ELEMENTARY PERSPECTIVE

CROSSKEY

ELEMENTARY PERSPECTIVE

ARRANGED TO MEET THE
REQUIREMENTS OF ARCHITECTS, DRAUGHTSMEN, AND STUDENTS

BY

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L.L.

PREFACE

This book explains the theory of perspective, and demonstrates how the perspective representation of objects is obtained from methods involving only the use of horizontal and vertical planes. The requirements of ordinary elementary examinations are fully considered, and solutions of numerous examples from examination papers are given.

In order to make the work useful to architectural students, the methods employed by architects in preparing perspective views of buildings have received attention. It is presumed that students studying perspective have a knowledge of geometrical drawing (both Plane and Solid), or are studying that subject in conjunction with perspective.

I have to acknowledge the assistance rendered to me in the preparation of this book by Mr. James Thaw of Allan Glen's School, who has taught the subject there with success.

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INTRODUCTORY NOTE.

The object of perspective projection is to suggest to the mind, by means of drawings on a plane surface, the conception of external objects which, in virtue of established associations and unconscious processes of inference, we receive from the objects themselves. The artist in pictorial representation has necessarily this aim before him, and therefore the presence of the subject of perspective in a preparatory curriculum for Art work is of unquestionable utility. Further, it may be urged that in schools where orthographic projection forms a part of the general course of training, the subject of perspective would be taught advantageously, as in a sense supplying a necessary supplement to the solid geometry treatment.

It is hardly necessary to refer to the interest which pupils find in perspective problems, and all teachers of drawing must have experienced pleasure in observing the fascination with which "Perspective" appealed to a class previously disciplined in model drawing, This text-book, which covers the course of study arranged for senior classes in Allan Glen's School, should prove of service in the upper forms of secondary schools and in day and evening Art classes.

If I may be permitted to make a general remark regarding the book, I should say that a successful endeavour has been made to present the subject of perspective in a simple and scientific manner, regard being most particularly paid to the exposition and application of the principles that underlie what are called the "Rules of Perspective".

JOHN G. KERR,
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ALLAN GLEN'S SCHOOL.

CONTENTS.

NOTATION

OBSERVED THROUGHOUT THE VARIOUS DRAWINGS.

Capital letters, as **A B C**, &c., indicate the actual points of an object.

Capital letters, as A_1 B_1 C_1, &c., indicate the perspective representation of the corresponding points **A B C**, &c.

Small letters, as **a b c**, &c., indicate the plans of **A B C**, &c.

Small letters, as a_1 b_1 c_1, &c., indicate the plans of A_1 B_1 C_1, &c.

Small letters, as a' b' c', &c., indicate the elevations of **A B C**, &c.

Small letters, as a'_1 b'_1 c'_1, &c., indicate the elevations of A_1 B_1 C_1, &c.

ABBREVIATIONS USED.

E.	Eye.
E_1.	Position of Eye when rotated into the Picture Plane.
e.	Plan of Eye.
S.P.	Station Point.
P.P.	Picture Plane.
p.p.	Plan of Picture Plane.
G.P.	Ground Plane.
V.P.	Vanishing Point.
v.p.	Plan of Vanishing Point.
V.L.	Vanishing Line.
H.L.	Horizontal Line.
G.L.	Ground Line.
M.P.	Measuring Point.
C.V.	Centre of Vision.
c.v.	Plan of Centre of Vision.
C.V.R.	Central Visual Ray.
P.D.	Point of Distance.
I.L.	Intersecting Line.

ELEMENTARY PERSPECTIVE.

INTRODUCTION.

1. A good idea as to what is meant by the perspective representation of an object is obtained by looking through a window, one eye being closed and the other remaining fixed while an object outside is traced on the glass as it appears to the spectator. The drawing or tracing thus made is the perspective representation of the object.

The appearance of objects is conveyed to the mind by rays of light proceeding from them in straight lines to the retina of each eye. There will thus be two distinct pictures conveyed to the mind, but by an unconscious action of the brain one representation is realized. In working perspective problems the action of one eye only is considered as the result is sufficiently correct for all practical purposes. The perspective representation of the object on the window has been made by indicating the intersection with the glass of the rays of light from the object to the eye.

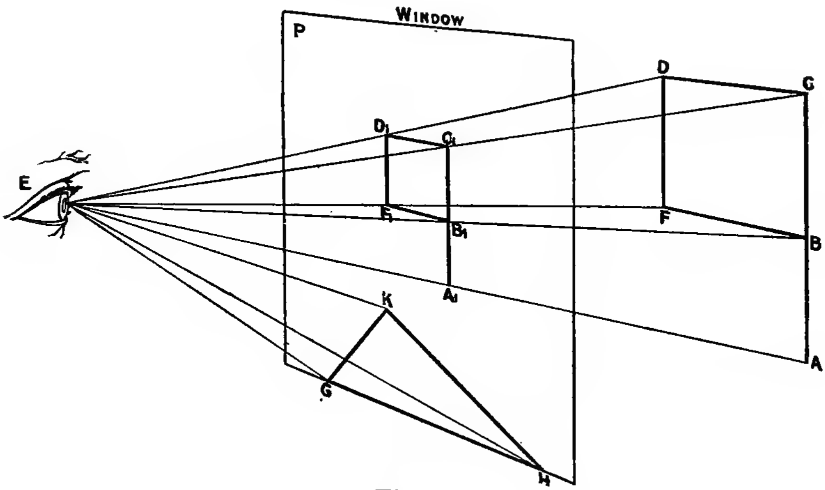


Fig. 1.

In fig. 1 E represents the eye of the spectator, P the window, and A B C D F an object.

$A_1B_1C_1D_1F_1$ is the perspective representation of the object, or the drawing obtained by marking the intersection with the glass of the rays of light from the object to the eye. The rays are represented by the straight lines A E, B E, C E, D E, F E, and these rays intersect the glass at points $A_1B_1C_1D_1F_1$. The rays of light proceed from the whole surface of the object that is visible to the eye; but as the figure is composed of straight lines, it is only necessary to show the rays from the points indicated.

In fig. 1 G H K is a triangle with its three sides on the glass. It will be obvious that the perspective representation of the triangle coincides with the triangle itself; therefore the *perspective representation of all lines and points lying on the glass coincides with the lines and points themselves.*

2. The term *plane* is frequently used in perspective, as the different lines and faces composing an object are considered to lie in planes for the purpose of working the problems. A plane surface is such that the straight line joining any two points which can possibly be assumed on it, lies entirely in the surface (Euc. I. Def. 7). A *plane* is a plane surface of indefinite extent, having only two dimensions, viz., length and breadth. For practical purposes limited portions of planes are usually shown by rectangles.

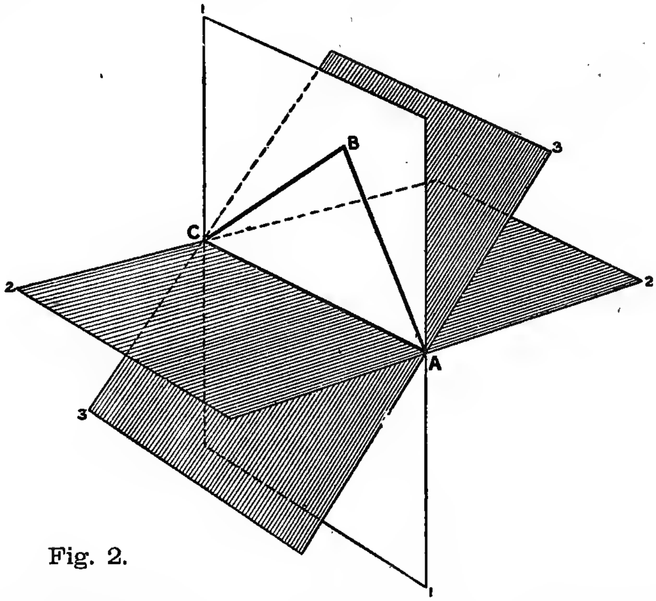


Fig. 2.

In fig. 2 the triangle **A B C** lies in plane 1, or as it is usually expressed, plane 1 contains the triangle **A B C**.

The line **A C** can be contained by an indefinite number of planes.

The line **A C** is shown as being contained by planes 1, 2, and 3.

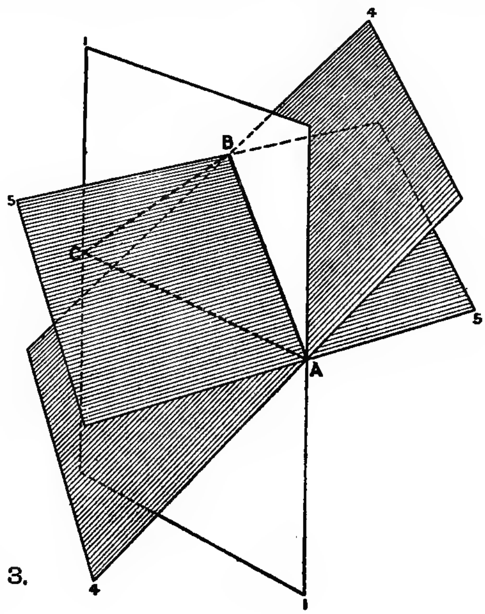


Fig. 3.

In fig. 3 the side **A B** of the triangle is shown as being contained by planes 1, 4, and 5.

From figs. 2 and 3 it will be evident that a straight line can be contained by an indefinite number of planes, and that one of these can always be a vertical one.

A plane figure can only be contained by one plane.

The intersection of two planes is always a straight line and lies in both planes.

The intersection of two vertical planes is always a vertical straight line.

In perspective the plane corresponding to the window is called the *Picture Plane* (**P.P.**), and the plane on which the spectator and objects are placed is called the *Ground Plane* (**G.P.**). The **G.P.** is a horizontal plane and the **P.P.** is generally taken at right angles to it. The line of intersection of the **P.P.** with the **G.P.** is called the *Ground Line* (**G.L.**).

VANISHING POINTS OF LINES.

3. The perspective representation of all straight lines (except those parallel to the **P.P.**) appear to terminate at a point, which is called the *Vanishing Point* (**V.P.**), of the line; the actual vanishing point being on the line produced at an infinite distance from the **P.P.** The perspective view of the vanishing point, generally called the **V.P.** of the line, is the intersection with the **P.P.** of a line drawn through the eye parallel to the line whose vanishing point is required. This line drawn through the eye is called the *Vanishing Parallel* of the original line.

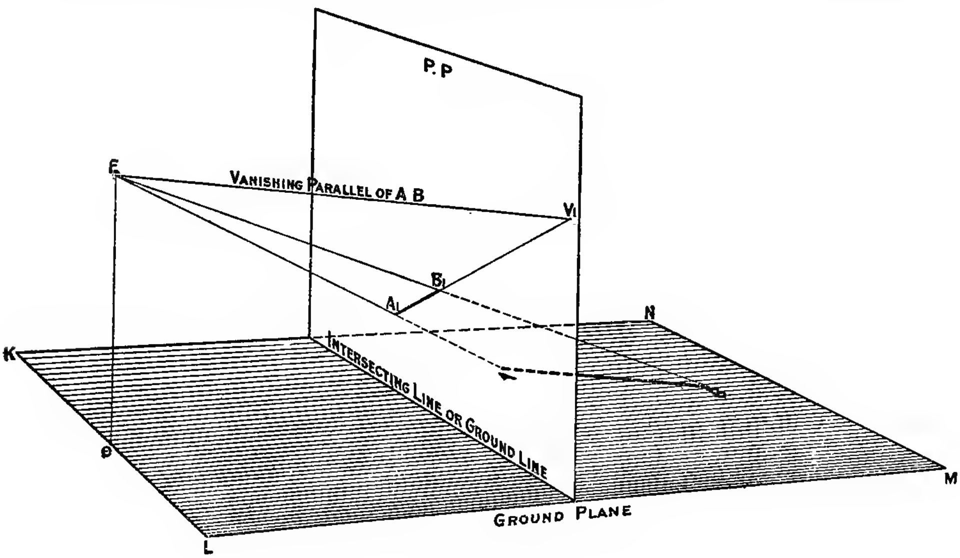


Fig. 4.

In fig. 4, **A B** is a line lying on a plane **K L M N** (in this figure **K L M N** represents the **G.P.**). **P.P.** is the Picture Plane.

A line is drawn from the eye **E**, parallel to **A B** and intersects the **P.P.** at V_1, V_1 is the **V.P.** of **A B**. A_1 and B_1 are the points of intersection with the **P.P.** of the rays of light from **A** and **B** to the eye, and the line A_1B_1 is therefore the perspective representation of **A B**.

It will be found that if A_1B_1 is produced the line will pass through the point V_1, A_1V_1 being the perspective representation of **A B** produced infinitely.

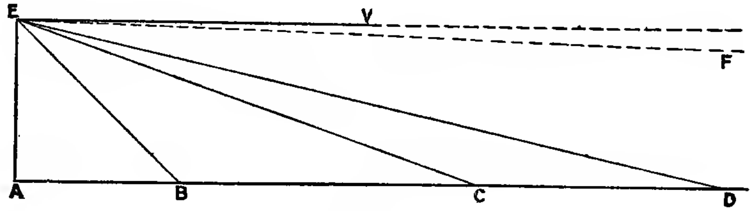


Fig. 5.

Draw any straight line **A D** (fig. 5) and take any point **E** (not in that line). Let **E A** be drawn perpendicular to **A D**. On **A D** take any points **B** and **C** and join **B, C,** and **D** to **E**. It is evident that

E B is nearer parallel to **A D** than **E A**
E C is nearer parallel to **A D** than **E B** or **E A**
E D is nearer parallel to **A D** than **E C, E B**, or **E A**.

If a point very distant from **A** (say 20 miles) is taken in **A D** produced, the line joining that point with **E** would be practically parallel to **A D**.

Therefore it may be concluded that if a point is taken in **A D** infinitely distant from **A**, the line joining that point to **E** will be parallel to **A D**.

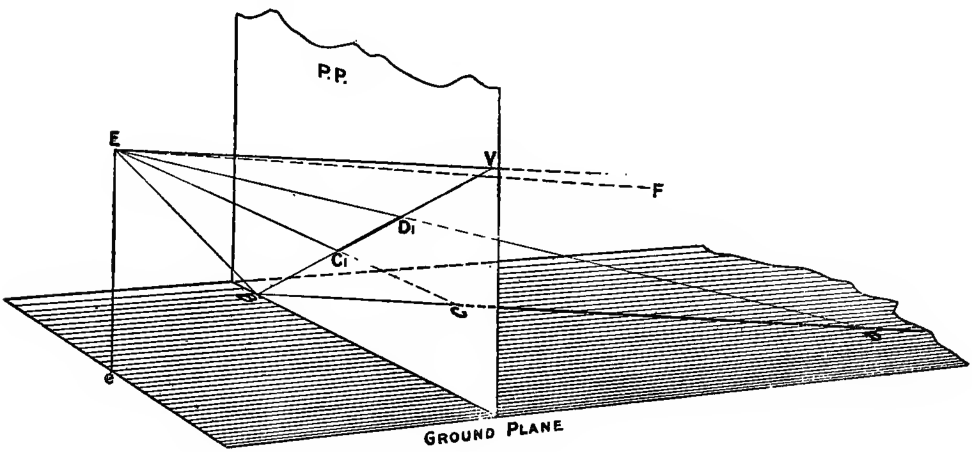


Fig. 6.

In fig. 6, **P.P.** is the picture plane, **E** the eye, **B C D** any straight line lying on the **G.P.** with one end **B** touching the **P.P.**, and **B $C_1$$D_1$** is the perspective representation of this line **B C D**; this representation has been determined by finding the intersection of the rays **C E** and **D E** with the **P.P.** From fig. 5 it is evident that the farther away a point on **B D** is from the **P.P.** the ray to the eye becomes nearer parallel to **B D**, and that we may obtain the ray from a point in **B D** infinitely distant by drawing a ray **E V** parallel to **B D** (the vanishing parallel of **B D**). The intersection of **E V** with the **P.P.** gives the **V.P.** of **B D**, viz., the perspective representation of a point infinitely distant in **B D**.

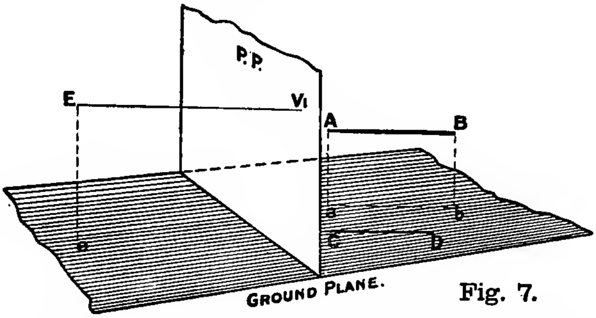


Fig. 7.

4. Parallel Lines, when drawn in Perspective, have the same Vanishing Point.

Let **C D** (fig. 7) be a line on the **G.P.** and **A B** a line parallel to **C D**, but not necessarily on the **G. P.** (In the present case **A B** is in a horizontal plane above the **G.P.**) As previously explained, the **V.P.** of **A B** is obtained by drawing through the eye (**E**) a line **E** V_1 parallel to **A B**. The **V.P.** of **C D** can be obtained in a similar manner. **A B** and **C D** being parallel, and the lines through **E** having been drawn parallel to **A B** and **C D**, they will necessarily coincide, and hence they will intersect the **P.P.** at the same point (V_1).

5. The perspective representation of all Horizontal Lines (except those parallel to the **P.P.**), appear to vanish on a horizontal line drawn on the **P.P.** the height of the eye above the **G.L.** This line is called the *Horizontal Line* (**H.L.**).

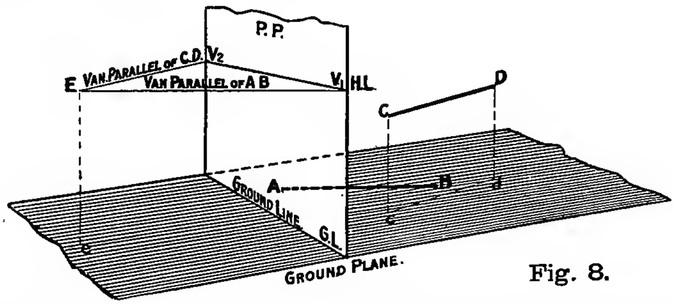


Fig. 8.

Let **A B** and **C D** (fig. 8) be any two horizontal lines (not parallel), **A B** lying on the **G.P.** and **C D** above **c d** on the **G.P.** The vanishing parallels of **A B** and **C D** intersect the **P.P.** at points V_1 and V_2 respectively. As **A B** is on the **G.P.** and **E** V_1 is parallel to **A B** any point on **E** V_1 is the same height above the **G.P.** as **E.** Therefore V_1 is the same height above the **G.L.** as the eye is above the **G.P.** Similarly it can be shown that the height of V_2 (the vanishing point of **C D**) is the same height as **E** above the **G.L.** Now as **A B** and **C D** are any two lines (not parallel) it can be concluded that the vanishing points of all horizontal lines (except those parallel to the **P.P.**) lie on the **P.P.** the height of the eye above the ground.

6. *Horizontal lines parallel to the* **P.P.** have no vanishing points and are represented horizontally in perspective. *Vertical lines* have also no vanishing point and are represented vertically.

It has been shown that the **V.P.** of a line is the intersection of its vanishing parallel with the **P.P.** It is therefore evident that the vanishing parallel of lines parallel to the **P.P.** cannot intersect the **P.P.** They will therefore be represented by parallel horizontal lines, and similarly vertical lines will be represented vertically.

The Vanishing Line of Horizontal Planes.

7. It has been shown that the **H.L.** contains the vanishing points of all horizontal lines (see par. 5). If the vanishing parallels of an infinite number of horizontal lines are drawn, they will form a horizontal plane passing through the eye. It will be evident that this plane will intersect the **P.P.** on the **H.L.**, and that the **H.L.** might have been obtained by finding the intersection with the **P.P.** of a horizontal plane containing the eye. This plane is called the *Vanishing Plane* of all horizontal planes, and its line of intersection with the **P.P.** is called their **V.L.**, *i.e.*, the perspective representation of the line in which the **G.P.** and all horizontal planes seem to vanish when infinitely produced.

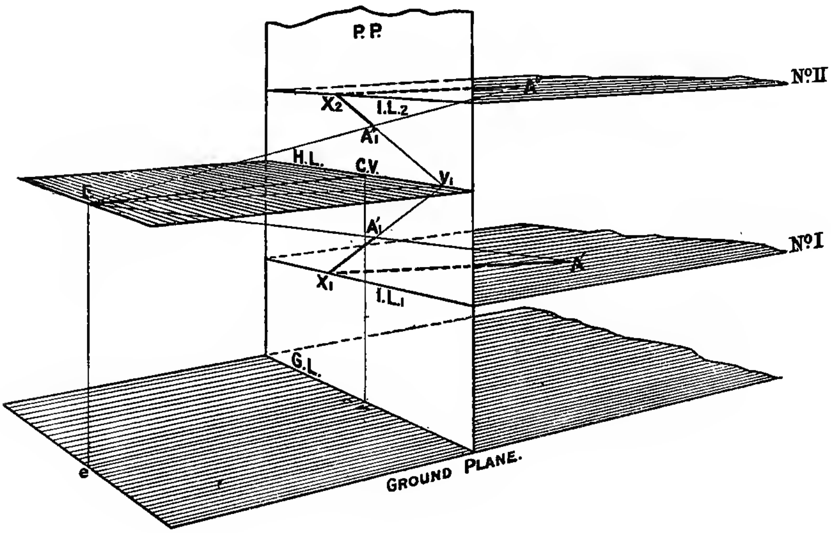


Fig. 9.

The vanishing plane of horizontal planes is represented in fig. 9 by **E** V_1 **H.L.**, which is parallel to the **G.P.** and intersects the **P.P.** in the **H.L.** (the vanishing line of all horizontal planes). In the same figure, two lines, X_1A' and X_2A'' are represented lying on horizontal Planes I. and II. respectively, and each line makes the same angle with the **P.P.**, and they therefore have the same **V.P.**, *i.e.*, V_1. It will be seen that the line X_1A' on Plane I. intersects the **P.P.** on the line of intersection of this plane with the **P.P.**, *viz.*, the Intersecting Line, (**I.L.**$_1$). The intersecting line is sometimes called the *Picture Line* of the plane. The line X_2A'' on Plane II. intersects the **P.P.** at X_2 on the Intersecting Line of Plane II. (**I.L.**$_2$); but note that both the lines vanish at V_1. X_2A'' is above the eye and appears to drop towards V_1, and as X_1A' is below the eye it will appear to rise towards V_1. It should be noted that all horizontal planes have the same Vanishing Line, but that each has a different Picture Line.

THE CONE OF VISION.

8. The rays of light received by the eye when looking in any given direction form a cone, the vertex of which is the eye, and its vertical angle about 60°.[1] The cone is called *Cone of Vision* and its axis is called the *Central Visual Ray* (**C.V.R.**) or *Line of Direction.* The extent of view is called the *Field of Vision.* The central visual ray is the line along which the spectator's sight is most acute, and an object is only seen when it comes within the cone of rays.

In perspective the spectator is invariably supposed to look in a direction perpendicular to the **P.P.**, hence the spectator's **C.V.R.** is perpendicular to the **P.P.** It follows that the intersection of the **P.P.** with the cone of rays is a circle; the centre of this circle is where the **C.V.R.** intersects the **P.P.**, and is termed the *Centre of Vision* (**C.V.**) or *Centre of Field of Vision* (**C.F.V.**).

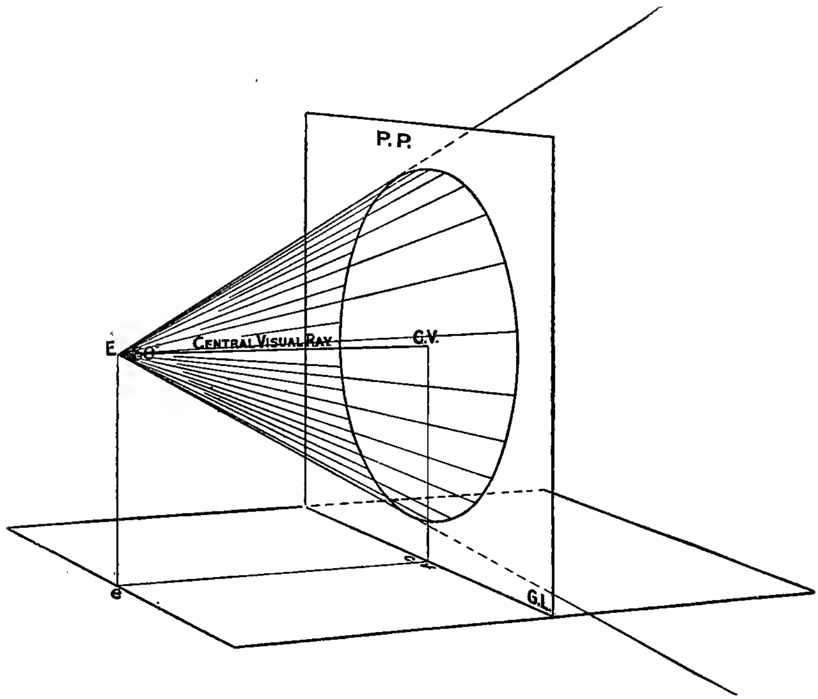


Fig. 10.

Fig. 10 illustrates the cone of rays intersected by the **P.P.**

[1] The size of this angle varies with different people, but by experiment it has been found that 60° is the greatest angle that can be employed in order to distinctly see all the field within the angle.

PLANS AND ELEVATIONS OF OBJECTS, PLANES, &c.

9. The figures hitherto represented have been drawings in perspective of an actual model illustrating the positions of the objects, planes, &c., and the perspective representations have been shown on the **P.P.** as they would have appeared when working by means of that model.

It would be very inconvenient to construct a model for the purpose of giving any required perspective representation; but by the geometrical treatment of the objects, planes, &c., a perspective drawing of an object can be obtained without constructing a model. Fig. 11 is a perspective sketch of a model, and by studying it carefully a method can be devised for drawing an object in perspective.

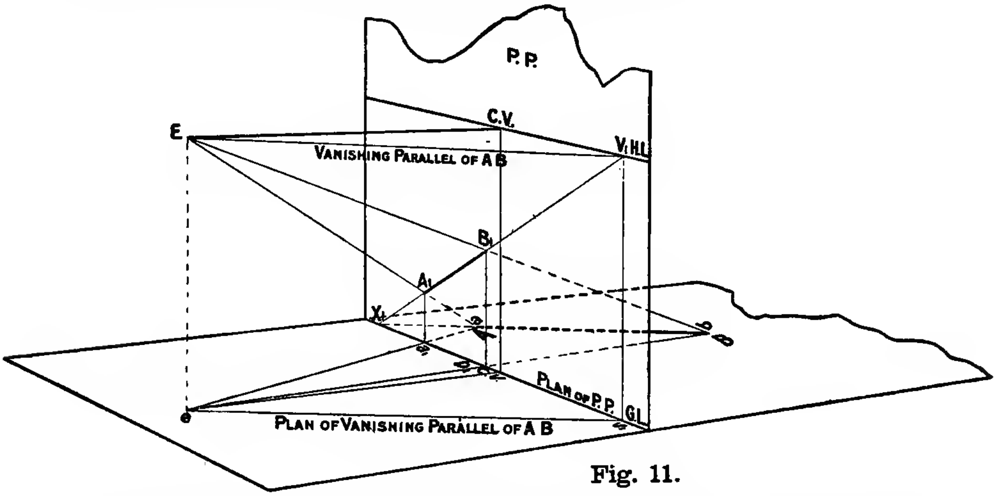


Fig. 11.

10. In fig. 11 **E** is the eye, **P.P.** the picture plane, **H.L.** the horizontal line, **C.V.** the centre of vision, **E** $\mathbf{V_1}$ the vanishing parallel of **A B**, **G.L.** the ground line, and **A B** a line lying on the ground.

The perspective representation of **A B** is $\mathbf{A_1B_1}$, which has been obtained by finding (by the use of the model) the intersection with the **P.P.** of the rays from points **A** and **B** to the eye.

The *Plan of the perspective representation* of a straight line will be first considered:—

In the same figure **a** is the plan of **A** and coincides with **A** since **A** is on the ground, **b** is the plan of **B** and coincides with **B** since **B** is on the ground, **e** is the plan of **E**, $\mathbf{v_1}$ is the plan of $\mathbf{V_1}$, therefore **e** $\mathbf{v_1}$ is the plan of **E** $\mathbf{V_1}$; **c.v.** is the plan of **C.V.**, therefore **e c.v.** is the plan of the central visual ray.

$\mathbf{X_1}$ is the intersection of **A B** with the **P.P.**

It is evident that **e c.v.** will be perpendicular to the **P.P.** and equal in length to **E C.V.**, which is the distance of the eye in front of the **P.P.**; it is also evident that **e** $\mathbf{v_1}$ is parallel to **a b** (which coincides with **A B**).

The plan of any vertical plane is a straight line, and that straight line contains the plan of every point in the plane. This may easily be verified by holding a sheet of paper vertically with an edge on a table and finding the plans of various points marked on the paper.

The plan of the **P.P.** is a straight line coinciding with the line **G.L.**

If the line **G.L.** is considered also as the plan of the **P.P.**, then as the perspective representation of an object necessarily lies on the **P.P.**, the plan of this representation must therefore lie on the line **G.L.**, which, as has been pointed out, coincides with the plan of the **P.P.**

Again, A_1, the perspective representation of **A** lies on the ray **E A**, consequently a_1 (the plan of its perspective representation) lies on the plan of the ray **e a**, therefore the plan of the perspective representation of **A** is at a_1, the intersection of **e a** with the plan of the **P.P.** In a similar way it may be shown that the plan of the perspective view of **B** is at b_1, the point of intersection of **e b** with the plan of the **P.P.**

It will now be evident that *the plan of the perspective representation of any point is the point of intersection with the plan of the* **P.P.** *of the plan of the ray from the point to the eye.*

In every perspective problem data as to the position of the object must be given, because the perspective view varies with the position of the object. When the positions of the **P.P.** and the eye are settled, the **C.V.** becomes a fixed point, and the position of the object is always referred to it.

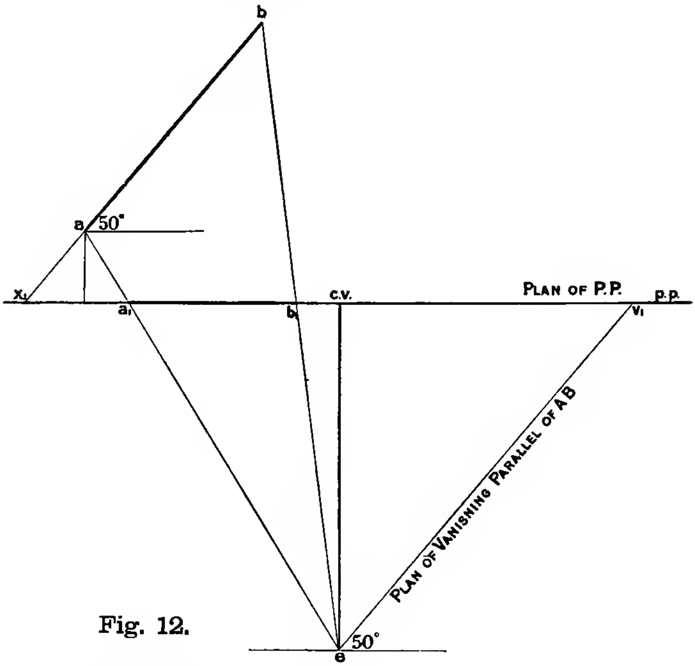


Fig. 12.

Fig. 12 is the plan of fig. 11.

Perspective problems are generally worked to scale; in fig. 12 the scale used is $\frac{1}{2}$ inch to 1 foot.

In drawing this plan the eye has been assumed as situated 3 ft. 9 ins. in front of the **P.P.**; point **A** as 2 ft. 9 ins. on the spectator's left and 9 ins. beyond the **P.P.**; the line **A B** as 3 ft. long lying on the ground and inclined at 50° to the **P.P.** towards the spectator's right. It will be evident from the figure and foregoing remarks that in the plan (fig. 12) **e c.v.** is 3 ft. 9 ins. long and **a** is 2 ft. 9 ins. on left of **c.v.** and 9 ins. measured perpendicularly from **p.p.** **a b** makes 50° with **p.p.** towards the right and is 3 ft. long. x_1 is the plan of the intersection of **A B** with the **p.p.** **e a** and **e b** are the plans of the rays from **A** and **B** to **E** intersecting the plan of the picture plane (**p.p.**) at a_1 and b_1 respectively, hence $a_1 b_1$ is the plan of the perspective representation of **A B** in the above position (corresponding to the similar points in fig. 11.) **e** v_1 is parallel to **a b** and is therefore the plan of the vanishing parallel of **A B.**

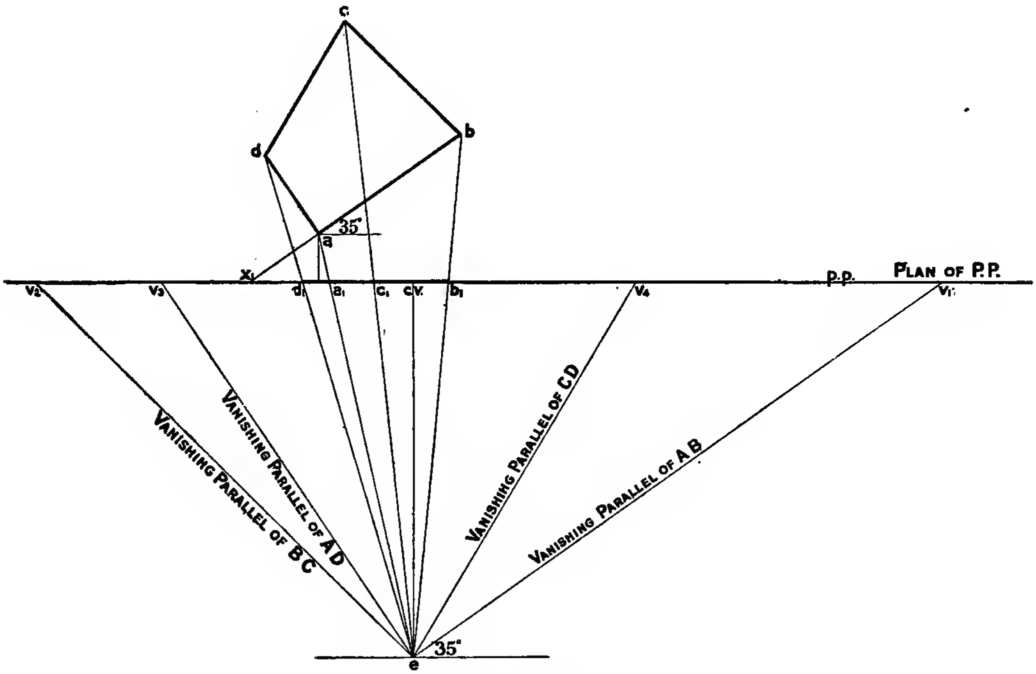


Fig. 13.

In fig. 13 $a_1b_1c_1d_1$ is the plan of a perspective representation of an irregular figure **A B C D** lying on the ground. The plan of each vanishing parallel is shown, from which the plan of each vanishing point ($v_1v_2v_3$ and v_4) has been obtained. Point **A** is 1 ft. on the left and 6 ins. beyond the **P.P.** **A B** recedes from **P.P.** at 35° towards spectator's right, and the distance of eye from **P.P.** is 4 ft.

The scale used is ½ inch to 1 foot.

PLAN METHOD.

11. There are several methods for obtaining the perspective representation of objects. As the plan method is the one now generally adopted in elementary perspective it will be considered first.

The impracticability of using models has already been pointed out; it is therefore necessary that all the construction connected with working out a problem must be contained in one plane, *viz.*, the plane of the paper. For this purpose the lines and points on the **P.P.** in the Plan Method are generally rotated into the **G.P.**, which is the plane represented by the paper.

In figure 14 the dotted lines represent a picture plane in position, and X_1V_1 is the perspective representation on that **P.P.** of a line lying on the ground and intersecting the **P.P.** at X_1. In order to avoid a confusion of lines this line on the ground is not shown, V_1 is the vanishing point of **A B** obtained by drawing its vanishing parallel **E** V_1. e v_1 is the plan of this vanishing parallel, and **c.v.** is the plan of the **C.V.**

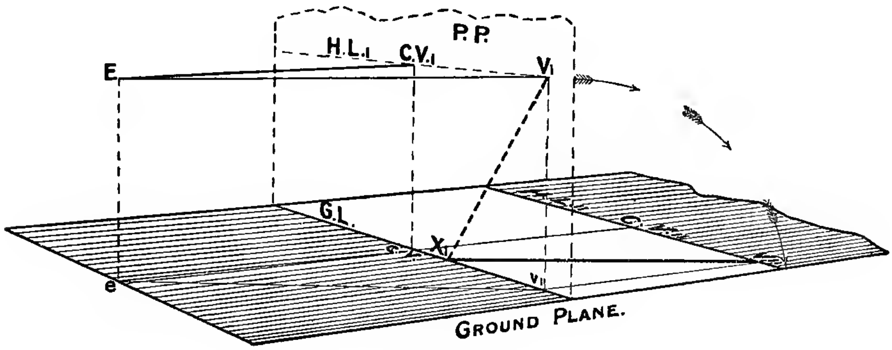


Fig. 14.

Suppose the **P.P.** to be rotated in the direction of the arrows into the **G.P.**, its intersection with the **G.P.** (*viz.* X_1v_1) remaining fixed. The point X_1 would retain its position. The line **H.L.**$_{11}$ marks the new position of the line **H.L.**$_1$, and the points **C.V.**$_{11}$ V_{11} indicate the corresponding position of the points **C.V.**$_1$ and V_1: X_1V_1 would coincide with the line X_1V_{11}, therefore the perspective representation of the required line has been rotated into the **P.P.** at X_1V_{11}.

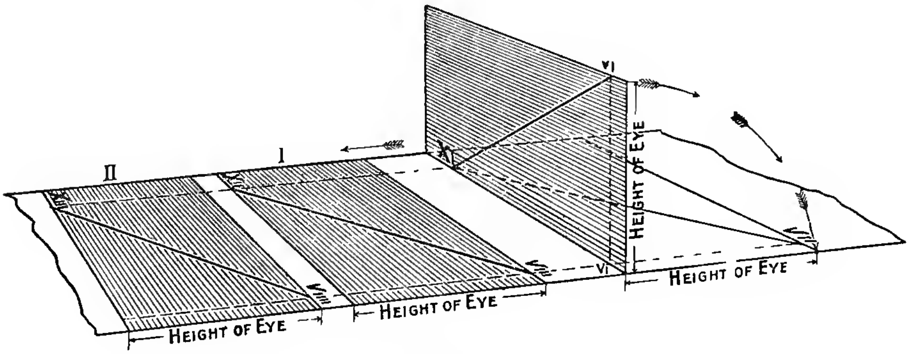


Fig. 15.

In fig. 15 the shaded upright plane, supposed to be made of cardboard, represents the **P.P.** from the **G.L.** to the **H.L.**

X_1V_1 is a line terminated by the two horizontal edges of the cardboard.

If the cardboard is hinged along the lower edge X_1v_1 and rotated into the ground as in fig. 15, the upper edge would be parallel to X_1v_1, and its distance from X_1v_1 will be unchanged. Suppose this cardboard to be brought forward on the ground in the direction of the arrow the long edge X_1v_1 of the cardboard always being kept parallel to its original position (two such positions are shown in the figure at I. and II.).

X_1 and the points which correspond to V_1 in the **G.P.** will always lie in lines at right angles to X_1v_1 irrespective of its distance from X_1v_1, and the long edges will both be parallel to X_1v_1.

Points on the **G.P.** corresponding to X_1 will always be on one of the long edges, and points on the **G.P.** corresponding to V_1 will be on the other, as shown at X_1 and V_{11}, X_{11} and V_{111}, X_{111} and V_{1111}, and if X_{11} and V_{111} or X_{111} and V_{1111} are joined, lines on the **G.P.** corresponding to X_1V_1 on the upright plane are obtained.

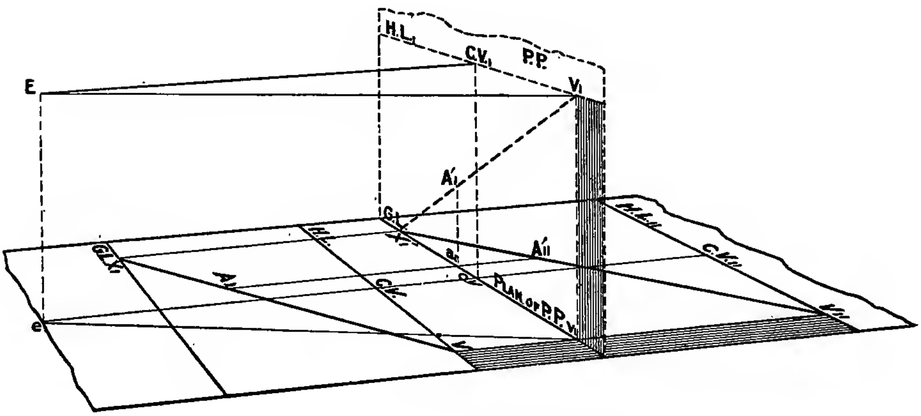


Fig. 16.

Fig. 16 is the application of the two preceding figures. The only additional point is $\mathbf{A}'_1$, which is the perspective representation of a point $\mathbf{A}'$ situated in the line whose perspective representation is $\mathbf{X}'_1\mathbf{V}_1$. To avoid confusion, as in the previous figure the original line is not shown.

$\mathbf{a}_1$ is the plan of $\mathbf{A}'_1$, which is the perspective view of $\mathbf{A}'$. When the **P.P.** has been rotated into the **G.P.**, $\mathbf{A}'_{11}$ will be found by drawing $\mathbf{a}_1\mathbf{A}'_{11}$ perpendicular to the plan of the **P.P.** (**p.p.**) through $\mathbf{a}_1$, intersecting $\mathbf{X}'_1\mathbf{V}_{11}$ at $\mathbf{A}'_{11}$.

If the diagram is brought forward in the same manner as indicated in fig. 15, two parallel lines **G.L.** and **H.L.** will be obtained, the distance between them being equal to the height of the eye above the ground. The **C.V.** will be on the **H.L.**, and $\mathbf{X}_1$ will be on the **G.L.** in the perpendicular to **p.p.** through **c.v.** and $\mathbf{X}'_1$ respectively.

$\mathbf{X}_1\mathbf{V}$ corresponds to $\mathbf{X}'_1\mathbf{V}_1$ on the upright plane. The path of $\mathbf{A}'_{11}$ will be along $\mathbf{A}'_{11}\mathbf{a}_1$ (which is perpendicular to the plan of the **P.P.**) and $\mathbf{A}_1$ will be at the intersection of this line with $\mathbf{X}_1\mathbf{V}$.

If the foregoing figures are thoroughly understood there will be no difficulty in following the working of the various problems which are given on the plan method.

PROBLEMS.

Before any attempt is made to work out the problems that now follow, the notation and abbreviations given on page viii. should be thoroughly understood and the following terms carefully studied and reference made to the paragraphs in the Introduction where they are explained.

	PAR.
Eye (**E.**) or Station Point (**S.P.**) is the point from which the spectator is supposed to view the object, ...	1
Picture Plane (**P.P.**) is an imaginary transparent plane, generally vertical, situated between the eye and the object, and upon which the drawing is supposed to be executed, ...	1, 2
Central Visual Ray (**C.V.R.**) or Line of Direction is the ray through the eye perpendicular to the **P.P.**, ...	8
Centre of Vision (**C.V.**) is the intersection of the **C.V.R.** with the **P.P.**, .	8
Ground Plane (**G.P.**) is a horizontal plane upon which the spectator and objects are placed, ...	1, 2
Ground Line (**G.L.**) is the line of intersection of the **P.P.** with the **G.P.**, .	2
Horizontal Line (**H.L.**) is the perspective representation of the horizon and is shown as a line drawn on the **P.P.** the height of the eye above the **G.L.**	5
Vanishing Point (**V.P.** or **V.**) is the perspective view of that point at which a line would seem to terminate if produced infinitely, ...	3

The principal facts to be remembered are:

(1) The perspective representation of all lines and points lying on the picture plane coincide with the lines and points themselves.

(2) All horizontal straight lines, except those parallel to the picture plane, will appear to vanish on the horizontal line.

(3) Parallel lines appear to have the same vanishing points.

(4) Horizontal lines parallel to the picture plane have no vanishing points and are represented horizontally in perspective.

Vertical lines have also no vanishing points and are represented vertically.

(5) The vanishing line of all horizontal planes is the horizontal line.

(6) The centre of vision is the vanishing point of all lines perpendicular to the picture plane.

