Dark Energy and the Accelerated Expansion of the Universe

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This Dissertation is dedicated to my parents

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Abstract

In this dissertation the accelerated expansion of the Universe and its causes are discussed. It is thought that dark energy is responsible for the negative pressure which results in the cosmic accelerated expansion. Various candidates of dark energy including the cosmological constant, scalar fields, the Chaplygin gas, holographic dark energy and Ricci dark energy models are discussed in detail.

In the first chapter a brief review of some basic concepts of differential geometry is given then an introduction of the Einstein field equations and their derivation is included and at the end some basic concepts of cosmology are discussed.

The second chapter contains the introduction of dark energy then its candidates are introduced. Furthermore, holographic dark energy and some of its versions, including the entropy corrected and power law entropy corrected holographic dark energy models are reviewed in the remaining part.

In the third chapter we have extended the work of a recently proposed model of Ricci entropy corrected holographic dark energy. In particular, the model *Ricci Power Law Entropy Corrected Holographic Dark Energy* is proposed. Also the dynamics of some scalar fields corresponding to this model are studied.

The dissertation is concluded in the last chapter and some further lines to extend the work are presented.

Chapter 1

Introduction

The expansion of the Universe is an interesting phenomenon in cosmology. The observations of Edwin Hubble opened new ways of thinking about the origin and the evolution of the Universe. By extending the observed rate of the speedily moving apart galaxies, backward in time, the big bang theory originated.

In 1905 Albert Einstein showed that all inertial frames are equivalent and the speed of light is constant in every inertial frame. Later he extended these postulates to non inertial frames as well. In 1915 he published a set of differential equations known as the Einstein field equations (EFEs). These equations relate the bending or curvature of the spacetime with the presence of mass, energy and momentum. These quantities are collectively called stress energy or mass energy density. Einstein believed that the Universe is static and to balance the gravitational pull he added a cosmological constant to the field equations.

After the discovery of the expanding Universe by Hubble, Einstein dropped the cosmological constant from the EFEs calling it the biggest blunder of his career. Later in early 1980s while studying the accelerated expansion of the Universe, the cosmological constant re-entered the EFEs, but this time in the sense of dark energy: a *mysterious* type of energy causing this accelerating expansion [1]. After the experimental acceptance of the Einstein theory of relativity it was used as the standard theory of gravitation. Since gravitation is a universal effect on all kinds of energy and matter, it is important in cosmology and has a dominant role in neutron stars, black holes etc. The evolutionary equations of cosmology are derived with the help of EFEs.

To overcome some other problems in cosmology, particularly the horizon and flatness problems the (important) idea of inflationary cosmology was introduced by Alan Guth in 1980 and later by Katsukiko Sato and Andrei Linde [2]. This hypothesis is about the rapid, exponential expansion of the Universe caused by the vacuum energy density. The vacuum energy density, or dark energy, has become the most important issue under discussion in this century.

The remaining part of this chapter consists of the derivation of the EFEs by using the variational principle, the derivation of evolutionary equations of the standard cosmological model based on the Friedmann-Robertson-Walker (FRW) metric and some problems in cosmology with their possible solutions in the form of inflationary cosmology.

Throughout the dissertation the signature of the metric is $(-,+,+,+)$, unless stated otherwise. The Planck mass and the reduced Planck mass are represented by $m_{pl} = (\sqrt{G})^{-1}$ and $M_{pl} = (\sqrt{8\pi G})^{-1}$ respectively, here G is Newton's gravitational constant and we have taken $c = \hbar \; (= \frac{h}{2\pi}) = 1$. c is the speed of light and h the Planck's constant. These constants are related as $\kappa^2 = 8\pi G = 8\pi m_{pl}^{-2} = M_{pl}^{-2}$.

Since differential geometry is the tool for deriving the EFEs so the basic concepts of differential geometry are reviewed first.

1.1 Review of Differential Geometry

This review has been restricted to the formulations which are important in the study of general relativity. Supposing that reader has basic knowledge of vectors and tensors, we have not included the basics in this dissertation. A discussion of the metric tensor, curvature tensor and other related tensors and some techniques including covariant derivative and affine connections is given in this section.

1.1.1 Minkowski Space and the Metric Tensor

Minkowski space or Minkowski spacetime is a flat four dimensional manifold having three ordinary dimensions of space combined with a single dimension of time. Individual points inside a Minkowski spacetime are known as *events*. An event is represented by a vector having three spatial coordinates and one time coordinate. Such vectors are known as *four vectors*.

Consider any two events A and B in Minkoski spacetime with coordinates (t_A, x_A, y_A, z_A) and (t_B, x_B, y_B, z_B) respectively. The distance between these two events denoted by ds^2 is defined as

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2. \tag{1.1}$$

This is known as the line element of Minkoski spacetime. and it is an invariant second rank tensor. Depending on the sign of ds^2 the interval separating any two events can be categorized in three types:

$$\begin{array}{lll} \text{timelike} & \text{if } ds^2 & < 0, \\ \text{spacelike} & \text{if } ds^2 & > 0, \\ \text{lightlike or null} & \text{if } ds^2 & = 0. \end{array} \tag{1.2}$$

Physically it means that if the interval between two events say A and B is timelike then an inertial frame can be found in which the events occur at the same spatial coordinate. For the spacelike interval we can find an inertial frame in which the events occur at the same time coordinate. Eq. (1.1) can be written as

$$\begin{aligned} ds^2 &= g_{\alpha\beta}dx^\alpha dx^\beta, \qquad \alpha, \beta{=}0,1,2,3 \\ &= g_{00}dx^0dx^0 + g_{ij}dx^idx^j, \qquad i, j{=}1,2,3, \end{aligned} \tag{1.3}$$

where dx^0 and dx^i are its time and spatial components respectively.

In Eq. (1.1) $g_{00} = -c^2$, $g_{11} = g_{22} = g_{33} = 1$. Here $\mathbf{g}$ is a position dependent, non degenerate able second rank covariant symmetric tensor called the metric tensor. It maps any two vectors $\mathbf{u}$ and $\mathbf{v}$ into $\mathbb{R}$ [5] i.e.

$$\begin{aligned} \mathbf{g}(\mathbf{u}, \mathbf{v}) &= g_{ab}u^av^b \\ &= \mathbf{u.v} \end{aligned} \tag{1.4}$$

1.1.2 Covariant Differentiation

The partial derivatives of a tensor of rank one or higher are not tensor. To reserve the invariance of the derivatives more complex rule of differentiation is used, known as *covariant differentiation*. In this process there is a guarantee for the obtained result, to ba a tensor, satisfying the condition of invariance under the coordinate transformation.
The covariant derivative of a contravariant vector $\mathbf{V}$ is given by

$$V^a_{;c} \;=\; V^a_{\;,c} + \Gamma^a_{\;bc} V^b. \tag{1.5}$$

It is denoted by a semicolon below the corresponding vector. Similarly the covariant derivative of a one-form V_a can be written as [5],

$$V_{a;b} = V_{a,b} - \Gamma^c_{ab} V_c, \tag{1.6}$$

and for a mixed tensor it is

$$V^a_{b;d} = V^a_{b,d} + \Gamma^a_{cd} V^c_b - \Gamma^c_{bd} V^{\;a}_c, \tag{1.7}$$

and the covariant derivative of a second rank tensor $A_{ab;c}$ is

$$A_{ab;c} = A_{ab,c} - \Gamma^d_{ac} A_{db} - \Gamma^d_{bc} A_{ad}. \tag{1.8}$$

Here Γ^e_{bd} is the christoffel symbol and its expression in terms of partial derivatives of the components of metric tensor is as follow

$$\Gamma^e_{bd} = \frac{1}{2} g^{ae}(g_{ab,d} + g_{ad,b} - g_{bd,a}). \tag{1.9}$$

Furthermore, the covariant derivative of a metric tensor is zero.

$$g_{ac;b} = 0, \tag{1.10}$$

1.1.3 The Curvature Tensor

The curvature tensor R^a_{bcd} is constructed out of a metric and its first and second derivatives. It is also called the Riemann tensor [5] and is related to the curvature of a manifold. Here

$$R^a_{bcd} = \Gamma^a_{bd,c} - \Gamma^a_{bc,d} + \Gamma^e_{bd}\Gamma^a_{ec} - \Gamma^e_{bc}\Gamma^a_{ed}. \tag{1.11}$$

It can be proved very easily that $R^a_{bcd} = 0$ is a necessary and sufficient condition for a manifold to be flat. The Riemann tensor R^a_{bcd} can also be written as

$$R_{abcd} = g_{ae} R^e_{bcd}. \tag{1.12}$$

It satisfies the following identity, known as the first Bianchi identity [5].

$$R^a_{bcd} + R^a_{cdb} + R^a_{dbc} = 0. \tag{1.13}$$

The covariant derivative of R_{abcd} also satisfies the identity

$$R_{abcd;e} + R_{abde;c} + R_{abec;d} = 0, \tag{1.14}$$

which is called the second Bianchi identity.

The Ricci Tensor: By contracting R^a_{bcd} of the first and third indices a new symmetric tensor called the Ricci tensor is obtained [5] i.e.

$$R_{ab} = R^c_{acb}. \tag{1.15}$$

Also

$$R^c_{cab} = 0. \tag{1.16}$$

$$R^c_{abc} = -R_{ab} \tag{1.17}$$

The Ricci Scalar: Contracting R_{ab} with g_{ab} one gets a scalar 'R', known as the Ricci scalar [5],

$$g^{ab} R_{ab} = R. \tag{1.18}$$

1.2 Einstein's Theory of General Relativity as a Foundation of Cosmology

The EFEs are required to understand the evolutionary behavior of the Universe. Before studying cosmology it is necessary to understand the basic components of these equations.

1.2.1 The Einstein Tensor

There is a very important tensor obtained from a unique combination of Ricci tensor and the curvature tensor, known as the Einstein tensor. It is a symmetric and divergence free tensor, used by Einstein in developing the gravitational field equations.

We start from the second Bianchi Identity, contracting Eq. (1.14) with g^{ab} and using the fact that $g^{ac}_{\ ;\ e} = 0$ from Eq. (1.10) one gets

$$R_{bd;\ e} + R^{c}_{bde;c} - R_{be;d} = 0. \tag{1.19}$$

Contracting again the above obtained equation with g^{bd} we get

$$(R^{ed} - \frac{1}{2}g^{ed}R)_{;d} \quad = \quad 0. \tag{1.20}$$

Where $R^{ed} - \frac{1}{2}g^{ed}R$ can be replaced by a special tensor G^{ed}, called the Einstein tensor [5], i.e.

$$G^{ed} = R^{ed} - \frac{1}{2}g^{ed}R. \tag{1.21}$$

This tensor specifies the geometric properties of a spacetime.

1.2.2 The Energy Momentum Tensor (EMT)

The energy momentum tensor (EMT) is the second most important tensor used in the EFEs describing the density and flux of energy and momentum in the spacetime. It plays an important role as a source of the gravitational field, just as mass is responsible for this field in Newtonian physics. EMT is a second rank, symmetric tensor, describing matter and energy distribution at each point of the spacetime. For the derivation of EMTs, the Lagrangian for matter and energy is used. The action integral is given by

$$I_M \quad = \quad \frac{1}{2\kappa}\int \mathcal{L}_M\sqrt{-g}d^4x. \tag{1.22}$$

Here κ is constant and $\mathcal{L}_M$ represent the Lagrangian density for all the fields except the gravitational field[6]. Using the variation principle, $\delta I = 0$, in particular

$$\delta I_M = 0, \tag{1.23}$$

one can derive the result. As $\mathcal{L}_M$ is a function of metric and its first derivative only, therefore, varying the action with respect to $g^{\mu\nu}$ and its derivative

$$\begin{aligned}\delta I_M &= \int \delta(\mathcal{L}_M\sqrt{-g})dx^4 \\ &= \int \frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial g^{\mu\nu}}\delta g^{\mu\nu}dx^4 + \int \frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}\delta {g^{\mu\nu}}_{,\lambda}dx^4. \end{aligned} \tag{1.24}$$

But we know that

$$\Big[\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}\delta g^{\mu\nu}\Big]_{,\lambda} = \Big[\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}\Big]_{,\lambda}\delta g^{\mu\nu} + \frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}\delta {g^{\mu\nu}}_{,\lambda},$$

or

$$\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}\delta {g^{\mu\nu}}_{,\lambda} = [\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}\delta g^{\mu\nu}]\ _{,\lambda} - [\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}]_{,\lambda}\delta g^{\mu\nu}. \tag{1.25}$$

Using Eq. (1.25) in Eq. (1.24), one gets

$$\begin{aligned}\delta I_M &= \int \frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial g^{\mu\nu}}\delta g^{\mu\nu}dx^4 + \int [\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}\delta g^{\mu\nu}]\ _{,\lambda}dx^4 \\ &- \int [\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}]_{,\lambda}\delta g^{\mu\nu}dx^4. \end{aligned} \tag{1.26}$$

Last term is negligible by Gauss divergence theorem so

$$\begin{aligned}\delta I_M &= \int \left(\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial g^{\mu\nu}} - [\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}]_{,\lambda}\right)\delta g^{\mu\nu}dx^4 \\ &= -\frac{1}{2}\int -\frac{2}{\sqrt{-g}}\left(\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial g^{\mu\nu}} - [\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}]_{,\lambda}\right)\sqrt{-g}\delta g^{\mu\nu}dx^4. \end{aligned} \tag{1.27}$$

The EMT, $T_{\mu\nu}$, is defined by

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\left(\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial g^{\mu\nu}} - [\frac{\partial(\mathcal{L}_M\sqrt{-g})}{\partial {g^{\mu\nu}}_{,\lambda}}]_{,\lambda}\right). \tag{1.28}$$

Using the Eq. (1.28) in Eq. (1.27), it becomes

$$\delta I_M = -\frac{1}{2}\int T_{\mu\nu}\sqrt{-g}\delta g^{\mu\nu}dx^4. \tag{1.29}$$

The EMT for a frictionless fluid, in thermal equilibrium, described by the energy density ρ_o and pressure p reduces to the form [5],

$$T^{\mu\nu} = (\rho_o + p)u^\mu u^\nu - pg^{\mu\nu}, \tag{1.30}$$

here u^μ is the four velocity of the fluid. And

$$T^{\mu\nu}{}_{;\nu} = o, \tag{1.31}$$

1.2.3 The Einstein Field Equations (EFEs)

Einstein's theory of general relativity is based on the fact that we are living in a 4 dimensional space. Einstein related the mass and energy of each point in this space with the help of some useful equations, known as EFEs. These equations describe the geometry and the structure of the space. Using the action integral I_G for the gravitational field [6],

$$I_G = \frac{1}{2\kappa}\int \mathcal{L}_G(g_{\mu\nu})\sqrt{-g}d^4x, \tag{1.32}$$

these equations can be derived. The field Lagrangian is

$$\mathcal{L}_G = R - 2\Lambda, \tag{1.33}$$

here Λ is called the cosmological constant and it is a dimensional parameter with units of $(length)^{-2}$. Using this Lagrangian in Eq. (1.32) one gets

$$\begin{aligned} I_G &= \frac{1}{2\kappa}\int (R-2\Lambda)\sqrt{-g}d^4x \\ &= \frac{1}{2\kappa}\int (g^{\mu\nu}R_{\mu\nu}-2\Lambda)\sqrt{-g}d^4x \\ &= \frac{1}{2\kappa}\int (g^{\mu\nu}R_{\mu\nu}\sqrt{-g}-2\Lambda\sqrt{-g})d^4x. \end{aligned} \tag{1.34}$$

