83]. This method minimizes objective function based on modal frequencies using BA & frequency response function (FRF) algorithm [84]. Location & depth of damage was determined by minimization of objective function based on modal frequency and used PSO algorithm [85]. A methodology for damage investigation was developed using mode shape & modal frequency. The l-1 regularization method makes use of structural sparsity to evaluate the percentage of localized damage [86]. Method was proposed that updated frequency-based finite element models to identify damage in plate and shell structures by using the power spectral density function [87].

2.3 General problems and possible scope of work

This literature review highlights the importance of damage detection algorithms. Almost every structure gets damaged during its service life which can cause a threat to human safety. Considering the importance of SHM in every domain, many techniques are proposed, validated, and applied to different structures. This study develops a damage detection algorithm that precisely locates, measures, and identifies damage to a cantilevered beam. Multistage methodology is proposed which suggests locating the damage in first step using MSECR. Localization of damage reduces solution space of GA. In the second step, GA is applied with reduced solution space is used for calculating true damage percentage. GA uses combined natural frequency and mode shape based objective function which is able to quantify the damage in localized and distributed damage scenarios. The proposed technique is applicable to large scale structures because of considerably reduced computational cost. Moreover, It is also a reliable method for locating damage and estimating its severity while using fewer mode shapes and modal frequencies, which results in reduction of computation time and cost.

CHAPTER 3: METHODOLOGY

The damage detection problem in current study requires an elaborate strategy for damage identification, localization, and quantification at a reduced computational and time cost [88]. The proposed methodology involves establishing the framework for damage identification using mode shapes and modal frequencies [89]. The current study uses an evolutionary optimization algorithm and MSECR in a multi-stage methodology to identify and quantify damage [90]. In the first step, damage is located using MSECR, and the true damage percentage is determined using GA based on combined natural frequency and MAC based objective function in second step.

The proposed damage detection method is tested on a cantilever beam using simulated natural frequencies and mode shapes. The damage detection process is broken down into three steps.

- Damage identification
- Damage localization
- Damage quantification

3.1 DAMAGE IDENTIFICATION

The phase of damage identification includes extracting vibration testing parameters. The structure's mode shapes and modal frequencies are represented by these parameters [91]. To investigate structural damage, modal frequencies & mode shapes of damaged structures as its primary focus of this methodology.

3.1.1 Natural frequency

Every structure has its own geometry and material characteristics whether it’s a bridge, building, aircraft or an automotive. Vibrations in any structure are sometimes desired and sometimes fatal to the lives. Whenever any structure vibrates, there is a certain frequency of its oscillation which is dependent on its geometric and material characteristics. There is a limit of vibration in any structure, above which structure enters in its resonance frequency domain, where higher resonance for a long period may result in fatigue failure to the structure.

There is a certain damping ratio associated with natural frequency as already mentioned, vibrations in a controlled limit are beneficial and exceeding that limit may be harmful to structural health. The collapse of Tacoma bridge in 1940 and aircraft structural failure [92] emphasis on importance of natural frequencies of structure and molding its design for better load carrying and flight dynamics characteristics in both cases. Structural damage and failure are shown in Fig 3.1 and Fig. 3.2.

Figure 3-1: Structural failure of Tacoma bridge in 1940

Figure 3-2: Structural failure of aircraft

3.1.2 Frequency response function (FRF)

Vibrating testing of any structure results in a certain output which requires transformation in frequency domain. FRF is one of the tools used in modal testing to obtain results in frequency domain. With output in the form of its magnitude and phase, a frequency response function (FRF) can be split into its real and imaginary parts. During the design phase of any structure, FRF is performed to obtain natural frequencies of structure. This FRF is analytical in design phase and can be measured either in prototype phase of structure. FRF is basically the response of structure in which every peak shows the natural frequency for structure. The structure is subjected to any external excitation in the form of force, and any response may take the form of displacement, acceleration, velocity, or frequency [91, 93]. FRF of any structure is critical to its sustainability as prediction of right FRF helps to develop structural damage detection approach. Any change in FRF corresponds to change in its stiffness and subsequent life of the structure.

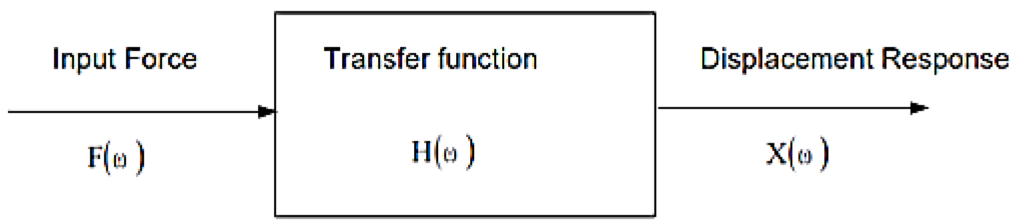

Figure 3-3: Frequency function model

Here F(ω) presents the external excitation force, H(ω) corresponds to transfer function applied and X(ω) is the displacement response.

3.2 MODAL ANALYSIS

First phase of proposed damage detection technique provides the basis for damage localization. Comparison between natural frequencies and modal displacements of damaged and undamaged structures indicates presence of damage in structure.

3.2.1 Mathematical framework for eigenvalue analysis of beam

Any damaged structure experiences changes in its dynamic properties, such as mode shapes, modal frequencies, and damping ratios, which affect the stiffness and flexibility matrices of the structure. A fundamental concept in structural mechanics called Euler Bernoulli's beam equation describes how beams behave when they are bent. Engineers can use the equation to analyze how beams change shape and internal forces in response to external loads. The fundamental assumption of the equation is that when a beam bends, its fibers either elongate or contract. Internal stresses are produced as a result, preventing deformation. The geometry, composition, and loading conditions of the beam are taken into consideration by the Euler-Bernoulli equation. An expression for the dynamic response of a linear, undamped, and n-degree of freedom (DOF) system is:

$$M\ddot{y}(t)+k\mathrm{y}(\mathrm{t})=\mathrm{Y}(\mathrm{t}) \tag{1}$$

where,

y(t)=Deviation from the mean position vector of order (nx1)

Y(t)=The load vector of order (nx1)

M=the mass matrix, K is the stiffness matrix.

Elemental stiffness matrix k of beam with 2 DOF per node is given as:

$$k = \frac{6EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix} \tag{2}$$

where,

E=Young's Modulus of beam

I=Moment of Inertia of beam element

L=Length of the beam element

The following defines the global stiffness matrix K:

$$K = \sum_{i=1}^{m} k_i \tag{3}$$

Elemental mass matrix *m* of beam with 2 DOF per node is given as:

$$m = \frac{\rho AL}{420}\begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix} \tag{4}$$

where,

ρ=Density of beam

A=Cross-sectional area of the beam

L=Length of beam element

The following defines the global mass matrix M:

$$M = \sum_{i=1}^{m} m_i \tag{5}$$

By identifying the natural frequencies and associated mode shapes, eigenvalue analysis is a mathematical method used to investigate the dynamic behavior of structures, such as beams. Eigenvalue analysis in the context of a beam aids in our comprehension of the vibrational behavior of the beam under various loads or disturbances. An overview of the mathematical foundation for a beam's eigenvalue analysis is given below. The associated j_{th} eigenvalue equation is:

$$K\phi_j - \lambda_j M\phi_j = 0 \tag{6}$$

where,

ϕ_j=Eigenvector (mode shapes)

λ_j=Eigenvalue (natural frequencies)

Thus, the stiffness matrix k^d, the i_{th} eigenvalue $\lambda_i{}^d$and the i_{th} mode shape $\phi_i{}^d$ of the damaged system can be expressed as:

$$\phi_i{}^d = \phi_i + \Delta\phi_i \tag{7}$$

$$\lambda_i{}^d = \lambda_i + \Delta\lambda_i \tag{8}$$

$$k^d = k + \sum_{j=1}^{m} \Delta k_j = \sum_{j=1}^{m} \beta_j k_j \tag{9}$$

If the eigenvalue equation is satisfied by damaged structure's finite element (FE)-based frequencies and modes:

$$k^d \phi_i{}^d - \lambda_i{}^d M\phi_i{}^d = 0 \tag{10}$$

where,

K^d=The stiffness matrix of damaged structure

$\lambda_i{}^d$= The i_{th} eigenvalue (Natural frequency) of damaged structure

$\phi_i{}^d$= The i_{th} eigenvector (Mode shape) of damaged structure

β_j=SRFs (j=1, 2, . . ., m) for 'm' elements

Δ=Small perturbation in properties of beam element

3.2.2 Noise effect

The measured modal parameters and the simulated FE parameters diverge because of noise. When using the damage detection algorithm, noise is used to simulate this impact on the structure's dynamic response [94]. Natural frequencies are the fundamental vibrational frequencies at which a structure, such as a beam, naturally tends to vibrate when disturbed. Natural frequency noise is the variation or uncertainty in these frequencies caused by things like material properties, geometric flaws, and measurement errors. The simplified mathematical models used for analysis may not exactly reflect the real-world circumstances in practical applications. As a result, minute differences can cause variations in expected natural frequencies, which is what we refer to as noise.