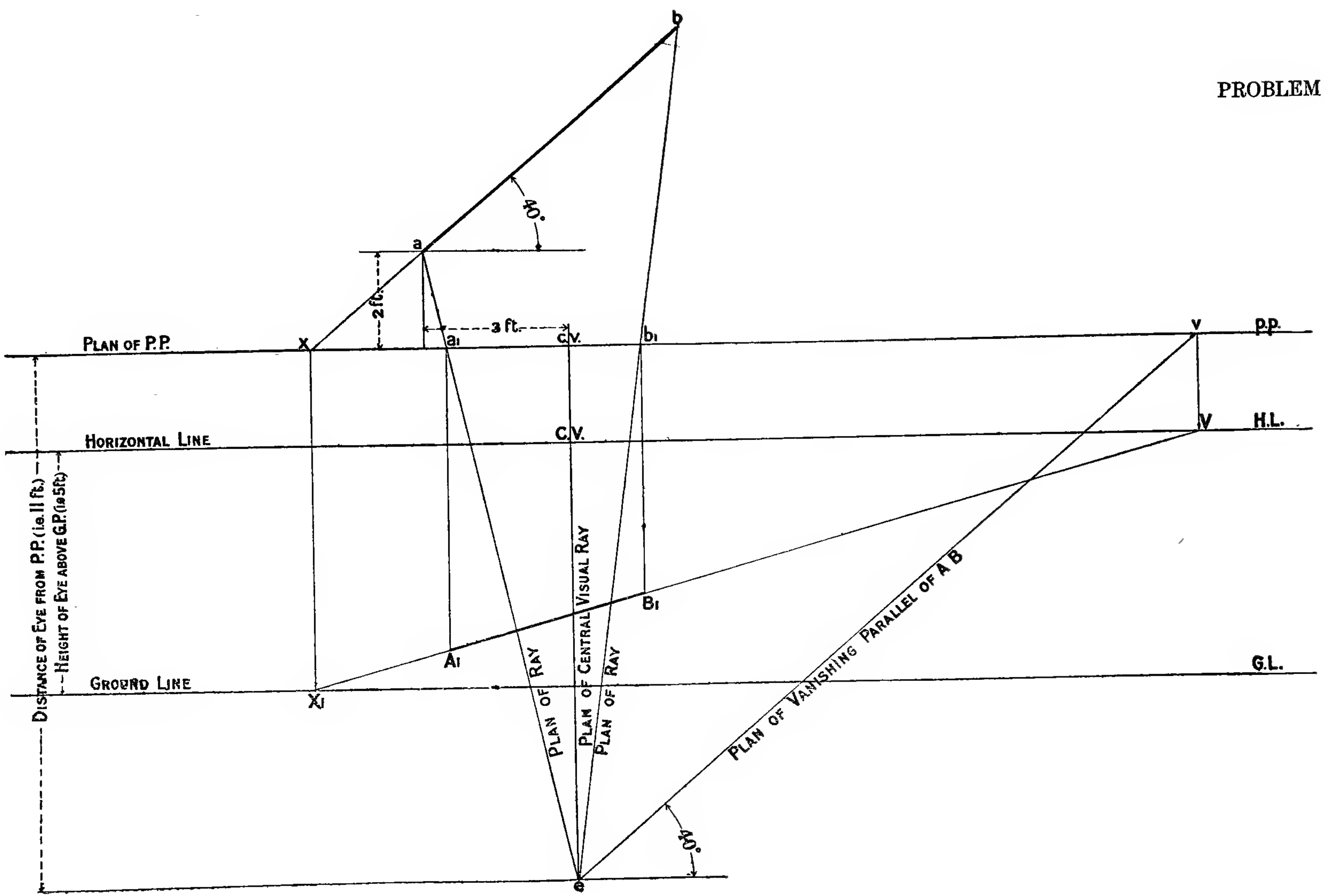
(7) In working problems by the Plan Method the **P.P.** is considered to be rotated into the **G.P.**, which is the plane represented by the paper (par. 11 of Introduction).

A difficulty may be found at first in following the references made in the text to the actual points and lines that have to be placed in perspective, when they do not appear in the figure that accompanies the explanatory matter of the problem.

For example, in Problem I. the line **A B** is referred to; this line is not drawn in the figure, but is represented by its plan **a b** (on referring to page viii., it will be seen that capital letters are used to express the actual points and lines to be placed in perspective, and the plans of these points and lines are expressed by corresponding small letters). The line **a b** is not the line that is being placed in perspective, but it is the plan of the line **A B** (which is not shown), **a b** being used as a means for obtaining the perspective representation of the actual line **A B**. It should therefore be thoroughly realized that the points in the plan (which are expressed by small letters) are not those which are placed in perspective, and that the actual points whose perspective views are required may not always be shown in the figure, but can be referred to by using capital letters corresponding to the small letters in the plan.

The problems are worked to various scales, but students are recommended to work to a scale of not less than $\frac{1}{2}$ in. to 1 ft.

PROBLEM I.



b
40°
a
2 ft.
3 ft.
X
a1
c.v.
b1
v
P.P.
PLAN OF P.P.
C.V.
V
H.L.
HORIZONTAL LINE
B1
A1
G.L.
GROUND LINE
X1
PLAN OF RAY
PLAN OF CENTRAL VISUAL RAY
PLAN OF RAY
PLAN OF VANISHING PARALLEL OF A B
40°
e
DISTANCE OF EYE FROM P.P. (i.e. 11 ft.)
HEIGHT OF EYE ABOVE G.P. (i.e. 5 ft.)

LINES LYING ON THE GROUND PLANE.

(i) Lines at any angle to the **P.P.** when the **V.P.** is accessible.
(ii) Lines at right angles to the **P.P.**
(iii) Lines parallel to the **P.P.**
(iv) Lines at any angle to the **P.P.** when the **V.P.** is inaccessible.

In the following problems, I. II. III. IV. and V., the eye is 11 ft. from the **P.P.** and 5 ft. above the **G.P.**

(i) PROBLEM I.

A right line **A B**, 7 ft. in length, lies upon the ground plane and is inclined at 40° to the **P.P.** towards the spectator's right. The nearer end of the line (**A**) is 3 ft. on the left of the spectator and 2 ft. beyond the ground line. Draw this line in perspective.

Draw a horizontal line and consider it to represent the plan of the **P.P.** and on this line fix the plan of the **C.V.** in any convenient position; letter this point **c.v.** Now draw the plans of the eye and the central visual ray. As the eye is 11 feet distant from the **P.P.** and the central visual ray is horizontal, therefore the plan of the **C.V.R.** will be 11 ft. long.

Through **c.v.** draw a line at right angles to the plan of **P.P.** to represent the plan of the central visual ray and on this line obtain a point **e**, 11 ft. from **c.v.**; this point represents the plan of the eye.

It is to be remembered that the paper on which the drawing is made corresponds to the **G.P.** Draw the **H.L.** any convenient distance below and parallel to the plan of the **P.P.**

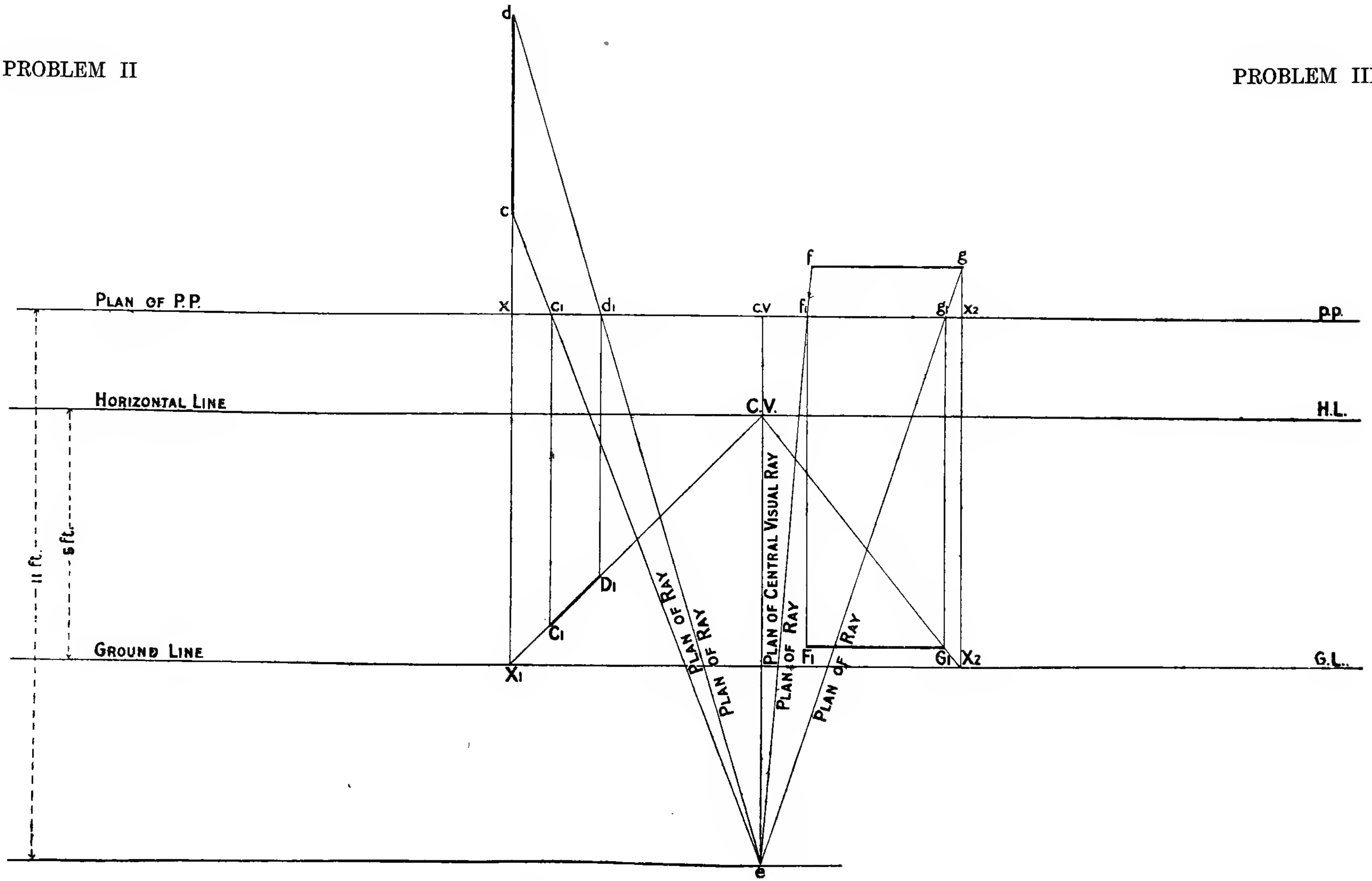
The **H.L.** can be drawn to coincide with the plan of the **P.P.** (**p.p.**), but although a little time and space is sometimes saved by doing so, it is advisable to use two lines, as beginners generally find it difficult to realize that the same line can be considered to represent both the **H.L.** and **p.p.** It also happens that, when objects have to be drawn in perspective that are higher than the height of the eye above the **G.P.**, the construction lines of the problem are apt to interfere with the plan when **H.L.** and **p.p.** are taken coincident. The **G.L.** must now be represented on the paper (*i.e.* the ground plane). Referring to paragraph 11 and fig. 16 of the Introduction, it will be seen how the **P.P.** is rotated into the **G.P.**

As the eye is 5 ft. above the **G.P.**, draw a line 5 ft. below and parallel to the **H.L.**; this line represents the **G.L.** Now draw the plan of **A B** in its proper position with regard to the **c.v.** To obtain this find a point (**a**) 3 ft. to the left of the **c.v.** and 2 ft. from **p.p.** From **a** draw a line **a b**, 7 ft. long, making an angle of 40° with the **p.p.** towards the right. It will be evident that the line **a b** is the plan of the line in the required position. From the plan of the eye, **e**, draw the plan of the vanishing parallel of **A B**, *i.e.*, a line **e v** parallel to **a b** cutting **p.p.** at **v**. Produce **b a** to intersect **p.p.** at **x**, and from **x** draw a projector, cutting **G.L.** at X_1. (In the problems throughout this book the word *projector* is used to denote a line perpendicular to the **G.L.** As a rule a projector has been formed by sliding the **P.P.** on the **G.P.**, keeping the **G.L.** parallel to **P.P.** as shown at fig. 16 in the Introduction.) From **v** draw a projector intersecting **H.L.** at **V**. **V** is the **V.P.** of **A B**. Join X_1 to **V**. X_1V contains the perspective representation of **A B**.

Now draw the plan of the rays from **A** and **B** to the eye by joining **a e** and **b e** which intersect **p.p.** at a_1 and b_1 respectively. a_1b_1 is the plan of the perspective representation of **A B**. From a_1 and b_1 draw projectors which intersect X_1V at A_1 and B_1 respectively. A_1B_1 is the required line.

PROBLEM II

PROBLEM III.



(ii) PROBLEM II.

A line **C D**, 4 ft. in length, lies upon the **G.P.** and is perpendicular to the **P.P.** The nearer end **C** of the line is 5 ft. on the left of spectator and 2 ft. from the **G.L.** Draw this line in perspective.

Obtain the positions of the **p.p.**, **c.v.**, **e**, **H.L.**, **C.V.**, and **G.L.** in a similar way to that shown in Problem I. In the problems that follow it will be assumed that these lines and points are placed in their required positions without any further explanation. As stated before, care must be taken when measuring the distance of the plan of the eye (**e**) from the plan of the centre of vision (**c.v.**) that it is measured from **c.v.** (*not from* **C.V.**).

Obtain a point **c**, 5 ft. on the left of **c.v.**, 2 ft. from **p.p.**, then from **c** draw a line **c d** 4 ft. long at right angles to **p.p.** This gives the position of the plan of **C.D.**

Draw the plan of the rays from **C** and **D** to **E** by joining **c e** and **d e** intersecting **p.p.** at c_1 and d_1 respectively. As **C D** is perpendicular to **G.L.** the plan of its vanishing parallel will coincide with the plan of the **C.V.R.**, *viz.*, **e c.v.** and the plan of its **V.P.** therefore coincides with the plan of the **C.V.** (**c.v.**). The projector from **c.v.** coincides with the **c.v. e**; this projector cuts **H.L.** at **C.V.**, so that **C.V.** is the **V.P.** of **C D**. Produce **d c** to cut **p.p.** at **x**. Obtain the perspective representation of **x** at X_1 by drawing the projector $x X_1$, which cuts **G.L.** at X_1. Join X_1 to **C.V.** This line X_1 **C.V.** is the perspective representation of a line from **X** (the plan of which is **x**) perpendicular to the **P.P.** and produced infinitely. The perspective representation of **C** and **D** must lie in this line, and is found by drawing projectors through c_1 and d_1 to cut X_1 **C.V.** at C_1 and D_1 respectively. $C_1 D_1$ is the required perspective representation of **C D**.

When a line is perpendicular to the **G.L.** and exactly opposite the eye, it cannot be drawn in perspective by the above method for reasons that will be explained in Problem IV.

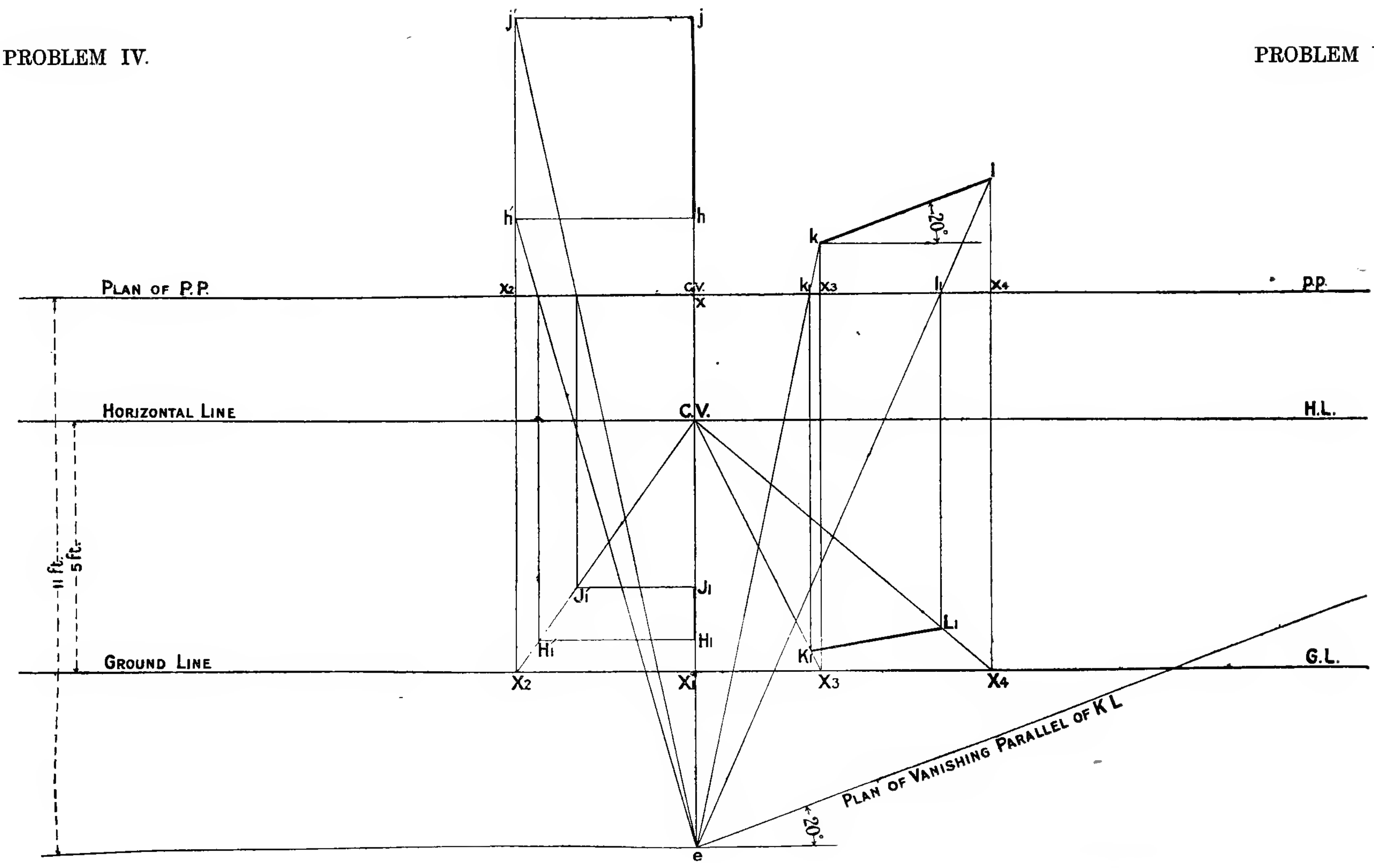
(iii) PROBLEM III.

A line **F G**, 3 ft. in length, lies upon the **G.P.** and is parallel to the **G.L.** It is 1 ft. beyond the **P.P.**, and the nearer end of the line **F** is 1 ft. on the right of the spectator, the other end, **G**, being also on the right. Place this line in perspective. Draw the plan (**f g**) of the line in its relative position to the **c.v.**

As **F G** is parallel to the **P.P.** it has no vanishing point (see Introduction, par. 6). The perspective representation of **F G** is obtained by considering **G** to lie in another line, whose **V.P.** can be obtained. From **G** draw a line perpendicular to **p.p.** and meeting it at x_2. The **V.P.** of x_2 **G** is **C.V.** The perspective view of **G** can now be obtained in a similar manner to that shown in Problem II. Draw the projector x_2X_2 to cut **G.L.** at X_2. Join X_2 to **C.V.** Draw the plan of the ray from **G** to the eye, *i.e.*, **g e**, and from its point of intersection with **p.p.** (g_1) draw a projector cutting X_2 **C.V.** at G_1.

G_1 is the perspective view of **G**. From G_1 draw a line parallel to **G.L.** The perspective view of **F** will lie in this line. Draw the plan of the ray from **F** to the eye, *i.e.*, **f e**, and from its point of intersection with **p.p.** (f_1) draw a projector. The perspective view of **F** will also lie in this projector, so that its intersection at F_1 with the line from G_1 parallel to **G.L.** will give the perspective view of **F**. F_1G_1 is therefore the perspective representation of **F G**.

PROBLEM IV. PROBLEM V.



(ii) PROBLEM IV.

A line **H J**, 4 ft. in length, lies upon the **G.P.** and is perpendicular to the **G.L.** This line is exactly opposite to the spectator, and the nearer end, **H**, is 1½ ft. from the **G.L.** Draw the line in perspective.

Draw **h j**, the plan of **H J**, in position. As the plan of the rays from the line to the eye coincides with the **C.V.R.** the method adopted in Problem II. cannot be used.

Draw the plan of a line at right angles to **P.P.** from any point x_2 in the plan of the **P.P.** Through **h** and **j** draw lines **h h′** and **j j′** parallel to the **P.P.** cutting $x_2 j'$ at h′ and j′ respectively. Find the perspective representation of $X_2 J'$ at $H_1'J_1'$ as in Problem II. (X_2, **J′**, and **H′** are the actual points, the plans of which are x_2, j′, and h′.) Since **h h′** and **j j′** are parallel to the **P.P.**, hence horizontal lines through J_1' and H_1' will pass through the perspective views of **J** and **H**.

It will be evident, as the **V.P.** of **H J** is the **C.V.**, the perspective representation of **H J** will lie on the line X_1 **C.V.**, and as the perspective representations of points **h** and **j** also lie in the lines H_1H_1' and J_1J_1' respectively, it follows that H_1J_1 is the perspective view of **H J**.

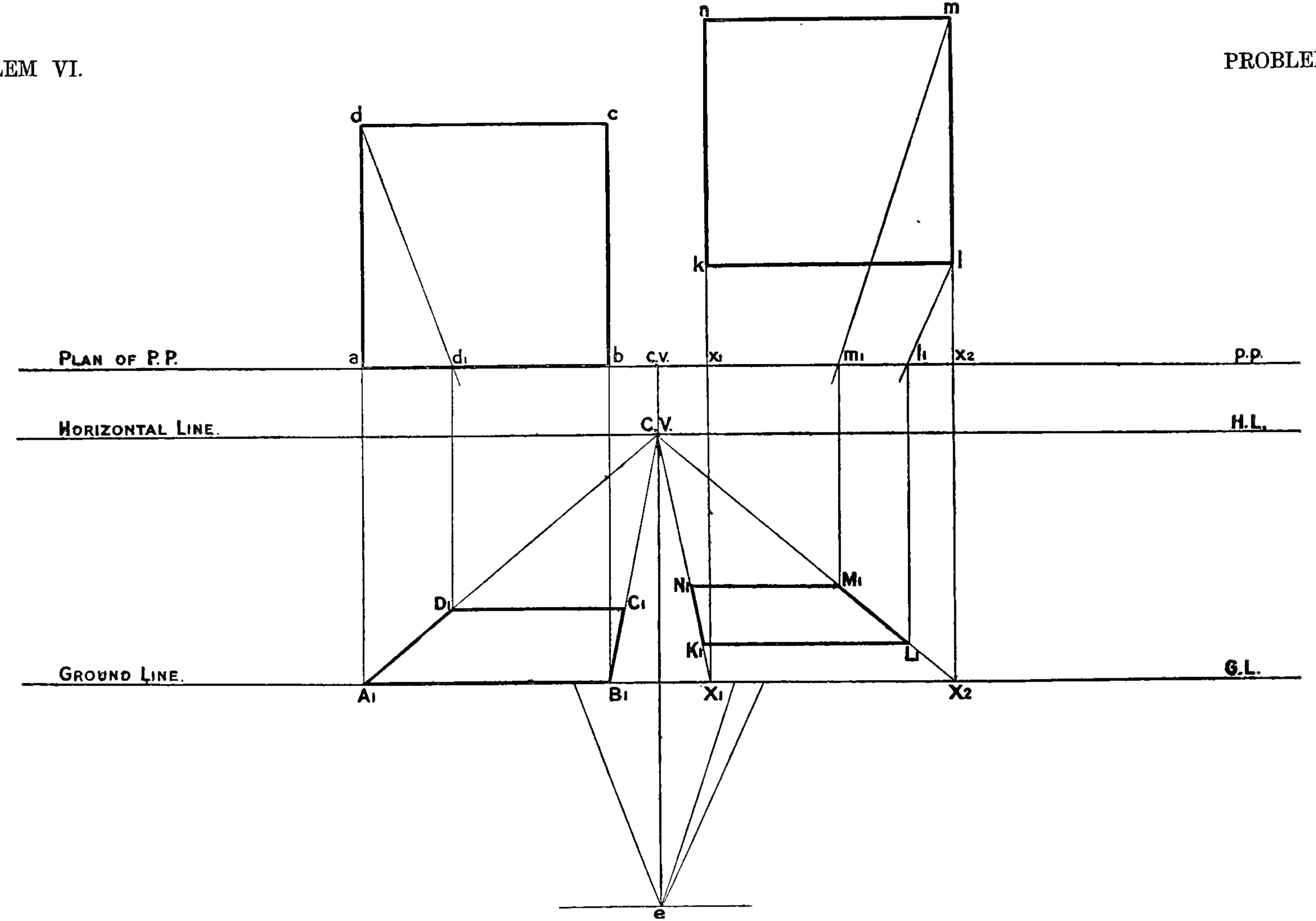
(iv) PROBLEM V.

A line **K L**, 3 ft. 6 ins. long, lies upon the **G.P.** and makes 20° with the **P.P.** towards the right. The nearer end of the line (**K**) is 2 ft. 6 ins. on the right and 1 ft. beyond the **P.P.** Draw the line in perspective.

Draw **k l**, the plan of the line, in its required position. The line is to be drawn in perspective without the use of its **V.P.** as that point is beyond the limits of the paper. From **k** and **l** draw lines **k** x_3 and **l** x_4 perpendicular to **p.p.** intersecting it at x_3 and x_4 respectively. Draw the projectors x_3X_3 and x_4X_4 to cut **G.L.** at X_3 and X_4 respectively. The perspective representation of points **K** and **L** are obtained by considering them to lie in the lines X_3K and X_4L respectively, as these lines vanish at **C.V.**, join X_3 **C.V.** and X_4 **C.V.** From **k** and **l** draw the plan of the rays (**k e** and **l e**) intersecting **p.p.** at k_1 and l_1 respectively. From k_1 and l_1 draw projectors intersecting X_3 **C.V.** and X_4 **C.V.** at K_1 and L_1 respectively. Join K_1L_1. K_1L_1 is the perspective representation of **K L**.

PROBLEM VI.

PROBLEM VII.



PLANE FIGURES ON THE GROUND PLANE.

(i) A square with one side touching the **P.P.**

(ii) A square not touching the **P.P.**, but with a side parallel to the **P.P.**

(iii) A rectangle with a side at any angle to the **P.P.**, and the **V.P.**'s are accessible.

(iv) An irregular figure.

(v) A circle.

(i) PROBLEM VI.

A square **A B C D** of 5 ft. side lies on the ground with one side **A B** touching the **P.P.** The nearest corner to the **C.V.** is 1 ft. to the left of the spectator. The height of the eye above the **G.P.** is 6 ft., and its distance from the **P.P.** 11 ft. Draw this square in perspective.

Draw and letter the **p.p., c.v., e, H.L., C.V., G.L.,** and the plan of the square in position (**a b c d**) (refer to Problem I.).

From **a** and **b** draw projectors to intersect the **G.L.** at $\mathbf{A_1}$ and $\mathbf{B_1}$ respectively. Join $\mathbf{A_1}$ and $\mathbf{B_1}$ to **C.V.** (the vanishing point of **A D** and **B C**). Draw the plan of the ray **D E** by joining **d** to **e** (the ray is shown broken off in order to avoid confusion), intersecting **p.p.** at $\mathbf{d_1}$. From this point draw a projector to intersect $\mathbf{A_1}$ **C.V.** at $\mathbf{D_1}$.

From $\mathbf{D_1}$ draw a horizontal line intersecting $\mathbf{B_1}$ **C.V.** at $\mathbf{C_1}$.

$\mathbf{A_1B_1C_1D_1}$ is the required perspective representation.

(ii) PROBLEM VII.

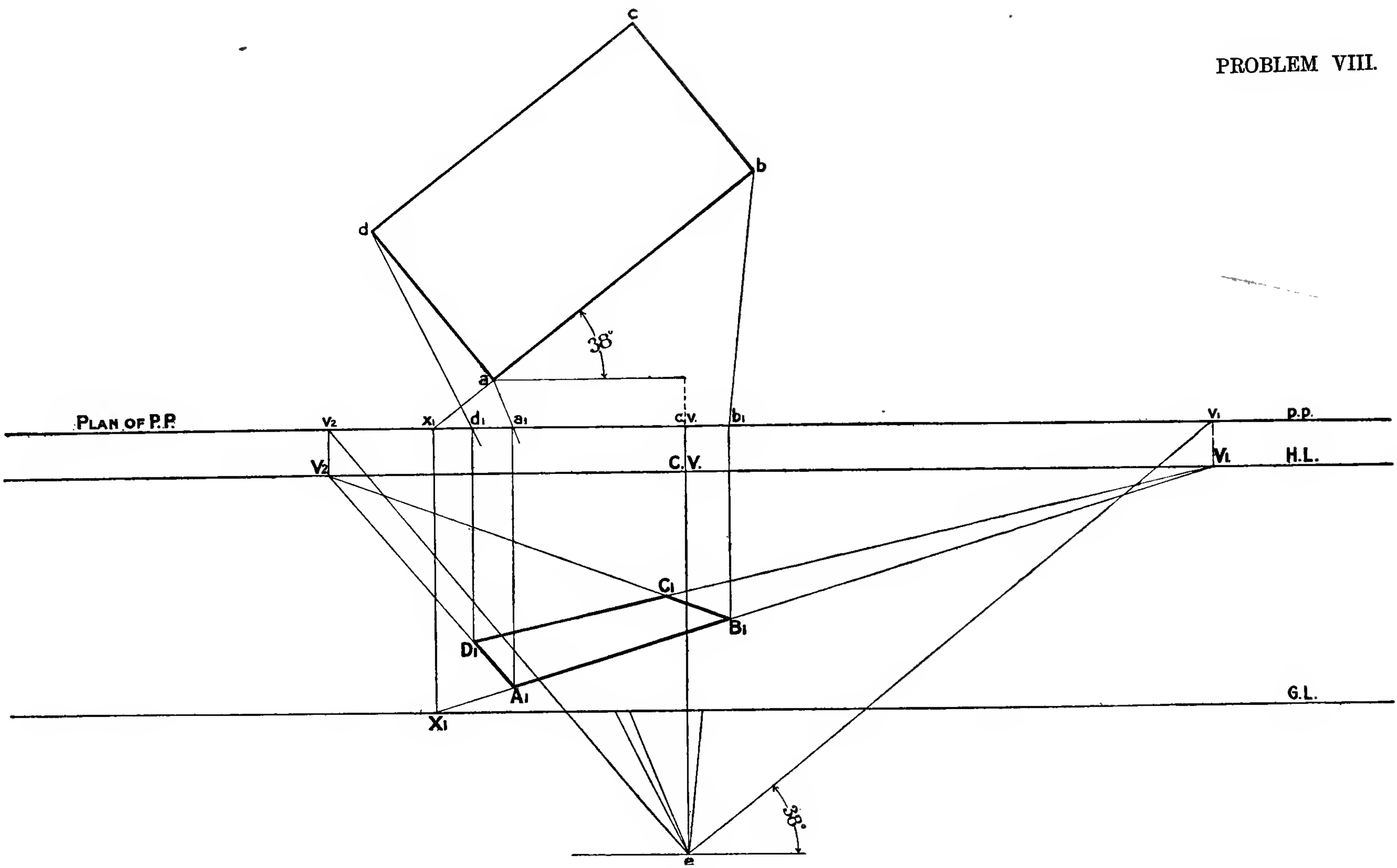
A square, **K L M N**, of 5 ft. side, lies on the ground with one side, **K L** parallel to the **P.P.** The nearest corner (**K**) to the spectator is 1 ft. on the right and 2 ft. from the **P.P.**

The height of eye and its distance from **P.P.** is the same as in previous problem. Place this square in perspective.

Draw the plan of the square in position. Produce **n k** and **m l** to intersect **p.p.** at $\mathbf{x_1}$ and $\mathbf{x_2}$ respectively, and from $\mathbf{x_1}$ and $\mathbf{x_2}$ draw projectors to intersect **G.L.** at $\mathbf{X_1}$ and $\mathbf{X_2}$ respectively. Join $\mathbf{X_1}$ and $\mathbf{X_2}$ to **C.V.** Draw **l e** and **m e** (plans of rays) intersecting **p.p.** at $\mathbf{l_1}$ and $\mathbf{m_1}$ respectively. (For brevity in the problems that follow on the plan method, when the term draw "*the ray*" appears thus, it must be understood that it is the PLAN of the ray which is drawn.)

From $\mathbf{l_1}$ and $\mathbf{m_1}$ draw projectors intersecting $\mathbf{X_2}$ **C.V.** at $\mathbf{L_1}$ and $\mathbf{M_1}$ respectively. From $\mathbf{L_1}$ and $\mathbf{M_1}$ draw horizontal lines intersecting $\mathbf{X_1}$ **C.V.** at $\mathbf{K_1}$ and $\mathbf{N_1}$ respectively. $\mathbf{K_1L_1M_1N_1}$ is the required perspective representation of the square.

PROBLEM VIII.



(iii) PROBLEM VIII.

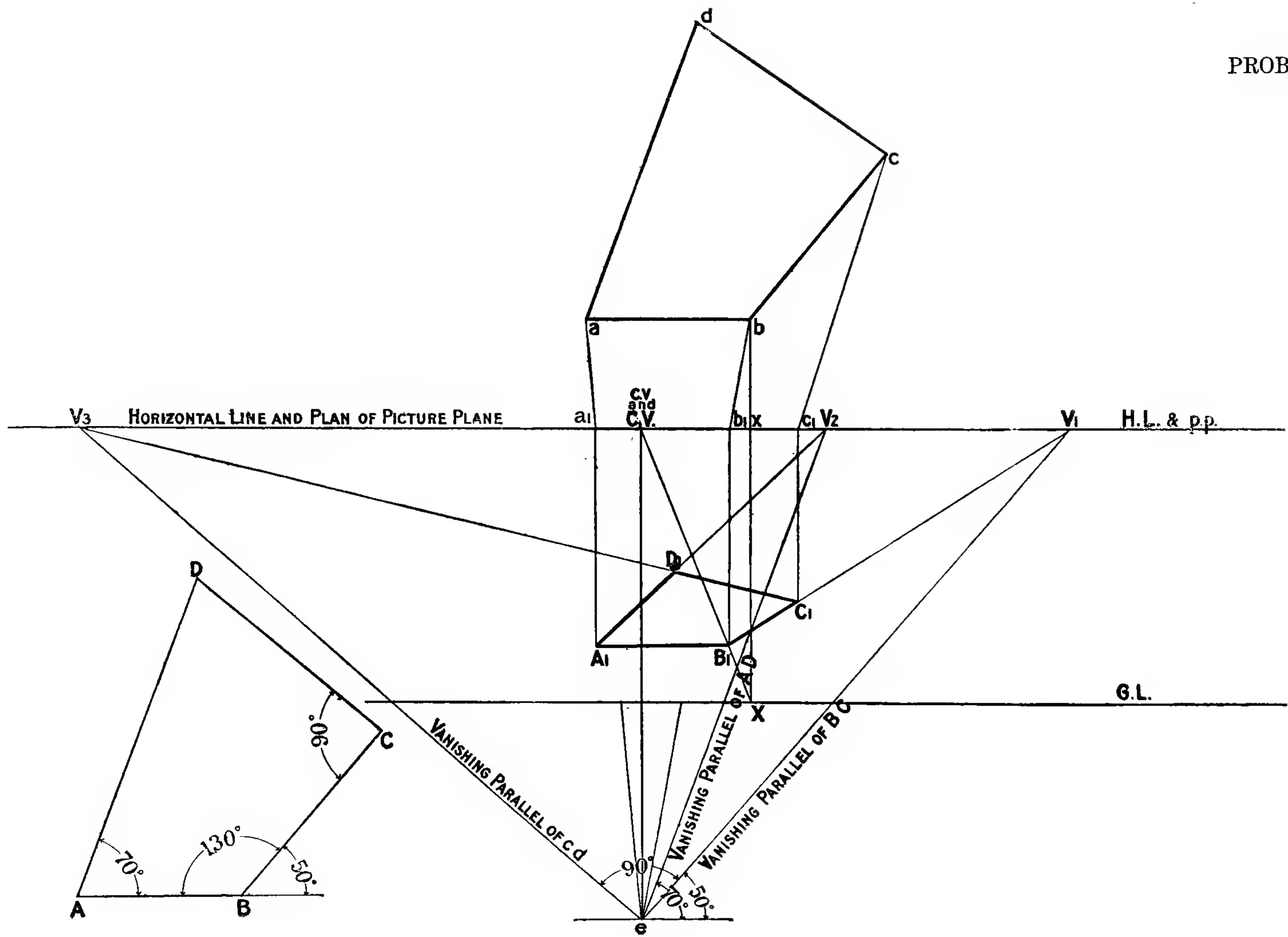
A rectangle 2 ft. × 3 ft. lies on the ground, with one corner 2 ft. on the spectator's left and 6 ins. beyond the **P.P.** Represent it in perspective, when one of its long edges recedes at an angle of 38° towards the spectator's right; the height of the eye being 2 ft. 6 ins. and its distance from **P.P.** 4 ft. 6 ins.

Set down **H.L.**, **G.L.**, **p.p.**, **e**, and the plan of the rectangle (**a b c d**) in position. Produce **b a** to cut **p.p.** at x_1; draw a projector from x_1 to cut **G.L.** at X_1. Draw the plan of the vanishing parallel of **A B** ($e\,v_1$). From v_1 draw a projector to intersect **H.L.** at V_1. V_1 is the **V.P.** of **A B**. Join X_1 to V_1. Draw *the rays* **a e** and **b e** cutting **p.p.** at a_1 and b_1 respectively; then from a_1 and b_1 draw projectors cutting X_1V_1 at A_1 and B_1 respectively.

Obtain V_2, the **V.P.** of **A D**, and join A_1 to V_2. Draw *the ray* **d e** cutting **p.p.** at d_1, then from d_1 draw a projector cutting A_1V_2 at D_1.

Since **D C** is parallel to **A B** it will have the same **V.P.**, *i.e.*, V_1. Therefore join D_1 to V_1. For a similar reason join B_1V_2, intersecting D_1V_1 at C_1. $A_1B_1C_1D_1$ is the required representation.

PROBLEM IX.



d
c
a
b
CV and C.V.
V3
HORIZONTAL LINE AND PLAN OF PICTURE PLANE
a1
b1 X
c1 V2
V1
H.L. & p.p.
D1
C1
A1
B1
G.L.
X
D
C
90°
A
B
70°
130°
50°
VANISHING PARALLEL OF c d
VANISHING PARALLEL OF A D
VANISHING PARALLEL OF B C
90°
70°
50°
e

(iv) PROBLEM IX.

In the accompanying plate **A B C D** is an irregular figure, **A B** = 3 ft.; **B C** = 4 ft.; ∠ **A B C** = 130°; ∠ **B C D** = 90°; ∠ **B A D** = 70°. Put this figure into perspective when it lies upon the ground plane with **A** 1 ft. to the left of the spectator and 2 ft. within the picture plane. **A B** being parallel to the ground line.

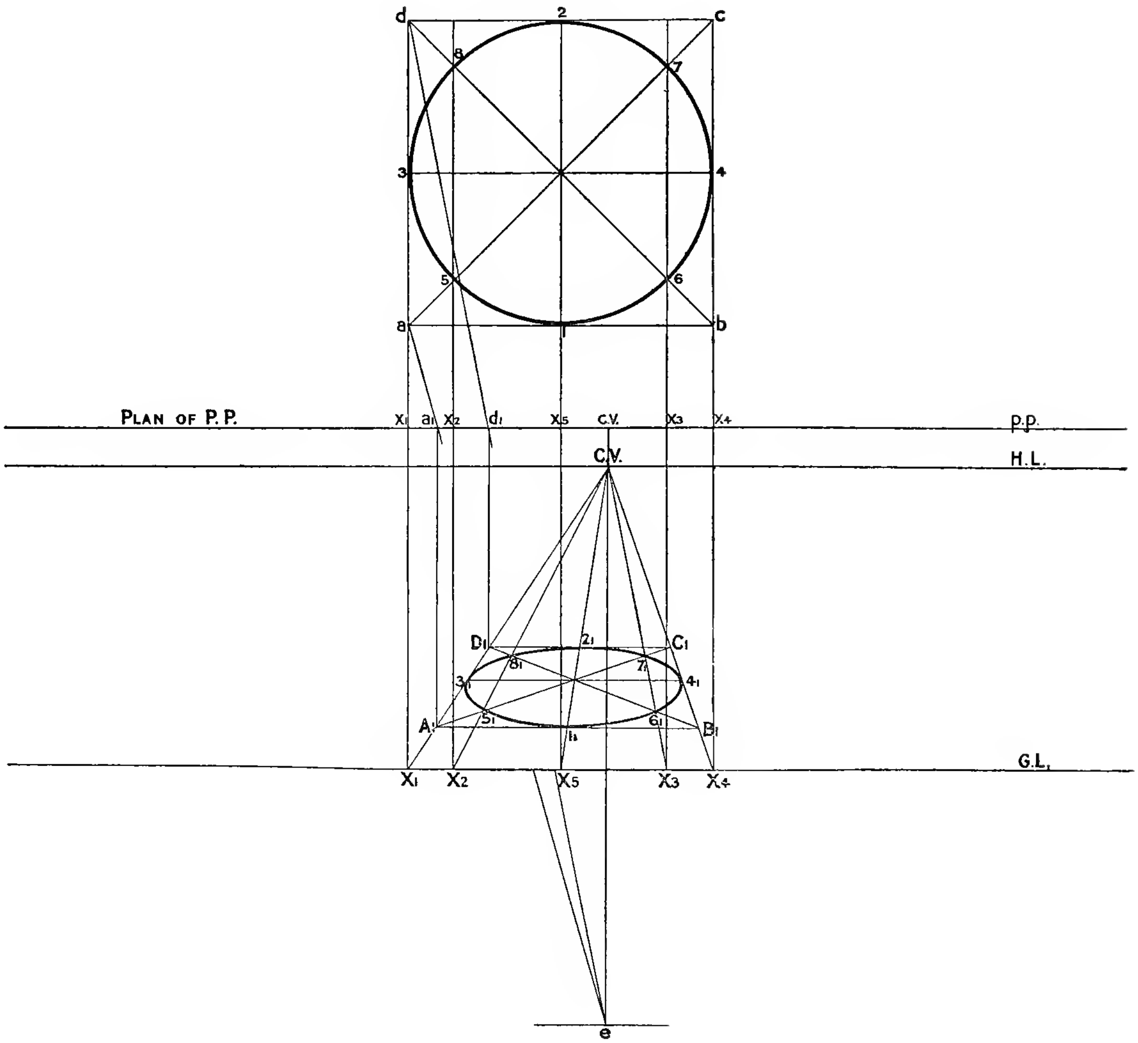
Height of eye above the ground 5 ft., and the distance of the eye from the picture plane 9 ft.

In the figure the plan of the **P.P.** and the **H.L.** are drawn coincident (which may be done when desirable, as pointed out in Problem I.). Place the plan of the figure in position. From **b** draw a perpendicular to **p.p.** intersecting it at **x**, and from **x** a projector to cut **G.L.** at **X.** Join **X** to **C.V.**; obtain *the ray* **b e** intersecting **p.p.** at b_1. From b_1 draw a projector intersecting **X C.V.** at B_1. B_1 is the perspective view of **B**.

Obtain *the ray* **a e** intersecting **p.p.** at a_1, and from a_1 draw a projector to intersect a horizontal line drawn through B_1 at A_1. A_1B_1 is the perspective view of **A B**.

From **e** draw the vanishing parallel of **b c** to intersect **p.p.** at V_1. In a similar manner obtain V_2 and V_3, the **V.P.**'s of **A D** and **C D** respectively. Join B_1 to V_1 and obtain *the ray* **c e** intersecting **p.p.** at c_1. From c_1 draw a projector intersecting B_1V_1 at C_1. Join C_1 to V_3 and A_1 to V_2, these lines intersect at D_1. D_1 is the perspective representation of **D**, and $A_1B_1C_1D_1$ is the perspective view of the figure.

PROBLEM X.



d
2
c
8
7
3
4
5
6
a
1
b
PLAN OF P. P.
X1
a1
X2
d1
X5
C.V.
X3
X4
P.P.
C.V.
H.L.
D1
21
C1
81
71
31
41
A1
51
61
B1
11
G.L.
X1
X2
X5
X3
X4
e

(v) PROBLEM X.

A circle of 3 ft. radius lies on the ground with its centre 1 foot on the left of the spectator and 5 ft. within the picture.

The height of the eye above the **G.P.** is 6 ft. and its distance from the **P.P.** 11 ft. 9 ins. Give a perspective representation of this circle.

The perspective view of any curve can only be obtained by finding the perspective view of a series of points in it and drawing a curve through them, by freehand or by the use of curves. The more numerous the points that are taken the more exact will be the curve's perspective representation.

In obtaining the perspective representation of a circle the circumference is divided into 8 equal parts. By drawing a good ellipse through the perspective representation of these 8 points a fair representation of the circle may be obtained.

Having placed the circle in the required position, draw a diameter **1,2** perpendicular to **p.p.** and a diameter **3,4** parallel to **p.p.** Through **1** and **2** draw lines **a b** and **c d** parallel to **3,4**, and through **3** and **4** draw lines parallel to **1,2.**

Note that **a b c d** will be a square having two sides parallel to **P.P.** and having 4 points of the circle on the sides of the square.