By the least action principle we know that $\delta I = 0$ i.e.

$$\delta I_G = \frac{1}{2\kappa}\int \left(g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g} + R_{\mu\nu}\delta[g^{\mu\nu}\sqrt{-g}] - 2\Lambda\delta\sqrt{-g}\right)d^4x = 0. \tag{1.35}$$

Consider a small volume element V such that on its boundary the variation of the metric and its first derivative are zero. Now introducing local coordinate system in this volume element V, we get $\Gamma^{\mu}_{\nu\gamma} = 0$. So in such a frame the components of the Ricci tensor are

$$R_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu,\lambda} - \Gamma^{\lambda}_{\mu\lambda,\nu},$$

and

$$\begin{aligned} \delta R_{\mu\nu} &= \delta\Gamma^{\lambda}_{\mu\nu,\lambda} - \delta\Gamma^{\lambda}_{\mu\lambda,\nu} \\ &= (\delta\Gamma^{\lambda}_{\mu\nu})_{,\lambda} - (\delta\Gamma^{\lambda}_{\mu\lambda})_{,\nu}. \end{aligned} \tag{1.36}$$

In our proposed frame $g_{\mu\nu,\lambda} = 0$, therefore

$$g^{\mu\nu}\delta R_{\mu\nu} = (g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu} - g^{\mu\lambda}\nu\Gamma^{\nu}_{\mu\nu})_{,\lambda}. \tag{1.37}$$

Integrating over the volume element i.e.

$$\int g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}d^4x = \int (g^{\mu\nu}\delta\Gamma^{\lambda}_{\mu\nu} - g^{\mu\lambda}\nu\Gamma^{b}_{\mu\nu})_{,\lambda}\sqrt{-g}d^4x, \tag{1.38}$$

only boundary term of the integral on the right hand side of Eq. (1.38) will contribute. Since the metric and its derivative vanish at the boundary of V, therefore

$$\int (g^{\mu\nu}\sqrt{-g}\delta R_{\mu\nu})d^4x = 0. \tag{1.39}$$

Using Eq. (1.39) in Eq. (1.35) gives

$$\delta I_G = \frac{1}{2\kappa}\int (R_{\mu\nu}\delta[g^{\mu\nu}\sqrt{-g}] - 2\Lambda\delta\sqrt{-g})d^4x = 0. \tag{1.40}$$

Now

$$\delta\sqrt{-g} = \frac{\partial\sqrt{-g}}{\partial g_{\mu\nu}}\delta g_{\mu\nu} = \frac{-1}{2\sqrt{-g}}(\frac{\partial g}{\partial g_{\mu\nu}})\delta g_{\mu\nu}. \tag{1.41}$$

But

$$g = \sum_{\mu} g_{\mu\nu}\text{Cofac}(g^{\mu\nu}) = \frac{\text{Cofac}(g^{\mu\nu})}{g^{\mu\nu}}, \tag{1.42}$$

where $\text{Cofac}(g^{\mu\nu})$ is the cofactor matrix of the element $g_{\mu\nu}$ in the matrix made of the components of the metric tensor, differentiating Eq. (1.42) with $g_{\mu\nu}$, we get

$$\frac{\partial g}{\partial g_{\mu\nu}} = \text{Cofac}(g^{\mu\nu}) = gg^{\mu\nu}. \tag{1.43}$$

Using Eq. (1.43) in Eq. (1.41) we get

$$\delta\sqrt{-g} = \frac{1}{2}\sqrt{-g}g^{\mu\nu}\delta g_{\mu\nu}. \tag{1.44}$$

To calculate $\delta g_{\mu\nu}$, we have

$$\begin{aligned} g^{\gamma\mu}g_{\mu\nu} &= \delta^{\gamma}_{\nu}, \\ \delta(g^{\gamma\mu}g_{\mu\nu}) &= 0, && (1.45) \\ g^{\gamma\mu}\delta g_{\mu\nu} &= -g_{\mu\nu}\delta g^{\gamma\mu}, && (1.46) \\ \delta g_{\mu\nu} &= -g_{\mu\gamma}g_{\nu\delta}\delta g^{\gamma\delta}. && (1.47) \end{aligned}$$

Using Eq. (1.46) in Eq. (1.44), to get

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g}g_{\mu\nu}\delta g^{\mu\nu}. \tag{1.48}$$

Now

$$\delta(g^{\mu\nu}\sqrt{-g}) = \sqrt{-g}\delta g^{\mu\nu} + g^{\mu\nu}\delta\sqrt{-g}. \tag{1.49}$$

By substituting Eq. (1.48) in Eq. (1.49), it becomes

$$\delta(g^{\mu\nu}\sqrt{-g}) = \sqrt{-g}\left(\delta g^{\mu\nu} - \frac{1}{2}g^{\mu\nu}g_{\mu\nu}\delta g^{\mu\nu}\right). \tag{1.50}$$

Putting Eq. (1.48) and Eq. (1.50) in Eq. (1.40), we have

$$\delta I_G = \frac{1}{2\kappa}\int\sqrt{-g}\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu}\right)\delta g^{\mu\nu}d^4x = 0. \tag{1.51}$$

This is true only if

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 0, \tag{1.52}$$

which are the required vacuum Einstein field equations. To obtain the full version of EFEs we consider that there are other fields present beside the gravitational field. These fields can be described by an appropriate Lagrangian density $\mathcal{L}_M$. The action integral for this case is [6],

$$\begin{aligned} I &= \frac{1}{2\kappa}\int(\mathcal{L}_G + \mathcal{L}_M)\sqrt{-g}d^4x \\ &= \frac{1}{2\kappa}\int\mathcal{L}_G\sqrt{-g}d^4x + \frac{1}{2\kappa}\int\mathcal{L}_M\sqrt{-g}d^4x \\ &= I_G + I_M, \end{aligned} \tag{1.53}$$

the expressions for I_M and I_G are given in Eq. (1.22) and Eq. (1.32), Inserting Eq. (1.51) and Eq. (1.29) in the variational principle $\delta I = \delta I_G + \delta I_M = 0$, we get

$$\delta I = -\frac{1}{2}\int\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} - \kappa T_{\mu\nu}\right)\sqrt{-g}\delta g^{\mu\nu}dx^4 = 0. \tag{1.54}$$

It is true only if

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}, \tag{1.55}$$

which are the required EFEs in the presence of matter. These are 16 coupled highly nonlinear partial differential equations. Using the symmetry of $G_{\mu\nu}$ and $T_{\mu\nu}$ the number of equations reduces to 10. These equations show the principle that *matter tells spacetime how to curve, and curved space tells matter how to move*, (John Wheeler, University of taxes at Austin). The curve of the space is clear from the L.H.S of Eq. (1.55) and the R.H.S explains the location and motion of the matter.

1.3 Basics of Cosmology

Cosmology is the study of the Universe, unlike atomic physics it is the study of the large-scale structure of the Universe and deals with the evolution of these structures over long periods of time. The origin of the Universe, the stages through which our Universe passed to arrive at its present form and its future are studied in this field. When we look at the large scale the matter distribution is very uniform. There is evidence showing that the Universe is isotropic on the large scale to high accuracy for example the Cosmic Microwave Background (CMB) radiation, to be discussed later.

1.3.1 Preliminaries

Here are some very fundamental definitions required for our purpose [1].

1. **Red Shift**: The term red shift is used to describe the situation when an astronomical object is observed moving away from the observer. The light coming from the object that is moving away is increased in wavelength and it shifts to the red part of spectrum. We denote it by z given by $z = (\lambda_1 - \lambda_o)/(\lambda_0)$, here λ_o and λ_1 are emitted and observed wavelengths of the radiation.

2. **Hubble Law**: It is the direct correlation between the distance to a galaxy and its recessional velocity as determined by the red shift. Mathematically, it is written as: $V = H_o r$ here V, H_o and, r are the recessional speed of the galaxy, Hubble constant and the distance of the galaxy from the observer respectively. Hubble constant is used to estimate the size and age of the Universe. Currently its value, measured by the WMAP survey, is $71 km/s/Mpc$. Another way of defining the Hubble constant is to emphasize

on the fact that the space itself is expanding, then $H = \dot{a(t)}/a(t)=$ ratio of the rate of change of the scale factor $a(t)$, to the current value of the scale factor.

3. **Deceleration parameter**: It is the measure of how fast the cosmological expansion is speeding up or slowing down and is denoted by q_o. A positive value of the deceleration parameter implies that the Universe is slowing down, if it is zero it means the Universe is expanding at a constant speed, and a negative value means it is accelerating.

4. **Equation of State (EoS)**: Equation of state is a formula that provides the connection between various macroscopic properties of the system. The EoS of a perfect fluid is given by $p = \omega\rho$, it is characterized by a dimensionless number ω, called EoS parameter. Here p and ρ are the pressure and density of the fluid respectively.

5. **Cosmological Principle**: Cosmological principle is the hypothesis that the Universe is spatially homogeneous and isotropic, i.e. at any particular time, the Universe looks the same in all directions from all positions in space. The homogeneity and the isotropy of the Universe does not apply to the Universe in general, but only to a smeared out Universe i.e. over the cells of very large diameter including galaxies, clusters and super clusters etc .

1.3.2 A Brief History of the Universe

Our Universe is estimated to be 13.7 billion years old. Initially it was supposed that the Universe is static, with no beginning, no end and unchanging. But now research has proved that we are living in a dynamical Universe. The Big Bang model is the standard model scientifically for explaining the origin of every thing. For all practical purposes we consider the Big Bang as the beginning of every thing. The Big Bang model has its beginning with the Hubble Law discovery in 1929. If the Universe is currently expanding then by running time backwards we conclude that the Universe must have been like a point in the past. So a long time ago all matter and energy existed in an infinitely small point of infinite density and has been expanding as our Universe. Since the Big Bang was not an explosion in the Universe, it was an explosion of the Universe, so it has no center from where the Big Bang started. *The Planck epoch*: when the Universe was $10^{-43}sec$ old, i.e. at $t = 10^{-43}sec$ it was $10^{32}K$ hot.

Electrons did not exist at this age, this age is studied by the quantum cosmologists. It was radiation dominated era. When it was $10^{-35} sec$ old, it cooled by a factor of 10^4 to $10^{28}K$. Now the subatomic particles were able to be generated and survive. At $t = 10^{-35} sec$, there occurred a rapid expansion called inflation. When it has been in existence for $1 sec$, it had cooled to $10^{10}K$(100 times hotter than the sun core). When it was a few minutes old the temperature had dropped to 10^9K and light atoms like **H** and **He** could form. Initially it was radiation dominated, then at $t = 44,000\ yrs$ transition from the radiation dominated Universe to a dust dominated model occurred. The dynamics of the Universe was studied by matter and vacuum energy at that time. At $t = 400,000\ yrs$ radiations cooled down and it did not have enough energy to keep the atoms ionized. When free electrons do not have enough energy to overcome the attraction, they get bound to the nuclei and form a neutral atom. So the photons moved freely as there were no free electrons to cause the *Compton scattering*. Thus the Universe became essentially transparent. This time is called the *recombination era*. These photons make out the cosmic microwave background (CMB) radiations. These radiations emitted after approximately 300,000 years after the Big Bang. The formation of stars came to being when the Universe continued to cool and expand. The gigantic clouds of atoms formed a filament like network throughout the Universe, inside these clouds the first stars formed. After the generation of the stars the Universe started to look much the same as it is today.

1.3.3 The Cosmic Microwave Background Radiation

There is whelming evidence the Big Bang. In 1964 the existence of the CMB radiations was predicted by Penzias and Wilson [7]. They noticed that a Dicke radio meter used for satellite communication experiments, had an excess of temperature$3.5K$ which they could not account for. Later observations lower the estimate to about $2.72548 \pm 0.0005K$. It lies primarily in the microwave portion of the electromagnetic spectrum and is invisible to the naked eye. But it fills the Universe and can be detected every where and its distribution is uniform. This uniformity is one compelling reason to interpret the radiation as the remnant heat from the Big Bang, because it would be very difficult to imagine a local source of such uniform radiation. As it is believed that CMB radiation was emitted 13.7 billion years ago, only a $10^5 years$ after the Big Bang so it has traveled over such a long distance. By studying the detailed properties of these radiations we learn about the conditions of very early stages of the Universe at very early times.

1.3.4 The Components of the Cosmological Fluid

In general the Universe is assumed to contain both matter and radiations and a non zero cosmological constant (Λ). Thus the cosmological fluid consist of three components namely matter, radiation, and the vacuum energy (because the modern interpretation of Λ is in terms of energy density of the vacuum). Each component has a different EoS. The total mass density of the Universe is simply the sum of the individual contributions

$$\rho(t) = \rho_m(t) + \rho_r(t) + \rho_\Lambda(t), \tag{1.56}$$

where t is the cosmic time. Although matter and radiation interacted in the early Universe but in this chapter we shall assume that these components do not interact (unless stated otherwise). Each component of the cosmological fluid is modeled as a perfect fluid with EoS of the form $p_i = \omega_i \rho_i$, for pressureless dust $\omega = 0$, for radiation $\omega = 1/3$, and for vacuum $\omega = -1$. There have been many proposed models for the Universe like, Lamda-CDM, deSitter model and some others. But we will discuss only the most accepted and the best model for the observable Universe, the Friedmann-Robertson-Walker (FRW) model.

1.3.5 Friedmann-Robertson-Walker (FRW) Universe

As discussed earlier, the EFEs are used to describe the dynamics of the Universe. The equations are very complicated and non-linear, but by making the assumptions of homogeneity and isotropy the FRW Universe is obtained. The FRW metric is given by [8]

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \tag{1.57}$$

where $a(t)$ is the scale factor that determines the scale of the expansion of the Universe. k measures the spatial curvature, its value can be $k > 0$, $k = 0$ or $k < 0$. For $k > 0$ the curvature of the spatial surfaces is a positive constant and are usually called closed models. For $k = 0$ the spatial surfaces have zero curvature and are called flat models, lastly for $k < 0$ the spatial surfaces have constant negative curvature and are called open models.It is convenient to write Eq. (1.57) in the following

$$ds^2 = -dt^2 + a^2(t)[d\chi^2 + f_k^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)], \tag{1.58}$$

where

$$f_k(\chi) = \begin{cases} \sin^{-1}\chi & k = +1 \\ \chi & k = 0 \\ \sinh\chi & k = -1. \end{cases} \tag{1.59}$$

In the background of the FRW Universe, the components of the Ricci tensor are obtained as:

$$R^0_0 = 3\frac{\ddot{a}}{a}, \tag{1.60}$$

$$R^i_j = \Big(\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{k}{a^2}\Big)\delta^i_j, \tag{1.61}$$

while the Ricci scalar is

$$R = 6\Big(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\Big), \tag{1.62}$$

where $\dot{a}$ denotes the derivative with respect to t. Considering the perfect fluid approximation for the source of energy momentum tensor in the FRW spacetime, from Eq. (1.30), we can write

$$T^\mu_\nu = (\rho + p)u^\mu u_\nu - p\delta^\mu_\nu, \tag{1.63}$$

or

$$T^\mu_\nu = diag(-\rho, p, p, p). \tag{1.64}$$

Here ρ and p are the density and pressure of all the species present in the Universe at a specific epoch.