$$\omega_j^* = \omega_j \left(1 + n\gamma_j\right) \tag{8}$$

where,

ω_j^*= Noise affected natural frequencies

ω_j =Noise free natural frequencies

n=The percentage of Noise

γ_j=The random number with of standard deviation of 1 and mean of 0

A structure's motion patterns at its natural frequencies are represented by mode shapes. Each mode shape represents a particular natural frequency and shows how the various components of the structure move in relation to one another. Noise in mode shapes is a term used to describe uncertainties or irregularities in these patterns that can be caused by things like manufacturing tolerances, material heterogeneities, and measurement constraints. Due to various noise sources, obtaining precise mode shapes in real-world situations can be difficult. To account for these uncertainties and improve the accuracy of the mode shapes obtained, engineers frequently use techniques like experimental modal analysis, finite element analysis, and statistical methods.

$$\varphi_j^* = \varphi_j\left(1 + n\gamma_j\right) \tag{9}$$

where,

φ_j^*= Noise affected mode shapes

φ_j =Noise free mode shapes

n=The percentage of Noise

γ_j=The random number with of standard deviation of 1 and mean of 0

3.3 DAMAGE LOCALIZATION

3.3.1Ratio of change in modal strain energy

Modal strain energy is a structural analysis concept that evaluates the distribution of strain energy within a vibrating structure. It clarifies how various vibration modes contribute to the overall energy of the system. This analysis is especially useful for designing structures that can withstand dynamic loads and vibrations. When a structure vibrates, it undergoes deformation, which necessitates the use of energy. This energy is stored in the material as strain energy. This stored energy is broken down into individual modes of vibration by modal strain energy. Each vibration mode has a distinct pattern of deformation known as a mode shape. The amount of strain energy associated with each mode shape is quantified by modal strain energy. Localization of the damage is performed using the method suggested by Shi et al. [45, 95]. Multiplying the elemental stiffness matrix and its square mode shape yields the MSE [44, 96].

$$MSE_{ij} = \frac{1}{2}\phi_i{}^T k_j \phi_i \tag{10}$$

$$MSE_{ij}{}^d = \frac{1}{2}\phi_i{}^{d^T} k_j \phi_i{}^d \tag{11}$$

where,

MSE_{ij}=Modal strain energy of undamaged structure

ϕ_i=Mode shapes of undamaged structure

k_j=Elemental stiffness matrix of undamaged structure

${MSE_{ij}}^d$= Modal strain energy of damaged structure

${\phi_i}^d$= Mode shapes of damaged structure

The ratio of change in modal strain energy is a technique used in SHM detect and localize damage within a structure. This method takes advantage of the fact that structural damage, such as cracks or defects, frequently causes changes in the distribution of strain energy across different vibration modes. Damage localization has been found to be well-indicated by the MSECR [97].

$$MSECR_{ij} = \frac{{MSE_{ij}}^d - MSE_{ij}}{MSE_{ij}} \tag{12}$$

where,

$MSECR_{ij}$=Ratio of change in modal strain energy

Damage identification indicator is determined by averaging the sum of $MSECR_i$ values for the first 'm' modes and normalizing the result to the highest $MSECR_{i,\,max}$value for each mode [1].

$$MSECR_J = \frac{1}{m}\sum_{i=1}^{m}\frac{MSECR_{ij}}{MSECR_{i,\,max}} \qquad 13$$

where,

$MSECR_J$=Damage localization indicator

$MSECR_{i,\,max}$= Highest $MSECR$ value for each mode

3.4 DAMAGE QUANTIFICATION

3.4.1 Objective function for damage quantification using GA

Structural damage can be of various types i-e an open crack, a breathing crack, a corrosion, a sediment, an internal crack, and a de-lamination. Whatever the type of damage, structural damage results in changes in stiffness and vibrational responses i-e frequencies and mode shapes. GA minimizes these changes based on an objective function to estimate the true damage percentage. Furthermore, experimental work can be carried out in future to validate this technique for the aforesaid different types of damages. Distributed damage results slight changes in mode shapes as compared to undamaged structure, therefore use of MAC based objective function is not feasible [97]. While localized damage results slight changes in natural frequencies as compared to undamaged structure, therefore use of natural frequency based objective function is not feasible [98]. Hence, both localized and distributed damage cases can be assessed by combined natural frequency and MAC based objective function. A combined mode shape and frequency-based objective function is a crucial component of a GA used in the context of SHM and damage quantification. The process of natural selection, in which a population of potential solutions evolves over successive generations toward an optimal solution, served as the inspiration for the optimization technique known as the GA.

The objective function specifies what the optimization process should achieve. The objective function in damage quantification using a GA is made to measure and locate structural damage within a system based on changes in the dynamic behavior of the system, particularly its natural frequencies and mode shapes.

$$max_{\beta} F = max_{\beta} \frac{C_1}{C_2+F_f} + max_{\beta} C.F_{MAC} \quad (14)$$

$$F_f = \sum_{i=1}^{m} \left(\frac{\lambda^d{}_{ei} - \lambda^d{}_{ai}}{\lambda^d{}_{ei}}\right)^2 \quad (15)$$

$$F_{MAC} = \sum_{i=1}^{m} MAC(\phi_e, \phi_a) \quad (16)$$

$$MAC(\phi_{e,}\phi_a) = \frac{\left|\phi_e{}^T \phi_a\right|^2}{(\phi_e{}^T \phi_e)(\phi_a{}^T \phi_a)} \quad (17)$$

where,

$max_{\beta} F$= Fitness of combined objective function

F_f= Frequencies correlation indicator

F_{MAC}= Mode shapes correlation indicator

$MAC(\phi_{e,}\phi_a)$= MAC

$\lambda^d{}_{ei}$ = Damaged structure’s natural frequencies

$\lambda^d{}_{ai}$ = Iterative natural frequencies

ϕ_e= Damaged structure's mode shapes

ϕ_a= Iterative mode shapes

C, C1 and C2 are taken as unity.

Value 1 denotes the perfect correlation among the measured and the analytical mode shape and natural frequency. In the genetic representation of a possible solution, SRFs are frequently encoded. Each SRF identifies a particular part or element of the structure and describes the stiffness loss that would occur if that part were to sustain damage. Through processes like crossover and mutation, SRFs are susceptible to genetic variation. These operations enable the algorithm to explore a variety of potential damage scenarios by allowing the values of SRFs to be changed. After the algorithm converges, it is possible to identify and measure the degree of damage by interpreting the SRFs in the chosen solution. Greater damage severity is correlated with higher values of the SRFs. Accurate damage detection and localization are facilitated by this method, which enables the algorithm to look for solutions that best match observed structural behavior. SRF's (Stiffness reduction factors) take the value between 0 and 1 converted into 15-digit binary number for operations of GA for n number of damaged elements [1].

$$\beta = \{\beta_1, \beta_2. \beta_3, \beta_4, \dots \dots .. \beta_n\}^T \tag{18}$$

where,

β = Solution parameters of GA (SRFs)

3.4.2 Damage detection algorithm

An algorithm for the detection of damage is recommended based on the methodology developed in the earlier sections. Single as well as multiple-damage cases can be solved using the aforesaid algorithm. Natural frequencies and modal displacements will change when structural damage is present. Damage is localized using MSECR. The subsequent step involves applying the GA method to quantify single and multiple damages [12]. GA is based on an objective function with a combined natural frequency and mode shape. An initial population of potential solutions representing various damage configurations within a structure is compiled during the initialization phase of a GA for damage detection. The algorithm iteratively evolves from this population as its starting point.

The goal is to explore a diverse range of solutions to improve the chances of finding optimal or near-optimal damage scenarios. In the context of damage detection, the fitness function assesses how effectively a specific solution represents the extent and location of structural damage. By evaluating fitness using a specialized objective function, the GA progressively identifies solutions that effectively represent the extent and location of structural damage. A selection method used in a GA for damage detection is called tournament selection. It works based on competition, where a group of potential solutions, known as a "tournament," compete to be selected as parents for the following generation. By using this technique, the algorithm is better able to explore various solutions and pick the best ones for further evolution. In conclusion, tournament selection is a technique that encourages diversity and robustness in the choice of a GA for detecting damage. It aids in the discovery of promising damage scenarios while avoiding premature convergence to suboptimal solutions by allowing a variety of solutions to compete. A fundamental genetic operator called two-point crossover is used in GA to solve optimization problems. By fusing the genetic material from two parent solutions, it acts as a mechanism to produce new potential solutions. The procedure involves selecting two random locations along the parents' genetic representation and transferring genetic material between these locations to produce offspring. A key genetic operator used in genetic algorithms is mutation, which introduces random, small changes into the genetic code of each solution. It keeps the population's diversity intact and delays the emergence of suboptimal solutions. A few specific parts of an individual's genetic representation are altered by mutation. The objective function is used to reassess the individual's fitness following the mutation. The altered genetic information is helpful and advances the algorithm if the mutation increases the fitness of the affected individual. The algorithm continues unless and until stopping criteria of evaluation of fitness is achieved. If the stopping criterion of fitness of objective function is achieved, GA is ended, and values of solution parameters of GA expected to represent optimal solution. In this way, true damage percentage of damaged elements of the structure calculated. Figure 3.4 represents a flowchart of GA which is explained above.

