Efficient Modeling and Simulation of Power Networks Using Model Order Reduction

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This thesis is dedicated to *my beloved parents & my friends*.

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Acknowledgments

"Read. Read in the name of thy Lord who created; [He] created the human being from a blood clot. Read in the name of thy Lord who taught by the pen: [He] taught the human being what he did not know"

The profound importance of education is embedded in the very first revelation of the Quran, and it has been the guiding light and motivation throughout my academic journey. All praises to ALLAH, the Most Gracious and Merciful, who bestowed upon me the knowledge, patience, health, and ability to embark on and complete this thesis.

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Abstract

Electricity is distributed through a network of transmission and distribution lines to industrial, commercial and residential load. These networks cover a large geographical area making their analysis and monitoring a complex and expensive task. An alternate way is to use mathematical modeling and simulation of power distribution network where mathematical model involves differential equations and algebraic equations leading to a system of differential algebraic equations (DAEs). It is also known that simulation of such large complex systems is computationally expensive and takes a lot of time to simulate such large models for observing their behavior. To resolve this problem, the concept of Model Order Reduction (MOR) can be used in which a reduced order model is constructed from the original large scale model in a way that the behavior remains approximately same. Various MOR methodologies have been developed in the literature for linear systems such as Balanced Truncation, Moment Matching Methods, Krylov Projection techniques and Proper orthogonal decomposition. Most of these techniques are based on the transfer function of the system that are identified by the assumption that the initial condition is zero. This means that the performance of the MOR method may not be accurate when the initial condition is non-zero. In this research, some well-used model order reduction techniques have been applied including Balanced Truncation (BT), Iterative Rational Krylov Algorithm (IRKA), and their repeated implementation (BT-BT) to a model with non-zero initial condition. It is shown that the performance of the repeated implementation is better as compared to the direct reduction methods (BT and IRKA). A variant of the repeated MOR implementation (IRKA-IRKA) has been proposed and implemented on a comprehensive 100 km transmission line model. It is shown that the repeated use of IRKA improve the absolute error to 10^{-4} range for the specific transmission line model with 70% reduction in size. The convergence and accuracy of the results has been demonstrated specifically for BT-BT and IRKA-IRKA

in terms of actual and reduced system responses, absolute error between actual and reduced system and computational time for simulation of actual and reduced system.

Keywords: *Model Order Reduction, Power Systems, Transmission Lines, BT, IRKA*

Chapter 1

Introduction

This chapter includes an outline of power networks and the process of power distribution from power plants to end-users. The analysis of such systems through mathematical modeling and simulation is also discussed along with the associated issues under large scale settings. The problem statement is then introduced and the motivation and objectives of this research work are presented. Finally an overview of the complete thesis is given.

1.1 Power Distribution Networks

The first power distribution system was introduced by Thomas Edison in 1882 transmitting 110V direct current (DC) from the Pearl Street Power Station in Manhattan, USA. Since the network was based on direct current, the distance from the point of production to the consumer was very small and its expansion was costly. In 1888, Nicola Tesla introduced alternating current (AC) which was used by George Westinghouse to built an 11000V AC distribution system between Niagara Fall and Buffalo New York covering a distance of 20 miles. From this point on three main factors in distribution networks grew rapidly:

- Power generation
- Voltage capacity on Transmission line
- Distance from generation to end-user

In the modern landscape of technology, instead of having the facility of AC electricity sources, various objects operate on DC flow of electricity. Examples of such objects are solar cells, processors, LEDs, cars etc. Moreover, various techniques have been made that are capable of transforming the direct current (DC) into different higher and lower tier voltages. In addition, engineers and technicians are working hard to attain methods to utilise High Voltage Direct Current (HVDC) for the purpose of transmission of electricity over large distances along with minimum power loss.

1.2 Why Mathematical Modeling of transmission line is important?

Transmission line modeling is a fundamental practice in electrical engineering and power systems for various critical purposes. It serves as the cornerstone for power system analysis, enabling engineers to comprehensively understand the dynamics of energy transmission and distribution. By accurately modeling transmission lines, engineers can conduct stability studies to predict and mitigate potential instabilities, analyze faults to enhance system reliability, and optimize power flow for efficient energy transfer. The distribution of voltage and current along transmission lines is closely examined to maintain stable power levels and minimize losses.

Moreover, transmission line models play a pivotal role in grid planning, helping engineers assess the need for expansions, integrate renewable energy sources, and determine optimal operating conditions. Dynamic studies, such as transient stability analysis, are facilitated through these models, enabling a thorough understanding of the power system's response to sudden changes. Overall, transmission line modeling ensures the robust design, operation, and maintenance of power grids, contributing to a reliable, efficient, and resilient electrical infrastructure.

1.3 Mathematical Analysis of Power Networks

Power networks can be described mathematically through a set of equations for analysis. These models describe their behavior, such as the flow of electricity throughout the network, the voltage and current levels at different points, and the power losses. The mathematical models used to represent power networks are based on the laws of physics and electronics such as Kirchhoff's laws, Ohm's law, and power flow equations. Different equations such as PDEs, ODEs, linear equation, algebraic equations represent any physical system. Analytical time-dependent solution for these models is possible for simple, structured equations but in majority cases, the solution is either approximated or numerically computed.

Mathematical modeling of power networks and their response to different inputs allow us to get insight about the power network resulting in improvement in power network design and performance optimization. In the following, the mathematical representation of power transmission lines are discussed.

1.4 Mathematical Modeling of Transmission lines

Transmission lines is the backbone of the whole power network by connecting power plants to substations and other loads. They are responsible for transferring electrical energy from one point to another and can be categorized on the basis of voltage and distance or length. There are three types of transmission lines: *Long, medium and short.* The general equivalent circuitry of transmission lines remains the same but it is slightly different for short-transmission lines. It is because they have to deal with short lengths and low voltage level. Meanwhile, medium and long transmission lines are used to cover long distances and they carry very high voltages.

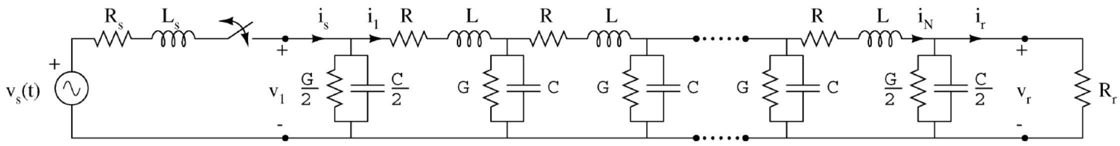


Figure 1.1: Transmission line model

Figure (1.1) shows the equivalent circuitry of transmission lines in which there are 'n' number of lumped parameters and

- a source delivering power
- current and voltage at every node
- capacitance and inductance uniformly distributed throughout the line
- and a load connected with transmission line

Implementing Kirchhof's voltage law (KVL) over first and primary node of the transmission line model along with the representation of the voltage across resistor and inductor as $v_R = iR$ and $v_L = L\frac{d_i}{dt}$ respectively, we obtain

$$v_s - i_s R_s - L_s \frac{di_s}{dt} - v_1 = 0$$

$$L_s \frac{di_s}{dt} = v_s - i_s R_s - v_1$$

$$\frac{di_s}{dt} = -\frac{i_s R_s}{L_s} - \frac{v_1}{L_s} + \frac{v_s}{L_s} \tag{1.4.1}$$

Now by applying Kirchhoff's Current Law (KCL) on the first node of the transmission line model along with the use of conductance G for reciprocal of resistance and current across capacitor as $i = C\frac{dv}{dt}$ we get

$$i_s = i_1 + \frac{G}{2}v_1 + \frac{C}{2}\frac{dv_1}{dt}$$

$$\frac{C}{2}\frac{dv_1}{dt} = i_s - \frac{G}{2}v_1 - i_1$$

$$\frac{dv_1}{dt} = \frac{2}{C}i_s - \frac{G}{C}v_1 - \frac{2}{C}i_1 \tag{1.4.2}$$

By applying KVL and KCL on Figure (1.1) yields "N" number of ordinary differential equations for current and voltage, which can be generalized. The generalized form is given as,

$$\frac{di_n}{dt} = \frac{1}{L_n}v_n - \frac{R_n}{L_n}i_n - \frac{1}{L_n}V_{n+1} \tag{1.4.3}$$

$$\frac{dv_k}{dt} = \frac{1}{C_{k-1}}i_{k-1} - \frac{G_{k-1}}{C_{k-1}}v_k - \frac{1}{C_{k-1}}i_k \tag{1.4.4}$$

Where i_n and i_k are the currents at 'N' nodes, L_n is the number of inductors used in the circuit, R_n represent resistance, G_n shows inductance, C_n depicts capacitors and v_n,v_k are the voltages. Generalized equations also provide an effective way to represent the behavior of a circuit and analyze its performance. This is especially useful when dealing with complex circuits that involve multiple components and connections. By using generalized equations, it is possible to represent the circuit's behavior in a simplified form and analyze it accurately.