The purpose of including Λ in the equation was to explain the static model of the Universe: the negative pressure induced by the cosmological constant could balance the equal gravitational pull of the matter without disturbing the spherically symmetric distribution of matter. Using Eq. (1.55), without Λ, and replacing κ by $8\pi G$ we can obtain

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2}, \tag{1.65}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{1.66}$$

Here H is the Hubble parameter. Since the energy momentum tensor is preserved by virtue of Bianchi identities, $T^{\mu\nu}_{;\nu} = 0$, we have the equation of continuity (energy conservation equation) for the FRW Universe

$$\dot{\rho} + 3H(\rho + p) = 0. \tag{1.67}$$

This equation can be derived by using Eqs. (1.65) and (1.66), so two of the Eqs. (1.65)-(1.67) are independent. By using the expression of dimensionless density parameter $\Omega = \rho/\rho_{cr}$, Eq. (1.65) can be written as

$$\Omega - 1 = \frac{k}{(aH)^2}, \tag{1.68}$$

here

$$\rho_{cr} = 3H^2/8\pi G, \tag{1.69}$$

is the critical density, the average density of spacetime required for the flat Universe, and its value is approximately 2×10^{-26} kg/m^3. The spatial geometry of the Universe is determined by the matter distribution, i.e.

$$\Omega > 1 \quad \text{or} \quad \rho > \rho_c \Leftrightarrow k = +1, \tag{1.70}$$

$$\Omega = 1 \quad \text{or} \quad \rho = \rho_c \Leftrightarrow k = 0, \tag{1.71}$$

$$\Omega < 1 \quad \text{or} \quad \rho < \rho_c \Leftrightarrow k = -1. \tag{1.72}$$

As mentioned earlier, from Eqs. (1.65)-(1.67) two equations are independent in three dependent variables p, ρ and $a(t)$. So to solve these equations we need one more equation relating pressure and density, commonly this equation is called equation of state (EoS). Now we shall consider the evolution of the Universe filled with a perfect fluid. The observations show that our Universe is spatially flat, so we shall consider $k = 0$ from here to onwards unless otherwise stated.

Solving the EFEs. (1.65) and (1.67) with EoS, one gets

$$H = \frac{1}{3(1+\omega)(t - t_o)}, \tag{1.73}$$

$$a(t) = a_o(t - t_o)^{\frac{2}{3(1+\omega)}}, \tag{1.74}$$

$$\rho = \rho_o a^{-3(1+\omega)}. \tag{1.75}$$

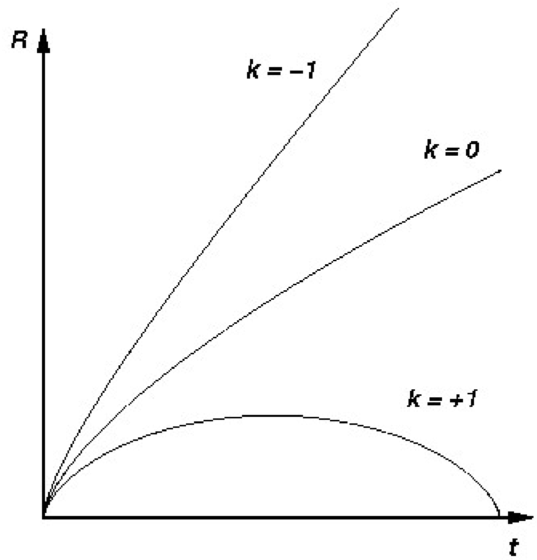

Figure 1.1: Evolution of a Friedmann Universe (scale factor vs time) for different choices of curvature parameter k

Using the values for the EoS parameters of radiation and dust dominated Universe, the above equations reduce to

$$Radiation: \quad a(t) \propto (t - t_o)^{1/2}, \quad \rho \propto a^{-4}, \tag{1.76}$$

$$Dust: \quad a(t) \propto (t - t_o)^{2/3}, \quad \rho \propto a^{-3}. \tag{1.77}$$

These are evolutionary equations of the Universe.

1.3.6 Some Problems in Cosmology

Although the modern cosmology has explained various aspects of the Universe successfully. It has effectively explained the origin of light chemical elements, formation of structures (galaxies and galaxy clusters) and the black body spectrum of CMB radiation etc. But there are still some challenging open problems in cosmology, like the Horizon problem, the Flatness problem and the Monopole problem. The details of the first two problems will be discussed in the next subsection. Restricting to the definition of the *Monopole problem* it can be stated as *Some cosmologists predict that in the early Universe a very large number of heavy, stable magnetic monopoles should have been produced.* But still these are not observed, and even if they are discovered, definitely they will be in a minor amount. The details of this problem are not included.

The Horizon Problem:

Thermal radiation emitted from opposite directions of the sky seem to be at the same temperature. This extreme isotropy of the CMB radiation is the origin of the horizon problem. There is an explanation required for the cause of this interesting observation. It can be explained in the way that the Universe has indeed reached a state of thermal equilibrium by interactions between its different parts in the past, but it is not possible in the Big Bang theory, even if these regions are separated by 1^o on the CMB radiation sky. So the problem is to explain how the regions which have not been in causal contact have nevertheless thermalized to give the same background temperature with an accuracy of $\leq 10^{-4}$. The particle horizon of each photon in the last scattering surface covers only a small patch of the sky and at the time of decoupling its volume $(V_{ph})_d$ can be determined as

$$(V_{ph})_d = \left(\frac{t_d}{t_o}\right)^3 V_o, \tag{1.78}$$

where t_d and t_o are the time of decoupling and the present time respectively, and V_o is the current horizon volume. The radiation and everything else inside the $(V_{ph})_d$ had been in thermal contact and in thermal equilibrium because of the fact that the things occurring inside the particle horizon are in causal contact with each other. The magnitude of V_o at the time of decoupling, denoted by $(V_o)_d$, can be calculated as

$$(V_o)_d = \frac{a^3(t_d)}{a^3(t_o)} V_o = \left(\frac{t_d}{t_o}\right)^2 V_o. \tag{1.79}$$

By comparing the last two equations, we have

$$\frac{(V_{ph})_d}{(V_o)_d} = \frac{t_d}{t_o}. \tag{1.80}$$

The approximated values are: $t_d = 3 \times 10^5$ years and $t_o = 15 \times 10^9$ years. Using these values we find that at the time of decoupling the horizon was a small patch of size 2×10^{-5} of the observable part of our Universe. The problem is to understand how this small connected region be responsible for the same microwave background.

The Flatness Problem:

While studying the Friedmann equation without contribution from the cosmological constant and using the density parameter, $\Omega(t) = \rho/\rho_{cr}$ we have

$$\Omega_{tot} - 1 = \frac{k}{aH^2} = \frac{k}{\dot{a}^2} = \Omega_k. \tag{1.81}$$

If Ω_{tot} is a constant equal to unity, it will remain that forever, otherwise it evolves with time i.e. in the matter dominated Universe we know that $a(t) \propto t^{2/3}$, so $|\Omega_{tot} - 1| \propto t^{2/3}$ while in the radiation dominated phase, $a(t) \propto t^{1/3}$ so, we have $|\Omega_{tot} - 1| \propto t$. Using the last relation we can estimate that at the time of big bang nucleosynthesis, we have $|\Omega_{tot} - 1| \sim 10^{-16}$ and near the Planck time, $|\Omega_{tot} - 1| \sim 10^{-60}$. From this relation we conclude that the Universe was spatially flat in its start. The recent observations show that we are again in the spatially flat phase of the Universe. Therefore, the flatness problem is to find out the reason that why Ω_{tot} is so close to unity or why the spatial curvature k is almost zero.

Cosmic Coincidence Problem:

Recent observations show that density parameters for both, matter and dark energy are almost the same at present; another puzzle in the FRW Universe model, called the coincident problem. From this inspection it is expected that the cosmological constant must be negligible in the previous history of the Universe and in the future Ω_Λ should be near unity. Hence a de Sitter type Universe is predicted. In the history of the Universe, present epoch is very special time in the sense being having $\Omega_\Lambda \sim \Omega_m$ [9]. But we know that matter and dark energy have independent evolutionary equations, $\rho_m \propto a^{-3}$ and $\rho_\Lambda \propto a^{-3(1+\omega_\Lambda)}$. For the justification of the observed data, it is required that $\rho_\Lambda \propto \rho_m$. Hence a modified model is required to fulfill these requirements or to solve these problems.

1.3.7 Cosmological Inflation: A Proposed Solution to the Cosmological Problems

In 1981 Alan Guth proposed a solution for cosmological problems [10]. This is the currently favored candidate for explaining the origin of the Universe structure. The problems faced by the Big Bang model are actually the motivation for the inflationary cosmology. It gives a best solution to the flatness problems and the horizon problem. The fundamental idea behind the inflation is that the Universe undergoes exponential accelerated expansion at some time in the past.

Chapter 2

Causes of Accelerated Expansion of the Universe

Apart from the evidence for acceleration of the Universe, observations also show that there is a mismatch between the gravitational mass and the luminous mass of galaxies and clusters of galaxies. Hence there must be some non-luminous matter in galaxies, called, *dark matter*. Current dark matter models show that it interacts weakly and has gravitational effects.

2.1 Dark Energy (DE)

The efforts made to understand the accelerated expansion of the Universe shows that there is a possibility that this expansion is a consequence of *dark energy* but the existence of DE is not proven yet. The concept of DE was introduced in 1998 [1].

2.1.1 Observational Evidence for Dark Energy

Since it is already discussed that the proposal of DE is favored by the observations of the expanding Universe [1] i.e. this energy is introduced to explain the phenomenon of accelerated expansion of the Universe. Luminosity distances to observe supernova, age of the Universe, and CMB radiations, for example.

Luminosity Distance:

Luminosity distance d_L is defined with the help of luminosity of stellar objects. It is very important in the supernova observations. The luminosity L_s of the source and its energy flux

F at a distance d_L are related by

$$d_L^2 = \frac{L_s}{4\pi F}. \tag{2.1}$$

Let us consider an observer at $\chi = 0$ and a source object with a luminosity L_s located at a coordinate distance χ_s from the observer. The relation between the energy ΔE_1 of the light emitted from the source object with the time interval Δt_1 and the luminosity L_s is:

$$L_s = \frac{\Delta E_1}{\Delta t_1}, \tag{2.2}$$

$$L_0 = \frac{\Delta E_0}{\Delta t_0}, \tag{2.3}$$

where ΔE_0 is the energy reaching the sphere of radius χ_s and L_0 is the luminosity at $\chi = 0$. These energies are proportional to the frequencies of the light at corresponding coordinate distances, i.e. $\Delta E_1 \propto \nu_1$ and $\Delta E_0 \propto \nu_0$. Using the frequency wavelength relation, the speed of light at $\chi = \chi_s$ and $\chi = 0$ is given by $c = \nu_1\lambda_1$ and $c = \nu_0\lambda_0$, since speed of light is constant so

$$\frac{\lambda_0}{\lambda_1} = \frac{\nu_1}{\nu_0} = \frac{\Delta E_1}{\Delta E_0} = \frac{\Delta t_0}{\Delta t_1} = 1 + z. \tag{2.4}$$

here λ denotes the wavelength of the light. Eq. (2.4) along with Eqs. (2.2) and (2.3) yields

$$L_s = L_0(1 + z)^2. \tag{2.5}$$

Since the light moves along the geodesics

$$ds^2 = -dt^2 + a^2(t)d\chi^2 = 0, \tag{2.6}$$

while traveling along the χ-direction, so using Eq. (2.6) we obtain

$$\chi_s = \int_0^{\chi_s} d\chi = \int_{t_1}^{t_0} \frac{dt}{a(t)} = \frac{1}{a_0 H_0}\int_0^z \frac{dz'}{h(z')}, \tag{2.7}$$

here $h(z) = \frac{H(z)}{H_0}$. Using the FRW metric given in Eq. (1.57) the area of the sphere at $t = t_0$ is given by

$$S = 4\pi(a_0 f_K(\chi_s))^2, \tag{2.8}$$

where $f_K(\chi_s)$ is defined in Eq. (1.59). Thus the observed energy flux becomes

$$F = \frac{L_0}{4\pi(a_0 f_K(\chi_s))^2}, \tag{2.9}$$

Eq. (2.1) with Eqs. (2.7) and (2.9) gives the luminosity distance for an expanding Universe as

$$d_L = a_0 f_K(\chi_s)(1+z), \tag{2.10}$$

which for a flat FRW background ($f_K(\chi) = \chi$) and with a use of Eq. (2.7) becomes

$$d_L = \frac{1+z}{H_0} \int_0^z \frac{dz'}{h(z')}. \tag{2.11}$$

Using this equation the Hubble parameter $H(z)$ can be written as

$$\left\{ \frac{d}{dz} \left(\frac{d_L(z)}{1+z} \right) \right\}^{-1}. \tag{2.12}$$

This relationship shows that the expansion rate of the Universe can be determined if we are able to make the measurements of the luminosity distance by observations. The evolutionary equation of density ρ given in Eq. (1.75) can be written by adding all the components present in the Universe and using $1+z = a/a_0$, as follow:

$$\rho = \Sigma_i \rho^{(0)} (a/a_0)^{-3(1+\omega_i)} = \Sigma_i \rho^{(0)} (1+z)^{3(1+\omega_i)}, \tag{2.13}$$

where $\rho^{(0)}$ is present energy density and $i = 1, 2, 3$ correspond to matter, radiation, and dark energy respectively. Using Eq. (1.73) the Hubble parameter can be written as

$$H^2 = H_0^2 \Sigma_i \Omega_i^{(0)} (1+z)^{3(1+\omega_i)}, \tag{2.14}$$

where $\Omega_i^{(0)}$ is the density parameter for each component of the present Universe. Thus the expression of luminosity distance in the flat FRW Universe becomes

$$d_L = \frac{1+z}{H_0} \int_0^z \frac{dz'}{\sqrt{\Sigma_i \Omega_i^{(0)} (1+z')^{3(1+\omega_i)}}}. \tag{2.15}$$

Plotting Eq. (2.15) for two components Universe: matter and the cosmological constant, shows that luminosity distance increases when the cosmological constant is present

Age of the Universe:

Age of the Universe, t_0, is calculated by comparing it with the age of the oldest stellar objects, t_s, subject to $t_0 > t_s$. The age of *Globular Cluster in Milkiway* determined separately by two cosmologist *Jimenez* and *Hensen* is $13.5 \pm 2Gyrs$ and $12.7 \pm 0.7Gyrs$ respectively [14]. Thus the lower bound of the age of the Universe becomes $t_0 > 11 - 12Gyrs$.

