Surface impedance and skin depth for transverse waves in bi-kappa distributed plasma.

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Abstract

The skin depth has been calculated using the surface impedance for the transverse waves in bi-kappa distributed plasma. The effects of temperature anisotropy on the surface impedance and the skin depth have been studied using the kinetic model for an electromagnetic wave striking on a plasma surface. It is noted that the real part of the surface impedance has direct relation with temperature anisotropy and kappa parameter, while changes inversely with the wave frequency. On the other hand the imaginary part, however, is not affected by kappa parameter and temperature anisotropy significantly but changes directly with the frequency. It also been calculated that the skin depth is inversely related to the frequency in both resonant and non-resonant case. It has been found that in low frequency regime (resonant case) the skin depth first increases with increasing temperature anisotropy and then remains constant, while in high frequency regime (non-resonant) the skin depth increases linearly by increasing the temperature anisotropy. It has been calculated that the skin depth in both high and low frequency regime increases by increasing the kappa parameter. The comparison between the skin depth at high and low frequency shows that skin depth is greater in low frequency regime/resonant case.

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1 INTRODUCTION

1.1 What is Plasma?

A plasma is a quasineutral gas of charged particles that shows collective behavior. Plasma is neutral enough that electron density is almost equal to density of ions but not so neutral that all the electromagnetic forces vanishes. Collective behavior means that plasma behavior depends on each individual particles. Plasma is the most abundant form of matter in the known universe. Everything in the early universe was made up of plasma. Stars, nebulae, and even interstellar space are all currently filled with plasma. Plasma is also propagated across the solar system in the form of the solar wind, the Earth is entirely wrapped by plasma, trapped within its magnetic field. There are also lots of terrestrial plasmas to be observed. They appear in scientific experiments of many kinds, lightning, fluorescent bulbs, and more [1].

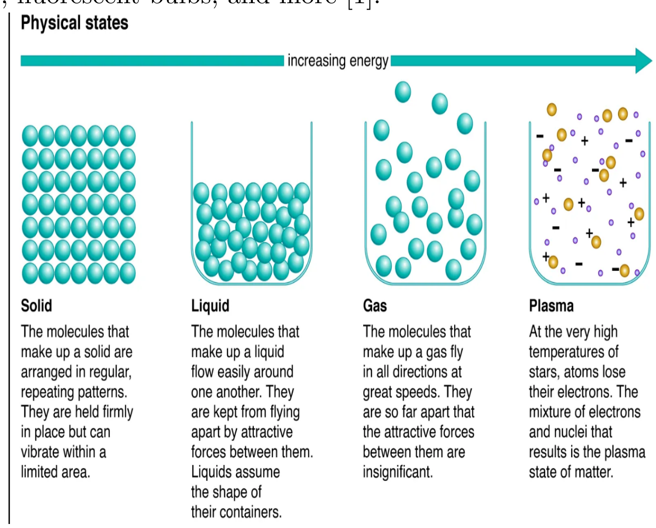

Fig. 1.1. States of matter [2].

1.2 Debye shielding

Debye shielding is the characteristic behavior of plasma that describe the screening of electric field by plasma particles. Assume we intended to create an electric field within plasma by introducing two charged balls connected to a battery. The balls would attract particles of opposite charge, resulting in a cloud of ions surrounding the negative ball and a cloud of electrons surrounding the positive ball. If the plasma was cold and there were no thermal motions, there would be the same number of charges in the cloud as there were in the ball; the shielding would be perfect, and there would be no electric field in the plasma's body beyond the clouds. If the temperature is finite, however, the particles at the cloud's edge, where the electric field is weak, have enough thermal energy to escape the electrostatic potential well. The cloud's "edge" then appears at the radius where the potential energy is approximately equal to the thermal energy KT of the particles, indicating that the shielding is not complete. Potentials of the order of KT/e can leak into the plasma, creating inadequate electric fields. The approximate thickness of such a charge clouds is given by,

$$\lambda_D = \sqrt{\frac{\epsilon_o K T_e}{n e^2}} \tag{1.1}$$

Where, λ_D is called Debye length and it's the measure of length over which the electric field is screening out by plasma [3]

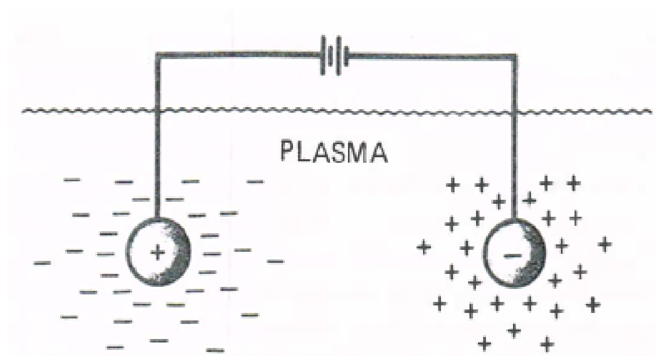

Fig. 1.2. The Debye shielding.

From here we can defined quasi-neutrality. If the debye length is much shorter than dimension L, then whenever the potential introduced to the plasma, it will shielded out in a short distance as compared to L, which will left the bulk of plasma free of potentials.

1.3 Criteria for plasma

Every ionized gas is not plasma, there is certain conditions that must satisfy for an ionized gas to be called a plasma.

1.3.1 Quasineutrality

If the dimension L of a system is larger than Debye length, then potential will be shielded out at a short distance, leaving the bulk of plasma free of fields.

$$\lambda_D << L$$

1.3.2 Numbers of particles

Debye shielding is possible only if there are large numbers of particles. If there are few particles then debye shielding will not be a valid concept.

$$N_D >> 1$$

1.3.3 Collision time

If collision between the particles are large, then the motion of particles is controlled by hydrodynamic forces rather than electromagnetic forces. If ω is the frequency of plasma oscillation and τ is the mean time between the collision with neutral atoms. then following condition is need to be satisfied for gas to behave like plasma [3].

$$\omega\tau >> 1$$

1.4 Plasma models

There are different plasma mathematical models that are used to examine the plasma, its properties and phenomenon associated with it e.g. particle orbit model, fluid model and kinetic model. We will discuss it one by one.

1.4.1 Particle orbit model

It is a simple approach in which we study the motion of individual particle of plasma i.e. its interaction with the electric and magnetic field. This gives us an understanding that how charged particle behave in the presence of electromagnetic field. It is applicable in low density plasma.

The basic equation of this model is Newton's equation;

$$\vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} \tag{1.2}$$

Where F is the Lorentz force defined as;

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1.3}$$

Thus;

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1.4}$$

But it failed in the case of collisional plasma, because of collisions it's very difficult to observe the particles trajectories and orbits [4].

1.4.2 Fluid Model

The presence of numerous fluid-like features in plasma, such as coherent motion has been observed. Plasma can be treated as a fluid since it has a wide range of velocities and particle collisions, which helps to maintain the local equilibrium distribution of particles. Thus, the dynamics of plasma can be explained by macroscopic quantities that are directly related to average values, such as temperature, densities, and velocities [1].

1.3.2.1 Equations of fluid model The equations that are use in a fluid model are,

1) Maxwell's equations
2) Equation of continuity
3) Equation for momentum transport

Maxwell's equations Maxwell equation are given as:

$$\nabla.\vec{E} = \frac{\rho}{\epsilon_\circ} \tag{1.5}$$

$$\nabla.\vec{B} = 0 \tag{1.6}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{1.7}$$

$$\nabla \times \vec{B} = \mu_\circ \vec{J} + \mu_\circ \epsilon_\circ \frac{\partial \vec{E}}{\partial t} \tag{1.8}$$

Here $\vec{E}$ and $\vec{B}$ are the electric and magnetic field respectively, $\vec{J}$ is the current density and ρ is the charge density. These quantities indicate the effect of particle position and mobility on electromagnetic fields.

Equation [1.5] is the Gauss law for electrostatic which tells us that the electric flux across any closed surface is proportional to the electric charge enclosed by the surface.

Equation [1.6] is the Gauss law for electromagnetism which says that magnetic monopole does not exist.

Equation [1.7] is the Faraday law of electromagnetic induction which states that "An emf will be induced in a coil by changing the magnetic flux".

Equation [1.8] is the Maxwell-Ampere's law, it states that by changing the electric fields or currents will generates circulating magnetic fields [5].

Equation of continuity It tells us that flow rate is constant which means that mass in a given volume of space changes only if there is a net mass flux into or out of that volume.

$$\frac{\partial n}{\partial t} = \nabla.(n\vec{u}) \tag{1.9}$$

In term of charge and current density, we can write it as,

$$\frac{\partial \rho}{\partial t} = \nabla.\vec{J} \tag{1.10}$$

Equation for momentum transfer As the velocity in fluid model is a function of both space and time, so the time derivative of velocity is,

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u_x}{\partial x}\frac{dx}{dt}\hat{x} + \frac{\partial u_y}{\partial y}\frac{dy}{dt}\hat{y} + \frac{\partial u_z}{\partial z}\frac{dy}{dt}\hat{z} \tag{1.11}$$

or

$$\frac{d\vec{u}}{dt} = \frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} \tag{1.12}$$

The equation of motion for n number of particles can be written as,

$$mn\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = nq(\vec{E} + \vec{v} \times \vec{B}) \tag{1.13}$$

The above equation is for collisionless plasma, if we consider collision and thermal effects than we add the pressure gradient term i.e. -$\nabla\vec{P}$ to the right side of the equation.

i.e.

$$mn\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = nq(\vec{E} + \vec{v} \times \vec{B}) - \nabla\vec{P} \tag{1.14}$$

The above equation is called momentum transport equation. All these equations are used to describe the plasma dynamics.

1.4.3 Kinetic model

The fluid model which is the microscopic description of plasma has some limitation that makes it unable to fully describe some phenomenon. For example we cannot study the wave-particles interactions, temperature anisotropic and non-thermal plasma. Kinetic theory is the microscopic description of plasma, it gives us more accurate and clear picture of the plasma as compared to fluid model.

In the kinetic model, the distribution function of the particles is depending upon position, velocity and time given as;

$$f(r, v, t) = f(x, y, z, v_x, v_y, v_z, t) \tag{1.15}$$

There are seven variables, three spatial, three velocity vectors and one temporal [3].

1.3.3.1 Equation of kinetic model As the distribution function is a function of position, velocity and time i.e. $f(r, v, t)$. So, the time derivative of distribution function can be written as;

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial f}{\partial v_x}\frac{\partial v_x}{\partial t} + \frac{\partial f}{\partial v_y}\frac{\partial v_y}{\partial t} + \frac{\partial f}{\partial v_z}\frac{\partial v_z}{\partial t} \quad (1.16)$$

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \vec{v}.\nabla f + \vec{a}.\nabla f \quad (1.17)$$

As,

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

also,

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \quad (1.18)$$

So, it can be written as;

$$\frac{df_c}{dt} = \frac{\partial f}{\partial t} + \vec{v}.\nabla f + \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) \quad (1.19)$$

In case of collisionless plasma $\frac{df_c}{dt} = 0$

$$\Rightarrow \frac{\partial f}{\partial t} + \vec{v}.\nabla f + \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) = 0 \quad (1.20)$$

The above equation is called Vlasov-Equation [6].

1.5 Classification of distribution functions

There are different types of distributions functions which are used to study the different plasma environments e.g. maxwellian distribution, kappa distribution etc.

1.5.1 Maxwellian distribution

The Maxwellian distribution is also known as Maxwell-Boltzmann distribution. A gas in thermal equilibrium contains particles of all velocities, and the most probable distribution for all these velocities is Maxwellian distribution.

which is given by;

$$f(u) = A\exp(-\frac{\frac{1}{2}m|u|^2}{KT}) \quad (1.21)$$

Here A is normalization constant given by;

$$A = n(\frac{m}{2\pi KT})^{1/2} \quad (1.22)$$

Here f(u)du shows the numbers of particles per m^3 with velocities between u and u+du. $\frac{1}{2}mu^2$ is the kinetic energy while K is the Boltzmann's constant $K=1.32\times10^{-23}J/K$.

The width of the distribution tells us about temperature T [4].

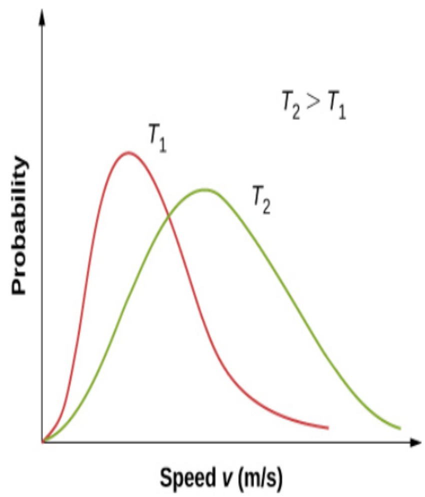

Fig. 1.3. Maxwell distribution at different temperatures.

Bi-Maxwellian distribution When temperature anisotropy is taken into account, then we have two different thermal velocities of charged particles i.e. parallel and perpendicular in the direction of the magnetic field. Mathematically,

$$f(v) = \frac{n}{\pi^{3/2}} \frac{1}{\theta_\parallel \theta_\perp^2} \exp\left(-\frac{v_\perp^2}{\theta_\perp^2} + \frac{v_\parallel^2}{\theta_\parallel^2}\right) \tag{1.23}$$

1.5.2 Non-Maxwellian distributions

It is observed experimentally that natural occurring plasma is not in thermodynamic equilibrium. So non-maxwellian distribution is required to deal with such type of plasma. These type of distributions are common in space and laboratory plasmas.

Kappa distribution Nonthermal particle distributions occur frequently in the solar wind and many space plasmas, their presence has been generally determined through spacecraft data. Such variations from Maxwellian distributions are likely to exist in every low-density plasma in the universe where binary charge collisions are rare. These suprathermal population are well described by kappa velocity distribution functions. These distribution have high energy tails that deviated from maxwellian distribution. It is defined as;

$$f(r,v) = \frac{n}{2\pi(\kappa\theta^2)^{3/2}} \frac{\Gamma(\kappa+1)}{\Gamma(\kappa-\frac{1}{2})\Gamma(\frac{3}{2})} \left(1+\frac{v^2}{\kappa\theta^2}\right)^{-\kappa-1} \tag{1.24}$$

Where θ is the thermal velocity,

$$\theta = \sqrt{\frac{2\kappa-3}{\kappa}\frac{k_B T}{m}}$$

Here κ is the spectral index. It must take the values from $\kappa > \frac{3}{2}$ because at $\frac{3}{2}$ the thermal velocity is not defined and the distribution function collapses. As $\kappa \to \infty$, the distribution function reduces to Maxwellian as shown in the figure [7].

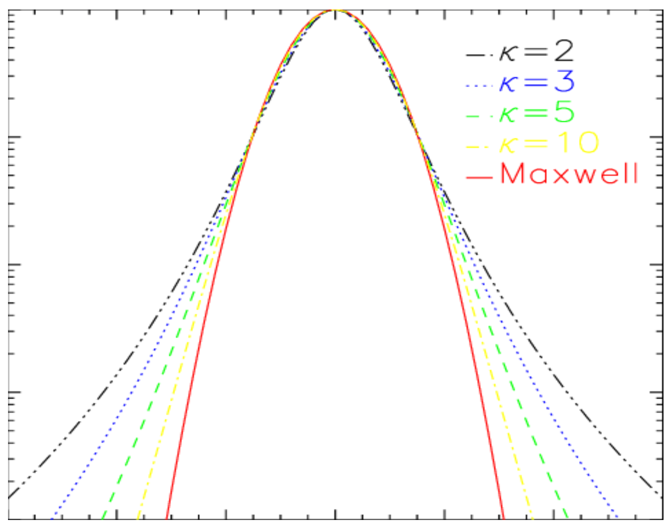

Fig. 1.4. The kappa velocity distribution function for the different values of kappa parameters.

Bi-kappa distribution In temperature anisotropic plasmas like non-thermal emission in astrophysical sources and the magnetic field fluctuations in space plasma, where the plasma have different temperatures in different directions, then we bi-kappa distribution function, which is given by,

$$f(v_\perp, v_\parallel) = \frac{n}{\pi^{3/2}} \frac{1}{\theta_\perp^2 \theta_\parallel} \frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})} \left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-(\kappa+1)} \tag{1.25}$$

It is very useful for studying the instabilities that arises from temperature anisotropy [8].

1.6 Waves in plasma

There are many types of waves depending on the direction of wave vector $\vec{k}$ to the electric field $\vec{E}$ and magnetic field $\vec{B}$. The waves exist in plasma are perpendicular propagating ($\vec{k} \perp \vec{B}_o$), parallel propagating ($\vec{k} \parallel \vec{B}_o$), transverse propagating ($\vec{k} \perp \vec{E}_1$), longitudinal propagating ($\vec{k} \parallel \vec{E}_1$), electrostatic propagating ($\vec{B}_1 = 0$) and electromagnetic propagating waves. Some of the electromagnetic propagating waves are, Ordinary-mode (O-mode), Extra Ordinary-mode (X-mode), R-L waves, etc., are produced when there is a magnetic field perturbation [9].

1.7 Surface impedance

It characterizes the interaction of electromagnetic waves with plasma boundary. It is defined as the ratio of tangential electric field to that of tangential magnetic field at the plasma surface. Surface impedance is a complex quantity whose real part shows the power absorption inside the plasma while the imaginary part give us the phase of the reflected wave [10].