1.5 Problem Statement

A large class of power networks is modelled as LTI systems, considering and supposing that initial conditions of the particular system is zero. If the initial condition is non-zero, the simulation process and the approach of reducing model complexity require special treatment. The research problem in this thesis is to explore how the approaches of model order reduction for linear state space models can be extended to power systems with in-homogeneous initial conditions. The reduced order model should not only ensure accuracy but also the properties of the true model such as stability of the system. In addition, the complete reduction and simulation process should be computationally cheap. To visualize the simulation results, geographic information system (GIS) can be used to show the calculation of the node on the power distribution network.

1.6 Model Order Reduction

Model Order Reduction (MOR) is considered and proved to be reliable and most used technique in engineering field to simulate large and complex models. MOR has recently undergone immense development for more complicated dynamical systems. Complex high-order dynamic models are typically produced when physical systems are modelled. Often, simpler models with lower orders are preferred to replace these models. It is essential to construct the reduced model in this procedure such that it retains the key elements of the original high-order model. Describing physical system through differential equations (mathematical model) is a helpful analytical technique. This model is attained via experiments or physical principles.

Constructing a state-space model for a transmission line involves representing the electrical behavior of the line using a series of first-order differential equations. A transmission line can be described using a distributed-parameter model, which accounts for the distributed capacitance and inductance along its length. Here's a general approach to constructing a state-space model:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1.6.1}$$

$$y(t) = Cx(t) + Du(t) \tag{1.6.2}$$

where $A \in R^{n\times n}, C \in R^{p\times n}, B \in R^{n\times m}$ and $D \in R^{p\times m}$ and $x(t) \in R^n$ is the state vector of model, $u(t) \in R^m$ represent input of the system and $y(t) \in R^p$ is the output of the model. In this case, the ROM will be represented as:

$$\dot{\hat{x}}(t) = \hat{A}x(t) + \hat{B}u(t) \tag{1.6.3}$$
$$\hat{y}(t) = \hat{C}x(t) + \hat{D}u(t)$$

where $\hat{A} \in R^{r\times r}, \hat{B} \in R^{r\times m}, \hat{C} \in R^{p\times r}$ and $\hat{D} \in R^{p\times m}$. The number of states of ROM is much less than that of the full order model (i.e. $r << n$). The output of reduced model is approximation of the output of the original model $\hat{y}(t) \cong y(t)$.

1.7 Research Objectives

The following objectives have been designed to conduct the research work:

- Implementation of Model Order Reduction techniques for efficient simulation of linear systems
- MOR techniques for power systems with non-homogeneous initial conditions
- Analysis of Reduced Order Model in terms of power transmission lines

1.8 Thesis Layout

This thesis embarks on a comprehensive exploration of power systems and transmission line networks, delving into the intricacies of Model Order Reduction (MOR). The initial chapter provides a foundational understanding, introducing power systems, transmission line networks, and laying the groundwork for MOR. Chapter 2 meticulously reviews the literature on various MOR techniques applicable to large-scale Linear Time-Invariant (LTI) systems, with a focus on those previously employed in power systems. Building on this foundation, Chapter 3 unfolds the methodology, elucidating the conversion of differential equations into state space models. The core of the research resides in Chapter 4, where diverse MOR techniques are applied, and their outcomes are rigorously examined. This chapter delves into results, conducting error analyses and offering insightful discussions. The conclusive Chapter 5 encapsulates the findings, draws overarching conclusions, and sets the stage for future research trajectories. The seamless progression through these chapters not only enhances our understanding of MOR but also contributes valuable insights to the broader landscape of power system modeling and optimization.

Chapter 2

Literature Review

In the previous chapter, we discussed about power networks and the mathematical representation of transmission lines. We now examine the methods used in Model Order Reduction (MOR). Model Order Reduction is a computational approach that lowers the order of a dynamical system specified by a set of differential or ordinary algebraic equations to allow for its design and optimization in the simulated physical system, its simulation, or the controller's model. This preserves the integrity of the input-to-output function by enabling the substitution of the large-scale set of describing ODEs/DAEs with a considerably smaller number of ODEs/DAEs.

2.1 Model Order Reduction for Linear Systems

Large-scale, linear systems can be found in many real-world scenarios, such as control problems and circuit simulations where partial differential equations are used to represent the underlying physical process [2]. MOR methods have already been extensively developed for linear systems. In essence, the MOR techniques for linear systems are just a continuation of the non-linear system techniques.

2.1.1 Balanced Truncation

Balanced Truncation (BT), one of the many MOR approaches, is the most often employed. Eliminating the states that have minimal effect on the response of the model or system results in a stable ROM with an existing frequency response error bound. This technique is known as balanced truncation. This method transforms a complex (higher) order system into a lower/simple order model and makes it easy enough to analyse its numerical simulations. BT assures stability of a system (bounded output for a given bounded input) along with an error bound. As this method of MOR is highly used and approached in Linear Time Invariant (LTI) systems such as power network systems, therefore it can provide lower order models for easy simulation, design and analysis of power networks.

Transfer function compromises of various realizations (state space). Internally balanced realizations (having controllability and observability gramians equal and diagonal) can differentiate most effective and least effective states of a model. Using Balance Truncation, less effective states can be truncated and eliminated, hence a lower approximation error is obtained for a frequency range giving appreciative performance. It must be noted that the states truncated are least controllable and observable. Balanced Truncation works best for higher frequencies, therefore other methods are considered for lower frequencies. BT is capable of maintaining basic properties of the original system after reducing its complexity by reducing its order along with error bound. It means that BT formulates an error bound too. Let's consider a higher order LTI system. A transfer function of a higher model complex system is represented by

$$G(s) = C(sI - A)^{-1}B + D$$

$A \in R^{n\times n}, C \in R^{p\times n}, B \in R^{n\times m}$ and $D \in R^{p\times m}$ and $x(t) \in R^n$. It must be noted that n = order of the system and p and m denotes number of outputs and inputs, respectively. After carrying on BT, the transfer function for the reduced order model (ROM) is given by

$$G_r(s) = C_r(sI_r - A_r)^{-1}B + D_r$$

Order for this model is r and ($r << n$). Moreover, the approximation error is given by $\parallel G(s) - G_r(s) \parallel_\infty$ by having A_{11}, B_1, C_1, and D as r^{th} order minimal realizations. Introducing a Controllability Gramian P and Observability Gramian Q to satisfy the following Lyapunov's equation:

$$AP + PA^T + BB^T = 0 \tag{2.1.1}$$

$$A^TQ + QA + C^TC = 0 \tag{2.1.2}$$

Now by defining a transformation matrix T we can transform the system in balanced form:

$$T^{-T}QT^{-1} = TPT^{T} = \Sigma = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix}$$

where $\Sigma_1 = diag\ [\alpha_1,\alpha_2,\ldots,\alpha_r]$, $\Sigma_2 = diag\ [\alpha_{r+1},\alpha_{r+2},\ldots,\alpha_n]$, in which α_i represent i-th Hankel singular value. Hence, obtaining new realizations as:

$$\bar{A} = T^{-1}AT = \begin{bmatrix} A_{11} & A_{1_2} \\ A_{21} & A_{22} \end{bmatrix}, \quad \bar{B} = T^{-1}B = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$\bar{C} = CT = \begin{bmatrix} C_1 \ C_2 \end{bmatrix}$$

The error expression is bounded and can be expressed as:

$$||G(s) - G_r(s)||_{\infty} \leq \ 2 \sum_{i=r+1}^{n} \alpha_i \tag{2.1.3}$$

2.1.2 Interpolation Based Model Order Reduction

A dynamical system can be treated as a problem of rational interpolation in which a system's transfer function is considered as n degree rational function H(s). Hr(s) represents the approximation of original transfer function with respect to H2 norm and is known as transfer function of ROM. Hr(s) is obtained by Petrov-Galerkin projection in which basis matrices (V and W) are constructed as discussed in [3]. The interpolation based model reduction gives good approximation of model but the error in output of original and reduced model depends on the selection of interpolation points and tangential directions. Apart from this, interpolation based model order reduction is possible only if there exists original model of a physical system.