Using ρ given in Eq. (2.13) and considering the contributions from radiation ($\omega = 1/3$), dust ($\omega = 0$) and cosmological constant ($\omega = -1$), the Friedmann equation given in Eq. (1.65) becomes

$$H^2 = H_0^2[\Omega_r^{(0)}(a/a_0)^{-4} + \Omega_m^{(0)}(a/a_0)^{-3} + \Omega_\Lambda^{(0)} - \Omega_k^{(0)}(a/a_0)^{-2}]. \tag{2.16}$$

Here Ω_m, Ω_r, and and Ω_Λ correspond to energy densities of matter, radiation and dark energy respectively. The age of the Universe can be obtained from the integral

$$\int_0^{t_0} dt = \int_0^\infty \frac{dz}{H(1+z)} \tag{2.17}$$

$$= \int_0^\infty \frac{dz}{H_0 x[\Omega_r^{(0)}x^4 + \Omega_m^{(0)}x^3 + \Omega_\Lambda^{(0)} - \Omega_k^{(0)}x^2]^{1/2}}. \tag{2.18}$$

In these equations we have used $1+z = a_0/a$ and $x(z) \equiv 1+z$ respectively. Since this integral is hardly effected by $z \geq 1,000$ it is reasonable to neglect the radiation dominated Universe while calculating the age of the Universe, i.e. using $\Omega_r^{(0)} = 0$. Further, we start from the case when the cosmological constant $\Omega_\Lambda^{(0)} = 0$, so $\Omega_m^{(0)} - 1 = \Omega_k^{(0)}$, and Eq. (2.18) gives

$$t_0 = \int_0^\infty \frac{dz}{(1+z)^2\sqrt{1+\Omega_m^{(0)}z}}. \tag{2.19}$$

For a flat Universe $\Omega_m^{(0)} = 1$ and $\Omega_k^{(0)} = 0$, in that case above expressions reduces to

$$t_0 = \frac{2}{3H_0}. \tag{2.20}$$

Using the value of the Hubble parameter obtained from the recent observations of the Hubble telescope key project [15], $H_0^{-1} = 9.776h^{-1}Gyr, \quad 0.64 < h < 0.80$, age of the Universe is obtained as $t_0 = 8 - 10Gyrs$. This value is not consistent with the lower bound set by the age of stellar objects.

From Eq. (2.19) we can see that in case of an open Universe ($\Omega_m^{(0)} < 1$) the age of the Universe is larger than the flat case. Observations of the CMB radiations [16] show that curvature of the Universe is very small, however even in this case its age remains less than the age of the oldest stellar object. For the flat case and $\Omega_\Lambda^{(0)} \neq 0$, Eq. (2.18) gives

$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)\sqrt{(1+z)^3\Omega_m^{(0)} + \Omega_\Lambda^{(0)}}} = \frac{2}{3\sqrt{\Omega_\Lambda^{(0)}}} \ln\left(\frac{1+\sqrt{\Omega_\Lambda^{(0)}}}{\Omega_m^{(0)}}\right). \tag{2.21}$$

Here $\Omega_\Lambda^{(0)} + \Omega_m^{(0)} = 1$ and the age increases as $\Omega_m^{(0)}$ decreases. For the recently observed energy density parameters, $\Omega_\Lambda^{(0)} = 0.7$ and $\Omega_m^{(0)} = 0.3$, the above equation gives $t_0 H_0 = 0.96$ for $h = 0.72$ and the corresponding age becomes $13.1Gyrs$, satisfying the lower age limit set by the age of the oldest stellar object. Thus to solve problem of age of the Universe presence of Λ, which corresponds to the dark energy, is needed. There are some other age estimates that are more precise. The best estimate is $13.76 \pm 0.02Gyrs$.

2.2 Candidates of DE

In this subsection some of the most accepted candidates of dark energy are included. At present there are a lot of proposals for theoretical candidates of DE including cosmological constant, scalar fields and Chaplygin gas to name some of them.

Cosmological Constant (Λ):

The oldest candidate of DE is Cosmological Constant. It is the simplest candidate having constant energy density in space and time [1]. It was originally introduced by Einstein in 1917 to achieve a static Universe. For the FRW Universe, defined in Eq. (1.57), the modified EFEs with cosmological constant give the following equations

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}, \tag{2.22}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}. \tag{2.23}$$

It is clear from Eq. (2.23) that Λ contributes positively to the pressure term and causes a repulsive effect. In the pressureless ($p = 0$), matter dominated Universe, the static Universe, corresponds to

$$\rho = \Lambda/4\pi G, \quad \Lambda = k/a^2. \tag{2.24}$$

Eq. (2.24) shows that the density ρ in the Universe is determined by Λ. In fact, if $\Lambda/3 > 4\pi G\rho/3$ then a growing a in Eq. (2.23), shows that the Universe is not static. If $\Lambda/3 < 4\pi G\rho/3$, the Universe moves away from the static point with decreasing a.

Cosmological Constant Problem:

If the cosmological constant energy (ρ_Λ) is the vacuum energy density then the observed energy density related to ρ_Λ is very small compared to the theoretical one. This problem asks

why is the energy density today so small compared to the typical particle physics scale?
For an accelerated expansion of the Universe the observed value of cosmological constant Λ is, of the order of square of the present the Hubble parameter H_0, equal to the Hubble parameter [1]

$$\Lambda \approx H_0^2 = (2.13 \times 10^{-42})^2 GeV^4. \tag{2.25}$$

Interpreting it as an energy density, we get

$$\rho_\Lambda = \frac{\Lambda m_{pl}^2}{8\pi} \approx 10^{-47} GeV^4. \tag{2.26}$$

The vacuum energy density can be calculated by summing the zero-point energies of the quantum fields of an arbitrary mass m is estimated as [1]

$$\rho_{vac} \approx 10^{74} GeV^4. \tag{2.27}$$

Comparison of Eqs. (2.26) and (2.27) shows that theoretical energy density is 10^{121} orders of magnitude greater than the observed one. Due to this cosmological problem many take a cosmological constant zero, and introduce other mechanisms like scalar fields to explain DE [1].

Scalar Fields and Dark Energy:
A class of scalar fields is one of the most important and promising candidate of DE. Experimentally there is no evidence for the scalar fields. but they are required in all unification theories. Due to the cosmological constant problem it has been proposed that, due to coupling with other matter fields, the vacuum energy density can be a time-dependent function, instead of a constant quantity [1]. There is need of a model for the description of dark energy, which could solve the cosmological problem.

In general we can take the EoS of dark energy to change with time. A variety of scalar field DE models have been proposed in the literature. Some of these are: quintessence, k-essence, phantom energy, dilatonic dark energy and tachyon scalar field. While studying the scalar fields we assume that Λ is zero due to some unknown mechanism and DE is caused by the dynamics of the scalar field. Although the cosmological constant problem remains unsolved even in this case, but it is another way of dealing with DE and its problems. The best possibility for this model is a dynamical, time dependent scalar field Φ whose potential

$V(\Phi)$ evolves slowly. For a homogeneous, time dependent, scalar field Φ we can take the energy density and pressure to be defined by:

$$\rho_\Phi = \frac{\sigma}{2}\dot{\Phi}^2 + V(\Phi), \tag{2.28}$$

$$p_\Phi = \frac{\sigma}{2}\dot{\Phi}^2 - V(\Phi). \tag{2.29}$$

$\sigma = -1$ corresponds to the phantom, while $\sigma = +1$ represents the standard scalar field known as quintessence field, and $V(\Phi)$ is the potential. In this case w_Φ is given by

$$w_\Phi = \frac{p_\Phi}{\rho_\Phi} = \frac{\sigma\dot{\Phi}^2 - 2V(\Phi)}{\sigma\dot{\Phi}^2 + 2V(\Phi)}. \tag{2.30}$$

Using Eqs. (2.28) and (2.29) we get the kinetic energy and the scalar potential terms as

$$\dot{\Phi}^2 = \frac{1}{\sigma}(1 + \omega_\Phi)\rho_\Phi, \tag{2.31}$$

$$V(\Phi) = \frac{1}{2}(1 - \omega_\Phi)\rho_\Phi. \tag{2.32}$$

Quintessence Scalar Field Model (Q):

Its EoS is dependent on the redshift $\omega(z)$, particularly $-1 < \omega(z) \leq 0$ and it evolves with time. Hence the cosmological constant model and Q model have different expansion history of the Universe. The action for Q is given by [17, 18]

$$S = \int d^4x\sqrt{-g}[\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi)], \tag{2.33}$$

here $V(\Phi)$ is potential of the field that leads to the accelerating Universe. In the flat Friedmann background, using $\sigma = +1$ in Eq. (2.30) shows that the EoS parameter for the field Φ lies in the range $-1 \leq \omega \leq 1$. So the continuity equation

$$\dot{\rho} + 3H(\rho + p) = 0, \tag{2.34}$$

becomes

$$\rho = \rho_o \exp\left[-\int 3(1 + \omega_\Phi)\frac{da}{a}\right], \tag{2.35}$$

or

$$\rho = \rho_o a^{-3(1+\omega_\Phi)}, \tag{2.36}$$

where ρ_o is the constant of integration. If $\omega_\Phi = -1$ then Eq. (2.36) gives $\rho = const$, which corresponds to the slow roll inflation i.e. $\dot{\Phi}^2 \ll V(\Phi)$, and if $\omega_\Phi = +1$, then $\dot{\Phi}^2 \gg V(\Phi)$, which makes $\rho \propto a^{-6}$. For all other values of ω the energy density behaves as

$$\rho \propto a^{-m}, \quad 0 < m < 6. \tag{2.37}$$

For $0 \leq m < 2$ the Universe is in an accelerated expansion, since $\omega_\Phi = -1/3$, is a separation between the accelerating and decelerating Universes. When $-1 < \omega < -1/3$ the Universe is in quintessence phase, for $\omega < -1$ it is in phantom phase and at $\omega = -1$ the Universe is dominated by the cosmological constant i.e. for accelerating Universe, $\dot{H} > 0$ is required and vice versa. But there is a need of theoretical explanation for the transition from $\dot{H} < 0$ to $\dot{H} > 0$.

k-essence:

k-essence was first introduced as a possible model of inflation. Later it was noted that k-essence can also yield interesting models for DE. The negative pressure of k-essence model is from the non-linear kinetic energy of scalar field. The speed of evolution of the k-essence changes in dynamics. The general form of the scalar field action S_K for k-essence scalar field as a function of ϕ and $\chi = \dot{\phi}/2$ is given by [19]

$$S_K = \int d^4x \sqrt{-g}\, p\left(\phi, \chi\right). \tag{2.38}$$

From the Lagrangian given in Eq. (2.38), the pressure, $p\left(\phi, \chi\right)$, and the energy density, $\rho_\Lambda\left(\phi, \chi\right)$, of the k-essence can be obtained respectively as:

$$p\left(\phi, \chi\right) = f\left(\phi\right)\left(-\chi + \chi^2\right), \tag{2.39}$$

$$\rho\left(\phi, \chi\right) = f\left(\phi\right)\left(-\chi + 3\chi^2\right). \tag{2.40}$$

The EoS parameter ω_K of k-essence scalar field is

$$\omega_K = \frac{p\left(\phi, \chi\right)}{\rho\left(\phi, \chi\right)} = \frac{\chi - 1}{3\chi - 1}. \tag{2.41}$$

It is clear from the above equation that $\omega_K < -1$ (the phantom behavior of k-essence scalar field) is possible if χ lies in the range $1/3 < \chi < 1/2$. Hence the kinetic term χ plays an important role in finding the EoS of the scalar field.

Tachyon Field:

Tachyon scalar field is one of the possible sources which can provide negative pressure for acceleration of the Universe. The EoS parameter of a rolling tachyon smoothly changes between -1 and 0 [22]. Here we shall start from the Lagrangian L_{tach} without worrying about its origin. Particularly without making attempts to connect the form of $V(\phi)$ with the string theoretic models. The effective Lagrangian for the tachyon field was taken to be [24]

$$L = -V(\phi)\sqrt{1 - g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}, \tag{2.42}$$

where $V(\phi)$ represents the potential of tachyon and $g^{\mu\nu}$ is the metric tensor. The energy density ρ_ϕ and pressure p_ϕ for the tachyon field are respectively given by:

$$\rho_\phi = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \tag{2.43}$$

$$p_\phi = -V(\phi)\sqrt{1-\dot{\phi}^2}. \tag{2.44}$$

The EoS parameter of tachyon scalar field reads

$$\omega_\phi = \frac{p_\phi}{\rho_\phi} = \dot{\phi}^2 - 1. \tag{2.45}$$

In the FRW background we have derived the Friedmann equations. Using Eqs. (1.65) and (1.67) we obtain the equations of motion as

$$H^2 = \frac{8\pi G V(\phi)}{3\sqrt{1-\dot{\phi}^2}}, \tag{2.46}$$

and

$$\frac{\ddot{\phi}}{1-\dot{\phi}} + 3H\dot{\phi} + \frac{1}{V}\frac{dV}{d\phi} = 0. \tag{2.47}$$

Combining the above equations one gets

$$\frac{\ddot{a}}{a} = \frac{8\pi G V(\phi)}{3\sqrt{1-\dot{\phi}^2}}\left(1 - \frac{3}{2}\dot{\phi}^2\right). \tag{2.48}$$

So for an accelerated expansion $\dot{\phi}^2 < \frac{2}{3}$ is required.

Phantom (ghost) Field:

So far we have discussed about the scalar field models with $\omega \geq -1$, but recently it is observed that the EoS parameter lies in the narrow strip around $\omega = -1$. The region where the EoS parameter is less than -1 is called Phantom (ghost) dark energy.

