MHD Flow Heat and Mass Transfer over an Oscillating and Translating Porous Surface

by

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A thesis submitted to the

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National University of Sciences and Technology,
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for the Degree of Master of Philosophy

Aug 2016

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A thesis
submitted for the M.phil Degree
in
Mathematics

Supervised by
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School of Natural Sciences,
National University of Sciences and Technology,
H-12, Islamabad, Pakistan

FORM TH-4

National University of Sciences & Technology

MASTER'S THESIS WORK

We hereby recommend that the dissertation prepared under our supervision by: Arshad Alam Khan, Regn No. NUST201463595MSNS78014F Titled: MHD Flow Heat and Mass Transfer over an Oscillating and Translating Porous Surface be accepted in partial fulfillment of the requirements for the award of **MS** degree.

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Dedicated to my lovely Parents,

brother and sisters

Acknowledgement

In the Name of Allah, The Most Beneficent, The Merciful. All the praise to His Prophet Muhammad (Peace be upon him), who gave us vision, wisdom and courage to complete this dissertation.

I am greatly thankful to Dr. Yasir Ali, Assistant Professor at the College of Electrical and Mechanical Engineering, National University of Sciences and Technology, Islamabad Pakistan, for his supervision and guidance throughout my thesis. His sympathetic attitude and friendly behavior is a beacon of guidance for my whole life.

I am very thankful to my friends Mehar Ali Malik and Muhammad Usman Sharif for their help.

Special thanks and appreciation goes to my parents for their endless love and prayers. It is with the support they provided that this work came into existence.

Arshad Alam Khan

Abstract

In this thesis, our focus is to study the flow of heat and mass transfer in electrically conducting fluid over oscillatory and translatory porous plate. The plate is immersed in the porous medium with linear heat source and an external magnetic field is applied in transverse direction to the flow. Finally the wall slip conditions are employed at the boundary. The governing system of partial differential equations, representing flow, energy and concentration of fluid are first separated into oscillatory and non-oscillatory parts using suitable transformations and as a result we have a system of ordinary homogeneous and non-homogeneous differential equations. These differential equations are then solved by finding the roots of auxiliary equations. The effects of the various physical parameters like translation, magnetic parameter, heat source parameter, Grashof number for heat and mass transfer, permeability parameter, schmidth number, Prandtl number and suction/injection parameter on the velocity, temperature and concentration profile. It is found that the magnetic field, Prandtl number and Schmidth number slow down the velocity of the flow while the translation parameter, heat source parameter, Grashof number for mass and heat transfer, permeability parameter will accelerats the flow through boundary layer. The heat source parameter having an increasing effect on temperature whereas suction parameter and prandtl number reduces the temperature. Both suction parameter and Schmidth number having a reduction effect on the concentration.

Contents

List of Symbols

Symbols	Name of physical parameters
G_r	Grashof number for heat transfer
G_c	Grashof number for mass transfer
β	Volumetric coefficient of expansion for heat
β^*	Volumetric coefficient of expansion for mass
T	Temperature of the fluid
T_∞	Temperature of fluid at a large distance from the plate
C	Concentration of fluid
C_∞	Concentration of fluid at a large distance from the plate
ρ	Density of fluid
ν	Kinematic viscosity
g	Gravitational acceleration
M	Hartmann number
K_p	Permeability parameter

Symbols	Name of physical parameters
P_r	Prandtl number
S_c	Schmidth number
D	Molecular diffusivity
σ	Electrical conductivity
ω	Frequency of the oscillation of plate
a	Translation of plate
υ_0	Suction velocity at the plate

Chapter 1

Introduction and Literature Review

Natural convection heat and mass transfer concept is used in many engineering problems, for example in fibrous insulation for the migration of moisture, in saturated soil, spreading of chemical pollutants and for the underground disposal of nuclear wastes. Similarly magneto-hydrodynamic flows has variety of applications in thermal physics, meteorology, motion of earth core, field of planetary magnetosphere, electronics, aeronautics and in chemical engineering. In astrophysics and geophysics the free convection flow is applied to the study of solar structures, radio propagation through iono sphere. The applications of convective flow through porous media are in thermal energy storage, geothermal energy recovery, and in production of oil etc. In filtration processes and to keep the temperature constant for heated body, permeable porous plates are used.

Unsteady hydromagnetic natural convective flow through a porous medium bounded by a vertical infinite plate is studied by Raptis and Vlahos [1]. They introduced some non-dimensional parameters and reduce the governing equations of motion to non-dimensional form, solved analytically by Laplace transform technique. They obtain expression for the velocity in term of physical parameter involved in the governing equations of motion and also discuss the effects of variation parameter entering into the governing equation of motion graphically.

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Singh and Dikshit [2] work on hydromagnetic fluid past over a semi-infinite plate moving continuously for large suction. Similarity transforms was used to reduce governing PDE's to ODE's. A perturbation method is then used to find analytical solution of resulting equation. Buoyancy-driven fluid flow and heat transfer from a vertical embedded in a porous medium was analyzed by Kim and Vafai [3], for the constant plate temperature and for constant plate heat flux. The governing equations are solved analytically using the method of matched asymptotic expansions along with the modified Oseen method. The numerical solution of the governing equations is also obtained based on the similarity transformations and physical description of the problem is presented. A flow model for the natural convection flow with heat and mass transfer in porous medium is analyzed by Bestman [4], for the moving boundary with suction. An asymptotic approximate solutions are obtained for the flow. Lia and Kulacki [5] obtained a similarity solution for the heat and mass transfer due to natural convection from a vertical plate embedded in a saturated porous medium. The study is done for two cases, for constant plate temperature and concentration and for uniform plate heat and mass flux. Dash and Das [6] studied the effect of hall current on the natural convection flow along an infinite vertical accelerated plate.

Free convection flow in thermally stratified media with buoyancy effects because of mass and thermal diffusion was discussed by Saha and Hossian [7]. They solved the dimensionless form of equation of momentum, concentration of species and energy by using method of local non-similarity and implicit finite difference, further they discuss graphically aspects of the solution and the dependence of various parameter involve in the solution on the flow

Unsteady natural convection flow and mass transfer past over an accelerated infinite vertical porous plate with large suction was discussed by Das *et al.* [8], using finite difference scheme solved numerically the governing equation of motion. Discuss the parameters effect such as Grashof number for heat and mass transfer, schmidt number, Prandtl number, porosity parameter, suction parameter on the velocity, temperature and concentration distribution and presented the result graphically.

Chaudhary and Jain [9] study the effects of mass and heat transfer on magneto-hydrodynamic natural convection flow over an oscillating plate immerse in a porous medium. Muthucumaraswamy and Manivannan [10] discuss the mass transfer effects on an oscillating vertical plate with heat flux. The partial differential equation that represent the flow converting to non-dimensional form by introducing some suitable dimensionless parameters, the equations obtain are then solved by Laplace transformation procedure. An exact solution is obtained in term of physical parameter and discuss graphically the effects of these parameter on the flow.

Natural convection and mass transfer hydromagnetic fluid past over a porous vertical infinite plate with viscous dissipation was analyzed by Poonia and Chaudhary [11]. The dimensionless equations that governs the flow are solve analytically using harmonic and non-harmonic function and graphically discuss the effects of physical parameters i.e. Eckert number and Schmidth number on velocity, concentration and temperature distribution with in the boundary layer. Das [12] presented exact solutions of mass transfer and free convection flow of hydromagnetic fluid near a plate moving vertically under the action of thermal radiation.

The effects of radiation on free convection near a plate vertically embedded in a porous medium with ramped wall temperature was discuss by Das *et al.* [13]. The method use for solving the governing equations was Laplace transform and obtain an analytical expressions for skin friction, velocity, concentration and temperature. Das *et al.* [14] study the combined effects of mass transfer and natural convection on unsteady fluid flow past a porous vertical infinite plate with heat source embedded in a porous medium. A multi parameter perturbation method is used and solve the governing equation of motion. An analytic expressions was obtain for temperature and velocity distribution, skin friction in term of some physical non dimensional parameter and in term of Nusselt number for rate of heat transfer. Present graphically the result and discuss the effects of physical parameters on the flow.

Kesavaiah *et al* [15] work on natural convection heat transfer in oscillatory flow of an elastico-viscous fluid from a vertical plate and solve the governing equation of motion in non-dimensional form by perturbation technique, an analytical expression for velocity, temperature distribution and skin friction was obtain. The effects of variation of parameter on the flow are also discuss graphically .

A numerical solution for unsteady fluid flow past over a semi-infinite vertically oscillating plate with uniform mass flux and variable surface temperature was given by Muthucumaraswamy and B. Saravanan [16] by using implicit finite difference scheme of Crank-Nicolson method. The result is shown with the help of graphs. The effects of the parameter such as thermal Grashof number, mass Grshof number, Schmidt number and thermal radiation on velocity, temperature and concentration distribution are analyzed.

Amit and Srivastava [17] obtain the exact solution of mass and heat transfer effects on flow past over a vertical oscillating infinite plate with an inconstant temperature through porous media. Das *et al.* [18] examine natural convection mass transfer in hydromagnetic fluid flow over a porous oscillating plate in the porous medium with heat source. The momentum, energy and concentration equation was solved analytically. An analytic solution obtained for velocity, temperature, concentration distribution, skin friction and in term of Nusselts number for heat flux. Observe the action of physical quantities like Hartmann number, Grashof number for mass and heat transfer, permeability parameter, Prandtl number and Schmidth number on the velocity, temperature and concentration graphically.

The effects of combine natural convection and mass transfer of incompressible viscous fluid past over vertical infinite porous plate embedded in a porous medium with heat source was carried out by Das *et al.* [19]. The solution of momentum, concentration and energy equations was given analytically by using multi-parameter perturbation technique and also graphically discuss the parameters effects such as Grashof number for heat and mass transfer, Heat source parameter, Schmidth number and prandtal number. Mukhopadhyay and Mandal [20] reported mixed convection and heat transfer in hydromagnetic fluid slip flow past a porous vertical plate. They use similarity transform the governing non-linear PDE's to non-linear ODE's. The resulting ODE's are numerically solved by shooting method and analyzed the result graphically.

Natural convection mass and heat transfer in magneto-hydrodynamic fluid past over a plate moving vertically with variable concentration and surface temperature in a porous medium was carried out by Javaherdeh *et al.* [21]. They used fully implicit finite difference method to solve the governing equation of motion in dimensionless form and analyzed with the help of graphs. Khalid and his co-workers [22] discuss unsteady MHD natural convective flow of cason fluid over a vertical oscillating plate embedded in a porous medium.

From the above literature review we note that work on natural convection magneto-hydrodynamics flow over a plate is done on either by considering accelerating plate or oscillating plate. To the best of our knowledge no work is done to study the combined effects of oscillation and translation. In the present work we consider natural convection and mass transfer of hydromagnetic fluid flow. We study the flow behavior over oscillating and translating porous plate in the presence of external magnetic field. This thesis is arranged as follows: Chapter 2 is about some basis definition of fluid mechanics including some general mathematical model equations. In chapter 3 the review of S. S. Das *et al.* [18] paper is presented. His is work on natural convection flow heat and mass transfer in hydromagnetic flow of viscous, incompressible and electrical conducting fluid over oscillating porous plate. At the boundary wall slip conditions are used. In chapter 4 we study the natural convection flow and heat transfer in hydromagnetic fluid flow over oscillating and translating porous plate by considering that there is no diffusing species in fluid. In chapter 5 we consider the natural convection flow heat and mass transfer in hydromagnetic fluid over an oscillating and translating plate assuming that the fluid contain other species.

Chapter 2

Prelaminaries

2.1 Fluid

Any substance that under the influence of shear stress continuously deform is termed as fluid, no matter how small is the shear stress.

2.2 Viscosity

It is the measure of resistance offers to the gradual deformation by shear stress when the fluid flow. It is the property of fluid which arises from the collision between neighboring particles that are moving with different velocities. The rate of deformation of different fluids under the same shear stress are different, so different fluids have different viscosities. No fluid with zero viscosity exist i.e. ideal fluid and thus there is a viscous effects of some degree in all fluids.

2.2.1 Types of Viscosity

Dynamic viscosity: The shear stress to strain rate ratio of fluid is known as dynamics viscosity. Mathematically, $\mu = \frac{\tau}{\gamma}$, where γ is rate of deformation. In SI system its unit is Kg/ms.

Kinematic viscosity: The dynamic viscosity μ and fluid density ρ ratio is known as kinematic viscosity. It is also called momentum diffusivity and denoted by ν. Mathematically, $\nu = \frac{\mu}{\rho}$. In SI system its unit is m^2/s.

2.3 Classification of Fluids

Fluid with zero viscosity is ideal fluid otherwise called real fluid. On the basis of the relation between viscous stress and deformation rate fluids are classified into two i.e. Newtonian and non-Newtonian fluids. The fluids in which viscous stress are in linear relationship to the deformation rate is known as Newtonian fluids. Mathematically, it can be write as

$$\tau = \mu \frac{du}{dy} \tag{2.1}$$

where μ is coefficient of viscosity called dynamic viscosity and eq (2.1) is known as Newton's law of viscosity. In non-Newtonian fluid there is a non linear relation among viscous stress and deformation rate. In mathematical way for uni-directional flow it can be express as

$$\tau_{xy} = \eta \left(\frac{du}{dy}\right). \tag{2.2}$$

where $\eta = N\left(\frac{du}{dy}\right)^{n-1}$, $n \neq 1$, is apparent viscosity, n is the flow behaviour index (measure of how the fluid deviates from the Newtonian fluid) and N is the consistancy index (measure fluid consistency) Newtonian for $n = 1$ and Non-Newtonian for $n > 1$.

2.4 Steady and Unsteady Flows

If all the properties of fluid flow is independent of time such a flow is said to be steady. Mathematically, $\frac{\partial\psi}{\partial t} = 0$, where ψ represents any fluid properties. If $\frac{\partial\psi}{\partial t} \neq 0$ the flow is said to be unsteady.

2.5 Laminar and Turbulent Flows

The motion of fluid in which each fluid particle move in a regular path and does not change its path That is parallel to each other is termed as laminar flow. Otherwise the flow is said to be turbulent flow. In turbulent flow the fluid particles cannot maintain their regular path and direction. The high viscous fluid like oil when flow with low velocity is laminar flow and the fluid with low viscosity like air at high velocity is turbulent.

2.6 Velocity Boundary Layer

It is define as a flow region where the effects of friction forces due to the viscosity of the fluid is maximum. In this region the fluid velocity varies from zero to $0.99u_\infty$, where u_∞ is free stream velocity in which viscous forces are relatively negligible.

2.7 Thermal Boundary Layer

The thermal boundary layer is develop due to the temperature difference between the surface and the fluid flow over it. We define it as the region of the flow above a surface where temperature varies in normal direction of the surface.

The thermal boundary layer thickness is the distance at which temperature difference $T - T_s$ equals $0.99(T_\infty - T_s)$ and as special case if $T_s = 0$ we have $T = 0.99T_\infty$ from the surface. Where T is temperature of fluid, T_s is temperature of the surface and T_∞ is fluid temperature at sufficiently large distance from the surface. The heat transfer effects are felt at a large distances from the surface as we go down in stream flow direction. This is because of the boundary layer thickness increase in flow direction.

2.8 Compressible and Incompressible Flow

A fluid is said to be incompressible in which density variation does not occur, otherwise it is compressible. Mathematically, for incompressible fluid

$$\frac{d\rho}{dt} = 0, \tag{2.3}$$

and for compressible fluid

$$\frac{d\rho}{dt} \neq 0, \tag{2.4}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V \cdot \nabla, \tag{2.5}$$

called material time derivative and ρ is density of the fluid. In above equation $\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$ called differential operator in cartesian form.

2.9 Viscous and Inviscid Fluids

Fluid which have significant viscosity are viscous fluid and an inviscid fluid is an ideal fluid which has no viscosity. In general there are no fluids which have zero viscosity. However, in many fluid flows of practical interest, the viscous forces relatively small as compared to inertial or pressure forces are considered as inviscid fluids.

2.10 Mathematical Equations

The generalized mathematical models that governs the flow are namely continuity equation and momentum equation. These models based on two basic laws: conservation of mass and conservation of momentum.