In case of linear systems (where Q and N are null matrices), there are several techniques in the literature to compute ROMs, cf., [4]. Among these methods, projection-based moment-matching methods [5, 6] are well used. Using projection matrices $V \in R^{n\times n}$ and $W \in R^{n\times r}$, we approximate $x(t) \approx Vx_r(t)$ such that the Petrov-Galerkin orthogonality condition holds:

$$W^T\Big(V\dot{x}_r(t) - \big(AVx_r(t) + NVx_r(t)u(t) + Q(Vx_r(t)\otimes Vx_r(t)) + Bu(t)\big)\Big) = 0, \tag{2.1.4}$$

$$y_r(t) = CVx_r(t).$$

The projection is called one-sided projection, if W = V . Otherwise it is known as two sided which gives ROM of the form:

$$E_r = W^T V, A_r = W^T AV, Q_r = W^T Q(V \otimes V), N_r = W^T NV, \tag{2.1.5}$$

$$B_r = W^T B, C_r = CV.$$

In case of linear time invariant systems, a suitable choice of V and W, implicitly ensure moment-matching, where moments are the coefficients of the Taylor series expansion of the transfer function at some predefined shift frequencies. Thus for projection-based moment-matching, the choice of V and W can be linked with the transfer function of the system and often they are biorthogonal, that is E_r is identity matrix.

2.1.3 Iterative Rational Krylov Algorithm (IRKA)

The Krylov Algorithm, also known as Iterative Rational IRKA, is a model order reduction method. It is used extensively in control theory and is also discussed in linear algebra. It also simplifies any dynamic systems and aids in the solution of challenging differential equations, perhaps in line with other MOR approaches. IRKA is helpful since it converts a higher dimensional model into a lower dimensional model while maintaining the fundamental qualities (input, output). This method approximates the behaviour of any system by using knowledge of Krylov sub-spaces to simulate its algorithm. When a system matrix is applied to some initial vectors, such sub-spaces are reached. IRKA employs an iterative methodology and context. For the iteration process, a starting model that is less complex than the original system under consideration is chosen, either arbitrarily or via a heuristic [7].

Let us consider a single-input-single-output (SISO) linear dynamical system in state space:

$$\dot{x}(t) = Ax(t) + Bu(t), \quad y = Cx(t) \tag{2.1.6}$$

The states, input, and output of the dynamical system are $A \in \mathbb{R}^{n\times n}$ and $B, C \in \mathbb{R}^n$, respectively; x(t)$\in R^n$, u(t)$\in \mathbb{R}$, and y(t) $\in \mathbb{R}$. The underlying system's transfer function is expressed as $H(s) = c^T(sI - A) - 1b$. We will refer to the system and its transfer function as H(s).

Dynamic systems of form (2.1.6) with a large state-space dimension n are used in many applications; see, for instance, [8]. Large-scale simulations like such put an enormous burden on the computing system. Model order reduction aims to create a system that is equivalent,

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t), \quad y(t) = C_r x_r(t) \tag{2.1.7}$$

of much smaller dimension ($r \ll n$), with $A_r \in R^{r \times r}$ and B_r, $C_r \in R^r$ such that $y_r(t)$ approximates y(t) well in a certain norm. Similar to H(s), the transfer function $H_r(s)$ of the reduced model (2.1.7) is given by $H_r(s) = c_r^T$ $(sI_r - Ar)$ -1B_r. Taking into account the reduced model $\mathrm{H}_r(s)$, which is acquired by projection. Assuming that $\mathrm{W}_r{}^T$ and V_r are invertible, we select full rank matrices V_r, $\mathrm{W}_r \in R^{r \times r}$. We next define the reduced-order state-space realization with (2.1.7) and,

$$Ar = (W_r^T V_r)^{-1} W_r^T A V_r, \quad B_r = (W_r^T V_r)^{-1} W_r^T B, \quad C_r = V_r^T C \tag{2.1.8}$$

The reduced system is entirely determined by the choice of Wr and Vr inside this "projection framework"; in fact, determining Hr(s) just requires defining the ranges of W_r and V_r. IRKA iteratively adjusts the rational function parameters of the reduced-order model to improve fit the spectrum and transfer function of the full-order system. This refining procedure continues until a good reduced-order model is obtained [9]. Iterative Rational Krylov Algorithm also known as IRKA has a competitive edge as it gives a simpler approach and algorithm. This technique has been put forward and proposed by Gugercin, Beattie and Antoulas [10, 11]. It is a MOR technique that gives an H2 optimal approximation problem approach related to interpolatory model reduction [12].

2.1.4 Proper Orthogonal Decomposition

Proper orthogonal decomposition, or POD for short, is a widely used technique for principal component analysis (PCA) and model order reduction. POD is a mathematical technique that can be used to reduce the number of dimensions in complex data or high-dimensional systems. To find the dominant modes or patterns in the data, it projects the data or system behaviour onto a lower-dimensional subspace. In the context of model order reduction, POD is used to reduce the degrees of freedom of a mathematical model while preserving its essential characteristics [13–15].

In this method, the inputs which consist of essential behaviour of the system are given to a certain model which builds outputs. These outputs are called ‘snapshots’ which consist of column vector [16]. These snapshots describe the state of model at some moment.

Consider Y is a matrix which contains snapshots of the output and belongs to $\mathbb{R}^{m\times n}$, then there exist

$$U = (u_1, u_2, ..., u_m), \quad V = (v_1, v_2, ..., v_n), \tag{2.1.9}$$

where $U \in \mathbb{R}^{m\times m}$ and $V \in \mathbb{R}^{n\times n}$. Using singular value decomposition (SVD)

$$Y = U\Sigma V^* \quad or \quad U^*YV = \left(\begin{smallmatrix} D & 0 \\ 0 & 0 \end{smallmatrix}\right) := \Sigma \in \mathbb{R}^{m\times n} \tag{2.1.10}$$

The Y can be written as:

$$Y = (y_1, y_2, ..., y_n) = U^d D(V^d)^T = \sum_{i=1}^{d} \quad < u_i, y_j > R^m u^i \tag{2.1.11}$$

Model order reduction using POD generates ROMs with good approximation of the output. POD has the advantage of being able to solve nonlinear PDEs. This method works well for the fixed input, but if input keeps changing, there will be need of designing ROM every time the input changes. The disadvantage of this method is that this method is structured (input) dependent.

2.1.5 Modal Reduction

Using the prominent eigenmodes or resonant frequencies of a linear time-invariant (LTI) system, a technique known as modal reduction can lower the system's order. The main principle of modal reduction is to find the most important system modes and keep them while eliminating the less important ones. This results in a reduced-order model that captures the fundamental dynamics of the original system [17, 18].

As previously mentioned, systems and control theory is the foundation of model order reduction. In this field, techniques that are very different from Krylov-based techniques have been created. The fundamental concept is to eventually end the dynamical system studies. To demonstrate its operation, let's revisit the linear dynamic system (1.6.1) and (1.6.2), Applying a state space transformation

$$T\tilde{x} = T$$

has no effect on the system's input-output behavior. Based on the A matrix eigenvalue decomposition, the following transformation might be selected:

$$TA = \Lambda T$$

T^{-1}, a diagonal matrix obtained from eigenvalues of A matrix when T is non-singular, can be ordered so that the eigenvalues on the diagonal appear in decreasing order of magnitude. Next, by limiting the matrix T to the dominating eigenvalues, the system can be terminated. We call this procedure "Modal Truncation."

2.2 MOR for Power Systems

Model Order Reduction (MOR) has made a significant impact in power systems by addressing the computational challenges associated with the associated large-scale and complex models. The scale and complexity of these networks will probably keep rising due to the growing tendency of enabling more connections to neighboring systems, which will pose difficulties for their development, management, and control. Therefore, reduced-order power system models are used for a variety of applications and research, especially for fast and inexpensive stability analysis [19]. In general, the primary goal of model reduction for large power networks—also referred to as Power System Dynamic Equivalency [20] is to produce a system equivalent model that can replicate the aggregated steady-state [21] and dynamic characteristics of the full-order network [22] while also working with the computation tools that are currently available for power system analysis [23, 24].

Simplified network models are frequently used in reliability analyses of distribution networks [25, 26]. This practice is becoming even more important as power systems get more complex [27], particularly with the incorporation of renewable energy sources as demand response actuators, electric cars, and photovoltaic solar power and energy storage (ES). In an increasingly complex environment, simplified benchmark models will be necessary to accurately assess network reliability and determine the optimal course of action for future investments in a reliable, secure, and flexible power grid that meets the necessary dependability criteria. [28].

For power systems, a model reduction method based on Krylov subspaces has been explained in [1]. The reduced model, which is seen as an input-output system during the reduction process, approximates the behavior of the exterior area of the power system by matching the leading coefficients of a power series expansion of the transfer function around pre-specified frequencies. The method was demonstrated using time domain simulations, and it was found that the trajectories of the reduced model match those of the original model for sufficiently large Krylov subspaces. According to their effect on the input-output behavior of the system, this method thus seems to successfully mimic the dominating modes of the original system without necessitating the possibly expensive eigenvalue decomposition. The method was tested using a 50-machine power system that functioned as the external region [29].