Join **a c** cutting the circle at **5** and **7**, join **b d** cutting the circle at **6** and **8**.

To find the perspective representation of the 8 points **1, 2, 3, 4, 5, 6, 7,** and **8,** produce **d a** and **c b** to cut **p.p.** at x_1 and x_4 respectively; from x_1 and x_4 draw projectors to cut **G.L.** at X_1 and X_4 respectively. Join X_1 and X_4 to **C.V.**

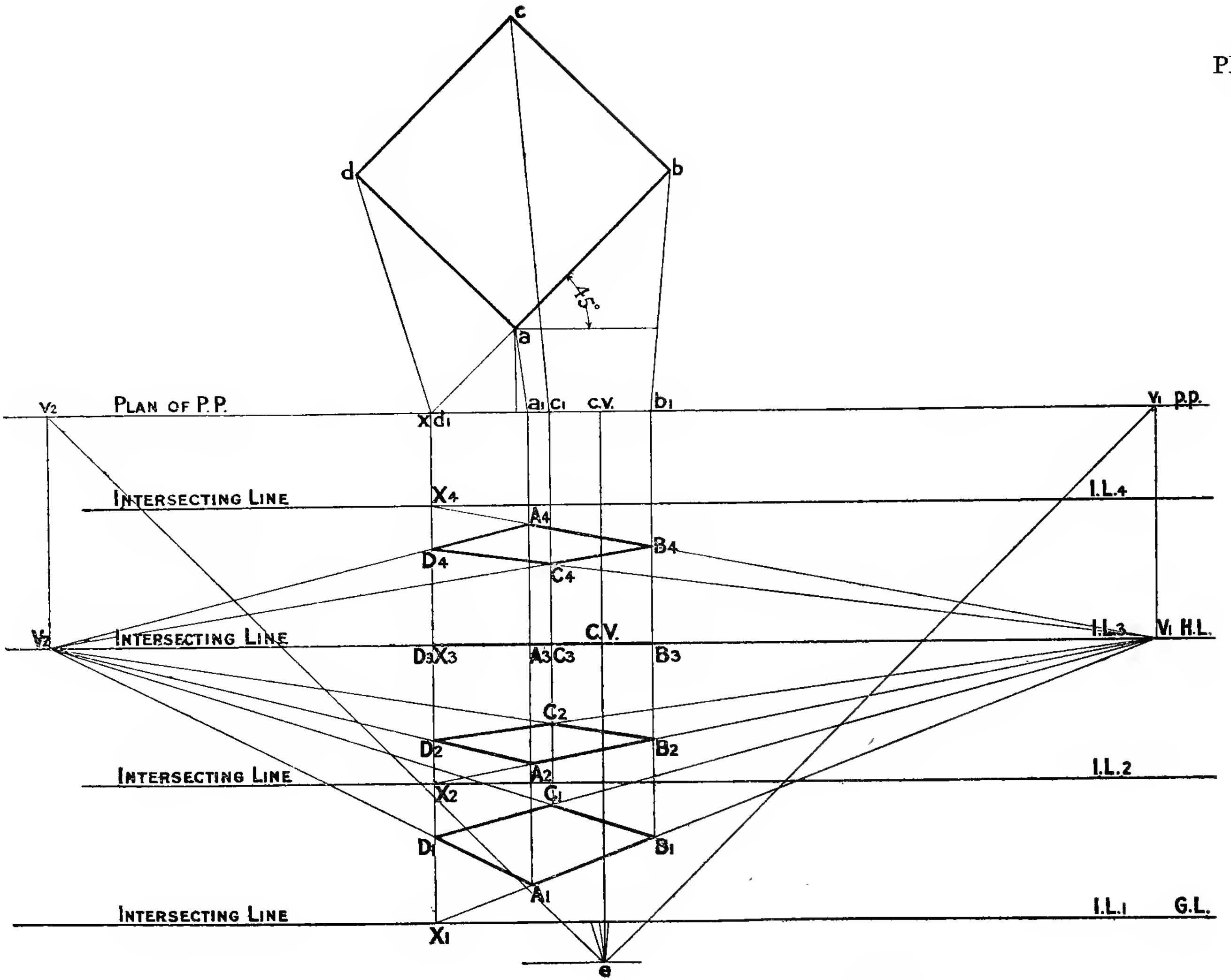
Draw *the rays* **a e** and **d e** cutting **p.p.** at a_1 and d_1 respectively. From a_1 and d_1 draw projectors cutting X_1**C.V.** at A_1 and D_1 respectively. Through A_1D_1 draw horizontal lines cutting X_4**C.V.** at B_1 and C_1 respectively. Then $A_1B_1C_1D_1$ is the perspective representation of the square circumscribing the circle. Produce **2,1** to cut **p.p.** at x_5, from x_5 draw a projector to cut **G.L.**, at X_5, join X_5**C.V.** cutting A_1B_1 and D_1C_1 at I_1 and 2_1 respectively.

Join A_1C_1 and B_1D_1. These lines represent the perspective view of the diagonals of the square. (The intersection of the diagonals, *i.e.*, the centre of the circle, should lie on X_5**C.V.**). Through the perspective representation of the circle's centre draw $3_1,4_1$ parallel to **G.L.,** cutting A_1D_1 and B_1C_1 at 3_1 and 4_1 respectively.

Produce **8,5** and **7,6** to cut **p.p.** at x_2 and x_3; from x_2 and x_3 draw projectors cutting **G.L.** at X_2 and X_3 respectively.

Join X_2 and X_3 to **C.V.**, and let the lines thus obtained cut the perspective view of the diagonals of the square at 5_1, 8_1, 6_1, and 7_1.

Draw an ellipse passing through 1_1, 6_1, 4_1, 7_1, 2_1, 8_1, 3_1, 5_1. In drawing a horizontal circle in perspective the square circumscribing the circle should, whenever practical, be taken with two sides parallel to the **P.P.** as has been done in the above problem.


c
d
b
45°
a
V2
PLAN OF P.P.
x d1
a1 c1
c.v.
b1
v1 p.p.
INTERSECTING LINE
X4
A4
B4
D4
C4
I.L.4
INTERSECTING LINE
V2
C.V.
D3 X3
A3 C3
B3
I.L.3
V1 H.L.
C2
D2
B2
A2
INTERSECTING LINE
X2
C1
I.L.2
D1
B1
A1
INTERSECTING LINE
X1
I.L.1
G.L.
e

PLANE FIGURES ON THE GROUND PLANE AND ON HORIZONTAL PLANES AT VARIOUS HEIGHTS ABOVE THE GROUND.

PROBLEM XI.

A square **A B C D** of 4 ft. side lies—

(i) On the ground.
(ii) On a horizontal plane 2 ft. 6 ins. above the **G.P.**
(iii) On a horizontal plane the height of the eye above the **G.P.**
(iv) On a horizontal plane 7 ft. 6 ins. above the **G.P.**

The edges of the square make angles of 45° with the **P.P.**, and the nearest corner to the spectator is 1 ft. 6 ins. on the left of **C.V.** and 1 ft. 6 in. within the picture. The height of the eye to be taken as 5 ft. and the distance of the eye from **P.P.** as 10 ft.

In working the above squares refer to the Introduction, par. 7, fig. 9.

(i) Place **p.p.**, **H.L.**, **G.L.**, **e**, and the plan of the square (**a b c d**) in their relative positions as shown. Produce **b a** to intersect **p.p.** at **x** and draw a projector from **x** intersecting **G.L.** at $\mathbf{X_1}$. Join $\mathbf{X_1}$ to $\mathbf{V_1}$ (the **V.P.** of **A B** and **D C**). Obtain *the ray* **a e** intersecting **p.p.** at $\mathbf{a_1}$. From $\mathbf{a_1}$ draw a projector to intersect $\mathbf{X_1V_1}$ at $\mathbf{A_1}$.

Join $\mathbf{A_1}$ to $\mathbf{V_2}$ (the **V.P.** of **A D** and **B C**). Obtain *the ray* **b e**, intersecting **p.p.** at $\mathbf{b_1}$ and *the ray* **d e** intersecting **p.p.** at $\mathbf{d_1}$ (in the figure $\mathbf{d_1}$ and **x** coincide). Draw projectors from $\mathbf{d_1}$ intersecting $\mathbf{A_1V_2}$ at $\mathbf{D_1}$, and from $\mathbf{b_1}$, intersecting $\mathbf{A_1V_1}$ at $\mathbf{B_1}$. Join $\mathbf{D_1}$ to $\mathbf{V_1}$ and $\mathbf{B_1}$ to $\mathbf{V_2}$ intersecting $\mathbf{D_1V_1}$ at $\mathbf{C_1}$.

$\mathbf{A_1B_1C_1D_1}$ is the perspective representation of the square on the ground.

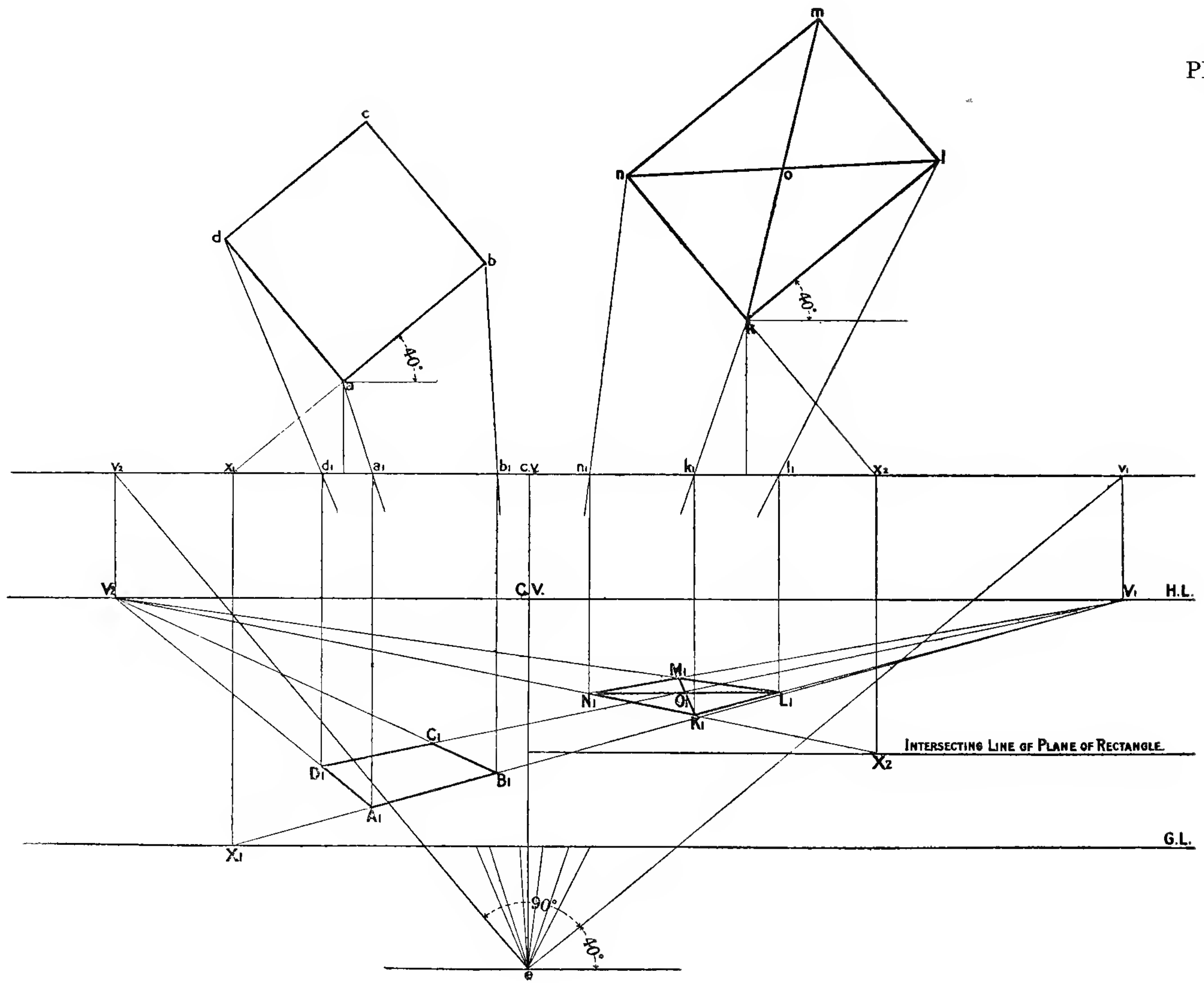
(ii) Draw a horizontal line, $\mathbf{I.L._2}$, 2 ft. 6 ins. above **G.P.**; this line is the intersection with the **P.P.** of a horizontal plane containing the square. The vertical line $\mathbf{x\,X_1}$ intersects $\mathbf{I.L._2}$ at $\mathbf{X_2}$. Join $\mathbf{X_2}$ to $\mathbf{V_1}$ ($\mathbf{I.L._2}$ is used instead of $\mathbf{I.L._1}$ or **G.L.** as the square is not on the **G.P.**). Complete the square in a similar way to that used when drawing the perspective view of the square on the ground. $\mathbf{A_2B_2C_2D_2}$ is the perspective representation of the square when it is 2 ft. 6 ins. above the **G.P.**

(iii) When the square lies on a horizontal plane the height of the eye above the ground, it is evident that the intersection of that plane with the **P.P.** coincides with the **H.L.**, and the perspective view of the square is a straight line. Draw *the ray* **c e** intersecting **p.p.** at $\mathbf{c_1}$. From $\mathbf{c_1}$ draw a projector intersecting $\mathbf{I.L._3}$ or **H.L.** at $\mathbf{C_3}$. The projectors from $\mathbf{a_1b_1d_1}$ intersect $\mathbf{I.L._3}$ at $\mathbf{A_3}$, $\mathbf{B_3}$, $\mathbf{D_3}$ respectively. $\mathbf{A_3}$, $\mathbf{B_3}$, $\mathbf{C_3}$, and $\mathbf{D_3}$ are the perspective representation of the points **A B C D** of the square.

(iv) Draw a horizontal line, $\mathbf{I.L._4}$, 7 ft. 6 ins. above the **G.L.** This is the intersection with the **P.P.** of a horizontal plane containing the square, and must be used instead of **G.L.**, $\mathbf{I.L._2}$, or **H.L.**, when obtaining its perspective representation. $\mathbf{X_4}$ is the intersection with **p.p.** of the vertical line drawn from **x**. Join $\mathbf{X_4}$ to $\mathbf{V_1}$ and obtain the perspective view of the square in a similar manner to that previously employed. In examples (i) and (ii) the lines of the perspective view of the square incline upwards as they recede from **P.P.**, while in the perspective view of the square situated above the eye the corresponding lines incline downwards as they recede.

PROBLEM XII.

PROBLEM XIII.



PROBLEM XII.

Draw the perspective representation of a square (9 ft. edge) when it lies on the **G.P.** with one edge making an angle of 40° with the **P.P.** towards the right. One corner of the square is situated 9 ft. on the spectator's left and 4 ft. 6 ins. beyond the **G.L.** The height of the eye to be taken as 12 ft. and the distance of the eye from the **P.P.** as 24 ft.

Place **p.p.**, **H.L.**, **G.L.**, **e**, and the plan of the square (**a b c d**) in their relative positions as shown. Produce **b a** to cut **p.p.** at x_1; project from x_1 to cut **G.L.** at X_1, then join X_1 to V_1 (the vanishing point of **A B**).

Draw *the rays* **a e** and **b e** cutting **p.p.** at a_1 and b_1 respectively, then draw projectors from a_1 and b_1 to cut X_1V_1 at A_1 and B_1 respectively.

Join A_1 to V_2 (the **V.P.** of **A D**); draw *the ray* **d e** cutting **p.p.** at d_1; from d_1 draw a projector cutting A_1V_2 at D_1.

Since **D C** is parallel to **A B** it must have the same **V.P.**, *i.e.* V_1; therefore join D_1V_1. For a similar reason join B_1V_2 cutting D_1V_1 at C_1.

$A_1B_1C_1D_1$ is the required perspective drawing.

PROBLEM XIII.

A figure which is a rectangle (9 ft. by 12 ft.) with the diagonals drawn lies on a horizontal plane 7 ft. 6 ins. below a spectator's eye which is 12 ft. above the ground. One corner of the rectangle is 10 ft. 6 ins. on the spectator's right and 7 ft. 6 ins. within the picture. Show its perspective representation when the eye is 24 ft. in front of the **P.P.**

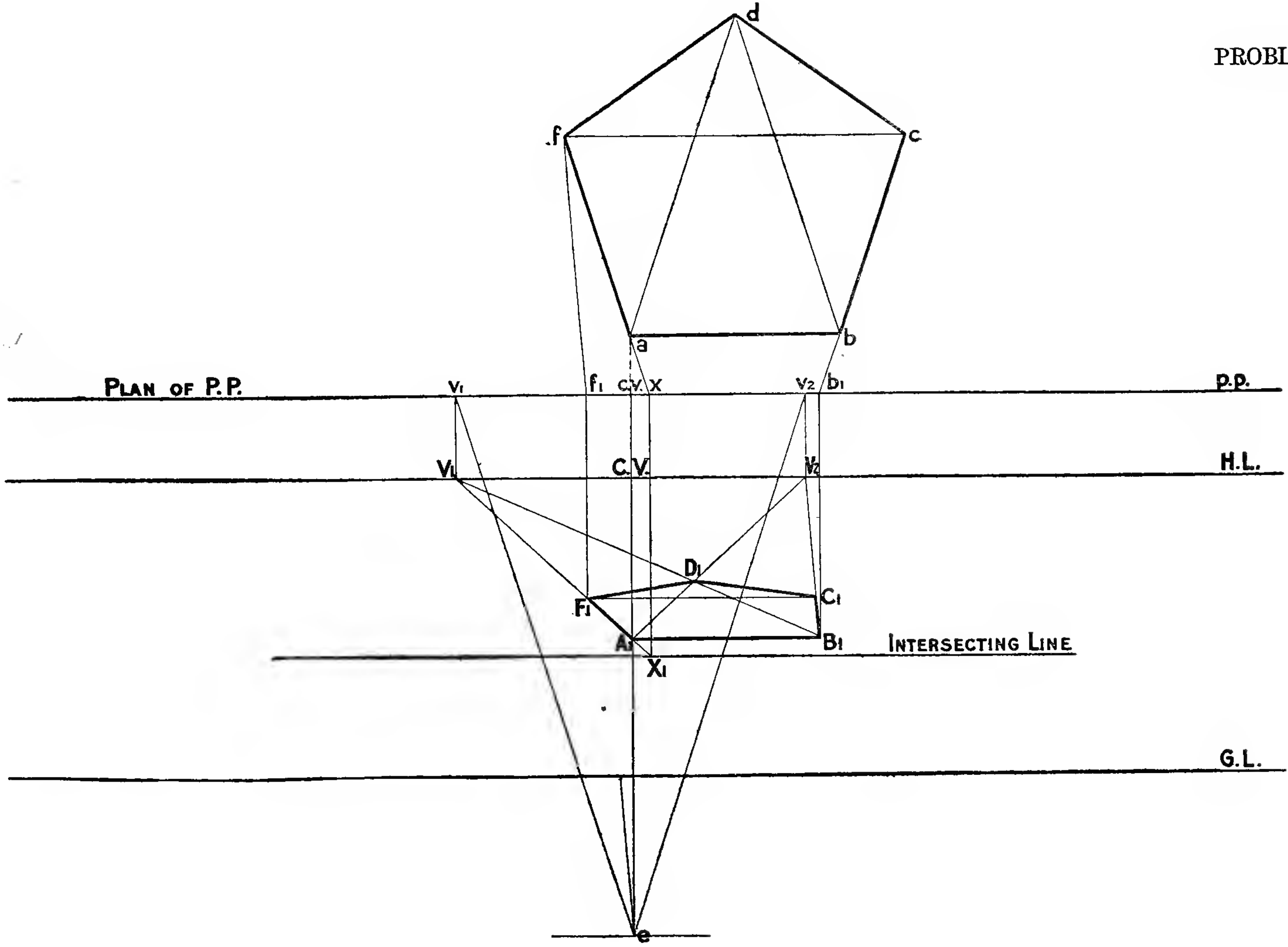
It should be observed that the height of the eye and its distance from the **P.P.** are the same as in the last figure, and as the two plans do not cross one another, the same **p.p.**, **H.L.**, **G.L.**, and **e** are used. Place the figure in its relative position to **c.v.** as at (**k l m n**).

Suppose the horizontal plane upon which **K L M N** rests to be produced to cut the **P.P.**, it must do so in a horizontal line 4 ft. 6 ins. above **G.L.** (*i.e.* 7 ft. 6 ins. below **H.L.**). This line is called the Intersecting Line of the Plane of the Rectangle; this line may be used for lines lying 4 ft. 6 ins. above **G.P.** in a similar way to the **G.L.** for lines lying on the **G.P.**

Produce **n k** to cut **p.p.** at x_2, from x_2 draw a projector cutting the Intersecting Line at X_2. (X_2 is the point of intersection of **K N** produced with **P.P.**)

Join X_2 to V_2. Proceed as in the last problem to draw the perspective representation of the rectangle as at $K_1L_1M_1N_1$. If K_1M_1 and L_1N_1 are joined and intersect at O_1, then O_1 will be the perspective representation of **O**.

PROBLEM XIV.


d
f
c
a
b
PLAN OF P.P.
v1
f1
C.V.
X
v2
b1
P.P.
V1
C.V.
V2
H.L.
D1
C1
F1
A1
B1
INTERSECTING LINE
X1
G.L.
e

PROBLEM XIV.

A pentagon **A B C D F** of 3 ft. 6 ins. side lies upon a horizontal plane 2 ft. above the ground. The nearest point (**A**) of the pentagon to **P.P.** is opposite the spectator and 1 ft. within the picture. One side **A B** is parallel to the **P.P.** and lies wholly on the right of the spectator. Draw its perspective representation.

Draw a horizontal line 2 ft. above the **G.L.** This is the intersection with the **P.P.** of a horizontal plane which contains the pentagon. Place the plan **a b c d f** in position and join **a** to **d**, **b** to **d**, and **f** to **c**. Produce **f a** to intersect **p.p.** at **x**. From **x** draw a projector to intersect the Intersecting Line at X_1 (X_1 is the point of intersection of **F A** with the **P.P.** Find V_1 and V_2 the **V.P.**'s of **a f** and **b c** respectively. Join X_1 to V_1 intersecting **e c.v. a** at A_1. As **A** is exactly opposite to the spectator, A_1 will be the perspective representation of **A**. From A_1 draw A_1B_1 a horizontal line, and draw *the ray* **b e** intersecting **p.p.** at b_1. From b_1 draw a projector to intersect A_1B_1 at B_1. Draw *the ray* **f e** intersecting **p.p.** at f_1, and from f_1 draw a projector intersecting X_1V_1 at F_1. Observe that **B D** is parallel to **A F**, **A D** is parallel to **B C**, **D** is the point of intersection of **A D** and **B D**, and that **F C** is parallel to **A B**. Join B_1 to V_1 and A_1 to V_2, these lines intersect at D_1. Join B_1 to V_2, and from F_1 draw F_1C_1 a horizontal line intersecting B_1V_2 at C_1.

$A_1B_1C_1D_1F_1$ is the perspective representation of the pentagon.

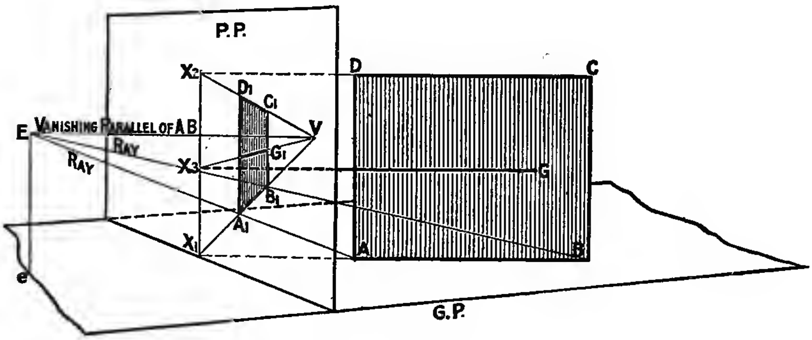


Fig. 17.

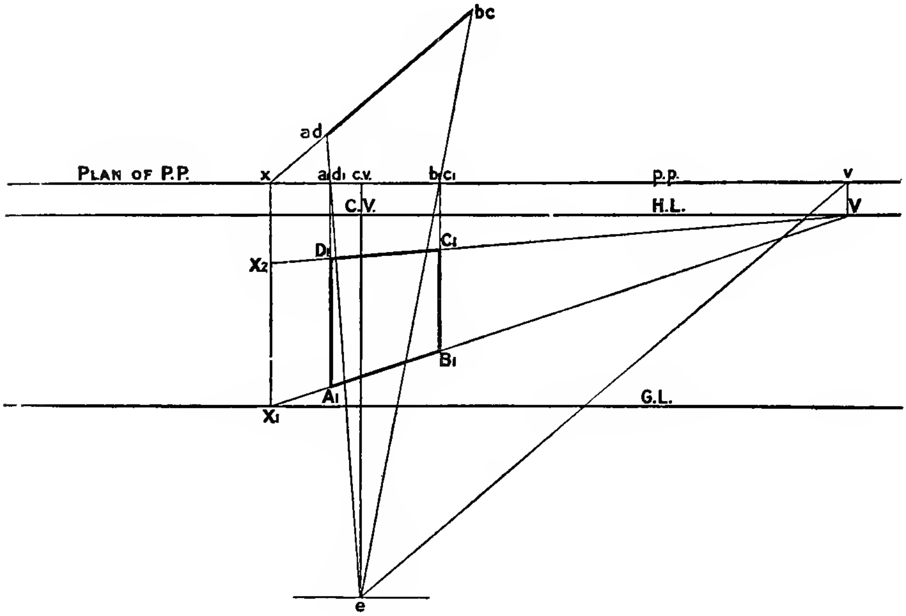
HEIGHT LINE.

The intersection of a vertical plane with the **P.P.** is called a height line when it is used as a means of obtaining the perspective view of points that are above the **G.P.**

Fig. 17 is a drawing of a model, and **A B C D** is a vertical rectangle with its lower edge, **A B**, on the **G.P.** **V** is the **V.P.** of **A B** (it is also the **V.P.** of all horizontal lines contained by the rectangle). If **B A** and **C D** are produced to cut the **P.P.** at X_1 and X_2 respectively, the line joining X_1 and X_2 will be vertical. Join X_1 and X_2 to **V**, also **A** and **B** to **E**. Let **A E** and **B E** cut X_1V at A_1 and B_1 respectively. Draw vertical lines A_1D_1 and C_1D_1 until they cut X_2V at D_1 and C_1 respectively. $A_1B_1C_1D_1$ is the perspective representation of **A B C D**.

It will be evident that X_1X_2 is the same length as **A D** or any vertical line terminated by **D C** and **A B**. X_1X_2 is called the height line of these vertical lines. It will also be the height line for any point in the plane of the rectangle **A B C D**. For instance, take any point **G** in **A B C D**, and from **G** draw a line parallel to **B A** to intersect the **P.P.** at X_3; it will be evident that X_3 will lie on the vertical line X_1X_2, and that X_3 and **G** will be the same height above the **G.P.**

X_3V is the perspective representation of X_3G produced infinitely, and all vertical lines terminated by X_3V and X_1V will be the perspective representation of vertical lines that are the same length as from the height of **G**.



PROBLEM XV.

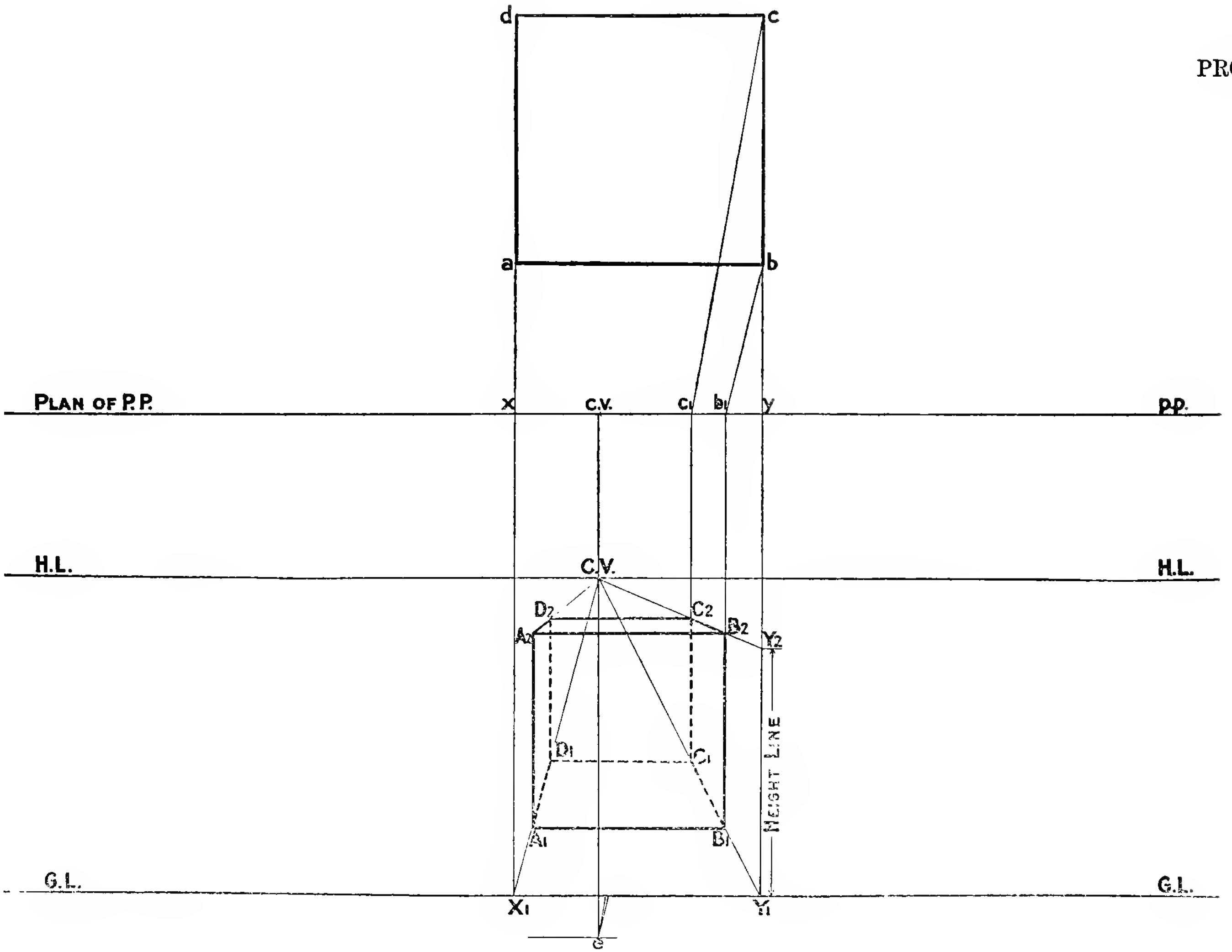
A rectangle **A B C D** (4 ft. × 3 ft.) rests, with **A B**, one of its larger edges, on the ground. The nearest corner (**A**) is 9 ins. on the spectator's left and 1 ft. from the ground line. Draw it in perspective when the plane of the rectangle is vertical and recedes at an angle of 40° with the **P.P.** towards the right; the height of the eye being 4 ft. and the distance of eye from **P.P.** 8 ft. 8 ins.

Draw **p.p.**, **e**, **G.L.**, **H.L.**, and the plan of the rectangle (**a b c d**) in their required positions. Find **V**, the **V.P.** of **A B** and produce **b a** to intersect **p.p.** at **x**.

Draw a projector from **x** intersecting the **G.L.** at X_1. Join X_1V. Draw **e a** and **e b** the plans of the rays **E A** and **E B** intersecting **p.p.** at a_1c_1 and b_1d_1 respectively. From a_1b_1 draw projectors intersecting X_1V at A_1 and B_1 respectively. A_1B_1 is the perspective representation of the lower edge **A B** of the rectangle.

Referring to fig. 17 it will be evident that X_1x indicates the intersection with the **P.P.** of the vertical plane containing the rectangle. Therefore obtain a point X_2 on X_1x the height of the rectangle above the **G.L.** (*i.e.* 3 ft.). Join X_2V. Then the intersection of A_1a_1 and B_1b_1 with X_2V is the perspective representation of D_1 and C_1 respectively. Hence $A_1B_1C_1D_1$ is the required perspective representation of **A B C D**, and X_1X_2 is the height line for the points C_1 and D_1.

PROBLEM XVI.



d
c
a
b
PLAN OF P.P.
x
C.V.
c1
b1
y
P.P.
H.L.
C.V.
H.L.
D2
C2
A2
B2
Y2
D1
C1
A1
B1
HEIGHT LINE
G.L.
X1
Y1
G.L.
e

RECTILINEAL SOLIDS.

PROBLEM XVI.

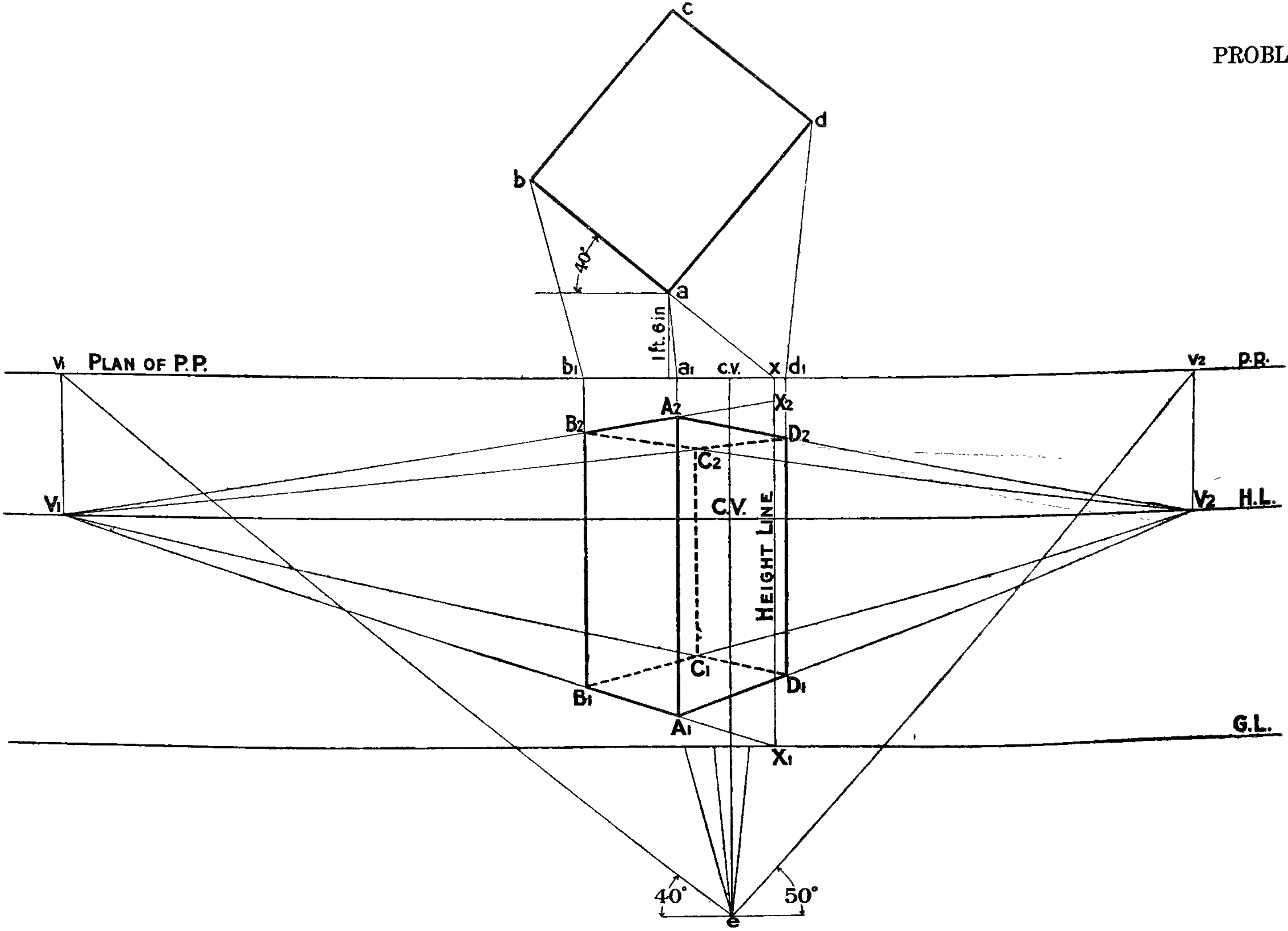
A cube of 4 ft. 6 ins. edge stands with one of its faces on the ground and with another face parallel to the picture plane. **A B**, its nearest edge to **P.P.** on the **G.P.**, is 2 ft. 6 in. beyond the **P.P.** and **A** is 1 ft. 6 ins. on the left of the spectator, **B** being on the right. Height of eye 5 ft. 9 ins., and its distance from **P.P.** 9 ft. 6 ins. Draw this cube in perspective.

a b c d is the plan of the cube in position.

Produce **d a** and **c b** to intersect **p.p.** at **x** and **y** respectively. From **x** and **y** draw projectors to intersect **G.L.** at X_1 and Y_1 respectively. Join X_1 and Y_1 to **C.V.** Draw *the ray* **b e** intersecting **p.p.** at b_1. From b_1 draw a projector intersecting Y_1 **C.V.** at B_1. From B_1 draw a horizontal line intersecting X_1 **C.V.** at A_1. A_1B_1 is the perspective view of the nearest edge of the cube on the ground. Obtain a point Y_2 on Y_1y, 4 ft. 6 ins. above Y_1. Y_1Y_2 is the height line for all points on the vertical face of the cube $B_1B_2C_2C_1$. Join Y_2 to **C.V.** intersecting B_1b_1 at B_2. From A_1 draw a vertical line to intersect a horizontal line drawn from B_2 at A_2.

$A_1B_1B_2A_2$ is the perspective view of the front face of the cube. Join A_2 to **C.V.** Draw *the ray* **c e** intersecting **p.p.** at c_1. From c_1 draw a projector intersecting Y_2 **C.V.** at C_2, and from C_2 draw a horizontal line intersecting A_2 **C.V.** at D_2. $A_2B_2C_2D_2$ is the perspective view of the top face of the cube.

From C_2 draw a vertical line to intersect Y_1 **C.V.** at C_1 and from D_2 draw a vertical line to intersect X_1 **C.V.** at D_1. Join C_1 to D_1 (if the working of the problem is correct this line will be horizontal). On referring to the figure the drawing may easily be completed.


c
d
b
40°
a
1 ft. 6 in
v_1
PLAN OF P.P.
b_1
a_1
C.V.
x
d_1
v_2
P.P.
A_2
X_2
B_2
D_2
C_2
V_1
C.V.
HEIGHT LINE
V_2
H.L.
C_1
B_1
D_1
A_1
G.L.
X_1
40°
50°
e

PROBLEM XVII.

A rectangular prism 5 ft. × 2 ft. 9 ins. × 1 ft. 6 ins. stands on the **G.P.** with its long edges vertical. Draw this prism in perspective when its nearest corner to **P.P.** (**A**) is 1 ft. on the left and 1 ft. 6 ins. from the **G.L.**, and its shortest edges recede at 40° with the **P.P.** towards the left. The height of the eye above the ground to be taken as 4 ft. and its distance from **P.P.** as 9 ft. 6 ins. Draw its perspective representation.

a b c d is the plan of the prism in position. Produce **b a** to intersect **p.p.** at **x**. From **x** draw a projector to intersect the **G.L.** at X_1. Obtain V_1, the vanishing point of the shortest edges of the prism. Join X_1 to V_1. Draw *the rays* **a e** and **b e** intersecting **p.p.** at a_1 and b_1 respectively. From a_1 draw a projector intersecting $X_1 V_1$ at A_1, this point is the perspective view of the nearest corner of the prism on the ground. Draw a projector from b_1 intersecting X_1V_1 at B_1. Obtain V_2, the **V.P.** of the edges of the prism that are 2 ft. 9 ins. long; as the shortest edges make 40° with the **P.P.** to the left, these will make 50° with the **P.P.** towards the right.

Join A_1 to V_2. Draw *the ray* **d e** intersecting **p.p.** at d_1, from d_1 draw a projector intersecting A_1V_2 at D_1. Join D_1 to V_1 and B_1 to V_2, these lines intersecting at C_1. Express the lines B_1C_1 and D_1C_1 by dotted lines.

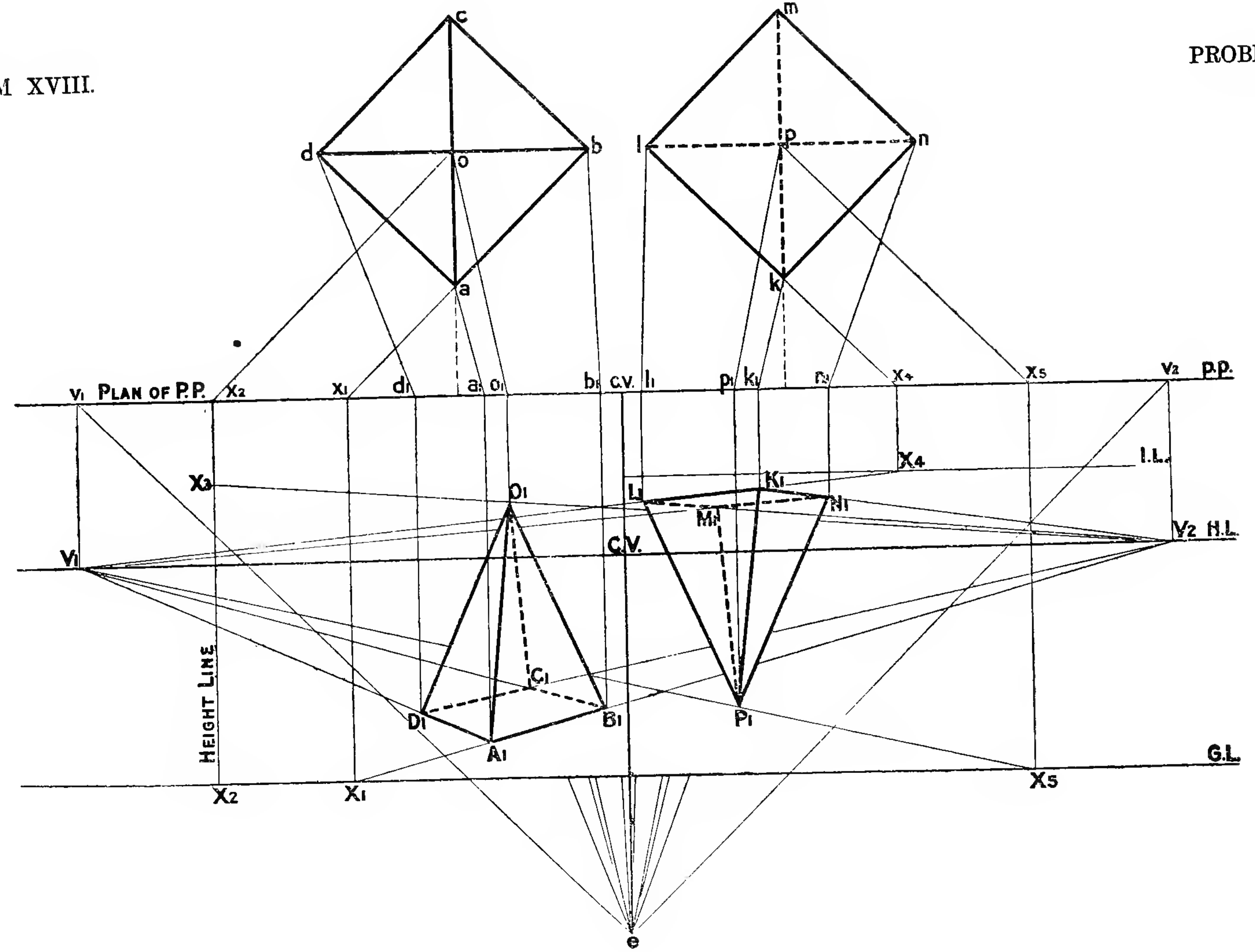
$A_1D_1C_1B_1$ is the perspective view of the face of the prism which lies upon the ground.

Obtain a point X_2 on X_1x 5 ft. above X_1. X_1X_2 is the height line for the vertical face of the prism in which the edge A_1B_1 lies.

Join X_2 to V_1 intersecting the projector $a_1 A_1$ at A_2, and the projector b_1B_1 at B_2. Join A_2 to V_2 intersecting d_1D_1 at D_2. Join B_2 V_2 and D_2V_1 intersecting at C_2.

Join C_1 to C_2. (C_1C_2 will be vertical if the working is accurate.) B_1C_1, D_1C_1, B_2C_2, D_2C_2, and C_1C_2 are to be expressed by dotted lines, as these edges of the prism would not be visible to the spectator. The perspective view of the prism has now been completed.

PROBLEM XVIII. PROBLEM XIX.



PROBLEM XVIII.

