Some Results of Variable Viscosity For MHD Boundary Layer Flow Over an Exponentially Stretching Sheet

Laraib Hanif

Thesis submitted for the degree of

Master of Philosophy in Mathematics

Supervisor: Dr. Muhammad Asif Farooq

School of Natural Sciences(SNS)

National University of Sciences and Technology(NUST)

Islamabad, Pakistan

FORM TH-4

National University of Sciences & Technology

MS THESIS WORK

We hereby recommend that the dissertation prepared under our supervision by: MS. LARAIB HANIF, Regn No. 00000117042 Titled: Some Results Of Variable Viscosity For Mhd Boundary Layer Flow Over An Exponentially Stretching Sheet be accepted in partial fulfillment of the requirements for the award of **MS** degree.

Examination Committee Members

1. Name: DR. MUJEEB-UR-REHMAN Signature: ____________

2. Name: DR. ADNAN MAQSOOD Signature: ____________

External Examiner: DR. MASOOD KHAN Signature: ____________

Supervisor's Name DR. M. ASIF FAROOQ Signature: ____________

Head of Department

Date 18/3/2019

COUNTERSINGED

Date: 18/03/19

Dean/Principal

THESIS ACCEPTANCE CERTIFICATE

Certified that final copy of MS thesis written by **Ms. Laraib Hanif** (Registration No. **00000117042**), of **School of Natural Sciences** has been vetted by undersigned, found complete in all respects as per NUST statutes/regulations, is free of plagiarism, errors, and mistakes and is accepted as partial fulfillment for award of MS/M.Phil degree. It is further certified that necessary amendments as pointed out by GEC members and external examiner of the scholar have also been incorporated in the said thesis.

Signature: ______________________

Name of Supervisor: Dr. M. Asif Farooq

Date: 18/03/19

Signature (HoD): ______________________

Date: 18/03/2019

Signature (Dean/Principal): ______________________

Date: 18/03/19

Dedicated to

My loving Parents.

Abstract

In this thesis we will present some results of variable viscosity for boundary layer flow and heat transfer over an exponentially stretching sheet.Also some results of MHD stagnation point flow with variable viscosity and variable thermal conductivity will also be discussed. Three different cases ie. constant fluid properties, variable viscosity and the exponential temperature dependency will be considered to see the solutions of the flow problems.

By using similarity parameters the non-linear partial differential equations are converted in to non-linear odinary differential equations. The numerical solutions are calculated by using a shooting technique and results will be compared by usnig a MATLAB built-in solver bvp4c. Velocity and temperature curves are analyzed by varying different values of prandtl number. Graphs have been developed to investigate the impact of parameters on temperature and velocity profiles.

Acknowledgement

First of all I would like to thank Allah Almighty for His guidance and courage to complete my thesis.

I would like to express my special thank to my supervisor, Dr. Asif Farooq, for his supervision and guidance throughout my thesis. His sympathetic attitude and friendly behaviour positively influenced my thesis. My sincere gratitude is to my GEC members, Dr. Mujeeb ur Rehman (SNS) NUST and Dr. Adnan Maqsood(RCMS) NUST for their helpful comments that encouraged me in completing this thesis in time.

I am very thankful to my friends Rida Ahmad, Azqa Arif, Iqra Zulfiqar and Razia for their valuable assistance.

Special thanks and appreciation goes to my parents, siblings and my husband for their prayers, support and encouragement that this work came in to an existence .

Laraib Hanif

Contents

Chapter 1

Introduction and Preliminaries

This chapter includes literature review and some basic definitions. Boundary layer equations, relevant definitions to this research work and few details about numerical methods will also be presented.

1.1 Literature review

The momentum and thermal boundary layers developing along a moving plate and over a stretching sheet in a quiescent ambient is considered. While considering magnetohydrodynamic (MHD), temperature's effect on viscosity and variable thermal conductivity are observed.

Many industrial processes are dependent on the comprehension of fluid flow over a surface.The first person who investigated the behaviour of boundary layer flow over solid surface moving continuously with persistent rate of speed was none other than Sakiadis [1]. Andersson and Aarseth [4] analyzed the effects of temperature on variable fluid properties in the Sakidis flow problems. Elbashbeshy and Bazid [5] investigated the influence of variable viscosity on heat transfer over a continuous surface in motion. Hayat et al. [14] examined stagnation point flow of Jaffrey fluid while incorporating thermal radiation and magnetic field effects. Swain et al. [26]

analyzed the stagnation point flow with variable fluid properties under the general setup of viscous, incompressible conducting fluid. Adnan et al. [27] studied boundary layer flow and heat analysis along with convected heat, partial slip and suction over a shrinking surface. Mukhopadhyay [28] studied MHD flow and analyzed heat transfer while considering exponentially stretching sheet inserted in thermally layered medium. Parasad et al. [29] examined the influence of temperature dependent fluid properties on hydrodynamic flow and heat transfer over a non-linear stretching surface. Ishak [30] analyzed how radiation effect the MHD flow when an exponentially stretching sheet is considered. Mustafa et al. [31] calculated the numerical and series solutions for stagnation-point flow past an exponentially stretching sheet. Subhas et al. [32] carried out an analysis to study the influence of variable viscosity and variable thermal conductivity on mixed convection heat transfer due to an exponentially stretching sheet with external magnetic field.

1.2 Preliminaries and basic definitions

1.2.1 Fluid

A material which can flow and deform continuously, no matter how small the shear stress may be applied. The fluid flow and assume the shape of the container they are poured into.

Although different in many respects both liquids and gases are classified as fluids because of their common characteristic that they offer a permanent resistance to a shearing force.