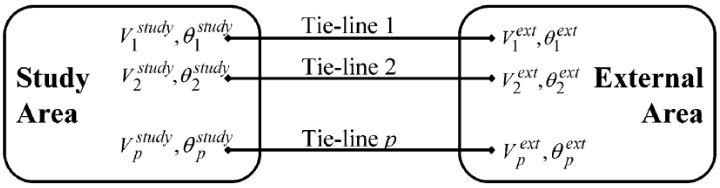


Figure 2.1: System configuration of the study and the external areas. [1]

Krylov subspace methods appear to be highly promising for reducing the power system model. Subsequent investigations would concentrate on addressing the issue of unstable reduced models and explicitly evaluating these techniques on big systems.

Network reduction in the context of reliability is typically achieved by methodically replacing some of the interconnected components of the chosen reliability model (e.g., series and parallel configurations in reliability block diagrams) with fewer equivalent components that share the same reliability properties. This simplifies the system representation. The following are the main drawbacks of this network reduction technique: The impact of critical or unreliable areas and components on the system reliability metrics becomes increasingly difficult to distinguish; a) it can only be applied to networks with very basic structure [30]; b) it cannot be used to calculate customer-based reliability indices, such as customer average interruption duration and frequency indices (CAIDI and CAIFI) [31]. Despite these limitations, this approach is practical, especially for straightforward investigations where further analytical modifications are not needed.

Because of these drawbacks, substitute strategies have been created, such as the decomposition technique, which depends on conditioning a complex system on the condition of a crucial power component (PC) [30]. However, because the model soon becomes unmanageable as the number of critical PCs increases, this strategy is not appropriate for big systems. There are other analytic methods that rely on evaluating minimum pathways or the minimal cut set technique [30], but their primary flaw is that when the number of paths and cut sets increases for big systems, a combinatorial explosion occurs. MOR for LTI systems with non-zero initial conditions is a complex but essential process, especially in fields like electrical engineering where transmission lines are common.

Conventional MOR techniques primarily focus on the system's transfer function, often overlooking initial conditions. For systems with non-zero initial conditions, it's crucial to incorporate these into the reduced model. These conditions significantly impact the system's response, especially in transient analysis [32]. In power systems, The reduced model must be thoroughly tested to ensure it accurately represents the original system under various conditions, including non-zero initial states. According to [Model reduction for systems with inhomogeneous initial conditions], the authors' approach is novel in that it decomposes the system's output into two separate mappings: one from input to output ($S_{u \to y}$) and another from initial condition to output ($S_{x \to y}$). This allows for the independent reduction of each mapping, offering greater control over the accuracy and complexity of the reduced models [33].

Chapter 3

Model Order Reduction for Power Transmission Lines

In the electrical power systems, transmission line models play a crucial role. Since simulations are necessary for the design and control of modern power systems, modeling such a system is still difficult. In order to analyze the numerical approach for a benchmark collection of some needful real-world examples, which can be utilized to evaluate and compare mathematical approaches for model reduction. The technique aims to preserve the main modes of the system while truncating the less significant ones. The stability is ensured by the reduction as the dominant modes of the large-scale stable system were retained in the development of the lower order model. The main drawback of many MOR techniques is that the reduced order model's steady state values do not match those of the higher order systems. This makes it possible for a new assessment of the error system offered that the Observability Gramian of the original system, an $H\infty$ and H_2 error bound can be calculated with minimal numerical effort for any minimized model attributable to the reduced order model (ROM) of a large-scale dynamical system is essential to effortlessness the study of the system utilizing approximation Algorithms.

3.1 Transmission Lines: An Overview

Electrical systems use transmission lines because they are essential for transferring electrical energy from one place to another. They have a unique function in the long-distance delivery of high-voltage power from power plants to distribution centres and, eventually, to final consumers [34]. Conductors (usually made of copper or aluminium), insulators, supporting structures, and grounding systems make up the majority of transmission lines. Although some more recent lines use direct current (DC), they are made to transport power using alternating current (AC). Some crucial elements are Resistance(R): The conductor material's natural resistance to current flow [35], Inductance (L): This is the result of a magnetic field created by current flow, which induces a voltage in opposition to the current change [36], Capacitance (C): The ability of a line to retain electrical charge, which is influenced by the conductors' proximity to one another and the insulating material [37] and Conductance (G): This represents the leakage current via the insulator [38].

Equations for transmission lines explain how voltage and current behave along the line. They provide a thorough mathematical foundation for examining transmission line characteristics and are derived from Maxwell's equations [39]. The Telegrapher's Equations are the basic formulas used in transmission line analysis. According to these formulas, the line is represented as an endless chain of segments with varying differential lengths, each having a unique conductance, capacitance, inductance, and resistance [40].

3.2 Theoretical Background

Understanding the fundamentals of electricity is required in order to be able to model telegraph equations. Ohm's law describes the relationship between voltage, current, and resistance in an electrical circuit. Ohm's law states that when one volt is applied to a one-ohm resistance, it will conduct one ampere (A) of current.

$$V = I.R$$

where R is resistance measured in ohms (Ω), I is current measured in amps, and V is voltage measured in volts. The current entering a junction in a circuit or node must be equal to the current leaving the junction or node, according to Kirchhoff's first law,

$$I_{total} = I_1 + I_2 + I_3$$

Kirchhoff's second law states that for every closed loop path in a circuit, the sum of the voltage gains and drops equals zero. This implies that there can be no overall voltage change because the circuit cannot gain or lose any energy. The formula for voltage in a closed circuit is given as,

$$V_{in} = V_1 + V_2 + V_3$$

Since an infinitely small piece of telegraph wire has a source and a load, as indicated by Kirchhoff's and Ohm's laws, modeling it as an electric circuit unveils a challenge. Modelling an indefinitely small bit of telegraph wire is adequate to replicate a transmission line across a distance since the features of a long transmission line and a small piece of telegraph wire are the same [41].

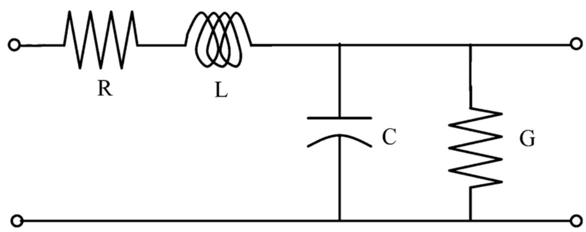


Figure 3.1: Equivalent Circuit of Transmission Lines

Extending the circuit to 'n' number of lumped parameters and applying Kirchhoff's current and voltage law on 3.1 yields current and voltage equations on each node and mesh respectively. The generalized equations at each node and loop are given as:

$$\frac{di_n}{dt} = \frac{1}{L_n}v_n - \frac{R_n}{L_n}i_n - \frac{1}{L_n}V_{n+1} \tag{3.2.1}$$

$$\frac{dv_k}{dt} = \frac{1}{C_{k-1}}i_{k-1} - \frac{G_{k-1}}{C_{k-1}}v_k - \frac{1}{C_{k-1}}i_k \tag{3.2.2}$$

3.3 State-Space Representation

The minimum number of variables, or "state variables," that adequately describe a dynamic system and its reaction to a certain set of inputs is known as the system's "state". In particular, a state-determined system model has the following property: the state of the system and its output at $t > t_0$ can be predicted using a basic understanding of the variables at initial time 't_0' and the system inputs for time $t \geq t_0$, as well as a fundamental set of variables used to mathematically describe the system, i.e. $x_i(t)$, $i = 1, ..., n,$.

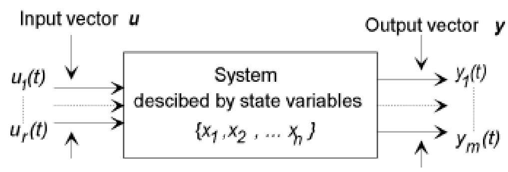


Figure 3.2: System Inputs and Outputs

Two inputs, $u_1(t)$ and $u_2(t)$, and four output variables, $y_1(t), ..., y_4(t)$, make up the system depicted in (3.2). In the event that the system is state-determined, all future behaviour of the system may be predicted from knowledge of its state variables $(x_1(t_0), x_2(t_0), ..., x_n(t_0))$ at some starting time t_0 and the inputs $u_1(t)$ and $u_2(t)$ for $t \geq t_0$. Any output variable $y_i(t)$ of the system can be computed using the state variables, which act as an internal system description that accurately represents the state of the system at any given time 't' [42].