2.10.1 Generalized Continuity Equation

The law of conservation of mass provides a base for the equation of continuity. This law states that the total mass entering and leaving the control volume must be equal.
$\dot{m}_{in} = \dot{m}_{out}$, where $\dot{m}$ stands for mass flow rate. The generalized continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho V = 0, \tag{2.6}$$

where ρ denotes density of fluid and V denotes fluid velocity and ∇ is the differential operator. Eq (2.6) is called differential form of equation of continuity. For incompressible fluid it becomes

$$\nabla \cdot V = 0. \tag{2.7}$$

2.10.2 Generalized Momentum Equation

The law of conservation of momentum provides base for momentum equation commonly known as Navier-Stokes equation. In vector form

$$\rho \frac{dV}{dt} = \nabla \cdot \tau + \rho F, \tag{2.8}$$

where $\frac{d}{dt}$ is differential operator given by eq (2.5), $\tau = -pI + \mu A$ is the Cauchy stress tensor, p denote pressure, I is the unit matrix of order 3, F is the sum of body forces per unit mass, A is the Rivlin Erickson tensor given by $A = \nabla V + \nabla V^T$. So in matrix form τ is given by

$$\begin{pmatrix} \tau_{xx} & \tau_{yx} & \tau_{zx} \\ \tau_{xy} & \tau_{yy} & \tau_{zy} \\ \tau_{xz} & \tau_{yz} & \tau_{zz} \end{pmatrix} = \begin{pmatrix} -p + 2\mu\frac{\partial u}{\partial x} & \mu(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}) & \mu(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}) \\ \mu(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}) & -p + 2\mu\frac{\partial v}{\partial y} & \mu(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}) \\ \mu(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}) & \mu(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}) & -p + 2\mu\frac{\partial w}{\partial z} \end{pmatrix},$$

which is for Newtonian incompressible viscous fluids.

$$= \begin{pmatrix} -p+2\mu\frac{\partial u}{\partial x}+\lambda(\nabla\cdot V) & \mu(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}) & \mu(\frac{\partial u}{\partial z}+\frac{\partial w}{\partial x}) \\ \mu(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}) & -p+2\mu\frac{\partial v}{\partial y}+\lambda(\nabla\cdot V) & \mu(\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}) \\ \mu(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}) & \mu(\frac{\partial w}{\partial y}+\frac{\partial v}{\partial z}) & -p+2\mu\frac{\partial w}{\partial z}+\lambda(\nabla\cdot V) \end{pmatrix}.$$

Here λ is bulk viscosity and $\lambda = -\frac{2}{3}\mu$, for Newtonian compressible viscous fluids. In vector form Navier stoke equation for viscous compressible Newtonian fluid

$$\rho(\frac{\partial V}{\partial t} + V \cdot \nabla V) = -\nabla p + \mu\nabla^2 V + \nabla(-\frac{2}{3}\mu\nabla \cdot V) + \rho F. \tag{2.9}$$

For incompressible viscous Newtonian fluid, we have

$$\rho(\frac{\partial V}{\partial t} + V \cdot \nabla V) = -\nabla p + \mu\nabla^2 V + \rho F. \tag{2.10}$$

In component form Navier Stoke equation for incompressible flow are

x-direction

$$\rho\frac{du}{dt} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + \rho F_x, \tag{2.11}$$

y-direction

$$\rho\frac{dv}{dt} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z} + \rho F_y, \tag{2.12}$$

z-direction

$$\rho\frac{dw}{dt} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \rho F_z. \tag{2.13}$$

2.11 Mechanism of Heat Transfer

Heat: The form of energy, which can be transfered from one system to another system as a result of temperature difference is known as heat. A thermodynamic analysis deals with heat transfer amount when a system goes form one equilibrium state to another during a processes. Heat transfer has three different forms, conduction, convection and radiation.

2.11.1 Conduction

The energy transfered between adjacent more energetic and less energetic particles of a substance as a result of interaction between them is known as conduction. The heat conduction rate through a surface is proportional to the area of the surface A and difference of temperature ΔT across the surface and inversely proportional to the surface thickness Δx.

$$\dot{Q}_{cond} = -KA\frac{\Delta T}{\Delta x}, \tag{2.14}$$

where K is the proportionality constant called thermal conductivity. It measure the heat conduction ability of the material. In case when the thickness of surface is very small (i.e $\Delta x \to 0$), eq (2.14) becomes

$$\dot{Q}_{cond} = -KA\frac{dT}{dx}. \tag{2.15}$$

called **Fourier's law of heat conduction.** Where the negative sign indicates that heat transfer occurs in decreasing temperature direction.

Thermal diffusivity: The property of a substance that appears in heat conduction analysis which represents that how heat diffuses fast through a substance. Mathematically,

$$\alpha = \frac{K}{\rho C_p}. \tag{2.16}$$

where K is thermal conductivity and the quantity, ρC_p is heat capacity and C_p is specific heat capacity of a material.

2.11.2 Convection

The energy transfer between a solid surface and the fluid adjacent to it which are in motion is known as convection.

Types of convection

Force convection A transport phenomena in which an external source (like fan, pump, suction device etc) is responsible for the generation of fluid motion or a heat transfer mechanism through fluid in the presence of bulk fluid motion.

Natural convection: A transport phenomena in which the buoyancy force cause the fluid motion. The buoyancy force generates due to the density differences which is caused by the temperature variation. It is also called free convection. The heat transfer rate in convection is proportional to the difference of temperature in fluid. Expressed by **Newton's law of cooling**. Mathematically,

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty). \tag{2.17}$$

where h is convection heat transfer coefficient its unit is $W/m^2 \cdot \mathring{C}$, A_s is the area of surface through which convection heat transfer take place, T_s is temperature of the surface and T_∞ is temperature of far away from the surface. Note that the coefficient of convective heat transfer (h) is not fluid property. It is a parameter that is find experimentally. Its value effect on all the variable that influencing convection like geometry of the surface, the fluid motion nature, bulk fluid velocity and fluid properties.

2.11.3 Radiation

The change of electronic of atom and molecules of a matter results emition of energy in form of electromagnetic waves or photon is called radiation. The energy transfer by radiation does not required medium like convection and conduction. In study of transfer of heat energy our interest is in thermal energy which is a form of radiation due to the temperature of the body. It is different from the other forms of radiation like X-rays, gamma rays, radio waves, microwaves that are not related to the temperature.

2.12 Energy Equation

First law of thermodynamics provides a base for the energy equation. This law states that during a processes the change (decrease or increase) in total energy of a system is equal to the energy difference between energy entering and leaving the system, that is

$E_{in} - E_{out} = \Delta E_{system}$.

The generalized energy equation is

$$\rho C_p \left(\frac{\partial T}{\partial t} + (V \cdot \nabla)T \right) = \nabla \cdot (\alpha \nabla T) + H. \tag{2.18}$$

where T is temperature, α is thermal diffusivity, C_p is specific heat capacity and H is heat source term.

2.13 Diffusion Equation

The general diffusion equation is

$$\frac{\partial P}{\partial t} + \nabla \cdot VP = \nabla \cdot (D \nabla P) + H_1. \tag{2.19}$$

where P is a variable represents species concentration for mass transfer or temperature for heat transfer. D is diffusivity (also called diffusion coefficient). V is velocity field. H_1 describe sources or sink of the quantity P.

2.14 Porous Medium

The word Porous is refers to the material that contains pores or holes in its surface. Porous medium is a substance that contains holes in its surface through which fluid can passes. examples include sponge, pumics stone, cork etc. The properties of porous medium through we can distinguish between porous mediums are porosity and permeability. Porosity measures the size of pores in the porous medium and permeability measures the ability of the porous medium to transmit fluids through them.

2.15 Physical Parameters

Grashof number A dimensionless number, define as the ratio between buoyancy force and viscous force acting on the fluid. For heat transfer and mass transfer it is denoted by

G_r and G_c respectively. In case of heat over transfer over a plate

$$G_r = g\beta\frac{(T_s - T_\infty)L^3}{\nu^2},$$

and mass transfer transfer over a plate

$$G_c = g\beta^*\frac{(C_s - C_\infty)L^3}{\nu^2},$$

where, g is gravitational acceleration (m/s^2), β is volumetric coefficient of expansion for heat transfer ($\beta = 1/T$ for ideal gases), β^* is coefficient of volumetric expansion for mass transfer, T_s is temperature of the surface, T_∞ is temperature of the fluid sufficiently far from the surface, L is characteristic length of the geometry (m), ν is kinematic viscosity of the fluid (m^2/s), C_s is concentration at surface and C_∞ is concentration sufficiently far away from the surface. Grashof number provides criteria for determining whether the flow is laminar or turbulent. The critical Grashof number obsreved to be 10^9. The flow is turbulent if Grashof number is greater than 10^9.

Prandtl number The dimensionless parameter, defined by the ratio of momentum diffusivity to thermal diffusivity. Mathematically,

$$\begin{aligned} P_r &= \frac{\nu}{\alpha}, \\ or \quad P_r &= \frac{\mu C_p}{K}, \end{aligned}$$

where ν is kinematic viscosity, α is thermal diffusivity, C_p specific heat capacity, μ is dynamic viscosity, K is thermal conductivity. Prandlt number describe the thickness of velocity and thermal boundary layers.

Schmidt number A dimensionless number denoted by S_c and defined by the ratio of momentum diffusivity(kimenatic viscosity) and mass diffusivity is called Schmidt number. Mathematically,

$$\begin{aligned} S_c &= \frac{\nu}{D}, \\ or \quad S_c &= \frac{\mu}{\rho D}, \end{aligned}$$

where ν kinematic viscosity, μ is dynamic viscosity, D is molecular diffusivity(m^2/s) Schmidt number describe the fluid flows in which simultaneous momentum and mass diffusion processes occurs.

Hartmann number It is the ratio of electromagnetic force to viscous force, denoted by Ha. Mathematically,

$$Ha = BL\sqrt{\frac{\sigma}{\mu}},$$

where B is magnetic field strength, L is the characteristic length scale, σ is electrical conductivity. μ is kimematic viscosity.

Magnetic Reynolds number The ratio of the induction of magnetic field by the motion of conducting fluid to the magnetic diffusion, denoted by R_m. Mathematically,

$$R_m = \frac{UL}{\theta},$$

where, U is the flow velocity, L is the characteristic length of the geometry and θ is magnetic diffusivity.

The dimensionless physical parameters discussed in the above section help us in converting the modeling equations of the flow to the dimensionless form.

Chapter 3

Natural Convection Hydromagnetic Fluid Flow over an Oscillating Porous Plate

3.1 Introduction

Natural convection and hydromagnetic flows attract the interest of many researchers because of its diverse implementations in many areas of science as we discuss in Chapter 1. Our interest is to observe the combine effects of acceleration and oscillation on natural convection fluid flows over porous plate. In 2014 S. S. Das *et al.* [18], considered the free convection flow and mass transfer of fluid which is incompressible, viscous and electrical conducting. The fluid flows over a moving porous plate in porous medium with heat source, keeping motion of the plate is oscillatory.

3.2 Formulation of Problem

We consider the two dimensional unsteady free convective flow of incompressible, viscous and electrical conducting fluid a porous plate embedded in a porous medium. The motion of the plate is oscillatory. A transverse magnetic field of strength B_0 is applied in the transverse direction of the flow. In the presence of external applied magnetic field, the magnetic field which is induced due to the flow is very small so we can neglect it because the magnetic Reynolds number is assumed to be much smaller than 1. All the fluid properties are assumed to be constant except the variation of density with variation of temperature and the concentration of species in fluid.

The components of velocity in x and y directions are u and v respectively. All physical variables are functions of y and t. So the velocity flow field is $V = [u(y,t), v(y,t), 0]$. Let v_0 denotes the injection/suction velocity at the plate. The continuity equation for incompressible fluid flow is given as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{3.1}$$

which implies that

$$\frac{\partial v}{\partial y} = 0. \tag{3.2}$$

From eq (3.2) we have, $v(y,t) = -v_0$, under the condition $y = 0$, $v = -v_0$ Thus the velocity flow field becomes

$$V = [u(y,t), -v_0, 0].$$

The momentum, energy and diffusion equations given by eq (2.8), eq (2.18) and eq (2.19) for the present model under boundary layer approximation becomes

$$\frac{\partial u}{\partial t} - v_0\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\rho\beta(T - T_\infty) + g\rho\beta^*(C - C_\infty) - \frac{\nu}{K_0}u - \frac{\sigma B_0^2 u}{\rho}, \tag{3.3}$$

$$\frac{\partial T}{\partial t} - v_0\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + S(T - T_\infty), \tag{3.4}$$

$$\frac{\partial C}{\partial t} - v_0\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2}. \tag{3.5}$$

In eq (3.3), $g\beta(T - T_\infty)$ and $g\beta^*(C - C_\infty)$ are the thermal and concentration buoyancy forces respectively, g is gravitational acceleration, β is volumetric coefficient of thermal expansion and β^* is volumetric coefficient of expansion for mass transfer, ν is kinematic viscosity, K is porous medium permeability, $-\sigma u B_0^2$ is magnetic force due to the applied magnetic field, σ is fluid electrical conductivity. In eq (3.4), T is temperature of the fluid, T_∞ is temperature of the fluid far away from the plate, α is thermal diffusivity and S is heat source parameter. In eq (3.5), C is concentration, C_∞ is concentration of fluid far away from the plate and D is molecular diffusivity.

When the plate start oscillation first order velocity slip boundary conditions are

$$\begin{aligned} u &= U_0 e^{i\omega t} + L'\frac{\partial u}{\partial y}, \quad T = T_w, \quad C = C_w \quad at \quad y = 0, \\ u &\to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad as \quad y \to \infty. \end{aligned} \tag{3.6}$$

Here $L' = \frac{2-m_r}{m_r}L$ and $L = \mu\left(\frac{\pi}{2\rho p}\right)^{1/2}$ is mean free path and m_r is Maxwell's reflection coefficient.

To solve eq (3.3) to eq (3.6). We reduce the equations and boundary condition into dimensionless form by using dimensionless parameters

$$\begin{aligned} y^* &= \frac{U_0}{\nu}y,\ u^* = \frac{u}{U_0},\ T^* = \frac{T - T_\infty}{T_w - T_\infty},\ C^* = \frac{C - C_\infty}{C_w - C_\infty},\ t^* = \frac{U_0^2}{\nu}t,\ \upsilon_0^* = \frac{\upsilon_0}{U_0}, \\ \omega^* &= \frac{\nu}{U_0^2}\omega,\ S^* = \frac{\nu}{U_0^2}S. \end{aligned}$$

The equations in dimensionless form are

$$\frac{\partial u^*}{\partial t^*} - \upsilon_0^*\frac{\partial u^*}{\partial y^*} = \frac{\partial^2 u^*}{\partial y^{*2}} + G_r T^* + G_c C^* - (\frac{1}{K_p} + M^2)u^*, \tag{3.7}$$

$$\frac{\partial T^*}{\partial t^*} - \upsilon_0^*\frac{\partial T^*}{\partial y^*} = \frac{1}{P_r}\frac{\partial^2 T^*}{\partial y^{*2}} + S^* T^*, \tag{3.8}$$

$$\frac{\partial C^*}{\partial t^*} - \upsilon_0^*\frac{\partial C^*}{\partial y^*} = \frac{1}{S_c}\frac{\partial^2 C^*}{\partial y^{*2}}. \tag{3.9}$$

and dimensionless boundary conditions are

$$\begin{aligned} u^* &= e^{\iota\omega^* t^*} + R\frac{\partial u^*}{\partial y^*}, \quad T^* = 1, \quad C^* = 1 \quad at \quad y^* = 0, \\ u^* &\to 0, \quad T^* \to 0, \quad C^* \to 0 \quad as \quad y^* \to \infty. \end{aligned} \tag{3.10}$$

where

$$\begin{aligned}
G_r &= \nu\beta g\frac{T_w - T_\infty}{U_0^3}\ (Grashofnumber\ for\ heat\ transfer),\\
G_c &= \nu\beta^* g\frac{C_w - C_\infty}{U_0^3}\ (Grashof\ number\ for\ mass\ transfer),\\
K_p &= \frac{K_0}{\nu^2}U_o^2\ (Permeability\ parameter),\\
M &= \frac{B_0}{U_0}\left(\frac{\nu\sigma}{\rho}\right)^{1/2}(Hartmann\ number\quad or\quad magnetic\ parameter),\\
P_r &= \frac{\nu}{\alpha}\ (Prandtl\ number),\\
S_c &= \frac{\nu}{D}\ (Schmidt\ number),\\
R &= \frac{U_0}{\nu}L_1(Rarefaction\ parameter).
\end{aligned}$$