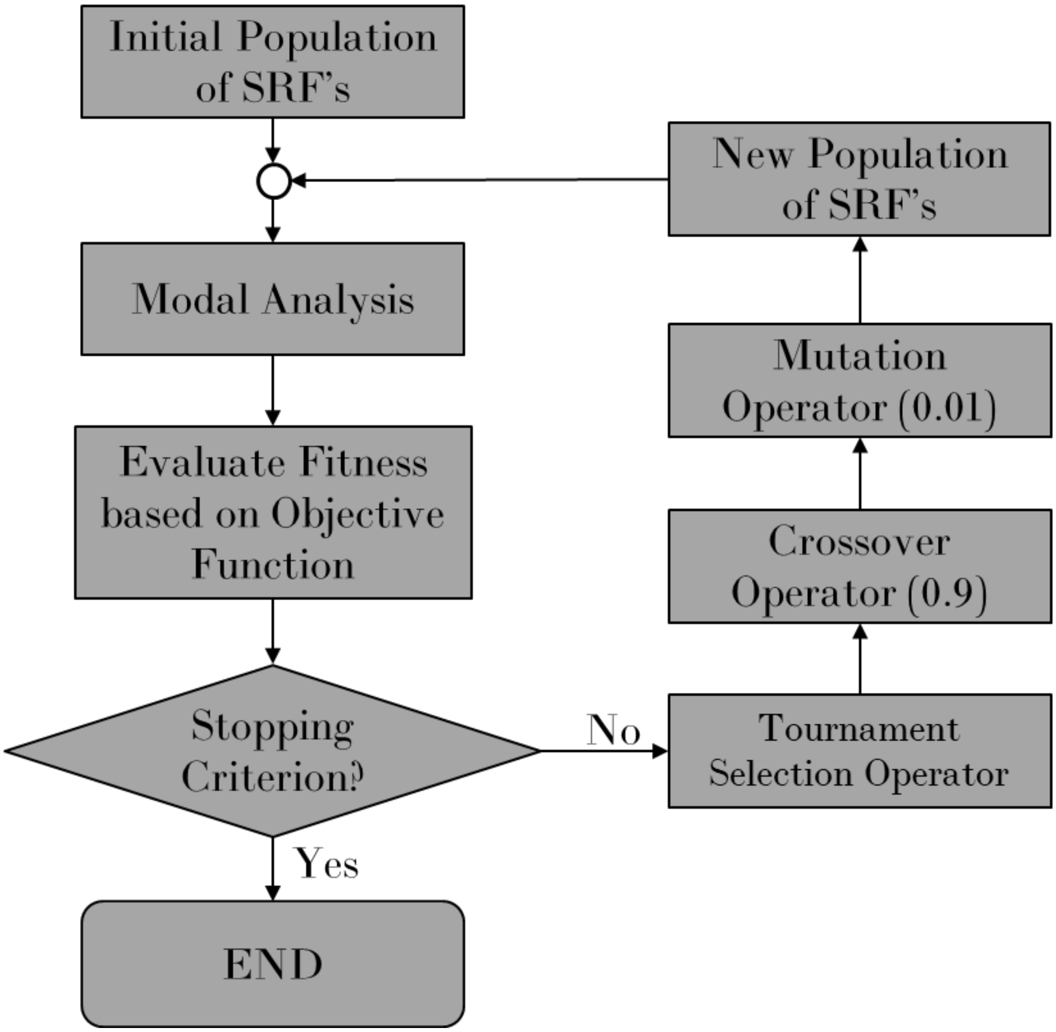

Figure 3-4: Damage detection algorithm for single or multiple damage

CHAPTER 4: RESULTS AND DISCUSSION

4.1 VALIDATION OF ANALYTICAL MODEL

Analytical model is validated with results by Kahya et al. [99]. In this research article, a cantilever beam of mild steel of length 0.9 meters is considered and studied for change in vibrational parameters i-e mode shapes and natural frequencies for different damage scenarios. Transfer matrix method (TMM) was used in aforesaid research article for analytical modeling of the beam. Moreover, experimental analysis was also performed for the validation of TMM method for different damage scenarios of the cantilever beam [99]. He concluded that when damage occurs, vibrational responses of cantilever beam changed significantly [99]. In current study, eigenvalue analysis of Euler's beam theory was performed for the calculation of vibrational responses i-e. mode shapes and natural frequencies. Table 4-1 provides a comparison of the first six natural frequencies for the undamaged case. Comparison of values of natural frequencies of current study with TMM method suggests that matlab coded analytical model is reliable and efficient to simulate cantilever beams.

Table 4-1: Validation of natural frequencies

Sr. no.	**Kahya et al.** [99] **(Hz)**	**Current study (Hz)**
1	10.25	10.25
2	64.24	64.23
3	179.83	179.87
4	352.42	352.59
5	582.60	583.31
6	870.24	872.64

4.2 NUMERICAL SIMULATION FOR CANTILEVERED BEAM

The beam is composed of 12 Euler Bernoulli beam elements and 13 nodes, each with two DOF. Vertical bending and rotation about out of the plane axis are taken as DOFs. Elemental stiffness consists of 4x4 matrix while global stiffness matrix consists of 24x24 matrix after application of boundary condition. Natural frequencies and mode shapes are calculated for intact and damaged structures. As shown in Table 4-2, three damage cases are taken into consideration. In Figure 2, a cantilever beam is shown which is considered for validation of proposed methodology with properties of:

Length=0.5 m

Width=0.05 m

Thickness=0.005 m

Elastic modulus=73.1 GPa

Density=2780 kg/m^3.

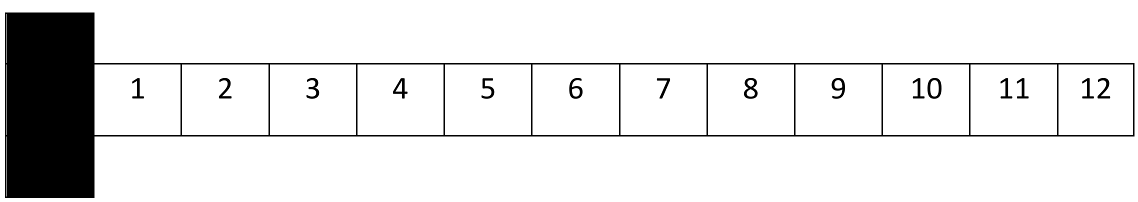

Figure 4-1: FE model of cantilevered beam

Table 4-2: Damage cases under consideration

Damage scenario	**Without noise**			**With noise**		
	Element no	**Damage %**	**Symbol**	**Element no**	**Damage %**	**Symbol**
Undamaged	N/A	N/A	N/A	N/A	N/A	N/A
Single damage	5	11 %	D_{11}^{5}	7	25%	D_{25}^{7}
Multiple damage	6,10	29%, 35%	$D_{29,35}^{6,10}$	4,8	31%, 28%	$D_{31,27}^{4,8}$

4.2.1 Modal analysis

Eigenvalue analysis known as modal analysis is performed to calculate natural frequencies and mode shapes for undamaged, single damaged, and multiple damaged cases using a Matlab code. Eigenvalues represent natural frequencies while eigenvectors represent mode shapes. Matlab code provides flexibility to induce damage in different elements. That's why preferred over FE software's where we would have to simulate beam for each damage case separately. Comparison of modal parameters of undamaged and damaged structures provide basis for identification of damage. Table 4-3 lists the first five natural frequencies for undamaged, single damaged, and multiple damaged cases with consideration of noise and without noise. It is clearly observed from table 4-3 that natural frequencies of damaged cases are less than undamaged case. Such comparison of natural frequencies indicates presence of damage.

Table 4-3: Damage cases under consideration

Mode	**Undamaged**		**Single damaged**		**Multiple damaged**	
	Without noise (Hz)	**With noise (Hz)**	D_{11}^{5} **Without noise (Hz)**	D_{25}^{7} **With noise (Hz)**	$D_{29,35}^{6,10}$ **With out noise (Hz)**	$D_{31,27}^{4,8}$ **With noise (Hz)**
1	16.5671	16.5703	16.4132	16.5393	16.3253	16.1647
2	103.8259	104.2705	102.7224	100.4075	98.9717	100.1339
3	290.7470	290.6671	287.9506	289.0730	276.8041	274.5504
4	569.9444	570.0754	566.3317	557.7398	531.5875	556.1508
5	942.8977	943.2710	933.0236	930.6272	898.4242	910.9128

Figure 4.2-4.4 show the first three mode shapes for undamaged, single damaged and multiple damaged cases. Modal displacements are plotted at y-axis and element number is plotted at x-axis. Comparison of mode shapes of damaged and undamaged cases also indicates presence of damage.