3.4 State Equation Based Modeling Procedure

The complete system model of a linear time-invariant system is composed of (i) a set of n state equations defined in terms of matrices A and B, and (ii) a set of output equations expressed in terms of matrices C and D that relate the inputs and state variables to any pertinent output variables. The tasks involved in modelling the system include creating the matrix elements and putting the system model in the state space form.

$$\dot{x} = Ax + Bu \tag{3.4.1}$$

$$y = Cx + Du \tag{3.4.2}$$

The matrices A and B, which are characteristics of the system, are defined by the components and structure of the system. The specific selection of output variables determines the output equation matrices C and D. Using matrix algebra and the Laplace transform, classical representations of higher-order systems may be generated in a similar manner. The standard form linear state and output equations are arranged, and (3.4.1) and (3.4.2) are subjected to the Laplace transformation.

$$sX(s) = AX(s) + BU(s) \tag{3.4.3}$$

$$Y(s) = CX(s) + DU(s) \tag{3.4.4}$$

and the state equations may be rewritten:

$$sx(s) - Ax(s) = [sI - A]x(s) = Bu(s). \tag{3.4.5}$$

where s is on the leading diagonal and zeros are everywhere else in an $n \times n$ matrix created by the phrase sI. The matrix [sI - A] is a square $n \times n$ matrix whose components are directly related to the A matrix throughout linear system theory.

$$[sI - A] = \begin{bmatrix} (s - a_{11}) & -a_{12} & \cdots & -a_{1n} \\ -a_{21} & (s - a_{22}) & \cdots & -a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ -a_{n1} & -a_{n2} & \cdots & (s - a_{nn}) \end{bmatrix}$$

The state equations consist of n simultaneous operational expressions and are expressed in the manner of (3.4.5). Other linear operational equations may be solved directly using standard techniques for solving linear algebraic equations, including as substitution, elimination, and matrix inverse, as well as Gaussian elimination. The Laplace transform of the state equations for a linear time-invariant system yields expressions involving the state variables, inputs, and initial conditions. Substituting this expression into the output equation and assuming zero initial conditions, the output can be represented in operational form with a transfer function matrix [42]. The input-output relationship of the system in the Laplace domain is captured by the transfer function. The dynamic behaviour of the system in the time domain is concisely captured by a single differential equation that is generated using the transfer function between the output and the system input.

3.5 State Space Representation of Transmission Lines

Modern power systems require transmission line models for simulations in order to be designed and controlled. The outdated electrical systems of today need to be replaced with a better energy grid. These models have very massive orders, and it is known that their simulations need a lot of computing power. To calculate surge responses and Corona effects, transmission line transients induced by faults and switching activities must be simulated [43]. One common classification for transmission overhead line models used in transient simulations is lumped and distributed parameters models.

When a lumped parameters line model is selected, state space techniques are frequently employed to analyse the currents and voltages along a line. This eliminates the requirement for explicit inverse transforms and enables the model to be easily created and run simulations in the time domain [44]. The lumped parameters approach has the same features for simulating electromagnetic transients on lines with nonlinear components, such as fault arcs and corona effects, or when a detailed voltage and current profile is needed.

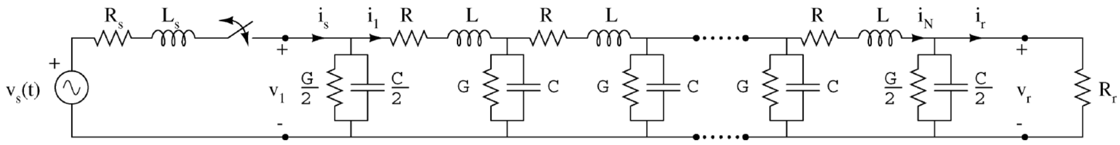


Figure 3.3: Transmission line model based on N lumped-parameter π networks.

In contrast to the distributed parameter line model, the cascaded connection of nominal circuits produces a linear state space model with a finite number of states. Each nominal network contains a series resistance, an inductance, a shunt conductance, and a capacitance, as illustrated in Figure 3.3.

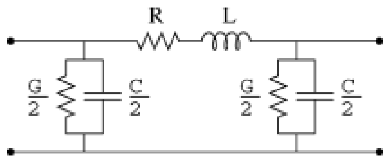


Figure 3.4: π section networks.

State variables in this model are capacitor voltages and inductor currents. Additionally, the state vector needs to have the source inductance current added to it for a typical arrangement such as the one seen in Figure 3.3 (a similar inductance might be applied to the load, which is excluded for simplicity). For the cascaded nominal circuits seen in Figure 3.3, the resultant state equations may be expressed as [45],

$$\dot{x} = A_l x + B_l v_s \tag{3.5.1}$$

where $x^T = [i_s \quad v_1 \quad i_1 \quad v_2 \quad i_2 \quad v_3 \quad ... \quad i_N \quad v_r]$ is the state vector and A_l is a tridiagonal matrix whose eigenvalues can be calculated in a straightforward manner. Consider a linear-time invariant system,

$$\dot{x} = Ax + Bu \quad ; \quad x(0) = x_0, \tag{3.5.2}$$

where x$\in \mathbb{R}^n$ is the state vector, u$\in \mathbb{R}^m$ is the excitation vector, and A $\in \mathbb{R}^{n\times n}$, B $\in \mathbb{R}^{n\times m}$ are known coefficient matrices.

$$B_l^T = [\frac{1}{L_s} \quad 0 \quad ... \quad 0] \tag{3.5.3}$$

$$A = \begin{bmatrix} -\frac{R_s}{L_s} & \frac{-1}{L_s} & & & & & & & \\ \frac{2}{C} & \frac{-G}{C} & \frac{-2}{C} & & & & & & \\ & \frac{1}{L} & \frac{-R}{L} & \frac{-1}{L} & & & & & \\ & & \frac{1}{C} & \frac{-G}{C} & \frac{-1}{C} & & & & \\ & & & \frac{1}{L} & \frac{-R}{L} & \frac{-1}{L} & & & \\ & & & & \frac{1}{C} & \frac{-G}{C} & \frac{-1}{C} & & \\ & & & & & \ddots & \ddots & \ddots & \\ & & & & & & \frac{1}{L} & \frac{-R}{L} & \frac{-1}{L} \\ & & & & & & & \frac{2}{C} & \frac{-2}{CR_r} - \frac{G}{C} \end{bmatrix}$$

The test case, shown in Figure 3.3, refers to a 220-kV system comprising a resistive load $R_r = 96\Omega$, source resistance with $R_s = 2\Omega$, $L_s = 0.06H$, and a 100-km transmission line with $R = 0.05\Omega/Km$, $L = 10^{-3}H/Km$, $C = 1219^{-9}F/Km$. The transmission line's conductivity is neglected. Now, we have all the information required for state-space analysis of transmission lines except initial conditions. Generally, we take initial condition zero i.e. $x_0 = 0$ but for this specific case initial conditions are specified i.e.

$$x_0 = [73902.6 \quad -31.6 \quad 43680 \quad -58.2 \quad 15574 \quad -112.3 \quad -10778]$$

3.6 MOR with zero Initial Conditions

We briefly navigate the conventional methods of Balanced Truncation (BT) and Iterative Rational Krylov Algorithm (IRKA), both of which are crucial to the proposed model reduction framework. Numerous methods have already been developed to produce high-fidelity reduced models with zero initial conditions ($x_0 = 0$). These methods include data-driven methods like the Loewner framework [46] and Vector Fitting [47], Lyapunov-based methods like Balanced Truncation (BT) and optimal Hankel Norm Approximation [48], and interpolatory methods like the Iterative Rational Krylov Algorithm (IRKA) [49] and spectral zero interpolation [50, 51]. For instance, [48] provides a comprehensive analysis of the relative advantages and disadvantages of some of these techniques. In chapter 2, a few reduction techniques are also covered.

3.7 MOR with Inhomogeneous Initial Conditions

When the initial condition is non-zero, the situation changes. One natural method (see, for example, to [52]) takes into account a translated state vector, $\hat{x}(t)$ = x(t) - x_0. Rewriting (2.1.6) in terms of $\hat{x}(t)$ permits the use of reduction techniques intended for null initial conditions since it now has a null initial condition, x(0) = 0. The reduced state trajectory is translated back to obtain an approximate version of the original state trajectory resulting from the nontrivial initial conditions (see, for example, [53]). This method might work well if there is only one initial condition that is known a priori. Note that an asymptotic state bias has been created from the temporary (but potentially large) effect of a nontrivial initial condition. This bias is likely to remain in the final reduced model even after the final reverse translation [54].Thus, this approach may overemphasise the significance of the initial conditions, perhaps resulting in imprecise response approximations from the associated reduced models.

If different initial conditions are used than those used in the initial reduction, it is not recommended to expect that reduced models produced in this way would yield accurate approximations. A new method was proposed in [55] to address this issue. It builds a reduced model that yields a good approximation to the true output, y(t), for a range of input functions, u(t), as well as for a range of initial conditions, x_0. This means that the model reduction process is largely independent of the specification of specific initial conditions. This method assumes that the initial conditions lie in a known n_0-dimensional subspace, x_0, which is covered by the columns of a matrix $X_0 \in \mathbb{R}^{n \times n_0}$, rather than that the initial circumstances are known a priori. The method of [55] proceeds by appending the basis X_0 to the input-to-state matrix, B and then performs balanced truncation using the augmented input-to-state matrix, [B X_0].