It is given as;

$$Z_s = \frac{4\pi}{c}\frac{\vec{E}_y(0)}{\vec{B}_x(0)} \tag{1.26}$$

1.8 Skin depth

When the frequency of the electromagnetic (EM) wave is less than the frequency of the plasma, it attenuates when it interacts with the plasma. This phenomenon is called skin depth. It is the measure of how much an electromagnetic wave travels inside the plasma. It depends on electrons thermal motion, if the thermal motion of an electron is weak, then it is called normal skin depth and if the electron thermal motion is taken into account the skin depth is called anomalous skin depth. The penetration of an electromagnetic wave depends on plasma frequency. if $\omega > \omega_p$ then plasma is called underdense plasma and electromagnetic wave can pass through the plasma and if $\omega < \omega_p$ then plasma is called overdense plasma and electromagnetic waves cannot pass through the plasma [11].

Let consider the dispersion relation of an O-mode,

$$c^2k^2 = \omega^2 - \omega_p^2$$

As in overdense plasma $\omega_p > \omega$, so the above equation will reduce to,

$$k_i = \frac{\omega_p}{c}$$

As, skin depth is related with k$_i$ as $\delta = \frac{1}{k_i}$

$$\Rightarrow \delta = \frac{c}{\omega_p} \tag{1.27}$$

where c is the speed of light and ω_p is the plasma frequency given as,

$$\omega_p = \sqrt{\frac{n_o e^2}{m_e \epsilon_o}}$$

1.9 Application of surface impedance and skin depth

Surface impedance and skin depth have numerous applications in plasma physics. Some of the applications are;

1.9.1 Designing plasma antennas

A plasma antenna is a new type of radio antenna in which plasma replaces the metal parts of a typical antenna. A type of plasma antenna " gas plasma antenna" uses a discharge tube instead of metal elements. A gas plasma antenna, as compared to metal elements, is a form of plasma antenna. As current travels into the tube, the gas partially or completely ionizes to plasma, becomes conductive, and behaves as a mirror, eventually transmitting and receiving signals. Plasma antennas are nearly transparent to a wide range of electromagnetic waves above the plasma frequency and becomes invisible when the apparatus is turned off and the gas de-ionizes. Plasma antennas have several advantage over metallic antennas like, plasma has extremely high electrical conductivity, which aids in the receiving direction, and transmission of various radio signals. Plasma antennas may be electronically modified, which implies that their frequency, bandwidth, and directivity can be changed without physically modifying the antenna. As a result, they are extremely flexible and responsive to a wide range of applications. The surface impedance of a plasma antenna is critical to the antenna's design and functioning. It determines the antenna's efficiency, bandwidth, and directivity. The surface impedance influences how the antenna interacts with the plasma. The skin depth of an antenna controls how much of its surface is efficiently employed to emit or receive electromagnetic waves [12].

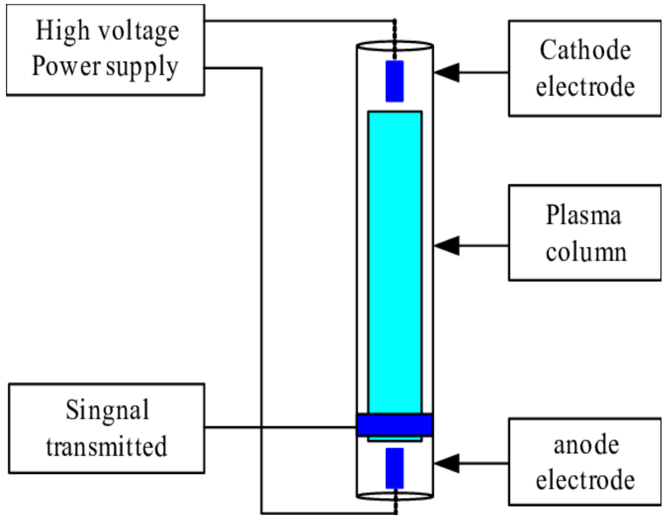

Fig. 1.5. Structure of plasma antenna.

1.9.2 Plasma wall interaction

The plasma core is well separated from the first wall materials in magnetically confined fusion plasmas. Highly energetic particles, on the other hand, can escape the contained plasma and incident with the surrounding walls. These collisions result in transfer of energy to the walls causing localizing heat. Surface impedance plays an important role in plasma wall interaction like, it can influence the efficiency of energy transfer from the plasma to the wall and hence the heat load on the material. Surface impedance can affects the magnetic fields that confine plasma as well as the behavior of instabilities caused by plasma-wall interactions. Surface impedance affects the reflection and absorptions of particles which causes sputtering, impurities and stability of plasma. The skin depth is significant in plasma wall interaction because it controls how far the plasma's electromagnetic fields can penetrate inside the wall [13].

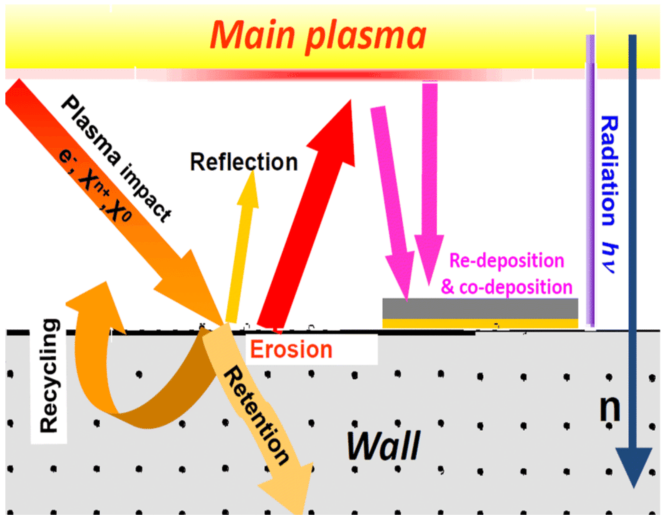

Fig. 1.6. Plasma wall interaction.

1.9.3 Plasma heating

The process of raising the temperature of a plasma is known as plasma heating. It needs to achieve high-temperature fusion processes. This is done by several methods like Ohmic heating, neutral beam injection, radio frequency heating and magnetic compression. Surface impedance is an important factor in plasma heating because it determines how effectively the plasma can be heated. The skin depth affects how far the radio waves can penetrate into the plasma [14].

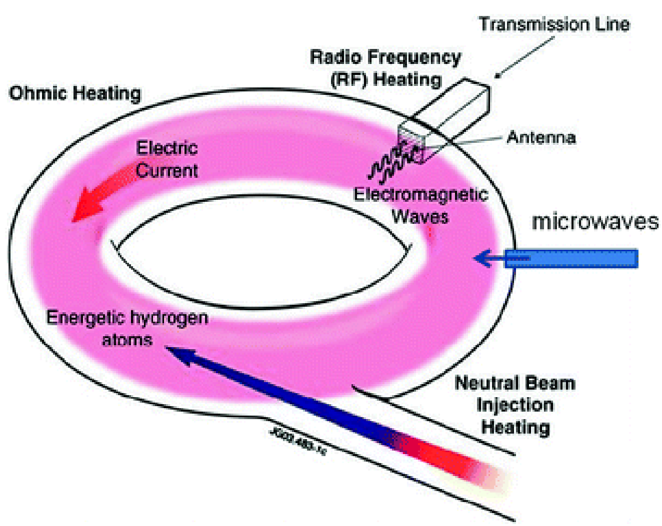

Fig. 1.7. Plasma heating

2 PLASMA KINETIC MODEL

The fluid model has some limitation that makes it unable to fully describe some phenomenon. For example, we can't use fluid model to study wave particle interaction, it fails in non-thermal equilibrium plasma. The instabilities arises in plasma due to different causes cannot be studied by using the fluid model. Moreover, even though the structure is different, as seen below, the fluid model cannot distinguish between distributions that have the same area under the curve (which will give us the total number of particles).

Kinetic theory is the microscopic description of plasma, it gives us more accurate and clear picture of the plasma as compared to fluid model.

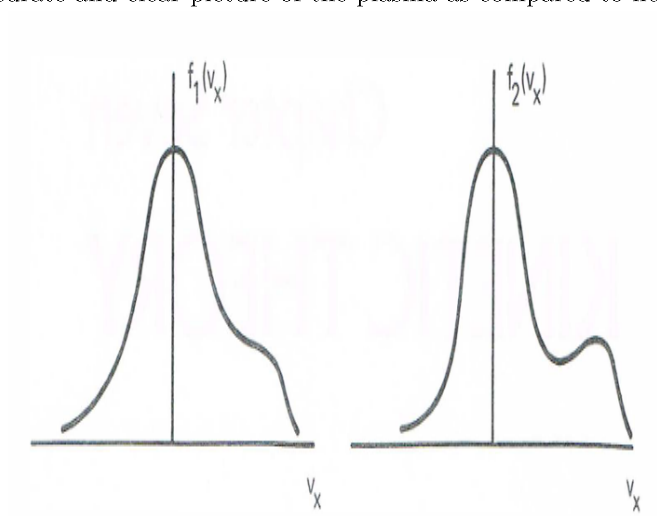

Fig. 2.1. Examples of two non-maxwellian distributions:

In the kinetic model, the distribution function of the particles is depending upon position , velocity and time given as;

$$f(r, v, t) = f(x, y, z, v_x, v_y, v_z, t) \tag{2.1}$$

There are seven variables, three spatial, three velocity vectors and one temporal.

2.1 Equation of kinetic model

As the distribution function is a function of position, velocity and time i.e. $f(r, v, t)$. So, the time derivative of distribution function can be written as;

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial f}{\partial z}\frac{\partial z}{\partial t} + \frac{\partial f}{\partial v_x}\frac{\partial v_x}{\partial t} + \frac{\partial f}{\partial v_y}\frac{\partial v_y}{\partial t} + \frac{\partial f}{\partial v_z}\frac{\partial v_z}{\partial t} \tag{2.2}$$

As,

$$\vec{F} = m\frac{d\vec{v}}{dt} \tag{2.3}$$

So, it can be written as;

$$\frac{df_c}{dt} = \frac{\partial f}{\partial t} + \vec{v}.\nabla_x f + \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}).\nabla_v f \tag{2.4}$$

In case of collisionless plasma $\frac{df_c}{dt} = 0$

$$\frac{\partial f}{\partial t} + \vec{v}.\nabla_x f + \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}).\nabla_v f = 0 \tag{2.5}$$

The above equation is called Vlasov-Equation.

2.2 Generalized dielectric tensor

The dielectric tensor is a matrix that describes the electric permittivity of plasma in different direction. It tells us how an electromagnetic waves propagate and interacts within a plasma. The dielectric tensor of a plasma is typically anisotropic, meaning that its components vary depending on the direction of the electric and magnetic fields. This anisotropy is due to the fact that plasmas are made up of charged particles that are free to move. The motion of these charged particles can cause the plasma to respond differently to electric and magnetic fields that are applied in different directions.

2.3 Derivation of dielectric tensor for magnetized plasma

The dielectric tensor matrix is derived from Vlasov's equation;

$$\frac{\partial f}{\partial t} + \vec{v}.\frac{\partial f}{\partial x} + \vec{a}.\frac{\partial f}{\partial v} = 0 \tag{2.6}$$

We can write the above equation in term of relativistic momentum. The relativistic momentum defined as,

$$\vec{p} = \gamma m \vec{v}$$

where;

$$\gamma = (1 - \frac{v^2}{c^2})$$

Also,

$$\vec{F} = \frac{d\vec{p}}{dt} = e(\vec{E} + \vec{v} \times \vec{B})$$

$$\frac{\partial F}{\partial v} = \frac{\partial F}{\partial p}\frac{\partial p}{\partial v} = \gamma m \frac{\partial F}{\partial p} \tag{i}$$

and

$$\frac{\partial p}{\partial t} = \gamma m \frac{\partial v}{\partial t} = \gamma m a$$

$$\Rightarrow \vec{a} = \frac{1}{\gamma m}\frac{\partial \vec{P}}{\partial t} = \frac{1}{\gamma m} e[\vec{E} + \vec{v} \times \vec{B}] \tag{ii}$$

Put Eq [i] and [ii] in Eq [2.6], we have;

$$\frac{\partial f}{\partial t} + \vec{v}.\frac{\partial f}{\partial x} + \frac{1}{\gamma m} e[\vec{E} + \vec{v} \times \vec{B}] \times \gamma m \frac{\partial f}{\partial \vec{p}} = 0$$

$$\frac{\partial f}{\partial t} + \vec{v}.\frac{\partial f}{\partial x} + e[\vec{E} + \vec{v} \times \vec{B}].\frac{\partial f}{\partial \vec{p}} = 0 \tag{2.7}$$

Now, on linearizing;

$$f = f_o + f_1$$

$$\vec{B} = \vec{B}_o + \vec{B}_1$$

$$\vec{E} = \vec{E}_1$$

Eq [2.7]⇒

$$\frac{\partial f_1}{\partial t} + \vec{v}.\frac{\partial f_1}{\partial x} + e\left(\vec{E}_1 + \frac{\vec{v} \times \vec{B}_1}{c}\right).\frac{\partial f_o}{\partial \vec{p}} + \frac{e}{c}\left(\vec{v} \times \vec{B}_o\right).\frac{\partial f_1}{\partial \vec{p}} = 0 \tag{2.8}$$

the Laplace Transform of derivative is;

$$L(\frac{\partial f_1}{\partial t}) = \int_0^\infty \frac{\partial f_1}{\partial t} e^{-st} dt$$

$$= f_1 e^{-st}|_o^\infty - \int_0^\infty f_1(-s)e^{-st} dt$$

⇒

$$= -f_1(t=0) + sL(f_1)$$

$$\frac{\partial f_1}{\partial t} = sL[f_1] \tag{iii}$$

Fourier transform of derivative is;

$$F(\frac{\partial f_1}{\partial x}) = \frac{1}{\sqrt{2\pi}}\left[\int_{-\infty}^{\infty} \frac{\partial f_1}{\partial x}(e^{-ikx})dx\right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[e^{-\imath kx} \int_{-\infty}^{\infty} \frac{\partial f_1}{\partial x} dx - \int_{-\infty}^{\infty} (-ik) e^{-ikx} \int \frac{\partial f_1}{\partial x} dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[e^{-ikx} f_1|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} (-ik) e^{-ikx} \times f_1 \times dx \right]$$

$$= \frac{1}{\sqrt{2\pi}} \left[(ik) \int_{-\infty}^{\infty} (e^{-ikx}) f_1 \times dx \right]$$

$$F[\frac{\partial f_1}{\partial x}] = (ik) F(f_1)$$

$$F[\frac{\partial f_1}{\partial x}] = ik \times F(f_1) \tag{iv}$$

Apply this Fourier-Laplace transform on Eq [2.8], we have;

$$(s + i\vec{k}.\vec{v}) f_1 + \frac{e}{c} (\vec{v} \times \vec{B}_o) \frac{\partial f_1}{\partial p} + e(\vec{E}_1 + \frac{1}{c} \vec{v} \times \vec{B}_1) \frac{\partial f_o}{\partial p} = g \tag{2.9}$$

where;

$$g = \int \frac{1}{\sqrt{2\pi}} e^{-ikx} f_1(t = 0) + \text{other terms} \tag{2.10}$$

Let's define the relativistic cyclotron frequency: Relativistic cyclotron frequency is defined as 'The frequency at which high speed charged particles (speed close to speed of light) gyrate around magnetic field lines.

i.e.;

$$\Omega = \frac{eB_o}{\gamma mc} = \frac{\Omega_o}{\gamma} \tag{v}$$

Put Eq [v] in Eq [2.9], we have;

$$(s + i\vec{k}.\vec{v}) f_1 - \Omega \frac{\partial f_1}{\partial \Phi} + \Phi_{(\Phi)} = 0 \tag{2.11}$$

where

$$\Phi_{(\Phi)} = e(\vec{E}_1 + \frac{1}{c} \vec{v} \times \vec{B}_1) \frac{\partial f_o}{\partial p} - g \tag{vi}$$

So, Eq [2.11] can be written as;

$$\frac{\partial f_1}{\partial \Phi} - \frac{s + i\vec{k}.\vec{v}}{\Omega} f_1 = \frac{\Phi_{(\Phi)}}{\Omega} \tag{2.12}$$

Eq [2.12] is the first order inhomogeneous differential equation where homogeneous part is;

$$\frac{\partial G_1}{\partial \Phi} - \frac{s + i\vec{k}.\vec{v}}{\Omega} G_1 = 0 \tag{vii}$$

Now let's define the coordinates for different parameters,
In cylindrical coordinates;

$$\vec{k} = (\vec{k}_\perp, 0, \vec{k}_\parallel)$$

$$\vec{v} = (\vec{v}_\perp \cos\Phi, \vec{v}_\perp \sin\Phi, \vec{v}_\parallel)$$

$\Rightarrow$

$$\vec{k}.\vec{v} = k_\perp v_\perp \cos\Phi + k_\parallel v_\parallel$$

So, Eq [vii] will becomes;

$$\frac{\partial G_1}{\partial \Phi} - \frac{s + ik_\perp v_\perp \cos\Phi + ik_\parallel v_\parallel}{\Omega} G_1 = 0$$

$$\int \frac{\partial G_1}{G_1} = \int \frac{s + ik_\perp v_\perp \cos\Phi + ik_\parallel v_\parallel}{\Omega} d\Phi$$

$$G_1^{'} = \exp\left[\frac{1}{\Omega}\int_{-\infty}^{\infty}\left(s + ik_\parallel v_\parallel + ik_\perp v_\perp \cos\Phi^{''}\right) d\Phi^{''}\right]$$

$$G_1^{'} = \exp\left[\frac{1}{\Omega} s + ik_\parallel v_\parallel (\Phi - \Phi^{'}) - ik_\perp v_\perp (\sin\Phi - \sin^{'})\right] \tag{viii}$$

The solution of inhomogeneous part is given as;

$$f_1 = \int \frac{G_{(\Phi')}\Phi_{(\Phi')}}{\Omega} d\Phi^{'} \tag{ix}$$

Now from Maxwell's curl equations;

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \times \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

By applying Fourier-Laplace transform to the above equation, we will get;

$$i\vec{k} \times \vec{E} = -\frac{s}{c}\vec{B} + X \;\Rightarrow\; \vec{B} = \frac{-ic}{s} k \times \vec{E} + X \tag{x}$$

$$i\vec{k} \times \vec{B} = \frac{s}{c}\vec{E} + \frac{4\pi}{c}\vec{J} - Y \tag{xi}$$

Here X and Y is the integration term.