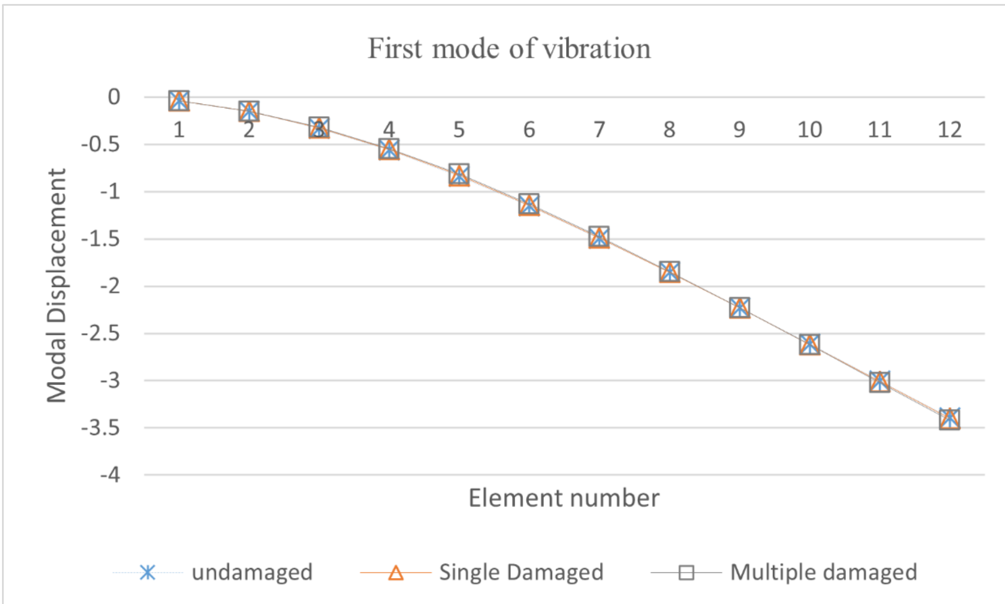

Figure 4-2: First mode shape

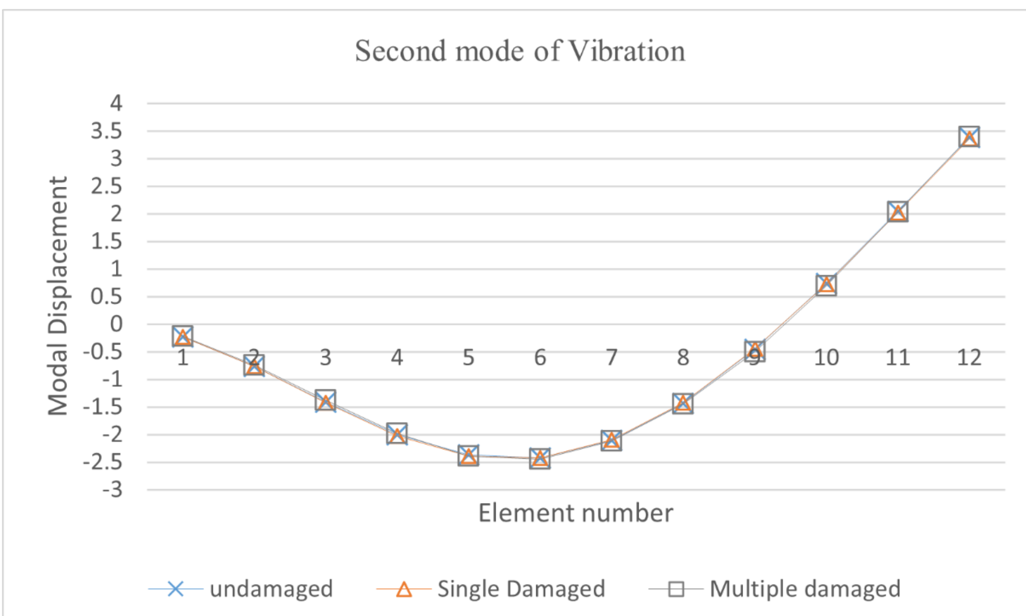

Figure 4-3: Second mode shape

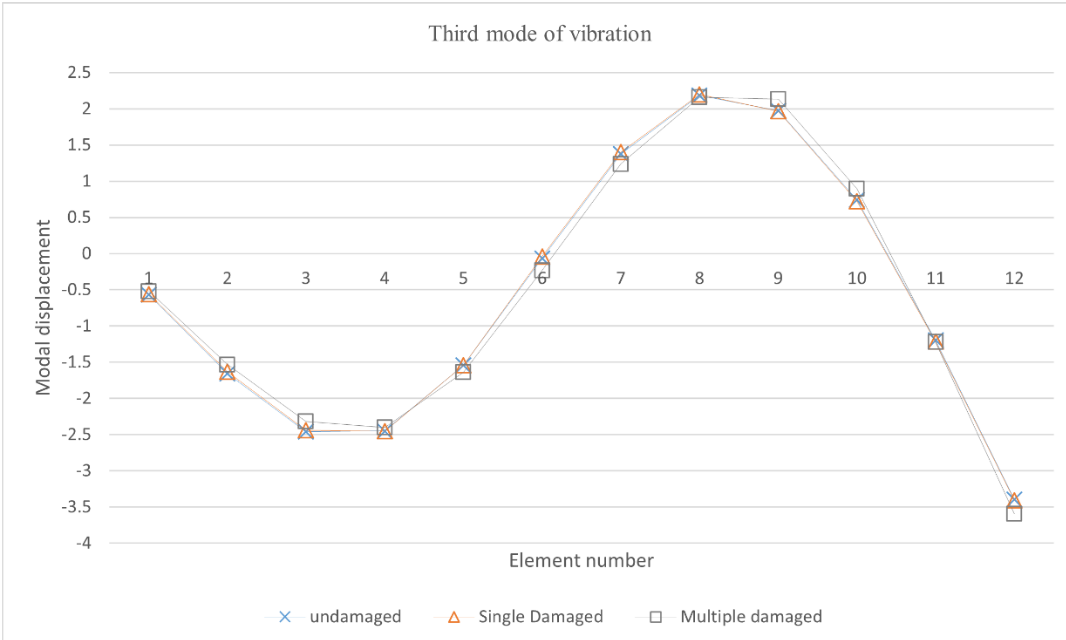

Figure 4-4: Third mode shape

4.2.2 Damage localization using ratio of change in MSE (MSECR)

Damage localization is achieved using damage location indicator MSECR. MSECR is calculated using equation 12. Elements of the beam with values of MSECR would be expected to have loss of stiffness. Purpose of localization of damage is to reduce solution space of GA which ultimately reduces computational cost of operation of GA. MSECR is calculated for each element for each damage scenario and plotted against element numbers. MSECR values are plotted at y-axis while element numbers are shown at x-axis. Higher values of MSECR indicate presence of damage. Indicated elements with higher MSECR would be considered as solution parameters along with their adjacent elements. Adjacent elements to the damaged elements are more likely to have presence of damage. Hence, they are included in solution space of GA and optimized for their true damage percentage. Undamaged elements show lower values of MSECR so, they are excluded from solution space of GA. Damage localization for single damage without noise is presented in Figure 4.5. Element number 5 shows a higher value of MSECR. Therefore, element number 4, 5 and 6 would be included in solution parameters of GA.

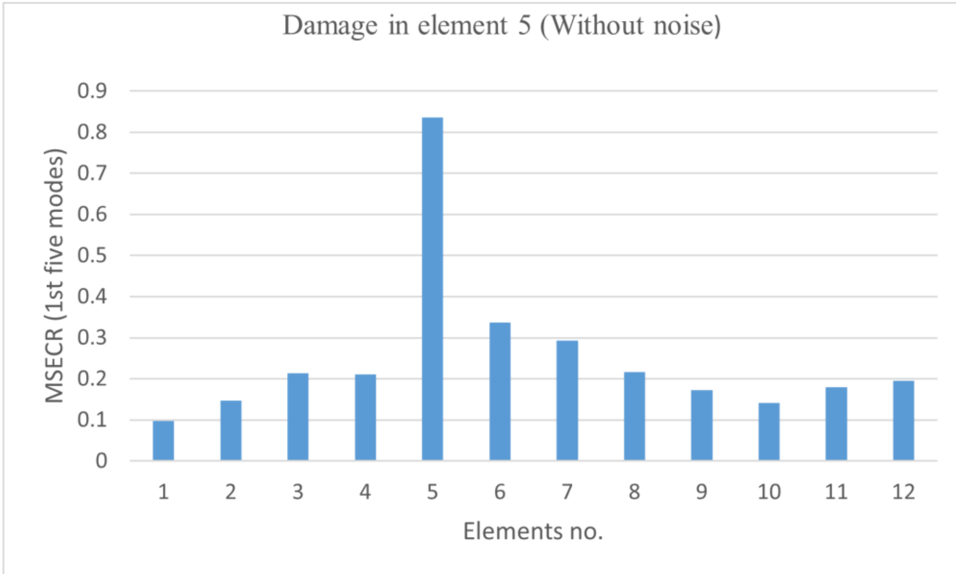

Figure 4-5: MSECR for single damage (Without noise)

Damage localization for multiple damage without noise is presented in Figure 4.6. Element number 6 and 10 shows higher value of MSECR Therefore, element number 5,6,7,9,10 and 11 would be considered as solution parameters of GA.