For convenience we will drop $\star$ sign that is used to indicate dimensionless quantities. Thus the dimensionless equations becomes

$$\frac{\partial u}{\partial t} - \upsilon_0\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r T + G_c C - (\frac{1}{K_p} + M^2)u, \tag{3.11}$$

$$\frac{\partial T}{\partial t} - \upsilon_0\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + ST, \tag{3.12}$$

$$\frac{\partial C}{\partial t} - \upsilon_0\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y} \tag{3.13}$$

and dimensionless boundary conditions are

$$\begin{aligned}
u &= e^{i\omega t} + R\frac{\partial u}{\partial y},\quad T = 1,\quad C = 1\quad at\quad y = 0,\\
u &\to 0,\quad T \to 0,\quad C \to 0\quad as\quad y \to \infty.
\end{aligned} \tag{3.14}$$

3.3 Solution Method

In this section, we solve equations (3.11)-(3.13) are solve for velocity, temperature and concentration by using the following transformations.

$$u(y,t) = u_0(y) + u_1(y)e^{i\omega t}, \tag{3.15}$$

$$T(y,t) = T_0(y) + T_1(y)e^{i\omega t}, \tag{3.16}$$

$$C(y,t) = C_0(y) + C_1(y)e^{i\omega t}, \tag{3.17}$$

which transform eq's (3.11) to (3.13) to the following ODE's

$$u_0'' + \upsilon_0 u_0' - \left(\frac{1}{K_p} + M^2\right) u_0 = -G_c T_0 - G_c C_0, \tag{3.18}$$

$$u_1'' + \upsilon_0 u_1' - \left(\frac{1}{K_p} + M^2 + i\omega\right) u_1 = -G_c T_1 - G_c C_1, \tag{3.19}$$

$$T_0'' + \upsilon_0 P_r T_0' + S P_r T_0 = 0, \tag{3.20}$$

$$T_1'' + \upsilon_0 P_r T_1' + P_r(S - i\omega)T_1 = 0, \tag{3.21}$$

$$C_0'' + S_c \upsilon_0 C_0' = 0, \tag{3.22}$$

$$C_1'' + \upsilon_0 S_c C_1' - i\omega S_c C_1 = 0. \tag{3.23}$$

Boundary conditions are transformed into

$$u_0 = R\frac{du_0}{dy}, \quad u_1 = 1 + R\frac{du_1}{dy}, \quad T_0 = 1, \quad T_1 = 0, \quad C_0 = 1, \quad C_1 = 0 \quad at \quad y = 0,$$
$$u_0 \to 0, \quad u_1 \to 0, \quad T_0 \to 0, \quad T_1 \to 0, \quad C_0 \to 0, \quad C_1 \to 0 \quad as \quad y \to \infty. \tag{3.24}$$

The eq's (3.20)-(3.23) are homogeneous ODE's solving with the boundary condition given by (3.24) we have

$$C_0(y) = e^{-\upsilon_0 S_c y}, \tag{3.25}$$

$$C_1(y) = 0, \tag{3.26}$$

$$T_0(y) = e^{\lambda_1 y}; \ \lambda_1 = \frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right], \tag{3.27}$$

$$T_1(y) = 0. \tag{3.28}$$

Eq (3.18) is a non homogeneous equation. Auxiliary equation for homogeneous part is

$$D^2 + \upsilon_0 D - \left(\frac{1}{K_p} + M^2\right) = 0, \tag{3.29}$$

having roots

$$D = \tfrac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\tfrac{1}{K_p} + M^2\right)}\;\right],\ D = \tfrac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\tfrac{1}{K_p} + M^2\right)}\;\right].$$

so the complementary solution is

$$u_{0(c)} = A_0 e^{\frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\right]y} + A_1 e^{\frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\right]y},$$

or

$$u_{0(c)} = A_0 e^{\lambda_3 y} + A_1 e^{-\lambda_2 y}.$$

where

$$\lambda_2 = \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\;\right], \quad \lambda_3 = \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\;\right].$$

As we know that from eq (3.18)

$$u_0'' + \upsilon_0 u_0' - (\frac{1}{K_p} + M^2)u_0 = -G_c T_0 - G_c C_0, \tag{3.30}$$

or

$$u_0'' + \upsilon_0 u_0' - (\frac{1}{K_p} + M^2)u_0 = -G_c e^{\lambda_1 y} - G_c e^{-\upsilon_0 S_c y}, \tag{3.31}$$

by using the values of C_0 and T_0 from eq (3.25) and eq (3.27) respectively. By applying method of variation of parameter we get the particular solution of the form

$$u_{0(p)} = -\frac{G_r}{\lambda_1^2 + \upsilon_0 \lambda_1^2 - (\frac{1}{K_p} + M^2)} e^{\lambda_1 y} - \frac{G_c}{\upsilon_0^2 S_c^2 - \upsilon_0^2 S_c - (\frac{1}{K_p} + M^2)} e^{-\upsilon_0 S_c y},$$

Therefore

$$\begin{aligned} u_0(y) &= A_0 e^{\frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\right]} + A_1 e^{\frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 - 4\left(\frac{1}{K_p} + M^2\right)}\right]} \\ &- \frac{G_r}{\lambda_1^2 + \upsilon_0 \lambda_1^2 - \left(\frac{1}{K_p} + M^2\right)} - \frac{G_c}{\upsilon_0^2 S_c^2 - \upsilon_0^2 S_c - \left(\frac{1}{K_p} + M^2\right)}, \end{aligned}$$

or

$$u_0(y) = A_0 e^{\lambda_3 y} + A_1 e^{-\lambda_2 y} - A_2 e^{\lambda_1 y} - A_3 e^{-\upsilon_0 S_c y}. \tag{3.32}$$

Here

$$\lambda_2 = \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\ \right], \tag{3.33}$$

$$\lambda_3 = \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\ \right], \tag{3.34}$$

$$A_2 = \frac{G_r}{\lambda_1^2 + \upsilon_0 \lambda_1^2 - \left(\frac{1}{K_p} + M^2\right)}, \tag{3.35}$$

$$A_3 = \frac{G_c}{\upsilon_0^2 S_c^2 - \upsilon_0^2 S_c - \left(\frac{1}{K_p} + M^2\right)}. \tag{3.36}$$

Using boundary condition $u_0 \to 0$ as $y \to \infty$ in eq (3.32) we put $A_0 = 0$ in order to satisfy boundness condition and by using the boundary condition $u_0 = R\frac{du_0}{dy}$ at $y = 0$, in eq (3.32) we get

$$A_1 = R\frac{du_0}{dy} + A_2 + A_3. \tag{3.37}$$

Adding eq (3.33) and eq (3.34) we have

$$\lambda_1 + \lambda_2 = \lambda_1 + \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\right], \tag{3.38}$$

Subtracting eq (3.33) and eq (3.34) we have

$$\lambda_1 - \lambda_3 = \lambda_1 - \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\right], \tag{3.39}$$

Multiplication of eq (3.38) and eq (3.39), gives

$$(\lambda_1 + \lambda_2)(\lambda_1 - \lambda_3) = \lambda_1^2 + \lambda_1 \upsilon_0 - 4\left(\frac{1}{K_p} + M^2\right). \tag{3.40}$$

Subtract $S_c \upsilon_0$ from eq (3.33) and add to eq (3.34) we have

$$-S_c \upsilon_0 + \lambda_2 = -S_c \upsilon_0 + \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\ \right], \tag{3.41}$$

and

$$
\begin{aligned}
S_c\upsilon_0 + \lambda_3 &= S_c\upsilon_0 + \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\,\right], \\
-(S_c\upsilon_0 + \lambda_3) &= -S_c\upsilon_0 + \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\,\right]. \qquad (3.42)
\end{aligned}
$$

Multiplication of eq (3.41) and eq (3.42), gives

$$
(S_c\upsilon_0 - \lambda_2)(S_c\upsilon_0 + \lambda_3) = S_c^2\upsilon_0^2 - S_c\upsilon_0^2 - 4\left(\frac{1}{K_p} + M^2\right). \qquad (3.43)
$$

Using eq (3.40) in eq (3.35) and (3.43) in eq (3.36). Therefore

$$
A_2 = \frac{G_r}{(\lambda_1 + \lambda_2)(\lambda_1 - \lambda_3)} \text{and} A_3 = \frac{G_c}{(S_c\upsilon_0 - \lambda_2)(S_c\upsilon_0 + \lambda_3)}.
$$

by differentiating eq (3.32) and putting $y = 0$ lead us to

$$
\frac{du_0}{dy} = -A_1\lambda_2 - A_2\lambda_1 + A_3\upsilon_0 S_c. \qquad (3.44)
$$

Substitute eq (3.44) in eq (3.37) Therefore

$$
A_1 = R(-\lambda_2 A_1 - \lambda_1 A_2 + \upsilon_0 S_c A_3) + A_2 + A_3.
$$

This implies that

$$
A_1 = \frac{1}{1 + \lambda_2 R}[(1 - \lambda_1 R)A_2 + (1 + \upsilon_0 S_c R)A_3]. \qquad (3.45)
$$

Finally we have

$$
u_0(y) = A_1 e^{-\lambda_2 y} - A_2 e^{\lambda_1 y} - A_3 e^{-\upsilon_0 S_c y}. \qquad (3.46)
$$

where

$$
\begin{aligned}
\lambda_1 &= \frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 + 4P_r S}\right],\ \lambda_2 = \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\,\right], \\
A_1 &= \frac{1}{1 + \lambda_2 R}[(1 - \lambda_1 R)A_2 + (1 + \upsilon_0 S_c R)A_3], \\
A_2 &= \frac{G_r}{(\lambda_1 + \lambda_2)(\lambda_1 - \lambda_3)},\ A_3 = \frac{G_c}{(S_c\upsilon_0 - \lambda_2)(S_c\upsilon_0 + \lambda_3)}.
\end{aligned}
$$

As $T_1(y) = 0$ and $C_1(y) = 0$ then eq (3.19) becomes

$$u_1'' + \upsilon_0 u_1' - \left(\frac{1}{K_p} + M^2 + i\omega\right) u_1 = 0, \tag{3.47}$$

$$D^2 + \upsilon_0 D - \left(\frac{1}{K_p} + M^2 + i\omega\right) = 0. \tag{3.48}$$

$$\Longrightarrow D = \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 + i\omega\right)}\ \right],$$

and

$$D = \frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 + i\omega\right)}\ \right].$$

Thus the general solution is

$$u_1(y) = A_5 e^{\frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 + i\omega\right)}\ \right]y} + A_4 e^{\frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 + i\omega\right)}\ \right]y}, \tag{3.49}$$

using boundary conditions that is $u_1 \to 0$ as $y \to \infty$ in eq (3.49) we put $A_5 = 0$ in order to satisfy boundary condition. Also by using the boundary condition $u_1 = 1 + R\frac{\partial u_1}{\partial y}$ at $y = 0$ in eq (3.49) we get

$$A_4 = 1 + R\frac{du_1}{dy}, \tag{3.50}$$

Therefore

$$u_1(y) = A_4 e^{\frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 + i\omega\right)}\ \right]y},$$

or

$$u_1(y) = A_4 e^{\lambda_4 y}, \tag{3.51}$$

where

$$\lambda_4 = \frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 + i\omega\right)}\ \right].$$

Differentiation of eq (3.51) and putting $y = 0$ leads us to

$$\frac{du_1}{dy} = A_4\lambda_4. \tag{3.52}$$

Now substitute in eq (3.52) in eq (3.50) we have

$$A_4 = 1 + R(A_{14}\lambda_4) \implies A_4 = \frac{1}{1 - R\lambda_4}. \tag{3.53}$$

Now substitute eq (3.46) and (3.51) in (3.15) we have

$$u(y,t) = A_1 e^{-\lambda_2 y} - A_2 e^{\lambda_1 y} - A_3 e^{-\upsilon_0 S_c y} + A_4 e^{\lambda_4 y + (i\omega)t}. \tag{3.54}$$

Now substitute eq $T_0(y) = e^{\lambda_1 y}$ and $T_1(y) = 0$ in eq (3.16) we have

$$T(y) = e^{\lambda_1 y}. \tag{3.55}$$

substitute $C_0(y) = e^{-\upsilon_0 S_c y}$ and $C_1(y) = 0$ in eq (3.17)

$$C(y) = e^{-\upsilon_0 S_c y}. \tag{3.56}$$

3.4 Graphical observation

In this section, we discuss graphically the effects of various physical parameter on the result we obtain in previous section.

3.4.1 Effects on Velocity

The velocity profile will effect by some physical parameter which are discuss below one by one.

Effect of Magnetic M and Heat Source Parameter S

In Figure 3.1, for different values of magnetic parameter different curves are plotted. It shows that the increase of magnetic parameter decrease the magnitude of velocity. It is due to the effect of Lorentz force. This effect is caused by the application of transverse magnetic field to the fluid flow. Figure 3.2, shows that the magnitude of velocity increase with increase in heat source parameter.

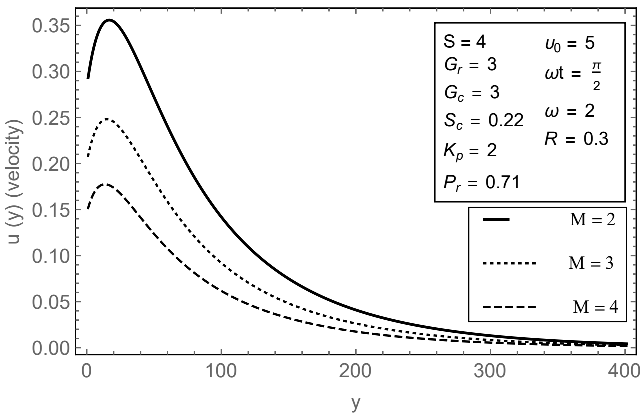

Figure 3.1: Velocity profile against y for different values of M.

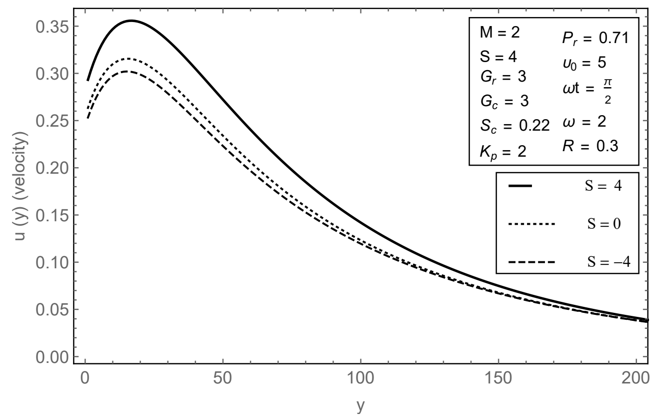

Figure 3.2: Velocity profile against y for different values of S.

Effect of Grashof number G_c and G_r for Mass and Heat Transfer, Permeability Parameter K_p and prandtl Number P_r

Figure 3.3 and 3.4, illustrate the effects of Grashof number G_c and G_r for mass and heat transfer. Considering the case of cooling of plate i.e. $G_c > 0$, $G_r > 0$. By varying G_c and G_r, mass and thermal buoyancy effect is increase, which causes increase in induced flow. Thus velocity increase corresponding to increase in both G_c and G_r. Figure 3.5 and 3.6, shows the effect permeability parameter K_p and Prandtl number P_r respectively. Increase permeability parameter, increase the magnitude of velocity while the increment in Prandtl number decrease the velocity, this fact is because of fluids with high Prandtl number having high viscosity and hence move slowly.

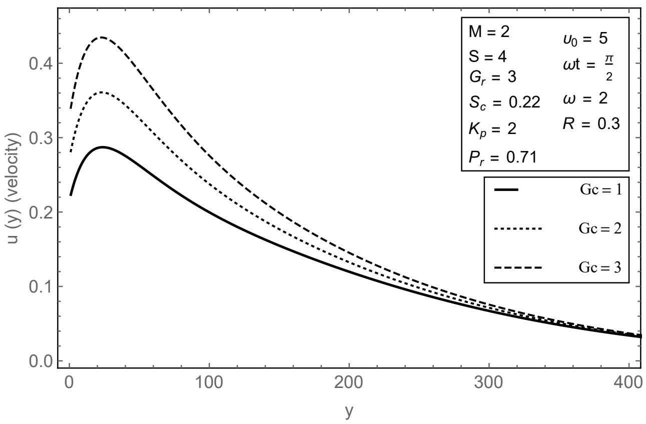

Figure 3.3: Velocity profile against y for different values of G_c.

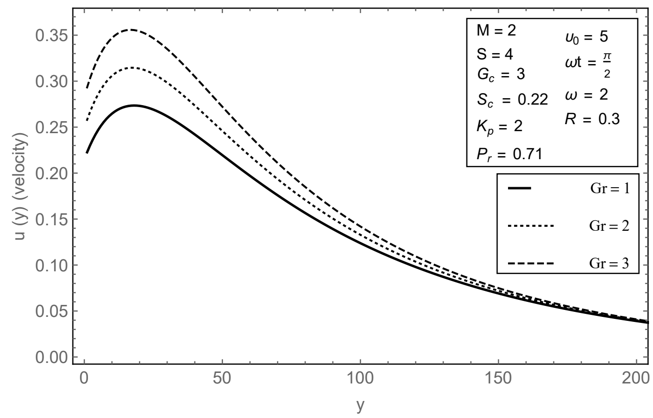

Figure 3.4: Velocity profile against y for different values of G_r.