Put Eq [x] in [xi];

$$i\vec{k} \times \frac{-ic}{s}\vec{k} \times \vec{E} = \frac{s}{c}\vec{E} + \frac{4\pi}{c}\vec{J}$$

$$c\left[\vec{k} \times c(\vec{k} \times \vec{E})\right] = s^2\vec{E} + 4\pi s\vec{J}$$

Applying the property

$$\vec{A} \times (\vec{B} \times \vec{C}) = (\vec{A}.\vec{C}).\vec{B} - (\vec{A}.\vec{B}).\vec{C}$$

$$(s^2 + c^2\vec{k}^2)\vec{E} - c^2\vec{k}(\vec{k}.\vec{E}) + 4\pi s\vec{J} = X - Y$$

$$(s^2 + c^2\vec{k}^2)E - c^2\vec{k}(\vec{k}.\vec{E}) + 4\pi s\vec{J} = I \tag{xii}$$

Here J is the current density and I is the integral term;

$$J = \sum_\alpha q_\alpha n_o \int f_\alpha v d^3p$$

put Eqns [vi] and [ix] in Eq [xii];

$$(s^2+c^2k^2)\vec{E}-c^2\vec{k}(\vec{k}.\vec{E})+4\pi s\sum_\alpha \frac{q_\alpha n_o}{m_\alpha \Omega}\int pdp\int_{\pm\infty}^{\Phi}\left[\begin{array}{c} e^{\frac{1}{\Omega}s+ik_\parallel v_\parallel(\Phi-\Phi')-ik_\perp v_\perp(\sin\Phi-\sin\Phi')} \\ \times\vec{E} - \frac{i}{s}\vec{v}\times(\vec{k}\times\vec{E}).\frac{\partial f_o}{\partial p} \end{array}\right]\times d\Phi = I \tag{2.13}$$

or we can written as;

$$(s^2 + c^2k^2)E - c^2\vec{k}(\vec{k}.\vec{E}) + 4\pi s(\delta.\vec{E}) = I$$

Here δ is the conductivity tensor, it tells us how in anisotropic plasma an electrical conductivity varies in different directions.

$$\vec{J} = \sigma\vec{E}$$

Now, we will solve the last term of Eq [2.13];
i-e:

$$\vec{E} - \frac{i}{s}\vec{v} \times (\vec{k} \times \vec{E}).\frac{\partial f_o}{\partial \vec{p}}$$

$$= \vec{E} - \frac{i}{s\gamma m}\left[\vec{p} \times (\vec{k} \times \vec{E})\right].\frac{\partial f_o}{\partial \vec{p}}$$

$$= \frac{\partial f_o}{\partial p} - \frac{i}{s\gamma m}\left[\vec{p} \times (\vec{k} \times \vec{E})\right].\frac{\partial f_o}{\partial \vec{p}} \tag{xiii}$$

$\Rightarrow$

$$\vec{E}.\frac{\partial f_o}{\partial \vec{p}} = E_x\frac{\partial f_o}{\partial p_x} + E_y\frac{\partial f_o}{\partial p_y} + E_z\frac{\partial f_o}{\partial p_z}$$

In cylindrical coordinates,

$$p_x = p_\perp \cos\Phi^{'},\ p_y = p_\perp \sin\Phi^{'},\ p_z = p_\parallel$$

$$\vec{E}.\frac{\partial f_o}{\partial \vec{p}} = E_x\cos\Phi^{'}\frac{\partial f_o}{\partial p_\perp} + E_y\sin\Phi^{'}\frac{\partial f_o}{\partial p_\perp} + E_z\frac{\partial f_o}{\partial p_\parallel}$$

Put the above value in Eq [xiii], we have;

$$= \left(E_x\cos\Phi^{'} + E_y\sin\Phi^{'}\right)\frac{\partial f_o}{\partial p_\perp} + E_z\frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s\gamma m}\left[\vec{p}\times(\vec{k}\times\vec{E})\right].\frac{\partial f_o}{\partial \vec{p}}$$

Applying the identity;

$$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}(\vec{A}.\vec{C}) - \vec{C}(\vec{A}.\vec{B})$$

$$\vec{p}\times(\vec{k}\times\vec{E}) = \vec{k}(\vec{p}.\vec{E}) - \vec{E}(\vec{p}.\vec{k})$$

So, the above equation will be;

$$= \left(E_x\cos\Phi^{'} + E_y\sin\Phi^{'}\right)\frac{\partial f_o}{\partial p_\perp} + E_z\frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s\gamma m}\left[(\vec{p}.\vec{E})(\vec{k}.\frac{\partial f_o}{\partial \vec{p}}) - (\vec{p}.\vec{k})(\vec{E}.\frac{\partial f_o}{\partial \vec{p}})\right] \tag{xiv}$$

Let solve each term separately;

$$\vec{p}.\vec{E} = p_\perp\cos\Phi^{'}E_x + p_\perp\sin\Phi^{'}E_y + p_\parallel E_z$$

$$\vec{k}.\frac{\partial f_o}{\partial \vec{p}} = k_\perp\cos\Phi^{'}\frac{\partial f_o}{\partial p_\perp} + k_\parallel\frac{\partial f_o}{\partial p_\parallel}$$

$$\vec{p}.\vec{k} = p_\perp\cos\Phi^{'}k_\perp + p_\parallel k_\parallel$$

$$\vec{E}.\frac{\partial f_o}{\partial \vec{p}} = E_x\frac{\partial f_o}{\partial p_\perp}\cos\Phi^{'} + E_y\frac{\partial f_o}{\partial p_\perp}\sin\Phi^{'} + E_z\frac{\partial f_o}{\partial p_\parallel}$$

By putting all the values in Eq [xiv], we will get;

$$= \left(E_x\cos\Phi^{'} + E_y\sin\Phi^{'}\right)\frac{\partial f_o}{\partial p_\perp} + E_z\frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s\gamma m}\begin{bmatrix} (p_\perp\cos\Phi^{'}E_x + p_\perp\sin\Phi^{'}E_y + p_\parallel E_z) \\ (k_\perp\cos\Phi^{'}\frac{\partial f_o}{\partial p_\perp} + k_\parallel\frac{\partial f_o}{\partial p_\parallel}) \\ -(p_\perp\cos\Phi^{'}k_\perp + p_\parallel k_\parallel) \\ (E_x\frac{\partial f_o}{\partial p_\perp}\cos\Phi^{'} + E_y\frac{\partial f_o}{\partial p_\perp}\sin\Phi^{'} + E_z\frac{\partial f_o}{\partial p_\parallel}) \end{bmatrix}$$

$$= \left(E_x \cos\Phi^{'} + E_y \sin\Phi^{'}\right) \frac{\partial f_o}{\partial p_\perp} + E_z \frac{\partial f_o}{\partial p_\parallel}$$

$$-\frac{i}{s\gamma m}\left[\begin{array}{c} p_\perp k_\perp \cos^2\Phi^{'} \frac{\partial f_o}{\partial \vec{p}_\perp} E_x + p_\perp k_\parallel \cos\Phi^{'} \frac{\partial f_o}{\partial p_\parallel} E_x + p_\perp k_\perp \cos\Phi^{'} \sin\Phi^{'} \frac{\partial f_o}{\partial p_\perp} E_y + p_\perp k_\parallel \sin\Phi^{'} \frac{\partial f_o}{\partial \vec{p}_\parallel} E_y \\ +p_\parallel k_\perp \cos\Phi^{'} \frac{\partial f_o}{\partial p_\perp} E_z + p_\parallel k_\parallel \frac{\partial f_o}{\partial p_\parallel} E_z - p_\perp k_\perp \cos^2\Phi^{'} E_x \frac{\partial f_o}{\partial \vec{p}_\perp} E_x - p_\perp k_\perp \cos\Phi^{'} \sin\Phi^{'} \frac{\partial f_o}{\partial \vec{p}_\perp} E_y \\ -p_\perp \cos\Phi^{'} k_\perp \frac{\partial f_o}{\partial p_\parallel} E_z - p_\parallel k_\parallel \sin\Phi^{'} \frac{\partial f_o}{\partial p_\perp} E_y - p_\parallel k_\parallel \sin\Phi^{'} \frac{\partial f_o}{\partial p_\perp} E_y - p_\parallel k_\parallel \frac{\partial f_o}{\partial \vec{p}_\parallel} E_z \end{array}\right]$$

After simplification:

$$= \left(E_x \cos\Phi^{'} E_y \sin\Phi^{'}\right) \frac{\partial f_o}{\partial p_\perp} + E_z \frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s\gamma m}\left[\begin{array}{c} E_x \cos\Phi^{'} k_\parallel \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right) \\ +E_y \sin\Phi^{'} k_\parallel \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right) \\ -E_z \cos\Phi^{'} k_\perp \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right) \end{array}\right]$$

$$\begin{array}{l} \left[\cos\Phi^{'} \frac{\partial f_o}{\partial p_\perp} - \frac{i}{s\gamma m} \cos\Phi^{'} k_\parallel \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right)\right] E_x \\ + \left[\sin\Phi^{'} \frac{\partial f_o}{\partial p_\perp} - \frac{i}{s\gamma m} \sin\Phi^{'} k_\parallel \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right)\right] E_y \\ + \left[\frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s\gamma m} \cos\Phi^{'} k_\perp \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right)\right] E_z \end{array}$$

$$\left[\begin{array}{c} \cos\Phi^{'} \frac{\partial f_o}{\partial p_\perp} - \frac{i}{s\gamma m} \cos\Phi^{'} k_\parallel \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right) \\ + \sin\Phi^{'} \frac{\partial f_o}{\partial p_\perp} - \frac{i}{s\gamma m} \sin\Phi^{'} k_\parallel \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right) \\ + \frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s\gamma m} \cos\Phi^{'} k_\perp \left(p_\perp \frac{\partial f_o}{\partial p_\parallel} - p_\parallel \frac{\partial f_o}{\partial p_\perp}\right) \end{array}\right] .E$$

$$= E - \frac{i}{s} \vec{v} \times (\vec{k} \times \vec{E}). \frac{\partial f_o}{\partial p} = \vec{A}.\vec{E}$$

As,

$$4\pi s(\vec{\sigma}.\vec{E}) = -s \sum_\alpha \frac{\omega_{p_\alpha}^2}{\Omega} \int pdp \int_{\pm\infty}^{\Phi} \left[e^{\frac{1}{\Omega}s + ik_\parallel v_\parallel (\Phi - \Phi^{'}) - ik_\perp v_\perp (\sin\Phi - \sin^{'})} \times E - \frac{i}{s} \vec{v} \times (\vec{k} \times \vec{E}). \frac{\partial f_o}{\partial \vec{p}}\right] d\Phi$$

So, by putting all the values in above equation we will get;

$$\begin{aligned} 4\pi s(\vec{\sigma}_{i,j}.\vec{E}) \quad = \quad & -s \sum_\alpha \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp dp_\perp \int_{-\infty}^{\infty} dp_\parallel \int_0^{2\pi} d\Phi \int_{\pm\infty}^{\Phi} d\Phi^{'} \qquad (2.14) \\ & \times \left(e^{\frac{1}{\Omega}s + ik_\parallel v_\parallel (\Phi - \Phi^{'}) - ik_\perp v_\perp (\sin\Phi - \sin\varphi^{'})}\right) \times \left(p_\perp \cos\Phi, p_\perp \sin\Phi, p_\parallel\right) (\vec{A}.\vec{E}) \end{aligned}$$

From the above equation, we can calculate the different elements of dielectric tensor.

Let's calculate σ_{xx}.

$$4\pi s\sigma_{xx} = -s\sum_{\alpha}\frac{\omega_{p_\alpha}^2}{\Omega}\int_0^\infty p_\perp dp_\perp\int_{-\infty}^\infty dp_\parallel\int_0^{2\pi}d\Phi\int_{\pm\infty}^{\Phi}d\Phi^{'} \tag{2.15}$$
$$\times\left(e^{\frac{1}{\Omega}s+ik_\parallel v_\parallel(\Phi-\Phi^{'})+ik_\perp v_\perp(\sin\Phi-\sin^{'})}\right)\times p_\perp\cos\Phi\cos\Phi^{'}\left[\frac{\partial f_o}{\partial p_\perp}-\frac{i}{s}k_\parallel\left(v_\perp\frac{\partial f_o}{\partial v_\parallel}-v_\parallel\frac{\partial f_o}{\partial v_\perp}\right)\right]$$

By change of variables;

$$\Phi-\Phi^{'}=\alpha$$

$$\Phi^{'}=\Phi-\alpha$$

$$d\Phi^{'}=-d\alpha$$

$$4\pi s\sigma_{xx} = -s\sum_{\alpha}\frac{\omega_{p_\alpha}^2}{\Omega}\int_0^\infty \vec{p}_\perp d\vec{p}_\perp\int_{-\infty}^\infty d\vec{p}_\parallel\int_0^{2\pi}d\Phi\int_{\pm\infty}^{0}d\Phi^{'} \tag{2.16}$$
$$\times\left(e^{\frac{1}{\Omega}s+i\vec{k}_\parallel\vec{v}_\parallel\alpha+i\vec{k}_\perp\vec{v}_\perp(\sin\Phi-\sin(\Phi-\alpha))}\right)\times\vec{p}_\perp\cos\Phi\cos(\Phi-\alpha)\left[\frac{\partial f_o}{\partial p_\perp}-\frac{i}{s}k_\parallel\left(v_\perp\frac{\partial f_o}{\partial v_\parallel}-v_\parallel\frac{\partial f_o}{\partial v_\perp}\right)\right]$$

Now, we will solve each term separately;

$$\int_0^{2\pi}e^{\frac{ik_\perp v_\perp}{\Omega}(\sin\Phi-\sin(\Phi-\alpha))}\cos\Phi\cos(\Phi-\alpha)d\Phi$$

By Bessel's identity:

$$e^{iz\sin\Phi}=\sum_{n\to-\infty}^{\infty}e^{\iota n\Phi}\times J_n(z)$$

$$e^{iz\sin(\Phi-\alpha)}=\sum_{n\to-\infty}^{\infty}e^{-\iota n(\Phi-\alpha)}\times J_n(z)$$

$$e^{iz(\sin\Phi-\sin(\Phi-\alpha))}=\sum_{n\to-\infty}^{\infty}e^{\iota n(\Phi-\Phi+\alpha)}\times J_n^2(z)$$

By using the above bessel property, we can write;

$$\int_0^{2\pi} e^{\frac{ik_\perp v_\perp}{\Omega}(\sin\Phi - \sin(\Phi-\alpha))} \cos\Phi \cos(\Phi - \alpha) d\Phi$$

$$= \int_0^{2\pi} \sum_{n\to-\infty}^{\infty} \left[e^{in\alpha} \times J_n^2(z)\right] \cos\Phi \cos(\Phi - \alpha) d\Phi$$

$$= \sum_{n\to-\infty}^{\infty} J_n^2(z) \int_0^{2\pi} e^{in\alpha} \times \cos\Phi \cos(\Phi - \alpha) d\Phi$$

$$= 2\pi \sum_{n\to-\infty}^{\infty} e^{in\alpha} \times J_n^2(z) \times \frac{n^2}{z^2} d\Phi \qquad \text{(xv)}$$

Put eq [xv] in Eq [2.16], we will get;

$$4\pi s\sigma_{xx} = -s \sum_\alpha \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp dp_\perp \int_{-\infty}^{\infty} dp_\parallel \int_{\pm\infty}^{0} e^{\frac{1}{\Omega}(s+ik_\parallel v_\parallel + in\Omega)\alpha} d\alpha \qquad (2.17)$$
$$\times 2\pi \left(j_n^2(z) \times \frac{n^2}{k_\perp^2 v_\perp^2}\right) \Omega^2 \times p_\perp \left[\frac{\partial f_o}{\partial p_\perp} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp}\right)\right]$$

Now,

$$\int_{\pm\infty}^{0} e^{\frac{1}{\Omega}(s+ik_\parallel v_\parallel + in\Omega)\alpha} d\alpha = \left[\Omega \times \frac{e^{\frac{1}{\Omega}(s+ik_\parallel v_\parallel + in)}}{s + ik_\parallel v_\parallel + in\Omega}\right]_{-\infty}^{0}$$

$$= \frac{\Omega}{s + ik_\parallel v_\parallel + in\Omega} \qquad \text{(xvi)}$$

Put Eq [xvi] in Eq [2.17], we have;

$$4\pi s\sigma_{xx} = -2\pi s \sum_\alpha \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp dp_\perp \int_{-\infty}^{\infty} dp_\parallel \frac{n^2 \times J_n^2(z) \times \Omega^3}{k_\perp^2 v_\perp^2 (s + ik_\parallel v_\parallel + in\Omega)} \left[\frac{\partial f_o}{\partial p_\perp} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp}\right)\right] \qquad (2.18)$$