A square pyramid, altitude 5 ft. 6 ins., stands on its base, the edges of which are 3 ft. 6 ins. long and make angles of 45° with the **P.P.** The nearest corner is 3 ft. to the left and 2 ft. from the **P.P.** Height of the eye above the ground 4 ft. Distance of eye in front of the picture plane 10 ft. Obtain its perspective representation.

a b c d o is the plan of the pyramid in position.

Produce **b a** to intersect **p.p.** at x_1. From x_1 draw a projector to intersect **G.L.** at X_1. Obtain V_2 the **V.P.** of **A B** and join X_1 to V_2. Draw *the rays* **a e** and **b e** intersecting **p.p.** at a_1 and b_1 respectively. From a_1 and b_1 draw projectors intersecting X_1 and V_2 at A_1 and B_1 respectively. Obtain V_1 the **V.P.** of **A D** and join A_1 to V_1.

Draw *the ray* **d e** intersecting A_1V_1 at D_1 and complete the perspective view of the base of the prism on the ground as at $A_1B_1C_1D_1$. From **o** draw a line parallel to **a b** intersecting **p.p.** at x_2, and from x_2 draw a projector intersecting **G.L.** at X_2. On X_2x_2 obtain a point X_3 the height of the apex of the pyramid above the ground (5 ft. 6 ins.).

Join X_3 to V_2. Draw *the ray* **o e** intersecting **p.p.** at o_1 and draw a projector from o_1 intersecting X_3V_3 at O_1.

O_1 is the perspective view of the apex of the pyramid. Join $A_1B_1C_1D_1$ to O_1 and the perspective representation of the pyramid is completed.

PROBLEM XIX.

A square pyramid of the same dimensions as the one in the previous problem stands with its apex on the ground 3 ft. to the right and 5 ft. 6 ins. from the **P.P.** The base is horizontal, and one side of it makes an angle of 45° with the **P.P.** towards the right. The positions of the **p.p., H.L., C.V.,** and eye are the same as in previous problem. Obtain its perspective representation.

k n m l is the plan of the pyramid in position. Draw a line **I.L.** parallel to **G.L.** the height of the base above the **G.P.,** *i.e.* 5 ft. 6 ins.; this line is the intersection with the **P.P.** of a horizontal plane which contains the base of the pyramid.

Produce **l k** to intersect **p.p.** at x_4. From x_4 draw a projector intersecting **I.L.** at X_4. Join X_4 to V_1. Draw the rays **k e** and **l e** intersecting **p.p.** at k_1 and l_1 respectively. From k_1 and l_1 draw projectors intersecting X_4V_1 at K_1 and L_1 respectively. Join K_1V_2.

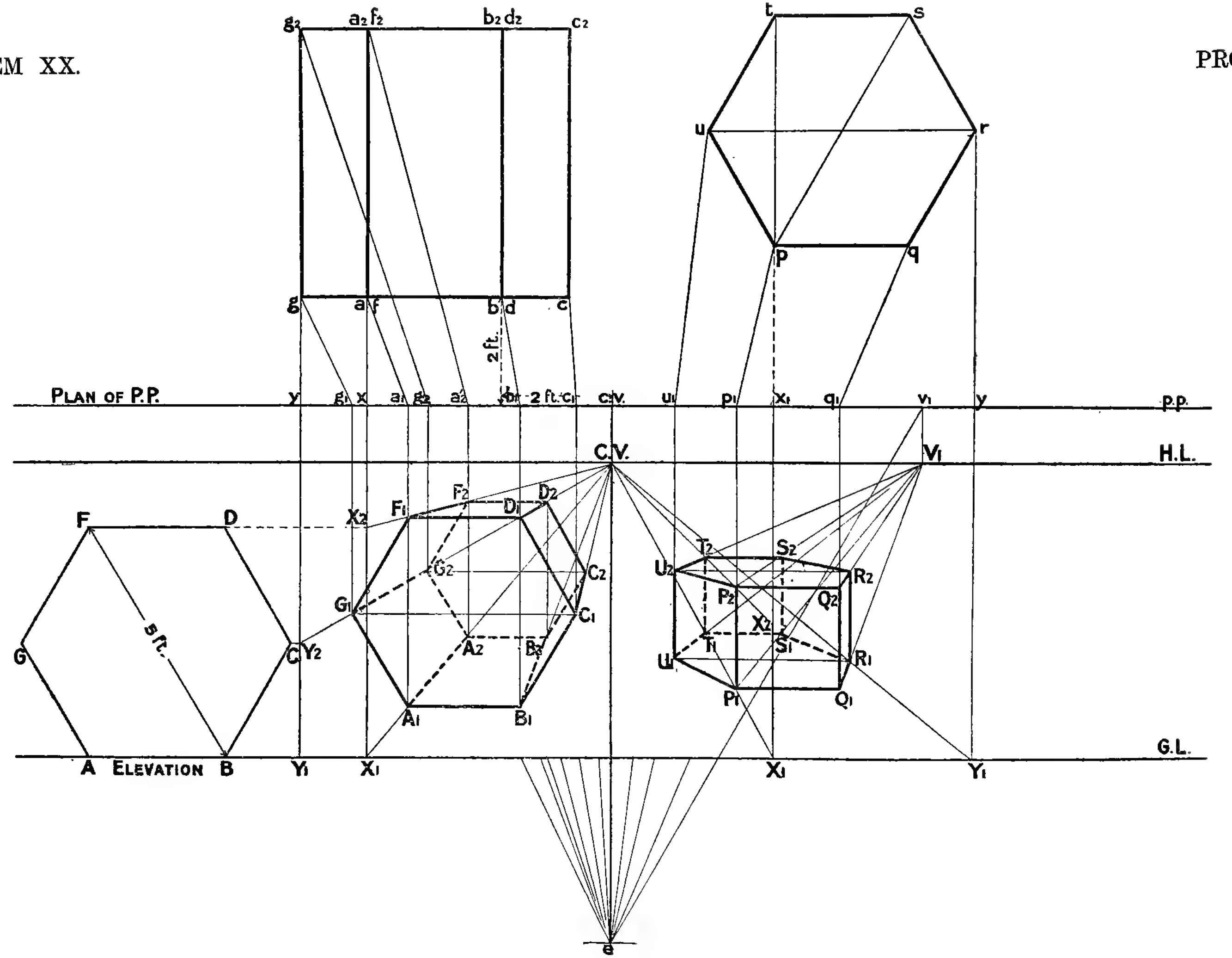
Through **n** draw *the ray* **n e** intersecting **p.p.** at n_1. From n_1 draw a projector intersecting K_1V_2 at N_1. Join N_1 to V_1 and L_1 to V_2 intersecting M_1.

$K_1N_1M_1L_1$ is the perspective view of the base of the pyramid.

From **p** draw a line parallel to **l k** intersecting **p.p.** at x_5, and from x_5 draw a projector intersecting **G.L.** at X_5. Join X_5 to V_1. Draw the ray **p e** intersecting **p.p.** at p_1, and from p_1 draw a projector intersecting X_5V_1 at P_1. Join $K_1N_1M_1L_1$ to P_1 and the perspective view of the pyramid is then completed.

PROBLEM XX.

PROBLEM XXI.



PROBLEM XX.

A hexagonal prism 5 ft. long rests on the ground on one of its rectangular faces. A hexagonal face, the diagonals of which are 5 ft. long, is parallel to the **P.P.** and lies on the left of the spectator. The corner **A** on the ground is 2 ft. on the spectator's left and 2 ft. within the picture.

Height of the eye above the ground 6 ft. 6 ins. Distance of the eye in front of **P.P.** 10 ft. Obtain the perspective representation of the prism.

$g\,c\,c_2g_2$ (on left side of plate) is the plan of the prism in position, and **A B C D F G** is an elevation of one of its hexagonal faces. Produce a_2a to intersect **p.p.** at **x**; project from **x** to intersect **G.L.** at X_1. Join X_1 to **C.V.**, then draw *the ray* **a e** intersecting **p.p.** at a_1. From a_1 draw a projector to intersect X_1 **C.V.** at A_1. Through A_1 draw a horizontal line A_1B_1 towards the right; join **b** to **e** intersecting **p.p.** at b_1, and from b_1 draw a projector to cut A_1B_1 at B_1.

Produce g_2g to intersect **p.p.** at **y**, project from **y** to **G.L.** at Y_1. Obtain a point Y_2 on Y_1y, the height of **G** above the ground (obtained from the elevation). Join Y_2 **C.V.** then **g e** intersecting **p.p.** at g_1. From g_1 draw a projector intersecting Y_2 **C.V.** at G_1. Join A_1G_1.

Obtain a point X_2 on X_1x, the same height above the ground as **F**. Join X_2 **C.V.** intersecting the projector A_1a_1 at F_1. Join F_1G_1.

From F_1 draw a horizontal line intersecting the projector B_1b_1 at D_1. Join **c** to **e** intersecting **p.p.** at c_1, and from c_1 draw a projector c_1C_1. From G_1 draw a horizontal line intersecting c_1C_1 at C_1. Join B_1C_1, C_1D_1, and D_1F_1. $A_1B_1C_1D_1F_1$ is the perspective view of the nearer hexagonal face. Join g_2e intersecting **p.p.** at g_2'; project from g_2' to intersect Y_2 **C.V.** at G_2. Draw a_2e_1 intersecting **p.p.** at a_2'; project from a_2' to intersect X_1 **C.V.** and X_2 **C.V.** at A_2 and F_2 respectively.

From A_2, F_2, and G_2 draw horizontal lines to intersect B_1 **C.V.**, D_1 **C.V.**, and C_1 **C.V.** at B_2, D_2, and C_2 respectively. (Note that B_2D_2 should be vertical.) Join C_2 to D_2 and B_2. G_2 to F_2 and A_2.

$A_2B_2C_2D_2F_2G_2$ is the perspective view of the back face of the prism, and A_1A_2, B_1B_2, C_1C_2, D_1D_2, F_1F_2, and G_1G_2 are the long edges.

PROBLEM XXI.

A hexagonal prism, side of hexagon 2 ft. 6 ins., axis 2 ft. 6 ins., stands on the ground on one of its hexagonal faces. The nearest corner to the spectator on the ground is 3 ft. within the picture and 5 ft. on the spectator's right.

The height of the eye and its distance from **P.P.** are the same as in Problem XV. Obtain the perspective representation of the prism when in this position.

p q r s t u (on right side of plate) is the plan of the prism in the required position.

Produce **t p** to intersect **p.p.** at x_1, project from x_1 to cut the **G.L.** at X_1; join X_1 **C.V.** Draw the ray **p e** cutting **p.p.** at p_1, and from p_1 draw a projector cutting X_1 **C.V.** at P_1. From P_1 draw a horizontal line P_1Q_1 (towards the right); join **q e** intersecting **p.p.** at q_1, and from q_1 draw a projector q_1Q_1 to intersect P_1Q_1 at Q_1.

Find V_1, the vanishing point of **Q R**, **P S**, and **U T**, then join Q_1V_1. From **r** draw **r y** perpendicular to **p.p.** and cutting it at **y**. Determine the perspective representation of **Y R** by drawing a projector from **y** to intersect **G.L.** at Y_1 and joining Y_1 **C.V.** Then R_1, the intersection of Y_1 **C.V.** with Q_1V_1, is the perspective view of **R**. Through R_1 draw a horizontal line R_1U_1, join **u e** intersecting **p.p.** at u_1 and from u_1 draw a projector to intersect R_1U_1 at U_1. Join P_1U_1.

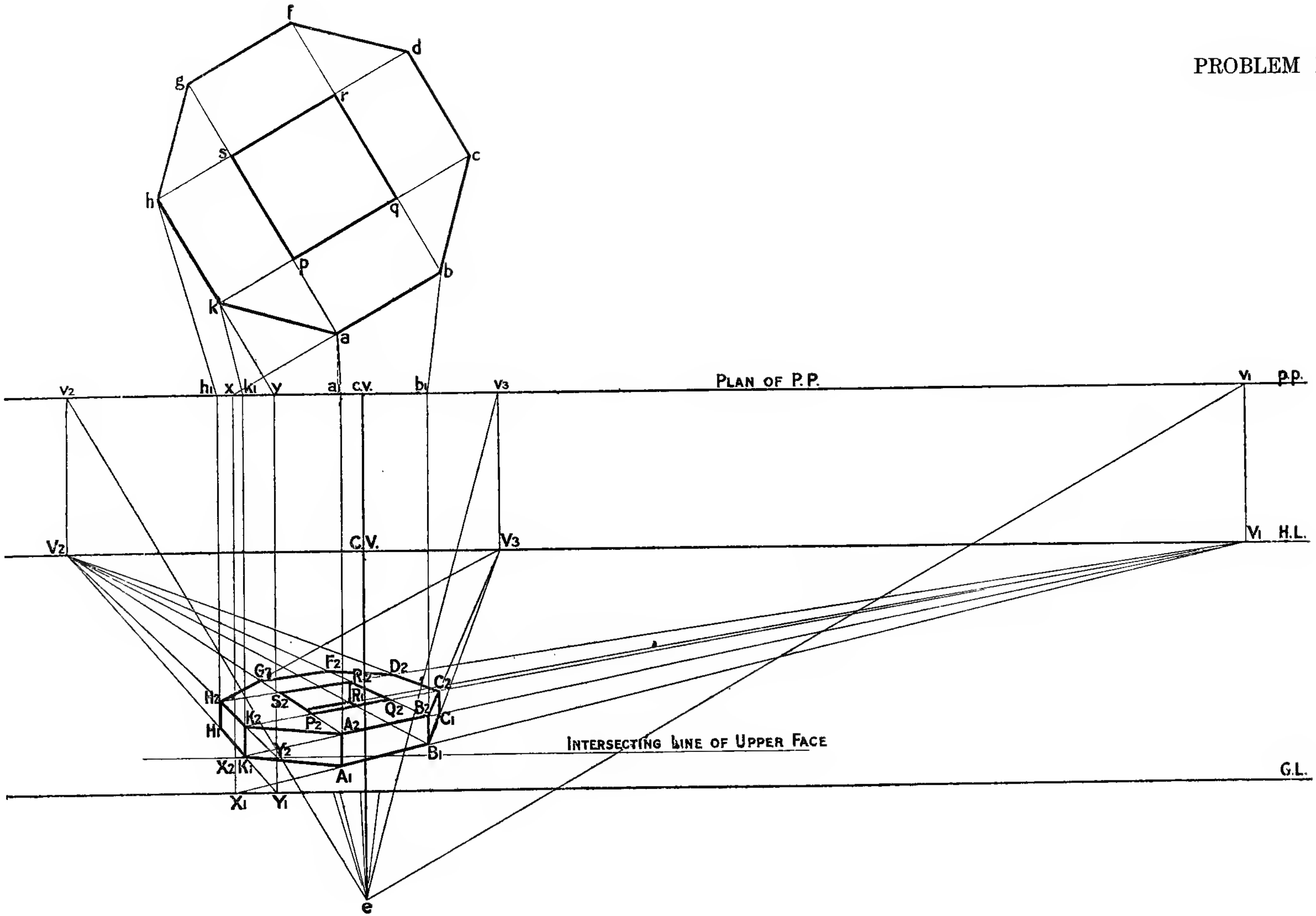
Join U_1 to V_1 intersecting X_1 **C.V.** at T_1. Join P_1 to V_1, and from T_1 draw a horizontal line intersecting P_1V_1 at S_1. Join S_1 to R_1.

$P_1Q_1R_1S_1T_1U_1$ is the perspective representation of the hexagonal face of the prism which is on the ground.

Obtain a point X_2 on X_1x_1 the height of the top face of the prism from the ground (*i.e.*, 2 ft. 6 ins.).

Join X_2 to **C.V.** intersecting the vertical line P_1p_1 at P_2. From P_2 draw a horizontal line intersecting Q_1q_1 at Q_2. Join Q_2 to V_1, and from R_1 draw a vertical line intersecting Q_2V_1 at R_2. Now from R_2 draw a horizontal line intersecting U_1u_1 at U_2. Join U_2 to V_1 intersecting P_2 **C.V.** at T_2. Join T_2 to T_1 (this line should be vertical). Join P_2 to V_1, and from T_2 draw a horizontal line intersecting P_2V_1 at S_2 and join S_2 to R_2. The perspective representation of the hexagonal prism is now easily completed.

PROBLEM XXII.



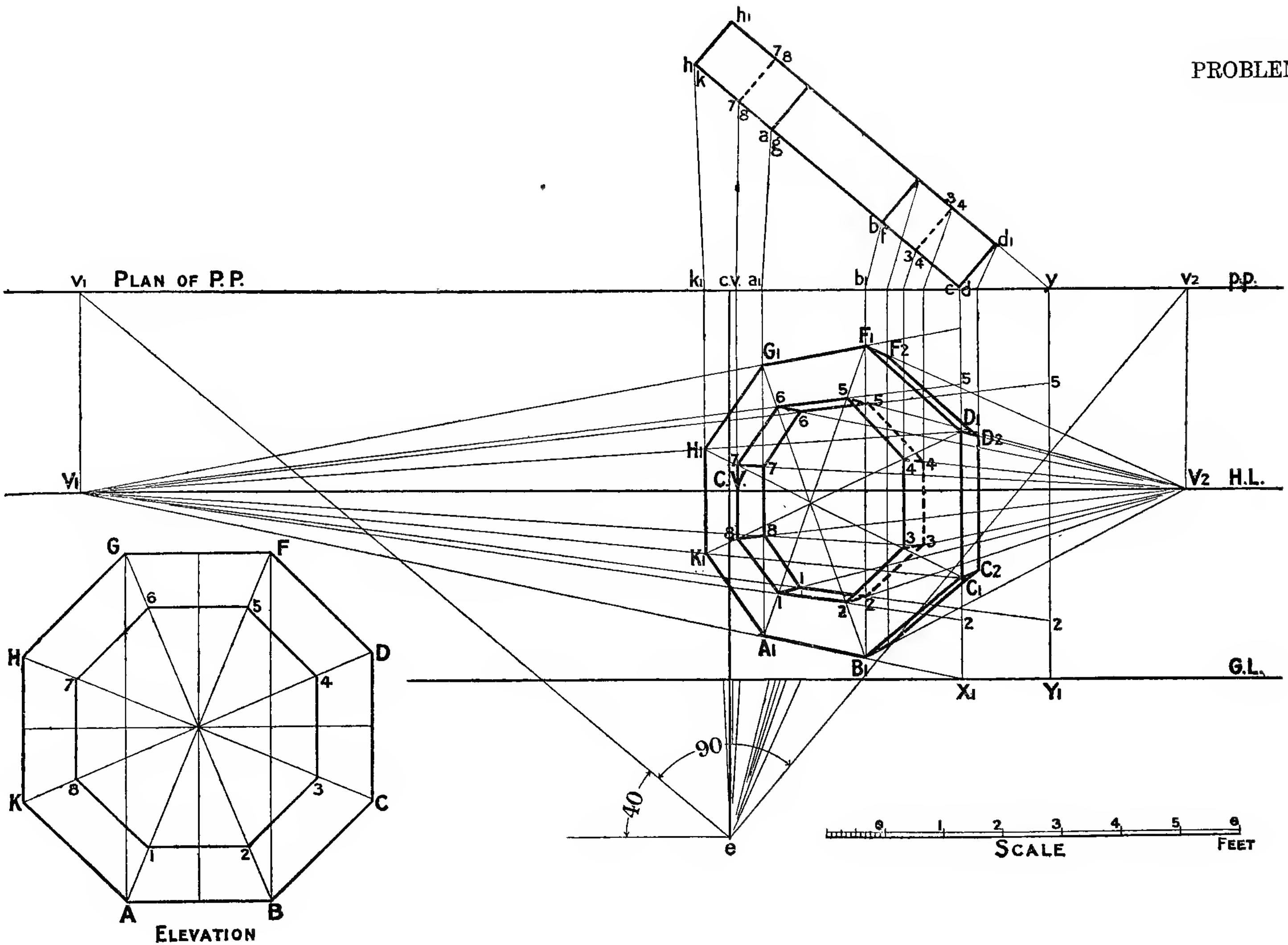
PROBLEM XXII.

The eye is 10 ft. 6 ins. distant from the **P.P.** and 5 ft. above the ground. Show the perspective representation of an octagonal slab **A B C D F G H** (side of octagon 2 ft. 6 ins.) when it lies on the ground with one corner (**A**) 6 ins. on spectator's left and 1 ft. 6 ins. within the picture. One edge **A B** makes 30° with **P.P.** towards the right. The slab is to be 9 ins. thick. Indicate also a hole 2 ft. 6 ins. side cut through the slab; the centres of the slab and hole being coincident, and one side of the hole being parallel to **A B**.

Place the slab in position as shown. Produce **b a** to cut **p.p.** at **x**, and draw a projector from **x** to intersect **G.L.** at X_1. Find V_1 the **V.P.** of **A B** and join X_1 to V_1. Draw *the rays* **a e** and **b e** cutting **p.p.** at a_1 and b_1 respectively. From a_1 and b_1 draw projectors cutting X_1V_1 at A_1 and B_1 respectively. Find V_2 the **V.P.** of **K H**, and V_3 the **V.P.** of **B C**. Produce **h k** to cut **p.p.** at **y**, from **y** draw a projector to cut **G.L.** at Y_1; join Y_1V_2. Draw *the rays* **h e** and **k e**, cutting **p.p.** at h_1 and k_1. From h_1 and k_1 draw projectors to cut Y_1V_2 at H_1 and K_1 respectively. Join A_1 to K_1, B_1 to V_3, and K_1 to V_1, intersecting B_1V_3 at C_1. Show the Intersecting Line of Upper Face (*i.e.*, a horizontal line 9 ins. above **G.L.**). Join X_2 (the intersection of $x\,X_1$ and Intersecting Line) to V_1 and Y_2 (the intersection of $y\,Y_1$ with Intersecting Line) to V_2. These lines determine $A_2\,B_2$ (above $A_1\,B_1$) and $K_2\,H_2$ (above $K_1\,H_1$) respectively. Join K_2 to A_2 then to V_1; now join $B_2\,V_3$ cutting K_2V_1 at C_2. From C_2 draw the vertical edge C_2C_1. Join H_2 to V_1, then C_2 to V_2, intersecting H_2V_1 at D_2. Join A_2 and B_2 to V_2, H_2 to V_3 cutting A_2V_2 at G_2, G_2 to V_1 cutting B_2V_2 at F_2, and finally F_2 to D_2. This completes the portion of the slab which is seen.

The lines indicating the plan of the hole will lie on **a g**, **b f**, **k c**, and **h d**. In the perspective drawing therefore $P_2Q_2R_2S_2$ is the upper edges of the hole. (These points have already been obtained.) Join B_1 to V_2; from R_2 draw a vertical line cutting B_1V_2 at R_1. Produce V_1R_1 until it meets P_2S_2.

All the visible portion of the figure has now been drawn.


PLAN OF P.P.
P.P.
H.L.
G.L.
C.V.
V1
V2
e
90
40
SCALE
FEET
ELEVATION

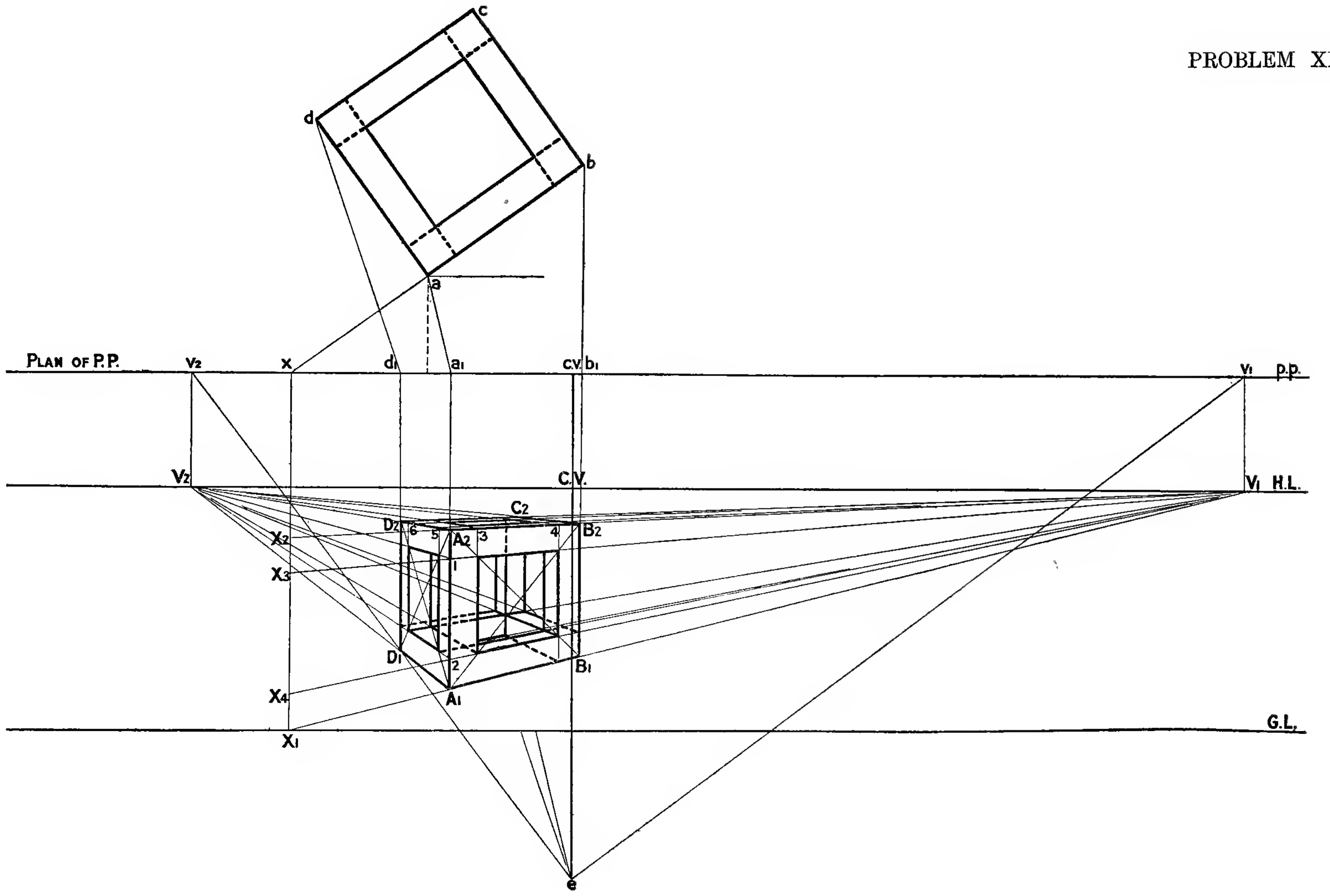
PROBLEM XXIII.

In the accompanying figure **A B C D F G H K** is the elevation of an octagonal frame the plan of which is shown in position. An edge of one of its rectangular faces touches the **P.P.** 4 ft. to the right of spectator, and the octagonal faces make an angle of 40° with the **P.P.** to the left. Height of the eye 3 ft. 3 ins., and its distance from the **P.P.** 9 ft. 6 in. Draw the perspective representation of the octagonal frame.

From **d** draw a vertical line intersecting **G.L.** at X_1. Obtain V_1, the vanishing point of the horizontal edges of the two octagonal faces of the frame. Join X_1 to V_1. Draw *the ray* **b e** intersecting **p.p.** at b_1, and from b_1 draw a projector intersecting X_1V_1 at B_1. Draw *the ray* **a e** intersecting **p.p.** at a_1, and from a_1 draw a projector intersecting X_1V_1 at A_1. Obtain a point C_1 on X_1d the height of **C** above the ground. (This is obtained from the elevation.) Join C_1 to V_1. Join B_1 to C_1. On X_1d obtain a point D_1 the height of **D** above the ground. Join D_1V_1, and complete the front octagonal face as shown.

Notice that the corners of the octagon, **1, 2, 3, 4, 5, 6, 7, 8,** lie on the diagonals of the larger octagon. As the short edges of the frame are perpendicular to the octagonal faces their vanishing point is the **V.P.** of lines at right angles to those vanishing at V_1. To find the height line for the points in the back face of the hexagonal frame, produce h_1d_1 to intersect **p.p.** at **y**, and from **y** draw a vertical line intersecting **G.L.** at Y_1. Y_1y is the required height line. From the knowledge obtained by working previous problems there should be no difficulty in completing the perspective view of the octagonal frame.

PROBLEM XXIV.



PROBLEM XXIV.

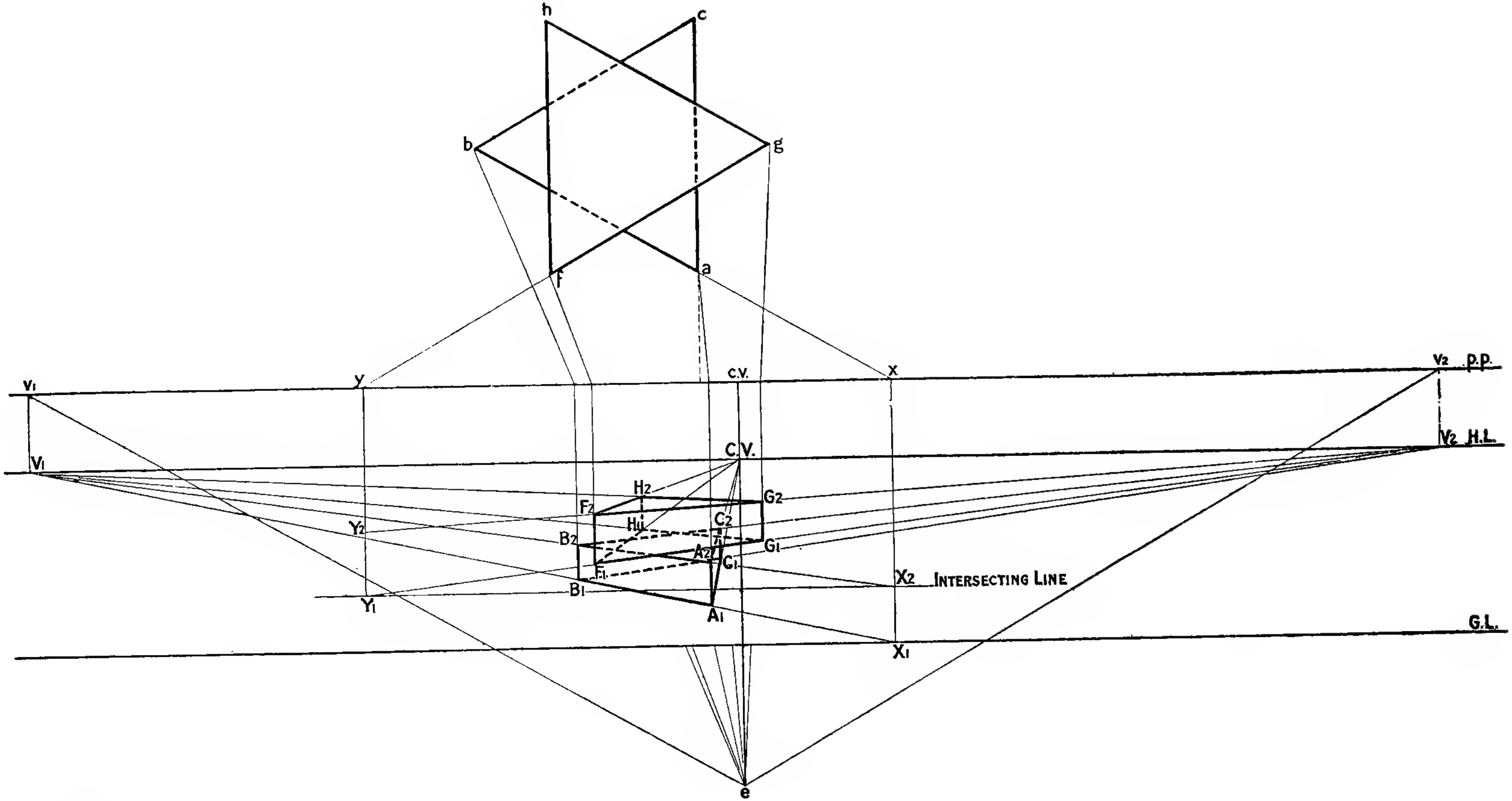
A skeleton cube (4 ft. edge, material 9 ins. square in section) stands with a face on a horizontal plane 5 ft. below the eye. The nearest corner to the **P.P.** is situated 3 ft. on the spectator's left and 2 ft. within the picture, one of its vertical faces making an angle of 37° with the **P.P.** towards the right. Put it in perspective, the eye to be 10 ft. 6 ins. in front of the **P.P.**

In the accompanying plate **a b c d** represents the plan of the cube in position. Find V_1 and V_2 the **V.P.**'s of **A B** and **A D** respectively. Draw the perspective representation of the cube as if it were solid (see Problem XVII.). Indicate the diagonals of each visible face. (These diagonals pass through the corners of the smaller square on the same face.)

X_1X_2 is used to obtain the cube's height. Make X_1X_4 and X_2X_3 each equal to the thickness of the material, *i.e.*, 9 ins. Join X_4V_1 and X_3V_1, from this the inner square of the face $A_1B_1B_2A_2$ is easily obtained. Let X_4V_1 and X_3V_1 cut A_1A_2 at **2** and **1** respectively. Join **1** V_2 and **2** V_2. From the intersection of these lines with the diagonals of $A_1A_2D_2D_1$, the inner squares on that face may be easily completed. The upright lines of the two inner squares on being produced fix the positions **3, 4, 5,** and **6.** Join **3** and **4** to V_2 and **5** and **6** to V_1.

The cube may now be completed by using the construction indicated in the plate.

PROBLEM XXV.



h
c
b
g
f
a
v1
y
c.v.
x
v2
P.P.
V1
C.V.
V2
H.L.
H2
F2
G2
Y2
H1
C2
B2
G1
A2
C1
F1
X2
INTERSECTING LINE
Y1
B1
A1
G.L.
X1
e

PROBLEM XXV.

The accompanying plate gives the plan of two equal and similar triangular slabs in position (sides of triangle 3 ft. 9 ins., thickness of slabs 1 ft. 3 ins.). The lower slab lies upon the ground plane, and the corner **A** is 9 ins. on the left of the spectator, and 2 ft. 3 ins. from the ground line, and the edge **A B** recedes towards the left at an angle of 30° with the ground line. Represent these two solids in perspective, the eye being 8 ft. 3 ins. distant from the **P.P.** and 3 ft. 9 ins. above the ground plane.

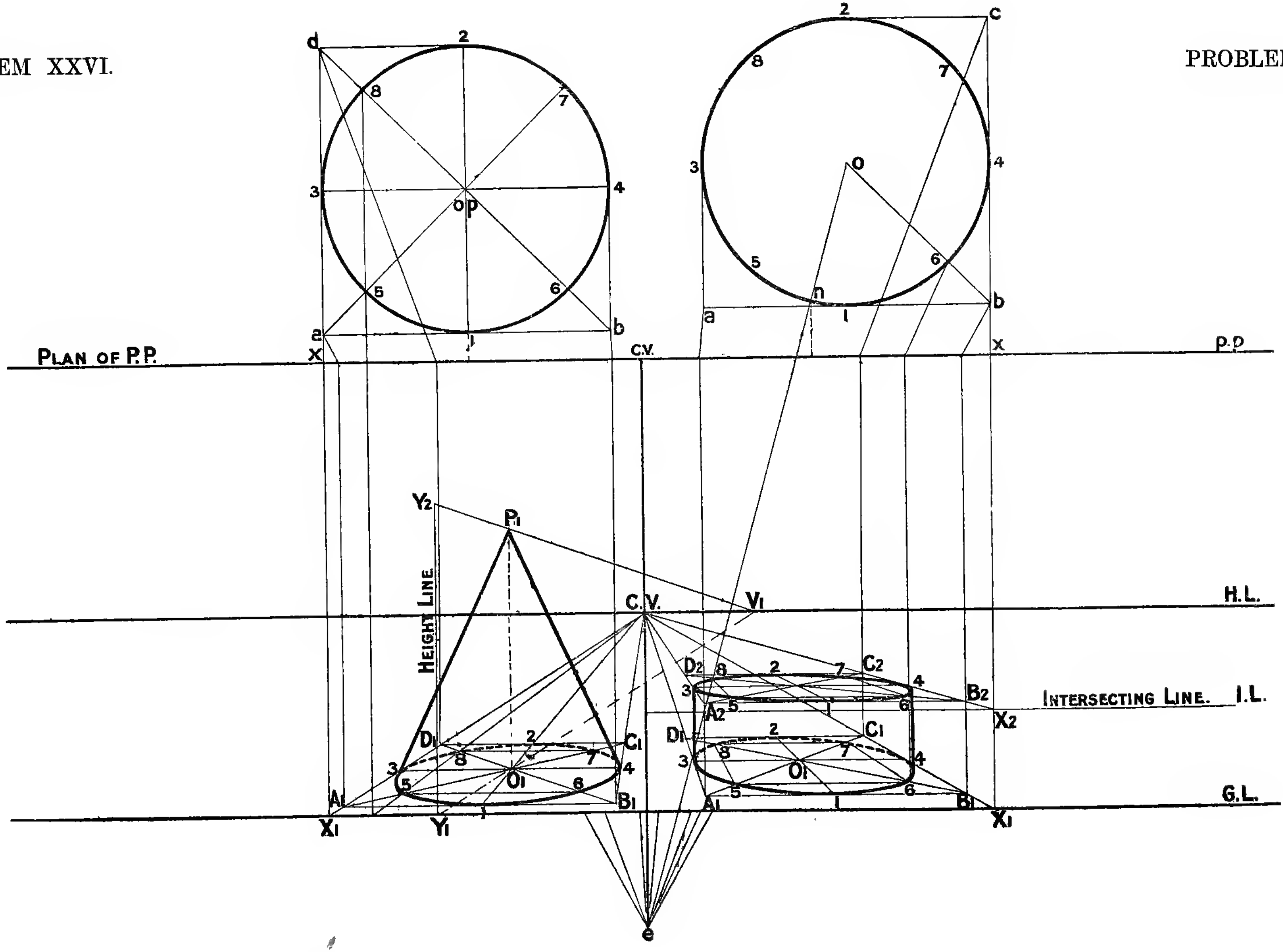
Observe that **AC** and **FH** are perpendicular to the **P.P.** and hence, in perspective, vanish at the **C.V.** Find V_1 and V_2 the **V.P.**'s of **A B** and **F G** respectively. Produce **b a** to cut **p.p.** at **x**; from **x** draw a projector to cut **G.L.** at X_1, join X_1V_1, draw *the rays* **a e** and **b e**, and from their intersections with the **p.p.** draw projectors cutting X_1V_1 at A_1 and B_1 respectively. Join A_1 **C.V.** and B_1V_2 intersecting at G_1. On X_1x make X_1X_2 equal to the thickness of one slab, *i.e.*, 1 ft. 3 ins., join X_2V_1 cutting the perpendiculars through A_1 and B_1 at A_2 B_2 respectively.

Join A_2 **C.V.** and B_2V_2 intersecting at C_2. This completes the lower block. The lower face of the upper block lies in a horizontal plane 1 ft. 3 ins. above the ground; Y_1X_2 is the intersecting line of this plane. Produce **g f** to cut **p.p.** at **y**; from **y** draw a projector to cut Y_1X_2 at Y_1, Y_1 is the intersection of **G F** produced with the **P.P.**

On Y_1y make Y_1Y_2 equal to the thickness of the upper block, *i.e.*, 1 ft. 3 ins. Join Y_1 and Y_2 to V_2. From **f** and **g** draw *the rays* to **e** and from their intersections with **p.p.** draw projectors cutting Y_1V_2 at F_1 and G_1 and Y_2V_2 at F_2 and G_2 respectively. Join F_1 **C.V.** and G_1V_1 cutting at H_1, also F_2 **C.V.** and G_2V_1 cutting at H_2. Join H_1H_2 by a dotted line and indicate those other portions of the figure that are not visible to the spectator by dotted lines.

PROBLEM XXVI.

PROBLEM XXVII.



CURVED SOLIDS.

PROBLEM XXVI.

A cone (diameter of base 5 ft., axis 5 ft. 6 in.) stands with its base upon the ground. The centre of its base is 3 ft. to the left of spectator and 3 ft. within the picture. Height of the eye 3 ft. 6 in. Distance of eye in front of **P.P.**, 10 ft.

Obtain the perspective view of the base of the cone on the ground (refer to Problem X.). Through O_1, the perspective view of the centre of the circle, draw the perspective representation of any line intersecting **G.L.** at Y_1 and vanishing at V_1 on **H.L.** From Y_1 draw a vertical line Y_1Y_2; so that Y_1Y_2 is equal in length to the height of the apex of the cone above the ground. Join Y_2V_1 and from O_1 draw a vertical line intersecting Y_2V_1 at P_1. P_1 is the perspective representation of the apex of the cone. To complete the perspective representation of the cone, from P_1 draw tangents to the ellipse **3514.**

PROBLEM XXVII.

A circular slab (1 ft. 9 ins. thick, circular faces 2 ft. 6 ins. radius) lies upon the ground. The nearest point to the eye of the face on the ground is situated 3 ft. on spectator's right and 1 ft. from the **G.L.** Draw its perspective representation, the height of the eye and its distance from **P.P.** being the same as in the previous problem.

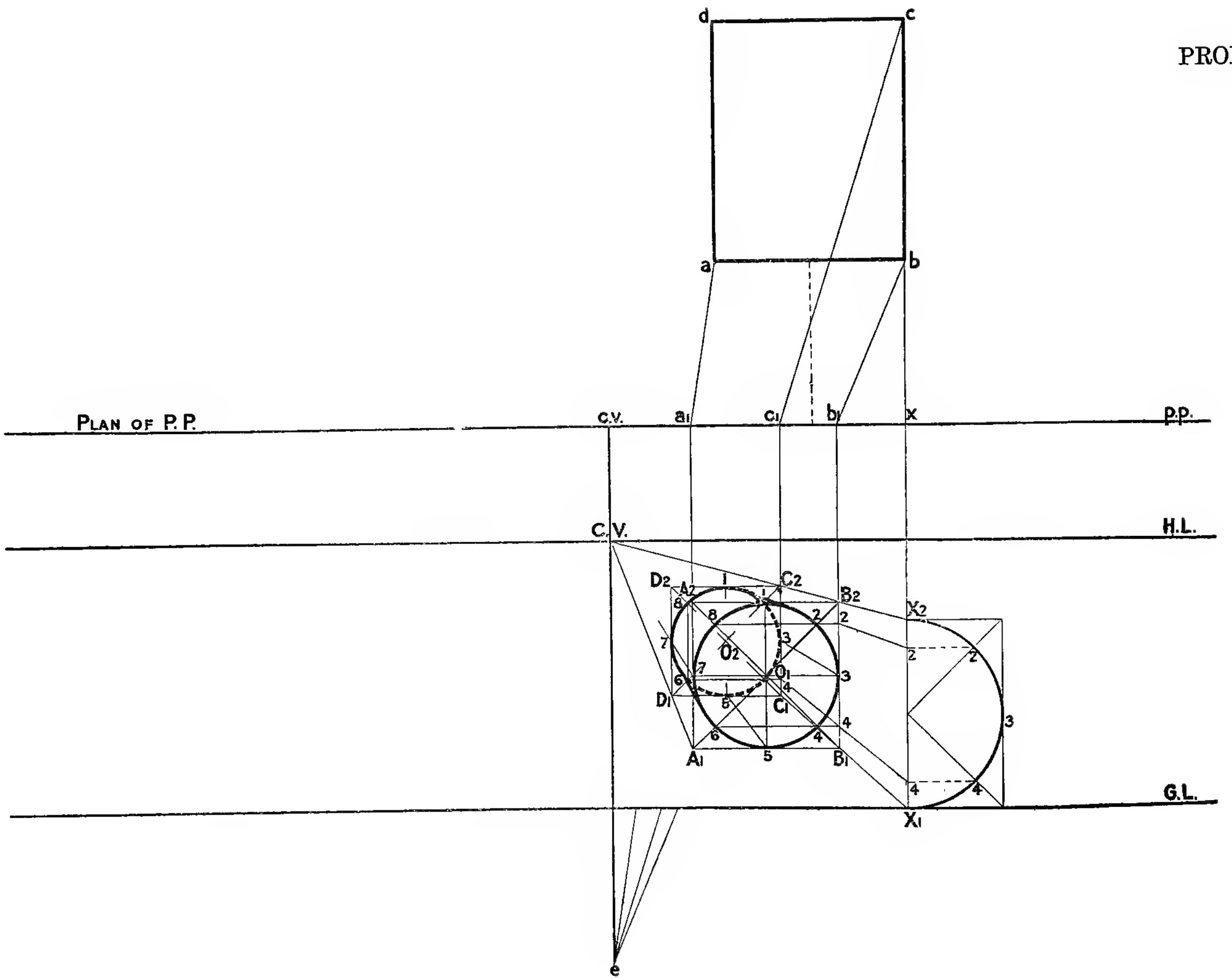
Obtain the plan of a point **N**, 3 ft. on spectator's right and 1 ft. beyond **P.P.** Carefully notice that this is the nearest point *to the eye* (not to the **P.P.**). Join **e** to **n** and produce **en** to **o** making **no** 2 ft. 6 ins. long. With **o** as centre and 2 ft. 6 ins. as radius, describe a circle 1, **4, 2, 3.** This completes the plan of the slab in position.