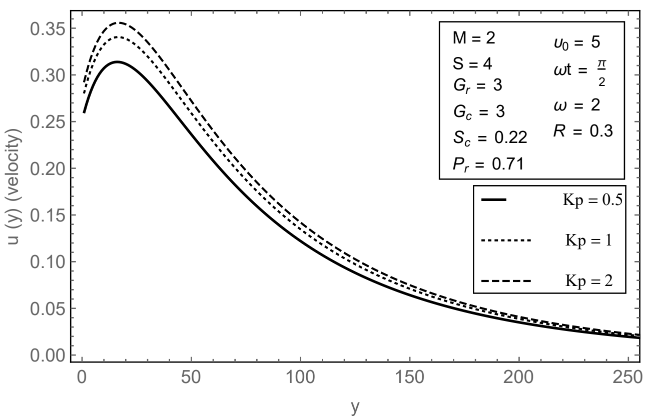

Figure 3.5: Velocity profile against y for different values of K_p.

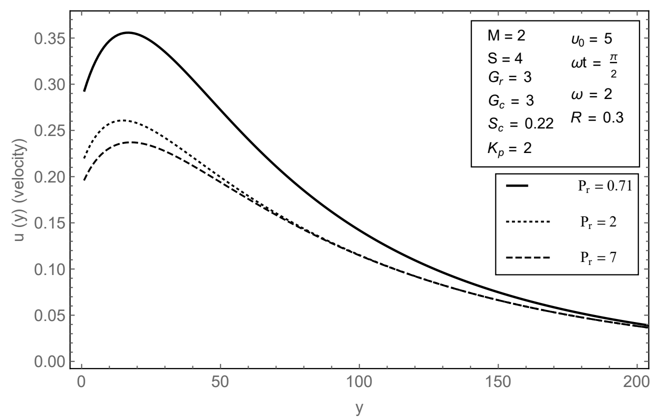

Figure 3.6: Velocity profile against y for different values of P_r.

Effect of Schmidth number S_c and Suction parameter υ_0.

Figure 3.7, Illustrate the effect of Schmidth number S_c on the velocity field. The observation of the curves shows that the growing Schmidth number result in slow down the velocity of the fluid. It is due to the presence of heavier diffusive species in the fluid. About the effect of suction parameter on the velocity is discuss in Figure 3.8, velocity decrease due to increase in suction parameter.

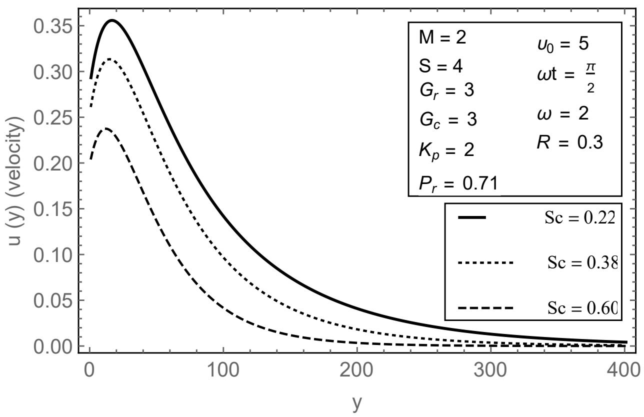

Figure 3.7: Velocity profile against y for different values of S_c.

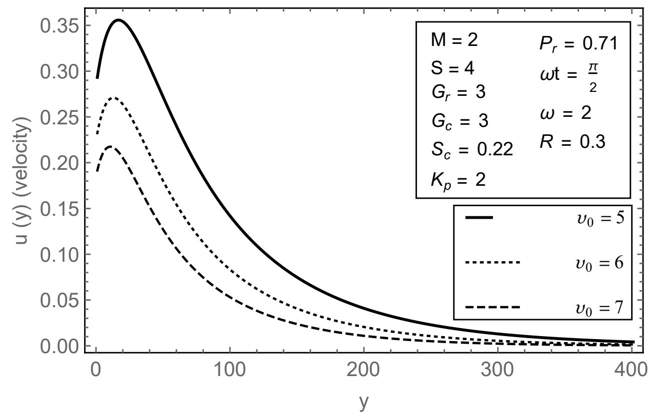

Figure 3.8: Velocity profile against y for different values of υ_0.

3.4.2 Effects on Temperature

Temperature distribution will effect only by three parameter namely Prandtl number P_r, heat source S and suction parameter υ_0. Heat source having an increasing effect on temperature, which is given by Figure 3.10. Both the Prandtl number and suction velocity will have a decreasing effect on temperature given by Figure 3.9 and 3.11 respectively. Thus greater will be P_r and υ_0, more will be cooling effect.

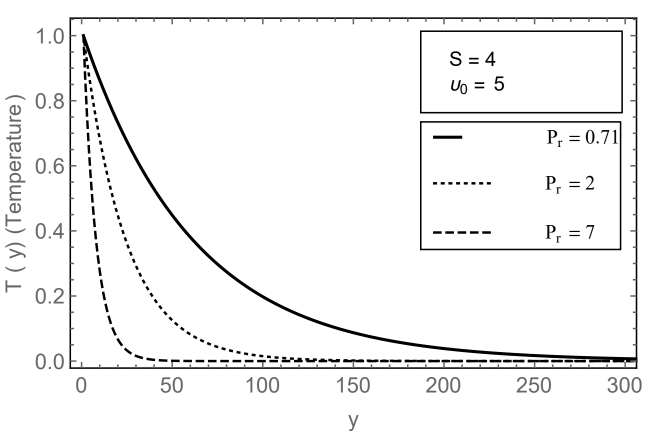

Figure 3.9: Temperature profile against y for different values of P_r.

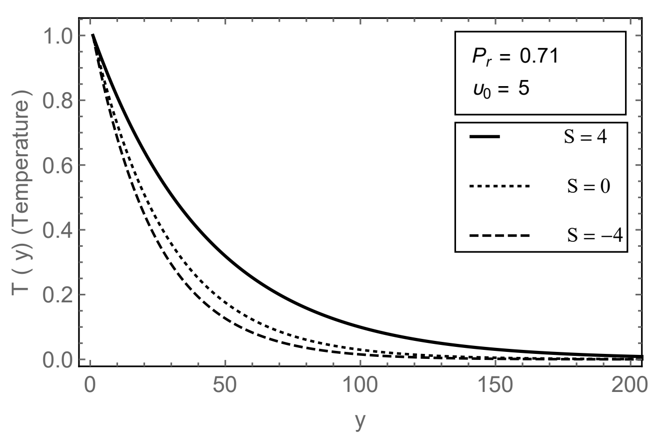

Figure 3.10: Velocity profile against y for different values of S.

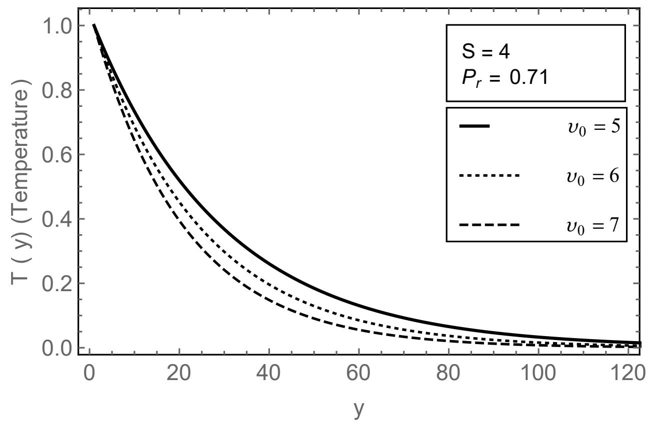

Figure 3.11: Velocity profile against y for different values of υ_0.

3.4.3 Effects on Concentration

Concentration distribution is effected only by Schmidth number S_c and Suction parameter υ_0. Figure 3.12, illustrate the effect of Schmidth number on concentration. It is found that the Schmidth number decrease the concentration in the fluid. This effect is because of the presence of heavier diffusive species in the fluid. The suction velocity will having a decreasing effect on the concentration given by Figure 3.13.

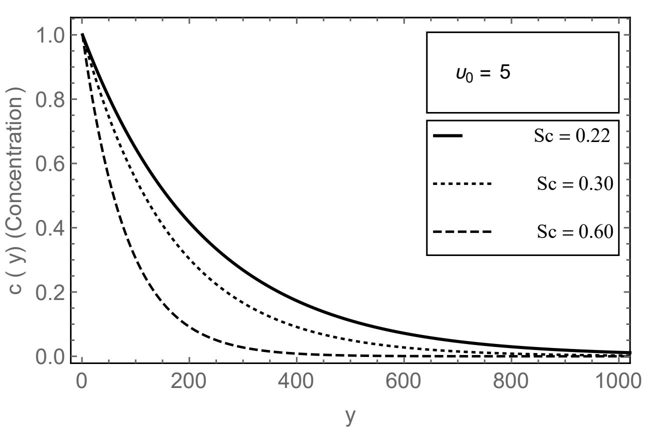

Figure 3.12: Concentration profile against y for different values of S_c.

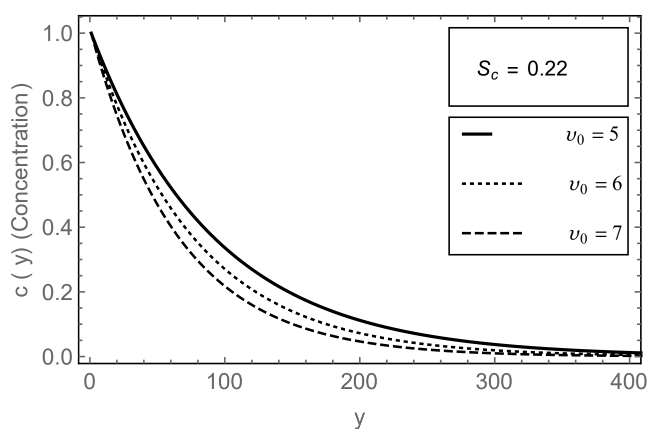

Figure 3.13: Velocity profile against y for different values of υ_0.

Chapter 4

Heat Transfer in Natural Convection Hydromagnetic Fluid Flow Over a Porous Plate

4.1 Introduction

In the previous chapter we discussed the natural convection hydromagnetic fluid flow over an oscillating plate and obtained an analytic solution which didn't match with the original result obtained in 2014 Das *et al.* [18] research paper due to some computational mistakes. We obtained an exact solution which is presented graphically. In the present study we extended the same problem being discussed in the previous chapter by considering oscillational and translational motion of the plate.

4.2 Formulation of Problem

Consider the two dimensional natural convection flow and heat transfer in an incompressible, viscous and electrical conducting fluid over a moving porous plate immersed in a porous medium. The motion of the plate is oscillatory and translatory. An external magnetic field of strength B_0 is applied in transverse direction of the flow. The induced magnetic field due to the flow is compared to the applied external magnetic field because of the magnetic Reynolds number is assume to be much less than 1. The pressure in the flow is assumed to be constant and variation in density is only due to variation in temperature.

The component of velocity in x and y directions are u and v respectively. All physical variable are functions of only y and t. So the velocity flow field is $V = [u(y,t), v(y,t), 0]$. v_0 is suction/injection velocity. According to the assumption the equation of continuity becomes

$$\frac{\partial v}{\partial y} = 0. \tag{4.1}$$

From eq (4.1) under the condition $y = 0$, $v = -v_0$ we have $v(y,t) = -v_0$. Thus the velocity flow field becomes

$$V = [u(y,t), -v_0, 0].$$

The momentum and energy equations given by eq (2.8) and eq (2.18) for the present model under boundary layer approximation becomes

$$\frac{\partial u}{\partial t} - v_0\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\rho\beta(T - T_\infty) - \frac{\nu}{K_0}u - \frac{\sigma B_0^2 u}{\rho}, \tag{4.2}$$

$$\frac{\partial T}{\partial t} - v_0\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + S(T - T_\infty). \tag{4.3}$$

In eq (4.2), $g\beta(T - T_\infty)$ is the thermal buoyancy force. The volumetric coefficients of thermal expansion is represented by β, g is acceleration due to gravitation and T is used to represent the temperature of the fluid and T_∞ represents the fluid's temperature far away from the plate. Other quantities, that are used in eq (4.2) include ν, K and $-\sigma u B_0^2$ represent kinematic viscosity, permeability of porous medium and magnetic force, respectively. In eq (4.3), α is thermal diffusivity and S is heat source parameter.

When the plate start oscillation and translation the first order velocity slip boundary conditions are

$$
\begin{aligned}
u &= U_0 e^{(i\omega - a)t} + L' \frac{\partial u}{\partial y}, \quad T = T_w \quad at \quad y = 0, \\
u &\to 0, \quad T \to T_\infty \quad as \quad y \to \infty.
\end{aligned} \tag{4.4}
$$

Here $a > 0$, $L^{'} = \frac{2-m_r}{m_r} L$ and $L = \mu \left(\frac{\pi}{2\rho p} \right)^{1/2}$ is mean free path and m_r is Maxwell's reflection coefficient.

To solve eq (4.2) and eq (4.3), We reduce the equations and boundary conditions into dimensionless form by using the dimensionless quantities

$$
\begin{aligned}
y^* &= \frac{U_0}{\nu} y, \ u^* = \frac{u}{U_0}, \ T^* = \frac{T - T_\infty}{T_w - T_\infty}, \ t^* = \frac{U_0^2}{\nu} t, \ \upsilon_0^* = \frac{\upsilon_0}{U_0}, \\
\omega^* &= \frac{\nu}{U_0^2} \omega, \ S^* = \frac{\nu}{U_0^2} S.
\end{aligned}
$$

The equations in dimensionless form are

$$
\frac{\partial u^*}{\partial t^*} - \upsilon_0^* \frac{\partial u^*}{\partial y^*} = \frac{\partial^2 u^*}{\partial y^{*2}} + G_r T^* - (\frac{1}{K_p} + M^2) u^*, \tag{4.5}
$$

$$
\frac{\partial T^*}{\partial t^*} - \upsilon_0^* \frac{\partial T^*}{\partial y^*} = \frac{1}{P_r} \frac{\partial^2 T^*}{\partial y^{*2}} + S^* T^*. \tag{4.6}
$$

and dimensionless boundary conditions are

$$
\begin{aligned}
u^* &= e^{(\iota\omega^* - a^*)t^*} + R \frac{\partial u^*}{\partial y^*}, \quad T^* = 1 \quad at \quad y^* = 0, \\
u^* &\to 0, \quad T^* \to 0 \quad as \quad y^* \to \infty.
\end{aligned} \tag{4.7}
$$

where

$$
\begin{aligned}
G_r &= \nu \beta g \frac{T_w - T_\infty}{U_0^3} \ (Grashof number\ for\ heat\ transfer), \\
K_p &= \frac{K_0}{\nu^2} U_o^2 \ (Permeability\ parameter), \\
M &= \frac{B_0}{U_0} \left(\frac{\nu \sigma}{\rho} \right)^{1/2} (Hartmann\ number \quad or \quad magnetic\ parameter), \\
P_r &= \frac{\nu}{\alpha} \ (Prandtl\ number), \\
R &= \frac{U_0}{\nu} L_1 (Rarefaction\ parameter).
\end{aligned}
$$

The sign $\star$ will drop here for convenience that is used to indicate dimensionless quantities. The above equations becomes

$$\frac{\partial u}{\partial t} - \upsilon_0 \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_r T - (\frac{1}{K_p} + M^2)u, \tag{4.8}$$

$$\frac{\partial T}{\partial t} - \upsilon_0 \frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + ST \tag{4.9}$$

and boundary conditions becomes

$$\begin{aligned} u &= e^{(i\omega - a)t} + R\frac{\partial u}{\partial y}, \quad T = 1 \quad at \quad y = 0, \\ u &\to 0, \quad T \to 0 \quad as \quad y \to \infty. \end{aligned} \tag{4.10}$$

4.3 Solution Method

In this section, we solve eq (4.8) and eq (4.9) for velocity and temperature according to the boundary conditions (4.10) by using the following transformations.