By using bessel identity, we can find the other elements of tensor as;

$$\int_0^{2\pi} e^{-iz[\sin\Phi - \sin(\Phi-\alpha)]} d\Phi \begin{bmatrix} \sin\Phi\sin(\Phi-\alpha) \\ \sin\Phi\cos(\Phi-\alpha) \\ \cos\Phi\sin(\Phi-\alpha) \\ \cos\Phi\cos(\Phi-\alpha) \\ 1 \\ \sin\Phi \\ \cos\Phi \\ \sin(\Phi-\alpha) \\ \cos(\Phi-\alpha) \end{bmatrix} = 2\pi \sum_{n\to-\infty}^{\infty} e^{in\alpha} \begin{bmatrix} J_n'^2 \\ -\frac{in}{z} J_n J_n' \\ \frac{in}{z} J_n J_n' \\ \frac{n^2}{z^2} J_n^2 \\ J_n^2 \\ -iJ_n J_n' \\ \frac{n}{z} J_n^2 \\ iJ_n J_n' \\ \frac{n}{z} J_n^2 \end{bmatrix}$$

Now Eq [xii] can be written as;

$$\left[(s^2+c^2k^2)I - c^2\vec{k}(\vec{k}.\vec{E}) + 4\pi s\sigma\right]\vec{E} = I \tag{xvii}$$

Here I is the generalized matrix. The above equation for different component of generalized dielectric tensors then reads as,

$$\begin{bmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{bmatrix}$$

By putting the value of σ_{xx} and doing some complicated algebra, we will get R_{xx} as,

$$R_{xx} = s^2 + c^2k_\perp^2 - 2\pi s \sum_\alpha \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp^2 dp_\perp \int_{-\infty}^{\infty} dp_\parallel \frac{n^2 \times J_n^2 \times \Omega^3}{k_\perp^2 v_\perp^2 (s + ik_\parallel v_\parallel + \imath n\Omega)} \left[\frac{\partial f_o}{\partial p_\perp} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp}\right)\right] \tag{2.19}$$

Similarly by doing the same calculations for σ_{xy}, σ_{xz}.....σ_{zz}, all other components of $R_{i,j}$ can be written as;

$$R_{xy} = -2i\pi s \sum_\alpha \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp^2 dp_\perp \int_{-\infty}^{\infty} dp_\parallel \frac{n^2 \times J_n J_n' \times \Omega^2}{k_\perp v_\perp (s + ik_\parallel v_\parallel + in\Omega)} \left[\frac{\partial f_o}{\partial p_\perp} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp}\right)\right] \tag{2.20}$$

$$R_{xz} = -c^2 k_\parallel k_\perp - 2\pi s \sum_\alpha \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp^2 dp_\perp \int_{-\infty}^{\infty} dp_\parallel \frac{n^2 \times J_n^2 \times \Omega^2}{k_\perp v_\perp (s + ik_\parallel v_\parallel + in\Omega)} \left[\frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp}\right)\right] \tag{2.21}$$

$$R_{yx} = 2\pi s \sum_\alpha \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp^2 dp_\perp \int_{-\infty}^{\infty} dp_\parallel \frac{n^2 \times J_n J_n' \times \Omega^2}{k_\perp v_\perp (s + ik_\parallel v_\parallel + in\Omega)} \left[\frac{\partial f_o}{\partial p_\perp} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp}\right)\right] \tag{2.22}$$

$$R_{yy} = s^2 + c^2k^2 - 2\pi s \sum_{\alpha} \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp^2 dp_\perp \int_{-\infty}^{\infty} dp_\parallel \frac{J_n^{'} \times \Omega^2}{k_\perp v_\perp (s + ik_\parallel v_\parallel + in\Omega)} \left[\frac{\partial f_o}{\partial p_\perp} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp} \right) \right] \tag{2.23}$$

$$R_{yz} = 2i\pi s \sum_{\alpha} \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp^2 d\vec{p}_\perp \int_{-\infty}^{\infty} dp_\parallel \frac{J_n J_n^{'} \times \Omega}{k_\perp v_\perp (s + ik_\parallel v_\parallel + in\Omega)} \left[\frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp} \right) \right] \tag{2.24}$$

$$R_{zx} = -c^2 k_\parallel k_\perp - 2\pi s \sum_{\alpha} \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp dp_\perp \int_{-\infty}^{\infty} p_\parallel dp_\parallel \frac{n \times J_n \times \Omega^2}{k_\perp v_\perp (s + ik_\parallel v_\parallel + in\Omega)} \left[\frac{\partial f_o}{\partial p_\perp} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp} \right) \right] \tag{2.25}$$

$$R_{zy} = -2i\pi s \sum_{\alpha} \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp dp_\perp \int_{-\infty}^{\infty} p_\parallel dp_\parallel \frac{J_n J_n^{'} \times \Omega}{(s + ik_\parallel v_\parallel + in\Omega)} \times \left[\frac{\partial f_o}{\partial p_\perp} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp} \right) \right] \tag{2.26}$$

$$R_{zz} = s^2 + c^2 k_\perp^2 - 2\pi s \sum_{\alpha} \frac{\omega_{p_\alpha}^2}{\Omega} \int_0^\infty p_\perp dp_\perp \int_{-\infty}^{\infty} p_\parallel dp_\parallel \frac{J_n^2 \times \Omega}{(s + ik_\parallel v_\parallel + in\Omega)} \left[\frac{\partial f_o}{\partial p_\parallel} - \frac{i}{s} k_\parallel \left(v_\perp \frac{\partial f_o}{\partial v_\parallel} - v_\parallel \frac{\partial f_o}{\partial v_\perp} \right) \right] \tag{2.27}$$

The above equations are the components of the dielectric tensor, each component of a tensor is defined for different types of waves.

3 MATHEMATICAL MODEL

The surface impedance and skin depth of a transverse waves have already been calculated by using the bi-maxwellian distribution function [11]. In this thesis we are extending this work discuss the surface impedance and skin depth for bi-kappa distributed plasmas in both the limits: that is resonant and non-resonant cases.

3.1 Generalized dispersion relation of transverse waves by kinetic model

Transverse waves propagate in such a way that its wave vector is perpendicular to electric field ($\vec{k} \perp \vec{E}$) as shown in the figure.

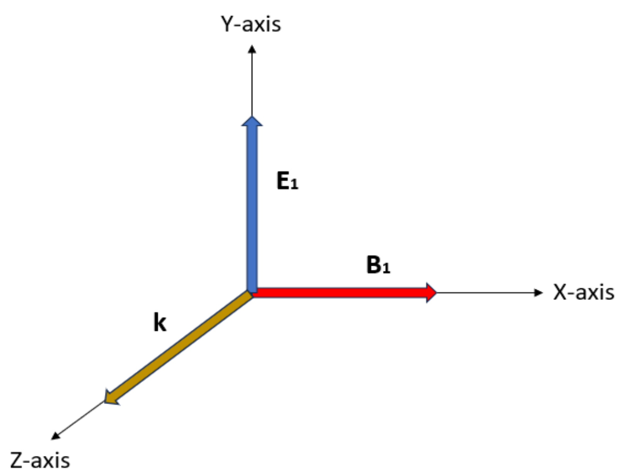

Fig. 3.1. Geometry of the wave.

The vlasov's equation is given by;

$$\frac{\partial f}{\partial t} + \vec{v}.\frac{\partial f}{\partial \vec{x}} + \frac{e}{m}[\vec{E} + \frac{1}{c}\vec{v} \times \vec{B}].\frac{\partial f}{\partial \vec{v}} = 0 \tag{3.1}$$

Upon linearization;

$$f = f_o + f_1$$

$$\vec{E} = \vec{E}_1$$

$$\vec{B} = \vec{B}_o + \vec{B}_1$$

In case of unmagnetized plasma $\vec{B}_o$=0

$$\frac{\partial f_1}{\partial t} + \vec{v}.\frac{\partial f_1}{\partial \vec{x}} + \frac{e}{m}[\vec{E}_1 + \frac{1}{c}\vec{v} \times \vec{B}_1].\frac{\partial f_o}{\partial \vec{v}} = 0 \tag{3.2}$$

By considering the sinusoidal perturbations,

$$\frac{\partial}{\partial t} = -i\omega, \ \frac{\partial}{\partial x} = ik$$

By applying Fourier-Laplace transform and put the above values in Eq [3.2];

$$-i\omega \times \delta f_1 + i\vec{v}.\vec{k} \times \delta f_1 + \frac{e}{m}[\vec{E}_1 \frac{\partial f_o}{\partial \vec{v}} + \frac{1}{c}\vec{v} \times \vec{B}_1 \frac{\partial f_o}{\partial \vec{v}}] = 0$$

$$i\left(-\omega + \vec{v}.\vec{k}\right)\delta f_1 + \frac{e}{m}[\vec{E}_1 \frac{\partial f_o}{\partial \vec{v}} + \frac{1}{c}\vec{v} \times \vec{B}_1 \frac{\partial f_o}{\partial \vec{v}}] = 0$$

$$\delta f_1 = i\frac{e}{m\left(\omega - \vec{v}.\vec{k}\right)}[\vec{E}_1 \frac{\partial f_o}{\partial \vec{v}} + \frac{1}{c}\vec{v} \times \vec{B}_1 \frac{\partial f_o}{\partial \vec{v}}] \tag{3.3}$$

Transverse permittivity

It is the component of the permittivity tensor which describe the response of a plasma to an electric field which is perpendicular to the direction of propagation ($\vec{E} \perp \vec{K}$). It is given as;

$$\varepsilon_t(\omega, k) = 1 - \frac{4\pi i}{\omega E_{(k)}} e \int v\delta\left(f\right)dv \tag{3.4}$$

Put Eq [3.3] in Eq [3.4],
$\Rightarrow$

$$\varepsilon_t(\omega, k) = 1 - \frac{4\pi i}{\omega E_{(k)}} e \int v \left[i\frac{e}{m\left(\omega - \vec{v}.\vec{k}\right)} \left(\vec{E}_1 \frac{\partial f_o}{\partial \vec{v}} + \frac{1}{c}\vec{v} \times \vec{B}_1 \frac{\partial f_o}{\partial \vec{v}} \right) \right] \tag{3.5}$$

Now from Faraday's law;

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\Rightarrow ik \times \vec{E} = -\frac{1}{c}(-i\omega)\vec{B}$$

$$\frac{c\vec{E}}{\omega} = \vec{B} \tag{3.6}$$

Put Eq [3.6] in Eq [3.5],

$$\varepsilon_t(\omega, k) = 1 - \frac{4\pi i}{\omega E_{(k)}} e \int v \left[i\frac{e}{m\left(\omega - \vec{v}.\vec{k}\right)} \left(E_1 \frac{\partial f_o}{\partial v} + \frac{1}{c}v \times \frac{ckE}{\omega}\frac{\partial f_o}{\partial v} \right) \right]$$

$$\varepsilon_t(\omega,k) = 1 + \frac{4\pi e^2}{m\omega}\int v\left[\frac{E_1}{\left(\omega - \vec{v}.\vec{k}\right)E_{(k)}}\frac{\partial f_o}{\partial v} + \frac{vk}{\left(\omega - \vec{v}.\vec{k}\right)}\frac{\partial f_o}{\partial v}\right]$$

Since our wave is transverse so $\vec{k}.\vec{E}$=0;

$$\varepsilon_t(\omega,k) = 1 + \frac{4\pi e^2}{m\omega^2}\int v_x\left[\frac{\partial f_o}{\partial v_\perp} + \frac{v_x^2 k}{\left(\omega - \vec{v}_\parallel.\vec{k}\right)}\frac{\partial f_o}{\partial v}\right]$$

$$\varepsilon_t(\omega,k) = 1 + \frac{4\pi e^2}{m\omega^2}\int v_x\frac{\partial f_o}{\partial v_\perp} + \frac{v_x^2 k}{\left(\omega - \vec{v}_\parallel.\vec{k}\right)}\frac{\partial f_o}{\partial v_\parallel} \tag{3.7}$$

The dispersion relation is;

$$\frac{c^2k^2}{\omega^2} = 1 + \frac{4\pi e^2}{m\omega^2}\int\left(v_\perp\frac{\partial f_o}{\partial v_\perp} + \frac{{v_\perp}^2 k}{\left(\omega - \vec{v}_\parallel.\vec{k}\right)}\frac{\partial f_o}{\partial v_\parallel}\right)dv \tag{3.8}$$

The above equation is the generalized dispersion relation of transverse waves in un-magnetized plasma.

3.2 Dispersion relation of transverse waves by bi-kappa distribution function

The generalized dispersion relation is given by;

$$\frac{c^2k^2}{\omega^2} = 1 + \frac{4\pi e^2}{m\omega^2}\int\left(v_\perp\frac{\partial f_o}{\partial v_\perp} + \frac{v_\perp^2 k}{\left(\omega - \vec{v}_\parallel.\vec{k}\right)}\frac{\partial f_o}{\partial v_\parallel}\right)dv \tag{3.9}$$

The bii-kappa distribution function is defined as;

$$f_o = \frac{1}{\pi^{3/2}\theta_\perp^2\theta_\parallel}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\left(1 + \frac{v_\parallel^2}{\kappa\theta_\parallel^2} + \frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-1}$$

$$f_o = A\left(1 + \frac{v_\parallel^2}{\kappa\theta_\parallel^2} + \frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-1} \tag{3.10}$$

Here;

$$A = \frac{1}{\pi^{3/2}\theta_\perp^2\theta_\parallel}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})},\ \theta_{\perp,\parallel}^2 = (\frac{2\kappa-3}{\kappa})v_{t_{\perp,\parallel}}^2$$

Now

$$\frac{\partial f_o}{\partial v_\perp} = A(-\kappa-1)\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2} \times \frac{2v_\perp}{\kappa\theta_\perp^2} \tag{3.11}$$

$$\frac{\partial f_o}{\partial v_\parallel} = A(-\kappa-1)\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2} \times \frac{2v_\parallel}{\kappa\theta_\parallel^2} \tag{3.12}$$

Put Eqns [3.10], [3.11], and [3.12] in Eq [3.9];

$$\frac{c^2k^2}{\omega^2} = 1+\frac{4\pi e^2}{m\omega^2}A\int dv\left[\begin{array}{c} v_\perp\left\{(-\kappa-1)\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2}\right\}\times\frac{2v_\perp}{\kappa\theta_\perp^2}+\frac{v_\perp^2 k}{(\omega-\vec{v}_\parallel.\vec{k})}(-\kappa-1) \\ \times\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2}\times\frac{2v_\parallel}{\kappa\theta_\parallel^2} \end{array}\right]$$

$$\frac{c^2k^2}{\omega^2} = 1+\frac{8\pi e^2}{m\omega^2}A\int dv\left[\frac{(-\kappa-1)}{\kappa\theta_\perp^2}v_\perp^2\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2}+\frac{k(-\kappa-1)v_\perp^2 v_\parallel}{\kappa\theta_\parallel^2(\omega-\vec{v}_\parallel\vec{k})}\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2}\right]$$

As;

$$\int dv = \int_0^\infty\int_{-\infty}^\infty\int_0^{2\pi} v_\perp dv_\perp dv_\parallel d\phi$$

$$\frac{c^2k^2}{\omega^2} = 1+\frac{8\pi e^2}{mw^2}A\left[\int_0^\infty\int_{-\infty}^\infty\int_0^{2\pi}\left(\begin{array}{c} [\frac{(-\kappa-1)}{\kappa\theta_\perp^2}v_\perp^2\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2} \\ +\frac{(-\kappa-1)v_\perp^2 k v_\parallel}{\kappa\theta_\parallel^2(\omega-\vec{v}_\parallel\vec{k})}\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2} \end{array}\right)v_\perp dv_\perp dv_\parallel d\phi\right]$$

as;

$$\int_0^{2\pi} d\phi = 2\pi$$

$$\frac{c^2k^2}{\omega^2} = 1+\frac{16\pi^2 e^2}{mw^2}A\left[\int_0^\infty\int_{-\infty}^\infty\left(\begin{array}{c} v_\perp^3\frac{(-\kappa-1)}{\kappa\theta_\perp^2}\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2} \\ +\frac{(-\kappa-1)v_\perp^2 k v_\parallel}{\kappa\theta_\parallel^2(\omega-\vec{v}_\parallel\vec{k})}\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2} \end{array}\right)dv_\perp dv_\parallel\right] \tag{3.13}$$

Now we will perform parallel and perpendicular integration separately;
Perpendicular Integration:

$$\int_0^\infty v_\perp^3\frac{(-\kappa-1)}{\kappa\theta_\perp^2}\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2} dv_\perp$$

$$= \frac{(-\kappa-1)}{\kappa\theta_\perp^2}\int_0^\infty v_\perp^3\left(1+\frac{v_\parallel^2}{\kappa\theta_\parallel^2}+\frac{v_\perp^2}{\kappa\theta_\perp^2}\right)^{-\kappa-2}dv_\perp$$

By change of variables;

$$= \frac{(-\kappa-1)}{\kappa\theta_\perp^2}\int_0^\infty x^3\left(1+\frac{v_\parallel^2}{\alpha^2}+\frac{v_\perp^2}{\beta^2}\right)^{-\kappa-2}dx$$

$$= \frac{(-\kappa-1)}{\kappa\theta_\perp^2}\left(\frac{(1+\frac{v^2}{\alpha^2})^{-\kappa}\beta^4}{2(\kappa+\kappa^2)}\right) \tag{3.14}$$

Put Eq [3.14] in Eq [3.13],

$$\frac{c^2k^2}{\omega^2} = 1+\frac{16\pi^2e^2}{m\omega^2}A\left[\int_{-\infty}^{\infty}\left\{\frac{(-\kappa-1)}{\kappa\theta_\perp^2}\left(\frac{(1+\frac{v^2}{\alpha^2})^{-\kappa}\beta^4}{2(\kappa+\kappa^2)}\right)+\frac{(-\kappa-1)kv_\parallel}{\kappa\theta_\parallel^2(\omega-\vec{v}_\parallel\vec{k})}\left(\frac{(1+\frac{v^2}{\alpha^2})^{-\kappa}\beta^4}{2(\kappa+\kappa^2)}\right)\right\}dv_\parallel\right]$$