Find the perspective representation of the lower circle as in Problem X.

Obtain the perspective representation of the upper face, but in finding it use the Intersecting Line, **I.L.**, instead of the **G.L.** (**I.L.** is the intersection with **P.P.** of a horizontal plane containing the cylinder's upper face, and is therefore 1 ft. 9 ins. above **G.L.**)

Notice that the various points of the upper face are vertically above the corresponding points of the lower face.

PROBLEM XXVIII.



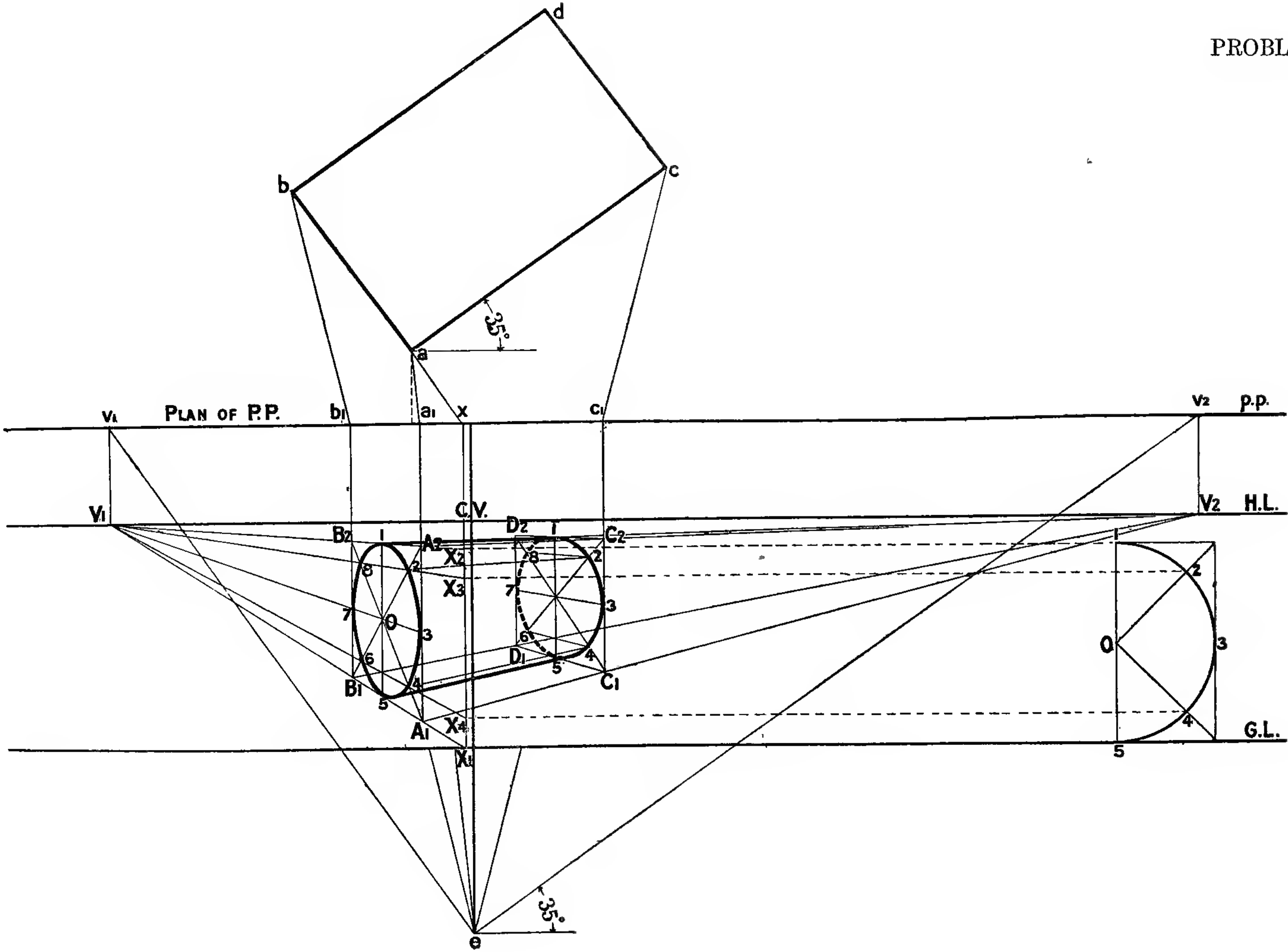
PROBLEM XXVIII.

A cylinder 4 ft. 6 in. long, diameter 3 ft. 6 ins., lies upon the ground with its axis horizontal and perpendicular to the **P.P.** The nearest point of contact with the ground of the nearer face is 3 ft. 9 ins. on the right and 3 ft. from the **P.P.** The eye is 5 ft. above the **G.P.** and 10 ft. from the **P.P.** Draw the cylinder in perspective.

In the accompanying plate **a b c d** is the plan of the cylinder in position. Produce **c b** to intersect **p.p.** at **x**, from **x** draw a projector cutting **G.L.** at X_1. Make X_1X_2 equal in length to the diameter of the cylinder (3 ft. 6 ins.). On this line describe a semicircle and on it obtain points **4**, **3**, and **2** as shown in figure. On X_1X_2 obtain the points **4** and **2** by drawing horizontal lines from points **4** and **2** on the semicircle. Join X_1, **4**, **2**, and X_2 to **C.V.**

Draw *the ray* **b e** intersecting **p.p.** at b_1, and from b_1 draw a projector intersecting X_2, **C.V.**, **2 C.V.**, **4 C.V.**, and X_1**C.V.** at B_2, **2**, **4**, and B_1 respectively.

Complete the perspective view of the square inclosing the front face of the cylinder ($A_1B_1B_2A_2$) and draw diagonals intersecting at O_1. Through O_1 draw a horizontal line intersecting A_1A_2 at **7** and B_1B_2 at **3** and also through the same point draw a vertical line intersecting A_1B_1 at **5**, and A_2B_2 at **1**. From **4** and **2** on B_1B_2 draw horizontal lines intersecting the diagonals; this determines the eight points for drawing the perspective view of the front face of the cylinder. Through these points draw an ellipse. Draw *the ray* **c e** intersecting **p.p.** at c_1, and from c_1 draw a projector intersecting B_2 **C.V.** at C_2 and B_1 **C.V.** at C_1. Now construct the perspective view of the square inclosing the more remote face of the cylinder. There will now be no difficulty in determining the eight points for drawing the ellipse forming the back face, and the perspective view of the cylinder will be completed by drawing the two common tangents to the ellipses as indicated in the plate.



d
b
c
a
35°
PLAN OF P.P.
v_1
b_1
a_1
x
c_1
v_2
p.p.
C.V.
V_1
V_2
H.L.
B_2
A_2
X_2
X_3
D_2
C_2
B_1
A_1
X_4
X_1
D_1
C_1
O
G.L.
e
35°

PROBLEM XXIX.

A cylinder 5 ft. 9 ins. long, diameter 3 ft. 6 ins., lies upon the ground with its axis horizontal and receding from the **P.P.** at 35° to the right. The nearest point to **P.P.** of the circumference of the end of the cylinder is 1 ft. to the left of **C.V.** and 1 ft. beyond the **P.P.** The eye is 4 ft. above the **G.P.** and 9 ft. from the **P.P.** Draw this cylinder in perspective.

In the accompanying plate **a c d b** is the plan of the cylinder in position. Produce **b a** to intersect **p.p.** at **x**, and from **x** draw a projector intersecting **G.L.** at X_1. Draw on **G.L.** an end elevation of cylinder (it is unnecessary to draw more than is shown on right of figure). Obtain points X_4, X_3, and X_2 on X_1x, the heights of **4**, **2**, and **1** respectively. These are found in the manner shown in figure.

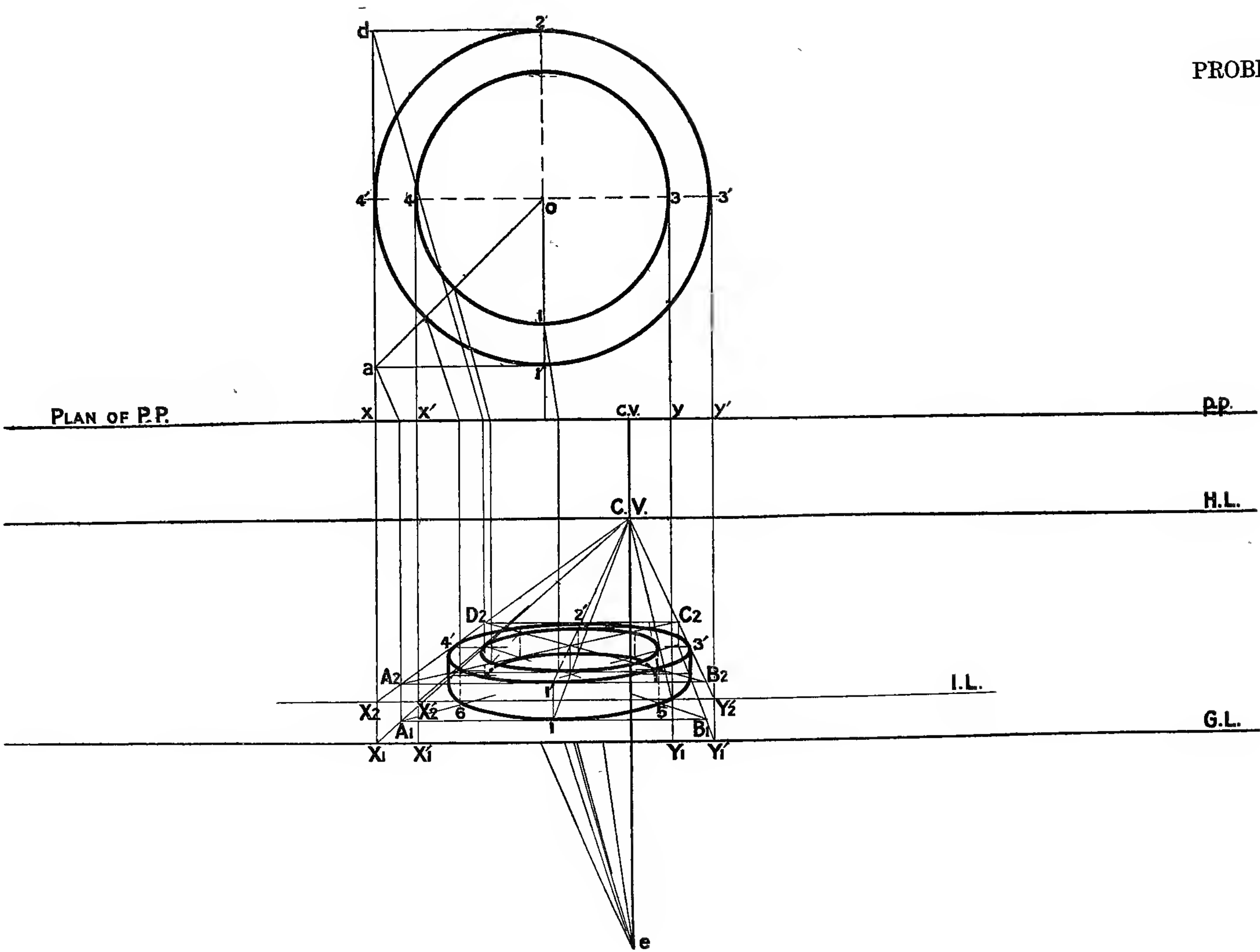
As the axis of the cylinder makes an angle of 35° with the **P.P.** to the right, the circular faces will make an angle of 55° with the **P.P.** to the left.

Find V_1 the **V.P.** of horizontal lines receding from the **P.P.** at 55° to the left, and V_2 the **V.P.** of horizontal lines receding at 35° to the right.

Join $X_1X_4X_3X_2$ to V_1. Draw *the ray* **a e** intersecting **p.p.** a_1, and from a_1 draw a projector intersecting X_2V_1 at A_2, and X_1V_1 at A_1.

Draw *the ray* **b e** intersecting **p.p.** at b_1, and from b_1 draw a projector intersecting X_2V_1 at B_2, and X_1V_1 at B_1. Join A_1B_2 intersecting X_3V_1 at **2** and **8**. Join A_2B_1 intersecting X_4V_1 at **4** and **6**, and A_2B_1 at **O**. Join **O** to V_1 intersecting B_1B_2 at **7**, and produce V_1O to intersect A_1A_2 at **3**. Through **O** draw a vertical line intersecting A_2B_2 at **1** and A_1B_1 at **5**. The perspective view of the front face of the cylinder is obtained by drawing an ellipse through points **1, 2, 3, 4, 5, 6, 7, 8**. Join A_1, B_1, and A_2 to V_2. Draw *the ray* **c e** intersecting **p.p.** at c_1; from c_1 draw a projector intersecting A_2V_2 at C_2 and A_1V_2 at C_1. Join C_1 to V_1 intersecting B_1V_2 at D_1 and join C_2 to V_1, and from D_1 draw a vertical line intersecting C_2V_1 at D_2. Join C_1D_2 and D_1C_2 and through the point of intersection of these diagonals draw a line to V_1 intersecting C_1C_2 at **3** and D_1D_2 at **7**. Join **2** in nearer face of cylinder to V_2 intersecting D_1D_2 at **2**, and join **4** in nearer face to V_2 intersecting C_1D_2 at **4**. Join **4** to V_1 intersecting D_1C_2 at **6**, and join **2** to V_1 intersecting C_1D_2 at **8**. Join **1** in front face to V_2 intersecting C_2D_2 at **1**, and from **1** draw a vertical line intersecting C_1D_1 at **5**. Through **1, 2, 3, 4, 5, 6, 7, 8**, draw an ellipse, and this will represent the second face of the cylinder. Draw two tangent lines to the curves as shown, and the perspective view of the cylinder is completed.

PROBLEM XXX.



d
2′
4′
4
o
3
3′
1
1′
a
PLAN OF P.P.
x
x′
C.V.
y
y′
P.P.
C.V.
H.L.
D2
2′
C2
4′
3′
A2
B2
I.L.
X2
X2′
6
5
Y2′
A1
1
B1
G.L.
X1
X1′
Y1
Y1′
e

PROBLEM XXX.

A ring 6 ft. in diameter, made of material 9 ins. square in section, lies with one of its faces on the ground. The nearest point of the ring to **P.P.** is 1 ft. 6 ins. on the spectator's left and 1 ft. from the **P.P.** The eye is 4 ft. above the **G.P.** and 9 ft. 6 ins. from the **P.P.** Draw the ring in perspective.

In the accompanying plate the plan of the ring is shown in position with the construction lines for obtaining the requisite points of the circles. In the plate a number of the lines used in working the problem are shown broken off in order to make the plate less confusing.

Draw a horizontal line **I.L.** 9 ins. above **G.L.** This is the intersection with the **P.P.** of the horizontal plane containing the top face of the ring.

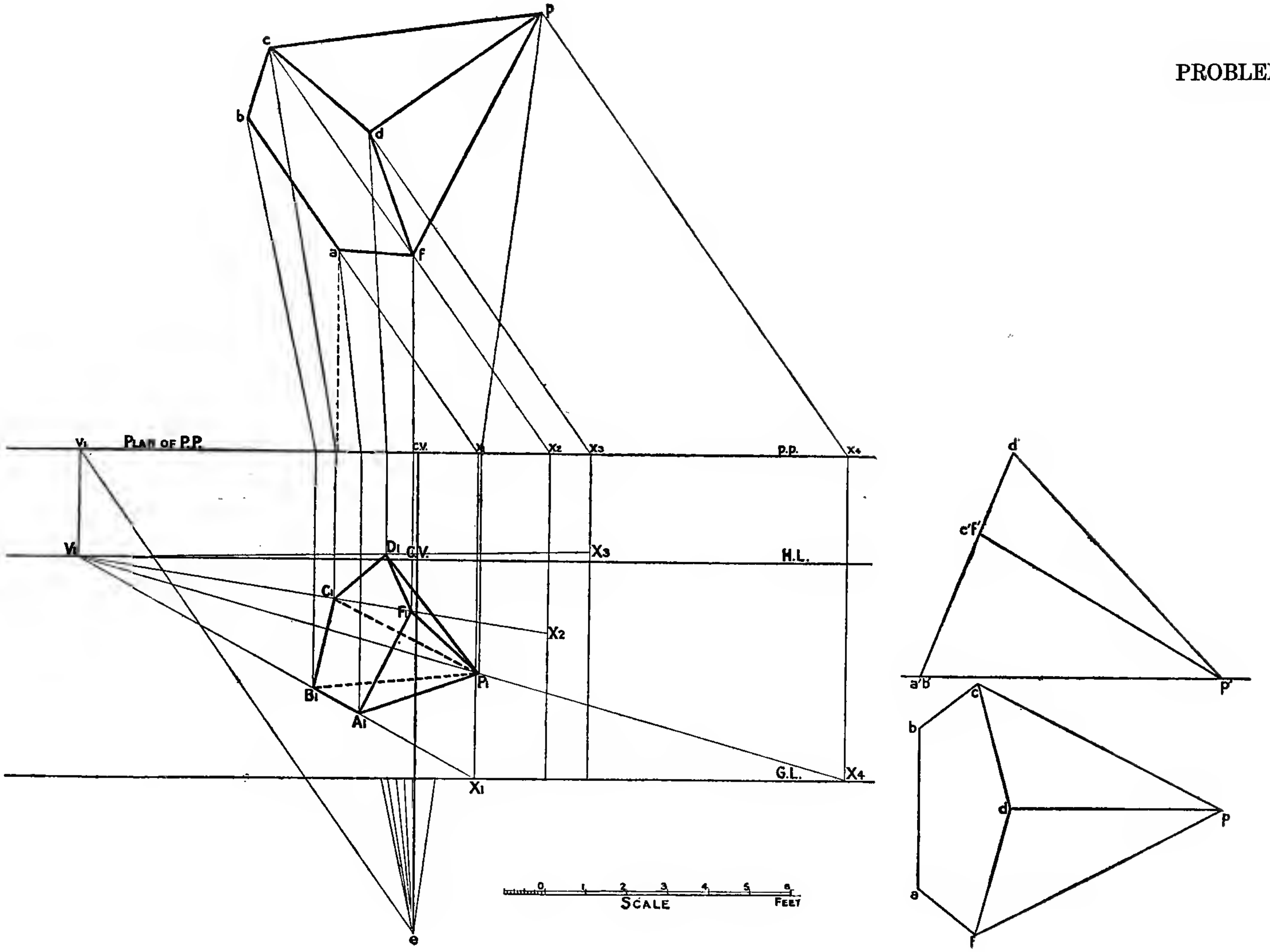
From 3′ in the plan draw a line at right angles to and intersecting **p.p.** at **y′**; from **y′** draw a projector cutting **I.L.** at Y_2 and **G.L.** at Y_1'. From **4′** draw a line perpendicular to and intersecting **p.p.** at **x**; from **x** draw a projector cutting **I.L.** at X_2 and **G.L.** at X_1. Join Y_2' and X_2 to **C.V.** and complete the perspective view of the outer top circle in a similar manner to that employed in Problem X., but using **I.L.** instead of **G.L.** Now draw the perspective view of the points of intersection of the top inner circle with the construction lines (**a o**, **1′o**, and **4′o**) in the plan. Draw an ellipse through these points and the perspective view of the top inner circle is obtained.

It is only necessary to draw those portions of the curves of the lower circles which are visible to the spectator.

Join X_1 to **C.V.**, and Y_1' to **C.V.**, and draw the requisite points on the **G.P.** corresponding to those obtained on the top face, and then draw the curves as shown in the figure.

Draw the tangents to the outer ellipses perpendicular to the **G.L.** and the perspective view of the ring is completed.

PROBLEM XXXI.



PROBLEMS TAKEN FROM RECENT EXAMINATION PAPERS.

The plates accompanying these problems are drawn to the appended scales, but students must work the problems to the required scales.

PROBLEM XXXI.

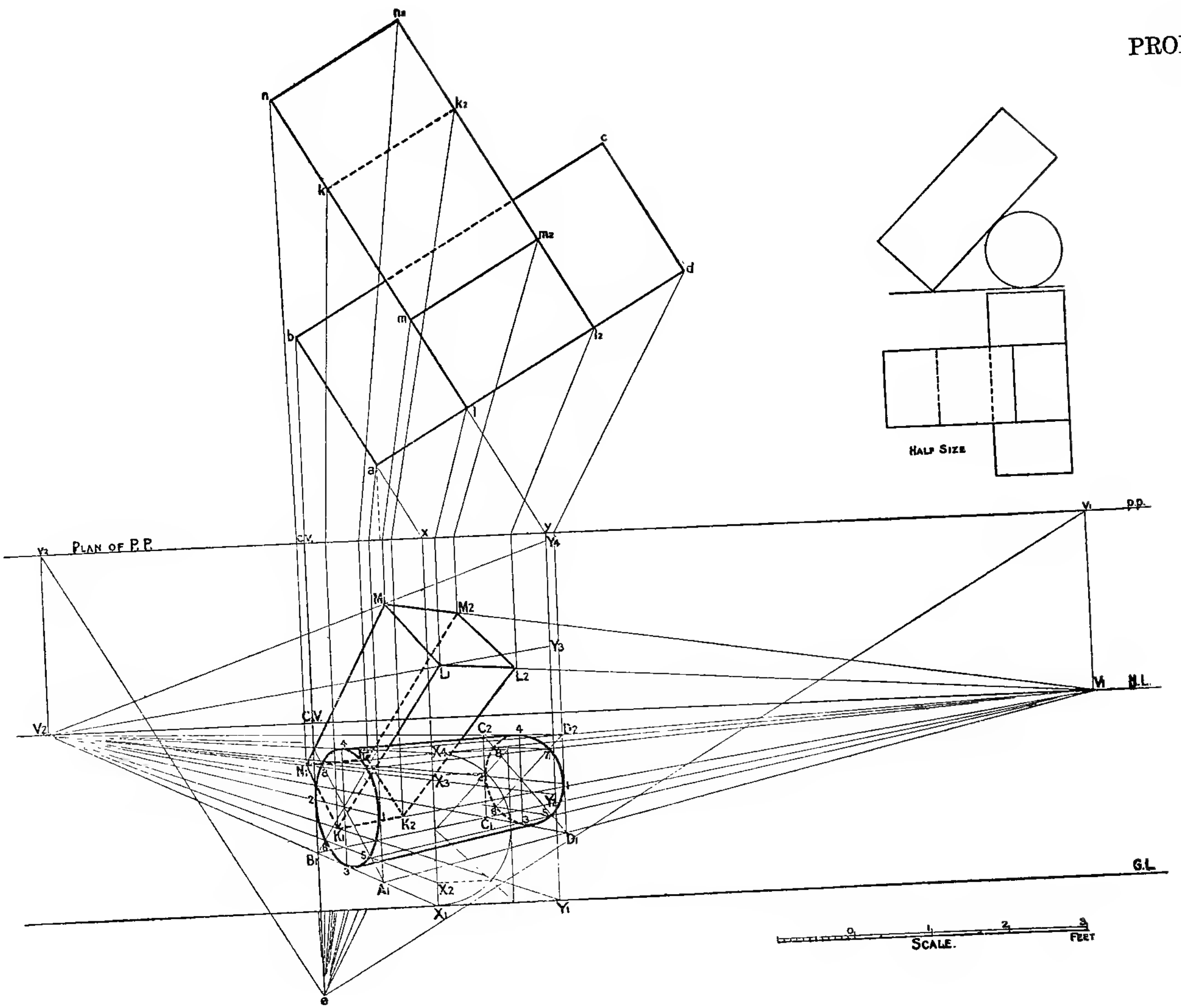
The accompanying plate gives the plan and elevation of a pentagonal pyramid lying on one of its sides. Show the same in perspective, when the point **A** is on the ground, 2 ft. to the left of the centre, and 5 ft. from the ground line; and the line **A B** is inclined at 55° to the picture plane towards your left.

The eye is to be 12 ft. distant from the picture plane, and 5½ ft. above the ground plane. Scale ½ in. to 1 ft.

Obtain the **V.P.** of lines vanishing at 55° to the left ($\mathbf{V}_1$). Produce **b a** to intersect **p.p.** at $\mathbf{x}_1$, and from $\mathbf{x}_1$ draw a projector to intersect **G.L.** at $\mathbf{X}_1$. Join $\mathbf{X}_1$ to $\mathbf{V}_1$ and obtain $\mathbf{A}_1\mathbf{B}_1$, the perspective view of **A B**.

Join **c** to **f** and produce **c f** to intersect **p.p.** at $\mathbf{x}_2$. From $\mathbf{x}_2$ draw a projector to intersect **G.L.**, and on the height line thus obtained mark a point $\mathbf{X}_2$, the height of **F** and **C** above the **G.P.** (this height is obtained from the elevation of the pentagon). Join $\mathbf{X}_2$ to $\mathbf{V}_1$. On $\mathbf{X}_2\mathbf{V}_1$ obtain the perspective views of **F** and **C**. From **d** draw a line parallel to **C F**, intersecting **p.p.** at $\mathbf{x}_3$, and from $\mathbf{x}_3$ draw a vertical line intersecting **G.L.** On that height line mark a point $\mathbf{X}_3$, the height of **D** above the **G.P.**, and join $\mathbf{X}_3$ to $\mathbf{V}_1$. On $\mathbf{X}_3\mathbf{V}_1$ obtain $\mathbf{D}_1$, the perspective view of **D**. Join $\mathbf{D}_1$ to $\mathbf{C}_1$ and $\mathbf{F}_1$ and join $\mathbf{C}_1$ to $\mathbf{B}_1$ and $\mathbf{F}_1$ to $\mathbf{A}_1$. $\mathbf{A}_1\mathbf{B}_1\mathbf{C}_1\mathbf{D}_1\mathbf{F}_1$ is the perspective view of the pentagonal base. From **p** draw a line parallel to **b a** intersecting **p.p.** at $\mathbf{x}_4$, and from $\mathbf{x}_4$ draw a projector intersecting **G.L.** at $\mathbf{X}_4$. Join $\mathbf{X}_4$ to $\mathbf{V}_1$. On $\mathbf{X}_4\mathbf{V}_1$ obtain $\mathbf{P}_1$, the perspective view of **P**. Join $\mathbf{P}_1$ to $\mathbf{A}_1\mathbf{B}_1\mathbf{C}_1\mathbf{D}_1\mathbf{F}_1$ and the perspective view of the pentagonal prism is completed.

PROBLEM XXXII.



PROBLEM XXXII.

The scale to be used in working this problem is 1 in. to 1 ft.

The accompanying plate gives the plan and elevation of a cylinder and a rectangular block; put these into perspective in their relative positions, with the cylinder vanishing at 30° to the picture plane towards the right, the block being behind the cylinder, and the nearest edge of the cylinder being 1 ft. to the right and 1 ft. within the picture, while it lies on a ground plane $2\frac{1}{2}$ ft. below the level of the eye.

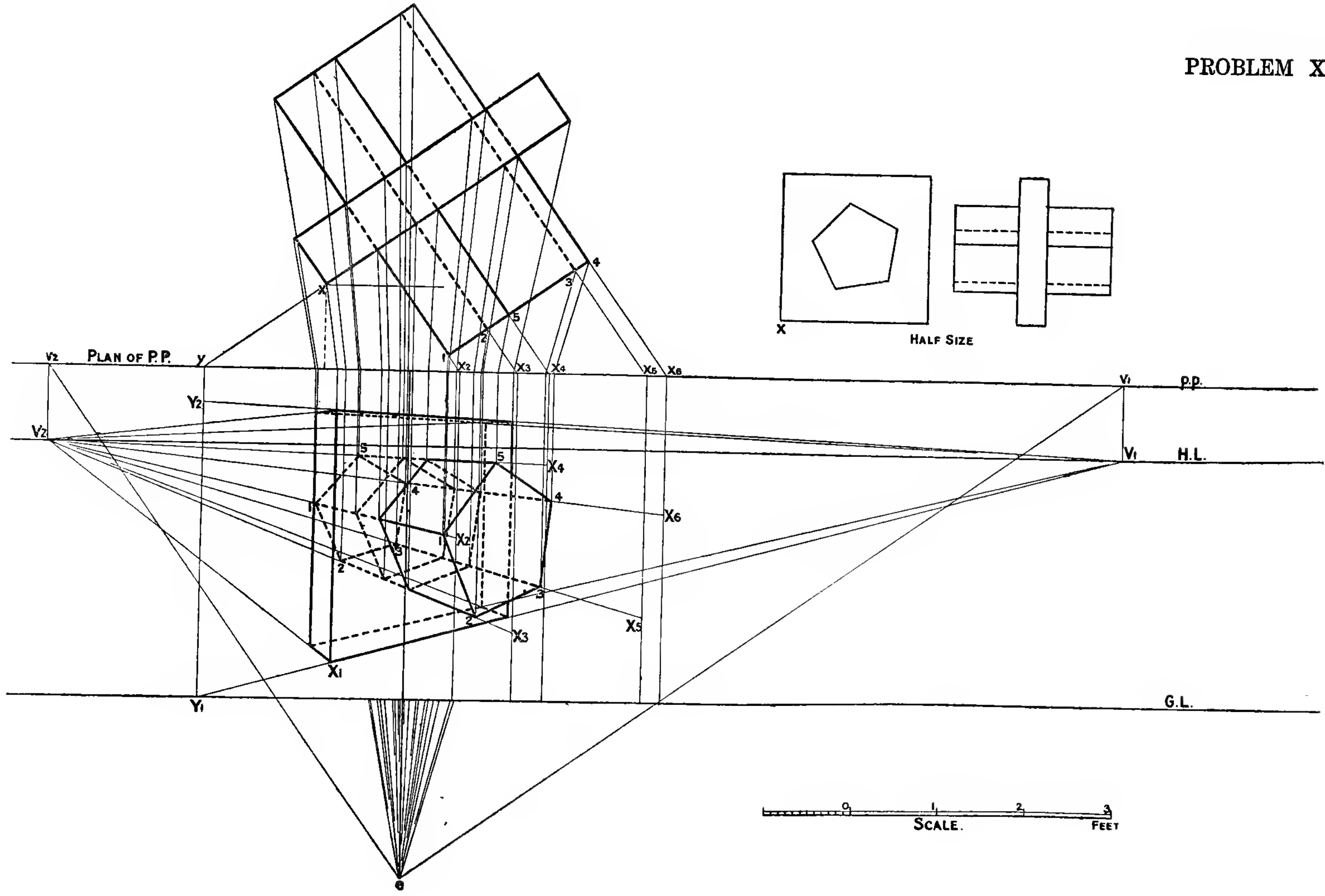
The distance of the spectator is 6 ft.

Begin this problem by obtaining the perspective representation of the nearer end of the cylinder (shown in plan at **a b**); in doing this, use the height line $x X_1$, obtained by producing **b a** to cut **p.p.** at **x**, and from this point drawing a projector.

Proceed to complete the cylinder by drawing in a vertical square surrounding the back circular end and then obtaining the usual eight points. In drawing the rectangular block, $K_1L_1M_1N_1$ should be first obtained, and this should be done in the following way. **K**, **L**, **M**, and **N** lie in the same vertical plane, which is shown in plan at **l m k n**. Produce **n l** to cut **p.p.** at **y**, and from **y** draw the projector $y Y_1$. $y Y_1$ is the intersection with the **P.P.** of the vertical plane containing **K L M N**. On this height line ($y Y_1$) mark off the heights of the various corners (obtained from the given elevation) as at Y_1, Y_2, Y_3, and Y_4; join these points to V_2. It will be evident that as the lines thus obtained have the same **V.P.**, they must be parallel to one another, and as they all intersect the **P.P.** on $y Y_1$, they will therefore pass through the corners **K L M N**. The position of the point in each of these lines is obtained by drawing the corresponding *ray* and projector.

As $K K_2$, $L L_2$, $M M_2$, and $N N_2$ are horizontal and parallel to the cylinder's axis, they must vanish at V_1; hence join K_1, L_1, M_1, and N_1 to V_1. The positions of K_2, L_2, M_2, and N_2 are settled by drawing *the rays* from k_2, l_2, m_2, and n_2, and from the intersection of each *ray* with **p.p.** drawing a projector until it intersects the corresponding line.

PROBLEM XXXIII.



PROBLEM XXXIII.

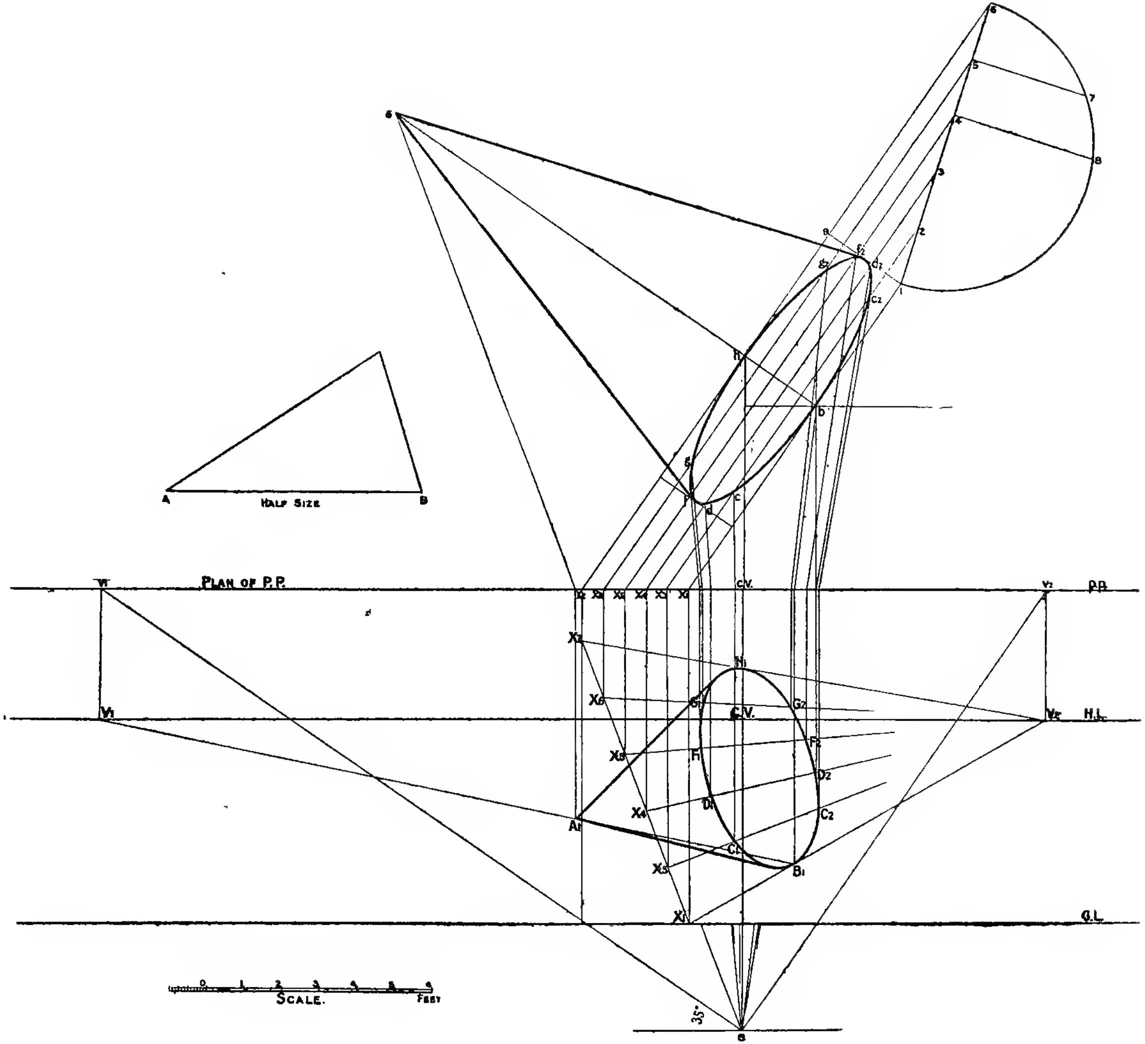
The accompanying plate gives the front and side elevations of a square slab penetrated by a pentagonal prism, put this into perspective, having the slab in vertical planes vanishing towards the right at 35° to the picture plane, the nearest point, **X**, being 1 ft. to the left, 1 ft. within, and 3 ft. below the eye. Distance of the eye 6 ft. The scale to be used in working the problem is 1 in. to 1 ft.

After placing the plan of the object in position, find the perspective representation of the slab, using Y_1Y_2 as a height line.

Produce the plans of the long edges of the pentagonal prism to cut **p.p.** at x_2, x_3, x_4, x_5, and x_6; from each of these points draw a projector. Using these projectors as height lines, on each measure the height of the corresponding point (obtained from the elevation).

Now join X_1, X_2, X_3, X_4, X_5, and X_6 to V_2. By drawing *rays* and projectors the front and back pentagonal faces may be obtained. The intersection of the prism with the slab can be found by drawing *the rays* from the plan of the intersection and drawing projectors to cut the corresponding long edge of the prism.

PROBLEM XXXIV.



PROBLEM XXXIV.

The accompanying plate gives the side elevation half-size of a right cone lying with its side on the ground. Put the same into perspective with the line of contact **A B** inclined towards the left at an angle of 35° to the **P.P.**; the nearer point **B** being 2 ft. to the right of the centre and 5 ft. from the ground line.

The scale to be used in working this problem is $\frac{1}{2}$-in. to 1 ft. The eye is to be 12 ft. by scale distant from **P.P.** and $5\frac{1}{2}$ ft. above the ground plane.

Obtain **b** 2 ft. to the right and 5 ft. from the **P.P.** From **b** draw **b a** at 35° to the left and equal in length to the slant edge of cone. **a** is the plan of the cone's vertex. The difficulty of the problem lies in finding the plan and then the perspective representation of the circle constituting the base of the cone.

In the figure the line **1,6** is an end view of the base, its inclination to **1,9** is obtained from the given elevation. **1,8,6** is a semicircle on **1,6** to aid in finding the plan of the base. Divide **1,6** into any number of parts, in the plate 5 equal parts have been taken. From each of these parts draw perpendiculars to **1,9**. The intersection of **6,9** with **b a** and **b** are the ends of the ellipse's minor axis. Points $c\,c_2$ and $g\,g_2$ are obtained by measuring off a length equal to **5,7** on each side of **a b** on $c\,c_2$ and $g\,g_2$ respectively. In a similar manner obtain the points $d\,d_3$ and $f\,f_2$. Draw the ellipse to pass through these points. The slant edges are obtained by drawing tangents to this ellipse from **a**.

Conceive a sheet of paper to be placed on the base of the cone. The lower edge of this paper would be on the ground along the line **1,b**, which, on being produced, cuts the **p.p.** at x_1, whose perspective representation is at X_1 on the **G.L.** The line $h\,x_2$ would lie on the paper and would cut **p.p.** at x_2 whose perspective representation is at X_2, its height being equal to **9,6**. It will be evident that as X_1 and X_2 are each on the **P.P.** and on the sheet of paper mentioned above, that the line in which this paper touches the **P.P.** must be along the line X_1X_2.

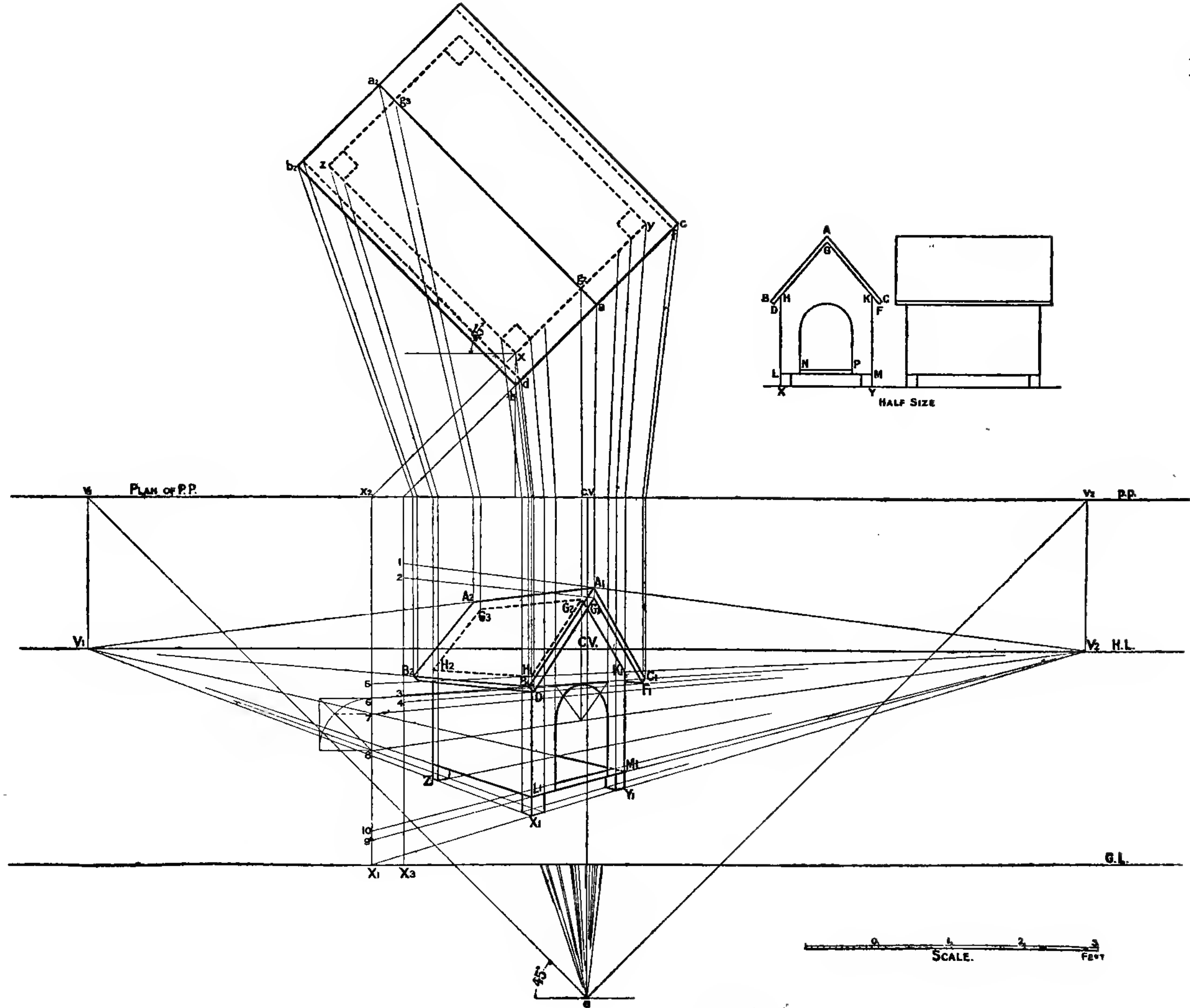
Again, as the lines $c\,c_2$, $d\,d_2$, $f\,f_2$, and $g\,g_2$ lie on the same sheet of paper they will also if produced cut the **P.P.** in X_1X_2. Hence produce c_2c, d_2d, f_2f, and g_2g to cut **p.p.** at x_3, x_4, x_5, x_6. Draw projectors to cut X_1X_2 at X_3, X_4, X_5, X_6 respectively. Then X_3, X_4, X_5, X_6 indicate where the above lines cut the **P.P.** Join each of these points to V_2 and thus obtain their perspective representation. Draw *the rays* **c e** and c_2e, and from their intersection with **p.p.** draw projectors to cut X_3V_2 at C_1 and C_2 respectively.

Similarly obtain the perspective representation of $B_1D_1D_2$, F_1F_2, G_1G_2, and H_1. Draw an ellipse through these points, and the perspective view of the cone's base will be complete.

Join B_1 to V_1 and **a** to **e**, and from the intersection of **a e** with **p.p.** project to cut B_1V_1 at A_1. A_1 is the perspective view of the vertex.

To complete the drawing, from A_1 draw two tangents to the base and the perspective view of the visible slant edges of the cone are obtained.

PROBLEM XXXV.