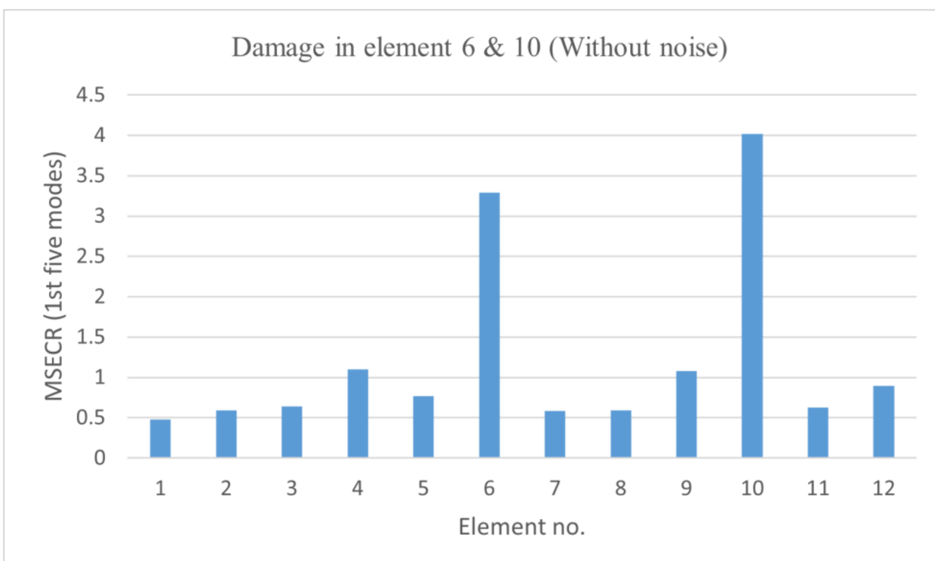

Figure 4-6: MSECR for multiple damage (Without

Damage localization for single damage with noise is presented in Figure 4.7. Element number 7 shows a higher value of MSECR. Therefore, element number 6, 7 and 8 would be considered as solution parameters of GA.

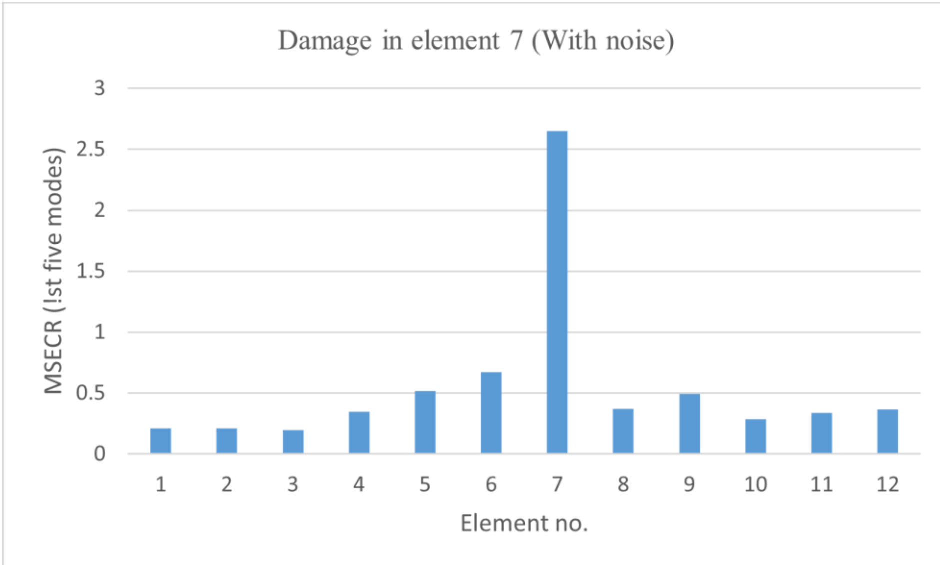

Figure 4-7: MSECR for single damage (With noise)

Damage localization for multiple damage with noise is presented in Figure 4.8. Element number 4 & 8 show higher value of MSECR. Therefore, element number 3, 4, 5, 7, 8 and 9 would be considered as solution parameters of GA.

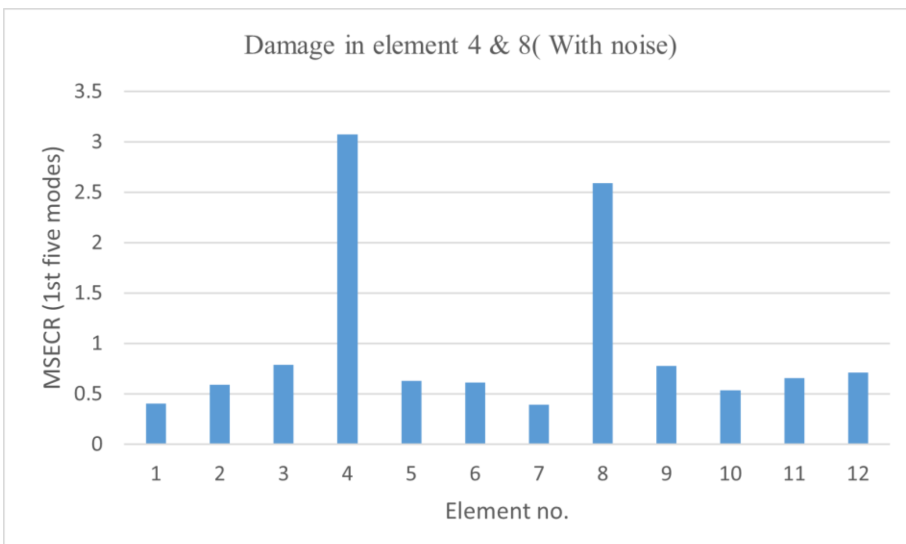

Figure 4-8: MSECR for multiple damage (With noise)

4.2.3 Application of genetic algorithm

The algorithm uses tournament selection criteria with probability of crossover 0.9 and rate of mutation 0.01. GA-based on combined natural frequency and modal assurance criteria is used [1]. Stiffness reduction factors (SRFs) take the value between 0 and 1 converted into 15-digit binary number taken as solution parameters of GA [1]. SRFs value of 1 corresponds to undamaged elements while damaged elements show SRFs value less than 1. SRFs of the elements identified by MSECR and their adjacent elements were taken as solution parameters of GA. Initial guess of SRFs for identified damaged elements is taken randomly, and modal analysis is performed. In the next step, the fitness of objective function is calculated using modal frequencies and mode shapes. If the stopping criterion for fitness is achieved, the process is stopped. But if the stopping criterion is not achieved, algorithm proceeds to GA. In the first step of GA, the initial population is converted to 15-digit binary number and tournament selection operator is applied. The second step of GA applies two-point crossover with probability of 0.9 to the population of SRFs generated from selection operator. Finally, mutation operator is applied that produces new population of SRFs. Values of SRFs and fitness of objective function improves at the end of each generation of GA. Process of GA is stopped when value of normalized fitness becomes equal to 1 which indicates exact correlation between mode shapes & modal frequencies.

4.2.4 Convergence history of GA

Convergence history of GA shows improvement of fitness and no. of generations for single and multiple damage cases and presented in Fig. 4.9-4.12. Fitness of objective function is evaluated at the end of each iteration of GA. GA proceeds until stopping criterion of fitness evaluation is achieved. Normalized fitness is average fitness relative to no. of modes & natural frequency based objective function. Normalized fitness is taken at y-axis and number of generations of GA are shown at x-axis respectively. Single damaged cases took 15 number of iterations of GA to converge. While multiple damaged cases took 30 number of iterations to converge. It shows robustness, rapid convergence and reliability of GA when applied with reduced solution parameters. Such ability of rapid convergence at a reduced computational cost makes this multistage methodology applicable for large scale mechanical and civil structures where solution space would be very large.