$$u(y,t) = u_0(y) + u_1(y)e^{(i\omega - a)t}, \tag{4.11}$$

$$T(y,t) = T_0(y) + T_1(y)e^{(i\omega - a)t}, \tag{4.12}$$

which transform eq (4.8) and eq (4.2) to the following ODE's

$$u_0'' + \upsilon_0 u_0' - (\frac{1}{K_p} + M^2)u_0 = -G_r T_0, \tag{4.13}$$

$$u_1'' + \upsilon_0 u_1' - (\frac{1}{K_p} + M^2 - a + i\omega)u_1 = -G_r T_1, \tag{4.14}$$

$$T_0'' + \upsilon_0 P_r T_0' + S P_r T_0 = 0, \tag{4.15}$$

$$T_1'' + \upsilon_0 P_r T_1' + P_r(S + a - i\omega)T_1 = 0. \tag{4.16}$$

and boundary conditions are transformed into

$$\begin{aligned} u_0 &= R\frac{du_0}{dy}, \quad u_1 = 1 + R\frac{du_1}{dy}, \quad T_0 = 1, \quad T_1 = 0 \quad at \quad y = 0, \\ u_0 &\to 0, \quad u_1 \to 0, \quad T_0 \to 0, \quad T_1 \to 0 \quad as \quad y \to \infty. \end{aligned} \tag{4.17}$$

Eq (4.15) and (4.16) are second order homogeneous ODE's which are solve with boundary condition given in (4.17) gives the following results

$$T_0(y) = e^{\lambda_1 y}, \quad \lambda_1 = \frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right], \tag{4.18}$$

$$T_1(y) = 0. \tag{4.19}$$

And eq (4.13) and (4.14) are solve with boundary given in (4.17) by similarly by the method already done in Section 3.3. Thus

$$u_0(y) = A_1 e^{-\lambda_2 y} - A_2 e^{\lambda_1 y}, \tag{4.20}$$

where

$$\begin{aligned}
\lambda_1 &= \frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right], \quad \lambda_2 = \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\,\right], \\
\lambda_3 &= \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\right], \\
A_1 &= \frac{1}{1+\lambda_2 R}[(1-\lambda_1 R)A_2], \quad A_2 = \frac{G_r}{(\lambda_1+\lambda_2)(\lambda_1-\lambda_3)},
\end{aligned}$$

and

$$u_1(y) = A_3 e^{\lambda_4 y}. \tag{4.21}$$

where

$$\lambda_4 = \frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 - a + i\omega\right)}\,\right], \quad A_3 = \frac{1}{1 - R\lambda_4}.$$

Now substitute eq (4.20) and (4.21) in (4.11) we have

$$u(y,t) = A_1 e^{-\lambda_2 y} - A_2 e^{\lambda_1 y} + A_3 e^{\lambda_4 y + (i\omega - a)t}, \tag{4.22}$$

and by substituting eq (4.18) and (4.19) in (4.12)

$$T(y) = e^{\lambda_1 y}. \tag{4.23}$$

4.4 Discussion

In this section, we discuss graphically of aspects velocity and temperature profile and the effects of the variation of various physical parameters on velocity and temperature.

4.4.1 Effects on Velocity

Effect of Translation a and magnetic parameter M

Figure 4.1, is about the velocity profile which is plotted for $a = 0,\ 2,\ 3,\ 4$. It shows that when translatory motion of the plate increase, its effect on the velocity is to decrease its magnitude. Figure 4.2, is about the effect of magnetic parameter on velocity distribution. The curves which is plotted for $M = 2,\ 3,\ 4$ indicates that increasing magnetic field oppose the velocity of the flow. This effect is nothing but only the effect of Lorentz force acting on the fluid.

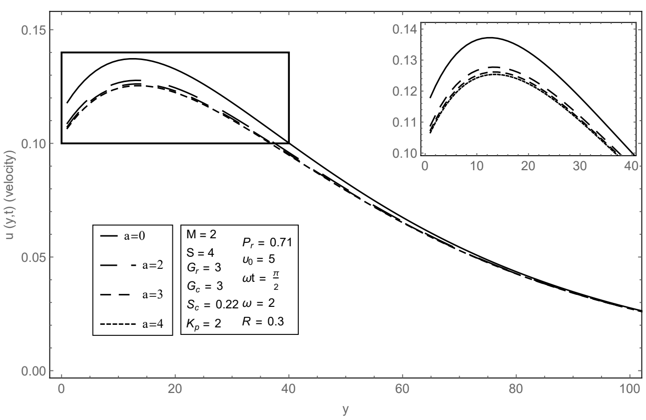

Figure 4.1: Velocity profile against y for different values of a.

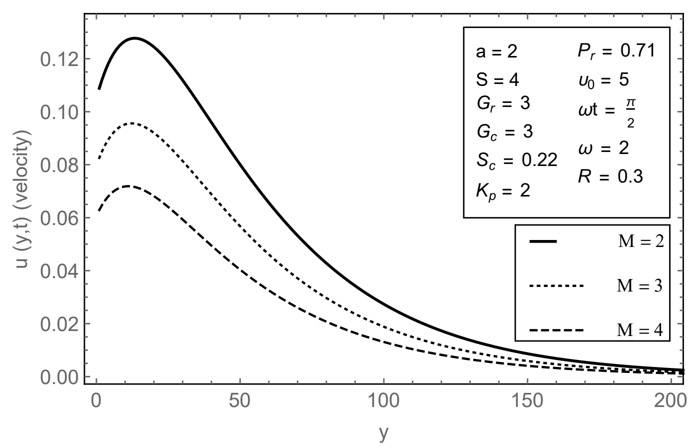

Figure 4.2: Velocity profile against y for different values of M.

Effect of Heat Source Parameter S and Grashof Number for Heat Transfer G_r

Figure 4.3, describe the effect of heat source parameter on the velocity. Velocity profile is plotted for $S = -4,\ 0,\ 4$. It has an increasing effect on the velocity. The fact is only that kinetic energy increase with increase in temperature, as a result fluid accelerates. Figure 4.4, illustrate the effect of thermal Grashof number on the velocity. Curves are plotted for $G_r > 0$ i.e. $G_r = 1,\ 2,\ 3$, which is the case of cooling of the plate. The increase in G_r will increase the thermal buoyancy effect, which results in rises the induced flow. Thus velocity increases with increase in thermal Grashof number.

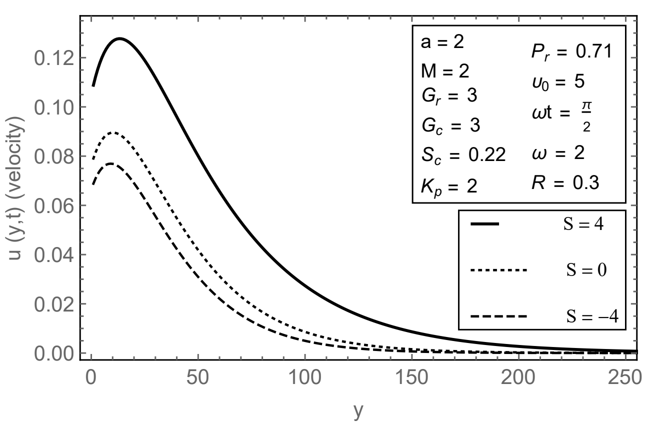

Figure 4.3: Velocity profile against y for different values of S.

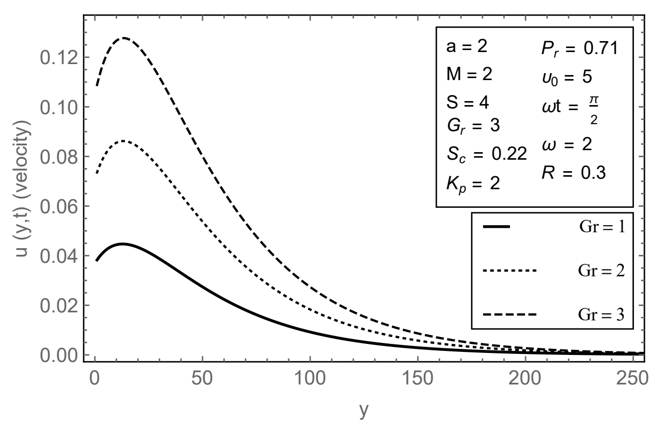

Figure 4.4: Velocity profile against y for different values of G_r.

Effect Permeability Parameter K_p and Prandtl Number and Suction Velocity υ_0

Figure 4.5, is about the permeability effect on the velocity. For $K_p = 0.5,\ 1,\ 2$ velocity profile is given, which indicate that permeability parameter having an increasing effect on velocity. For $P_r = 0.71,\ 2,\ 7$ different curves are plotted in Figure 4.6. Its shows that the greater Prandtl number decreases the velocity. The physical fact is that high Prandtl number having high viscosity and thus moves slowly. Figure 4.7, illustrate that higher suction parameter will decrease the fluid velocity.

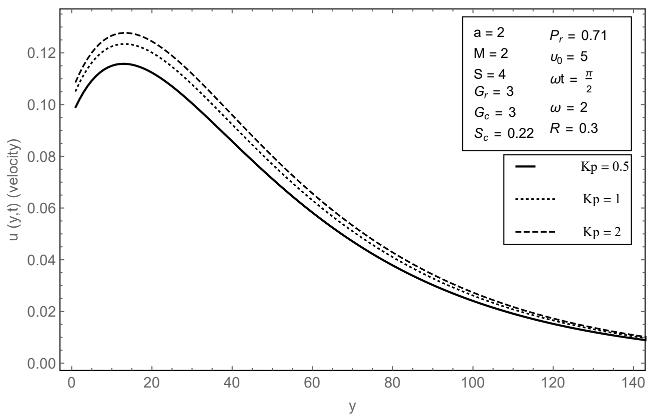

Figure 4.5: Velocity profile against y for different values of K_p.

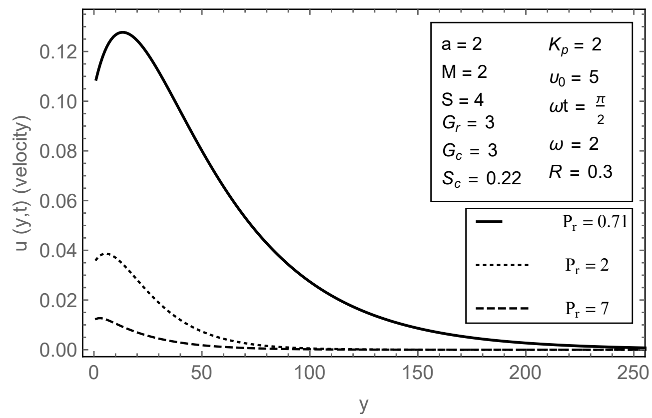

Figure 4.6: Velocity profile against y for different values of P_r.

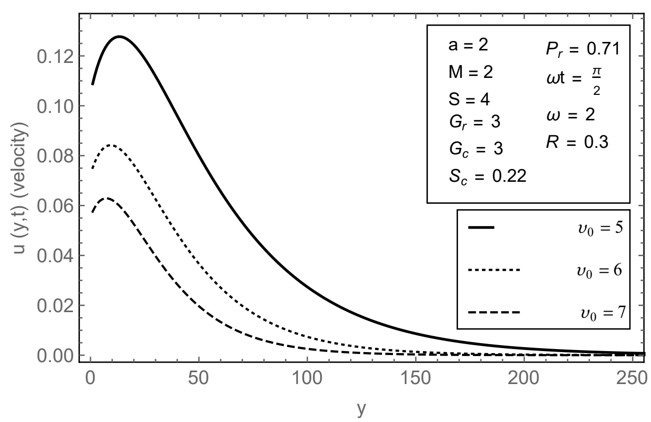

Figure 4.7: Velocity profile against y for different values of υ_0.

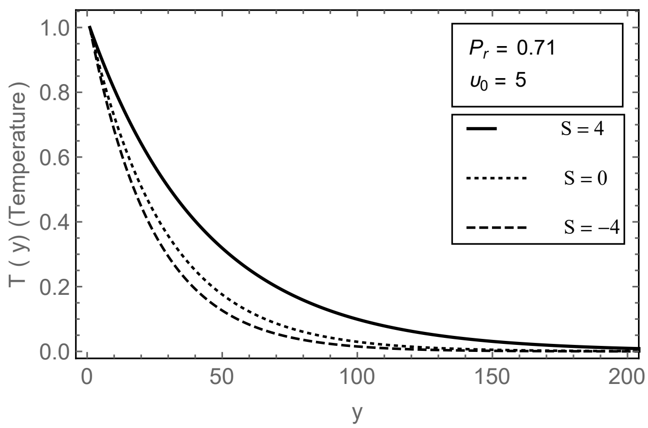

Figure 4.8: Temperature profile against y for different values of S.

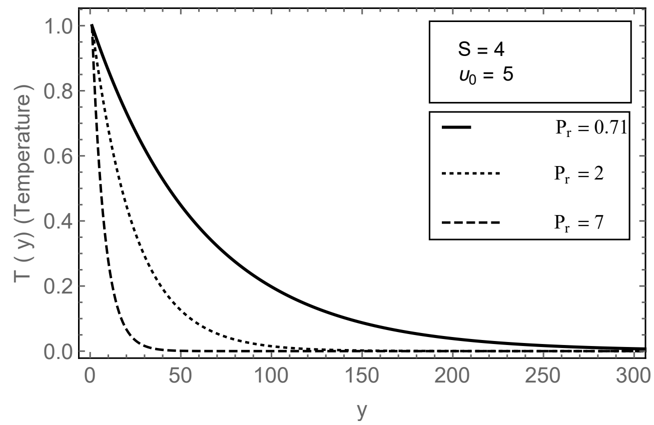

Figure 4.9: Temperature profile against y for different values of P_r.

4.4.2 Effects on Temperature

The only parameter which effect the temperature profile are heat source parameter S, Prandtl number P_r and suction parameter υ_0. Figure 4.8, describe the effect of heat source parameter on temperature distributions. The curves are plotted corresponding to $S = -4,\ 0,\ 4$. It shows that increase in heat source parameter will cause increase in temperature. The effect of other two parameter that is P_r and υ_0 is given in Figure 4.8 and 4.9 respectively The temperature decrease with both increase in Prandtl number and suction parameter. It means that greater P_r and υ_0, more will be the cooling effect.

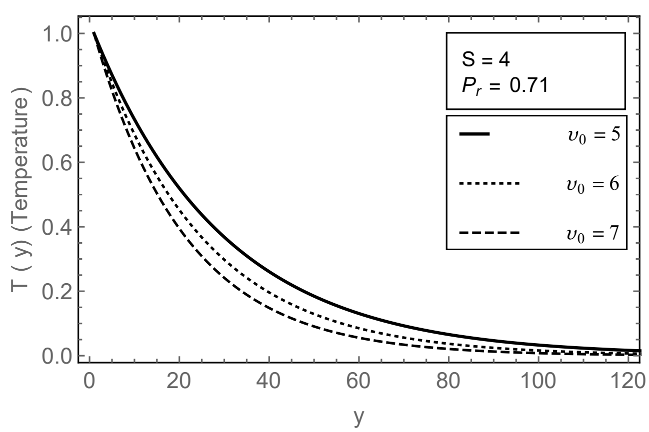

Figure 4.10: Temperature profile against y for different values of υ_0.

Chapter 5

Heat and Mass Transfer in Hydromagnetic Fluid Flow Over a Porous Plate

5.1 Introduction

In this chapter we consider natural convection and mass transfer of hydromagnetic fluid including the concentration of species in the fluid. Which flows over an oscillating and translating porous plate. The fluid is assume to be incompressible, viscous and electrical conducting. An analytic method is used to solve the modeling equations for the boundary layer flow. An analytic expression for velocity, temperature and concentration is obtained and presented graphically.

5.2 Formulation of Problem

We consider the two dimensional unsteady natural convection flow heat and mass transfer in an incompressible, viscous and electrical conducting fluid over a moving porous plate immersed in a porous medium with heat source. The motion of the plate is oscillating and translating. An external magnetic field of Strength B_0 is applied in transverse direction of the flow. In the presence of external applied magnetic field, the magnetic field which is induce due to the flow is neglect because of the magnetic Reynold number is assumed to be much smaller than 1. Assume that pressure in the flow is constant i.e $\nabla p = 0$. The variation of density is only with variation of temperature and concentration of species in the fluid.