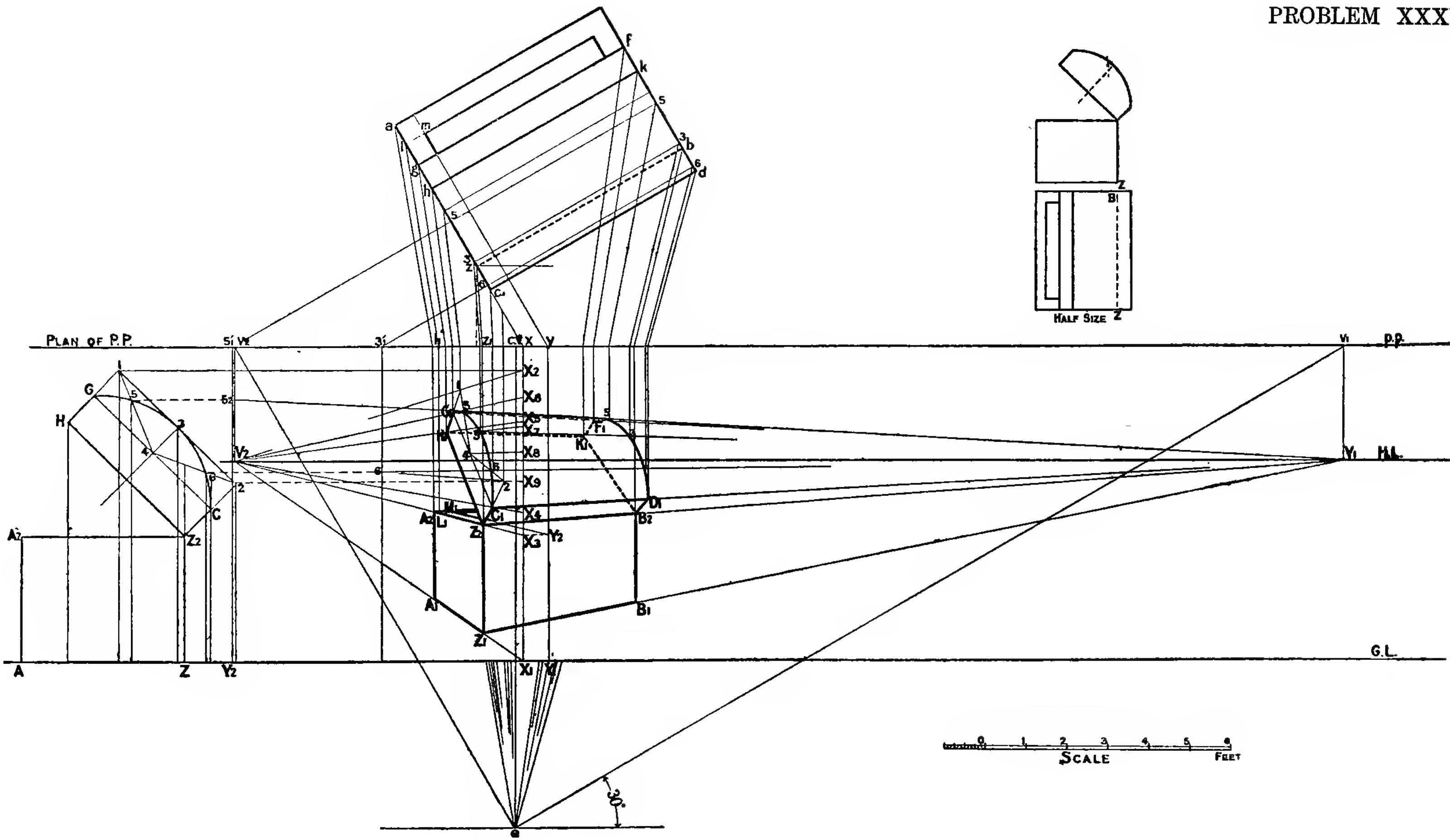
PROBLEM XXXV.

The accompanying plate gives the front and side elevations of a dog-kennel (half size); put this into perspective with the sides vanishing towards the left at 45° to the picture plane, the nearest point **X** on the ground to be 1 ft. to the left, 2 ft. within, and 3 ft. below the **H.L.** Distance of the spectator being 7 ft. Scale 1 in. to 1 ft.

Begin this problem by determining the perspective representation of the three supports that are visible to the spectator as at $\mathbf{X}_1$, $\mathbf{Y}_1$, and $\mathbf{Z}_1$. Let **c b** and **y x** be produced to intersect **p.p.**

$\mathbf{x}_2$ is the plan of the vertical straight line which is the intersection of the **P.P.** and the vertical plane which contains the front face of the kennel; it is on this line that the heights of the various points on that face must be marked. $\mathbf{X}_1\mathbf{9}$, $\mathbf{X}_1\mathbf{10}$, and $\mathbf{X}_1\mathbf{5}$ are the heights from the **G.P.** of **L M**, **N P**, and **H K** respectively. **8**, **7**, **6** on the height line are the heights of the points required for obtaining the perspective view of the semicircle at the top of the doorway. $\mathbf{X}_3$ is the intersection with the **P.P.** of the vertical plane containing the front edges of the roof, and **4**, **3**, **2**, and **1** are the heights of **D F**, **B C**, **G**, and **A** respectively. The perspective view of the sides of the kennel that are visible to the spectator are obtained by joining the points of the front face to $\mathbf{V}_1$, and on marking on these lines the perspective view of the required points at the back. Observe that one edge within the kennel is visible to the spectator as it is seen through the doorway. It is determined by joining $\mathbf{M}_1$ $\mathbf{V}_1$ and showing that portion which comes between the two upright sides of the doorway.

PROBLEM XXXVI.



PROBLEM XXXVI.

The accompanying plate gives the plan and elevation of a trunk (half size) with the lid partially opened. Put this trunk into perspective, with its back towards the spectator and its longest edges vanishing to the right at an angle of 30° to the picture plane, and the nearest corner **Z** on a ground plane 5 ft. below the eye, 1 ft. to the left of the centre, and 2 ft. within the picture. Make the distance of the eye from the picture plane 12 ft., working to a scale of $\frac{1}{2}$ in. to 1 ft.

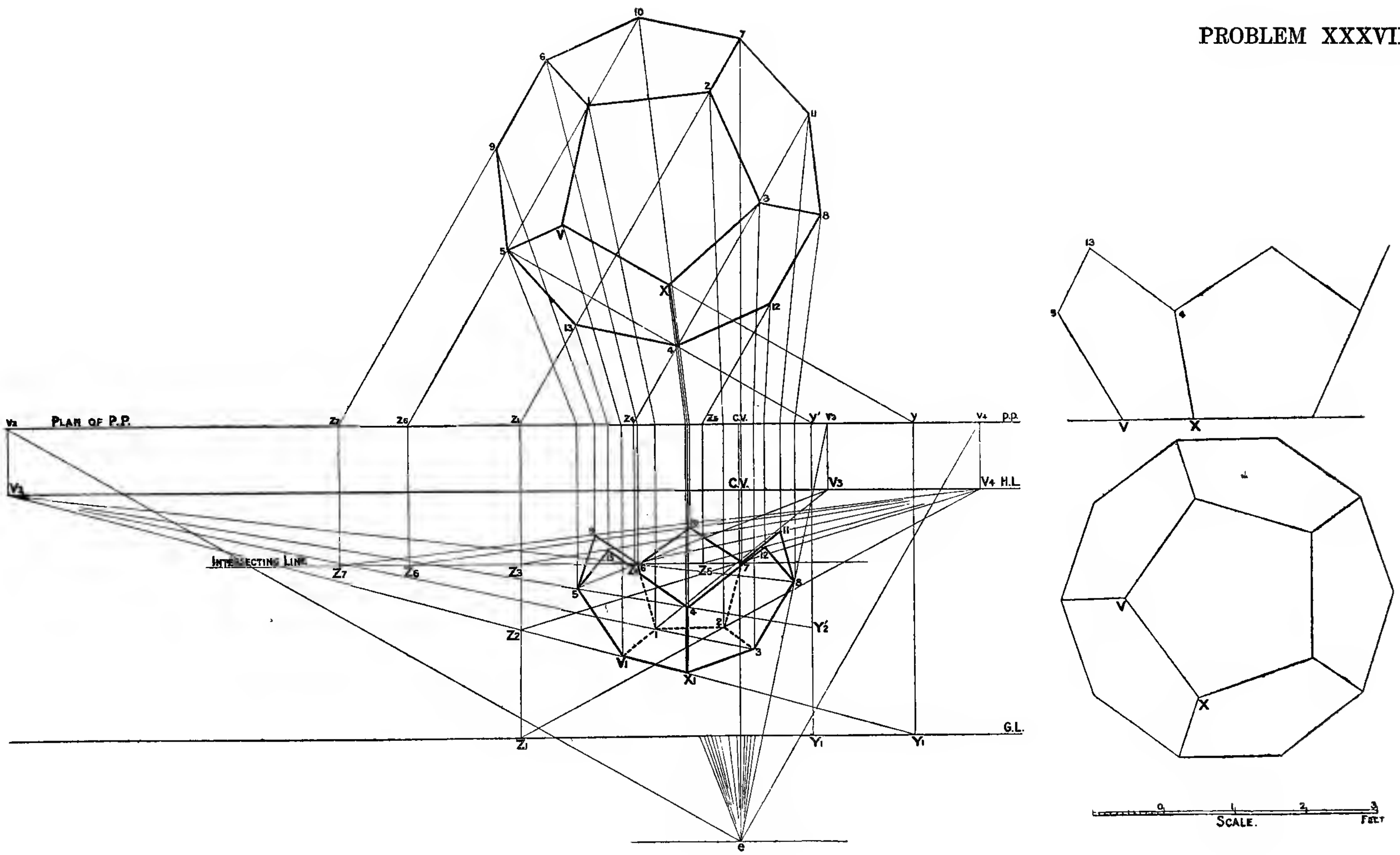
In the plate a plan of the trunk is shown in position, and on the left, upon the **G.L.**, is an elevation with the requisite construction lines from which the various heights of the curved portions of the lid are obtained. Find the vanishing points of the longer and shorter edges of the box and lid, *viz.*, V_1 and V_2 respectively. Produce **a c** to intersect **p.p.** at **x**, and from **x** draw a height line intersecting **G.L.** at X_1. $x X_1$ is the intersecting line of the **P.P.** with the vertical plane containing the near end of the box and lid inclined at 60° with the **P.P.** to the left. Join X_1, V_2, and on X_1x obtain X_3, X_4, X_9, X_8, X_7, X_5, X_6, and X_2, the respective heights of Z_2, **C**, **2**, **4**, **3**, **H**, **G**, and **1**; join these points to V_2, and draw the perspective view of the near shorter edges of the box as at $A_1Z_1Z_2A_2$. On X_4V_2, X_9V_2, X_8V_2, X_7V_2, X_5V_2, X_6V_2, and X_2V_2 obtain the perspective views of **C**, **2**, **4**, **3**, **H**, **5**, and **1** respectively (as shown in figure). Join Z_2H_1, Z_2C_1, H_11, cutting X_6V_2 at **G**, Z_22 cutting X_4V_2 at C_1.

To obtain **5** and **6** on the curve. Join **4**, **1** and **4**, **2**. From **5** and **6** in the plan draw lines parallel to the long edges of the box intersecting the plan of the curve on the opposite edge at **5** and **6** respectively. Produce **5**, **5** in the plan, intersecting **p.p.** at $5_1'$, and from $5_1'$ draw a height line intersecting **G.L.** at Y_2. On Y_25_1' obtain a point 5_2 the height of **5** from **G.L.** in the elevation, and join 5_2 to V_1. The intersection of 5_2V_1 with **4 1** will be the perspective view of **5**. In a similar manner obtain **6** (by producing **6,6** in the plan, &c.). Draw a curve from C_1 through **6,3,5** to **G**, and the perspective view of one end of the lid is completed.

Join Z_1, Z_2, C_1, **3**, G_1, H_1, A_2, to V_1, and there should now be no difficulty in finding the perspective view of the remaining parts of the box and lid by referring to the figure. The edge $G F_1$ of the lid is not visible to the spectator, so that a tangent to the curves must be drawn to represent a boundary of the visible portion of the lid. This tangent will vanish at V_1. The thickness of the box must now be shown.

From **l** in the plan draw *the ray* **l e** intersecting **p.p.** at l_1', and from l_1' draw a vertical line intersecting Z_2A_2 at L_1. Join L_1 to V_1. From **m** in the plan draw a line parallel to $a\,c_1$, intersecting **p.p.** at **y**, and from **y** draw a height line intersecting **G.L.** at Y_1. On Y_1y obtain a point Y_2, the height of the top edges of the box from the ground (X_1X_3 is the same as the required height). Join Y_2 to V_2, intersecting L_1V_1 at M_1, and the thickness is indicated as shown in figure.

PROBLEM XXXVII.



PROBLEM XXXVII.

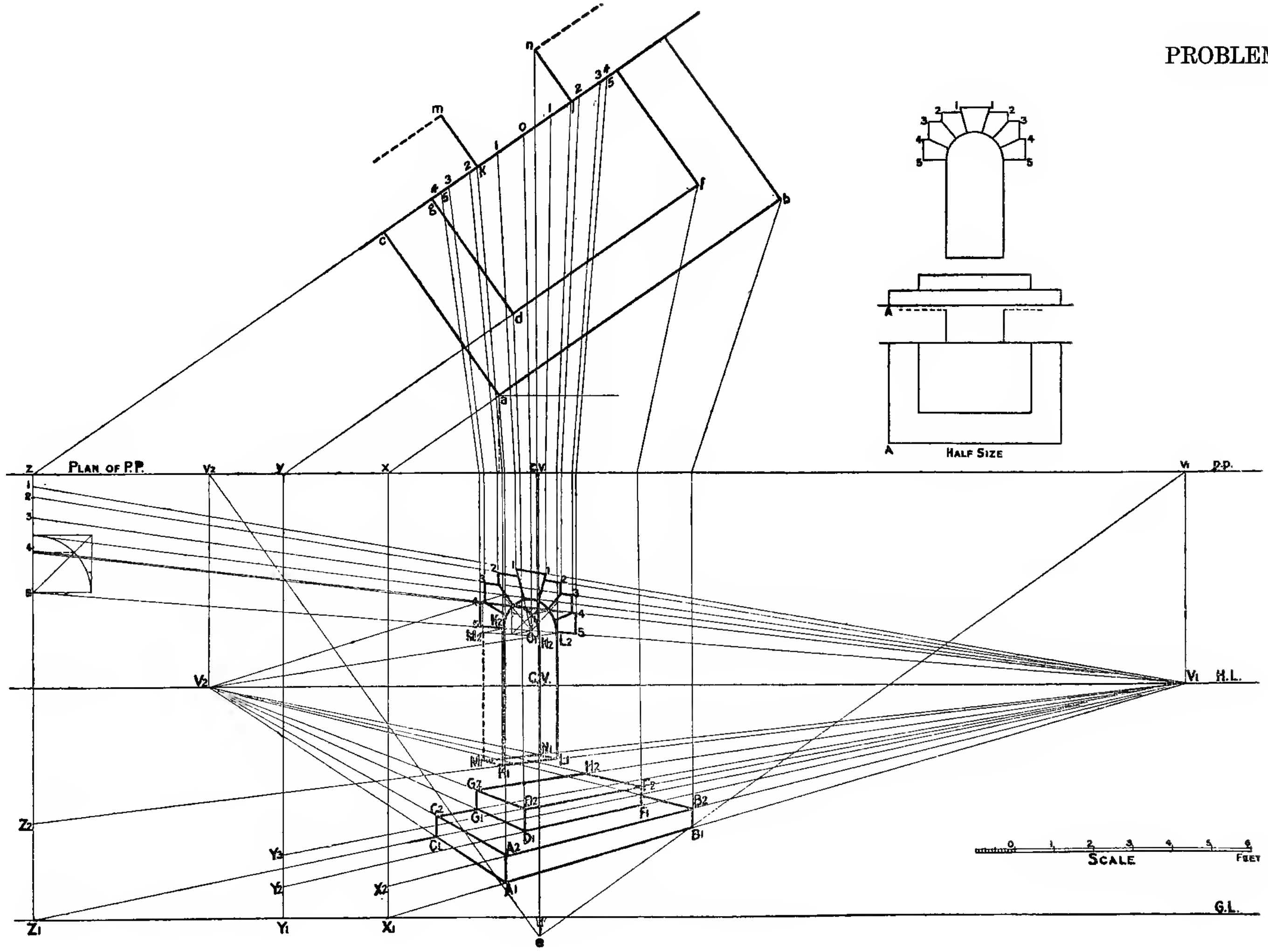
The accompanying plate gives the plan and elevation of a card-tray composed of six equal pentagons; put this into perspective, having the point **X** $3\frac{1}{2}$ ft. below the eye, 1 ft. to the left, and 2 ft. within the picture, and the side **X V** vanishing towards the left at 30° to the picture plane. The scale to be used in working this problem is 1 in. to 1 ft. The distance of the spectator is 6 ft.

The point **X** is $3\frac{1}{2}$ ft. below the eye, hence the eye may be considered as $3\frac{1}{2}$ ft. above the **G.P.** Having placed the plan in the required position, proceed to find the base **X, 3, 2, 1, V** in perspective. Produce **V X** to cut **p.p.** at **y**; from **y** draw a projector to cut **G.L.** at Y_1. Join Y_1 to V_2 (the **V.P.** of **X V**). Determine X_1 and V_1 by drawing *rays* and the corresponding projectors. Find V_3 the **V.P.** of **V,1**; join V_1V_3, then determine point **1** in perspective by drawing *the ray* and corresponding projector. As **1,3** is parallel to **V X**, therefore point **3** lies in $V_2 1$ produced, and may easily be obtained. Point **2** is obtained by drawing $2\,z_1$ perpendicular to **X V** cutting **p.p.** at z_1. V_4 is the **V.P.** of $z_1 2$; join Z_1 to V_4, and on this line obtain the perspective view of **2** by drawing *the ray* **2 e** and corresponding projector.

Now obtain **4** and **5** by producing **4,5** in the plan to cut **p.p.** at **y′** from which the height line $y'Y_1'$ is obtained. $Y_1'Y_2'$ is equal to the height of **4,5** above the **G.P.** **5,6** vanishes at V_3. A line joining **6 8** vanishes at V_2. From the points thus obtained the sides $4\,X_1$, $5\,V_1$, **6 1**, **7 2**, and **8 3** are determined. To obtain the top corners of the box **9, 10, 11, 12,** and **13** the height lines z_7Z_7, z_6Z_6, z_4Z_4, z_5Z_5, and z_3Z_3 respectively are used. Z_7, Z_6, Z_4, Z_5, and Z_3 lie in a horizontal line the same height above **G.L.** as **13** is above **X V** in the elevation, and this horizontal line indicates the intersection with the **P.P.** of a horizontal plane containing the top corners of the box.

The method for completing the figure is shown in the plate.

PROBLEM XXXVIII.



PROBLEM XXXVIII.

The accompanying plate gives a plan and elevation of a doorway and steps (half size); put these in perspective, having the long edges of the steps vanishing to the right at 35° to the picture plane, the point **A** being upon a ground plane 6 ft. below the eye, 1 ft. to the left of the centre, and 2 ft. within the picture. Make the distance of the eye from the picture plane 12 ft. Scale $\frac{1}{2}$ in. to 1 ft.

Produce **b a** and obtain $\mathbf{X_1x}$ the height line for the near step, and on it mark $\mathbf{X_2}$, the height of the step above the **G.P.** Join $\mathbf{X_2}$ to $\mathbf{V_1}$ (the **V.P.** of lines vanishing to the right at 35°) and complete the lower step. Produce **f d** in order to obtain the height line of the second step, and on that line mark a point $\mathbf{Y_2}$ the height of the lower edges of the second step from the ground, and mark $\mathbf{Y_3}$ the height of the upper edges of the same step. Draw the perspective view of the second step. Produce the plan of the face of the wall (**C**) and draw the height line $\mathbf{Z_1z}$. Obtain on this line the heights for drawing the curves of the front semicircular arch and the stones with which it is formed. By the aid of the figures and letters shown in the diagram the method by which this problem has been worked may be easily followed.

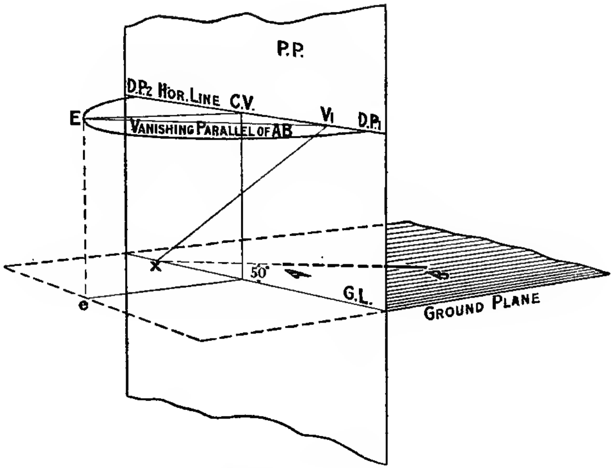

P.P.
D.P.2 Hor. Line C.V.
V1 D.P.1
E
Vanishing Parallel of AB
X
50°
G.L.
e
Ground Plane

Fig. 18.

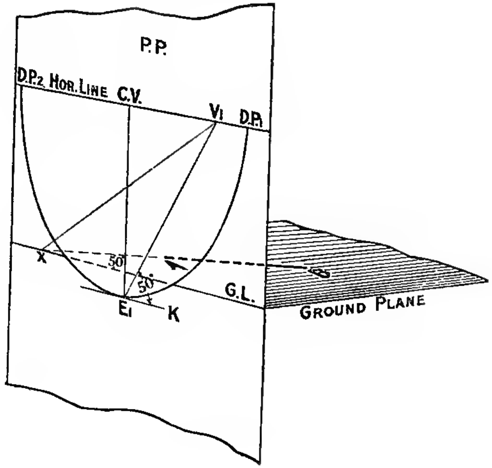

P.P.
D.P.2 Hor. Line C.V.
V1 D.P.1
X
50°
50°
G.L.
E1
K
Ground Plane

Fig. 19.

THE MEASURING POINT METHOD.

All the previous problems have been worked by the Plan Method, but they could have been worked by the Measuring Point Method. In elementary perspective, where the perspective representations of objects are obtained only by the use of horizontal and vertical planes, the plan method is the more convenient, but in advanced perspective, when other planes are introduced, the measuring point method is adopted.

It is advisable that in the elementary stage both methods should be thoroughly understood.

In fig. 18 the eye, **P.P.**, **G.P.**, and a line **A B** lying on the ground are shown in their relative position. **D** P_1 **E D** P_2 represents a semicircular piece of cardboard.

If this semicircular piece of cardboard is rotated about the **H.L.** into the **P.P.**, as in fig. 19, it is evident that **C.V.** E_1 would be equal to the distance of eye in front of the **P.P.**, and that if E_1K is a horizontal line, the angle V_1E_1K is equal to the angle that **A B** makes with the **P.P.** (*i.e.*, 50°). **X** V_1 is the perspective representation of **X A B** produced.

In the measuring point method the paper upon which the problems are worked represents the **P.P.**, the eye and vanishing parallels being rotated into the **P.P.**, and the plan of the **P.P.** is not used.

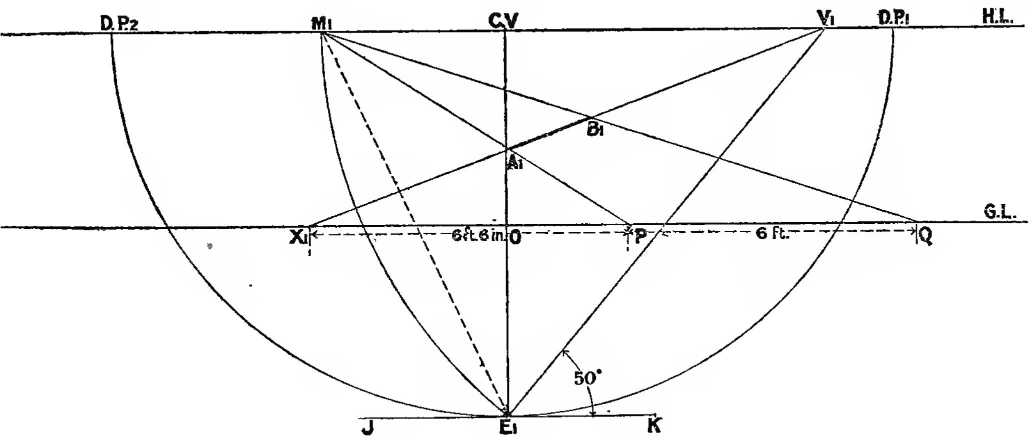

D.P.2
M1
C.V
V1
D.P.1
H.L.
B1
A1
G.L.
X1
6 ft. 6 in.
O
P
6 ft.
Q
50°
J
E1
K

Fig. 20.

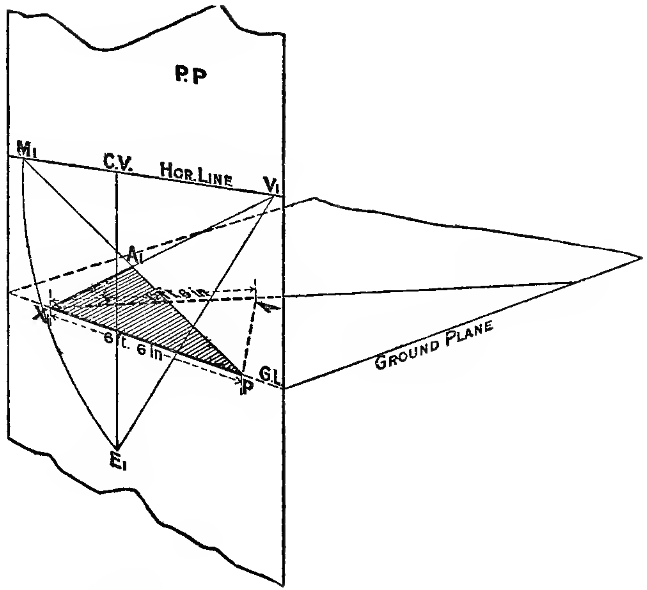

P.P
M1
C.V.
HOR. LINE
V1
A1
X1
6 ft. 6 in.
G.L.
P
E1
GROUND PLANE

Fig. 21.

In order to obtain the length of the perspective view of any portion of a line whose perspective representation has been found, it is necessary to find the Measuring Point (**M.P.**) of the **V.P.** of that line.

The Measuring Point of a **V.P.** *is that point by means of which lengths in perspective can be measured on lines having that* **V.P.**

To find the **M.P.** *of any* **V.P.**; *with the* **V.P.** *as centre and radius from* **V.P.** *to the eye draw an arc of a circle to cut the* **H.L.** *(for horizontal lines). The point of intersection of this arc with the* **H.L.** *is the required* **M.P.**

In fig. 20 X_1V_1 is the perspective view of a line which has been obtained in a similar manner to that in Problem XXXIX. It is required to give the perspective view of a point on the line 6 ft. 6 ins. from X_1, X_1 being the point in which the line cuts the **P.P.** Draw an arc with V_1 as centre and V_1E_1 as radius cutting the **H.L.** at M_1. M_1 is the measuring point of lines vanishing at V_1.

Obtain a point **P** on the **G.L.** 6 ft. 6 ins. from X_1 on the right. Join **P** to M_1, and let PM_1 intersect X_1V_1 at A_1. X_1A_1 is the perspective view of the line 6 ft. 6 ins. in length starting on the **P.P.**

In the method adopted for finding the measuring point M_1, the triangle X_1PA_1 has been obtained. This triangle is the perspective view of an isosceles triangle lying on the ground with one side X_1P on the **P.P.** PX_1 is equal in length to X_1A, a side of the triangle of which X_1PA_1 is the perspective representation. As X_1P lies in the **P.P.** its perspective view coincides with the line itself. It will be evident that as PX_1 is 6 ft. 6 ins. in length, A_1 represents the perspective view of a point 6 ft. 6 ins. from the **P.P.** on the line XA. Fig. 21 is a sketch showing the isosceles triangle X_1PA lying on the **G.P.**, the side X_1A being equal to X_1P, and X_1PA_1 is the perspective view of that triangle, X_1A_1 being the perspective view of X_1A.

If in fig. 20 another point **Q** is obtained on the **G.L.** 6 ft. distant from **P**, and a line drawn from **Q** to M_1 intersecting X_1V_1 at B_1; QX_1B_1 is the perspective view of an isosceles triangle, and as QX_1 is 12 ft. 6 ins. in length, B_1 is the perspective view of a point 12 ft. 6 ins. from X_1, and hence is the perspective view of a point 6 ft. distant from A_1.

It will be seen that by the use of the measuring point (M_1) the perspective representation of any point in the line of which X_1V_1 is the perspective representation can be obtained.

The proof that X_1A_1 is the perspective representation of a line equal in length to X_1P is as follows (refer to fig. 20):—

$\angle V_1E_1K + \angle V_1E_1M_1 + \angle M_1E_1J$ = two right angles $= \angle A_1X_1P + \angle X_1A_1P + \angle A_1PX_1$; but $\angle V_1E_1M_1 = \angle V_1M_1E_1$ (since $V_1M_1 = V_1E_1$, Euc. I. 5) $= \angle M_1E_1J$ (Alternate angles), and $\angle V_1E_1K = \angle AX_1P$ (the perspective representation of this angle being $\angle A_1X_1P$), and similarly $\angle M_1E_1J = \angle M_1PX_1$, therefore $\angle X_1PA_1 = \angle X_1A_1P$, therefore $X_1A_1 = X_1P = 6$ ft. 6 ins.

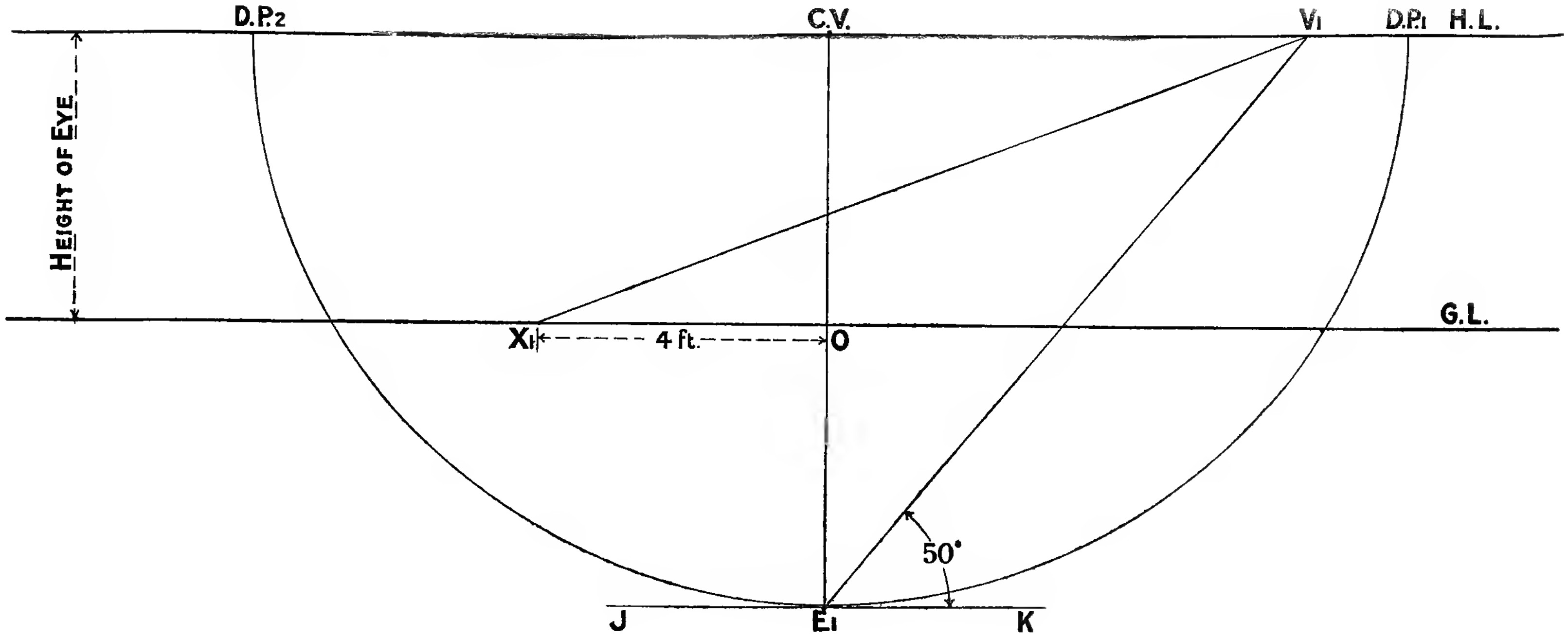
In a similar way it may be shown that the X_1B_1 is the perspective representation of a line equal to X_1Q. Hence by difference A_1B_1 is equal to PQ.

DISTANCE POINTS.

A distance point (**D.P.**) is the measuring point of lines perpendicular to the **P.P.**, *i.e.*, lines that vanish at the **C.V.**

With centre **C.V.** and radius E_1 (fig. 20) describe a semicircle intersecting the **H.L.** at points DP_1 and DP_2. DP_1 and DP_2 are the vanishing points of horizontal lines making 45° with the **P.P.** towards the right and left respectively, and at the same time the **M.P.**'s of lines perpendicular to the **P.P.** They are called *distance points* (**D.P.**).

PROBLEM XXXIX.

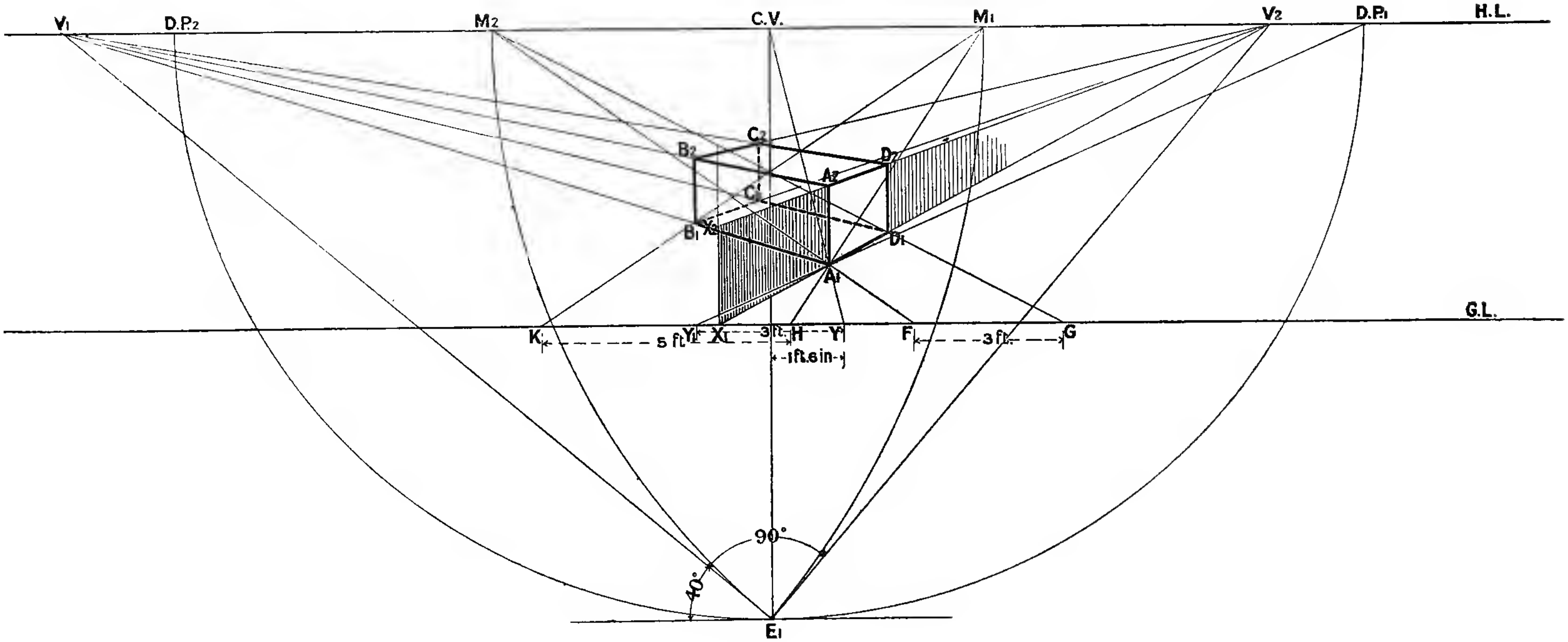


PROBLEM XXXIX.

It is required to find (without using the plan) the perspective representation of a line of infinite length, with one end touching the **P.P.** 4 ft. on the left of the spectator, the line making an angle of 50° with the **P.P.** towards the right. The height of the eye above the ground to be 4 ft., and its distance from the **P.P.** 8 ft.

Draw the **H.L.** and **G.L.**, the distance between them being equal to the height of the eye (see accompanying plate). Draw **C.V.** E_1 perpendicular to the **H.L.** and equal in distance to that of the eye in front of the **P.P.** Through E_1 draw E_1V_1, making 50° with E_1K (E_1K is parallel to the **H.L.**) to cut the **H.L.** at V_1. V_1 is the **V.P.** of the line whose perspective view is required. From **O** measure **O** X_1 on the **G.L.** 4 ft. to the left. Join X_1V_1. Then X_1V_1 is the perspective representation of the required line.

PROBLEM XL.


V1
D.P.2
M2
C.V.
M1
V2
D.P.1
H.L.
G.L.
B2
C2
D2
A2
C1
B1
X2
D1
A1
K
Y1
X1
3 ft.
H
Y
F
G
5 ft.
1 ft. 6 in.
3 ft.
90°
40°
E1

PROBLEM XL.

In working this problem the height of the eye is to be taken as 6 ft. and the distance of eye from **P.P.** as 12 ft.

A rectangular slab, 5 ft. long, 3 ft. wide, and 2 ft. thick, lies upon the **G.P.** on one of its largest faces. One of its long edges recedes from the **P.P.** at 40° towards the left. The nearest corner upon the **G.P.** is 1 ft. 6 in. to the right and 3 ft. within the picture. Draw the slab in perspective.

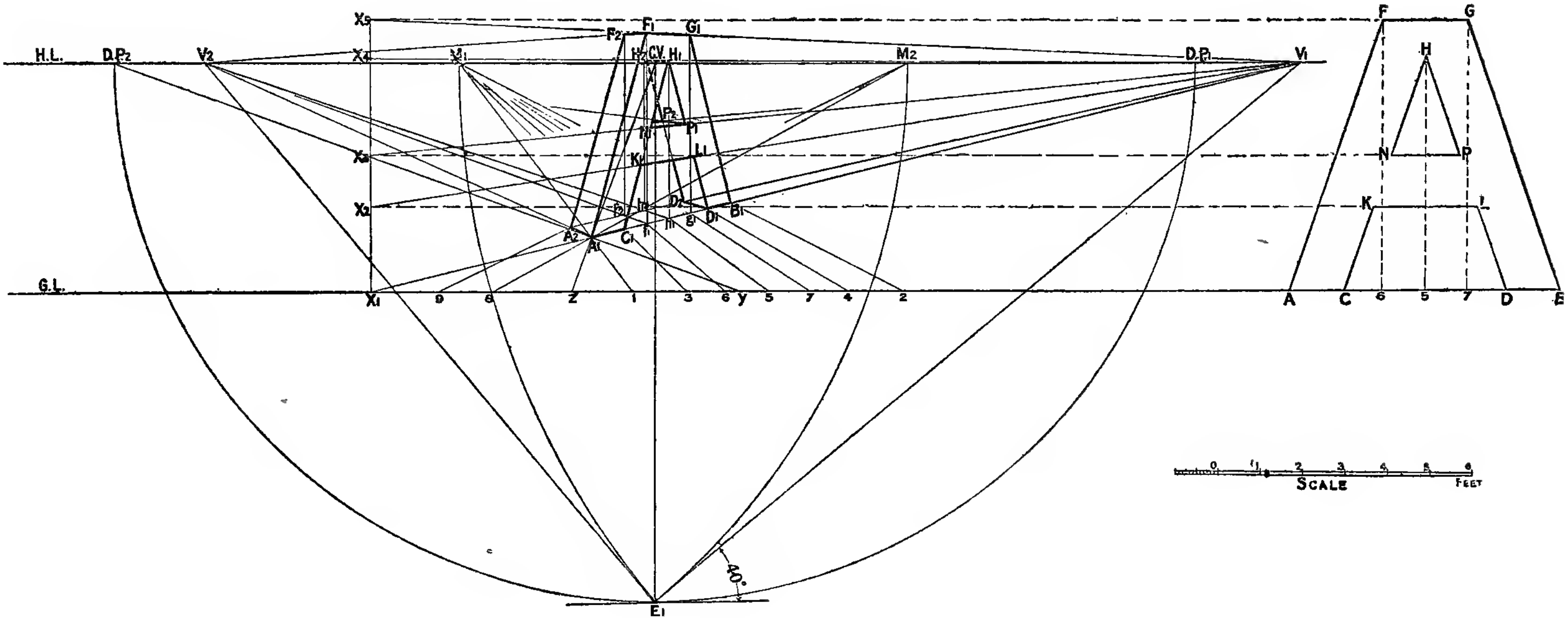
Draw the **H.L., G.L.**, and $\mathbf{E}_1$ in position (see accompanying plate). With **C.V.** as centre and radius from **C.V.** to $\mathbf{E}_1$, describe a semicircle cutting **H.L.** at the distance points $\mathbf{D.P.}_1$ and $\mathbf{D.P.}_2$. These points are the **M.P.**'s for lines vanishing at **C.V.**, *i.e.*, lines perpendicular to the **P.P.** On **G.L.** obtain a point **Y**, 1 ft. 6 ins. distant from $\mathbf{E}_1\mathbf{C.V.}$ on the right, and on **G.L.** obtain a point $\mathbf{Y}_1$, 3 ft. distant from **Y** on the left. Join **Y** to **C.V.** and $\mathbf{Y}_1$ to $\mathbf{D.P.}_1$, intersecting **Y C.V.** at $\mathbf{A}_1$. $\mathbf{D.P.}_2$ could be used to obtain $\mathbf{A}_1$, but $\mathbf{Y}_1$ would then have to be measured 3 ft. on the right of **Y**. Obtain $\mathbf{V}_1$, the **V.P.** of the long edges, and $\mathbf{V}_2$, the **V.P.** of the shorter edges. With centre $\mathbf{V}_1$ and radius $\mathbf{V}_1\mathbf{E}_1$ describe an arc intersecting **H.L.** at $\mathbf{M}_1$. $\mathbf{M}_1$ is the measuring point (**M.P.**) of lines vanishing at $\mathbf{V}_1$. With centre $\mathbf{V}_2$ and radius $\mathbf{E}_1$ describe an arc intersecting **H.L.** at $\mathbf{M}_2$. $\mathbf{M}_2$ is the **M.P.** of $\mathbf{V}_2$.

Join $\mathbf{A}_1$ to $\mathbf{V}_1$ and $\mathbf{V}_2$. From $\mathbf{M}_1$ draw a line through $\mathbf{A}_1$ intersecting **G.L.** at **H**, and on **G.L.** obtain a point **K**, 5 ft. from **H** (on the left). Join **K** to $\mathbf{M}_1$ intersecting $\mathbf{A}_1\mathbf{V}_1$ at $\mathbf{B}_1$. From $\mathbf{M}_2$ draw a line through $\mathbf{A}_1$, intersecting **G.L.** at **F**, and on **G.L.** obtain a point **G**, 3 ft. from **F** (on the right). Join **G** to $\mathbf{M}_2$ intersecting $\mathbf{A}_1\mathbf{V}_2$ at $\mathbf{D}_1$. Join $\mathbf{B}_1$ to $\mathbf{V}_2$ and $\mathbf{D}_1$ to $\mathbf{V}_1$, intersecting at $\mathbf{C}_1$. $\mathbf{A}_1\mathbf{B}_1\mathbf{C}_1\mathbf{D}_1$ is the perspective view of that face of the slab which is on the ground.

Produce $\mathbf{V}_2\mathbf{A}_1$ to intersect **G.L.** at $\mathbf{X}_1$. From $\mathbf{X}_1$ draw a vertical line upwards. On that line obtain a point $\mathbf{X}_2$ the height of the top edges of the slab above the **G.P.**, *i.e.*, 2 ft. Join $\mathbf{X}_2\mathbf{V}_2$, and from $\mathbf{A}_1$ and $\mathbf{D}_1$ draw vertical lines to intersect $\mathbf{X}_2\mathbf{V}_2$ at $\mathbf{A}_2$ and $\mathbf{D}_2$ respectively. $\mathbf{A}_1\mathbf{D}_1\mathbf{D}_2\mathbf{A}_2$ is the perspective view of one of the smallest faces of the slab. Join $\mathbf{A}_2$ and $\mathbf{D}_2$ to $\mathbf{V}_1$. From $\mathbf{B}_1$ draw a vertical line intersecting $\mathbf{A}_2\mathbf{V}_1$ at $\mathbf{B}_2$, and join $\mathbf{B}_2$ to $\mathbf{V}_2$ intersecting $\mathbf{D}_2\mathbf{V}_1$ at $\mathbf{C}_2$. Join $\mathbf{C}_1$ to $\mathbf{C}_2$, and the perspective view of the rectangular slab is completed.

N.B. The explanation of the use of the height line (as $\mathbf{X}_1\mathbf{X}_2$) is given in detail in the Plan Method.

PROBLEM XLI.



H.L.
G.L.
D.P2
V2
M1
M2
D.P1
V1
C.V.
E1
40°
X1
X2
X3
X4
X5
A
C
D
E
F
G
H
K
N
P
SCALE
FEET

PROBLEM XLI.

The accompanying plate gives the elevation of a block-letter **A**, cut out of material 1 ft. 3 ins. square in section. Represent this letter in perspective, standing upon the ground plane—the corner **A** being 2 ft. on the left of spectator and 4 ft. from the ground line. The front face of the letter recedes at an angle of 40° with the **P.P.** towards the right. The eye is situated 13 ft. from the **P.P.** and 5 ft. 6 ins. above the ground.

Determine DP_1 and DP_2 by drawing a semicircle with **C.V.** as centre and radius $C.V.E_1$. Obtain V_1, the **V.P.** of lines receding at 40° towards the right, and V_2 the **V.P.** of lines at right angles to these. Find M_1 and M_2, the **M.P.**'s of V_1 and V_2 respectively.