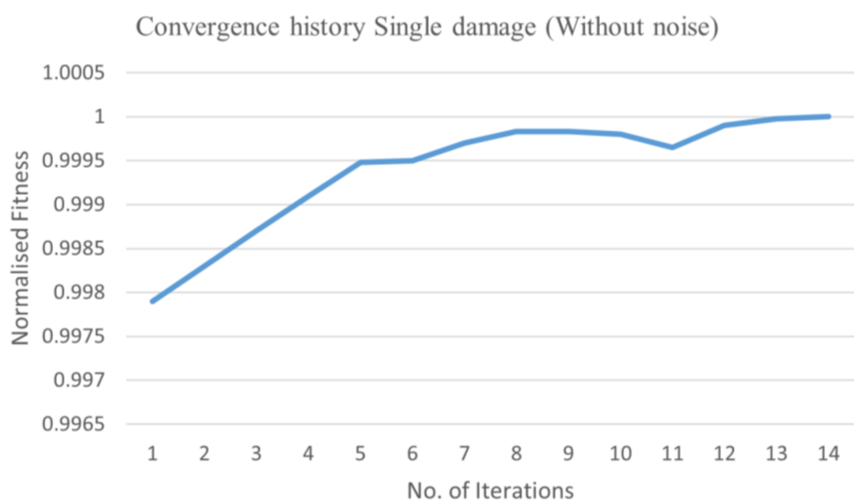

Figure 4-9: History of convergence of GA for single damage (Without

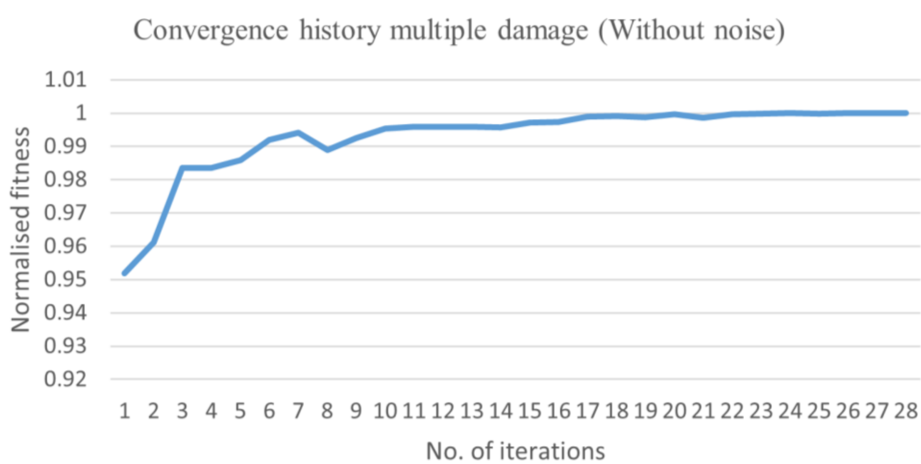

Figure 4-10: History of convergence of GA for multiple damage (Without

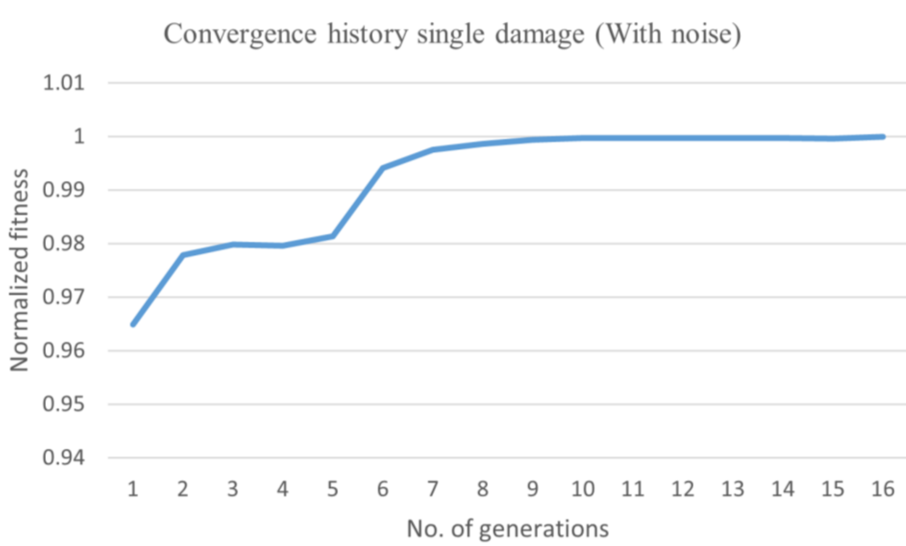

Figure 4-11: History of convergence of GA for single damage (With

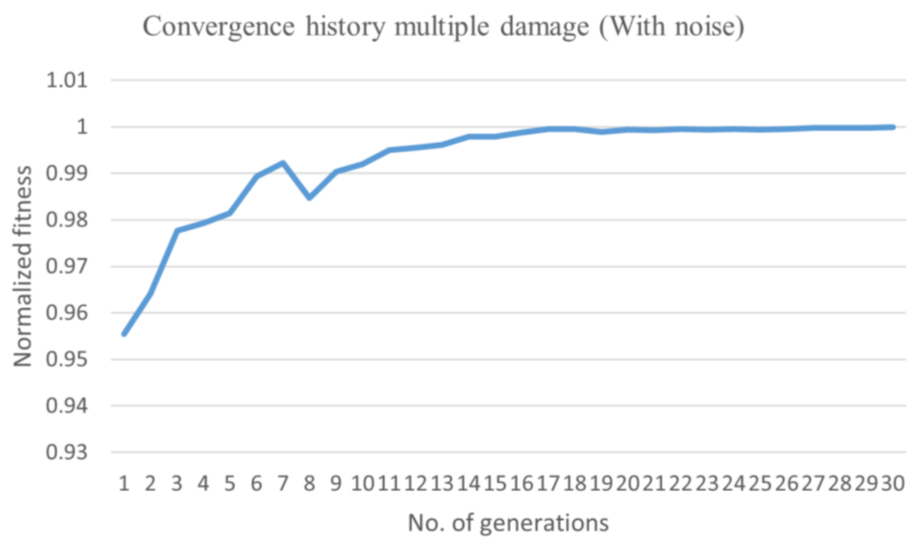

Figure 4-12: History of convergence of GA for multiple damage (With

4.2.5 Evidence of reduction of computational cost

Hao et al. [7] used GA comprising of minimization of a combined objective function based on mode shapes and modal frequencies to investigate damage in cantilever beams. They included all the beam's elements in the solution parameters of the GA because they did not localized damage using MSE. In the meantime, the solution space for GA was reduced in the current study by localizing the damage using MSE. As a result, the computational cost in the current study is significantly decreased due to the reduced solution space of GA. Figures 4.13 and 4.14 compare the convergence history of two techniques.

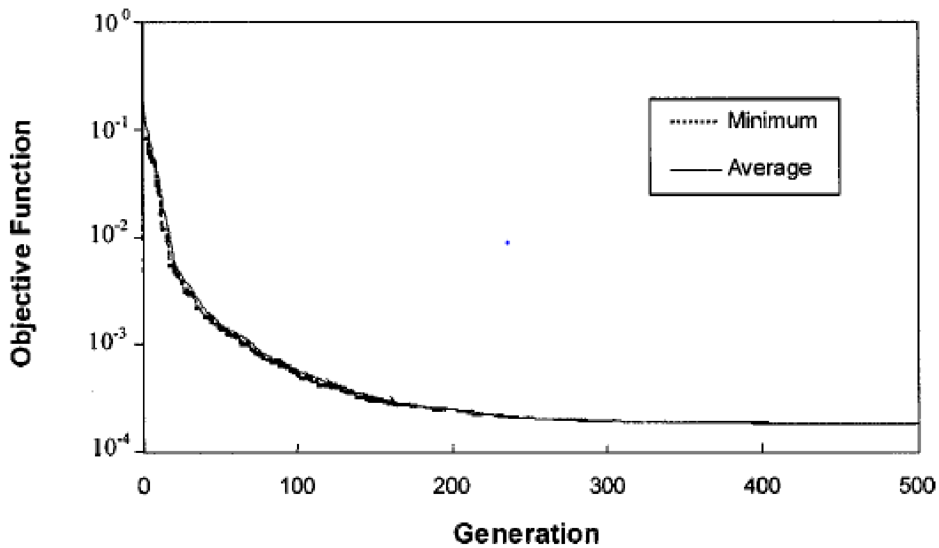

Figure 4-13: Convergence history of GA by Hong Hao

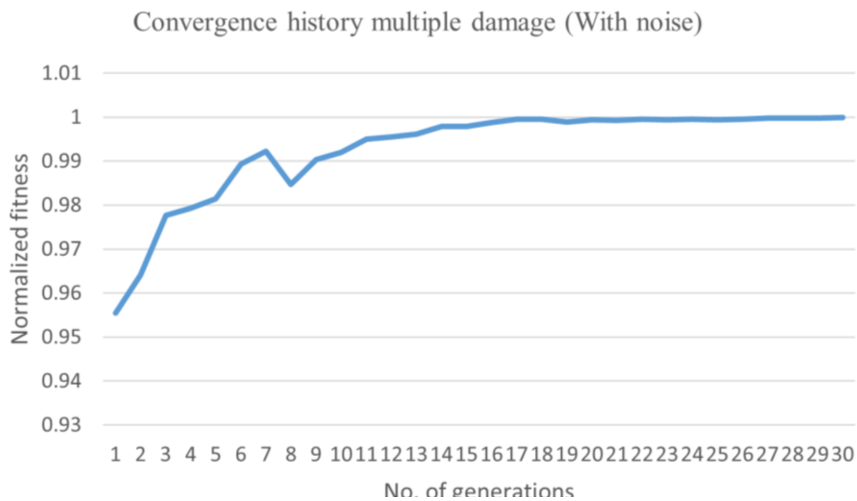

Figure 4-14: Convergence history of GA in current study

4.2.6 Results of damage quantification

Application of GA improves population of SRFs and fitness at the end of each iteration. Fitness is calculated using combined natural frequency and mode shape based objective function using equation 14. If the stopping criteria of fitness value of objective function is not achieved, algorithm shifts to tournament selection, two-point crossover and mutation to generate new population of SRFs. Steps of tournament selection, two-point crossover and mutation improve values of SRFs and fitness of objective function. But if the stopping criterion for fitness value of objective function is achieved, values of SRFs show real damage percentage of damaged elements. When process of GA ends and stopping criterion value of objective function is achieved, it is expected that solution parameters of GA have attained their optimal values. Table 4-4 shows stiffness reduction (SRFs) of elements of cantilever beam. Undamaged elements that excluded from solution space show value of 1 while damaged elements show their real damage percentages. Element number 4, 5 & 6 (without noise) & element number 6,7 & 10 (with noise) are included in solution space of GA for single damage. Element number 5,6,7,9,10 and 11 (without noise) & 3,4,5,7,8 and 9 (with noise) are included in solution space of GA for multiple damage. Results of damage quantification found to be accurate and satisfactory that confirms reliability of methodology for damage detection in various damage cases. Validation of methodology for noise contaminated cases confirms reliability of methodology in real life structural damage applications.