3.8 Proposed Methodology

This section presents a novel and adaptable model reduction framework for systems with nonzero initial conditions. It permits the reduction of the initial condition to output map as well as the reduction of input to output map separately. The idea is to incorporate the initial condition information into the model reduction process with a slightly different approach. The method is based on the well-known observation that the output $y(t)$ of the linear dynamical system (2.1.6) is a superposition of two outputs: one associated with u(t) = 0 and x(0) = x_0, and the other corresponding to the response of the system to u(t) with $x(0) = 0$. As a result, we have two separate model reduction processes: one on the initial condition-to-output map ($M_{x_0 \to y}$) with zero input and other on the input-to-output map ($M_{u \to y}$) with zero initial condition.

3.8.1 BT based reduction for $M_{u \to y}$ and $M_{x_0 \to y}$

For the system (1.6.2), the output y(t) can be expressed formally as follows using the Duhamel formula:

$$y(t) = \underbrace{Ce^{At}x_0}_{y_{x_0}(t)} + \underbrace{\int_0^t Ce^{A(t-\tau)}Bu(\tau)\,d\tau}_{y_u(t)} \tag{3.8.1}$$

where $e^{A(\tau)}$ is the matrix exponential, y_u(t) is the system's response to the input u(t) with zero initial conditions (x_0 = 0), and y_{x_0}(t) is the system's response to the initial condition x_0 without any input (u = 0). Consequently, the output is the superposition of these two signals because the underlying dynamics are linear. In order to fully infer the new technique, we express the initial condition of the system as a linear combination of a matrix X_0 as

$$x_0 = X_0 z_0 \tag{3.8.2}$$

in which $z_0 \in \mathbb{R}^{n_0}$ and $X_0 \in \mathbb{R}^{n \times n_0}$. This means that the response y_{x_0}(t) may be expressed as,

$$y_{x_0}(t) = Ce^{At}X_0 z_0. \tag{3.8.3}$$

This demonstrates that $y_{x_0}(t)$ is the response of a dynamical system with zero initial condition and an input of v = $z_0\delta$(t), where δ(t) stands for the Dirac delta function.

That is

$$\begin{aligned} \dot{x}_v(t) &= Ax_v(t) + X_0 v(t), \quad x_v(0) = 0, \\ y_{x_0}(t) &= Cx_v(t), \end{aligned} \tag{3.8.4}$$

Thus we can use the problem of model order reduction on the above dynamical system with zero initial condition to get approximation of $y_{x_0}(t)$. The reduced model can be obtained by using balanced truncation as discussed in Subsection 2.1.1 to obtain:

$$\begin{aligned} \dot{x}_{rv}(t) &= A_{rv}x_{rv}(t) + X_{rv0} v(t), \quad x_{rv}(0) = 0, \\ y_{rx_0}(t) &= C_{rv}x_{rv}(t), \end{aligned} \tag{3.8.5}$$

Similarly we can write another dynamical system with zero initial condition and input $u(t)$:

$$\begin{aligned} \dot{x}_u(t) &= Ax_u(t) + Bu(t), \quad x_u(0) = 0, \\ y_u(t) &= Cx_u(t), \end{aligned} \tag{3.8.6}$$

The above system can also be reduced separately from states n to r_u using the methodolgy discussed in Subsection 2.1.1, and the resulting reduced order model becomes:

$$\begin{aligned} \dot{x}_{ru}(t) &= A_{ru}x_{ru}(t) + B_{ru}u(t), \quad x_{ru}(0) = 0, \\ y_{ru}(t) &= C_{ru}x_{ru}(t) \end{aligned} \tag{3.8.7}$$

Thus we used balanced truncation two times in order to reduce the original model with non-zero initial condition and thus the response $y(t)$ can be approximated by using

$$y(t) \approx y_r(t) = y_{rx_0} + y_{ru}(t)$$

This method is called BT-BT because balanced truncation is used two times. It should be noted that the two components can be computed simultaneously in an offline phase prior to applying the simplified model to do simulation or perform controller design and testing. The complete framework of BT-BT is given in Algorithm 1 as a pseudo-code.

3.8.2 Error bounds for the BT-BT method

It is observed in [33] that the error in output functions $\| y_{x_0}(t) - y_{rx_0}(t) \|$ in reduction of $M_{x_0 \to y}$ to $M_{r_{x_0 \to y}}$ is directly dependent on the truncated Hankel singular values of the system. The error in output functions $\| y_u(t) - y_{ru}(t) \|$ is straight forward and well-known as it is the standard input-output linear system with zero initial condition. Thus the upper bound in this case is known from Balanced truncation method and is

Algorithm 1 BT-BT model reduction for systems with nonzero initial conditions

Offline Phase: Compute the two balanced and truncated models

Input: The system matrices **A, B, C,** and the initial condition basis $\mathbf{X}_0$.

Output: Reduced models for $M_{u\to y}$ and $M_{x_0\to y}$.

1: $M_{x_0\to y}$ to $M_{r_{x_0\to y}}$: Use BT on (3.8.4) to obtain the reduced model in (3.8.5)

2: $M_{u\to y}$ to $M_{r_{u\to y}}$: Use BT on (3.8.6) to obtain the reduced model in (3.8.7)

Online Phase: Simulate using the reduced models.

Input: The forcing term u(t) and the initial condition x_0.

Output: The estimated outputs of the two reduced models: $y_r(t)$

1: Identify z_0 such that $\mathrm{X}_0 z_0 = \mathrm{x}_0$.

2: Acquire output $y_{ru(t)}$ by simulating $M_{r_{u\to y}}$ using input u(t) and zero initial condition.

3: Acquire output y_{rx_0} by simulating $M_{r_{x_0\to y}}$ with zero input and the initial state z_0.

4: The approximate final output is as follows: $y_r(t) = y_{ru}(t) + y_{rx_0}(t)$.

related to the truncated Hankel singular values of $M_{u\to y}$. Thus the complete error in output function $\| y(t) - y_r(t) \|$ can be linked to the error bounds of the individual components and therefore to the corresponding truncated Hankel singular values.

3.8.3 IRKA-based MOR

It is easy to see that Algorithm 1 can be modified to use IRKA instead of BT for reducing the model $M_{x_0\to y}$ and $M_{u\to y}$. We can also use a combination of IRKA and BT, one for $M_{x_0\to y}$ and the other for $M_{u\to y}$ or vice versa. The error bound with IRKA is not known in the literature as in case of BT but when IRKA converges, it ensure the necessary conditions for optimality in terms of $\mathcal{H}_2$ norm. Thus approximating $M_{x_0\to y}$ and $M_{u\to y}$ by model reduction methods that seek to minimise the H_2 error norm will be beneficial. Note that the framework of IRKA based MOR is exactly similar to the one discussed in Algorithm 1. The only difference is to use IRKA as discussed in Subsection 2.1.3 at Step 1 and 2 instead of BT in the Offline Phase.

This approach is called IRKA-IRKA method as we are using IRKA two times now. As discussed in [49, 56], it is expected that IRKA will result in better approximations of the model especially in terms of $\mathcal{H}_2$ norm than BT. In case where IRKA and BT are used in combinations, we will call them IRKA-BT or BT-IRKA, depending on the sequence of the implementation.

3.9 NUST Power Network Integration with ArcGIS

The modeling and simulation of power network gives useful information about the power values on the transmission line at different nodes over a period of time, starting from an initial state of the system. The model order reduction framework discussed above can help in extending the scalability of the modeling and simulation of power networks. In the latter phase of this study, we sought to integrate the modeling and simulation results of power network of NUST University into ArcGIS, a geographic information system, by substituting some power values at different points in the network through our modeling and simulation framework. This integration aimed to assess the practical applicability of the Model Order Reduction (MOR) techniques applied earlier in a simulated environment. The process involved mapping the power network onto the GIS platform, allowing for spatial visualization and analysis. As a starting point, random values were assigned to the network that allowed us to provide a simulated environment in the GIS setup. The lack of real data and resources for real-time communication with the central system presented a limitation.

Figure 3.5: Nust Power Network in ArcGIS

The integration process allowed us to observe how the power network behaved within the GIS framework, providing insights into potential spatial correlations and fault identification. However, the absence of authentic, real-time data hindered the full realization of the system's operational capabilities. For optimal functionality, a seamless connection to real-time data sources and resources would be necessary to communicate live values with the central system. This limitation underscores the practical challenges faced in implementing these advanced modeling and GIS integration techniques in real-world scenarios, emphasizing the need for access to authentic data and adequate resources for comprehensive system operation and analysis. As we move forward, addressing these limitations will be pivotal in bridging the gap between simulation and practical application in power system management.