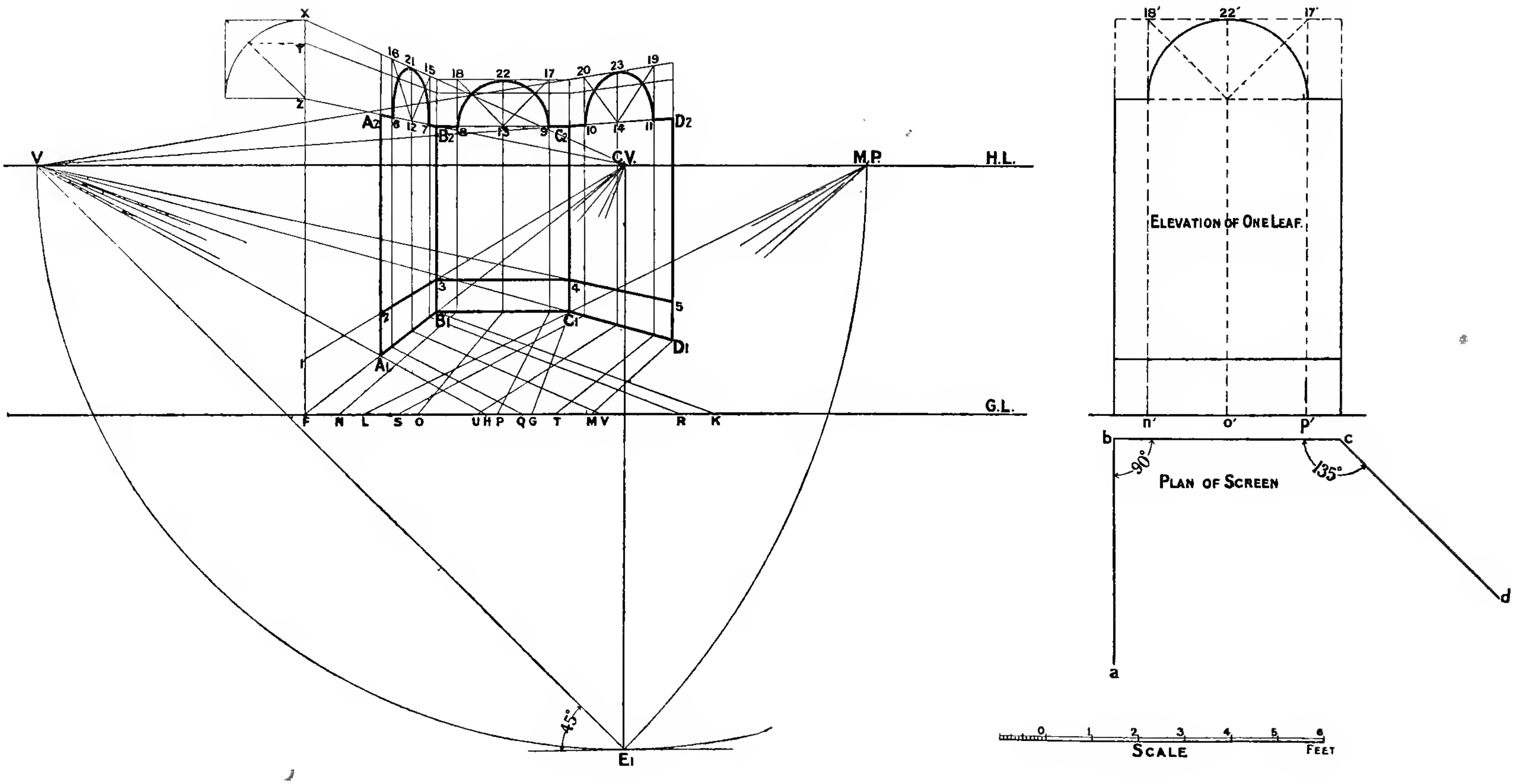
Mark a point **z** on the **G.L.**, 2 ft. on the left, and join **z C.V.** Make **z y** on **G.L.** 4 ft. long (*i.e.*, distance of **A** from the **G.L.**); join $y\,D.P._2$ intersecting **z C.V.** at A_1. Join A_1V_1; from M_1 draw M_1A_1 cutting the **G.L.** at **1**; measure from **1** to **3**, **6**, **5**, **7**, **4**, and **2** distances equal to that from **A** to **C**, **6**, **5**, **7**, **D**, and **B** respectively in the elevation; join the points obtained to M_1, and let the lines thus drawn intersect A_1V_1 at $C_1f_1h_1g_1D_1B_1$. A_1C_1 and D_1B_1 are portions of the required drawing.

Determine the height line for the nearer face of the letter by producing V_1A_1 to cut the **G.L.** at X_1, and from X_1 draw a vertical line X_1X_5. On X_1X_5 measure X_1X_2, X_1X_3, X_1X_4, and X_1X_5, equal in length to the height of **K L**, **N P**, **H**, and **F G** respectively above the ground (taken from the elevation); join X_2, X_3, X_4, and X_5 to V_1. From f_1 and g_1 draw vertical lines intersecting X_5V_1 at F_1 and G_1 respectively; join A_1F_1 and B_1G_1. From h_1 draw a vertical line intersecting X_4V_1 at H_1; join H_1C_1 and H_1D_1; H_1C_1 cuts X_3V_1 and X_2V_1 at N_1 and K_1 respectively, and H_1D_1 cuts the same two lines at P_1 and L_1 respectively. This completes the front face of the letter.

Join A_1V_2; from M_2 draw M_2A_1 and produce it to cut the **G.L.** at **8**; make **8, 9** equal to the thickness of the letter, *i.e.* 1 ft. 3 ins.; join $9\,M_2$ cutting A_1V_2 at A_2. Join A_2V_1. Now draw f_1V_2, h_1V_2, and D_1V_2 cutting A_2V_1 at f_2, h_2, and D_2 respectively. Raise a vertical line from f_2 to intersect the line joining F_1V_2 at F_2; then join F_2A_2.

Similarly from h_2 raise a vertical line to intersect H_1V_2 at H_2, and join H_2D_2 (a portion of this line is not visible). Join P_1V_2 intersecting H_2D_2 at P_2, and through P_2 produce V_1P_2 until it meets H_1C_1. This completes the visible portion of the figure.

PROBLEM XLII.



PROBLEM XLII.

The accompanying plate gives the plan of a folding screen, composed of three similar leaves of equal size, and an elevation of one of them. Put the screen in perspective, having the leaves standing vertically on a **G.P.** 5 ft. 6 ins. below the spectator's eye, and one of them being at right angles to the **P.P.**

The nearest point on the ground to the **P.P.** (**A**) is situated 7 ft. to the left of the **C.V.** and 4 ft. within the picture.

Distance from spectator to the **P.P.** to be taken as 13 ft.

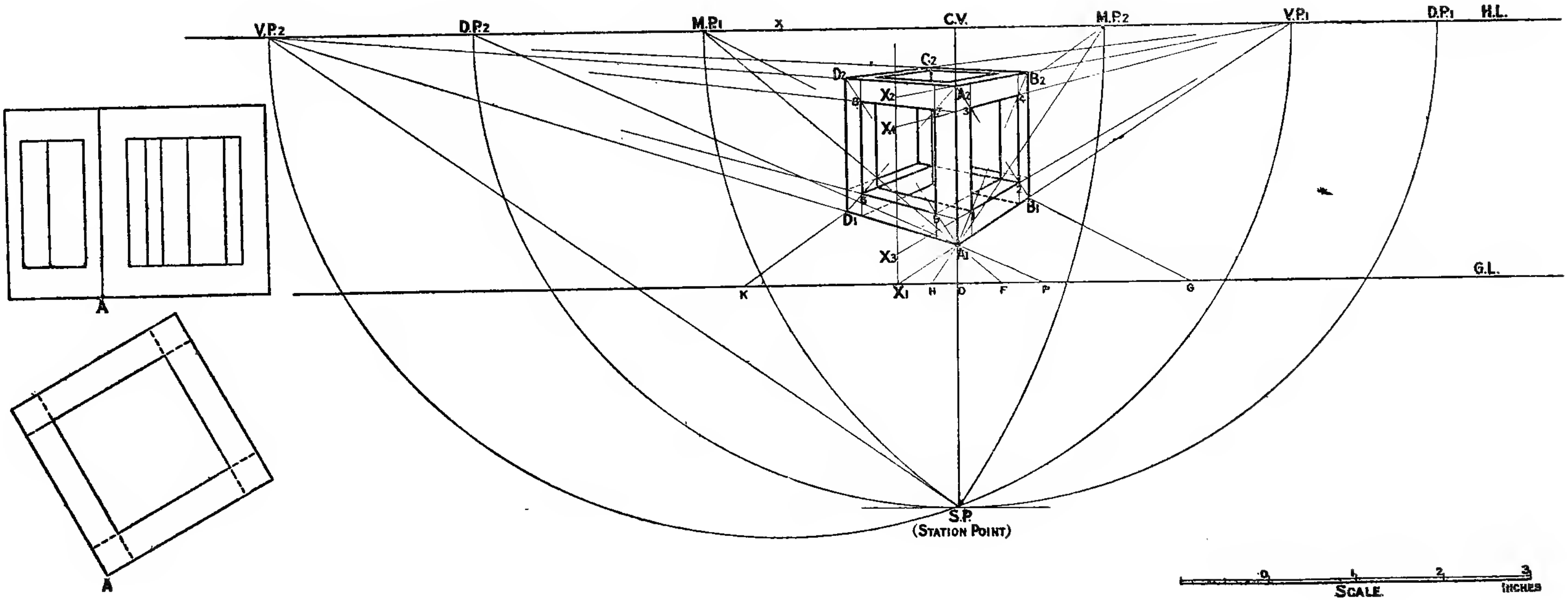
On **G.L.** obtain a point **F** 7 ft. to the left, and join **F** to **C.V.** With **C.V.** as centre and radius **C.V.** E_1 describe an arc intersecting **H.L.** on the left at **V**. **V** is the vanishing point of horizontal lines making an angle of 45° to the **P.P.** towards the left, and it is also the **M.P.** of lines vanishing at the **C.V.** With centre **V** and radius **V** E_1 describe an arc intersecting **H.L.** at **M.P.**; this point is the **M.P.** of lines vanishing at **V**. On **G.L.** obtain a point **H** 4 ft. from **F** on the right, and join **H** to **V** intersecting **F C.V.** at A_1. On **G.L.** obtain a point **K** on the right of **H** and the width of the screen (**a b**) distant from it. Join **K** to **V** intersecting **F C.V.** at B_1. On **G.L.** obtain a point **G** on the right of **F** and the width of the screen distant from it. Join **G** to **C.V.**, and from B_1 draw a horizontal line intersecting **G C.V.** at C_1. From **M.P.** draw a line through C_1 to intersect **G.L.** at **L**. Obtain a point **M** on **G.L.** to the right of **L** and the width of the screen distant from it. Join **M** to **M.P.**, and from **V** draw a line through C_1 to intersect **M M.P.** at D_1. $A_1B_1C_1D_1$ is the perspective view of the edges on the ground.

Draw the height line from **F** and complete the perspective view of the screen in the manner shown in figure.

It should be specially noted how horizontal lines on the screen $A_1B_1B_2A_2$ are carried round the other two leaves.

(The above problem is similar to one given in an examination paper.)

PROBLEM XLIII.



PROBLEMS TAKEN FROM RECENT EXAMINATION PAPERS.

PROBLEM XLIII.

The accompanying plate gives the plan and elevation of a skeleton cube; put this figure into perspective under the following conditions:—

Draw a Horizontal Line (**H.L.**) and upon it mark two points 1 ft. apart.

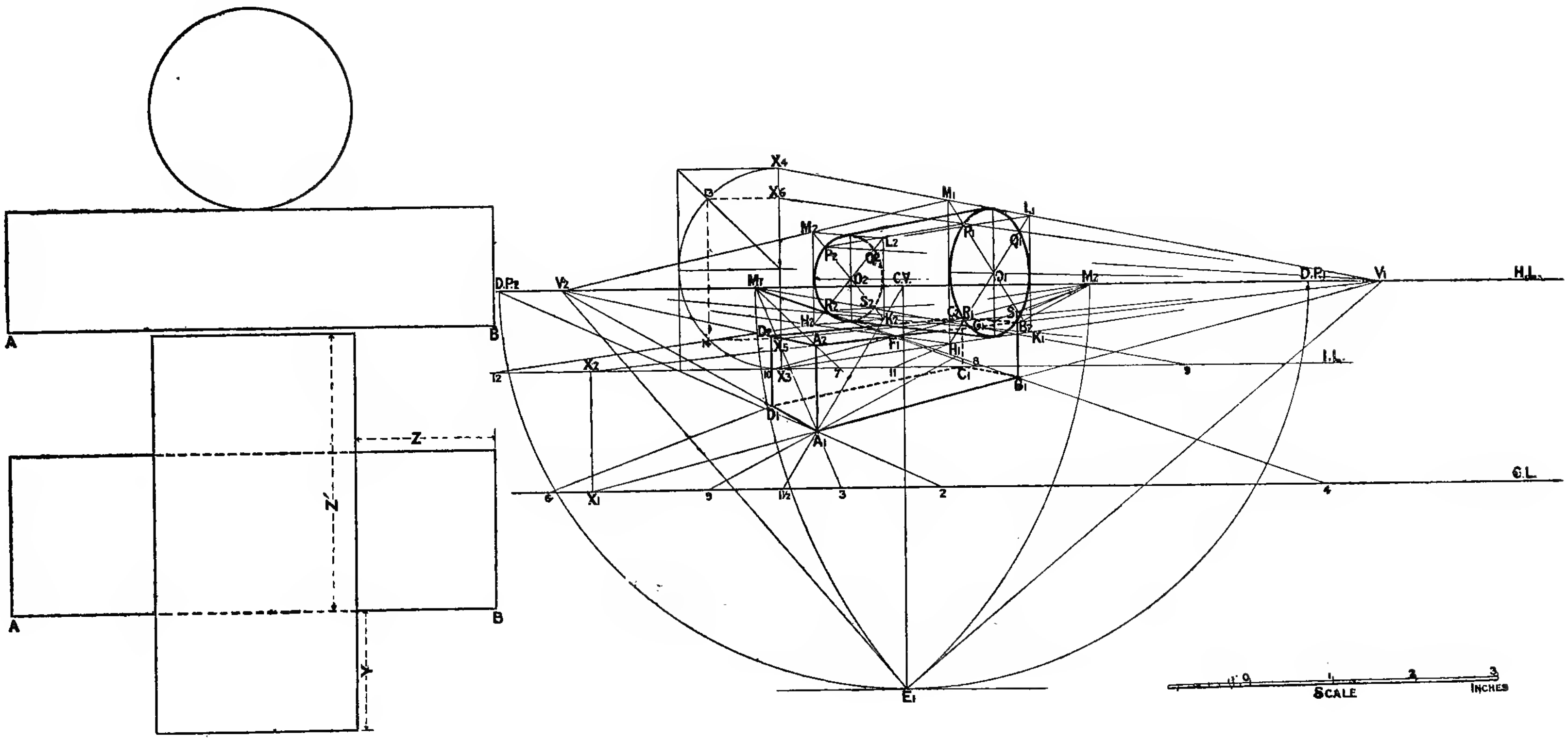
These points, marked **V.P.**$_{1}$, **V.P.**$_{2}$, shall be the vanishing points of edges of the cube, vanishing at angles of 55° and 35° to the picture, to the right and left respectively. Find the station point and the Centre of Vision (**C.V.**). Place point **A** opposite the centre and 1 in. within the picture, on a ground plane 3 ins. below the eye, and work the perspective view of the cube with the given vanishing points.

Obtain **V.P.**$_{1}$ and **V.P.**$_{2}$ on the **H.L.** as directed and bisect **V.P.**$_{1}$ **V.P.**$_{2}$ at **x**. With **x** as centre and **V.P.**$_{1}$ as radius describe a semicircle. From **V.P.**$_{2}$ draw a line making an angle of 35° with **V.P.**$_{1}$ **V.P.**$_{2}$, and intersecting the semicircle at **S.P.** **S.P.** is the required Station Point. From **S.P.** draw a line at right angles to **H.L.**; its intersection with **H.L.** is the **C.V.**

Draw the **G.L.** 3 ins. below **H.L.** and obtain the distant points **D.P.**$_{1}$ and **D.P.**$_{2}$. **O** is the intersection of **S.P. C.V.** with **G.L.** From **O** mark a point **P** 1 in. to the right on **G.L.** Join **P** to **D.P.**$_{2}$ intersecting **O C.V.** at **A**$_{1}$. Join **A**$_{1}$ to **V.P.**$_{1}$ and **V.P.**$_{2}$, and produce **V.P.**$_{1}$ **A**$_{1}$ to intersect **G.L.** at **X**$_{1}$. Draw a vertical line from **X**$_{1}$ and on this line mark the heights required.

Complete the representation of the cube as if it were solid. Join the diagonals of each visible face. Note that **X**$_{4}$ **V.P.**$_{1}$ and **X**$_{3}$ **V.P.**$_{1}$ intersect the diagonals **A**$_{1}$**B**$_{2}$ and **A**$_{2}$**B**$_{1}$ at the points **1**, **2**, **3**, **4**; and that if **2,1** is produced to cut **A**$_{2}$**A**$_{1}$, and this point of intersection joined to **V.P.**$_{2}$, the line **5,6** is obtained. The drawing of the cube should be finished as shown in the figure.

PROBLEM XLIV.



PROBLEM XLIV.

The accompanying plate gives the plan and elevation of a cylinder, resting upon a rectangular block. Put these solids into perspective in the same relative position to each other, resting horizontally upon a ground plane $2\frac{1}{2}$ ins. below the eye, with the corner **A** at a point $1\frac{1}{2}$ ins. to the left of the centre and 2 ins. within the picture, and the line **A B** inclined towards the right at an angle of 40° to the picture plane. Make the distance of the eye from the picture plane 5 ins.

Obtain $\mathbf{V}_1$ and $\mathbf{M}_1$, the **V.P.** and **M.P.** respectively of lines vanishing at 40° to the right, $\mathbf{V}_2$ and $\mathbf{M}_2$ the **V.P.** and **M.P.** respectively of lines vanishing at 50° to the left, and find the distance points $\mathbf{D.P.}_1$ and $\mathbf{D.P.}_2$. On **G.L.** mark a point $1\frac{1}{2}$ ins. on the left and join it to **C.V.**, also mark a point 2 ins. to the right from point $1\frac{1}{2}$ and join it to $\mathbf{D.P.}_2$, intersecting $1\frac{1}{2}$ **C.V.** at $\mathbf{A}_1$. Join $\mathbf{A}_1$ to $\mathbf{V}_1$ and $\mathbf{V}_2$, and with the respective measuring points of these **V.P.**'s obtain the lengths of the perspective view of the edges of the block on the ground.

Produce $\mathbf{V}_1\mathbf{A}_1$ to intersect **G.L.** at $\mathbf{X}_1$, and draw a vertical line ($\mathbf{X}_1\mathbf{X}_2$) the height of the block above the **G.P.** Join $\mathbf{X}_2$ to $\mathbf{V}_1$ and complete the view of the block.

Through $\mathbf{X}_2$ draw a horizontal line (**I.L.**); this line is the intersection with the **P.P.** of a horizontal plane containing the upper face of the block. Inclose the cylinder in a rectangular square prism and obtain its position on the block. From $\mathbf{M}_1$ draw a line through $\mathbf{A}_2$ to intersect **I.L.** at **7**, and on **I.L.** mark a point **8** at a distance **Z** (see plan) from **7** on the right, and also mark a point **9**, the diameter of the cylinder distant from **8** on the right. Join **8** and **9** to $\mathbf{M}_1$, intersecting $\mathbf{A}_2\mathbf{B}_2$ at $\mathbf{F}_1$ and $\mathbf{G}_1$ respectively. From $\mathbf{M}_2$ through $\mathbf{F}_1$ draw a line to intersect **I.L.** at **10**, and on **I.L.** mark a point **11** at a distance **Y** (see plan) from **10** on the right, and also mark a point **12** at distance $\mathbf{Z}_1$ (see plan) from **10** on the left. Join **11** and **12** to $\mathbf{M}_2$. From $\mathbf{V}_2$ draw a line through $\mathbf{F}_1$ to intersect **12** $\mathbf{M}_2$ at $\mathbf{H}_2$, and **11** $\mathbf{M}_2$ at $\mathbf{H}_1$.

Join $\mathbf{H}_1$ and $\mathbf{H}_2$ to $\mathbf{V}_1$. From $\mathbf{V}_2$ draw a line through $\mathbf{G}_1$ to intersect $\mathbf{H}_2\mathbf{V}_1$ at $\mathbf{K}_2$ and $\mathbf{H}_1\mathbf{V}_1$ at $\mathbf{K}_1$.

$\mathbf{H}_1\mathbf{K}_1\mathbf{K}_2\mathbf{H}_2$ is the face of the imaginary prism that lies on the top face of the block.

Produce $\mathbf{K}_1\mathbf{H}_1$ to intersect **I.L.** at $\mathbf{X}_3$, and from $\mathbf{X}_3$ draw a vertical line $\mathbf{X}_3\mathbf{X}_4$ the height of the diameter of cylinder. Join $\mathbf{X}_4$ to $\mathbf{V}_1$ and through $\mathbf{H}_1$ and $\mathbf{K}_1$ draw vertical lines to intersect $\mathbf{X}_4\mathbf{V}_1$ at $\mathbf{M}_1$ and $\mathbf{L}_1$ respectively, and proceed to draw the perspective view of the cylinder as shown in the figure.

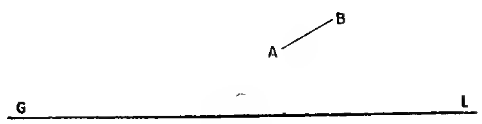

B
A
G
L

Fig. 1.

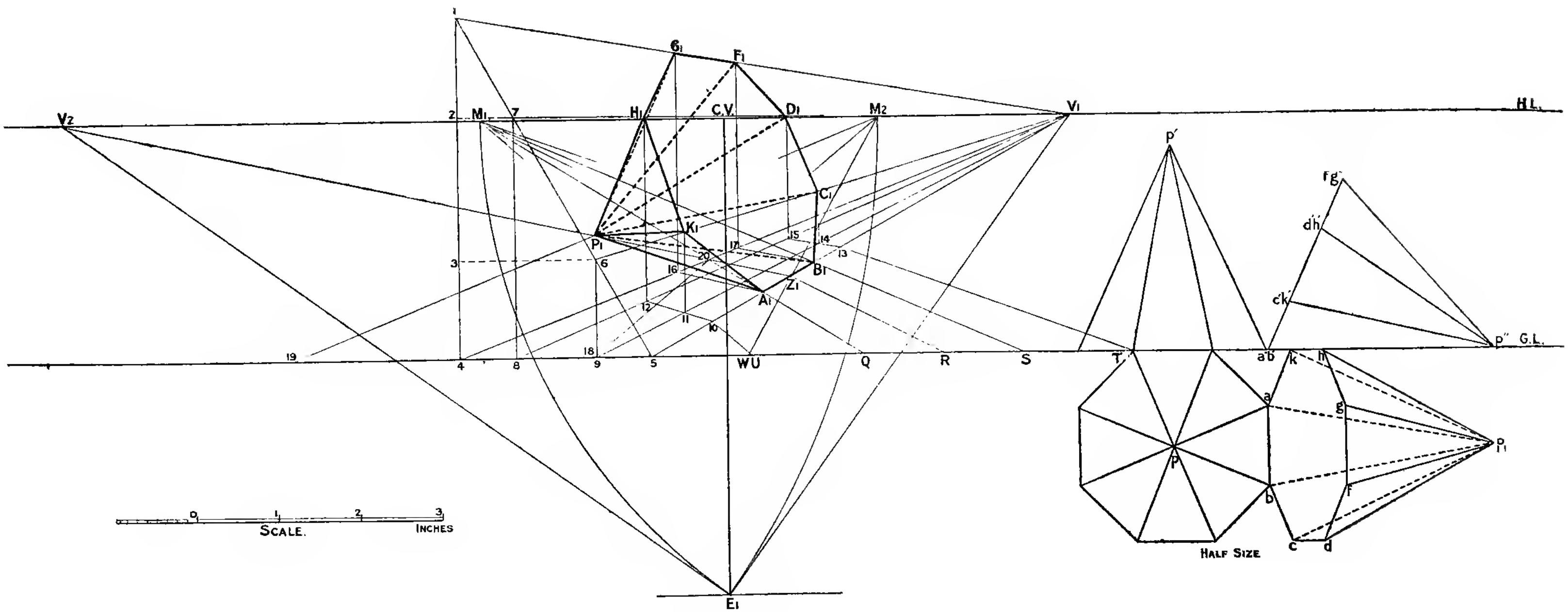

V2
V1
H.L.
G.L.
M1
M2
C.V.
E1
WU
Q
R
S
T
p'
p''
P
P1
HALF SIZE
SCALE.
INCHES

Fig. 2.

PROBLEM XLV.

In the accompanying plate fig. 1 gives the perspective representation of a horizontal line **A B**. This line **A B** represents one of the edges of the base of an octagonal pyramid, which it is required to complete in perspective projection as lying on one of its sides with its vertex pointing towards the left. The length of the axis of the pyramid is equal to the longest diagonal of the octagon. The diagram was to be pricked upon the paper, and the distance of the eye from the picture plane was 6 ins.

The diagram having been transferred (fig. 2). To find the length of **A B** from its perspective representation. Produce A_1B_1 to cut **H.L.** at V_1 the **V.P.** of **A B**; find M_1 the **M.P.** of V_1; through A_1 and B_1 draw lines from M_1 cutting the **G.L.** at **Q** and **S** respectively. **Q S** is the true length of **A B**.

Draw the plan of the octagonal pyramid when resting with its base on the ground, taking care to have an edge **A B** perpendicular to the **G.L.** as shown at the right side of the plate. The elevation may easily be found from this plan. The pyramid is now rotated about the edge, whose plan is **a b** until **P A B** strikes the ground; the new plan and elevation are shown, and should present no difficulty. These geometric figures are shown half size in the plate.

Proceed to find the perspective representation of points on the **G.P.** beneath the corners of the base. Find **R** the middle point of **Q S**; join **R** M_1 cutting A_1B_1 at Z_1. Find V_2 the **V.P.** of lines at right angles to **A B**, and join Z_1V_2.

Produce M_2Z_1 to cut the **G.L.** at **W**. From **W** measure off **W 18** and **W 19** equal in length to the distance of **a b** from **g f** and **a b** from p_1 respectively (obtained from the plan). Join **18** and **19** to M_2 cutting Z_1V_2 at **20** and P_1 respectively. P_1 is the perspective representation of the vertex. Produce V_1**20** to cut the **G.L.** at **4**, through **4** draw a vertical line **4,1** equal in length to the height of **F G** above the ground (obtained from the elevation). Join **1** to V_1.

Draw A_1V_2 and B_1V_2 cutting **4**,V_1 at **16** and **17**; from **16** and **17** raise perpendiculars to cut **1**,V_1 at F_1 and G_1 respectively. (Observe that **1, 4, 16, 17** V_1 is a vertical plane containing **F G**.) To obtain H_1, K_1, C_1, and D_1. On **G.L.** measure **Q U** and **S T** equal in length to **a,a′** (in the plan and elevation); join **U** M_1 and **T** M_1 cutting **5** $A_1B_1V_1$ at **10** and **13** respectively. If the base of the octagon was produced it would cut the **P.P.** along the line **1,5**. Measure off **4,3** and **4,2** on **4,1** equal to the height of **C K** and **D H** respectively above the ground (obtained from the elevation).

Through **2** and **3** draw horizontal lines to cut **1,5** at **7** and **6** respectively. (**7** and **6** are the points of intersection of **D H** and **C K** if produced, with the **P.P.**) Join **7** V_1 and **6** V_1; these lines pass through H_1D_1 and K_1C_1 respectively. Now draw **10** V_2 cutting **9** V_1 and **8** V_1 at **11** and **12** respectively, and **13** V_2 cutting **9** V_1 and **8** V_1 at **14** and **15** respectively.

(Observe that **9, 11, 14** V_1 and **8, 12, 15** V_1 are vertically below **6** $K_1C_1V_1$ and **7** $H_1D_1V_1$ respectively.) From **11** and **14** draw vertical lines to cut **6** V_1 at K_1 and C_1 respectively, and from **12** and **15** vertical lines to cut **7** V_1 at H_1 and D_1 respectfully. Join A_1K_1, K_1H_1, H_1G_1, B_1C_1, C_1D_1 and D_1F_1; this completes the base.

By joining the corners of the base to **P** the drawing is completed.

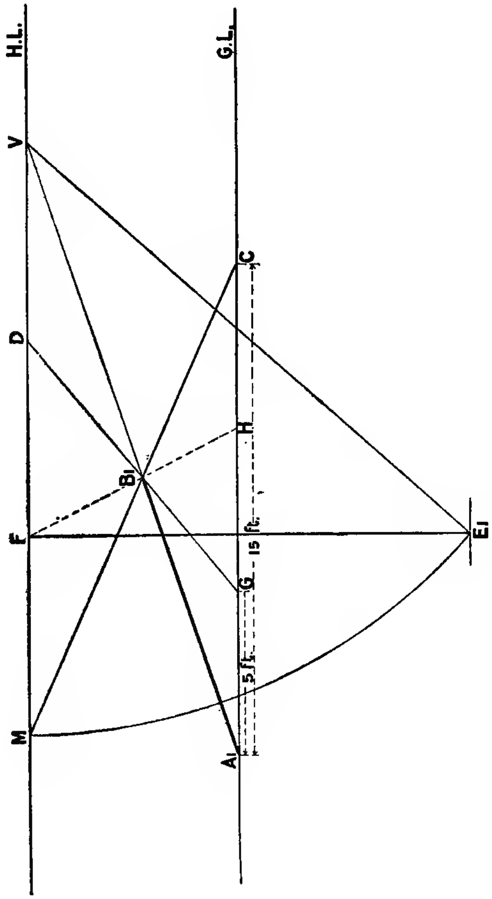


Fig. 22.

PROPORTIONAL MEASURING POINTS.

Proportional Measuring Points are used when the scale which is given for working a problem is too large to allow the use of the ordinary **M.P.**

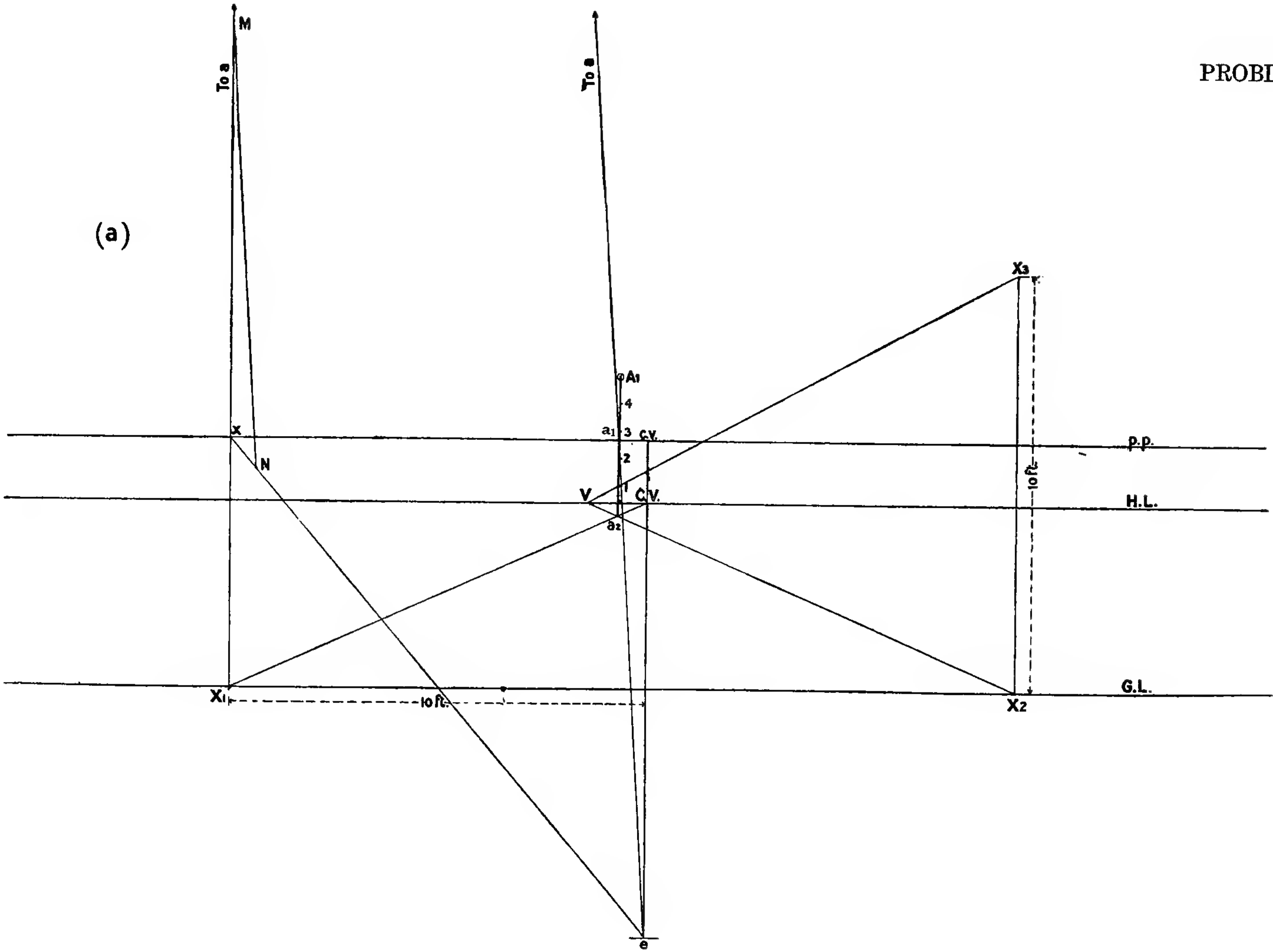
In figure 22 **V** is the vanishing point and **M** the measuring point of a line **A B** 15 ft. long inclined to **P.P.** and touching it at **A**. If A_1C is 15 ft. long and **C M** be joined, A_1B_1 will be the perspective representation of a line **A B** 15 ft. long.

Divide **V M** into three equal parts at **D** and **F**, then the distance from the **V.P.** to **D**, instead of being equal to the vanishing parallel VE_1, it is only equal to $\frac{1}{3}$ of VE_1. Now, if **A C** be divided into three equal parts at **G** and **H** (**A G** will be equal to 5 ft.) and **G D** be joined, then **G D** will pass through B_1. Hence it will be seen that B_1 could have been obtained by using **D** as the **M.P.** of **A B**, but in that case only $\frac{1}{3}$ of the length of **A B** is required to be measured. **D** is called a Proportional Measuring Point of **A B**. In the same plate, if **F** had been used as the proportional **M.P.**, since **V F** is $\frac{2}{3}$ of the vanishing parallel VE_1 it would be necessary to measure off $\frac{2}{3}$ of the dimension; this is done at **H**, and the construction line which would have been obtained is indicated by the dotted line **H F**.

Similarly, if **V M** had been divided into a greater number of parts a corresponding diminution of the length required to be measured on **G.L.** would result.

PROBLEM XLVIa.

(a)



TO OBTAIN THE PERSPECTIVE REPRESENTATION OF POINTS WHEN THE USUAL CONSTRUCTION WOULD EXTEND BEYOND THE LIMITS OF THE PAPER.

The following problems, XLVIa, XVLIb, and XLVII., are worked to various scales, but students must use a scale of $\frac{1}{2}$ in. to 1 ft. The height of the eye in each case is to be taken as 5 ft. and the distance of the eye from the **P.P.** 12 ft.

PROBLEM XLVIa.

Put in perspective a point (**A**) situated 10 ft. on the left of the spectator, 160 ft. within the picture, and 50 ft. above the ground.

BY THE PLAN METHOD.

Find **x** 10 ft. on the left of **c.v.**; from **x** draw **x M** at right angles to **p.p.** (the line **x M** *on being produced* would pass through **a**). Obtain X_1 the perspective representation of **x**; join X_1**C.V.** (X_1**C.V.** is the perspective representation of a line containing A_1).

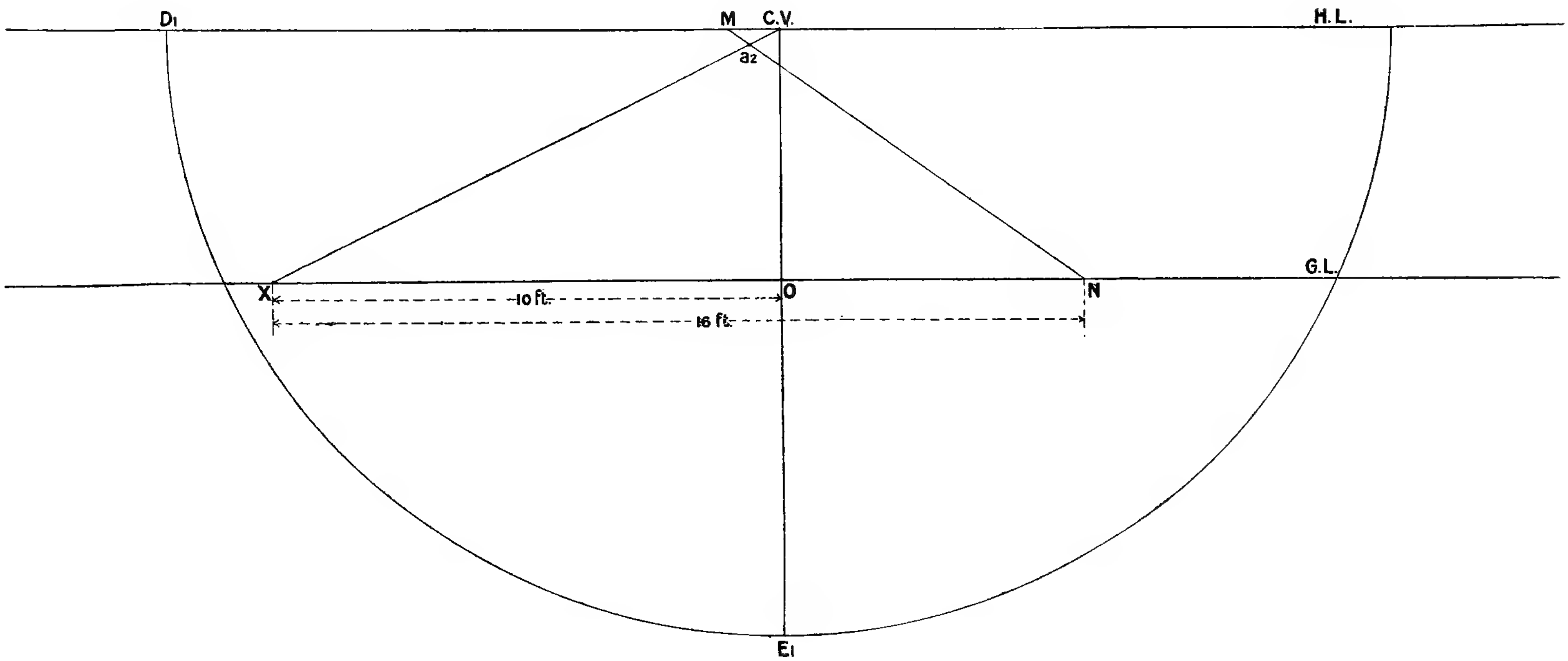
To obtain the plan of the ray **E A**.

Join **x e**; mark a point **M** on **x M** any convenient fraction of **x a**; let the fraction settled on be $\frac{1}{16}$, then **x M** will be 10 ft. (to scale). Find **N** in **x e**. **x N** being *the same* fraction of **x e** as **x M** is of **x a** (**x N** to equal $\frac{1}{16}$ of **x e**). Join **M N**. Through **e** draw **e** a_1 parallel to **N M** cutting **p.p.** at a_1 (as **e** a_1 on being produced would pass through **a**, hence a_1 is the intersection with **P.P.** of *the ray* from **a** to **e**). From a_1 draw a projector cutting X_1**C.V.** at a_2. a_2 is the perspective view of the point beneath **A** on the ground.

To measure on the projector a_2a_1 *a height of 50 feet.*

Draw $V a_2 X_2$ through a_2 in any convenient direction cutting **H.L.** at **V** and **G.L.** at X_2. Measure X_2X_3 (on a height line through X_2) any convenient fraction of **A**'s height. (In this case $\frac{1}{5}$ is taken.) Join X_3V intersecting a_2a_1 at **1**. Then as X_2X_3 is $\frac{1}{5}$ of the required height a_2**1** is $\frac{1}{5}$ of the corresponding height in perspective, hence make a_2A_1 equal in length to 5 times A_2**1**. A_1 is the required point.

PROBLEM XLVIb.



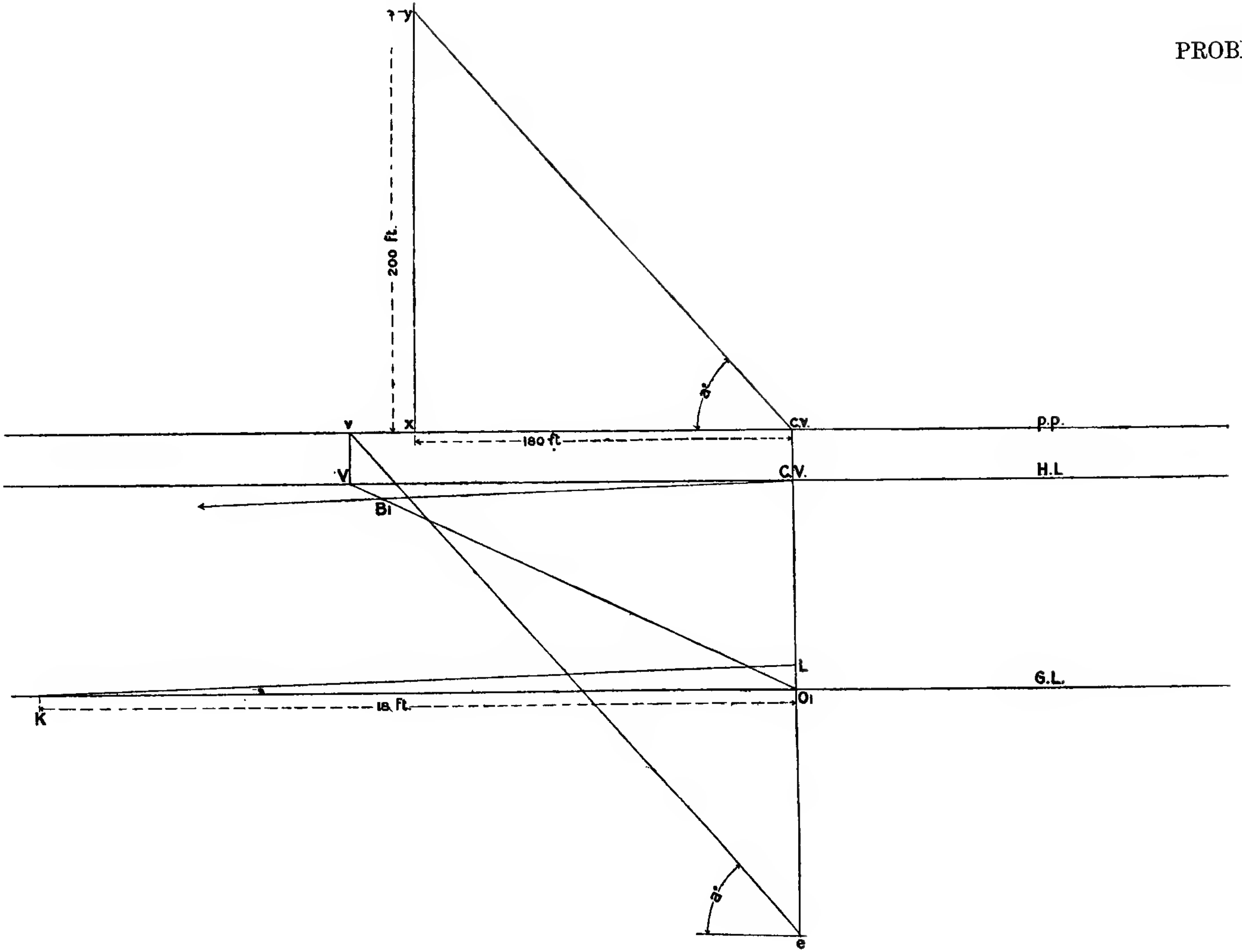
PROBLEM XLVIb.

BY THE USE OF A PROPORTIONAL MEASURING POINT.

Find **X** on the **G.L.** 10 ft. on the left. Join **X C.V.** (This line corresponds to **X C.V.** in the plan method, Problem XLVIa.) Find $\mathbf{D_1}$ the **M.P.** of **X C.V.**

Take a point **M** any convenient fraction of **C.V.** $\mathbf{D_1}$. (In this case $\frac{1}{10}$.) **M** is a proportional **M.P.** of lines vanishing at **C.V.** From **X** 160 ft. should be set off (along the **G.L.** towards the right) if $\mathbf{D_1}$ is used as **M.P.**; but $\frac{1}{10}$ of 160 ft., *i.e.* 16 ft., is only required if **M** is used, hence measure off **X N** 16 ft. on the right of **X**. Join **N M** cutting **X C.V.** at $\mathbf{a_2}$. The height of **A** may be measured in the same way as in the plan method. The point $\mathbf{a_2}$ corresponding to $\mathbf{a_2}$ in the preceding plate.