The velocity components in x and y-directions are u and v respectively. All physical variable are considered to be function of y and t. Let υ_0 denotes injection/suction velocity at the plate. In the presence of above assumptions, the continuity equation for incompressible fluid is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5.1}$$

which implies that

$$\frac{\partial v}{\partial y} = 0. \tag{5.2}$$

From eq (5.2) we have $v(y,t) = -\upsilon_0$ under the boundary condition $v = \upsilon_0$, at $y = 0$. Thus our velocity field become

$$V = [u(y,t), \upsilon_0, 0]. \tag{5.3}$$

The momentum, energy and diffusion equation given by eq (2.8), eq (2.18) and eq (2.19), for the present model under boundary layer approximation becomes

$$\frac{\partial u}{\partial t} - \upsilon_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\rho\beta(T - T_\infty) + g\rho\beta^*(C - C_\infty) - \frac{\nu}{K_0} u - \frac{\sigma B_0^2 u}{\rho}, \tag{5.4}$$

$$\frac{\partial T}{\partial t} - \upsilon_0 \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + S(T - T_\infty), \tag{5.5}$$

$$\frac{\partial C}{\partial t} - \upsilon_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2}. \tag{5.6}$$

In eq (5.4), $g\beta(T - T_\infty)$ and $g\beta^*(C - C_\infty)$ are the thermal and concentration buoyancy forces respectively, g is gravitational acceleration, β is volumetric coefficient of thermal expansion and β^* is volumetric coefficient of expansion for mass transfer, T is temperature of the fluid, T_∞ is temperature of the fluid at large distance from the plate. C is concentration, C_∞ is concentration of fluid far away from the plate, ν is kinematic viscosity, K is permeability of porous medium, $-\sigma u B_0^2$ is magnetic force caused by the applied magnetic field, σ is electrical conductivity of fluid. In eq (5.5), $\alpha = \frac{K}{\rho C_p}$ called thermal diffusivity, C_p is specific heat capacity, ρ is density and K is thermal conductivity the of fluid. T is temperature of the fluid, T_∞ is temperature of the fluid far away from the plate. where in eq (5.6) D is molecular diffusivity. when the plate start oscillatory and translatory motion the velocity slip first order boundary conditions are

$$\begin{aligned} u &= U_0 e^{(\iota\omega - a)t} + L'\frac{\partial u}{\partial y}, \quad T = T_w, \quad C = C_w \quad at \quad y = 0, \\ u &\to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad as \quad y \to \infty. \end{aligned} \tag{5.7}$$

where $a > 0$, $L^{'} = \frac{2-m_r}{m_r} L$ and $L = \mu \left(\frac{\pi}{2\rho p} \right)^{1/2}$ is mean free path and m_r is Maxwell's reflection coefficient.

To solve eq (5.4) to eq (5.6). We reduce the equations and boundary conditions into dimensionless form by using the dimensionless parameters

$$\begin{aligned} y^* &= \frac{U_0}{\nu} y, \; u^* = \frac{u}{U_0}, \; T^* = \frac{T - T_\infty}{T_w - T_\infty}, \; C^* = \frac{C - C_\infty}{C_w - C_\infty}, \; t^* = \frac{U_0^2}{\nu} t, \; \upsilon_0^* = \frac{\upsilon_0}{U_0}, \; \omega^* = \frac{\nu}{U_0^2}\omega \\ a^* &= \frac{\nu}{U_0^2} a, \; S^* = \frac{\nu}{U_0^2} S. \end{aligned}$$

Substitution of the above dimensionless values in eq (5.4) to (5.6) will lead to dimensionless form. The dimensionless equations are

$$\frac{\partial u^*}{\partial t^*} - \upsilon_0^* \frac{\partial u^*}{\partial y^*} = \frac{\partial^2 u^*}{\partial y^{*2}} + G_r T^* + G_c C^* - (\frac{1}{K_p} + M^2) u^*, \tag{5.8}$$

$$\frac{\partial T^*}{\partial t^*} - \upsilon_0^* \frac{\partial T^*}{\partial y^*} = \frac{1}{P_r} \frac{\partial^2 T^*}{\partial y^{*2}} + S^* T^*, \tag{5.9}$$

$$\frac{\partial C^*}{\partial t^*} - \upsilon_0^* \frac{\partial C^*}{\partial y^*} = \frac{1}{S_c} \frac{\partial^2 C^*}{\partial y^{*2}}. \tag{5.10}$$

and dimensionless boundary conditions are

$$\begin{aligned} u^* &= e^{(\iota\omega^* - a^*)t^*} + R\frac{\partial u^*}{\partial y^*}, \quad T^* = 1, \quad C^* = 1 \quad at \quad y^* = 0, \\ u^* &\to 0, \quad T^* \to 0, \quad C^* \to 0 \quad as \quad y^* \to \infty. \end{aligned} \tag{5.11}$$

Where

$$
\begin{aligned}
G_r &= \nu\beta g\frac{T_w - T_\infty}{U_0^3} \quad (Grashof\ number \quad for \quad heat \quad transfer),\\
G_c &= \nu\beta^* g\frac{C_w - C_\infty}{U_0^3} \quad (Grashof\ number \quad for \quad mass \quad transfer),\\
K_p &= \frac{K_0}{\nu^2}U_o^2 \quad (Permeability \quad parameter),\\
M &= \frac{B_0}{U_0}\left(\frac{\nu\sigma}{\rho}\right)^{1/2} (Hartmann \quad number \quad or \quad Magnetic \quad parameter),\\
P_r &= \frac{\nu}{\alpha} \quad (Prandtl \quad number),\\
S_c &= \frac{\nu}{D} \quad (Schmidt\ number),\\
R &= \frac{U_0}{\nu}L_1 \quad (Rarefaction \quad parameter).
\end{aligned}
$$

For convenience we will drop $\star$ sign that is used to indicate dimensionless quantities. so we write the above equations and boundary conditions with out $\star$ but it is understood that these are now dimensionless equations

$$
\frac{\partial u}{\partial t} - \upsilon_0\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G_rT + G_cC - (\frac{1}{K_p} + M^2)u, \tag{5.12}
$$

$$
\frac{\partial T}{\partial t} - \upsilon_0\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} + ST, \tag{5.13}
$$

$$
\frac{\partial C}{\partial t} - \upsilon_0\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \tag{5.14}
$$

and boundary conditions are

$$
\begin{aligned}
u &= e^{(\iota\omega - a)t} + R\frac{\partial u}{\partial y}, \quad T = 1, \quad C = 1 \quad at \quad y = 0,\\
u &\to 0, \quad T \to 0, \quad C \to 0 \quad as \quad y \to \infty.
\end{aligned} \tag{5.15}
$$

5.3 Solution Method

In this section, we solve equations (5.12)-(5.14) for velocity, temperature and concentration distributions of flow by using the following transformations.

$$u(y,t) = u_0(y) + u_1(y)e^{(i\omega - a)t}, \tag{5.16}$$

$$T(y,t) = T_0(y) + T_1(y)e^{(i\omega - a)t}, \tag{5.17}$$

$$C(y,t) = C_0(y) + C_1(y)e^{(i\omega - a)t}. \tag{5.18}$$

By differentiating partially eq (5.16) (5.17) and (5.18) with respect to y and t we have the following

$$\frac{\partial u}{\partial t} = u_1(i\omega - a)e^{(i\omega - a)t}, \tag{5.19}$$

$$\frac{\partial u}{\partial y} = u_0' + u_1'e^{(i\omega - a)t}, \tag{5.20}$$

$$\frac{\partial^2 u}{\partial y^2} = u_0'' + u_1''e^{(i\omega - a)t}, \tag{5.21}$$

$$\frac{\partial T}{\partial t} = T_1(i\omega - a)e^{(i\omega - a)t}, \tag{5.22}$$

$$\frac{\partial T}{\partial y} = T_0' + T_1'e^{(i\omega - a)t}, \tag{5.23}$$

$$\frac{\partial^2 u}{\partial y^2} = T_0'' + T_1'')e^{(i\omega - a)t}, \tag{5.24}$$

$$\frac{\partial C}{\partial t} = C_1(i\omega - a)e^{(i\omega - a)t}, \tag{5.25}$$

$$\frac{\partial C}{\partial y} = C_0' + C_1'e^{(\iota\omega - a)t}, \tag{5.26}$$

$$\frac{\partial^2 C}{\partial y^2} = C_0'' + C_1''e^{(i\omega - a)t}. \tag{5.27}$$

substituting eq (5.19)-(5.27) in eq (5.12) (5.13) (5.14) will transformed the PDE's to the following ODE's

$$
\begin{aligned}
u_0'' + \upsilon_0 u_0' - (\frac{1}{K_p} + M^2)u_0 &= -G_c T_0 - G_c C_0, & (5.28)\\
u_1'' + \upsilon_0 u_1' - (\frac{1}{K_p} + M^2 + i\omega - a)u_1 &= -G_c T_1 - G_c C_1, & (5.29)\\
T_0'' + \upsilon_0 P_r T_0' + S P_r T_0 &= 0, & (5.30)\\
T_1'' + \upsilon_0 P_r T_1' + P_r(S + a - i\omega)T_1 &= 0, & (5.31)\\
C_0'' + S_c \upsilon_0 C_0' &= 0, & (5.32)\\
C_1'' + \upsilon_0 S_c C_1' + S_c(a - i\omega)C_1 &= 0. & (5.33)
\end{aligned}
$$

Boundary conditions are transformed into

$$
\begin{aligned}
u_0 &= R\frac{du_0}{dy}, \quad u_1 = 1 + R\frac{du_1}{dy}, \quad T_0 = 1, \quad T_1 = 0, \quad C_0 = 1, \quad C_1 = 0 \quad at \quad y = 0,\\
u_0 &\to 0, \quad u_1 \to 0, \quad T_0 \to 0, \quad T_1 \to 0, \quad C_0 \to 0, \quad C_1 \to 0 \quad as \quad y \to \infty. \qquad (5.34)
\end{aligned}
$$

Auxiliary eq of eq (5.32)

$$D^2 + \upsilon_0 S_c D = 0. \qquad (5.35)$$

implies $D = 0$ and $D = -\upsilon_0 S_c$ where D here is a differential operator

Therefore the general solution is $C_0 = A_1 + A_2 e^{-\upsilon_0 S_c y}$.

Boundary condition $C_0 = 1 \quad$ at $\quad y = 0$ implies that $A_1 + A_2 = 1$ and $C_1 \to 0 \quad$ as $\quad y \to \infty$ insure that $A_1 = 0$ which implies $A_2 = 1$. Thus

$$C_0(y) = e^{-\upsilon_0 S_c y} \qquad (5.36)$$

Auxiliary eq of eq (5.33)

$$D^2 + \upsilon_0 S_c D + (a - i\omega)S_c = 0. \qquad (5.37)$$

$$\implies D = \frac{1}{2}[-\upsilon_0 S_c + \sqrt{\upsilon_0^2 S_c^2 - 4S_c(a - i\omega)}], \quad D = \frac{1}{2}\left[-\upsilon_0 S_c - \sqrt{\upsilon_0^2 S_c^2 - 4S_c\,(a - i\omega)}\right].$$

so the general solution is

$$C_1(y) = A_3 e^{\frac{1}{2}\left[-\upsilon_0 S_c + \sqrt{\upsilon_0^2 S_c^2 - 4S_c(a-i\omega)}\right]y} + A_4 e^{\frac{1}{2}\left[-\upsilon_0 S_c - \sqrt{\upsilon_0^2 S_c^2 - 4S_c(a-i\omega)}\right]y}.$$

Boundary condition $C_1 = 0 \quad \text{at} \quad y = 0$ implies that $A_3 + A_4 = 0$ and $C_1 \to 0 \quad \text{as} \quad y \to \infty$ insure that $A_3 = 0$ which implies $A_4 = 0$. Therefore $C_1(y) = 0$.

Auxiliary eq of (5.30)

$$D^2 + \upsilon_0 P_r D + P_r S = o. \tag{5.38}$$

implies that $D = \frac{1}{2}\left[-\upsilon_0 P_r + \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right], \quad D = \frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right].$

The general solution is

$$T_0(y) = A_5 e^{\frac{1}{2}\left[-\upsilon_0 P_r + \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right]y} + A_6 e^{\frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right]y}.$$

boundary conditions $T_0 = 1 \quad \text{at} \quad y = 0$ implies that $A_5 + A_6 = 1$ and $T_1 \to 0 \quad \text{as} \quad y \to \infty$ insure $A_5 = 0$ which implies $A_6 = 1$, so

$$T_0(y) = e^{\frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right]y},$$

or

$$T_0(y) = e^{\lambda_1 y}, \ \lambda_1 = \frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 - 4P_r S}\right]. \tag{5.39}$$

Auxiliary eq of (5.31)

$$D^2 + \upsilon_0 P_r D + (S + a - i\omega)P_r = 0, \tag{5.40}$$

$$\implies D = \frac{1}{2}\left[-\upsilon_0 P_r + \sqrt{\upsilon_0^2 P_r^2 + 4P_r\left(S + a - i\omega\right)}\right], \quad D = \frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 + 4P_r\left(S + a - i\omega\right)}\right].$$

The general solution is

$$T_1(y) = A_7 e^{\frac{1}{2}\left[-\upsilon_0 P_r + \sqrt{\upsilon_0^2 P_r^2 + 4P_r(S+a-i\omega)}\right]y} + A_8 e^{\frac{1}{2}\left[-\upsilon_0 P_r - \sqrt{\upsilon_0^2 P_r^2 + 4P_r(S+a-i\omega)}\right]y}.$$

Boundary conditions $T_1 = 0$ at $y = 0$ implies that $A_7 + A_8 = 0$ and $T_1 \to 0$ as $y \to \infty$ insure that A_7 must equal to 0 which implies $A_8 = 0$. Therefore $T_1(y) = 0$.
Eq (5.28) is a non homogeneous eq Auxiliary eq for homogeneous part

$$D^2 + \upsilon_0 D - \left(\frac{1}{K_p} + M^2\right) = 0, \tag{5.41}$$

having roots

$$D = \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\ \right],\ D = \frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\ \right],$$

so the complementary solution is

$$u_{0(c)} = A_9 e^{\frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\ \right]y} + A_{10} e^{\frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\ \right]y},$$

or

$$u_{0(c)} = A_9 e^{\lambda_3 y} + A_{10} e^{-\lambda_2 y},$$

Here

$$\lambda_2 = \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\ \right], \tag{5.42}$$

$$\lambda_3 = \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\ \right]. \tag{5.43}$$

For particular sol from eq (5.28) we have

$$u_0'' + \upsilon_0 u_0' - \left(\frac{1}{K_p} + M^2\right) u_0 = -G_c T_0 - G_c C_0,$$

or

$$u_0'' + \upsilon_0 u_0' - \left(\frac{1}{K_p} + M^2\right) u_0 = -G_c e^{\lambda_1 y} - G_c e^{-\upsilon_0 S_c y},$$

by using the values of C_0 and T_0 from eq (5.36) and eq (5.39) respectively. By applying method of variation of parameter we get the particular solution of

$$\left[D^2 + \upsilon_0 D - \left(\frac{1}{K_p} + M^2\right)\right] u_0 = -G_c e^{\lambda_1 y} - G_c e^{-\upsilon_0 S_c y}.$$

that is $\quad u_{0(p)} = -\dfrac{G_r}{\lambda_1^2 + \upsilon_0\lambda_1^2 - (\frac{1}{K_p} - M^2)} e^{\lambda_1 y} - \dfrac{G_c}{\upsilon_0^2 S_c^2 - \upsilon_0^2 S_c - (\frac{1}{K_p} + M^2)} e^{-\upsilon_0 S_c y}.$

Therefore,

$$\begin{aligned} u_0(y) &= A_9 e^{\frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\,\right]y} + A_{10} e^{\frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2\right)}\,\right]y} \\ &- \frac{G_r}{\lambda_1^2 + \upsilon_0\lambda_1^2 - \left(\frac{1}{K_p} + M^2\right)} e^{\lambda_1 y} - \frac{G_c}{\upsilon_0^2 S_c^2 - \upsilon_0^2 S_c - \left(\frac{1}{K_p} + M^2\right)} e^{-\upsilon_0 S_c y} \end{aligned} \quad (5.44)$$

and under the boundary conditions

$$\begin{aligned} u_0 &= R\frac{du_0}{dy} \quad \text{at} \quad y = 0 \quad \text{implies that} \\ R\frac{du_0}{dy} &= A_9 + A_{10} - \frac{G_r}{\lambda_1^2 + \upsilon_0\lambda_1 - (\frac{1}{K_p} + M^2)} - \frac{G_c}{\upsilon_0^2 S_c + \upsilon_0^2 S_c - (\frac{1}{K_p} + M^2)} \\ \text{and} \quad u_0 &\to 0 \quad \text{as} \quad y \to \infty \quad \text{insure that} \quad A_9 = 0 \end{aligned}$$