Here;

$$\beta^2 = \kappa\theta_\perp^2$$

$$\frac{c^2k^2}{\omega^2} = 1+\frac{16\pi^2e^2}{m\omega^2}A\left[\int_{-\infty}^{\infty}\left\{\frac{-(\kappa+1)}{\kappa\theta_\perp^2}\left(\frac{(1+\frac{v^2}{\alpha^2})^{-\kappa}\beta^4}{2\kappa(\kappa+1)}\right)+\frac{-(\kappa+1)kv_\parallel}{\kappa\theta_\parallel^2(\omega-\vec{v}_\parallel\vec{k})}\left(\frac{(1+\frac{v^2}{\alpha^2})^{-\kappa}\beta^4}{2\kappa(\kappa+1)}\right)\right\}dv_\parallel\right]$$

$$\frac{c^2k^2}{\omega^2} = 1-\frac{16\pi^2e^2}{m\omega^2}A\left[\int_{-\infty}^{\infty}\left\{\frac{(1+\frac{v^2}{\alpha^2})^{-\kappa}\beta^4}{2\kappa^2\theta_\perp^2}+\frac{(1+\frac{v^2}{\alpha^2})^{-\kappa}\beta^4}{2\kappa^2\theta_\parallel^2(\omega-v_\parallel k)}\right\}dv_\parallel\right] \tag{3.15}$$

As;

$$\int_{-\infty}^{+\infty}\left(1+\frac{v_\parallel^2}{\alpha^2}\right)^{-\kappa}dv_\parallel = \frac{\sqrt{\pi}\Gamma(\kappa-\frac{1}{2})}{\sqrt{\frac{1}{\alpha^2}}\Gamma(\kappa)} \tag{3.16}$$

and

$$\int_{-\infty}^{+\infty}\frac{\left(1+\frac{v_\parallel^2}{\alpha^2}\right)^{-\kappa}}{\omega-v_\parallel\kappa}dv_\parallel = -\frac{\omega}{\kappa}\int_{-\infty}^{+\infty}\frac{\left(1+\frac{v_\parallel^2}{\alpha^2}\right)^{-\kappa}}{v_\parallel-\frac{\omega}{\kappa}}$$

By change of variables;

$$x^2 = \frac{v_\parallel^2}{\theta_\parallel^2},\ v_\parallel^2 = x^2\theta_\parallel^2,\ dv_\parallel = dx\theta_\parallel$$

$$\Longrightarrow -\frac{\omega}{\kappa}\int_{-\infty}^{+\infty}\frac{\left(1+\frac{v_\parallel^2}{\kappa}\right)^{-\kappa}}{(x\theta_\parallel - \frac{\omega}{\kappa})}dx\theta_\parallel$$

let

$$g = \frac{\omega}{k\theta_\parallel}$$

$$-\frac{\omega}{\kappa}\int_{-\infty}^{+\infty}\frac{\left(1+\frac{x^2}{\kappa}\right)^{-\kappa}}{(x-g)}dx = I \tag{3.17}$$

Put Eq [3.16] and [3.17] in Eq [3.15];

$$\Rightarrow \frac{c^2k^2}{\omega^2} = 1 - \frac{16\pi^2e^2}{m\omega^2}A\left[\frac{\beta^4}{2\kappa^2\theta_\perp^2}\frac{\sqrt{\pi}\Gamma(-\frac{1}{2}+\kappa)}{\sqrt{\frac{1}{\alpha^2}}\Gamma(\kappa)} + \frac{\beta^4}{2\kappa^2\theta_\parallel^2}[\frac{\sqrt{\pi}\Gamma(-\frac{1}{2}+\kappa)}{\sqrt{\frac{1}{\alpha^2}}\Gamma(\kappa)} - \frac{\omega}{\kappa}\int_{-\infty}^{+\infty}\frac{(1+\frac{x^2}{\kappa})^{-\kappa}}{(x-g)}dx]\right]$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{16\pi^2e^2}{m\omega^2}A\left[\begin{array}{c}\frac{\beta^4}{2\kappa^2\theta_\perp^2}\frac{\sqrt{\pi}\Gamma(\kappa-\frac{1}{2})}{\Gamma(\kappa)}\theta_\parallel\sqrt{\kappa} \\ -\frac{\beta^4}{2\kappa^2\theta_\parallel^2}\left(\frac{\sqrt{\pi}\Gamma(\kappa-\frac{1}{2})}{\Gamma(\kappa)}\theta_\parallel\sqrt{\kappa} + \frac{\omega}{\kappa}\int_{-\infty}^{+\infty}\frac{(1+\frac{x^2}{\kappa})^{-\kappa}}{(x-g)}dx\right)\end{array}\right], \because \alpha^2 = \kappa\theta_\parallel^2$$

Now, by using modified plasma dispersion function;

$$Z_\kappa(g) = \frac{1}{\sqrt{\pi}\kappa^{1/2}}\frac{\Gamma(\kappa)}{\Gamma(\kappa-\frac{1}{2})}(I)$$

$$I = \frac{\sqrt{\pi}\kappa^{1/2}\Gamma(\kappa-\frac{1}{2})Z_\kappa(g)}{\Gamma(\kappa)}$$

Put the value of modified plasma dispersion function in above equation, we will have;

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{16\pi^2e^2}{m\omega^2}A\left[\frac{\beta^4}{2\kappa^2\theta_\perp^2}\frac{\sqrt{\pi}\Gamma(\kappa-\frac{1}{2})}{\Gamma(\kappa)}\theta_\parallel\sqrt{\kappa} - \frac{\beta^4}{2\kappa^2\theta_\parallel^2}\left(\frac{\sqrt{\pi}\Gamma(\kappa-\frac{1}{2})}{\Gamma(\kappa)}\theta_\parallel\sqrt{\kappa} + \frac{\omega}{\kappa}\frac{\sqrt{\pi}\kappa^{1/2}\Gamma(\kappa-\frac{1}{2})Z_\kappa(g)}{\Gamma(\kappa)}\right)\right]$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{16\pi^2e^2}{m\omega^2}A\frac{\beta^4}{2\kappa^2}\left[\frac{\sqrt{\pi}\Gamma(\kappa-\frac{1}{2})}{\Gamma(\kappa)}\theta_\parallel\sqrt{\kappa}\left\{\frac{1}{\theta_\perp^2} - \frac{1}{\theta_\parallel^2}\left(1 + \frac{\omega}{k\theta_\parallel}Z_\kappa(g)\right)\right\}\right]$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{8\pi e^2n_o}{m}\frac{A}{n_o\omega^2}\frac{\pi\beta^4}{2\kappa^2}\left[\frac{\sqrt{\pi}\Gamma(-\frac{1}{2}+\kappa)}{\Gamma(\kappa)}\theta_\parallel\sqrt{\kappa}\left\{\frac{1}{\theta_\perp^2} - \frac{1}{\theta_\parallel^2}\left(1 + \frac{\omega}{k\theta_\parallel}Z_\kappa(g)\right)\right\}\right]$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{4\pi e^2 n_o}{m}\frac{A}{n_o\omega^2}\frac{\pi\beta^4}{\kappa^2}\left[\frac{\sqrt{\pi}\Gamma(-\frac{1}{2}+\kappa)}{\Gamma(\kappa)}\theta_\parallel\sqrt{\kappa}\left\{\frac{1}{\theta_\perp^2} - \frac{1}{\theta_\parallel^2}\left(1+\frac{\omega}{k\theta_\parallel}Z_\kappa(g)\right)\right\}\right] \tag{3.19}$$

Put the value of A in Eq [3.19],

$$A = \frac{1}{\pi^{3/2}\theta_\perp^2\theta_\parallel}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})},\ \beta = \sqrt{\kappa}\theta_\perp^2$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{\pi\kappa^2\theta_\perp^4}{\kappa^2}\frac{1}{\pi^{3/2}\theta_\perp^2\theta_\parallel}\frac{\Gamma(\kappa+1)}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\left[\frac{\sqrt{\pi}\Gamma(-\frac{1}{2}+\kappa)}{\Gamma(\kappa)}\theta_\parallel\sqrt{\kappa}\left\{\frac{1}{\theta_\perp^2} - \frac{1}{\theta_\parallel^2}\left(1+\frac{\omega}{k\theta_\parallel}Z_\kappa(g)\right)\right\}\right]$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{\sqrt{\kappa}\Gamma(\kappa+1)}{\Gamma(\kappa)\kappa^{3/2}}\left[1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\left(1+\frac{\omega}{k\theta_\parallel}Z_\kappa(g)\right)\right]$$

By using Gamma function properties;

$$\Gamma(\kappa+1) = \kappa\Gamma(\kappa)$$

$$\Rightarrow \frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\frac{\sqrt{\kappa}\kappa\Gamma(\kappa)}{\Gamma(\kappa)\kappa^{3/2}}\left[1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\left(1+\frac{\omega}{k\theta_\parallel}Z_\kappa(g)\right)\right]$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\left[1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\left(1+\frac{\omega}{k\theta_\parallel}Z_\kappa(g)\right)\right]$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\left[1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\left(1+gZ_\kappa(g)\right)\right] \because g = \frac{\omega}{k\theta_\parallel} \tag{3.20}$$

The above equation is the dispersion relation of transverse wave by bi-kappa distribution function, where;

$Z_\kappa(g)$ is the modified plasma dispersion function

$$Z_\kappa(g) = \frac{\Gamma(\kappa)}{\sqrt{\pi}\Gamma(\kappa-\frac{1}{2})}\int_{-\infty}^{\infty}\frac{ds}{s-g_\kappa}\left(1+\frac{s^2}{\kappa}\right)^{-\kappa}$$

$$\theta_{\perp,\parallel}^2 = (\frac{2\kappa-3}{\kappa})v_{t_{\perp,\parallel}}^2 \text{ and } v_{t_{\perp,\parallel}} = \sqrt{\frac{T_{\perp,\parallel}}{m}}$$

3.3 Electric field equation

Now we will calculate the electric field equation from which we will obtain the electric field profile for resonant and non-resonant case when a transverse wave travel in bi-kappa distributed plasma and the sum of both electric fields equation will give us a surface impedance whose real part shows power absorption and imaginary part gives us phase of a reflected wave then we will connect the surface impedance equation with skin depth and will obtain the skin depth for high and low frequency regime.

By Maxwell's equation,

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}. \tag{3.21}$$

$$\nabla \times \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} = \frac{4\pi}{c}\vec{J} \tag{3.22}$$

Using the identity $\vec{A}.[\vec{B}\times\vec{C}] = \vec{B}.[\vec{C}\times\vec{A}] = \vec{C}[\vec{A}\times\vec{B}]$ on Eq [3.21],

$$\Rightarrow \nabla.[\nabla \times E] = -\frac{1}{c}\nabla.(\frac{\partial B}{\partial t})$$

$$[\nabla.\nabla]E = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla \times B)$$

$$\nabla^2 E = -\frac{1}{c}\frac{\partial}{\partial t}(\nabla \times B)$$

$$\nabla^2 E = -\frac{1}{c}\frac{\partial}{\partial t}\left(\frac{1}{c}\frac{\partial E}{\partial t} + \frac{4\pi}{c}J\right)$$

$$\nabla^2 E = -\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} - \frac{4\pi}{c^2}\frac{\partial}{\partial t}J$$

$$\frac{d^2 E_y(z)}{dz^2} + \frac{\omega^2}{c^2}E_y(z) = -\frac{4\pi i\omega}{c^2}J \tag{3.23}$$

Apply Fourier transform on Eq [3.23],

$$\int_{-\infty}^{+\infty}\frac{d^2E_y(z)}{dz^2}e^{-ikz}dz + \int_{-\infty}^{+\infty}\frac{\omega^2}{c^2}E_y(z)e^{-ikz}dz = \int_{-\infty}^{+\infty} -\frac{4\pi i\omega}{c^2}Je^{-ikz}dz$$

$$\left(e^{-ikz}E_y^{'}(z)\mid_{-\infty}^{+\infty}\right) - \left((-ik)e^{-ikz}E_y^{'}(z)\right) + \frac{\omega^2}{c^2}E_y(k) = \int_{-\infty}^{+\infty} -\frac{4\pi i\omega}{c^2}Je^{-ikz}dz$$

$$\left[e^{-ikz}E_y^{'}(z)\right]_{-\infty}^{+\infty} + ike^{-ikz}E_y^{'}(z) + \frac{\omega^2}{c^2}E_y(k) = \int_{-\infty}^{+\infty} -\frac{4\pi i\omega}{c^2}\partial E(z)e^{-ikz}dz, \because J = \partial E(z)$$

$$\left[e^{-ikz}E_y^{'}(z)\right]_{-\infty}^{0}+\left[e^{-ikz}E_y^{'}(z)\right]_{0}^{+\infty}+ik\left(ikE_{(k)}\right)+\frac{\omega^2}{c^2}E_y(k)=-\frac{4\pi i\omega}{c^2}E_y(k)$$

$$\left(e^{-ik0}E_{y(0)}^{'}-e^{-ik(-\infty)}E_{y(-\infty)}^{'}\right)+\left(e^{-ik\infty}E_{y(\infty)}^{'}-e^{-ik0'}E_{y(0)}\right)+i^2k^2E_{(k)}+\frac{\omega^2}{c^2}E_y(k)=-\frac{4\pi i\omega}{c^2}E_y(k)$$

$$2E^{'}(0)=k^2E_{(k)}-\frac{\omega^2}{c^2}E_y(k)-\frac{4\pi i\omega}{c^2}E_y(k)$$

Now;

$$\nabla\times\vec{E}=-\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\vec{E}^{'}=-\frac{i\omega}{c}\vec{B}\because\frac{\partial}{\partial t}=ik$$

Put in above equation, we have;

$$2\left(-\frac{i\omega B_x(z)}{c}\right)=E_y(z)\left(k^2-\frac{\omega^2}{c^2}-\frac{4\pi i\omega}{c^2}\right)$$

$$-\frac{2i\omega B_x(z)}{c}e^{ikz}=E_y(z)\left[k^2-\frac{\omega^2}{c^2}\left(1+\frac{4i\pi}{\omega}\right)\right]$$

$$-\frac{2i\omega B_x(z)}{c}e^{ikz}=E_y(z)\left(k^2-\frac{\omega^2}{c^2}[\varepsilon_t(\omega,k)]\right)$$

$$E_y(z)=-\frac{2i\omega B_x(z)}{c}\frac{e^{ikz}}{k^2-\frac{\omega^2}{c^2}[\varepsilon_t(\omega,k)]}$$

Now, on applying inverse Fourier transform we have;

$$E_y(z)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}-\frac{2i\omega B_x(z)}{c}\frac{e^{ikz}}{k^2-\frac{\omega^2}{c^2}\left[\varepsilon_t(\omega,k)\right]}$$

The electric field profile while entering the plasma is given as,

$$E_y(z)=\frac{-i\omega}{\pi c}B_x(z)\int_{-\infty}^{+\infty}\frac{e^{ikz}}{k^2-\frac{\omega^2}{c^2}[\varepsilon_t(\omega,k)]}dk \tag{3.24}$$

Here, $\varepsilon_t(\omega,k)$ is the transverse permittivity , which is define as,

$$\varepsilon_t(\omega,k)=1-\frac{\omega_p^2}{\omega^2}\left[1-\frac{\theta_\perp^2}{\theta_\parallel^2}\left(1+gZ_\kappa(g)\right)\right] \tag{3.25}$$

3.4 Resonant and Non-Resonant cases

The interaction of waves and particles in a plasma is referred to as the resonant and non-resonant case. The wave frequency in the resonant situation is the same as the particle's cyclotron frequency. As a result, the wave and particle are in phase and the wave is able to impart energy to the particle. While in non-resonant case both frequencies are not same due to which the wave and particles are out of phase and the wave does not transmit much amount of energy to the particles.