It is a scalar field with a negative kinetic energy, and Lagrangian,

$$L_{\Phi} = -\frac{1}{2}\partial_{\mu}\Phi\partial^{\mu}\Phi - V(\Phi). \tag{2.49}$$

The action for the phantom field minimally coupled to gravity is given by

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}(\nabla\phi)^2 - V(\phi)\right]. \tag{2.50}$$

For this field we consider the perfect fluid energy momentum tensor, with the Lagrangian given in Eq. (2.49), to express the energy density and the pressure respectively as:

$$\rho_{\Phi} = \frac{1}{2}\dot{\Phi}^2 + V(\Phi), \tag{2.51}$$

$$\rho_{\Phi} = \frac{1}{2}\dot{\Phi}^2 - V(\Phi). \tag{2.52}$$

Since $\omega < -1$, the energy density of phantom energy varies proportional to the power of scale factor $a(t)$

$$\rho \propto a^{3|1+\omega|}, \quad \omega < -1, \tag{2.53}$$

unlike the behavior of ordinary matter. Caldwell called this matter *phantom energy* [25]. The most remarkable feature of phantom energy model is that it predicts the end of the Universe with a *Big Rip*, breaking the fabric of the spacetime. The idea of Big Rip was first introduced by Caldwell *et al* [26]. Explaining the final singularity of this Universe in the form of Big Rip. The literature contains work on how the Big Rip concept could be avoided [25, 27]. The interaction between matter and energy is the best explanation for this purpose. Many models on the interacting DE and DM have been proposed [28, 29].

Chaplygin Gas (CG)

The Chaplygin gas interpolates the evolution of Universe from dust phase to the acceleration phase. It has been suggested as an interesting candidate of unified model of dark matter and dark energy and it fits best with the observational data [30]. The CG is defined as

$$p_{\Lambda} = -\frac{D}{\rho_{\Lambda}} \tag{2.54}$$

Here D is a positive constant. The density evolution of CG, calculated by using the density conservation equation, is given by

$$\rho_\Lambda = \left[D + \frac{B}{a^6}\right]^{1/2}, \tag{2.55}$$

where B is the constant of integration. There are some generalizations of this fluid, each different from the other, due to inclusion of new parameters.

Generalized Chplygin gas (GCG) is a commonly studied generalization of CG given by $p_\Lambda = -A\rho_\Lambda^{-\alpha}$, here $0 < \alpha \leq 1$ [31]. For $\alpha > 0$, the pressure decreased relative to the energy density in the early Universe. In late Universe the negative pressure becomes important for realizing the cosmic acceleration. Hence a fluid with EoS, like Chaplygin gas behaves like dust at early stages of Universe and as dark energy in late times. So it can in principle replace both. The corresponding equation of density evolution for GCG is

$$\rho_\Lambda = \left[D + \frac{B}{a^{3(1+\alpha)}}\right]^{\frac{1}{1+\alpha}}. \tag{2.56}$$

The kinetic energy and the scalar potential terms of Chaplygin gas are the same as for a homogeneous and time dependent scalar field 'Φ' , given in Eqs. (2.31) and (2.32). The more general form of CG, known as modified Chaplygin gas (MCG), was proposed by Debnath *et al* [32], with EoS given by:

$$p_\Lambda = A\rho - \frac{D}{\rho_\Lambda^\alpha}. \tag{2.57}$$

Here X and D are positive constants and α is an arbitrary constant. The density evolution of MCG, calculated by using the density conservation equation, is given by

$$\rho_\Lambda = \left[\frac{D}{1+A} + \frac{B}{a^{3(1+\alpha)(1+A)}}\right]^{\frac{1}{1+\alpha}}, \tag{2.58}$$

where B is the constant of integration. CMB radiation data shows that $-0.35 \leq A \leq 0.025$ and $-0.021 \leq \alpha \leq 0.54$ [33]. It is recently shown that for MCG stable attractor solution exists at $\omega = -1$ i.e. the EoS of MCG approaches to $\omega = -1$ from either $\omega > -1$ or $\omega < -1$, independent of the choice of its initial density parameter and the ratio of pressure to critical density [34]. This result suggests that the Universe would not end up in a Big Rip. MCG best fits with other cosmological parameters if $\alpha = 1/4$ and $A = 1/3$ [35]. Further a *new modified Chaplygin gas* is an other extension of CG.

Problems Related to DE:

There are some problems related to dark energy:

1. Observations show that the EoS for dark energy is not a constant but it varies with time.

2. Determine the EoS parameter such that $-1.38 < \omega < -0.82$

3. Explanation of the phantom divide: the transition from the quintessence regime to phantom regime.

4. The explanation for the same order of magnitude of energy densities of matter and dark energy in the present time of the Universe, (*cosmic coincidence problem*).

5. If we are sure about the existence of DE, its magnitude should be close to the cosmological energy density. Theoretical considerations roughly measure the value of ρ_{vac} as $10^{50} - 10^{120}$ times higher than is deduced from the empirical data, it is known as *Fine Tuning Problem*. Hence the other main problem in this scenario is *why observed vacuum energy is so small.*

6. If DE is not the source of accelerated expansion of the Universe then which agent is responsible for that? Is it explainable by the present knowledge of physics or some new physics is required for its explanation?

Recently many new models have been constructed, to overcome these problems.

2.3 Holographic Dark Energy(HDE)

It is proposed that a consistent theory of quantum gravity should give complete and correct description of DE.

Holographic Principle:

This principle was proposed by ‘t Hoofs and Susskind [36] it states that the entropy of a system does not change with its volume but it changes with its surface area.

Holographic Principle is about encoding information from (*D+1*)-dimensional space onto *D*-dimensional space. For a better understanding, we can take it as the interference pattern on a photographic plate from a *3*-dimensional object. Using the holographic principle a model of HDE was proposed [37].

2.3.1 Introduction of HDE

Recently a paradigm for DE has been constructed by using holographic principle of quantum gravity theory and it represents some interesting features of DE. It is considered as a form of gravitational DE. This paradigm may solve the coincidence problem also. Cohen has shown in [38] that in a quantum field theory a short distance cut-off Λ is related to a long distance cut-off L_Λ due to the limit set by forming a black hole. Since for a box of size L and ultraviolet cut-off Λ, the effective field theory gives the relation for entropy, S, scaling as: $S \sim L^3\Lambda^3$ [39]. Bekenstein's postulate gives the maximum entropy of this box growing only with the area of the box. So this postulate can hold in an effective field theory only if the following condition fulfills,

$$L^3\Lambda^3 \leq S_{BH} \equiv L^2 M_p^2. \tag{2.59}$$

It gives

$$\Lambda \leq \left(\frac{M_p^2}{L}\right)^{1/3}. \tag{2.60}$$

If $\rho_\Lambda = \Lambda^4$ is the quantum zero point energy density caused by a short distance cut-off, then use of Eq. (2.60) in the expression $L^3\Lambda^4$ gives $L^3\rho_\Lambda \leq LM_p^2$, which yields that the total energy of the system of size L_Λ should not exceed the mass of black hole of the same size. We can write it as $L_\Lambda^3\rho_\Lambda \leq L_\Lambda M_p^2$, here M_p^2 is the reduced planck's constant. When L is saturated the equality is obtained

$$\rho_\Lambda = 3c^2 M_p^2 L_\Lambda^{-2}, \tag{2.61}$$

it is the HDE for that L. Here $3c^2$ is introduced for convenience it is a numerical constant.

It is interesting to note that the derivation and definition of HDE depends on the entropy-area relation, $S \sim A \sim L^2$, but not only on holographic principle. Recent observational data shows that for a non-flat Universe $c = 0.815^{+0.179}_{-0.139}$ and for the flat case $c = 0.818^{+0.113}_{-0.097}$ [40, 41]. It is claimed that the reasonability and the simplicity of HDE model provide a suitable frame to investigate the DE problems compared to other models proposed in literature. The fundamental assumption that matter and HDE do not conserve separately solves the coincidence problem. In recent years HDE is studied as a possible candidate for DE and many models have been proposed by extending this idea.

2.3.2 Proposals for Horizon (L)

When HDE was introduced, the next target was to find an appropriate horizon L. In this subsection we shall discuss some important horizons proposed for the HDE.

$L =$ Hubble Horizon H^{-1}:

In the first attempt, size of the current Universe, Hubble scale was chosen as Horizon. This resulted in a DE density comparable to the present day DE density, while Hsu *et al* have recently pointed out that in this case the EoS does not match the experimental data Hsu *et al* [42]. His argument can be refined by replacing ρ in the Friedman equation $\rho = 3M_p^2H^2$ with $\rho_m + \rho_\Lambda$, and using ρ_Λ given in Eq. (2.61) with

$$L = H^{-1}, \tag{2.62}$$

we get

$$\rho_m = 3M_p^2H^2[1 - c^2], \tag{2.63}$$

which shows that the behavior of ρ_m varies as H^2, but Eq. (2.61) along with Eq. (2.62) shows that ρ_Λ also has the same behavior. But we know that ρ_m scales with the Universe scale factor as a^{-3}, so ρ_Λ will also scale similarly. It implies that DE is pressureless i.e. $P = \omega\rho$; $\omega = 0$. While $\omega < -1/3$ is the requirement of an accelerating Universe and recently obtained data indicates $\omega < -0.76$ with 95% confidence level [43]. Hence they find that the proposal for horizon given in Eq. (2.62) is not able to explain the DE dominated (present) Universe.

Particle Horizon $L = R_h$:

The particle horizon, R_h, is used by Fischler and Susskind [44] and is defined as

$$R_h = a\int_0^t \frac{dt}{a} = a\int_0^a \frac{da}{Ha^2}. \tag{2.64}$$

Replacing this L in Eq. (2.61) the solution of Friedmann equation with another energy component say matter is obtained. But unluckily the required results are not obtained in this case also. We shall briefly discuss it here. In the dark energy dominated Universe the Friedmann equation reduces to

$$\frac{1}{Ha^2} = c\frac{d}{da}(\frac{1}{Ha}). \tag{2.65}$$

One gets $H^{-1} = \alpha a^{1+1/c}$; $\alpha =$ constant. The dark energy becomes

$$\rho_\Lambda = 3\alpha^2 M_p^2 a^{-2(1+1/c)}. \tag{2.66}$$

Hence the EoS parameter becomes $\omega = \frac{-1}{3} + \frac{2}{3c} > \frac{-1}{3}$. Since $c = HR_h > 0$ always, so $\omega > \frac{-1}{3}$ and Eq. (2.65) shows that Hubble scale $1/H$ compared to scale factor a is always increasing. But from the inflationary cosmology we know that for an accelerating Universe a shrinking Hubble scale is required. Hence this proposal does not gives the required results [45].

Future Event Horizon:

In order to get the results which correspond to an accelerating Universe the particle horizon is replaced by the future event horizon

$$R_h = a\int_t^\infty \frac{dt}{a} = a\int_a^\infty \frac{da}{Ha^2}. \tag{2.67}$$

This Horizon is the boundary of the volume observable by a fixed observer. In a similar way as we have discussed in the particle horizon, here one gets $\omega = \frac{-1}{3} - \frac{2}{3c}$. This describes the DE with $\omega < \frac{-1}{3}$ for the specific case, $c = 1$, it becomes $\omega = -1$, a similar behavior as that of a cosmological constant. For $c < 1$ this horizon gives $\omega < -1$, a value obtained only in phantom model.

Granda-Oliveros Infrared Cut-Off Proposal for the Holographic Density:

Initially, it was proposed that the unknown vacuum DE density, ρ_Λ, is proportional to the square of the Hubble scale. It solves the fine tuning problem, leaving the problem of accelerated expansion unsolved. As discussed above, the use of the particle horizon as a length scale did not give the required results. The future event horizon faces the causality problem but gives the explanation for the acceleration regime. For dimensional reasons a new infra red cut-off for HDE has been proposed by L. N. Granda and A. Oliveros named as *Granda-Oliveros cut-off*. We shall review the useful results of their work [46] in this subsection. They proposed the holographic energy density as

$$\rho = 3(\alpha H^2 + \beta\dot{H}). \tag{2.68}$$

Here α and β are constants and $H = \dot{a}/a$. This expression of density includes time derivative of the Hubble parameter and fits best with the observational data. The new term is contained in the Ricci scalar which scales with L^{-2} (we will discuss later about Ricci scalar HDE). They have shown by plotting the EoS parameter against z that for $\beta = 0.5$ this model evolves like that of some scalar field models of DE. This model of HDE with new cut-off depending on local quantities avoids the causality problem.

2.4 Granda-Olivers Infrared Cut-Off and HDE Models

In this section few models of HDE with Granda-Olivers (GO)infrared (IR) Cut-Off are reviewed.

2.4.1 New Infrared Cut-Off for the Holographic Scalar Fields Model of Dark Energy

Holographic Scalar Fields have been reconstructed by L. N. Granda and A. Oliveros [48]. Their study is restricted to the DE dominated phase of the Universe (present age). The HDE is given by

$$\rho_\Lambda = 3M_p^2(\alpha H^2 + \beta \dot{H}). \tag{2.69}$$

The Friedmann equation becomes

$$H^2 = \alpha H^2 + \beta \dot{H}. \tag{2.70}$$

Integrating this equation with respect to cosmic time, t, one obtains

$$H = \frac{\beta}{(\alpha - 1)t}. \tag{2.71}$$

It gives

$$a \propto t^{\beta/\alpha - 1}. \tag{2.72}$$

For $\beta/\alpha - 1 = 2/3$, the parameter a given in the above equation shows a matter dominated Universe. Which indicates that this model solves the coincidence problem. Using the equation of energy conservation and EoS for the HDE and pressure densities $p_\Lambda = \omega_\Lambda \rho$, the EoS parameter for this HDE model is obtained as [48]

$$\omega_\Lambda = -1 - \frac{2\alpha H\dot{H} + \beta \ddot{H}}{3H(\alpha H^2 + \beta \dot{H})}. \tag{2.73}$$

Using H given in Eq. (2.71) it becomes

$$\omega_\Lambda = -1 + \frac{2(\alpha - 1)}{3\beta}. \tag{2.74}$$

Here α and β are constants. Eqs. (2.71) and (2.74) show that if one considers the phase $-1 < \omega_\Lambda < -1/3$ i.e. to obtain the accelerated expansion, it is required that $\beta > \alpha - 1$ if $\alpha > 1$ or $\beta < \alpha - 1$ if $\alpha < 1$. And for $\alpha < 1$; $\beta > 0$ or $\alpha > 1$; $\beta < 0$, a phantom like phase of evolution $\omega < -1$ can be obtained.
A correspondence has been established by L. N. Granda and A. Oliveros between their model of HDE and various scalar fields. The results are in the favor of accelerated expansion of the Universe. Hence the model with this infrared cut-off is viable [48].

2.4.2 HDE in a Non-Flat Universe with GO Cut-Off

Although an early inflation era leads to a flat Universe, but if the number of e-foldings is not very large it is not necessary that the Universe is perfectly flat [49] . So it is not wise to study only spatially flat Universe. Some experiments of CMB and WMAP favor this concept [50] - [52]. Motivated from these studies for the non-flat Universe K. Karami and J. Fehri [53] have generalized the Granda-Oliveros model [46] to the non-flat Universe.