Therefore, eq (5.44) implies

$$A_{10} = R\frac{du_0}{dy} + \frac{G_r}{\lambda_1^2 + \upsilon_0\lambda_1 - (\frac{1}{K_p} + M^2)} + \frac{G_c}{\upsilon_0^2 S_c^2 + \upsilon_0^2 S_c - (\frac{1}{K_p} + M^2)},$$

or

$$A_{10} = R\frac{du_0}{dy} + A_{11} + A_{12}, \quad (5.45)$$

here

$$A_{11} = \frac{G_r}{\lambda_1^2 + \upsilon_0\lambda_1 - \left(\frac{1}{K_p} + M^2\right)}, \quad (5.46)$$

$$A_{12} = \frac{G_c}{\upsilon_0^2 S_c^2 + \upsilon_0^2 S_c - \left(\frac{1}{K_p} + M^2\right)}. \quad (5.47)$$

Adding eq (5.42) and eq (5.43), we have

$$\lambda_1 + \lambda_2 = \lambda_1 + \frac{1}{2}\left[\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\,\right], \quad (5.48)$$

and subtraction of eq (5.42) and eq (5.43), gives

$$\lambda_1 - \lambda_3 = \lambda_1 - \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_P} + M^2\right)}\,\right]. \quad (5.49)$$

Multiplication of eq (5.48) and eq (5.49), gives

$$(\lambda_1+\lambda_2)(\lambda_1-\lambda_3)=\lambda_1^2+\lambda_1\upsilon_0-4\left(\frac{1}{K_p}+M^2\right). \tag{5.50}$$

Now add $S_c\upsilon_0$ to eq (5.42)

$$\begin{aligned} -S_c\upsilon_0+\lambda_2 &= -S_c\upsilon_0+\frac{1}{2}\left[\upsilon_0+\sqrt{\upsilon_0^2+4\left(\frac{1}{K_P}+M^2\right)}\ \right], \\ S_c\upsilon_0-\lambda_2 &= S_c\upsilon_0-\frac{1}{2}\left[\upsilon_0+\sqrt{\upsilon_0^2+4\left(\frac{1}{K_P}+M^2\right)}\ \right] \end{aligned} \tag{5.51}$$

and

$$S_c\upsilon_0+\lambda_3=S_c\upsilon_0+\frac{1}{2}\left[-\upsilon_0+\sqrt{\upsilon_0^2+4\left(\frac{1}{K_P}+M^2\right)}\ \right]. \tag{5.52}$$

Multiplication of eq (5.51) and eq (5.52), gives

$$(S_c\upsilon_0-\lambda_2)(\lambda_3+S_c\upsilon_0)=S_c^2\upsilon_0^2-S_c\upsilon_0^2-4\left(\frac{1}{K_p}+M^2\right). \tag{5.53}$$

Substitute eq(5.50) in eq (5.46) and eq (5.53) in eq (5.47), Therefore

$$A_{11}=\frac{G_r}{(\lambda_1+\lambda_2)(\lambda_1-\lambda_3)} \quad \text{and} \quad A_{12}=\frac{G_c}{(S_c\upsilon_0-\lambda_2)(S_c\upsilon_0+\lambda_3)}.$$

Thus eq (5.44) becomes

$$u_0(y)=A_{10}e^{-\lambda_2 y}-A_{11}e^{\lambda_1 y}-A_{12}e^{-\upsilon_0 S_c y}. \tag{5.54}$$

Differentiation of eq (5.54) lead us to

$$\frac{du_0}{dy}=-A_{10}\lambda_2e^{-\lambda_2 y}-A_{11}\lambda_1e^{\lambda_1 y}+A_{12}\upsilon_0S_ce^{-\upsilon_0S_cy}, \tag{5.55}$$

Put $y=0$ we have

$$\frac{du_0}{dy}=-A_{10}\lambda_2-A_{11}\lambda_1+A_{12}\upsilon_0S_c, \tag{5.56}$$

Substitute eq (5.56) in eq (5.45) Therefore

$$A_{10}=R(-\lambda_2A_{10}-\lambda_1A_{11}+\upsilon_0S_cA_{12})+A_{11}+A_{12}.$$

This implies that

$$A_{10} = \frac{1}{1+\lambda_2 R}[(1-\lambda_1 R)A_{11} + (1+\upsilon_0 S_c R)A_{12}]. \tag{5.57}$$

As $T_1(y) = 0$ and $C_1(y) = 0$ the eq (5.29) becomes

$$u_1'' + \upsilon_0 u_1' - \left(\frac{1}{K_p} + M^2 - a + i\omega\right) u_1 = 0. \tag{5.58}$$

Its auxiliary equation is

$$D^2 + \upsilon_0 D - \left(\frac{1}{K_p} + M^2 - a + i\omega\right) = 0, \tag{5.59}$$

having roots

$$D = \frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 - a + i\omega\right)}\ \right]$$

and

$$D = \frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 - a + i\omega\right)}\ \right].$$

Thus the general solution is

$$u_1(y) = A_{13} e^{\frac{1}{2}\left[-\upsilon_0 + \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 - a + i\omega\right)}\ \right]y} + A_{14} e^{\frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 - a + i\omega\right)}\ \right]y},$$

Boundary conditions

$$\begin{array}{rcl} u_1 & = & 1 + R\dfrac{du_1}{dy} \quad \text{at} \quad y = 0 \quad \text{implies} \quad A_{13} + A_{14} = 1 + R\dfrac{du_1}{dy} \\ \text{whereas} \quad u_1 & \to & 0 \quad \text{as} \quad y = 0 \quad \text{insure} \quad A_{13} = 0 \end{array}$$

,

$$\text{implies that} \quad A_{14} = 1 + R\frac{du_1}{dy}. \tag{5.60}$$

Therefore

$$u_1(y) = A_{14} e^{\frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 - a + i\omega\right)}\ \right]y},$$

or

$$u_1(y) = A_{14} e^{\lambda_4 y}, \tag{5.61}$$

here

$$\lambda_4 = \frac{1}{2}\left[-\upsilon_0 - \sqrt{\upsilon_0^2 + 4\left(\frac{1}{K_p} + M^2 - a + i\omega\right)}\,\right].$$

Differentiation of eq (5.61) leads us to

$$\frac{du_1}{dy} = A_{14}\lambda_4 e^{\lambda_4 y}, \tag{5.62}$$

At $y = 0$ eq (5.62) becomes

$$\frac{du_1}{dy} = A_{14}\lambda_4, \tag{5.63}$$

Now substitute eq (5.63) in eq (5.60) we have

$$A_{14} = 1 + R(A_{14}\lambda_4) \text{implies} A_{14} = \frac{1}{1 - R\lambda_4}. \tag{5.64}$$

Now substitute eq (5.54) and (5.61) in (5.16) we have

$$u(y,t) = A_{10}e^{-\lambda_2 y} - A_{11}e^{\lambda_1 y} - A_{12}e^{-\upsilon_0 S_c y} + A_{14}e^{\lambda_4 y + (i\omega - a)t}, \tag{5.65}$$

and substitution of $T_0(y) = e^{\lambda_1 y}$ and $T_1(y) = 0$ in eq (5.17) we have

$$T(y) = e^{\lambda_1 y}, \tag{5.66}$$

whereas the substitution of $C_0(y) = e^{-\upsilon_0 S_c y}$ and $C_1(y) = 0$ in eq (5.18) lead us to

$$C(y) = e^{-\upsilon_0 S_c y}. \tag{5.67}$$

Skin Friction

Skin friction at the wall is given by

$$\begin{aligned} \tau &= \left(\frac{\partial u}{\partial y}\right)_{y=0}, \\ &= -\lambda_2 A_{10} - \lambda_1 A_{11} - S_c \upsilon_0 A_{12} + \lambda_4 A_{14} e^{\lambda_4 y + (i\omega - a)t}. \end{aligned} \tag{5.68}$$

Heat Flux

The rate of heat transfer or the heat flux at the wall in terms of Nusselts number is given by

$$N_u = \left(\frac{dT}{yy}\right)_{y=0} = \lambda_1. \tag{5.69}$$

5.4 Graphical Observation

In this section, we discuss graphically the analytic solution which is obtained in the previous section 5.2, depend on various physical parameters, like translation a, magnetic parameter M, heat source parameter S, Grashof number for mass and heat transfer G_c, G_r, permeability parameter K_p, Prandtl number P_r, Schmidth number S_c and suction velocity υ_0. We observed the effect of the variation of these parameter on solution of momentum, energy and concentration equation.

5.4.1 Effect on Velocity

Velocity profile is effect by some physical parameters which is discuss as follow.

Effect of Translation Parameter a

Figure 5.1, Describe four different curves, that are plotted for different values of a i.e. $a = 0, 2, 3, 4$. This shows that by increasing a, velocity will decrease.

Effect of Magnetic Parameter M

Figure 5.2, Shows the effects of magnetic parameter M on velocity. Corresponding to $M = 2, 3, 4$ three curves are given in Figure 5.2, the greater magnetic parameter has a decreasing effect on velocity. This effect is because of the action of the Lorentz force acting on the electrical conducting fluid, which is caused due to the application of external

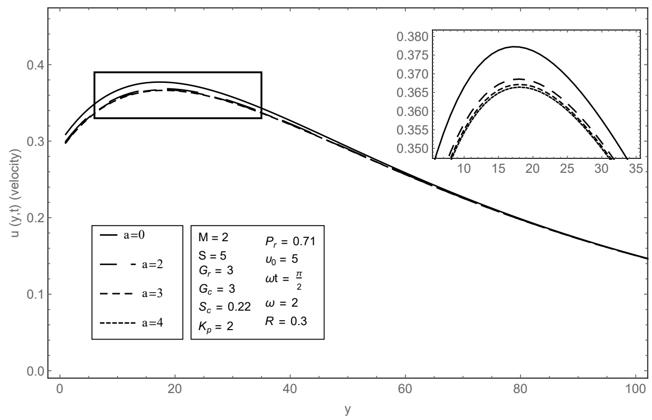

Figure 5.1: Velocity profile against y for different values of a.

magnetic field. In fact this force has the capability to slow down the motion of electrical conducting fluid.

Effect of Heat Source Parameter S

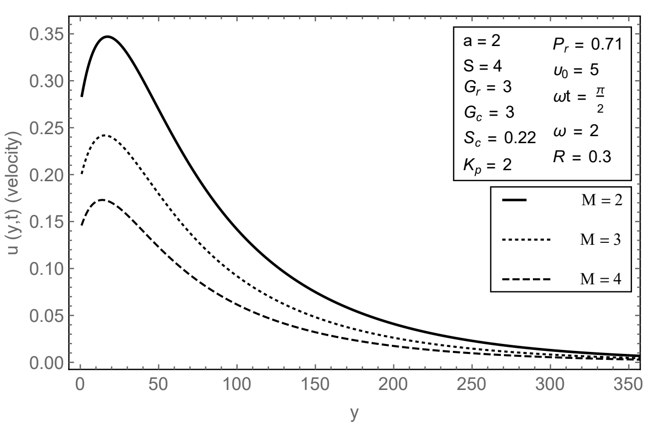

Figure 5.2: Velocity profile against y for different values of M.

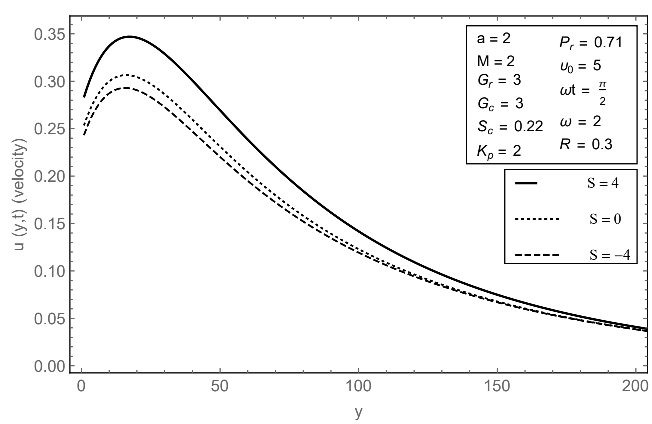

Figure 5.3: Velocity profile against y for different values of S.

Figure 5.3, is about the effect of heat source S on velocity. The curves will plotted for $S = -4, 0, 4$. The curves for $S = 0$ indicates the absence of heat source, while the curves with $S < 0$ and $S > 0$ is about the presence of heat sink and heat source in the flow respectively. From the observation of the curves it is found that larger heat source will rise the flow velocity. This is because of kinetic energy of molecules increase with increase in temperature as a result total velocity of the fluid will grown up.

Effect of Grashof number for Mass and Heat Transfer, G_C and G_r, permeability Parameter K_p, and prandtl number P_r

Figure 5.4 and 5.5, describe the effect of parameter G_c and G_r respectively. The result will plotted for $G_c > 0$ and $G_r > 0$, which mean for cooling of the plate. Keeping all the other parameters constant. It is found that the increasing value of G_c and G_r will cause increase in velocity. The fact is that the increase in G_c and G_r will increase mass and thermal buoyancy effect, through which induced flow increase. The permeability parameter K_p effect on velocity will discuss in Figure 5.6. It shows that increase in K_p, increase the velocity. Figure 5.7, shows that velocity decrease with increase in Prandtl number P_r. Physically, the fact is that fluid with high Prandtl number having high viscosity. Therefore it moves slowly.

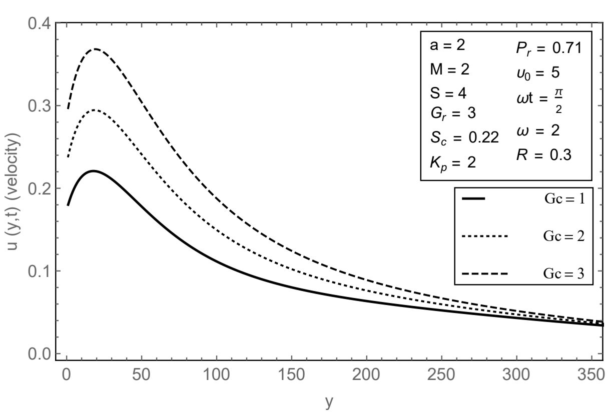

Figure 5.4: Velocity profile against y for different values of G_c.

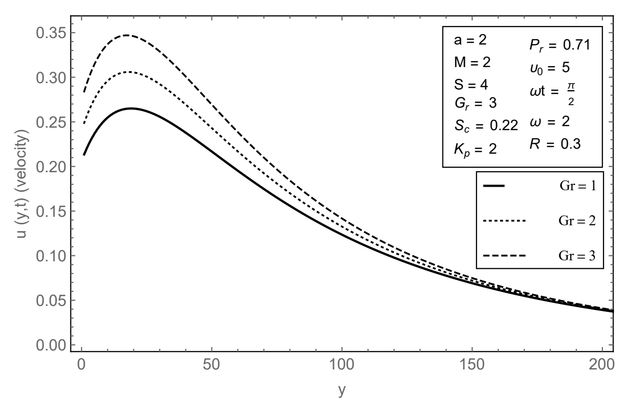

Figure 5.5: Velocity profile against y for different values of G_r.

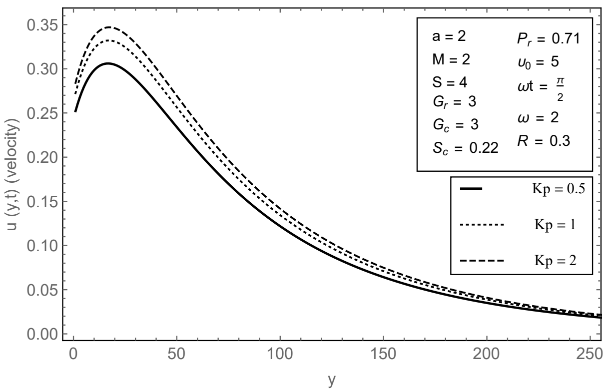

Figure 5.6: Velocity profile against y for different values of K_p.

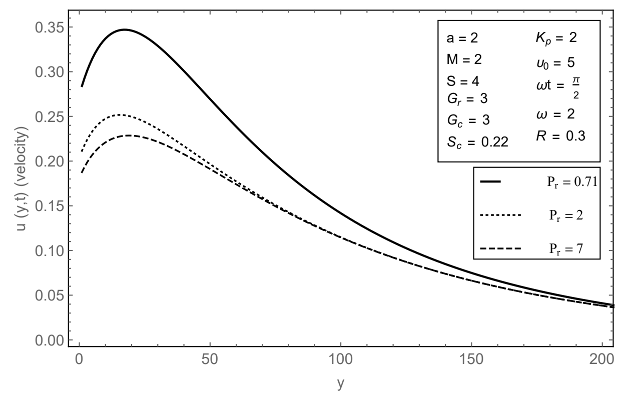

Figure 5.7: Velocity profile against y for different values of P_r.