PROBLEM XLVII.



PROBLEM XLVII.

Put in perspective by the plan method a point (**B**) situated on the ground 180 ft. on the spectator's left and 200 ft. within the picture.

Find a point **y** 180 ft. on the left and 200 ft. beyond the **P.P.** to any convenient scale; join **c.v. y**.

Note that **c.v. y** on being produced would pass through **b**. The perspective representation of **c.v.** is at $\mathbf{O_1}$ on the ground line.

Obtain **V** the **V.P.** of **c.v. y b**, and join $\mathbf{O_1V}$. $\mathbf{O_1V}$ is the perspective representation of **c.v. y b** produced infinitely, and hence $\mathbf{B_1}$ lies in $\mathbf{O_1V}$.

$\mathbf{B_1}$ would also lie on the line joining a point **Z** on the **G.L.** to **C.V.** when $\mathbf{O_1Z}$ is 180 ft. in length; this point **Z** comes beyond the paper, but the same line may be obtained in the following way.

Make $\mathbf{O_1K}$ any convenient fraction of $\mathbf{O_1Z}$ (in this case say $\frac{1}{10}$), thus $\mathbf{O_1K}$ will be 18 ft. to the scale used in working the problem, make $\mathbf{O_1L}$ the *same* fraction of $\mathbf{O_1C.V.}$ (in this case $\frac{1}{10}$). Join **K L**. Through **C.V.** draw $\mathbf{C.V.\ B_1}$ parallel to **L K** and let it cut $\mathbf{O_1V}$ at $\mathbf{B_1}$. $\mathbf{B_1}$ is the required point. (Note $\mathbf{C.V.B_1}$ if produced will intersect the **G.L.** 180 ft. to left, to scale.)

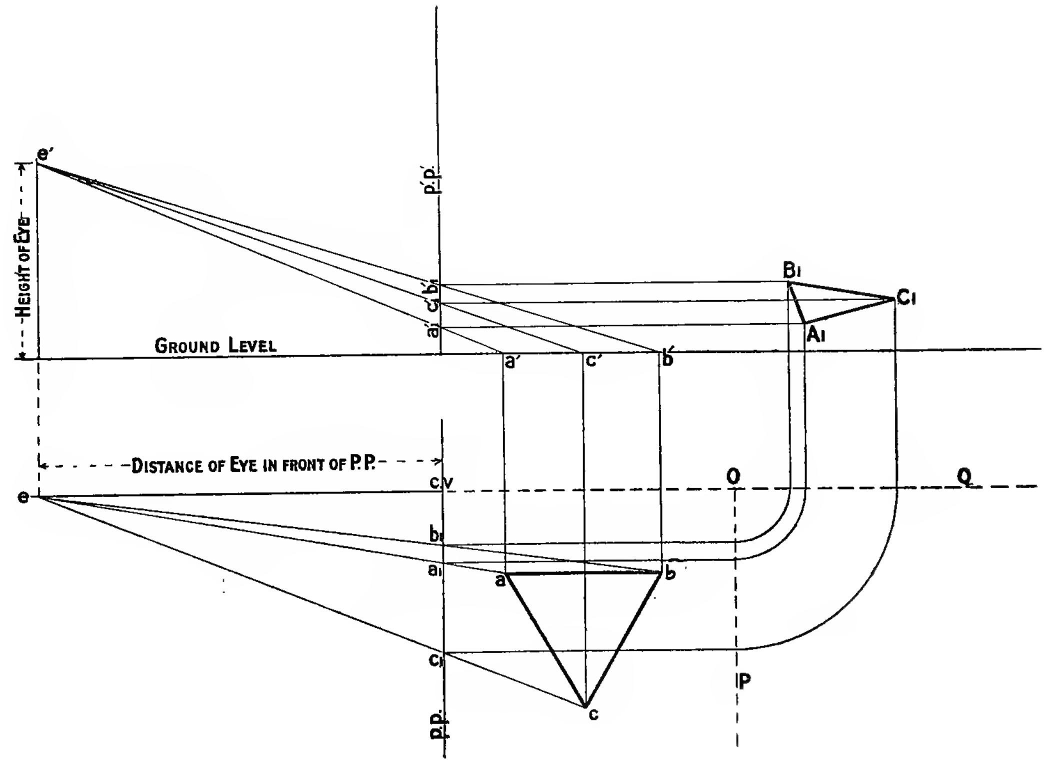

e′
HEIGHT OF EYE
GROUND LEVEL
p′p′
b₁′
c₁′
a₁′
a′
c′
b′
B₁
C₁
A₁
DISTANCE OF EYE IN FRONT OF P.P.
e
c.v
b₁
a₁
a
b
c
c₁
p.p.
O
Q
P

Fig. 23.

DIRECT METHOD.

Vanishing and Measuring Points are not used in this method; but it is not usually adopted in working perspective problems, as correct results are not so easily obtained as in the Plan and Measuring Point Methods.

In the lower half of fig. 23 the plan of an equilateral triangle lying on the **G.P.** with one side perpendicular to the **P.P.** is shown in position with reference to the plan of the eye (e).

The plan of the perspective representation has been obtained from this by drawing the plans of the rays, intersecting the plan of the **P.P.** at $a_1b_1c_1$ (as in the plan method).

The upper half of the figure is an elevation of the eye, &c., in position; e′ must be measured the height of the eye above the ground.

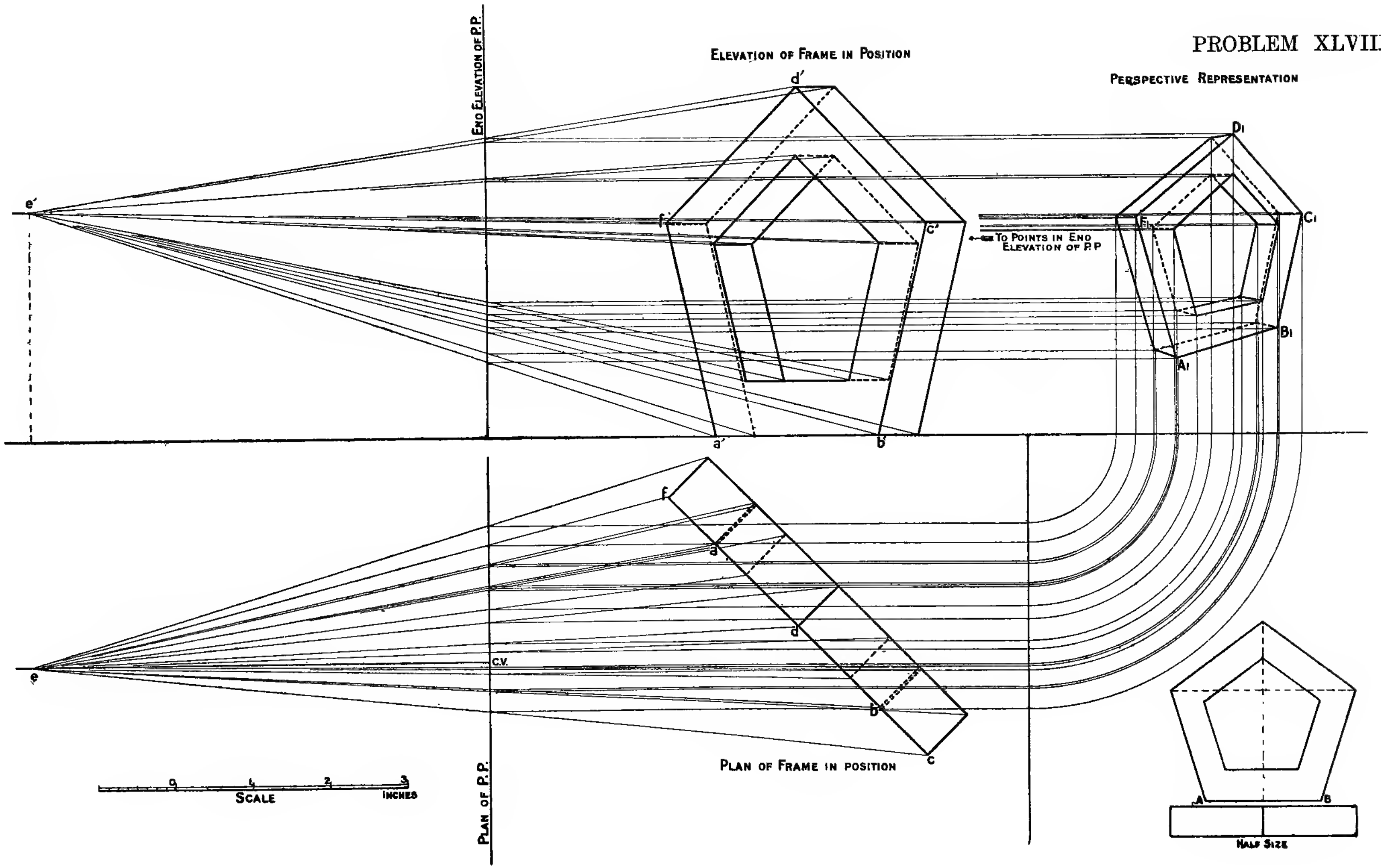
The end elevation of the **P.P.** is a vertical straight line (p.′p.′) and a′, b′, and c′ are the elevations of a, b, and c respectively. The elevation of each of the rays is obtained by joining e a′, e b′, and e c′, cutting p.′p.′ at a_1', b_1', and c_1' respectively. The heights of a_1', b_1', and c_1' determine the heights of the perspective representation of **A**, **B**, and **C** respectively.

Having the plan and the heights of the various points required for determining the perspective representation of the triangle, the drawing is transferred to a suitable position of the paper as shown.

The dotted lines **O P** and **O Q** may be taken in any suitable position (**O P** being parallel to p.p. and **O Q** perpendicular to **P.P.**).

a, b, and c have been projected to intersect **O P**; the points thus obtained have been rotated round **O** to the line **O Q** and projectors drawn to intersect the corresponding lines from a_1', b_1', and c_1', thus giving A_1, B_1, and C_1 the perspective representation of the triangle.

PROBLEM XLVIII.



PROBLEM TAKEN FROM A RECENT EXAMINATION.

PROBLEM XLVIII.

The accompanying plate gives the plan and elevation of a pentagonal frame; put this into perspective vertically, as shown in elevation, with a point **A** on a ground plane 3 ins. below the level of the eye, $1\frac{1}{2}$ ins. to the left of the centre, and 3 ins. within the picture, and the line **A B** vanishing horizontally towards your right at 45° to the picture plane. The distance from the eye to the picture plane to be 6 ins.

Place the plan of the object in position as shown in the lower half of the plate. From the plan obtain the elevation of the eye, **P.P.**, and object. By projectors obtain the perspective representation of the pentagon **A B C D F**. This is done as in the last problem by drawing the plan and elevation of the rays from **A**, **B**, **C**, **D**, and **F** to the eye, and transferring the drawing to a suitable position on the paper. Find the perspective representation of the figure, as if it were a solid slab, by drawing the rays corresponding to the above. Next obtain the nearer small pentagon and finally the more remote one.

EXERCISES.

The problems that follow are taken from recent examination papers, and students should work them by both the Plan and Measuring Point methods. The accompanying figures are drawn full size. If more exercises are desired, students are recommended to work any of the problems that have been previously given by a different method to that shown.

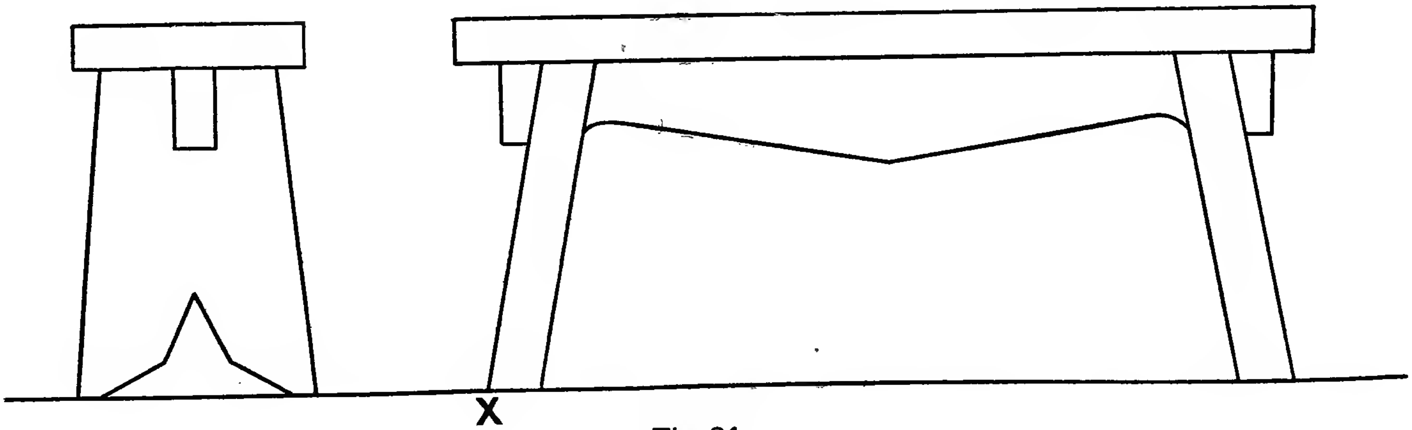


Fig. 24.

PROBLEM XLIX.

Fig. 24 gives front and side elevations of a long seat. Put this into perspective, having the point X 4 ft. below the eye, 1 ft. to the left of the centre, and 1 ft. within the picture; the long sides vanishing towards the right at 50° to the picture plane.

The distance of the spectator is 10 ft. The scale to be used in working the problem is 1 in. to 1 ft.

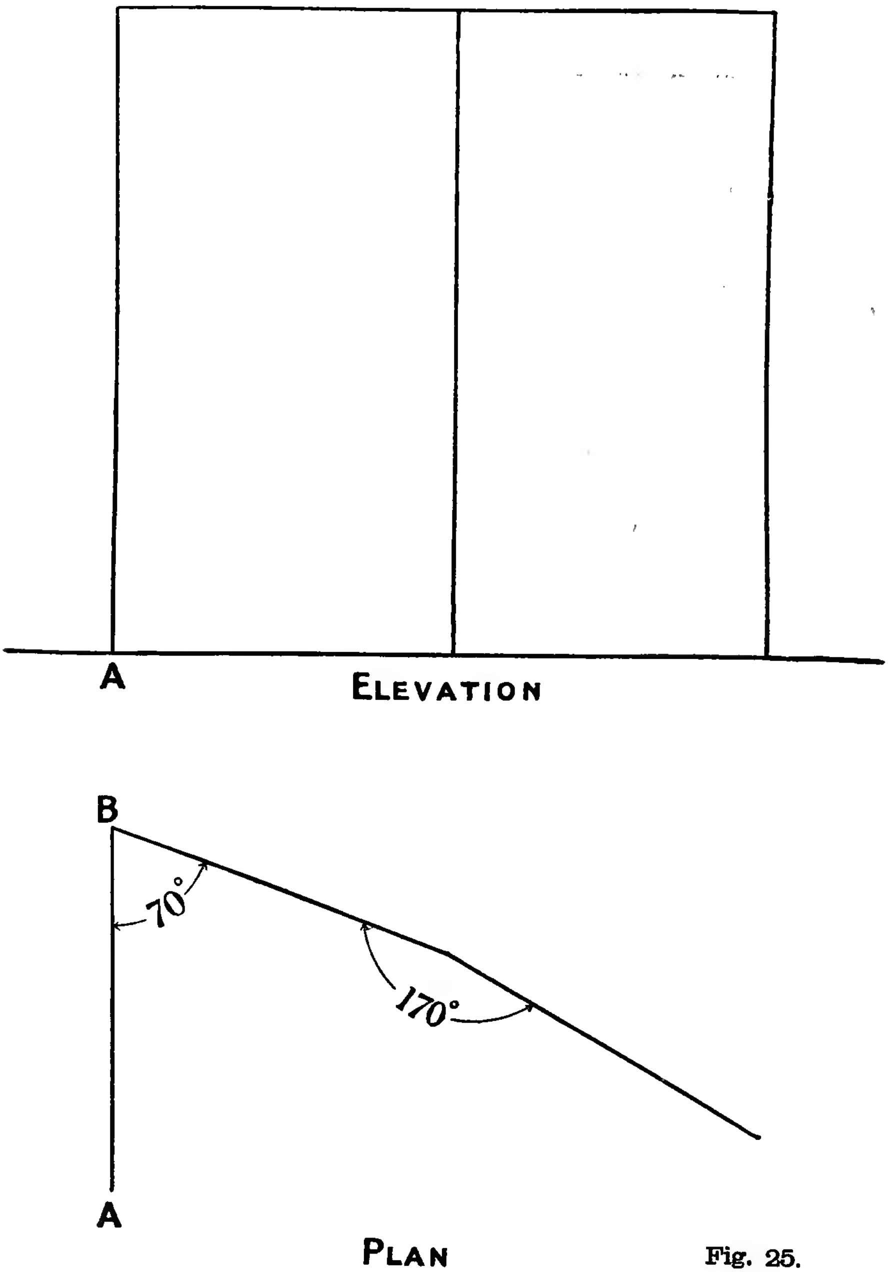


Fig. 25.

PROBLEM L.

Fig. 25 gives the plan and elevation of a folding screen of three equal leaves. The screen stands upon the ground plane, and the corner **A** is to be 3 ft. on the left of the spectator and 5 ft. from the ground line, and the line **A B** is to recede towards the right at an angle of 45° with the ground line. Represent the screen in perspective.

The scale to be used in working the problem is ½ in. to 1 ft. The eye is to be 11 ft. by scale distant from the picture plane, and 5 ft. above the ground plane.

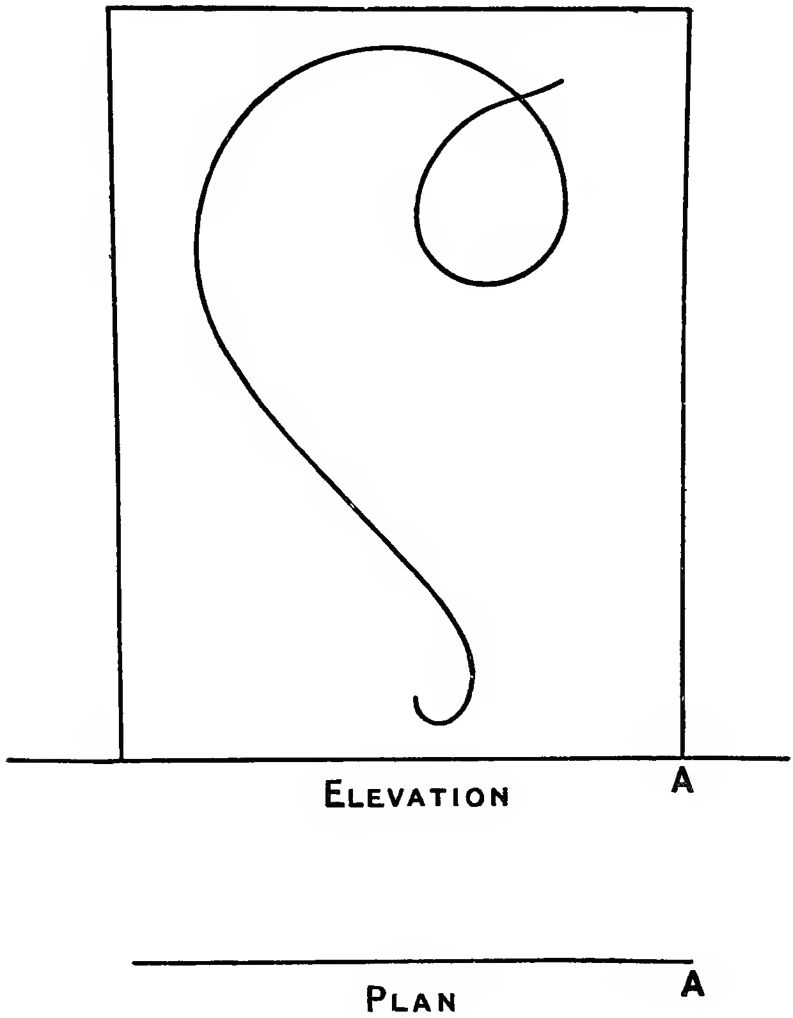


Fig. 26.

PROBLEM LI.

Fig. 26 gives the plan and elevation of a board with an irregular curve drawn upon it. The board stands upon the ground plane, and the plane of the board is a vertical plane which recedes towards the left at an angle of 40° with the picture plane. The corner **A** is to be 3 ft. on the right of the spectator and 2 ft. from the ground line. Represent the board and curve in perspective.

The scale to be used in working the problem is $\frac{1}{2}$ in. to 1 ft. The eye is to be 11 ft. by scale distant from the picture plane, and 5 ft. above the ground plane.

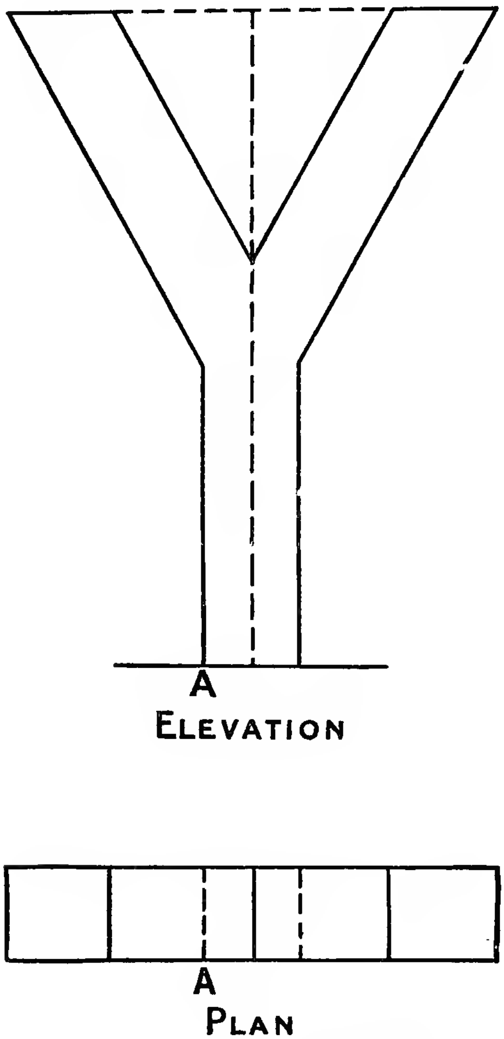


Fig. 27.

PROBLEM LII.

Fig. 27 gives the plan and elevation of a letter **Y** cut out of wood. Represent the letter in perspective standing upon the ground plane. The corner **A** is to be 1 ft. on the right of the spectator and 3 ft. from the ground line, and the face of the letter is to be in a vertical plane which recedes towards the right at an angle of 40° with the picture plane.

The scale to be used in working this problem is $\frac{1}{2}$ in. to 1 ft. The eye is to be 11 ft. by scale distant from the picture plane, and 5 ft. above the ground plane.

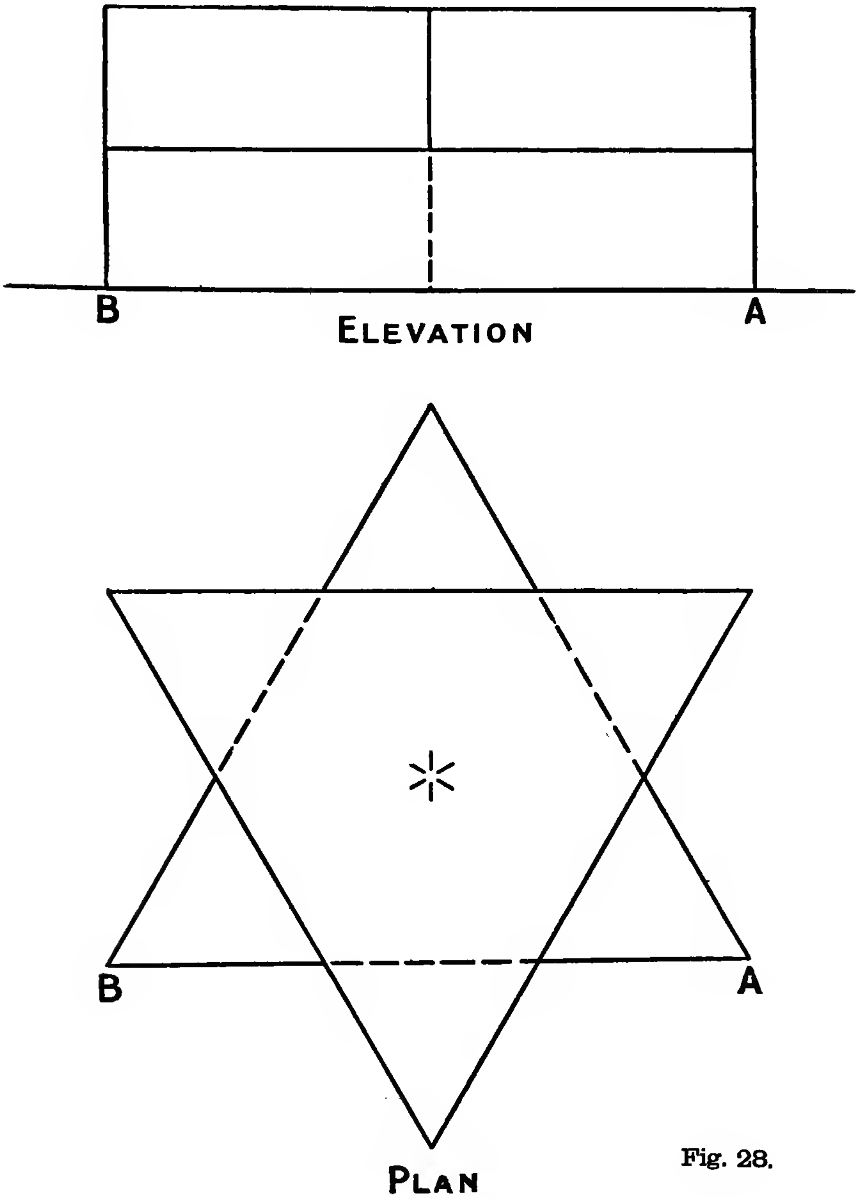


Fig. 28.

PROBLEM LIII.

Fig. 28 gives the plan and elevation of two equal and similar triangular slabs. The lower slab lies upon the ground plane, and the corner **A** is to be 1 ft. on the left of the spectator and 3 ft. from the ground line, and the edge **A B** is to recede towards the left at an angle of 30° with the ground line. Represent these two solids in perspective.

The scale to be used in working the problem is ½ in. to 1 ft. The eye is to be 11 ft. by scale distant from the picture plane, and 5 ft. above the ground plane.

METHODS EMPLOYED BY ARCHITECTS IN PREPARING PERSPECTIVE VIEWS OF BUILDINGS.

METHODS EMPLOYED BY ARCHITECTS.

In the perspective views of buildings that are required by architects, the amount of detail that has to be represented renders it almost impossible to obtain results with the same degree of accuracy that have been arrived at in the preceding problems. Drawings of this character need only give a good representation of the main features of a building and its general appearance when looked at from a certain point of view. Much can be drawn in merely by eye, and various devices may be used to facilitate the labour of the draughtsman, and although the perspective representation thus obtained is not theoretically correct it is sufficiently so for practical purposes.

When a perspective view has to be drawn from an actual building the knowledge of the principles of perspective is only of use in enabling the draughtsman to avoid or correct errors, and should not be used as a means for working out a drawing. The

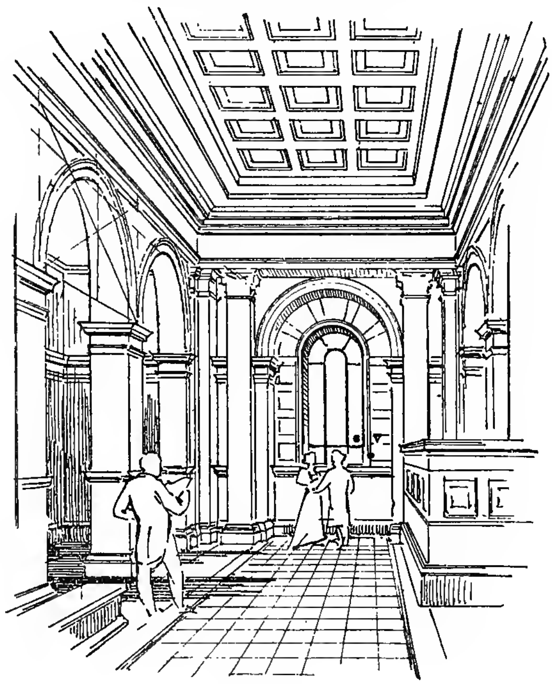


Fig. 1.

point of view from which a building is to be drawn is a matter of importance, and should be chosen with care so that the drawing may give an adequate representation of its principal features. Frequently the position of the spectator is determined by varying levels and the proximity of other buildings, but when there are no such considerations it is advisable for the spectator to stand well back if horizontal lines predominate, as in Classic buildings; when drawing Gothic buildings, where vertical lines predominate, it is better for the spectator to have a near position. When drawing an interior, such as is shown in fig. 1, a position should be chosen in which the principal lines are parallel and perpendicular to the **P.P.** It is often necessary to slightly alter the point of view during the progress of a drawing by taking a step to one side in order to reveal important architectural features that are hidden by those of minor importance. By doing so the relative positions of some of the features will be altered, so that great care must be taken not to impair the general truth of the drawing.

Perspective views of buildings which have to be drawn from particulars given by plans, elevations, and sections are worked out by the plan method in much the same manner as those problems that have been given, but certain deviations are made from a strict adherence to a theoretically correct method of working. The plan of the building has to be placed at an angle with the p.p. which will give the best view of

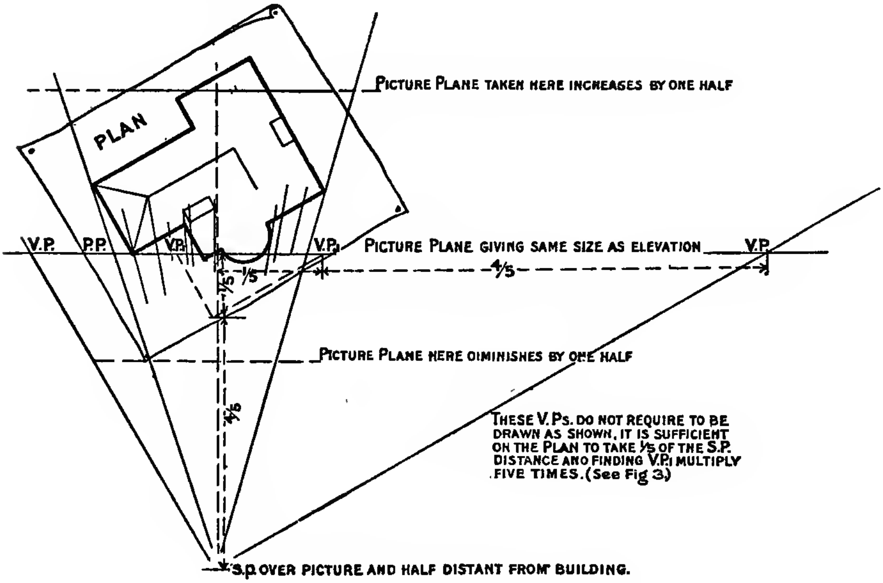


Fig. 2.

its salient features, and it is found advisable to draw the p.p. touching or even cutting a near corner of the building (fig. 2). It frequently happens that a larger or smaller view is required than that which is given when the p.p. touches or intersects the plan, this can be obtained by placing the p.p. either behind or in front of the plan (fig. 2), the distance from the plan being determined by the size of the required view.

It is better in many cases not to alter the position of the p.p. but to obtain the dimensions for the perspective view by enlarging or reducing by the aid of compasses. If, for example, the required perspective view is to be twice the size which is obtained on the p.p. which touches a corner of the building, each of the dimensions found on that first plan is doubled, and then a drawing may be made twice the size of the one which would have been obtained by using the dimensions given on the p.p. By halfing the dimensions a drawing may be made half size, and so on.

In drawings for competitions an exact limit for the width of the perspective view is often given, and the width which is required cannot be found by any simple multiple of the dimensions on the p.p. A scale should then be constructed by dividing the p.p. into suitable parts and the required width similarly. When enlarging or reducing care must be taken that the *elevation* dimensions correspond with the enlargement or reduction of the plan.

For drawing objects in the foreground that lie between the building and the spectator, another **p.p.** must be employed much nearer to the spectator than the one from which the dimensions of the building are obtained. In determining the position of the **S.P.** care must be taken that it is at a sufficient distance from the **p.p.** to enable the drawing to come within the cone of rays (60°). Usually the plan is wider than the building is high, so that the distance of the **S.P.** from the **P.P.** can be taken from the plan, but when the height is greater than the width the vertical

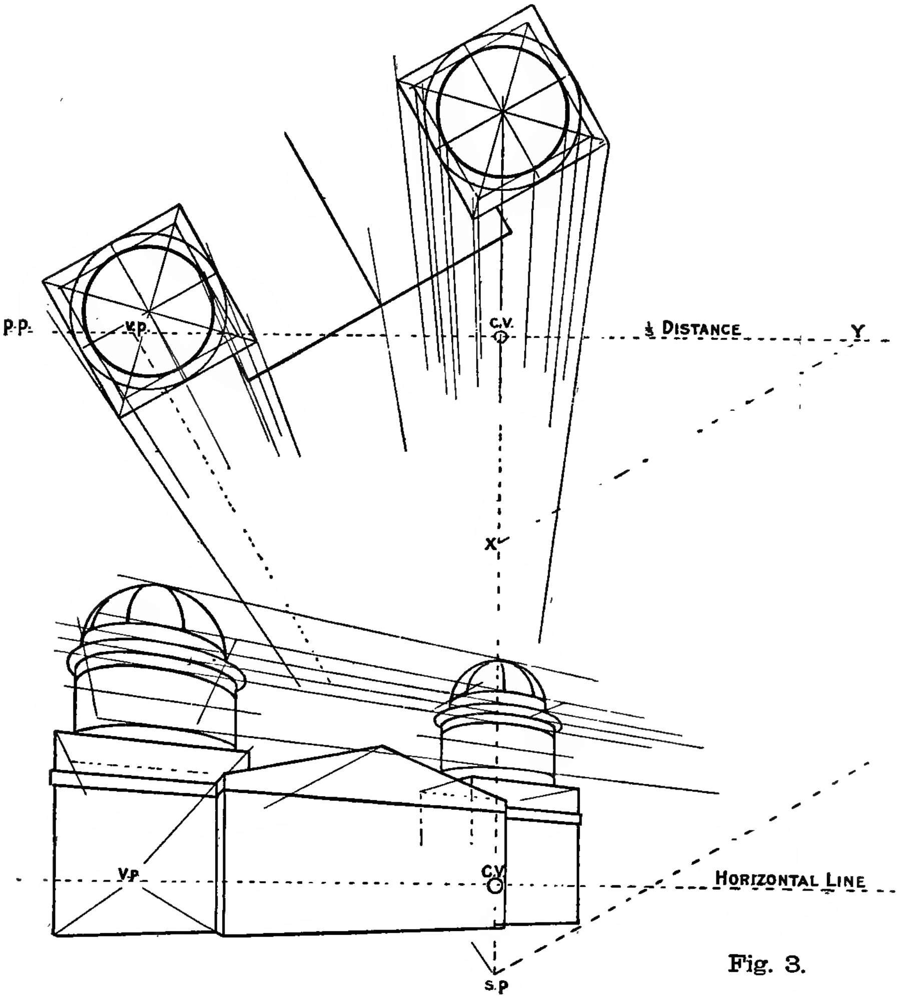


Fig. 3.

height must determine the position of the **S.P.** The best views are obtained if the **S.P.** is placed at a distance from the **P.P.** equal to a height and a half of the picture if the building is of a vertical type, but for a building of a horizontal character the distance may be two and a half times the width of the picture.

As has already been pointed out the **S.P.** should be opposite, or nearly so, to the centre of the building, except in views of interiors when the **S.P.** is better at the side of the picture (fig. 1). If a building contains a feature such as a dome or circular tower it is advisable to obtain the perspective representations of the circles by having the **S.P.** opposite to the centre of the dome or tower. This is done in order to avoid the distorted appearance which is given to a circular form when placed in perspective with the **S.P.** to the side of its centre (fig. 3).

The **V.P.**'s are found in the usual manner when they come within the limits of the paper. When the position of a **V.P.** is such that it cannot be obtained on the paper or drawing-board it would be extremely awkward to find the **V.P.** by drawing the vanishing parallel from the **S.P.**, so that is usually found in the following manner:—

In fig. 3 a third of the distance between **c.v.** and **s.p.** is obtained at **X**, and the vanishing parallel is drawn from **X** to intersect **p.p.** at **Y**. It will be evident that the point of intersection of the vanishing parallel drawn from **s.p.** with the **p.p.** will be found on the **p.p.** by measuring from **c.v.** three times the distance of **c.v.** to **Y**.

The vanishing point thus obtained can be marked by fixing a pin to a table or box

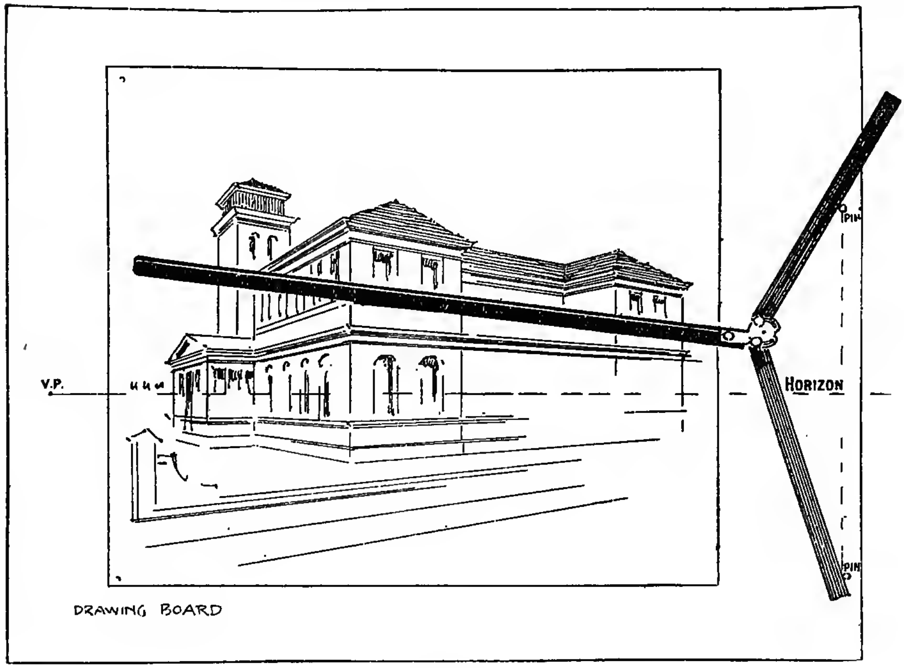


Fig. 4.

placed in a suitable position for that purpose. A piece of string is fastened to the pin and stretched from it, against which a small straight-edge can be placed, and by this means the lines are drawn with their true convergence.

As the use of a long straight-edge or string is cumbersome it is only necessary to use them at the beginning, drawing lightly over the whole sheet of paper a series of converging lines so closely together that any required intermediate lines can be drawn in by eye without much chance of error.

The Centrolinead (fig. 4) is an instrument designed to facilitate perspective drawing when a vanishing point lies outside the drawing board. It consists of three arms: the shorter two are set to varying angles and run against two studs or pins fixed clear of the drawing and equidistant from the horizontal line. The centre arm will then converge to a point near or far as the short arms are closed or open. The vanishing point has to be found, as previously explained, and then the centrolinead is set by arithmetical rule. According to the distance the two studs are from one another, and also their distance from the centre of vision, a point has to be found on the horizontal line over which the centre of the centrolinead must come, the short arms being sufficiently opened for the purpose and then clamped. Sometimes the instrument is set by drawing one converging line to the vanishing point by the straight-edge, or by using the string, well above or below the horizon, and the studs put in where convenient, and the arms manœuvred until the angle is found that will enable the long blade of the centrolinead to line truly both with the converging line that has been obtained and the horizon. The instrument can be set so that it can be worked either to the left or the right, but it cannot be set for both directions for the same drawing. So that if there is more than one vanishing point outside of the board another centrolinead will have to be used.

The centrolinead cannot be used when the station point has to be near the plan. A spire, for example, may require an angle of 60° or 20° and a bird's-eye view requires quite as quick an angle under the horizontal line.

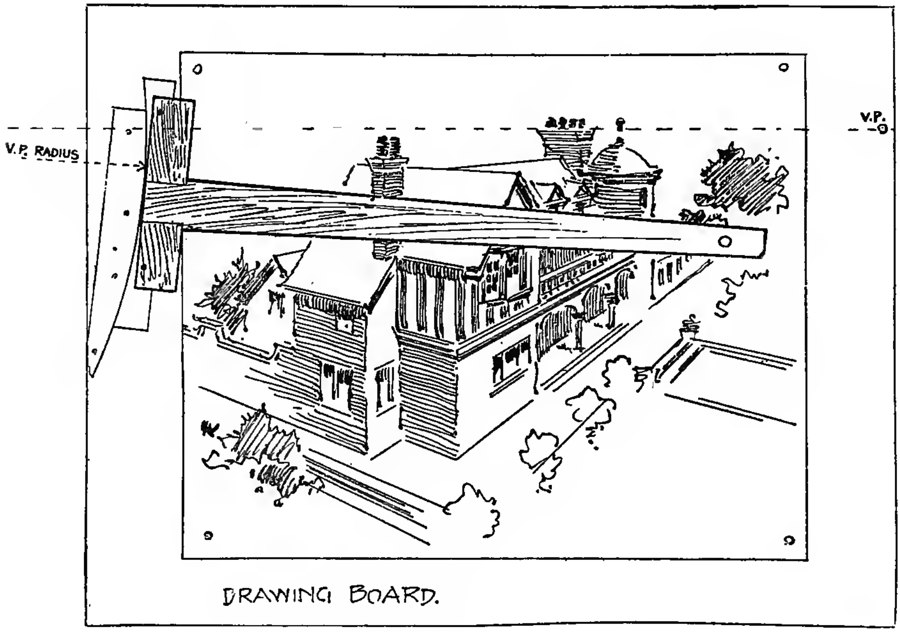


Fig. 5.

A T-square with a segmental head (fig. 5) is also used for working with distant vanishing points. Outside of the drawing, fix on the board a cardboard arc whose centre is the V.P., and fasten the concave cutting to a T-square or straight-edge. The position of the cardboard arc on the board is determined according to whether the angle of perspective is greater above or below the H.L. By this means a greater angle can be drawn than by the use of the centrolinead.

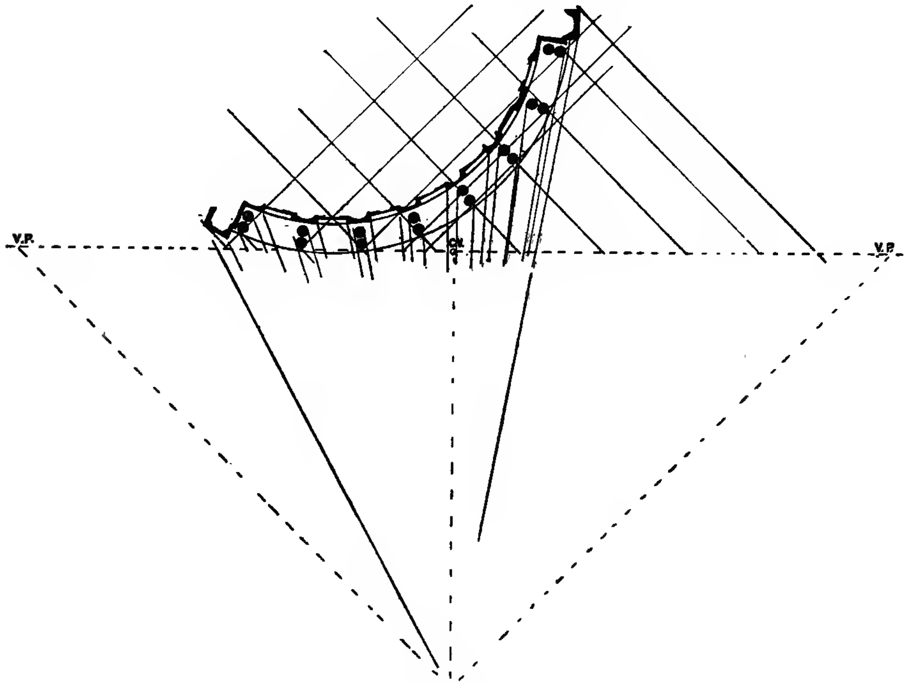


Fig. 6.

The most difficult building to draw in perspective is one with many curved features. In a crescent-shaped façade (fig. 6) there may not be a single straight line converging to a **V.P.** The perspective view is obtained by assuming the building to be intersected by an indefinite number of vertical planes making angles, say of 45°, to the right and left with the **P.P.** as shown in fig. 6, and the widths and heights for the perspective view obtained in the usual manner.

With a knowledge of the theory of perspective and the explanations that have just been given of the practical methods employed, little or no difficulty will be found by an architectural draughtsman in preparing any required perspective view of a building.