3.4.1 Non-resonant case (large argument)

The equation for electric field profile is,

$$E_y(z) = \frac{-i\omega}{\pi c} B_x(z) \int_{-\infty}^{+\infty} \frac{e^{ikz}}{k^2 - \frac{\omega^2}{c^2}[\varepsilon_t(\omega,k)]} dk \tag{3.26}$$

where $\epsilon_t(\omega,k)$ shows us the transverse permittivity given as,

$$\varepsilon_t(\omega,k) = 1 - \frac{\omega_p^2}{\omega^2}\left[1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\left(1 + gZ_\kappa(g)\right)\right] \tag{3.27}$$

As in the Eq [3.27] poles exist in the denominator, we will apply residue theorem.

$$E_y(z) = \frac{-i\omega}{\pi c} B_x(z) \left[e^{ikz} 2\pi i \operatorname{Re} s \left(\frac{e^{ikz}}{k^2 - \frac{\omega^2}{c^2}[\in_t(\omega,k)]}\right)\right] \tag{3.28}$$

Put the denominator equal to zero.

$$k^2 - \frac{\omega^2}{c^2}[\varepsilon_t(\omega,k)] = 0$$

Put Eq [3.27] in above equation, we will get.

$$k^2 - \frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega^2}\left\{1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\left(1 + gZ_\kappa(g)\right)\right\}\right] = 0$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\left[1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\left\{1 + \xi_k(Z_\kappa(g)\right\}\right]$$

Here $Z_\kappa(g)$ is the modified plasma dispersion function which we can expand as,

$$Z_\kappa(\xi_k \geq 1) = \frac{i\sqrt{\pi}\kappa!\kappa^{3/2}}{\Gamma\left(\kappa - \frac{1}{2}\right)\xi_k^{2\kappa}}\left(1 - \frac{\kappa^2}{\xi_k^2} + ...\right) - \frac{1}{\xi_k}\left(1 + \frac{1}{2\xi_k^2} + ...\right)$$

Put $-\frac{1}{\xi}(1 + \frac{1}{2\xi^2})$ term of modified plasma dispersion function expansion in above equation, we have;

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\left[1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\left\{1 + \xi_k\left(-\frac{1}{\xi_k}(1 + \frac{1}{2\xi_k^2})\right)\right\}\right]$$

Put

$$\theta_{\perp,\parallel}^2 = \frac{2\kappa - 3}{\kappa} v_{t_{\perp,\parallel}}^2;\ v_{t_{\perp,\parallel}}^2 = \frac{T_{\perp,\parallel}}{m}$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + \frac{T_\perp}{T_\parallel}\frac{1}{2\xi_k^2}\right)$$

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2}\left(1 + \frac{T_\perp}{T_\parallel}\frac{1}{2\frac{\omega^2}{k^2\theta_\parallel^2}}\right) \because \xi_k = \frac{\omega^2}{k^2\theta_\parallel^2}$$

$$k^2\left(\frac{c^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{\theta_\parallel^2}{2\omega^2}\right) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$k^2\left(\frac{c^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa - 3}{\kappa}\frac{v_{t_\parallel}^2}{2\omega^2}\right) = 1 - \frac{\omega_p^2}{\omega^2} \because \theta_\parallel^2 = \frac{2\kappa - 3}{\kappa} v_{t_\parallel}^2$$

$$k^2 = \frac{1 - \frac{\omega_p^2}{\omega^2}}{\frac{c^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{\omega^2}}$$

$$\frac{c^2}{\omega^2} \times k^2 = \frac{1 - \frac{\omega_p^2}{\omega^2}}{\frac{c^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{\omega^2}} \times \frac{c^2}{\omega^2}$$

$$\frac{c^2k^2}{\omega^2} = \frac{1 - \frac{\omega_p^2}{\omega^2}}{\frac{\omega^2}{c^2}\left(\frac{c^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{\omega^2}\right)}$$

$$\frac{c^2k^2}{\omega^2} = \frac{1 - \frac{\omega_p^2}{\omega^2}}{\frac{\omega^2}{c^2} \times \frac{c^2}{\omega^2} + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{\omega^2}{c^2}\frac{v_{t_\parallel}^2}{\omega^2}}$$

$$\frac{c^2k^2}{\omega^2} = \frac{1 - \frac{\omega_p^2}{\omega^2}}{1 + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{c^2}}$$

$$k = i\sqrt{\frac{\frac{\omega_p^2}{c^2} - \frac{\omega^2}{c^2}}{1 + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{c^2}}} \tag{3.29}$$

So the electric field profile for non-resonant case will be,

$$E_{y_1}(z) = \frac{\omega}{c} B_x(z) \frac{e^{ikz}}{k_{p_1}}$$

Put Eq [3.29] in above equation we will get,

$$E_{y_1}(z) = \frac{i\omega}{c} B_x(z) \frac{e^{ikz}}{\sqrt{\frac{\frac{\omega_p^2}{c^2} - \frac{\omega^2}{c^2}}{1 + \frac{\omega_p^2}{\omega^2} \frac{T_\perp}{T_\parallel} \frac{2\kappa-3}{2\kappa} \frac{v_{t\parallel}^2}{c^2}}}}$$

For the limit case; z→ 0,

$$e^{ikz} = e^{ik0} = 1$$

$$E_{y_1}(0) = \frac{i\omega}{c} B_x(0) \frac{\sqrt{1 + \frac{\omega_p^2}{\omega^2} \frac{T_\perp}{T_\parallel} \frac{2\kappa-3}{2\kappa} \frac{v_{t\parallel}^2}{c^2}}}{\frac{\omega}{c}\sqrt{(\frac{\omega_p^2}{\omega^2} - 1)}}$$

$$E_{y_1}(0) = iB_x(0) \sqrt{\frac{1 + \frac{\omega_p^2}{\omega^2} \frac{T_\perp}{T_\parallel} \frac{2\kappa-3}{2\kappa} \frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2} - 1)}} \tag{3.30}$$

The above equation [3.30] tells us about the electric field profile of a wave incident on a plasma boundary.

3.4.2 Resonant case (small argument)

$$E_y(z) = \frac{-i\omega}{\pi c} B_x(z) \int_{-\infty}^{+\infty} \frac{e^{ikz}}{[k^2 - \frac{\omega^2}{c^2}[\in_t (\omega, k)]}$$

As the poles are exist in the denominator so by residue theorem,

$$\Rightarrow \left[k^2 - \frac{\omega^2}{c^2} \in_t (\omega, k)\right] = 0$$

Put Eq [3.27] in above equation,

$$\Rightarrow k^2 - \frac{\omega^2}{c^2} \left[1 - \frac{\omega_p^2}{\omega^2} \left\{1 - \frac{\theta_\perp^2}{\theta_\parallel^2} (1 + \xi_k Z_k(\xi_k))\right\}\right] = 0$$

For small arguments the modified plasma dispersion relation can be expand as,

$$Z_\kappa(\xi_k \leq 1) = \frac{i\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})}(1 - \xi^2 + \frac{(\kappa^2+1)}{2\xi^2}\xi^4 + ...) - \frac{2\kappa-1}{\kappa}\xi(1 - \frac{2\kappa+1}{3\kappa}\xi^2 + ...).$$

Put $\frac{i\sqrt{\pi}\kappa}{\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})}$ in above equation,

$$k_{p_n}^2 - \frac{\omega^2}{c^2}\left[1-\frac{\omega_p^2}{\omega^2}\left\{1-\frac{\theta_\perp^2}{\theta_\parallel^2}\left(1+\frac{\omega}{k\theta_\parallel}\frac{i\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\right)\right\}\right]=0$$

$$k_r^2 + 2ik_r k_\iota - k_\iota^2 = \frac{\omega^2}{c^2}\left[1-\frac{\omega_p^2}{\omega^2}\left\{1-\frac{\theta_\perp^2}{\theta_\parallel^2}\left(1+\frac{\omega}{k\theta_\parallel}\frac{i\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\right)\right\}\right]$$

$$k_i^2 < k_r^2$$

$$\Rightarrow k_r^2 + 2ik_r k_i = \frac{\omega^2}{c^2}\left[1-\frac{\omega_p^2}{\omega^2}\left\{1-\frac{\theta_\perp^2}{\theta_\parallel^2}\left(1+\frac{\omega}{k\theta_\parallel}\frac{i\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}\right)\right\}\right]$$

Now, separating real and imaginary part,
Real part:

$$k_r^2 = \frac{\omega^2}{c^2}\left[1-\frac{\omega_p^2}{\omega^2}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right]$$

Put

$$\theta_{\perp,\parallel}^2 = \frac{2\kappa-3}{\kappa}v_{t_{\perp,\parallel}}^2, v_{t_{\perp,\parallel}}^2 = \frac{T_{\perp,\parallel}}{m}$$

$$k_r^2 = \frac{\omega^2}{c^2}\left[1-\frac{\omega_p^2}{\omega^2}\left(1-\frac{\frac{2\kappa-3}{\kappa}v_{t_\perp}^2}{\frac{2\kappa-3}{\kappa}v_{t_\parallel}^2}\right)\right]$$

$$k_r^2 = \frac{\omega^2}{c^2}\left[1-\frac{\omega_p^2}{\omega^2}\left(1-\frac{v_{t_\perp}^2}{v_{t_\parallel}^2}\right)\right]$$

$$k_r^2 = \frac{\omega^2}{c^2}\left[1-\frac{\omega_p^2}{\omega^2}\left(1-\frac{T_\perp}{T_\parallel}\right)\right] \tag{3.31}$$

Imaginary part:

$$2k_r k_i = \frac{\omega_p^2}{c^2}\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{k\theta_\parallel}\frac{\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}$$

$$k_i = \frac{1}{2k_r^2}\frac{\omega_p^2}{c^2}\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\theta_\parallel}\frac{\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})}$$

Put value of $\theta_\parallel^2 = \frac{2\kappa-3}{\kappa}v_{t_\parallel}^2$ and k$_r$ in above equation,

$$k_i = \frac{1}{2\left[\frac{\omega^2}{c^2}\left\{1-\frac{\omega_p^2}{\omega^2}\left(1-\frac{T_\perp}{T_\parallel}\right)\right\}\right]}\frac{\omega_p^2}{c^2}\frac{T_\perp}{T_\parallel}\frac{\omega}{v_{t_\parallel}}\frac{\sqrt{\pi}\kappa!}{\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}}$$

$$k_i = \frac{1}{\left(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel}\right)} \frac{T_\perp}{T_\parallel} \frac{\omega}{v_{t_\parallel}} \frac{\sqrt{\pi}\kappa!}{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}}$$

As,

$$k = k_r + ik_i$$

Put the value of k_r and k_i in above equation,

$$k_{p_2} = \sqrt{\frac{\omega^2}{c^2}\left[1 - \frac{\omega_p^2}{\omega^2}\left(1 - \frac{T_\perp}{T_\parallel}\right)\right]} + i\frac{1}{\left(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel}\right)} \frac{T_\perp}{T_\parallel} \frac{\omega}{v_{t_\parallel}} \frac{\sqrt{\pi}\kappa!}{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}}$$

$$k_{p_2} = \sqrt{\frac{\omega^2 - \omega_p^2}{c^2\left(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel}\right)}\left(1 - \frac{T_\perp}{T_\parallel}\right)} + i\frac{1}{\left(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel}\right)} \frac{T_\perp}{T_\parallel} \frac{\omega}{v_{t_\parallel}} \frac{\sqrt{\pi}\kappa!}{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}}$$

$$k_{p_2} = \sqrt{\frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2}\left(1 - \frac{T_\perp}{T_\parallel}\right)} + i\frac{1}{\left(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel}\right)} \frac{T_\perp}{T_\parallel} \frac{\omega}{v_{t_\parallel}} \frac{\sqrt{\pi}\kappa!}{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}} \tag{3.32}$$

So the electric field profile for resonant case will become,

$$E_{y_2}(z) = B_x(z)\frac{\omega}{c}\left[\frac{e^{ikz}}{\sqrt{\frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2}(1 - \frac{T_\perp}{T_\parallel})} + \frac{i\sqrt{\pi}\kappa! \times \frac{T_\perp}{T_\parallel}\frac{\omega}{v_{t_\parallel}}}{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel})}}\right]$$

$$E_{y_2}(z) = B_x(z)\frac{\omega}{c}\left[\frac{e^{ikz}}{\left(\frac{\omega}{c}\sqrt{1 - \frac{\omega_p^2}{\omega^2}(1 - \frac{T_\perp}{T_\parallel})} + \frac{\frac{c}{\omega} \times i\sqrt{\pi}\kappa! \times \frac{T_\perp}{T_\parallel}\frac{\omega}{v_{t_\parallel}}}{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel})}\right)}\right]$$

$$E_{y_2}(z) = B_x(z)\left[\frac{e^{ikz}}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}(1 - \frac{T_\perp}{T_\parallel})} + \frac{i\sqrt{\pi}\kappa! \times \frac{T_\perp}{T_\parallel}\frac{c}{v_{t_\parallel}}}{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel})}}\right]$$

As surface impedance define on the plasma boundary so at z$\rightarrow$ 0;

$$\sum_{z\to 0} e^{ikz} = 1$$

$$E_{y_2}(0) = B_x(0)\left[\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}\frac{c}{v_{t\parallel}}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}}\right]$$

$$E_{y_2}(0) = B_x(0)\left[\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}}\right] \tag{3.33}$$

The above equation tells us about the electric field for resonant case when a transverse wave penetrating the plasma

The complete profile of an electric field is the sum of two electric fields equations given by Eq [3.30] and [3.33];

$$E_{y(0)} = E_{y_1}(0) + E_{y_2}(0)$$

$$E_y(0) = iB_x(0)\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}+\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}}$$

$$\frac{E_y(0)}{B_x(0)} = i\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}+\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}}$$

The ratio of tangential electric field to that of magnetic field is called surface impedance. So the above equation can be written as,

$$Z_s = \frac{E_y(0)}{B_x(0)} = i\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}+\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}} \tag{3.34}$$

3.5 Surface impedance

The surface impedance tells us how an electromagnetic wave interacts with plasma boundary. It is a complex quantity. The real part of surface impedance gives us power absorption while the imaginary part tells us about the phase of the reflected wave. It is defined as;

$$Z_s = \frac{4\pi}{c}\frac{E_y(0)}{B_x(0)}$$

3.5.1 Real part of surface impedance

The real part of surface impedance tells us about how much a wave will absorb while entering the plasma. It is given as;

$$\begin{aligned}
\mathrm{Re}\,|Z_s| \;&=\; \frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}} \\
&\quad\times\frac{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}-\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}-\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}} \\
&= \frac{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-\mathbf{1}+\frac{T_\perp}{T_\parallel})}}{\left[\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right]^2+\left[\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right]^2}
\end{aligned}$$

$$\mathrm{Re}\,|Z_s| = \frac{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}}{\left[\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right]^2+\left[\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right]^2} \tag{3.35}$$

3.5.2 Imaginary part of Surface Impedance

The imaginary part of surface impedance tells us about the phase of the reflected wave from the plasma boundary. It is given as;

$$\mathrm{Im}\,|Z_s| = \sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}} - \frac{\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\left[\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right]^2+\left[\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right]^2} \tag{3.36}$$

3.6 Relationship between surface impedance and skin depth

The relationship between the surface impedance and skin depth is derived by using Faraday's law along side with the definition of surface impedance.

$$\vec{E}_y(z) = \vec{E}_y\,(0)\,e^{ikz}$$

$$\vec{E}_y(z) = \vec{E}_y\,(0)\left[e^{i(k_r+ik_i)z}\right]$$

Now, By Faraday Law;

$$\nabla\times\vec{E} = -\frac{1}{c}\frac{\partial\vec{B}}{\partial t}$$

$$\nabla\times\vec{E} = -\frac{i\omega}{c}\vec{B}$$

As,

$$\nabla\times\vec{E} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \vec{E}_y(z) & 0 \end{vmatrix}$$

$$-\frac{\partial}{\partial z}E_y(z) = -\frac{i\omega}{c}B_x(z)$$

$$(ik_r - k_i)\,E_y(z) = \frac{i\omega}{c}B_x(z)$$

$$B_x(z) = \frac{c\,(ik_r-k_i)}{i\omega}E_y(z)$$

$$\frac{E_y(z)}{B_x(z)} = -\frac{i\omega}{c\,(k_i-ik_r)}$$

As,

$$Z_s = \frac{4\pi}{c}\frac{E_y(z)}{B_x(z)}$$

$$Z_s = \frac{4\pi}{c}\left[-\frac{i\omega}{c\,(k_i - ik_r)}\right], \because \frac{E_{y(z)}}{B_{x(z)}} = -\frac{i\omega}{c\,(k_i - ik_r)}$$

$$Z_s = -\frac{4\pi i\omega}{c^2}\left(\frac{1}{k_i - ik_r} \times \frac{k_i + ik_r}{k_i + ik_r}\right)$$

$$Z_s = -\frac{4\pi i\omega}{c^2}\left(\frac{k_i + ik_r}{(k_i)^2 + (k_r)^2}\right)$$

$$Z_s = \frac{4\pi\omega}{c^2}\left(\frac{k_r - ik_i}{(k_i)^2 + (k_r)^2}\right)$$

$$|Z_s|^2 = \left|\frac{4\pi\omega}{c^2}[\frac{(k_r - ik_i)}{(k_i)^2 + (k_r)^2}]\right|^2$$

$$|Z_s|^2 = \frac{16\pi^2\omega^2}{c^4}\left|\frac{(k_r - ik_i)}{(k_i)^2 + (k_r)^2}\right|^2$$

$$\mathrm{Im}\,|Z_s| = -\frac{4\pi\omega}{c^2}\left[\frac{ik_i}{(k_i)^2 + (k_r)^2}\right]$$

$$\frac{|Z_s|^2}{\mathrm{Im}\,|Z_s|} = \frac{\frac{16\pi^2\omega^2}{c^4}\left[\frac{(k_r - ik_i)}{(\vec{k}_i)^2 + (\vec{k}_r)^2}\right]^2}{-\frac{4\pi\omega}{c^2}\left[\frac{ik_i}{(k_i)^2 + (k_r)^2}\right]}$$

$$\frac{|Z_s|^2}{\mathrm{Im}\,|Z_s|} = -\frac{4\pi\omega}{c^2}\frac{1}{k_i}$$

$$-\frac{c^2}{4\pi\omega}\frac{|Z_s|^2}{\mathrm{Im}\,|Z_s|} = \delta \because \delta = \frac{1}{k_i} \tag{3.37}$$

The above equation relates surface impedance with skin depth.