By comparing the HDE density for the GO cut-off given in Eq. (2.69) with Eq. (2.61) they obtained the corresponding IR cut-off L for their model as:

$$L = H^{-1}\left(1 + \frac{\beta\dot{H}}{\alpha H^2}\right)^{-1/2}. \tag{2.75}$$

Note that for $\beta = 0$ Hubble horizon is obtained. The first Friedmann equation in the non-flat Universe becomes:

$$H^2 = \frac{1}{3}(\rho_m + \rho_r + \rho_\Lambda + \rho_k), \tag{2.76}$$

here ρ_m , ρ_r, ρ_Λ and ρ_k are the contributions of non relativistic matter , radiation, dark energy and of curvature respectively. Here $\rho_k = -3k/a^2$ and k represents the curvature of the space, $k = 0, 1, -1$ for a flat, closed and open Universe respectively.

For the special case when $\Omega_{ko} = 0$, the results obtained for the deceleration parameter and EoS parameter, reduce to the equations obtained in case of flat Universe, discussed earlier.

They have plotted the DE density parameter, decelerating parameter and EoS parameter, versus redshift z. The numerical values of the evolutionary behavior of open, closed and flat Universe with the auxiliary parameters, $\beta = 0.5$, $\Omega_{ko} = \pm 0.015$ (for open and close case respectively) show that with this small curvature the non-flat and flat cases differ only by an order of 10^{-2}.

2.5 Entropy Corrected Holographic Dark Energy (ECHDE)

So far we have discussed the HDE models which have been studied widely in literature [54] - [68]. In this section we shall discuss Entropy Corrected Holographic Dark Energy model. The entropy area relation, $S = (A/4G)$, has been modified to

$$S = (A/4G) + \tilde{\gamma} \ln(A/4G) + \tilde{\beta}, \tag{2.77}$$

where $\tilde{\gamma}$ and $\tilde{\beta}$ are constants of order unity and their exact values are still unknown. These corrections appear in the black hole entropy due to the quantum fluctuations, thermal equilibrium fluctuations or mass and charge fluctuations in loop quantum gravity [69, 70, 71, 72]. Using the entropy-area relation given in Eq. (2.77) the relation for ECHDE is obtained as [73]

$$\rho_\Lambda = 3n^2 M_p^2 L^{-2} + \gamma_1 L^{-4} \ln(M_p^2 L^2) + \gamma_2 L^{-4}. \tag{2.78}$$

Here n^2, γ_1 and γ_2 are dimensionless constants of order unity. For the special case when $\gamma_1 = \gamma_2 = 0$, the well known HDE is obtained.

2.5.1 Interacting Holographic Dark Energy with Logarithmic Correction

Since observations show that the energy densities of matter and DE are almost same at present time, which is not expected in the DE dominated Universe, the coincidence problem. A possible solution for this problem was given by Wetterich in 1995. There is a possibility that these components interact directly with each other and exchange their energy during evolution, to keep this ratio roughly constant in the Universe [4]. Jamil and Farooq have studied this model [74] with IR cut-off proposed by Li. They also made a correspondence of this model with generalized Chaplygin gas in their work. Here we shall briefly discuss their work.

For a result compatible with the accelerated Universe, Li proposed that L should be the future event horizon defined as [45]

$$L = a(t)\frac{\sin(\sqrt{|k|}y)}{\sqrt{|k|}}, \quad y = \frac{R_h}{a(t)}, \tag{2.79}$$

where R_h is the size of the future event horizon defined as

$$R_h = a(t)\int_t^{\infty}\frac{dt'}{a(t')} = a(t)\int_0^{r_1}\frac{dr}{\sqrt{1-kr^2}}. \tag{2.80}$$

The integral can be written explicitly as

$$\int_0^{r_1}\frac{dr}{\sqrt{1-kr^2}} = \frac{1}{\sqrt{|k|}}\sin^{-1}(\sqrt{|k|}r_1) = \begin{cases} \sin^{-1}(r_1), & k=+1, \\ r_1, & k=0, \\ \sinh^{-1}(r_1), & k=-1. \end{cases} \tag{2.81}$$

To overcome the cosmic coincidence problem and the cosmic fine tuning problem, a model of interacting DE with DM has been proposed [74, 75, 76]. In the FRW back ground spacetime given in Eq. (1.57), the Einstein field equation is

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}\left[\rho_\Lambda + \rho_{\rm m}\right]. \tag{2.82}$$

In the dimensionless form it is written as

$$1 + \Omega_k = \Omega_\Lambda + \Omega_{\rm m}. \tag{2.83}$$

The density parameters are defined as

$$\Omega_{\rm m} = \frac{\rho_{\rm m}}{\rho_{\rm cr}} = \frac{\rho_{\rm m}}{3M_p^2H^2}, \quad \Omega_\Lambda = \frac{\rho_\Lambda}{\rho_{\rm cr}} = \frac{\rho_\Lambda}{3M_p^2H^2}, \quad \Omega_k = \frac{k}{(aH)^2}, \tag{2.84}$$

where $\rho_{\rm cr}$ is the critical density. For the interacting model the equations of energy conservation are given by [77]

$$\dot{\rho}_\Lambda + 3H(\rho_\Lambda + p_\Lambda) = -Q, \tag{2.85}$$

$$\dot{\rho}_{\rm m} + 3H\rho_{\rm m} = Q. \tag{2.86}$$

Here Q is an arbitrary function of cosmological parameters like the Hubble parameter and energy densities and is used as an interacting term. Up to linear order in energy densities

$Q \simeq H(\rho_\Lambda + \rho_{\rm m})$, by using a coupling parameter b^2 this term can be saturated as $Q = 3b^2 H(\rho_\Lambda + \rho_{\rm m})$ [78, 79, 80]. Observations of galactic clusters and CMB show that $b^2 < 0.025$, i.e. it is a small positive constant of order unity [81]. The effective EoS for DE and DM are defined as [82]

$$\omega_\Lambda^{\rm eff} = \omega_\Lambda + \frac{\Gamma}{3H}, \quad \omega_{\rm m}^{\rm eff} = -\frac{1}{r_{\rm m}}\frac{\Gamma}{3H}. \tag{2.87}$$

Using Eq. (2.87) in Eqs. (2.85) and (2.86) one gets

$$\dot{\rho}_\Lambda + 3H(1+\omega_\Lambda^{\rm eff})\rho_\Lambda = 0, \tag{2.88}$$

$$\dot{\rho}_{\rm m} + 3H(1+\omega_{\rm m}^{\rm eff})\rho_{\rm m} = 0. \tag{2.89}$$

the definitions of Ω_Λ and ρ_{cr} are used to get

$$HL = \sqrt{\frac{3n^2M_p^2 + \gamma_1 L^{-2}\ln(M_p^2L^2) + \gamma_2 L^{-2}}{3M_p^2\Omega_\Lambda}}. \tag{2.90}$$

Using Eq. (2.90), the differentiation of L gives

$$\dot{L} = \sqrt{\frac{3n^2M_p^2 + \gamma_1 L^{-2}\ln(M_p^2L^2) + \gamma_2 L^{-2}}{3M_p^2\Omega_\Lambda}} - \text{cosn}(\sqrt{|k|}y). \tag{2.91}$$

Using Eq. (2.91) the differentiation of Eq. (2.78) with respect to t is completed to get $\dot{\rho_\Lambda}$as

$$\dot{\rho}_\Lambda = \Big[2\gamma_1 L^{-5} - 4\gamma_1 L^{-5}\ln(M_p^2L^2) - 4\gamma_2 L^{-5} - 6n^2M_p^2L^{-3}\Big]\Big[\sqrt{h(a)} - \text{cosn}(\sqrt{|k|y})\Big], \tag{2.92}$$

where $h(a) = \frac{3n^2M_p^2 + \gamma_1 L^{-2}\ln(M_p^2L^2) + \gamma_2 L^{-2}}{3M_p^2\Omega_\Lambda}$.

Use of Eq. (2.92) in Eq. (2.85) gives the EoS parameter ω_Λ as

$$w_\Lambda = -1 - g(a)\Big[1 - \sqrt{\frac{3M_p^2\Omega_\Lambda}{3n^2M_p^2 + \gamma_1 L^{-2}\ln(M_p^2L^2) + \gamma_2 L^{-2}}} \times \text{cosn}(\sqrt{|k|}y)\Big] - \frac{b^2(1+\Omega_k)}{\Omega_\Lambda}, \tag{2.93}$$

where $g(a) = \frac{2\gamma_1 L^{-2} - 4\gamma_1 L^{-2}\ln(M_p^2L^2) - 4\gamma_2 L^{-2} - 6n^2M_p^2}{3(3n^2M_p^2 + \gamma_1 L^{-2}\ln(M_p^2L^2) + \gamma_2 L^{-2})}$.

The expression for the $w_\Lambda^{\rm eff}$ is obtained by using the above equation in Eq. (2.87) and is given by

$$w_\Lambda^{\rm eff} = -1 - g(a)\Big[1 - \sqrt{\frac{3M_p^2\Omega_\Lambda}{3n^2M_p^2 + \gamma_1 L^{-2}\ln(M_p^2L^2) + \gamma_2 L^{-2}}} \times \text{cosn}(\sqrt{|k|}y)\Big]. \tag{2.94}$$

The above equation is the effective equation of state parameter for ECHDE model, here $g(a)$ is the same one, used in previous equation. For the flat Universe this parameter reduces to -1 , for non-flat case phantom divide is possible for the suitable selection of parameters. They have also made a correspondence of their model with the Chaplygin gas.

Interacting Entropy Corrected Holographic Scalar Fields in Non-Flat Universe: The experimental evidences from the CMB radiation favor that we are living in a non-flat FRW Universe. In the light of above mentioned arguments Khodam-Mohammadi and Malekjani worked on a model including interacting DM and DE with a non-flat universe [83]). They started with the FRW metric containing DM and DE. The Friedmann equation is the same as given in Eq. (2.83) and the energy conservation equations for both components were considered as given in Eqs. (2.85) and (2.86). Using the L proposed by Li [45] as given in Eqs. (2.79)-(2.81) and following the similar steps as mentioned earlier in Eqs. (2.87)-(2.93), the EoS parameter for this model is obtained. In the limiting cases when ECHDE reduces to HDE and without interacting term i.e. by taking $b = 0$ the EoS parameter becomes similar to the one obtained by Setare [84]. Khodam-Mohammadi and Malekjani have established *correspondence between interacting ECHDE and scalar fields* including, k-essence, tachyon and dilaton scalar fields. The potentials and dynamics of these scalar fields are reconstructed in order to show that these scalar fields describe the evolutionary behavior of interacting ECHDE model. From this analysis it is shown that ECHDE tachyon model can not cross the phantom divide but other two models can cross it.

2.5.2 Reconstructing Interacting Entropy Corrected Holographic Scalar Fields of Dark Energy in Non-Flat Universe

Khaledian *et al* [85] have completed a reconstruction of the potentials and dynamics of the quintessence, tachyon, k-essence and dilaton scalar field models according to the evolutionary behavior of the interacting ECHDE model. In particular they discussed the explicit evolutionary forms of the corresponding scalar fields for both phases of the expansion of the Universe, for the inflationary age, i.e. by taking $L = R_h = H^{-1}: \quad H = \text{constant}$, and for the late-time acceleration, by taking $L = R_h \neq H^{-1}: \quad H \neq \text{constant}$, they have shown that the vacuum energy producing inflation at the early cosmic time and the one responsible for the late-time cosmic acceleration are fundamentally different. So at different times in the history of the Universe, same scalar fields move in different potentials.

2.6 Power Law Entropy Corrected Relation

By the power law corrections to the entropy area relation the power law corrected entropy has been given as [86]

$$S = \frac{A_h}{4G}\left(1 - K_\alpha A^{1-\frac{\alpha}{2}}\right), \tag{2.95}$$

where α is a dimensionless positive constant with unknown value, $A = 4\pi R^2$ is the area of the cosmological horizon of radius R and

$$K_\alpha = \frac{\alpha(4\pi)^{\frac{\alpha}{2}-1}}{(4-\alpha)r_c^{2-\alpha}}, \tag{2.96}$$

where r_c represents the crossover scale. The second term in Eq. (2.95) is a power law correction to the area law and it is due to the entanglement, when the wave function of the field is chosen to be a superposition of ground state and excited state [87]. So only excited state is responsible for the entropy correction. Also note that the correction term falls off rapidly with A. So the correction term contributes in the small black holes.

Sheykhi and Jamil [88] have proposed a new version of HDE called *Power Law Entropy Corrected HDE Model (PLECHDE).* Following the derivations of HDE [89] and ECHDE [73] they have obtained the PLECHDE as

$$\rho_\Lambda = 3\gamma M_p^2 L^{-2} - \beta M_p^2 L^{-\alpha}. \tag{2.97}$$

Note that when $\beta = 0$ or $\alpha = 2$ the above expression reduces to ordinary HDE density. For $\alpha > 2$ the corrected term with small L is compatible with the first term. Details are available in [88].

Chapter 3

Ricci Power Law Entropy Corrected Holographic Dark Energy and Scalar Fields

As already mentioned, recent observations favor an accelerating Universe with a present density of 70% exotic components having negative pressure and pushing the expansion of the Universe into an accelerating phase. Various candidates of DE have been put forward in order to find out the solution of this strange phenomenon. These proposals start from the cosmological constant and move on to scalar field theories of DE, but two fundamental problems of fine tuning and coincidence are faced. To alleviate these problems another model known as HDE model has been proposed (details are already discussed in previous chapter). HDE and its other versions including interacting HDE models are the proposals which fit best with observations. However, these models have some serious conceptual problems. Gao et al [90] pointed out that using the future event horizon as an infra-red cut-off for the HDE model leads to a causality problem. Since for a flat FRW Universe a future event horizon exists if and only if the Universe is accelerating, the HDE model has itself assumed the acceleration of the Universe in order to understand the cosmic acceleration.

3.1 Ricci Dark Energy (RDE) Model

Whenever there is a shortcoming in model under discussion, alternative solutions are tried. Motivated by the HDE models, Gao *et al* proposed a new model [90]. In this work the infrared cuf-off L is taken as the average radius of the Ricci scalar curvature, $R^{-1/2}$, giving $\rho \propto R$. This model is known as the RDE model. Gao *et al* investigated the phenomenological properties of the model and showed that their model not only resolve the causality problem but also solves the coincidence problem. Gao et al further pointed out that $\alpha \simeq 0.46$ yields the correct DE density and equation of state today. Moreover, the RDE model is compatible with observational data from supernova and CMB radiation [91]. However, there is some criticism on RDE model: Kim *et al* [92] pointed out that an accelerating phase of the RDE is that of a constant DE model. This implies that the RDE may not be a new model to explain the present accelerating Universe.

3.1.1 Interacting Ricci Dark Energy with Logarithmic Correction

Recently a model has been proposed by Pasqua *et al*, in which the entropy corrected HDE model in the non-flat FRW Universe with the Ricci scalar curvature as the infrared cut-off [93] has been studied. They calculated the EoS parameter, deceleration parameter and the density parameter. Further, a correspondence of this model with some scalar fields has been established in their work.

Following the idea we have extended their work for the power law entropy corrected version. In the following section this model is presented.