Chapter 4

Results and Discussion

This chapter presents the results of the approach used for model order reduction on a system of power networks. As discussed earlier, the focusing is to use an equivalent model of transmission lines derived from Kirchhoff's laws in terms of Ordinary Differential Equations (ODEs) and algebraic equations and subsequently transformed them into a state space model. The original model has been simulated without the application of model order reduction in order to compare the performance of the ROM techniques in mitigating computational costs. The accuracy of the reduced order model in time domain and the associated error are also observed.

4.1 Transmission Lines Model

In the vibrant urban landscape powered by a critical 100 km transmission line, this study undertakes a transformative exploration guided by the application of Model Order Reduction (MOR) techniques. The lumped parameter model considers a voltage source of 220 kV, source resistance (Rs) of 0.07 Ω/km, inductor (Ls) of 10^-3 H/km, capacitance (C) of 1219 * 10^-9 F/km, and a terminating load resistance (Rr) of 96 Ω. The protagonist, an adept electrical engineer, ventures into the heart of the lumped transmission line's intricacies, navigating the distributed resistance, inductance, and capacitance to construct a robust state-space model and implemented few techniques of model order reduction. This reduced-order model opens the door to previously unheard-of efficiency benefits in power system analysis while also illuminating the complex behavior under changing conditions. Considering the state space model in (3.5.1) of a π-model transmission line shown in figure [3.3],

$$A = \begin{bmatrix} 0 & -5\cdot 10^6 & & & & & & \\ 30 & -70 & -30 & & & & & \\ & 2.5\cdot 10^6 & 0 & -2.5\cdot 10^6 & & & & \\ & & 30 & -70 & -30 & & & \\ & & & 2.5\cdot 10^6 & 0 & -2.5\cdot 10^6 & & \\ & & & & 30 & -70 & -30 & \\ & & & & & 5\cdot 10^6 & -5.2\cdot 10^4 & \end{bmatrix}$$

$$X_0(t) = [73902.6 \quad -31.6 \quad 43680 \quad -58.2 \quad 15574 \quad -112.3 \quad -10778]$$

$$B^T = [\frac{1}{L_s} \quad 0 \quad ... \quad 0],$$

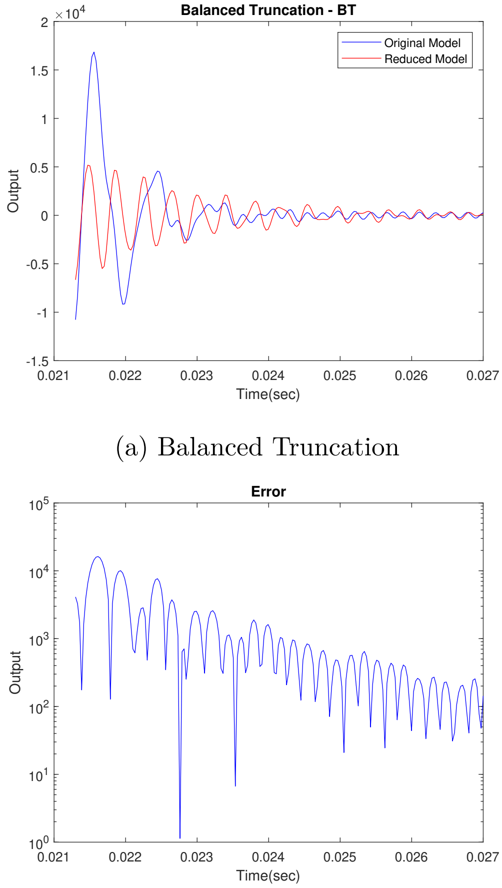
Where A is system matrix of 100Km transmission line network, u(t) is the 220-KV voltage source/excitation vector, X_0(t) is our state vector and C is our output matrix. All the parameters have been defined for state space model of the transmission line. In the next section, we will apply different techniques of model order reduction on it to analyse how it performs for different methods.

4.2 Implementation of MOR Techniques

Before delving into the specifics of each model order reduction (MOR) technique, a holistic overview of their application to the lumped parameter model of the 100 km transmission line reveals an intricate exploration into optimizing computational efficiency without compromising the precision of the system's dynamics. The collective goal across all techniques — Iterative Krylov Subspace Algorithm (IRKA), Balanced Truncation (BT), and their iterative combinations — is to distill the complexities of the original model while ensuring that the reduced representation faithfully captures the essential behavior of the transmission line. Each technique introduces its unique methodology, contributing to a comprehensive understanding of how MOR strategies impact error convergence and model fidelity. The forthcoming discussions will scrutinize the specific outcomes of applying IRKA, BT, BT-BT, and IRKA-IRKA techniques, shedding light on their individual strengths, convergence characteristics, and their collective implications on enhancing the computational efficiency of the lumped parameter transmission line model.

4.2.1 Balanced Truncation

The Balanced Truncation (BT) technique applied to our transmission line model offers a unique perspective on system reduction. The order of system matrix 'A' of original model of transmission line shown in figure (3.3) is 7×7. We want to reduce it to r=2, keeping all the dynamics of the system same.



(a) Balanced Truncation

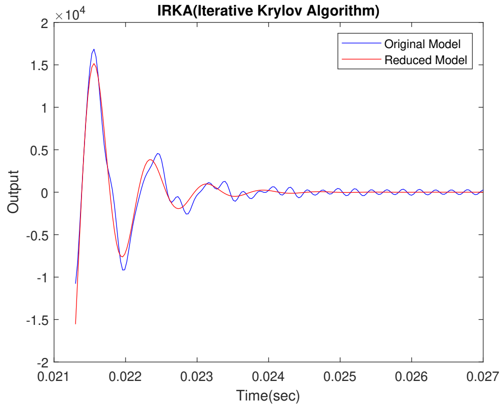
(b) Error BT

Figure 4.1: Original and Reduced Model of TL in case of BT

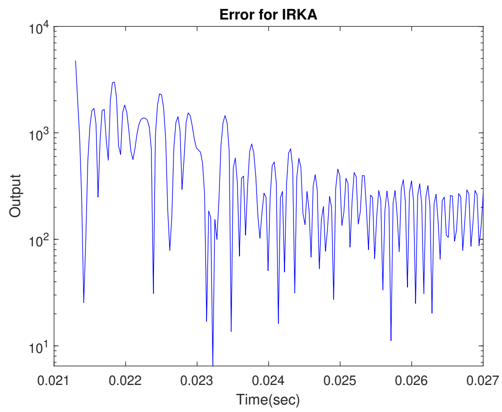
Analysing the graph shown in figure (4.1a), it can be clearly noticed that BT does not converge to the original model and its error $\| y_r - y \|$ can be viewed here in figure (4.1b).

4.2.2 Iterative Rational Krylov Algorithm (IRKA)

Model order reduction for the 100 km transmission line, the exploration extended from Balanced Truncation (BT) to the Iterative Rational Krylov Algorithm (IRKA) with a targeted rank (r=2). The original system, encapsulated in a 7×7 matrix, set the stage for examination of the IRKA method. The first graph (4.2a) in this section visually encapsulates the essence of the reduction, displaying the side-by-side comparison of the original and the reduced models. A striking visual alignment between the two underscores the efficiency of IRKA in distilling the complexity of the transmission line while retaining its intrinsic dynamics.



(a) IRKA based MOR of TL



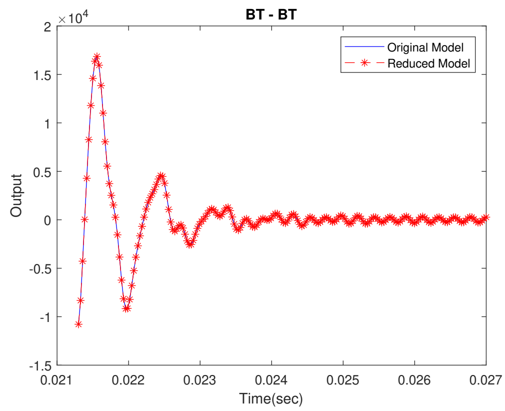
(b) Error for IRKA

Figure 4.2: Original and Reduced Model of TL in case of IRKA

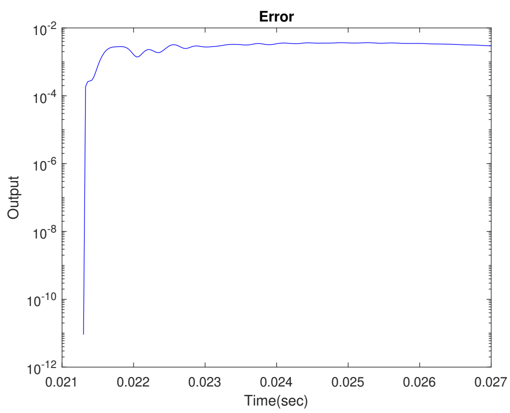
The second graph (4.2b), dedicated to error analysis $\parallel y_r - y \parallel$, meticulously charts the evolution of errors throughout the reduction process. This graph serves as a quantitative lens into the convergence behavior, revealing how the error systematically diminishes with each iteration, affirming the precision of the reduced model. The combination of these graphical representations affords a comprehensive understanding of the IRKA technique's impact on the 100 km transmission line. It can be noted that error has been reduced and convergence is better in case of IRKA but the error is still there.