Effect of Schmidth Number S_c and Suction Parameter υ_0

Figure 5.8, illustrate the effect of Schmidth number S_c on the velocity field. The observation of the curves shows that the growing Schmidth number result in slow down the velocity of the fluid. It is due to the presence of heavier diffusive species in the fluid. The effect of suction parameter on the velocity is discuss in Figure 5.9, velocity decrease due to increase in suction parameter.

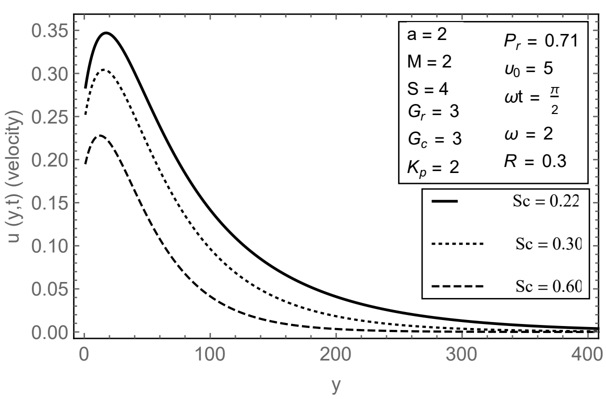

Figure 5.8: Velocity profile against y for different values of S_c.

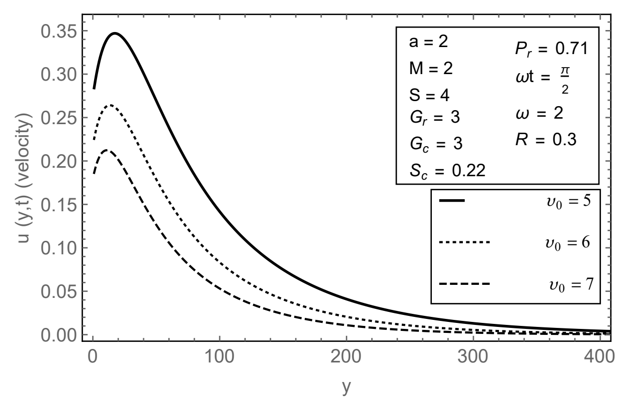

Figure 5.9: Velocity profile against y for different values of υ_0.

5.4.2 Effect on Temperature

Temperature profile will effect only with the effect of by the variation of heat source parameter, Prandtl number and suction velocity.

Effect of Heat Source Parameter S

Observations of the curves in Figure 5.10, shows that increase in heat source parameter will rise the temperature, due to which fluid move fastly. The variation of temperature with the variation of prandtl number is discuss in Figure 5.11. From the plotted curves for different values of P_r, we analyze that as a result of increase in Prandtl number temperature drops. it means that higher prandtl number, more is the cooling effect. In Figure 5.11, curves are plotted for different values of suction parameter υ_0, keeping Prandtl number P_r and heat source parameter S constant. From the analysis of the curves it is found that decrease in temperature occur due to increase in suction parameter υ_0. In other words, the higher the suction velocity, faster will be the cooling effect.

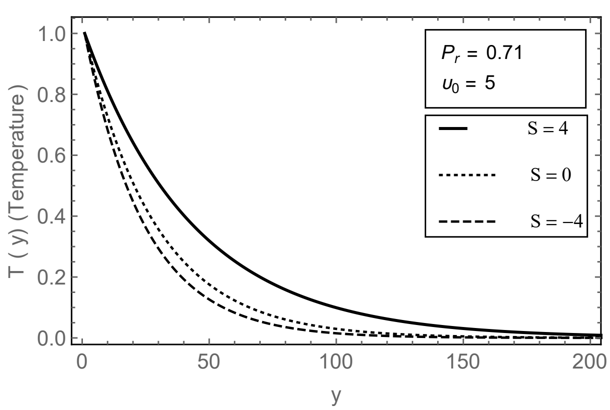

Figure 5.10: Temperature profile against y for different values of S.

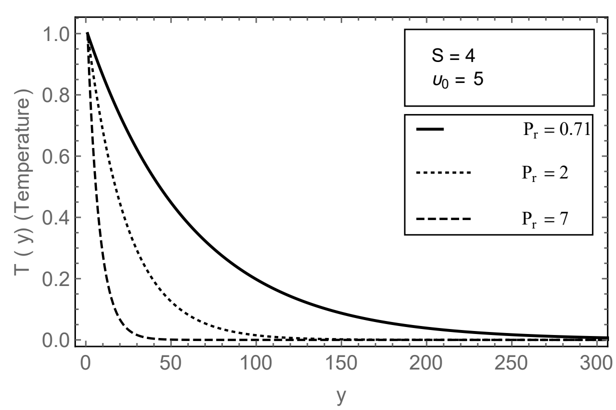

Figure 5.11: Temperature profile against y for different values of P_r.

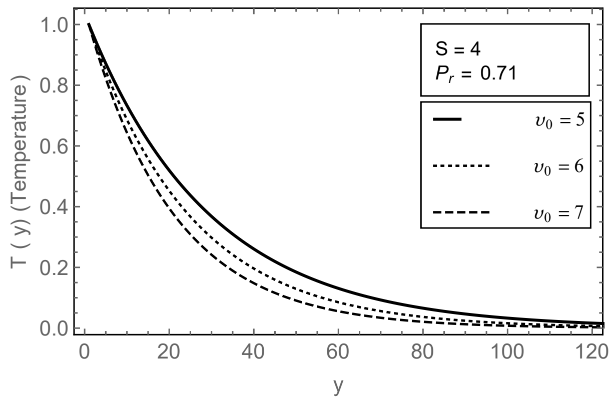

Figure 5.12: Temperature profile against y for different values of υ_0.

5.4.3 Effect on Concentration

The concentration profile is effected by only two parameters, namely Schmidth number and Suction parameter. Figure 5.13, shows that concentration decrease with increase S_c. This effect is due to the presence of heavier diffusing species in the fluid. where Figure 5.14, shows that concentration decrease with large suction.

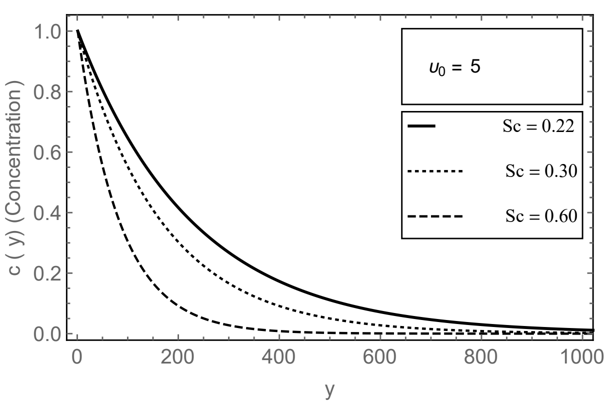

Figure 5.13: Concentration profile against y for different values of S_c.

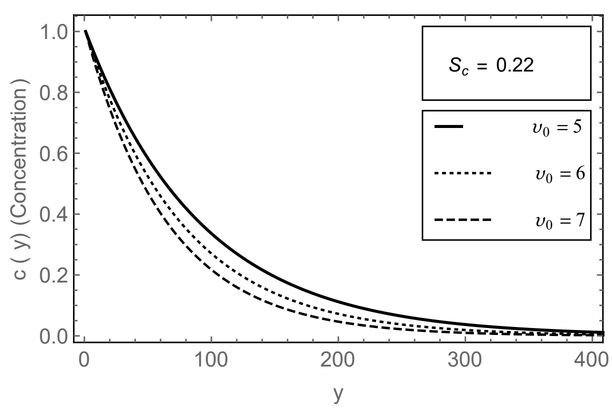

Figure 5.14: Concentration profile against y for different values of υ_0.

5.5 Comparison

The translation parameter a effect only the velocity profile. Therefore in this section we compare the velocity profiles given in chapter 3 and chapter 5 graphically. The result obtained in this chapter is the extension of the problem as we discussed earlier in chapter 3. If we eliminate the term translation a from the solution obtained in this chapter, we obtained the solution of the problem discussed in chapter 3. Thus by doing this we see that the the graphs obtain so are the same as for the solution of problem in Chapter 3 and thus both have the same profiles which are the following.

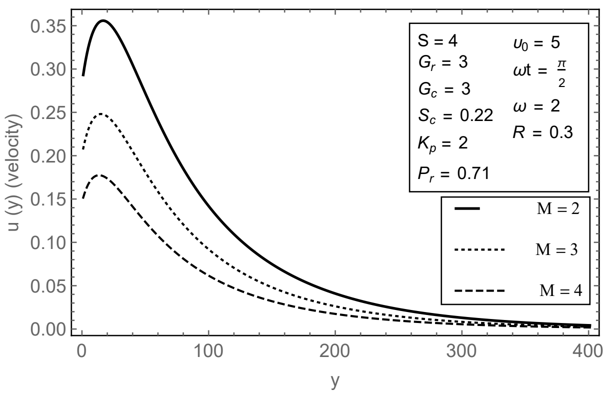

Figure 5.15: Velocity profile against y for different values of M.

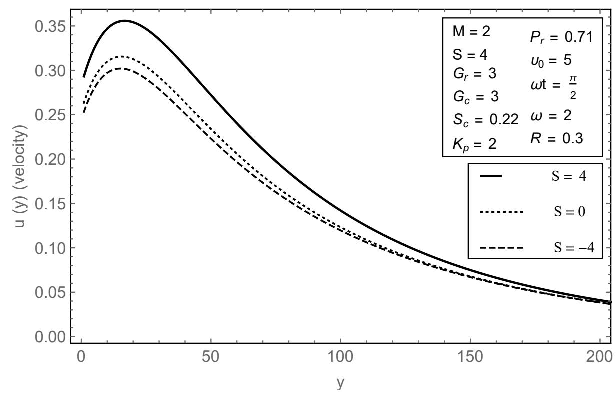

Figure 5.16: Velocity profile against y for different values of S.

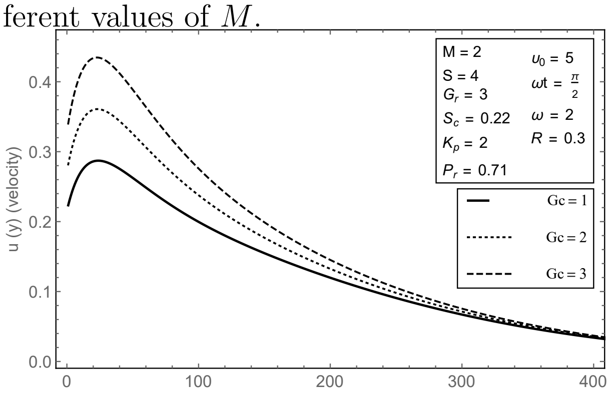

Figure 5.17: Velocity profile against y for different values of G_c.

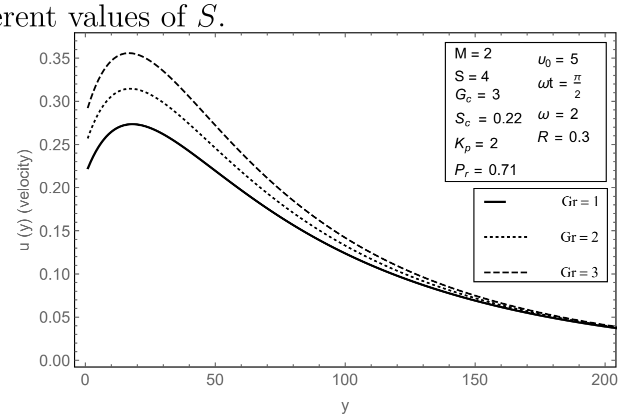

Figure 5.18: Velocity profile against y for different values of G_r.

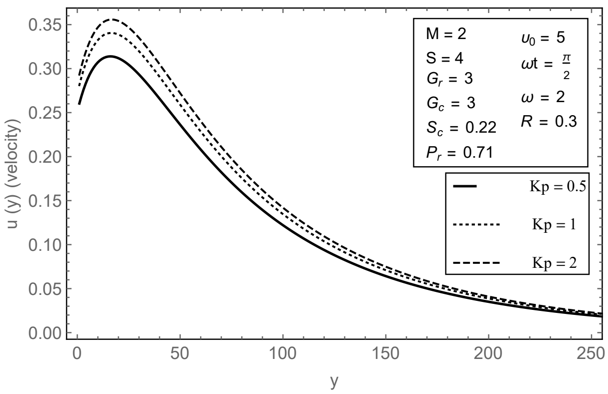

Figure 5.19: Velocity profile against y for different values of K_p.

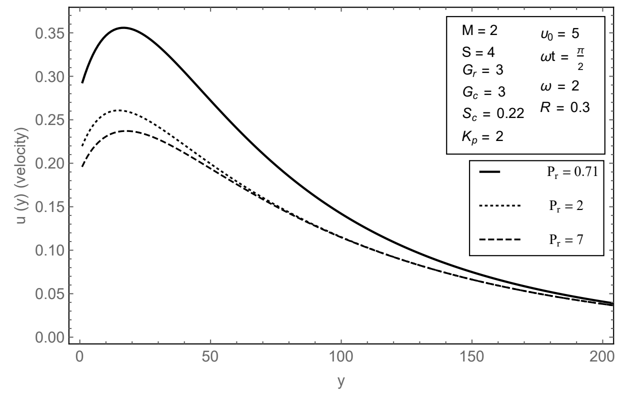

Figure 5.20: Velocity profile against y for different values of P_r.

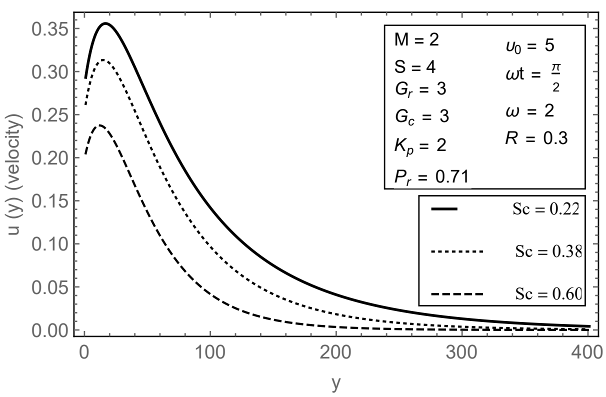

Figure 5.21: Velocity profile against y for different values of S_c.

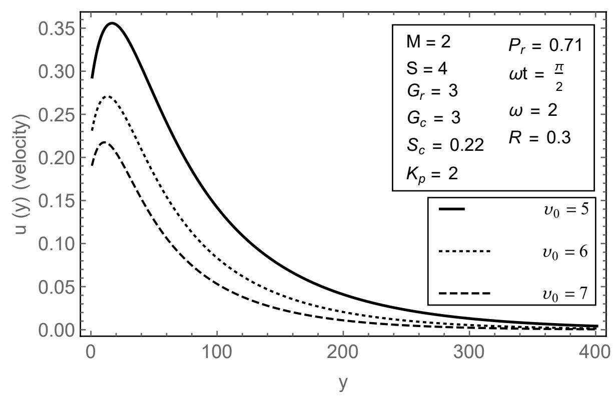

Figure 5.22: Velocity profile against y for different values of υ_0.

5.6 Conclusion

In this thesis we discussed about the natural convection and mass transfer of hydro magnetic fluid flow over an oscillating and translating porous plate, with heat source in the porous medium. We present a graphical representation for velocity, temperature and concentration distribution to see the behavior with the variation of various physical parameters. From the above graphical analysis the following conclusion are obtained.

The greater the translatory motion of the plate less will be the velocity. A strengthen magnetic field will offers more retardation to the flow due to the action fo Lorentz force.

The greater magnetic field less will be the fluid velocity. The greater magnetic field will cause increase in temperature as a result fluid fluid flow will accelerate due to the increase in kinetic energy of molecules. A growing Grashof number for mass and heat transfer will cause increase in the induced due to increase in thermal and mass buoyancy forces, which is responsible for increase in velocity. Where as permeability parameter increase while Prandtl number decrease the magnitude of velocity. The reason is that the fluids with high Prandtl number having high viscosity. Therefore the greater Prandtl number less will be the fluid velocity. The greater Schmidth indicates the presence of heavier diffusing species in the fluid due to which viscosity of the fluid increase. The greater Schmidth number will leads to deaccelerate the velocity. The large reduction occurs in the fluid velocity is cause by large suction. The increase in heat source parameter will cause increase in temperature while increase in Prandtl number and suction velocity will decrease the temperature. Concentration will decrease with both increase in Schmidth number and suction velocity.

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