Table 4-4: Stiffness reduction factors for a beam (MAC and frequency objective function)

Element number	Single damage scenario		Multiple damage scenario	
	D_{11}^{5} Without noise	D_{25}^{7} with noise	$D_{29,35}^{6,10}$ Without noise	$D_{31,27}^{4,8}$ With noise
1	1.0	1.0	1.0	1.0
2	1.0	1.0	1.0	1.0
3	1.0	1.0	1.0	0.9687
4	0.9511	1.0	1.0	0.6921
5	0.8906	1.0	0.9217	0.9531
6	0.9432	0.9725	0.7156	1.0
7	1.0	0.7489	0.9481	0.9218
8	1.0	0.9568	1.0	0.7265
9	1.0	1.0	0.9375	0.9589
10	1.0	1.0	0.6543	1.0
11	1.0	1.0	0.9531	1.0
12	1.0	1.0	1.0	1.0

CHAPTER 5: CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

In this thesis, damage detection and damage quantification in cantilevered beam is performed using multistage methodology. In first step, damage is located using ratio of change in MSE which reduces solution space of GA. Secondly, GA with reduced solution space based on natural frequency and MAC objective functions is used to calculate the true damage percentage. As far as novelty of study is concerned, this methodology was previously validated for fixed-fixed beam. In current study, methodology is validated for a cantilever beam for single and multiple damage cases without noise and with consideration of noise. GA proceeds at a reduced computational cost attributed to reduction in solution space using MSECR. This multistage methodology overcomes drawback of GA based damage detection in big scale structures where solution space is very large, and problem of convergence arises. Consideration of noise determines reliability of methodology in real life structural damage applications. GA applied with reduced solution space in a multistage methodology found to be robust and rapidly convergent at a reduced computational cost. Therefore, this technique can be experimentally validated for damage detection in large scale mechanical and civil big scale structures in future.

5.2 FUTURE WORK

As discussed in earlier sections, structural damage detection is subjected to reliability of its assessment techniques. This method provides a better approach for damage detection because of its robustness and reduced computational cost. We will also be performing experimental validation of this technique for cantilever beams. Moreover, this technique can also be validated for large scale mechanical and civil structures like bridges, offshore platforms, submarines, and aircrafts because of its considerable reduced computational cost.

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APPENDICES

MODE SHAPES OF CANTILEVER BEAM

Undamaged case (without noise)

Node	First mode	Second mode	Third mode	Fourth mode	Fifth mode
	mm	mm	mm	mm	mm
2	-0.03984	-0.22513	-0.56883	-0.99259	-1.44393
	-1.87414	-9.98025	-23.5315	-37.418	-48.0212
3	-0.15301	-0.76351	-1.6593	-2.35113	-2.56633
	-3.52051	-15.0594	-25.4814	-20.4255	4.553348
4	-0.33007	-1.4157	-2.45866	-2.32679	-0.97026
	-4.94049	-15.5152	-10.7615	22.60781	64.22398
5	-0.56162	-2.00057	-2.44978	-0.66391	1.68507
	-6.13752	-11.9601	11.54969	51.98855	48.51647
6	-0.83852	-2.36988	-1.54201	1.447192	2.216984
	-7.118	-5.35406	30.63112	42.33954	-25.8945
7	-1.15192	-2.42137	-0.06682	2.401342	-0.00256
	-7.89193	3.074895	37.68251	0.307564	-68.0002
8	-1.49351	-2.10705	1.378711	1.476524	-2.22703
	-8.47355	11.9804	29.15703	-41.4504	-26.153
9	-1.85564	-1.43419	2.184329	-0.57905	-1.71639
	-8.8818	20.08996	7.946885	-50.0285	47.64654
10	-2.23158	-0.45798	1.973219	-2.11027	0.87027
	-9.14063	26.4085	-18.0928	-17.8115	61.42698
11	-2.61569	0.733974	0.738339	-1.80818	2.243869
	-9.27929	30.40431	-39.7963	32.37967	-4.54276
12	-3.00366	2.044199	-1.19655	0.365545	0.397289
	-9.33244	32.15495	-51.1856	67.29342	-77.6217
13	-3.39276	3.392864	-3.39359	3.395867	-3.40093
	-9.3403	32.44107	-53.2706	74.68458	-96.178

Undamaged Case (With Noise)

Node	First mode	Second mode	Third mode	Fourth mode	Fifth mode
	mm	mm	mm	mm	mm
2	-0.06177	-0.22037	-0.58198	-0.99166	-1.43344
	-1.87748	-9.96613	-23.5357	-37.4218	-48.0146
3	-0.14588	-0.76329	-1.64706	-2.36596	-2.54125
	-3.51734	-15.0599	-25.4818	-20.4259	4.563983
4	-0.32593	-1.39869	-2.45284	-2.31718	-0.95869
	-4.94626	-15.5203	-10.7716	22.6252	64.22451
5	-0.56018	-2.0006	-2.44913	-0.66821	1.672186
	-6.15391	-11.9509	11.55569	51.97228	48.51276
6	-0.84612	-2.36838	-1.55563	1.448856	2.209406
	-7.12618	-5.34001	30.6346	42.3433	-25.9001
7	-1.14672	-2.41103	-0.06864	2.399073	0.002992
	-7.89207	3.077811	37.67311	0.296075	-68.0057
8	-1.50506	-2.11483	1.378336	1.496767	-2.23598
	-8.47365	11.98607	29.13806	-41.474	-26.1571
9	-1.86254	-1.44802	2.163049	-0.58415	-1.718
	-8.88847	20.09241	7.935115	-50.0418	47.65064
10	-2.22294	-0.4499	1.963314	-2.11663	0.860744
	-9.1395	26.41063	-18.1045	-17.8084	61.43016
11	-2.6117	0.74277	0.721084	-1.8068	2.244649
	-9.27045	30.4247	-39.7934	32.37256	-4.52952
12	-3.00186	2.053439	-1.21249	0.373315	0.395158
	-9.32693	32.15762	-51.1845	67.29965	-77.623
13	-3.38593	3.399281	-3.38572	3.40234	-3.41264
	-9.3286	32.44533	-53.2706	74.68033	-96.1918

Single damaged case D_{11}^{5} (without noise)

Node	First mode	Second mode	Third mode	Fourth mode	Fifth mode
	mm	mm	mm	mm	mm
2	-0.03906	-0.22529	-0.55761	-0.99013	-1.43299
	-1.83771	-9.99962	-23.1127	-37.3788	-47.8044
3	-0.15004	-0.76605	-1.63391	-2.35313	-2.56429
	-3.45241	-15.1649	-25.3233	-20.7153	3.901325
4	-0.32368	-1.42552	-2.44132	-2.34836	-0.99633
	-4.84542	-15.7683	-11.3683	21.99653	63.90676
5	-0.55209	-2.02196	-2.45569	-0.67488	1.697084
	-6.08046	-12.2397	11.04191	52.90978	50.29375
6	-0.82879	-2.39242	-1.53994	1.475821	2.21771
	-7.16183	-5.08508	31.36781	42.52774	-28.3673
7	-1.1447	-2.42813	-0.04118	2.406332	-0.06851
	-7.96749	3.572204	37.9685	-1.11866	-68.1526
8	-1.48921	-2.09869	1.404051	1.446608	-2.24318
	-8.53871	12.21041	28.91437	-41.6904	-24.2139
9	-1.8539	-1.42144	2.19468	-0.5988	-1.66812
	-8.93974	20.07686	7.526107	-49.4174	48.41018
10	-2.23215	-0.44999	1.967341	-2.10147	0.909992
	-9.19404	26.20383	-18.4181	-17.2003	60.44211
11	-2.61842	0.73063	0.723017	-1.78619	2.237688
	-9.33029	30.07682	-39.9226	32.34755	-5.42017
12	-3.0085	2.025862	-1.21383	0.372516	0.379411
	-9.38253	31.77298	-51.1725	66.6683	-77.3194
13	-3.39968	3.358334	-3.40944	3.372535	-3.39454
	-9.39026	32.05013	-53.2284	73.91992	-95.4881

Single damaged case D_{25}^{7} (with noise)

Node	First mode	Second mode	Third mode	Fourth mode	Fifth mode
	mm	mm	mm	mm	mm
2	-0.03935	-0.21673	-0.57298	-0.95106	-1.45916
	-1.85129	-9.62893	-23.7165	-35.9859	-48.7306
3	-0.15115	-0.73849	-1.67346	-2.27158	-2.61866
	-3.47795	-14.6624	-25.7598	-20.363	3.649025
4	-0.32608	-1.37811	-2.48436	-2.289	-1.04146
	-4.88133	-15.3518	-11.0136	20.67659	64.46074
5	-0.55488	-1.96506	-2.48272	-0.71665	1.662008
	-6.06481	-12.2546	11.48326	50.06361	50.50161
6	-0.82851	-2.35884	-1.5707	1.366734	2.283414
	-7.03461	-6.25097	30.93244	43.3671	-24.1171
7	-1.13873	-2.45689	-0.06635	2.436123	0.040969
	-7.82217	1.740405	38.73702	4.805811	-71.0156
8	-1.48152	-2.15363	1.399928	1.507079	-2.25186
	-8.5912	12.78598	28.28139	-45.2203	-22.4763
9	-1.84887	-1.45176	2.168872	-0.64316	-1.65092
	-9.01393	20.6848	7.018512	-50.7022	48.27915
10	-2.23027	-0.46012	1.938708	-2.16025	0.877629
	-9.27049	26.57903	-18.0851	-16.7624	58.82377
11	-2.61976	0.732798	0.721182	-1.80728	2.166045
	-9.408	30.30657	-39.0197	33.57638	-5.31781
12	-3.01309	2.036029	-1.17149	0.407368	0.3632
	-9.46073	31.9392	-50.0026	68.0627	-74.9382
13	-3.40753	3.375104	-3.31693	3.465528	-3.2933
	-9.46853	32.20596	-52.0127	75.31407	-92.5057