3.7 Calculations of skin depth

In this section we will derive the expressions of skin depth for high and low frequency regime. As skin depth is defined in eq (2.34) as;

$$\delta = -\frac{c^2}{4\pi\omega}\frac{|Z_s|^2}{\mathrm{Im}\,|Z_s|}$$

3.7.1 Skin depth (in low frequency regime)

The surface impedance is calculated as,

$$Z_s = \frac{E_y(0)}{B_x(0)} = i\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}+\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}$$

In the low frequency regime the second term of above equation is taken in consideration. Surface impedance is defined as;

$$Z_s = \frac{4\pi}{c}\frac{E_y(0)}{B_x(0)}$$

By putting the values,

$$Z_s = \frac{4\pi}{c}\left[\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}\right]$$

$$Z_s = \frac{4\pi}{c}\left[\begin{array}{c}\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}} \\ \times\frac{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}-\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}-\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}\end{array}\right]$$

$$Z_s = \frac{4\pi}{c}\left[\frac{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}-\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\left(\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right)^2+\left(\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right)^2}\right]$$

$$|Z_s|^2 = \frac{16\pi^2}{c^2}\left[\frac{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}-\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\left(\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right)^2+\left(\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right)^2}\right]^2 \tag{3.38}$$

Now,

$$\mathrm{Im}\,|Z_s| = -\frac{4\pi}{c}\left[\frac{\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\left(\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right)^2+\left(\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right)^2}\right] \tag{3.39}$$

Put the values of Eqns [3.38] and [3.39] in Eq [3.37], we will get;

$$\delta = -\frac{c^2}{4\pi\omega}\frac{\frac{16\pi^2}{c^2}\left[\frac{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}-\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\left(\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right)^2+\left(\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right)^2}\right]^2}{-\frac{4\pi}{c}\left[\frac{\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\left(\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right)^2+\left(\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right)^2}\right]}$$

$$\delta = \frac{c}{\omega}\times\frac{1}{\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}$$

$$\delta = \frac{c}{\omega}\times\frac{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}$$

$$\delta = \frac{\omega_p}{\omega}\times\frac{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}} \tag{3.40}$$

This is the equation for skin depth in low frequency regime.

3.7.2 Skin depth (in high frequency regime)

The surface impedance is given by Eq [3.34] ,

$$Z_s = \frac{E_y(0)}{B_x(0)} = i\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}+\frac{1}{\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}+\frac{i\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}$$

and it is defined as;

$$Z_s = \frac{4\pi}{c}\frac{E_y(0)}{B_x(0)}$$

we will only considered the first term of Eq [3.34] for calculations,

$$Z_s = \frac{4\pi}{c}\left[i\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}\right]$$

$$|Z_s|^2 = -\frac{16\pi^2}{c^2}\left[\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}\right] \tag{3.41}$$

While,

$$\mathrm{Im}\,|Z_s| = \frac{4\pi}{c}\left[\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}\right] \tag{3.42}$$

Put the Eqns [3.42] and [3.41] in Eq [3.37], we will get,

$$\delta = -\frac{c^2}{4\pi\omega}\frac{-\frac{16\pi^2}{c^2}\left[\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}\right]}{\frac{4\pi}{c}\left[\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}\right]}$$

$$\delta = \frac{c}{\omega}\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}}$$

$$\delta = \frac{\omega_p}{\omega}\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}} \tag{3.43}$$

This equation describes the skin depth in high frequency regime.

4 RESULTS AND DISCUSSION

We have calculated the surface impedance real and imaginary parts and skin depth both for resonant and non-resonant cases of transverse wave in bi-kappa distributed unmagnetized plasma by using the kinetic theory. The integral given in Eq. [11] has been solved for two limiting case high and low frequency regime to calculate the real and imaginary parts of the surface impedance and relates it with the skin depth. As previously mentioned, the plasma's density and the electromagnetic wave's frequency determine whether plasma is underdense ($\omega > \omega_{pe}$) or overdense ($\omega < \omega_{pe}$). The results have been plotted for normalized frequencies to maintain discussion validity over an extensive spectrum of frequencies and densities i.e., as a function of ω/ω_{pe}. In this discussion, all the results are for overdense plasma, where $\omega < \omega_{pe}$. As our plasma are bi-kappa distributed, which means that there will be high velocity particles called suprathermal particles and temperature anisotropy, which means that the temperature are different in different direction. It is represented by $\eta = \frac{T_\perp}{T_\parallel}$,Here $T_\perp$ is the temperature of the electron moving perpendicular to the direction of the propagation and $T_\parallel$ is the temperature of electrons parallel to the direction of the propagation. When plasma is isotropic then $\eta = 1$, the greater the value of η , the more the plasma will be anisotropic. In this work we consider temperature anisotropy $\eta > 1$. We investigate how the temperature anisotropy, wave and plasma frequencies and kappa parameter affects the surface impedance and skin depth. The detail discussion are given below;

4.1 Real part of surface impedance against wave frequency for the different values of temperature anisotropy

The real part of surface impedance tells us how much a wave absorb while interacting with the plasma surface. We have obtained the real part of surface impedance by using the dispersion relation of a transverse wave and solving the modified plasma dispersion function analytically.

$$\mathrm{Re}\,|Z_s| = \frac{\sqrt{1 - \frac{\omega_p^2}{\omega^2}(1 - \frac{T_\perp}{T_\parallel})}}{1 - \frac{\omega_p^2}{\omega^2}(1 - \frac{T_\perp}{T_\parallel}) + \left(\frac{\sqrt{\pi}\kappa! \frac{T_\perp}{T_\parallel}}{\frac{v_{t_\parallel}}{c} 2\kappa^{3/1}\Gamma(\kappa - \frac{1}{2})\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel})} \right)^2} \tag{4.1}$$

Where ω is the wave frequency, ω_p is the plasma frequency, $\frac{T_\perp}{T_\parallel}$ is the temperature anisotropy, κ is the spectral index which tells about the supra-thermal particles. By plotting the above Eq [4.1] by keeping $\kappa = 2$ and $\frac{v_{t_\parallel}}{c} = 0.01$ we will get the graph of real of surface impedance for different value of temperature anisotropy.

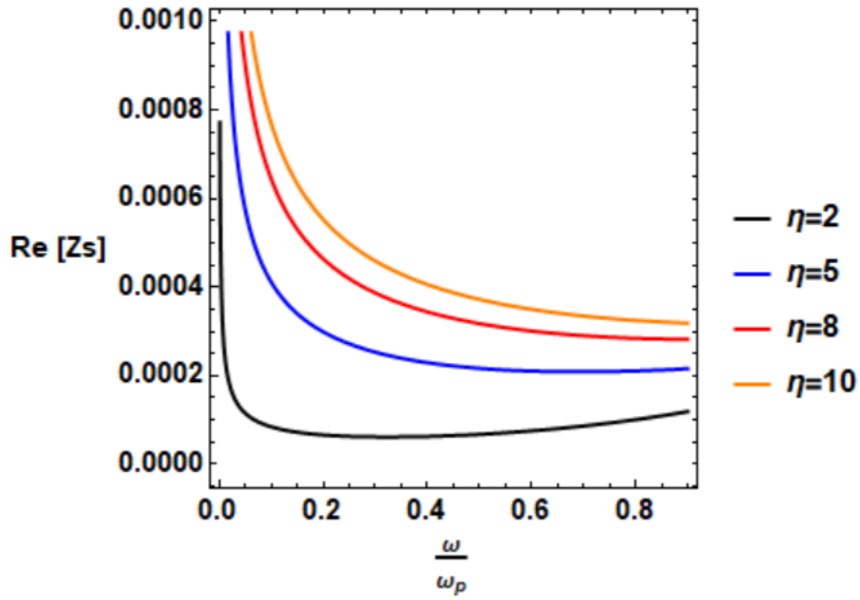

Fig. 4.1. Real part of surface impedance vs wave frequency for the different values of temperature anisotropy.

The graph shows that the real part of the surface impedance is inversely proportional to the wave frequency. This means that increasing the frequency the power absorption decreases. Additionally, keeping the spectral index "κ" constant and increasing the temperature anisotropy will increase the power absorption. In other words, the wave gains more energy as the temperature anisotropy increases.

4.1.1 Variation of real part of surface impedance with kappa parameter

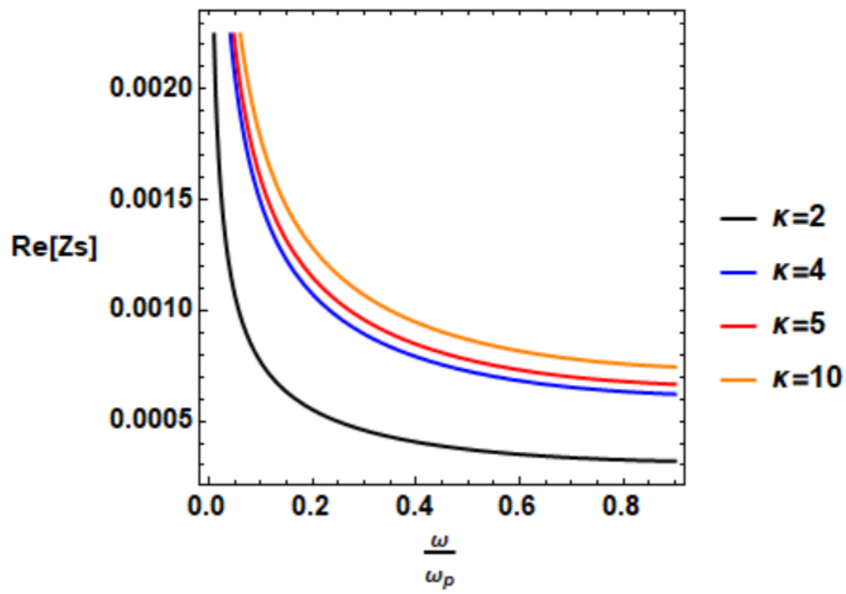

Fig. 4.2. Real part of surface impedance vs wave frequency for the different values of kappa parameter.

We can see that the power absorption increases as κ increases by varying the value of the spectral index κ while keeping the temperature anisotropy constant at η=2. However, it is noticeable from both graphs that the spectral index influences power absorption more than temperature anisotropy.

4.2 Imaginary part of surface impedance against wave frequency for the different values of temperature anisotropy

The imaginary part of surface impedance related with wave reflection. It has been calculated by using transverse waves dispersion relation and its electric field profile. The expression of the imaginary part of the surface impedance is;

$$\operatorname{Im}|Z_s| = \sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}} - \frac{\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}}{\left[\sqrt{1-\frac{\omega_p^2}{\omega^2}(1-\frac{T_\perp}{T_\parallel})}\right]^2+\left[\frac{\sqrt{\pi}\kappa!\times\frac{T_\perp}{T_\parallel}}{2\kappa^{3/2}\Gamma(\kappa-\frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2}-1+\frac{T_\perp}{T_\parallel})}\right]^2} \tag{4.2}$$

By plotting the above Eq [4.2] by keeping $\kappa = 2$ and $\frac{v_{t_\parallel}}{c} = 0.01$, we will get the graph of imaginary part of surface impedance for different value of temperature anisotropy.

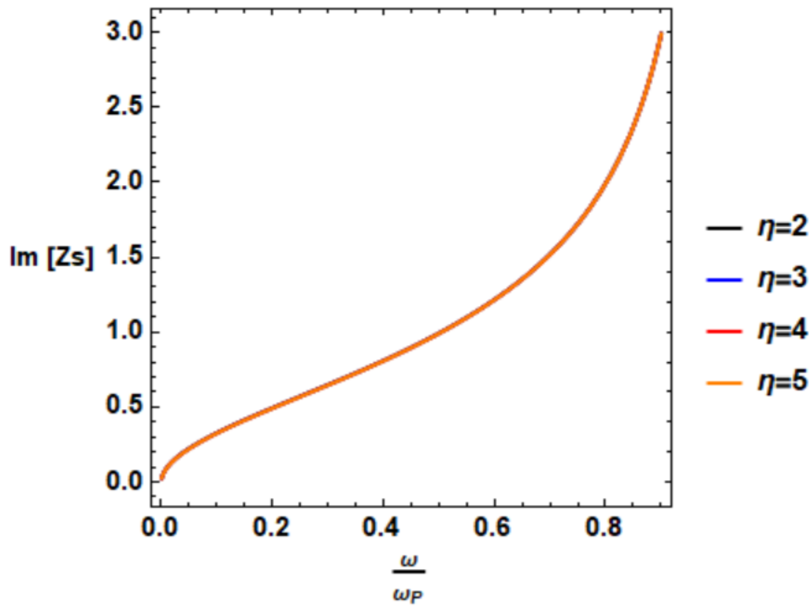

Fig. 4.3. Imaginary part of surface impedance vs wave frequency for the different values of temperature anisotropy.

The graph shows that the phase reflection of the wave is directly proportional to the wave frequency,which means that the wave reflection is greater at high frequencies Furthermore, the imaginary part of the surface impedance isn't significantly affected by temperature anisotropy.

4.2.1 Variation of imaginary part of surface impedance with kappa parameter

Now we will see how the imaginary of surface impedance changes with kappa parameter by keeping the temperature anisotropy $\frac{T_\perp}{T_\parallel} = 2$ and $\frac{v_{t_\parallel}}{c} = 0.01$ constant.

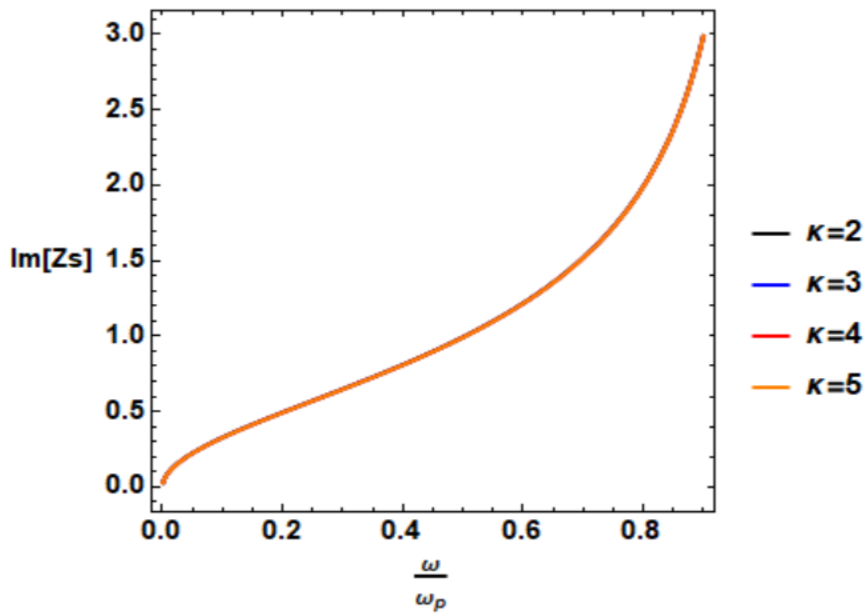

Fig. 4.4. Imaginary part of surface impedance vs wave frequency for the different values of kappa parameter.

The graph shows that by keeping the temperature anisotropy constant and changes the values of the spectral index kappa, the imaginary part of the surface impedance does not changes, which means that the spectral index kappa does not have any significant affect on the phase reflection of the wave.

4.3 Skin depth against wave frequency for different temperature anisotropy values

The mathematical expression for skin depth has been obtained both for high and low frequency, by using the relation between surface impedance and skin depth. The combined expression is given by;

$$\delta = \frac{\omega_p}{\omega} \frac{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel})}{\sqrt{\pi}\kappa! \times \frac{T_\perp}{T_\parallel}} + \frac{\omega_p}{\omega}\sqrt{\frac{1 + \frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2} - 1)}} \tag{4.3}$$

Where ω is the wave frequency, ω_p is the plasma frequency, $\frac{T_\perp}{T_\parallel}$ is the temperature anisotropy, $v_{t_\parallel}$ is the thermal velocity of particles,c is the speed of light κ is the kappa parameter which tells about high velocity particles.

Upon plotting the above Equation [4.3] by keeping $\kappa = 2$ and $\frac{v_{t_\parallel}}{c} = 0.01$, we get the graph of skin depth against the wave frequency for different value of temperature anisotropy.

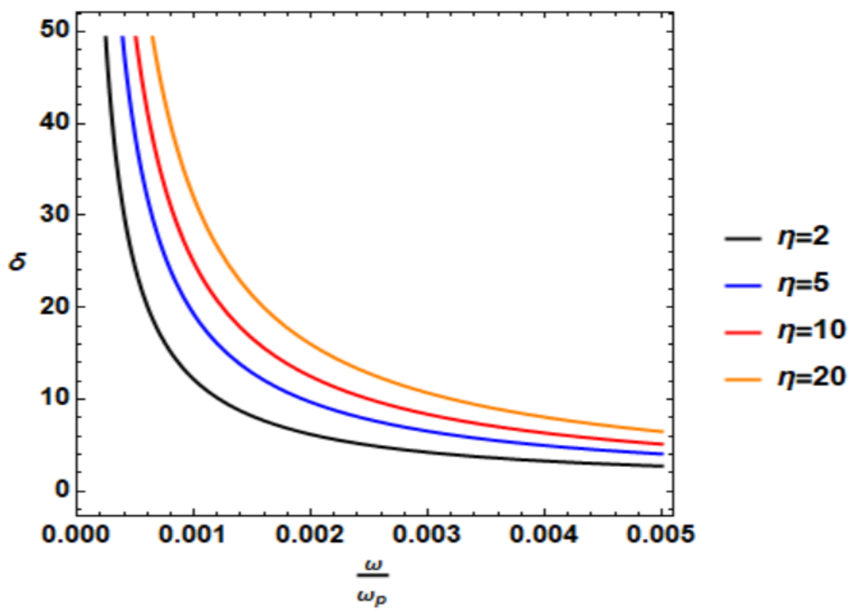

Fig. 4.5. Skin depth vs wave frequency for the different values of temperature anisotropy.

The plot shows the inverse relation between the skin depth and wave frequency becuase as frequency increases wave oscillate more rapidly and their energy dissipate more quickly as compared to the lower frequencies. It also indicates that by keeping all other parameters constant and changes only the temperature anisotropy, the skin depth increases by increasing the temperature anisotropy because the temperature anisotropy acts energy source, which means the greater the temperature anisotropy the more the distance will be travel by the wave inside the plasma.

4.3.1 Variation of skin depth with kappa parameter

The plot of skin depth for different values of kappa parameter has been plotted by keeping the temperature anisotropy $\frac{T_{\perp}}{T_{\parallel}} = 2$ and $\frac{v_{t_{\parallel}}}{c} = 0.01$.