3.2 Ricci Power Law Entropy Corrected HDE and Dynamics of scalar Fields

We consider the Rici power law entropy corrected holographic dark energy (R-PLECHDE) model in the non-flat FRW Universe, with the future event horizon replaced by $R^{-1/2}$. We derive the equation of state (EoS) parameter ω_Λ and the evolution of energy density parameter Ω'_D in presence of interaction between DE and DM. We consider the correspondence between our R-PLECHDE model and the GCG, the MCG and some scalar fields like tachyon, k-essence, dilaton and quintessence. The potential and the dynamics of the scalar field models according to the evolutionary behavior of the interacting power law entropy corrected Ricci holographic DE model have been reconstructed.

Motivated by the previous study of Gao *et al* [90] we consider the entropy corrected version of Ricci power law entropy corrected HDE as

$$\rho_\Lambda = 3\gamma M_p^2 R - \beta M_p^2 R^{\alpha/2}, \tag{3.1}$$

where R represents the Ricci scalar and is given by

$$R = 6\Big(\dot{H} + 2H^2 + \frac{k}{a^2}\Big). \tag{3.2}$$

3.2.1 Interacting Model in a Non-Flat Universe

We assume the background spacetime to be the FRW metric given in Eq. (1.57). The corresponding Friedmann equation in the dimensionless form is given in Eq. (2.83). In order to preserve the Bianchi identity or local energy-momentum conservation law, i.e. $\nabla_\mu T^{\mu\nu} = 0$, the total energy density $\rho_{tot} = \rho_\Lambda + \rho_m$ must satisfy the relation, given in Eq. (1.67), which represents the energy conservation equation. Since we are considering the interaction between DE and DM, the two energy densities ρ_Λ and ρ_m are preserved separately and the equations of conservation are as given in Eqs. (2.85) and (2.86) Now we derive the expression for EoS parameter ω_Λ for our R-PLECHDE model. Using the Friedmann equation given in (2.82), the Ricci scalar, R, can be rewritten in the following form

$$R = 6\left(\dot{H} + H^2 + \frac{\rho_m + \rho_\Lambda}{3M_p^2}\right). \tag{3.3}$$

Also

$$\dot{H} = \frac{k}{a^2} - \frac{1}{2M_p^2}\left[\rho_m + \rho_\Lambda\left(1 + \omega_\Lambda\right)\right]. \tag{3.4}$$

Adding Eqs. (2.82) and (3.4), we get

$$\dot{H} + H^2 = \frac{\rho_m + \rho_\Lambda}{3M_p^2} - \frac{1}{2M_p^2}\left[\rho_m + \rho_\Lambda\left(1 + \omega_\Lambda\right)\right]. \tag{3.5}$$

So, the Ricci scalar given in Eq. (3.3) can be rewritten as

$$R = \frac{\rho_m + \rho_\Lambda}{M_p^2} - \frac{3\rho_\Lambda\omega_\Lambda}{M_p^2}. \tag{3.6}$$

The EoS parameter ω_Λ can be derived from Eq. (3.6) as

$$\omega_\Lambda = -\frac{RM_p^2}{3\rho_\Lambda} + \frac{\Omega_\Lambda + \Omega_m}{3\Omega_\Lambda}. \tag{3.7}$$

where we used the relation $\frac{\rho_\Lambda+\rho_m}{3\rho_\Lambda} = \frac{\Omega_\Lambda+\Omega_m}{3\Omega_\Lambda}$.
Substituting the expression of the energy density ρ_Λ given in Eq. (3.1) in Eq. (3.7), also using Eq. (2.82) we get

$$\omega_\Lambda = -\frac{1}{3(3\gamma - \beta R^{\frac{\alpha}{2}-1})} + \frac{(1+\Omega_k)}{3\Omega_\Lambda}. \tag{3.8}$$

This is the EoS parameter for the R-PLECHDE model.

For completeness, we can also derive the expression of the deceleration parameter q, which is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}. \tag{3.9}$$

The deceleration parameter, combined with the Hubble parameter H and the dimensionless density parameters, form a set of very useful parameters for the description of the astrophysical observations. Taking the time derivative of the Friedmann given in Eq. (2.82) and using Eqs. (2.83), (2.85) and (2.86), it is possible to write the deceleration parameter q as

$$q = \frac{1}{2}\left[1 + \Omega_k + 3\Omega_\Lambda\omega_\Lambda\right]. \tag{3.10}$$

Substituting the expression of the EoS parameter ω_Λ given in Eq. (3.8) we obtain

$$q = 1 - \frac{1}{2}\left(\frac{\Omega_\Lambda}{3\gamma - \beta R^{\alpha/2-1}}\right) + \Omega_k. \tag{3.11}$$

We can now derive the important quantities of the R-PLECHDE model in the limiting case, for a flat dark dominated Universe, i.e. when $\beta = 0$, $\Omega_\Lambda = 1$ and Ω_k=0.
The energy density ρ_Λ given in Eq. (3.1) reduces to

$$\rho_\Lambda = 3\gamma M_p^2 R. \tag{3.12}$$

From the Friedmann equation given in (2.82), we can derive the following expressions for the Hubble parameter H and the Ricci scalar curvature R

$$H = \frac{6\gamma}{12\gamma - 1}\left(\frac{1}{t}\right), \tag{3.13}$$

$$R = \frac{36\gamma}{(12\gamma - 1)^2}\left(\frac{1}{t^2}\right). \tag{3.14}$$

Finally, the EoS parameter ω_Λ and deceleration parameter q respectively, reduce, to

$$\omega_\Lambda = \frac{1}{3} - \frac{1}{9\gamma}, \tag{3.15}$$

$$q = 1 - \frac{1}{6\gamma}. \tag{3.16}$$

Using Eq. (3.14) in (3.12) we can write the energy density as

$$\rho_\Lambda = 3\gamma M_p^2\Big(\frac{36\gamma}{(12\gamma-1)^2}\frac{1}{t^2}\Big). \tag{3.17}$$

From Eq. (3.15) we see that, the EoS parameter of DE becomes $\omega_\Lambda < -1$, for $\gamma < 1/12$, hence the phantom divide can be crossed. Since the Ricci scalar R diverges at $\gamma = 1/12$, this value of γ can not be taken into account. From Eq. (3.16), we obtain that the acceleration starts at $\gamma \leq 1/6$, where the quintessence regime is started ($\omega_\Lambda \leq -1/3$).

3.3 Correspondence between R-PLECHDE and Scalar Fields

In this Section we establish a correspondence between the interacting Ricci power law entropy corrected model and the tachyon, k-essence, dilaton and quintessence scalar field models, the GCG and the MCG. The importance of this correspondence is that the scalar field models are an effective description of an underlying theory of DE. Therefore, it is worthwhile to reconstruct the potential and the dynamics of scalar fields according the evolutionary form of Ricci scalar model. For this purpose, we first compare the energy density of Ricci scale model given in Eq. (3.1) with the energy density of corresponding scalar field model. Then we equate the EoS parameters of scalar field models with the EoS parameter of Ricci scalar model given in Eq. (3.8).

Interacting Tachyon Model:

The effective Lagrangian of the tachyon scalar field is motivated from open string field theory [94] and it is a successful candidate for cosmic acceleration. It has the Lagrangian given in Eq. (2.42). The energy density, ρ_Φ, and pressure, p_Φ, and the EoS parameter, ω_Φ, for the tachyon field are given, respectively, in Eqs. (2.43), (2.44), and (2.45). We derive the following expression for the potential $V(\Phi)$ of the tachyon field

$$V(\Phi) = \rho_\Lambda\sqrt{1-\dot{\Phi}^2}. \tag{3.18}$$

Instead, equating Eqs. (3.8) with Eq. (2.45), we obtain the expressions of the kinetic energy term $\dot{\Phi}^2$ for the R-PLECHDE as

$$\dot{\Phi}^2 = 1 + \omega_\Lambda = 1 - \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{3\Omega_\Lambda}. \tag{3.19}$$

Moreover, using Eq. (3.19) into Eq. (3.18), it is possible to write the potential of the tachyon as

$$V(\Phi) = \rho_\Lambda \sqrt{-\omega_\Lambda} = \rho_\Lambda \sqrt{\frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} - \frac{(1+\Omega_k)}{3\Omega_\Lambda}}. \tag{3.20}$$

Using $\dot{\Phi} = \Phi' H$, in Eq. (3.19) we get

$$\Phi' = \frac{1}{H}\sqrt{1 - \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{3\Omega_\Lambda}}. \tag{3.21}$$

Then, from Eq. (3.21), it is possible to derive the evolutionary form of the tachyon scalar field as

$$\Phi(a) - \Phi(a_0) = \int_{a_0}^{a} \frac{da}{aH}\sqrt{1 - \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{3\Omega_\Lambda}}, \tag{3.22}$$

where a_0 represents the present value of the scale factor $a(t)$. Here

$$\frac{da}{aH} = \frac{da}{a(\dot{a}/a)} = \frac{da}{da/dt} = dt,. \tag{3.23}$$

In the limiting case for flat DE dominated Universe i.e. when $\beta = 0$, $\Omega_\Lambda = 1$ and Ω_k=0, using Eq. (3.17) the scalar field expression and potential of the tachyon assume the following form:

$$\Phi(t) = \sqrt{\frac{12\gamma - 1}{9\gamma}}\,t, \tag{3.24}$$

$$V(\Phi) = \frac{4M_p^2}{(12\gamma - 1)}\sqrt{\gamma(1-3\gamma)}\frac{1}{\Phi^2}. \tag{3.25}$$

The above equation is obtained by replacing the expression of $1/t^2$ with the one given in Eq. (3.24). In this correspondence, the scalar field exist when $\gamma > 1/12$, which shows that the phantom divide can not be achieved.

Interacting k-essence Model:

Model of k-essence was proposed as a solution to the problem of small cosmological constant and late-time cosmic acceleration [95]. Its action is defined in Eq. (2.38). According to the Lagrangian (2.38), the pressure, $p(\Phi,\chi)$, the energy density, ρ, and EoS parameter, ω_K, of k-essence scalar field are defined in Eqs. (2.39) - (2.41) respectively. In order to consider the k-essence field as a description of the interacting R-PLECHDE density, we establish the correspondence between the k-essence EoS parameter, ω_K, and the interacting R-PLECHDE EoS parameters.

The expressions of χ for R-PLECHDE can be obtained by equating Eqs. (3.8) with Eq. (2.41) as

$$\chi=\frac{\omega_\Lambda-1}{3\omega_\Lambda-1}=\frac{-1-\frac{1}{3\left(3\gamma-\beta R^{\frac{\alpha}{2}-1}\right)}+\frac{(1+\Omega_k)}{3\Omega_\Lambda}}{-1-\frac{1}{\left(3\gamma-\beta R^{\frac{\alpha}{2}-1}\right)}+\frac{(1+\Omega_k)}{\Omega_\Lambda}}, \tag{3.26}$$

Equating Eqs. (3.1) and (2.40), we obtain

$$f\left(\Phi\right)=\frac{\rho_\Lambda}{\chi(3\chi-1)}. \tag{3.27}$$

Using $\dot{\Phi}^2=2\chi$ and remembering that $\dot{\Phi}=\Phi' H$, we can write

$$\Phi'=\frac{\sqrt{2}}{H}\sqrt{\frac{-1-\frac{1}{3\left(3\gamma-\beta R^{\frac{\alpha}{2}-1}\right)}+\frac{(1+\Omega_k)}{3\Omega_\Lambda}}{-1-\frac{1}{\left(3\gamma-\beta R^{\frac{\alpha}{2}-1}\right)}+\frac{(1+\Omega_k)}{\Omega_\Lambda}}}. \tag{3.28}$$

Integrating Eq. (3.28) we can find the evolutionary form of the k-essence scalar field as

$$\Phi\left(a\right)-\Phi\left(a_0\right)=\sqrt{2}\int_{a_0}^{a}\frac{da}{aH}\sqrt{\frac{-1-\frac{1}{3\left(3\gamma-\beta R^{\frac{\alpha}{2}-1}\right)}+\frac{(1+\Omega_k)}{3\Omega_\Lambda}}{-1-\frac{1}{\left(3\gamma-\beta R^{\frac{\alpha}{2}-1}\right)}+\frac{(1+\Omega_k)}{\Omega_\Lambda}}}. \tag{3.29}$$

In the limiting case for flat dark dominated Universe the scalar field and potential of k-essence field reduce to

$$\Phi(t)=\sqrt{\frac{12\gamma+2}{3}}t, \tag{3.30}$$

and

$$f(\Phi)=\frac{36\gamma M_p^2}{(12\gamma-1)^2}\frac{1}{\Phi^2}. \tag{3.31}$$

Notice that as γ is an arbitrary constant and can assume all real values, therefore, our Universe may behave in all accelerated regimes (phantom and quintessence).

Interacting Dilaton Model:

Dilaton model arises as a low-energy limit of string theory and is found to be a useful candidate of DE [96]. The expressions of its pressure and energy density are

$$p_D = -\chi + ce^{\lambda\Phi}\chi^2, \tag{3.32}$$

$$\rho_D = -\chi + 3ce^{\lambda\Phi}\chi^2, \tag{3.33}$$

where c and λ are positive constants and $2\chi = \dot{\Phi}^2$. The negative coefficient of the kinematic term of the dilaton field in the Einstein frame makes a phantom-like behavior for dilaton field. The EoS parameter for the dilaton scalar field is given by

$$\omega_D = \frac{p_D}{\rho_D} = \frac{-1 + ce^{\lambda\Phi}\chi}{-1 + 3ce^{\lambda\Phi}\chi}. \tag{3.34}$$

In order to consider the dilaton field as a description of the interacting R-PLECHDE density we now establish a correspondence between the dilaton EoS parameter and the EoS parameter of the R-PLECHDE model. By equating Eqs. (3.8) and (3.34), we obtain

$$ce^{\lambda\Phi}\chi = \frac{\omega_\Lambda - 1}{3\omega_\Lambda - 1} = \frac{-1 - \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{3\Omega_\Lambda}}{-1 - \frac{1}{\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{\Omega_\Lambda}}. \tag{3.35}$$

Since $\dot{\Phi}^2 = 2\chi$, Eq. (3.35) can be rewritten as

$$e^{\lambda\Phi/2}\dot{\Phi} = \frac{-1 - \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{3\Omega_\Lambda}}{-1 - \frac{1}{\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{\Omega_\Lambda}}. \tag{3.36}$$

Integrating Eq. (3.36) we obtain

$$e^{\lambda\Phi(a)/2} = e^{\lambda\Phi(a_0)/2} + \frac{\lambda}{2\sqrt{c}}\int_{a_0}^{a}\frac{da}{aH}\sqrt{\frac{-1 - \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{3\Omega_\Lambda}}{-1 - \frac{1}{\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{\Omega_\Lambda}}}. \tag{3.37}$$

The evolutionary form of the dilaton scalar field is given by

$$\Phi(a) = \frac{2}{\lambda}\log\left[e^{\lambda\Phi(a_0)/2} + \frac{\lambda}{\sqrt{2c}}\int_{a_0}^{a}\frac{da}{aH}\sqrt{\frac{-1 - \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{3\Omega_\Lambda}}{-1 - \frac{1}{\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{\Omega_\Lambda}}}\right]. \tag{3.38}$$

In the limiting case for flat dark dominated Universe with the use of Eq. (3.17) the scalar field of dilaton field reduces to the following form

$$\Phi(t) = \frac{2}{\lambda} \ln \left[\lambda t \sqrt{\frac{1+6\gamma}{6c}} \right]. \tag{3.39}$$

We see that γ can assume all the possible values. Therefore, by this correspondence the Universe may behave both in phantom and quintessence regime.