4.2.3 BT-BT method of MOR

In order to approximate the output y(t) for transmission line network with $x_0 = X_0 z_0$, we implement the BT-BT method on it. Again the system matrix for transmission line model shown in figue [3.3] is 7×7 and the target reduced size for it is (r=2).



(a) BT-BT method of MOR



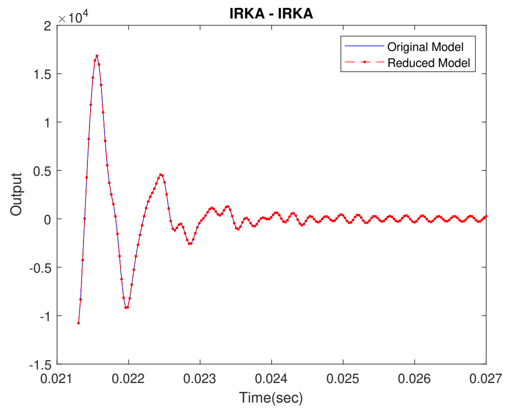
(b) Error

Figure 4.3: Original and Reduced Model of TL in case of BT-BT

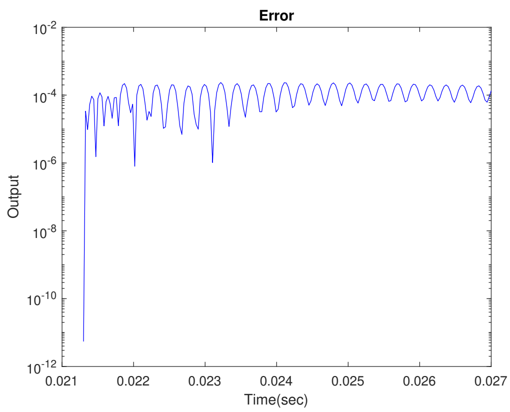
Implementing the BT-BT method involves a balance between achieving significant model order reduction and maintaining the fidelity of the system's response, particularly in the presence of non-zero initial conditions. Throughout the process, the BT-BT method accounts for the impact of non-zero initial conditions, ensuring that the reduced-order models capture the system's response to these conditions accurately. The error in case of BT-BT is shown in (4.3b).

4.2.4 IRKA-IRKA

The implementation of the IRKA-IRKA (Iterative Rational Krylov Algorithm) model order reduction technique on our transmission line network model, particularly considering non-zero initial conditions, has yielded commendable results. The original model, characterized by a higher-order system, underwent an iterative refinement process through IRKA-IRKA, resulting in a doubly-reduced model that remarkably outperforms other techniques. Figure (4.4a) in our results depicts the convergence behavior of both the original and doubly-reduced models. Visually comparing the responses of the two models highlights the striking alignment, affirming the capability of the IRKA-IRKA technique in faithfully preserving the essential dynamics of the transmission line network. This graphical representation underscores the successful reduction from the higher-order system to a more computationally efficient form while maintaining accuracy, particularly crucial in scenarios with non-zero initial conditions.



(a) IRKA-IRKA MOR

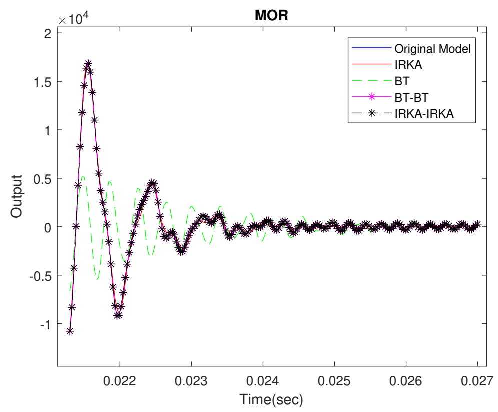


(b) Error for IRKA-IRKA

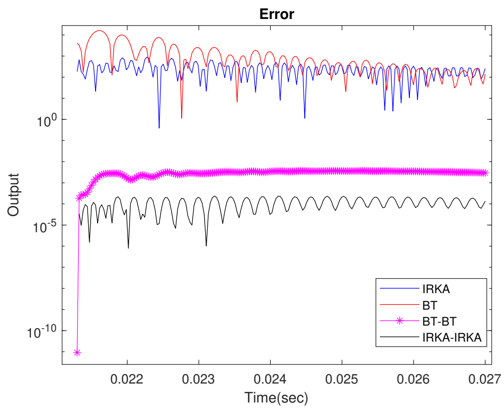
Figure 4.4: Original and Reduced Model of TL in case of IRKA-IRKA

The second graph (4.4b) focuses on the error $\parallel y_r - y \parallel$ evolution throughout the iterative application of IRKA-IRKA. This error plot showcases an impressive reduction in error, emphasizing the technique's efficacy in achieving significant model order reduction. The diminishing error signifies that the doubly-reduced model closely approximates the behavior of the original system, demonstrating the power of the Iterative Rational Krylov Algorithm in capturing essential dynamics while minimizing computational complexity.

The evaluation of Model Order Reduction (MOR) techniques, namely BT, IRKA, BT-BT, and IRKA-IRKA, on our transmission line model with a system matrix of 7×7 reduced to (r=2), has yielded compelling insights. As depicted in the (4.5a), which showcases the convergence and response of the original and reduced models, it is evident that both BT-BT and IRKA-IRKA techniques exhibit remarkable performance. The responses of the doubly-reduced models closely align with the original, affirming their effectiveness in capturing the essential dynamics of the transmission line network.



(a) Different Techniques of MOR on TL network



(b) Error

Figure 4.5: Combined techniques of MOR and Error plots

In the accompanying error plot (4.5b), it provides a quantitative perspective on the reduction achieved by each technique. Here, the substantial decrease in error $\| y_r - y \|$ for BT-BT and IRKA-IRKA becomes apparent, underscoring their proficiency in achieving significant model order reduction. This observation is particularly noteworthy in the presence of non-zero initial conditions, emphasizing the resilience of these techniques in accurately representing the system's behavior. The superiority of BT-BT and IRKA-IRKA in both the convergence and error aspects suggests their heightened efficacy for our specific transmission line model. This observation opens avenues for further exploration and underscores the potential of these techniques to streamline computational complexity while maintaining precision in scenarios involving non-zero initial conditions.

Chapter 5

Conclusion and Future Work

In this chapter, we presented the conclusion and future direction of our work by extending Model Order Reduction in power networks for its enhanced efficiency and coping with new world challenges. With growing technology, robustness in power networks is required for efficient flow of energy. In future, power distribution network of a whole city can be modeled and then integrated with ArcGIS. It will increase the efficiency of the network by immediate detection of faults, reduce line losses and quality management of electricity where load-shedding or power blackouts are major areas of concern. Through proper communication setup at different nodes of transmission or distribution network of transmission lines, real-time data can be optimized at one central system and monitored for further analysis accordingly. MOR can also be further applied to other non-linear parts of the power networks like generating side of it to increase its efficiency as well and especially dealing with systems having non-zero initial conditions.

5.1 Conclusion

This study presents an interesting development in the field of model order reduction with a specific application on the analysis of a π-model of power transmission lines. In particular, it is shown that the performance of standard model order reduction techniques may not be accurate for systems with nonzero initial condition. In such cases, it is shown that splitting the system into two parts and utilizing model order reduction on each of the subpart is more useful and accurate. The integrated response of the two reduced subsystems match very well with the actual system as compared to direct reduced system response. In terms of the computational cost, the simulation of the reduced system is much faster than the original large scale system. The study analyzes the intricate dynamics of the brief transmission line segment, offering insightful information about how the network behaves in various scenarios. The application of model order reduction methods, including IRKA, BT, BT-BT, and IRKA-IRKA, provides an extensive understanding of applications and constraints associated to power system modeling.

5.2 Future Work

Moving forward, the research agenda broadens to encompass various dimensions, ensuring a holistic approach that aligns with the complexities of real-world power networks. In terms of scalability and diversity, future research endeavors should delve into the scalability of the developed model order reduction framework to handle large scale nonlinear power networks. The challenge lies in ensuring that model order reduction techniques remain effective and accurate as they transition from localized linear segments to entire nonlinear power networks.

An essential aspect is to explore the robustness and resilience of the developed models under different scenarios. This includes analyzing the performance of reduced-order models under varying network conditions, ensuring adaptability to dynamic changes in the power grid. To address the multi-faceted nature of power systems, data science, machine learning, and data assimilation can be used to update physics-based models resulting in high-fidelity models.

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