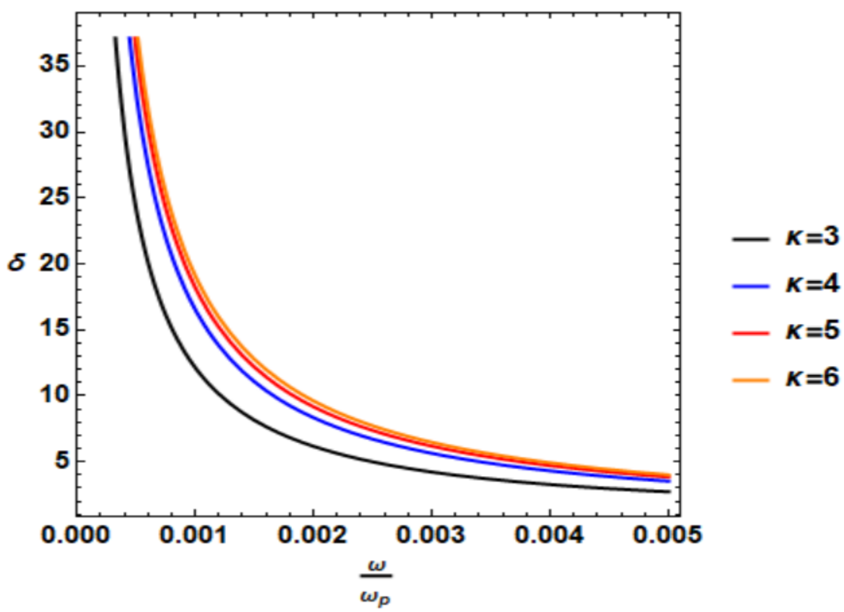

Fig. 4.6. Skin depth vs wave frequency for the different values of kappa parameter.

Skin depth varies directly with the kappa parameter. By keeping all other parameters constant, we observe that as the kappa value increases, the number of energetic particles decreases, which means that the resistance to the wave decreases. Therefore, the wave will penetrate more deeply into the plasma. It is clear from the above graphs that temperature anisotropy affects skin depth more than the spectral index.

4.4 Skin depth against temperature anisotropy for the different values of wave frequency (low frequency regime)

The skin depth in low frequency regime has been calculated by using the relation between surface impedance and skin depth. Mathematically;

$$\delta = \frac{\omega_p}{\omega} \frac{2\kappa^{3/2}\Gamma(\kappa - \frac{1}{2})\frac{v_{t_\parallel}}{c}\sqrt{\frac{2\kappa-3}{\kappa}}(\frac{\omega^2}{\omega_p^2} - 1 + \frac{T_\perp}{T_\parallel})}{\sqrt{\pi}\kappa! \times \frac{T_\perp}{T_\parallel}} \tag{4.4}$$

Where ω is the wave frequency, ω_p is the plasma frequency, $\frac{T_\perp}{T_\parallel}$ is the temperature anisotropy, $v_{t_\parallel}$ is the thermal velocity of particles, c is the speed of light κ is the kappa parameter. The skin depth has been plotted against the temperature anisotropy for the different values of wave frequency by keeping $\kappa = 2$ and $\frac{v_{t_\parallel}}{c} = 0.01$ constant.

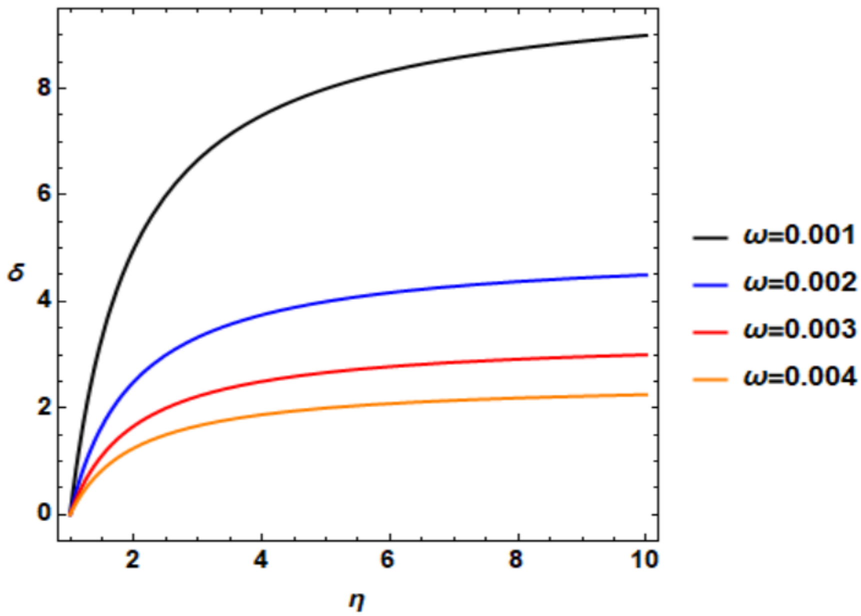

Fig. 4.7. Skin depth vs temperature anisotropy for different values of wave frequency (low frequency regime).

The graph shows that skin depth is inversely related to wave frequency because the wave dissipates its energy significantly more at high frequency than at low frequency. The skin depth directly proportional to the temperature anisotropy. This is due to temperature anisotropy providing more energy to the wave, causing the wave to go far deeper in plasma. The increase in skin depth is more dominant at low temperature anisotropy values.

4.4.1 Variation of skin depth with kappa parameter in low frequency regime

Now we will see how the skin depth in low frequency regime variates with kappa parameter by keeping the wave frequency $\omega = 0.001$ and $\frac{v_{t_\parallel}}{c} = 0.01$ constant.

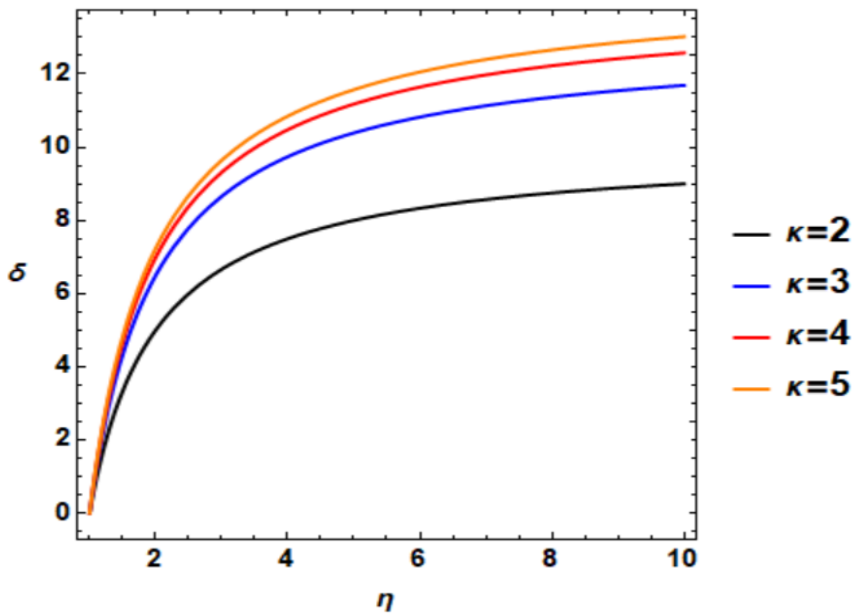

Fig. 4.8. Skin depth vs temperature anisotropy for different values of kappa parameter (low frequency regime).

Skin depth decreases for lower kappa values due to the presence of high-energy particles. These high-energy particles create more resistance to the wave, leading it to lose its energy quicker and penetrate less into the plasma. The number of high-velocity particles reduces as the kappa parameter increases, and the wave can travel further into the plasma.

4.5 Skin depth against temperature anisotropy for the different values of wave frequency (high frequency regime)

The skin depth in high frequency regime calculated numerically by using the expression that relates skin depth and surface impedance. Mathematically;

$$\delta = \frac{\omega_p}{\omega}\sqrt{\frac{1+\frac{\omega_p^2}{\omega^2}\frac{T_\perp}{T_\parallel}\frac{2\kappa-3}{2\kappa}\frac{v_{t_\parallel}^2}{c^2}}{(\frac{\omega_p^2}{\omega^2}-1)}} \tag{4.5}$$

Where ω is the wave frequency, ω_p is the plasma frequency, $\frac{T_\perp}{T_\parallel}$ is the temperature anisotropy,$v_{t_\parallel}$ is the thermal velocity of particles,c is the speed of light κ is the kappa parameter. The skin depth has been plotted against temperature anisotropy for different values of wave frequency where kappa parameter and speed of particles remains constant i.e.: $\kappa = 2$ and $\frac{v_{t_\parallel}}{c} = 0.01$.

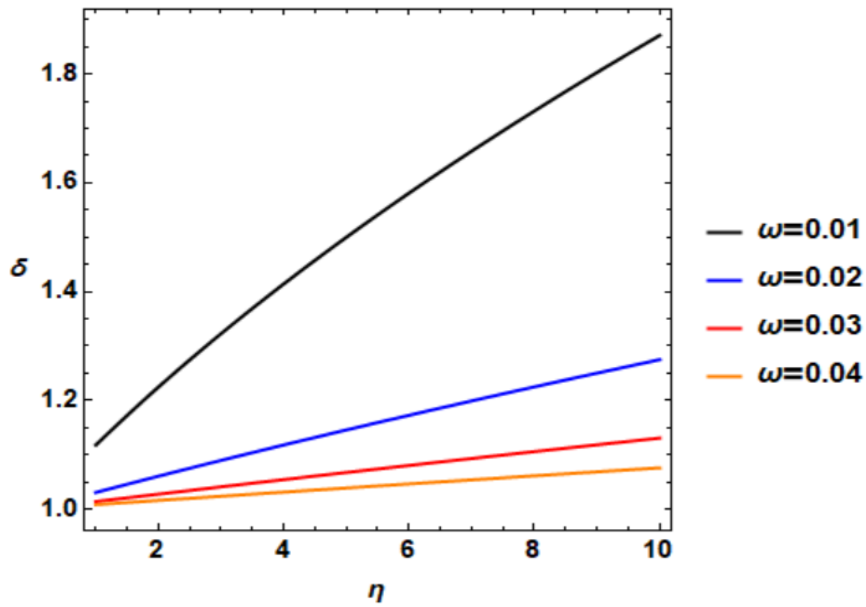

Fig. 4.9. Skin depth vs temperature anisotropy for different values of wave frequency (high frequency regime).

The skin depth increases linearly with increasing temperature anisotropy at high frequency by keeping all the other parameters constant. This is because the more the temperature anisotropy, the greater will be the energy source for the wave, and the wave will cover maximum distance . It also shows that skin depth has an inverse relationship with wave frequency. The higher the frequency of the wave, the more quickly it will dissipate its energy.

4.5.1 Variation of skin depth with kappa parameter in high frequency regime

Now we will see how skin depth changes by changing the kappa parameter and keeps the other variable constant. i.e.: $\omega = 0.001$" and $\frac{v_{t_\parallel}}{c} = 0.01$.

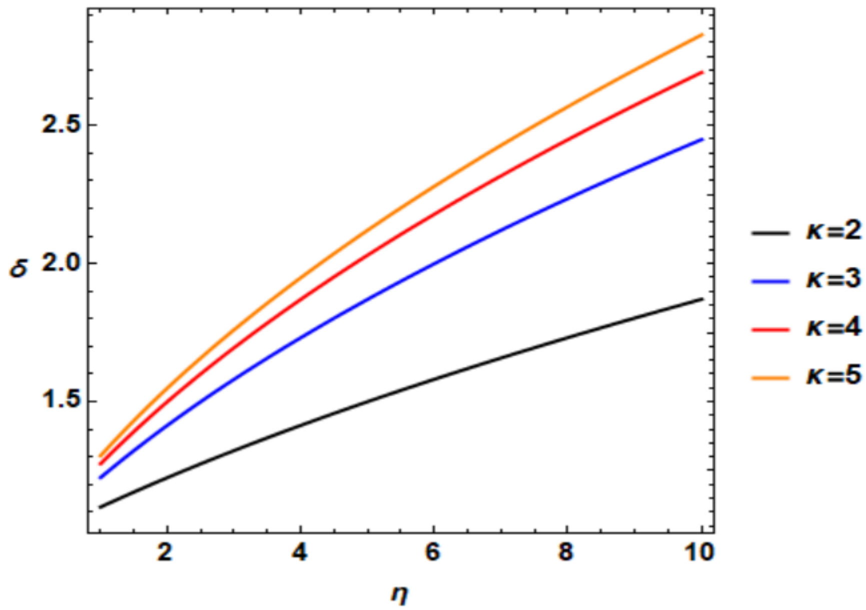

Fig. 4.10. Skin depth vs temperature anisotropy for different values of kappa parameter (high frequency regime).

The skin depth decreases in the presence of high-energy particles because the higher the energy of the particles, the greater the resistance to the wave, and smaller will be the skin depth. By increasing the value of the kappa parameter, the skin depth increases. It is because the high the value of the kappa the less will be the number of high-energy particles, allowing the wave to penetrate deeper into the plasma.

4.6 Comparison of skin depth at low and high frequency regime

As we obtained the expression of the skin depth for two limiting case i.e. high and low frequency cases. So we compared the skin depth plots for both low and high frequencies while keeping all other variables constant, to observe how an electromagnetic wave travel inside the plasma in case of high and low frequencies. The detail discussions are given below,

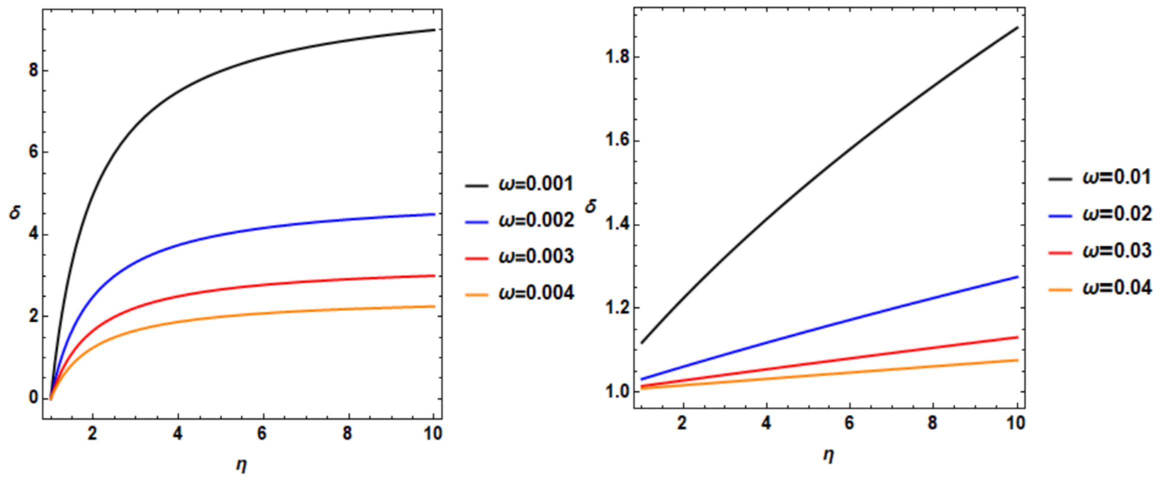

Fig. 4.11. Comparison between the skin depth at low and high frequency regime.

Skin depth has inverse relationship with the wave frequency, meaning higher frequencies penetrate less deeply than lower frequencies. This occurs because the faster oscillations of the electric field in high-frequency waves cause the free electrons in the plasma to respond more quickly and absorb energy more efficiently, limiting their penetration depth. The above graphs shows the comparison of the skin depth at low and high frequency regime by keeping all the other parameters constant. So we can see from the plots that because of the low frequency the skin depth in resonant case is greater than the skin depth in non-resonant case

5 CONCLUSION

This thesis is about the spatial damping of electromagnetic transverse waves in bi- Kappa distributed plasma, which is of significant interest to understand the surface impedance, absorption, reflection and heating mechanisms in both space and laboratory plasmas. The kinetic theory is used to calculate the expressions of the surface impedance and skin depth. The effects of the different parameters on surface impedance and skin depth has been studied. The results indicate that the real part of the surface impedance gives us the absorption which is inversely proportional to the frequency. Moreover, it is also observed that the real part of the surface impedance varies with both the temperature anisotropy and the kappa parameter. Notably, the kappa parameter has a greater influence on the real part of the surface impedance than the temperature anisotropy. On the other hand, the imaginary part of the surface impedance varies directly with the frequency. In contrast, the temperature anisotropy and the kappa parameter have no significant impact on the imaginary part of the surface impedance. The skin depth has also been calculated by using the relation between surface impedance and skin depth. The skin depth for both the resonant and non-resonant cases (i.e. low and high frequencies regimes respectively) has been studied. We also calculate the general expression of skin depth (both for resonant and non-resonant case) and observed that the skin depth is inversely proportional to the wave frequency and have direct relation with both the temperature anisotropy and kappa spectral index. It has been observed that in anisotropic plasma, the skin depth varies inversely with frequency in both the low and high frequency regimes. While the effect of the temperature anisotropy in low frequency regime is more significant in low value of temperature anisotropy as η increases the skin depth remains constant, In contrast to this in high frequency regime the skin depth varies linearly with the temperature anisotropy. It is also noted, the kappa spectral index have direct relation with the skin depth in both the frequency regimes. In low kappa distributed plasma the wave attenuates less as compared to the more kappa distributed plasma.

We also compared the resonant and non-resonant case and observe that the skin depth in resonant case (low frequency) is greater than non-resonant case(high frequency) it is because of skin depth has inverse relationship with the wave frequency, meaning higher frequencies penetrate less deeply than lower frequencies. This occurs because the faster oscillations of the electric field in high-frequency waves cause the free electrons in the plasma to respond more quickly and absorb energy more efficiently, limiting their penetration depth.

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