Quintessence:

Quintessence is described by a time dependent and homogeneous scalar field, Φ, which is minimally coupled to gravity and has a potential $V(\Phi)$ that leads to the accelerating Universe. Taking $\sigma = 1$ in Eqs. (2.28) - (2.32) one can get the expressions for energy density, pressure , EoS, kinetic energy, and scalar potential of this field. Substituting Eq. (3.8) into Eqs. (2.31) and (2.32), the kinetic energy term $\dot{\Phi}^2$ and the quintessence potential energy $V(\Phi)$ can be easily found as follow

$$\dot{\Phi}^2 = \rho_\Lambda \left(1 - \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} + \frac{(1+\Omega_k)}{3\Omega_\Lambda} \right), \tag{3.40}$$

$$V(\Phi) = \frac{\rho_\Lambda}{2} \left(1 + \frac{1}{3\left(3\gamma - \beta R^{\frac{\alpha}{2}-1}\right)} - \frac{(1+\Omega_k)}{3\Omega_\Lambda} \right). \tag{3.41}$$

From Eq. (3.40) we can obtain the evolutionary form of the quintessence scalar field as

$$\Phi(a) - \Phi(a_0) = \int_{a_0}^{a} \frac{da}{a} \sqrt{3M_p^2\Omega_\Lambda \left(1 - \frac{1}{3(3\gamma - \beta R^{\frac{\alpha}{2}-1})} + \frac{(1+\Omega_k)}{3\Omega_\Lambda} \right)}, \tag{3.42}$$

In the limiting case for flat dark dominated Universe and using Eq. (3.17) the scalar field and potential of quintessence reduces to

$$\Phi(t) = \frac{6\gamma M_p}{\sqrt{3\gamma(12\gamma - 1)}} \ln(t), \tag{3.43}$$

$$V(\Phi) = \frac{6\gamma(6\gamma+1)}{(12\gamma-1)^2} M_p^2 \exp\left[\frac{-\sqrt{3\gamma(12\gamma-1)}}{3\gamma M_p} \Phi \right]. \tag{3.44}$$

The potential exists for all values of $\gamma > 1/12$ (which correspond to the quintessence regime).

3.3.1 Generalized Chaplygin Gas (GCG)

In this Section, we want to obtain a correspondence between the GCG and the R-PLECHDE. The equations of pressure and density evolution of GCG are given in Eqs. (2.54) and (2.55) respectively. Using Eq. (2.54) along with EoS we know that

$$\omega_\Lambda = -\frac{D}{\rho_\Lambda^{\theta+1}}, \tag{3.45}$$

which corresponds to

$$D = -\omega_\Lambda \rho_\Lambda^{\theta+1}. \tag{3.46}$$

If we now substitute in Eq. (3.46), the EoS parameter of the R-PLECHDE given in Eq. (3.8) then D can we written as

$$D = \rho_\Lambda^{\theta+1}\left[\frac{1}{3\left(3\gamma - \beta R^{\alpha/2-1}\right)} - \frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right]. \tag{3.47}$$

From Eq. (2.55) we have $B = a^{3(\theta+1)}\left(\rho_\Lambda^{\theta+1} - D\right)$, which can be rewritten in the form

$$B = \left(a^3\rho_\Lambda\right)^{\theta+1}\left(1+\omega_\Lambda\right). \tag{3.48}$$

Substituing in Eq. (3.48) the expression of D given in Eq. (3.47), we obtain

$$B = \left[a^3\rho_\Lambda\right]^{\theta+1}\left[1 - \frac{1}{3\left(3\gamma - \beta R^{\alpha/2-1}\right)} + \frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right]. \tag{3.49}$$

Using Eqs. (3.45), (3.47), and (3.49) in Eqs. (2.31) and (2.32), along with $\sigma = 1$ we can derive the kinetic and the potential terms for the R-PLECHDE as

$$\sigma\dot{\Phi}^2 = \rho_\Lambda\left[-\frac{1}{3\left(3\gamma - \beta R^{\alpha/2-1}\right)} + \frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + 1\right], \tag{3.50}$$

$$2V\left(\Phi\right) = \rho_\Lambda\left[1 + \frac{1}{3\left(3\gamma - \beta R^{\alpha/2-1}\right)} - \frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right]. \tag{3.51}$$

We can obtain the evolutionary form of the GCG by integrating Eq. (3.50)

$$\Phi\left(a\right) - \Phi\left(a_0\right) = \int_{a_0}^{a}\left\{\left[\frac{3M_p^2\Omega_\Lambda}{\sigma}\left(-\frac{1}{3\left(3\gamma - \beta R^{\alpha/2-1}\right)} + \frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right) + 1\right)\right]\right\}^{1/2}\frac{da}{a}. \tag{3.52}$$

where we used the relation $\dot{\Phi} = \Phi' H$.

In the limiting case for flat dark dominated Universe with use of Eq. (3.17) the scalar field and the potential of the GCG reduce to

$$\Phi\left(t\right) = \frac{6\gamma M_p}{\sqrt{3\gamma\sigma\left(12\gamma-1\right)}} \ln\left(t\right), \tag{3.53}$$

and

$$V(\Phi) = \frac{6\gamma(6\gamma+1)}{(12\gamma-1)^2} M_p^2 \exp\left[\frac{-\sqrt{3\gamma(12\gamma-1)}}{3\gamma M_p}\Phi\right], \tag{3.54}$$

respectively.

Modified Chaplygin Gas (MCG):

The MCG is a generalization of the GCG with the addition of a barotropic term [97] -[100]. The MCG seems to be consistent with the 5-year WMAP data and supports the unified model with DE and matter based on GCG. The density evolution of the MCG, calculated by using the density conservation equation is given in Eq. (2.58). For a homogeneous and time dependent scalar field, Φ, energy density, pressure and EoS parameter are defined in Eqs. (2.28) - (2.29), along with $\sigma = 1$. Now we want to reconstruct the potential and dynamics of the scalar field in the light of R-PLECHDE model. We know that the EoS parameter can be written as

$$\omega_\Lambda = A - \frac{D}{\rho_\Lambda^{\alpha+1}}, \tag{3.55}$$

or

$$D = \rho_\Lambda^{\alpha+1}\Big(A - \omega_\Lambda\Big). \tag{3.56}$$

Inserting the EoS parameter ω_Λ of the R-PLECHDE given in Eq. (3.8) into Eq. (3.56) we obtain

$$D = [\rho_\Lambda]^{\theta+1}\left[A + \frac{1}{3\left(3\gamma - \beta R^{\alpha/2-1}\right)} - \frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right]. \tag{3.57}$$

From Eq. (2.55), we can derive $B = a^{3(\theta+1)(A+1)}(\rho_\Lambda^{\theta+1} - \frac{D}{A+1})$ which is equivalent to the equation

$$B = \left(a^{3(A+1)}\rho_\Lambda\right)^{1+\theta}\left(1 - \frac{A-\omega_\Lambda}{1+A}\right). \tag{3.58}$$

Inserting in Eq. (3.58) the EoS parameter ω_Λ of the R-PLECHDE given in Eq. (1.77), we have

$$B = [a^{3(\theta+1)(A+1)}\rho_\Lambda]^{\theta+1}\left[1-\frac{1}{A+1}\left(A+\frac{1}{3\left(3\gamma-\beta R^{\alpha/2-1}\right)}-\frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right)\right]. \quad (3.59)$$

Taking $\sigma = 1$ in Eqs. (2.31) and (2.32) and using the result with Eqs. (3.57) and (3.59) we obtain the kinetic and potential terms for the scalar field as:

$$\sigma\dot{\Phi}^2 = \rho_\Lambda\left[1-\frac{1}{1+A}\left(A+\frac{1}{3\left(3\gamma-\beta R^{\alpha/2-1}\right)}-\frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right)\right], \quad (3.60)$$

$$2V(\Phi) = \left[1+\frac{1}{A+1}\left(A+\frac{1}{3\left(3\gamma-\beta R^{\alpha/2-1}\right)}-\frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)\right)\right] \quad (3.61)$$

We can find the evolutionary form of the MCG scalar field by integrating Eq. (3.60)

$$\Phi(a)-\Phi(a_0) = \int_{a_0}^{a}\left\{\left[\frac{3M_p^2\Omega_\Lambda}{\sigma}\left(1-\frac{1}{1+A}\left(f(a)\right)\right)\right]\right\}^{1/2}\frac{da}{a}, \quad (3.62)$$

where $f(a) = A+\frac{1}{3\left(3\gamma-\beta R^{\alpha/2-1}\right)}-\frac{1}{3}\left(\frac{1+\Omega_k}{\Omega_\Lambda}\right)$, also we have used the relation $\dot{\Phi} = \Phi' H$. In the limiting case for flat dark dominated Universe and using Eq. (3.17) the scalar field and the potential of the MCG reduce to

$$\Phi(t) = \frac{6\gamma M_p}{(12\gamma-1)}\sqrt{\frac{12\gamma-1-9\gamma A}{3\gamma\sigma(1+A)}}\ln(t), \quad (3.63)$$

$$V(\Phi) = \frac{54\gamma^2 M_p^2}{(12\gamma-1)^2}\left(\frac{6\gamma+18\gamma A+1}{9\gamma(1+A)}\right)\frac{1}{t^2}. \quad (3.64)$$

In the limiting case of $A = 0$, Eqs. (3.63) and (3.64) reduce to

$$\Phi(t) = \frac{6\gamma M_p}{\sqrt{3\gamma\sigma(12\gamma-1)}}\ln(t), \quad (3.65)$$

$$V(\Phi) = \frac{6\gamma(6\gamma+1)}{(12\gamma-1)^2}M_p^2\exp\left[\frac{-\sqrt{3\gamma(12\gamma-1)}}{3\gamma M_p}\Phi\right]. \quad (3.66)$$

which are the same results obtained for the GCG in the previous paragraph.

Notice that a correspondence has been completed between the interacting R-PLECHDE model and the tachyon, k-essence, dilaton, GCG, MCG and quintessence scalar field models in the hypothesis of non-flat FRW Universe.

Chapter 4

Conclusion

In this dissertation we have studied holographic dark energy with power law entropy corrections replacing the infrared cut-off with $R^{-1/2}$.

In the first chapter the basics of differential geometry are studied. This is helpful in understanding the Einstein field equations (EFEs). These equations are the most important in the study of cosmology, describing the relation between mass and the curvature of the spacetime. We have restricted the study of the Einstein theory of general relativity only to the derivation of EFEs by variational principle.

In the remaining chapter an introduction of Cosmology is given. Inclusion of a brief description of history of the Universe, its components, and the basics of the evolutionary equations of the most accepted model of the Universe *the Friedmann-Robertson-Walker (FRW) Universe* make the reader able to understand the further work presented in this thesis. Some problems of cosmology are discussed in the end of this chapter. These problems are Horizon problem, Flatness Problem and the Fine tuning problem. Since inflation is the best proposed solution of most of the cosmological problems, so a brief introduction of inflation is also included.

The second chapter is devoted to the study of the most important discovery of the 20th century that our Universe is expanding and this expansion is at an accelerating rate. Some observational evidences of this hypothesis are supernova and cosmic microwave background. It is believed that there is some mysterious component of the Universe which causes this acceleration. A discussion on some candidates of dark energy is completed. The cosmological constant was introduced as the first candidate but it faces the fine tuning problem.

The fine tuning problem is that, the simplest form of dark energy, the cosmological constant has the same properties as the vacuum energy in quantum field theory. But the estimated size of the vacuum energy is $\rho \simeq \rho_p$ where $\rho_p \simeq m_p^4$ is the Planck density. While the observed value $\rho \simeq 10^{-123}\rho_p$ so the observed value is less than the exact value of the vacuum energy by the factor of 10^{123} hence the fine tuning of the vacuum energy is required.

As a possible alternative of cosmological constant, some scalar fields are introduced, these are quintessence, k-essence, tachyon and phantom fields to name some of them. These fields differ from the cosmological constant, in the sense that they have a time dependent EoS parameter. In this way they can correspond to cosmological constant if $\omega = -1$, and when $-1 < \omega < -1/3$ it corresponds to the quintessence phase and it will be phantom phase Universe for $\omega < -1$. But the phantom divide $\omega = -1$ can not be crossed by the quintessence or phantom alone.

Some progressive efforts are made by the cosmologists for obtaining a dark energy model with EoS crossing the phantom divide. The HDE is one of them, it is based on the holographic principle. by applying this principle the upper bound of the entropy of the Universe can be obtained. The literature review shows that HDE model is viable if the infrared cut-off is set as the future event horizon.

Since the HDE density depends on the entropy area relation of the black hole so there could appear a modification in this relation. A new version of the HDE, named as ECHDE model has been proposed with these corrections to the entropy of a black hole. For the ECHDE dominated Universe with $H > 0$ and $L = H^{-1}$ works and give the beneficial results for the inflation in the early Universe. But still there are two unsolved problem in the case of these scalar fields as well. These are the fine tuning problem and the coincidence problem.

The coincidence problem is that, why densities of dark energy and dark matter are comparable today? While they evolve differently in expansion of the Universe and if there is no interaction between two densities.

These problems are the motivations to study another model of dark energy. In the last sections of second chapter *holographic dark energy* and *some of its versions*, the entropy corrected and the power law entropy corrected holographic dark energy models, are discussed.

In the third chapter we have studied the power law entropy corrected version of the HDE model which is in interaction with DM in the non-flat FRW Universe. We considered the power law corrected term to the energy density of HDE model. Using the expression of this modified energy density, we obtained the EoS parameter, deceleration parameter and evolution of energy density parameter for the interacting R-PLECHDE model. Moreover, we established a correspondence between our model and the MCG, the tachyon, k-essence, dilaton and quintessence scalar field models in the hypothesis of non-flat FRW Universe. These correspondences are important because they allow us to understand how the various candidates of DE are mutually related to each other. The limiting case of flat dark dominated Universe without entropy correction were studied in each scalar field and we see that the EoS parameter is constant in this case and we calculate the scalar field and its potential which can be obtained by idea of power law expansion of scalar field.

Further, working in this area of research, a similar studies can be done for other versions of HDE. In particular some one can reconstruct the $f(R)$ theory of gravity (a theory dealing with studies of the accelerated expansion of the Universe) with the help of the under discussion models of HDE.

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