Multiple damaged case $D_{29,35}^{6,10}$ (without noise)

Node	First mode	Second mode	Third mode	Fourth mode	Fifth mode
	mm	mm	mm	mm	mm
2	-0.03863	-0.21554	-0.51869	-0.95935	-1.36298
	-1.8174	-9.58994	-21.5756	-36.6068	-46.0205
3	-0.14839	-0.73671	-1.53168	-2.33528	-2.50797
	-3.41465	-14.6897	-24.0932	-22.4095	1.102068
4	-0.32014	-1.38056	-2.31683	-2.4525	-1.11482
	-4.79306	-15.5425	-11.6472	18.30703	59.57466
5	-0.54482	-1.98029	-2.40032	-0.93662	1.505434
	-5.95588	-12.689	8.077554	50.24931	52.83154
6	-0.8153	-2.38889	-1.64093	1.260447	2.349695
	-6.99008	-6.50289	27.15764	48.2846	-16.3584
7	-1.12942	-2.44884	-0.23099	2.40594	0.193118
	-8.04248	3.886243	37.4705	1.849781	-71.581
8	-1.47762	-2.11221	1.237164	1.613026	-2.20785
	-8.63995	12.25137	30.53995	-37.7037	-30.5787
9	-1.84646	-1.44631	2.15603	-0.33407	-1.89503
	-9.03789	19.51453	12.03025	-49.9097	43.93617
10	-2.22914	-0.49974	2.134361	-2.02608	0.763492
	-9.30723	25.58042	-13.2238	-25.8973	69.03907
11	-2.62179	0.692634	0.898572	-1.8475	2.278614
	-9.51413	31.10346	-44.2285	35.11526	-9.93741
12	-3.01958	2.029915	-1.21955	0.322633	0.397717
	-9.56862	32.77099	-55.5747	65.09757	-74.7975
13	-3.41852	3.403449	-3.59665	3.221395	-3.19961
	-9.5763	33.03139	-57.5687	71.18176	-90.6173

Multiple damaged case $D^{4,8}_{31,27}$ (with noise)

Node	First mode	Second mode	Third mode	Fourth mode	Fifth mode
	mm	mm	mm	mm	mm
2	-0.07063	-0.04676	0.369341	-0.91626	-1.16594
	-1.49004	9.71623	23.21358	-34.7731	-47.5575
3	-0.12012	0.663471	1.85261	-1.90473	-2.56726
	-3.1395	14.57599	26.70723	-20.7245	1.587125
4	-0.33457	1.302394	2.967339	-2.23906	-0.85507
	-5.013	15.40045	13.84777	18.0471	63.12684
5	-0.83158	2.117688	2.498088	-0.6674	1.745269
	-6.41761	11.92322	-14.9403	53.76588	48.42912
6	-0.90554	2.492432	1.485946	1.562406	2.324074
	-7.11306	5.42333	-32.048	41.41734	-23.6973
7	-1.30259	2.692375	0.04401	2.285839	0.080153
	-8.07759	-1.96946	-38.3708	0.623087	-66.5529
8	-1.4288	1.710166	-1.69261	1.334086	-2.24513
	-8.65452	-10.7751	-30.0431	-42.3721	-31.8955
9	-2.09326	1.504734	-1.74572	-0.71976	-1.69374
	-9.17899	-20.7858	-4.41675	-50.9396	53.07493
10	-2.3672	0.452911	-1.86893	-2.107	1.210619
	-9.18281	-26.6825	19.14106	-16.1872	58.97419
11	-2.57818	-0.98529	-0.41579	-2.24195	2.263729
	-9.34183	-30.083	38.15748	34.16239	-6.89408
12	-3.10956	-1.76287	1.202227	0.297665	0.428861
	-9.07752	-32.2283	47.67423	68.32076	-76.3864
13	-3.39189	-3.32463	3.507126	3.66851	-3.55298
	-9.6922	-32.6158	49.56151	75.25976	-93.783

DAMAGE LOCATION INDICATOR (MSECR)

Single damaged case D_{11}^{5} (without noise)

Elements	First Mode	Second Mode	Third Mode	Fourth Mode	Fifth Mode	MSECR Total
1	0.037335	0.003172	0.036365	0.004088	0.015868	0.096827
2	0.036953	0.029094	0.025013	0.026694	0.028721	0.146476
3	0.036484	0.061335	0.095982	0.013816	0.006005	0.213622
4	0.061048	0.008957	0.007074	0.100306	0.033051	0.210437
5	0.212079	0.170538	0.13438	0.202849	0.115549	0.835394
6	0.080538	0.05262	0.0644	0.07505	0.064111	0.336719
7	0.03439	0.057515	0.088882	0.053216	0.05893	0.292933
8	0.033994	0.057443	0.01514	0.079973	0.029457	0.216006
9	0.03365	0.058074	0.007002	0.004375	0.069698	0.172798
10	0.033347	0.058851	0.017385	0.024749	0.006947	0.141278
11	0.03308	0.059553	0.023205	0.032349	0.030743	0.178931
12	0.032851	0.060104	0.026672	0.036063	0.040005	0.195695

Single damaged case D_{25}^{7} (with noise)

Elements	First Mode	Second Mode	Third Mode	Fourth Mode	Fifth Mode	MSECR Total
1	0.024791	0.072123807	0.01438	0.080789	0.015989	0.208073
2	0.024338	0.025929315	0.018877	0.130728	0.007944	0.207816
3	0.023763	0.048557891	0.002386	0.089808	0.031135	0.195649
4	0.023042	0.235231534	0.015648	0.014422	0.056568	0.344911
5	0.022157	0.176972906	0.03797	0.275652	0.002513	0.515265
6	0.041445	0.099065534	0.185901	0.151684	0.191768	0.669864
7	0.74226	0.528215224	0.536317	0.432604	0.409026	2.648422
8	0.095171	0.033817242	0.022501	0.154724	0.066039	0.372252
9	0.017874	0.132538613	0.074725	0.091607	0.17785	0.494594
10	0.016971	0.132395648	0.073661	0.005385	0.057216	0.285628
11	0.016168	0.132820182	0.07416	0.023809	0.091124	0.338081
12	0.01547	0.13330874	0.074772	0.037482	0.103375	0.364407

Multiple damaged case $D_{29,35}^{6,10}$ (without noise)

Elements	First Mode	Second Mode	Third Mode	Fourth Mode	Fifth Mode	MSECR Total
1	0.059276	0.078851	0.162718	0.060426	0.115426	0.476697
2	0.058418	0.004083	0.121305	0.222781	0.183682	0.590269
3	0.057339	0.151614	0.275652	0.100441	0.058368	0.643415
4	0.056015	0.338758	0.215902	0.136823	0.350046	1.097545
5	0.1046	0.127966	0.008972	0.387944	0.135672	0.765155
6	0.856445	0.525053	0.975466	0.247308	0.68278	3.287052
7	0.054467	0.118685	0.28194	0.111964	0.013862	0.580917
8	0.049561	0.19799	0.23453	0.092563	0.015692	0.590336
9	0.097392	0.066559	0.046364	0.401463	0.469	1.080778
10	1.194298	0.881138	1.007846	0.461153	0.473001	4.017436
11	0.050128	0.094304	0.010604	0.263022	0.20758	0.625638
12	0.043826	0.172232	0.086848	0.321807	0.267576	0.892288

Multiple damaged case $D_{31,27}^{4,8}$ (with noise)

Elements	First Mode	Second Mode	Third Mode	Fourth Mode	Fifth Mode	MSECR Total
1	0.131435	0.051528492	0.034884	0.143356	0.041411	0.402615
2	0.130472	0.020144504	0.065346	0.268998	0.106791	0.591751
3	0.069064	0.244757218	0.246329	0.182217	0.047936	0.790302
4	0.831462	0.453574592	0.61452	0.52873	0.64886	3.077146
5	0.038946	0.148756056	0.143565	0.233221	0.065265	0.629753
6	0.125599	0.195176153	0.243019	0.025684	0.024867	0.614345
7	0.000272	0.059826777	0.061139	0.031443	0.240111	0.392792
8	0.645237	0.566792791	0.481994	0.561849	0.33449	2.590363
9	0.07943	0.117339802	0.196984	0.12681	0.255258	0.775822
10	0.121247	0.155128448	0.253923	0.002814	0.002576	0.535688
11	0.120421	0.153577995	0.267831	0.024172	0.092618	0.65862
12	0.119709	0.152736354	0.276911	0.03743	0.12393	0.710716