1.2.2 Boundary layer flow

When the fluid flows over a solid boundary, the layer of the fluid immediately in contact with the surface, where the fluid flow is affected by the viscous forces is called boundary layer.

As the real fluid flows over a solid body the particles of the fluid cling on the surface of a solid boundary. Velocity of the fluid particles on the surface will have the same velocity as that of the surface of the solid boundary i.e. zero. As we start going up, the velocity starts to rise which is a witness that there is an existence of velocity gradient as we go away from the boundary of the sloid body. After certain height and certain time velocity will no longer vary with distance and this velocity is called the free stream velocity. So the veocity gradient exists in small layers above the surface and beyond this velocity gradient will vanish. The region in which velocity gradient is occurring i.e. velocity is varying with respect to the distance from the surface of a body. This small layer is called boundary layer. (see Fig.1.1)

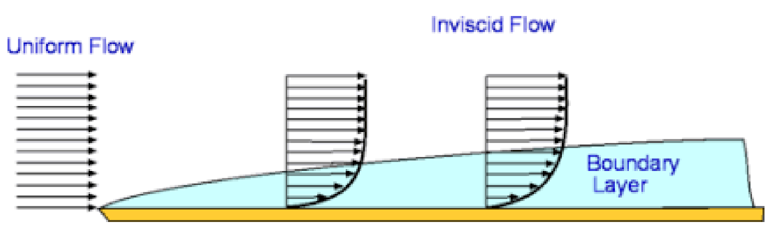


Figure 1.1: Boundary layer flow (Source: Internet).

1.2.3 Stagnation point flow

In a flow field stagnation point refers to a point where the fluid becomes stagnant by the solid body. (see Fig.1.2)

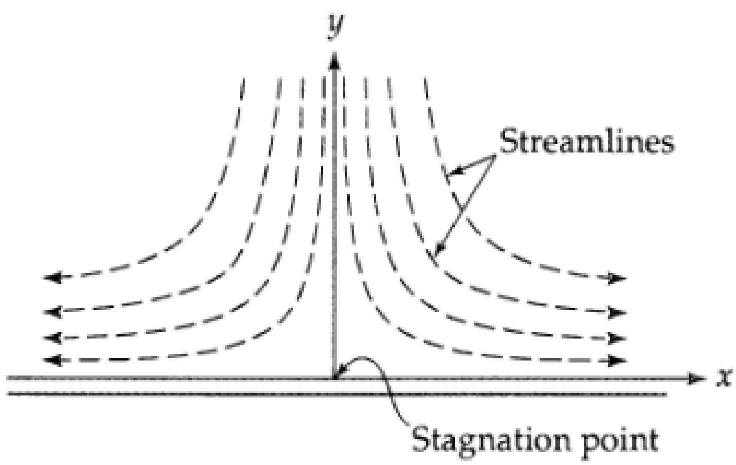


Figure 1.2: Stagnation point flow (Source: Internet).

1.2.4 MHD

The analysis of magnetic properties of electrically conducting fluids is called magnetohydrodynamic (MHD). Plasmas, liquid metals and salt water or electrolytes are some examples of magneto-fluids. The basic concept behind MHD is that magnetic field may generate currents in the flowing conductive fluid.

1.2.5 Nusselt number

The dimensionless number which is regarded as the ratio of the thermal energy convected to the fluid to the thermal energy conducted within the fluid. Mathematically defined as Mustafa [33]:

:

$$Nu_x = \frac{xq_w}{T_w - T_0} \tag{1.2.1}$$

where q_w is the heat flux given by

$$q_w = -k\frac{\partial T}{\partial y} \tag{1.2.2}$$

1.2.6 Skin friction coefficient

Measure of resistance between the fluid and solid surface is known as skin friction coefficient. It is defined as,

$$C_f = \frac{\tau_w}{\rho {U_w}^2} \tag{1.2.3}$$

where U_w is the surface velocity and τ_w is the total wall shear stress.

1.2.7 Prandtl number

A dimensionless number regarded as momentum diffusivity divided by thermal diffusivity. Mathematically it is defined as:

$$Pr = \frac{\nu}{\alpha} \tag{1.2.4}$$

where ν is momentum diffusivity defined as μ/ρ and α is thermal diffusivity defined as $\kappa/\rho c_p$. Here μ, κ, c_p and ρ represents dynamic viscosity, thermal conductivity, specific heat and fluid's density .

1.2.8 Reynolds number

A Reynolds number is regarded as a dimensionless number which is used to specify the behaviour of the fluid flow i.e. whether flow is laminar or turbulent. It is the fraction of inertial forces to viscous forces. Mathematically represented as:

$$Re = \frac{UL}{\nu} \tag{1.2.5}$$

where U, L and ν are velocity, reference length and viscosity of the fluid.

1.2.9 Thermal conductivity

The extent to which a specific material can transmit heat. It can be mathematically expressed as,

$$k = \frac{QL}{A\Delta T} \tag{1.2.6}$$

1.2.10 Steady flows and unsteady flows

Steady flow refers to the fluid flow in which the properties of the fluid (velocity, pressure, temperature etc.) within the control volume is independent of time i.e.

$$\frac{\partial v}{\partial t} = \frac{\partial P}{\partial t} = \frac{\partial T}{\partial t} = 0 \tag{1.2.7}$$

Non-steady flow refers to the fluid flow in which the properties of the fluid (velocity, pressure, temperature etc.) within the control volume is time dependent i.e.

$$\frac{\partial v}{\partial t} \neq 0, \frac{\partial P}{\partial t} \neq 0, \frac{\partial T}{\partial t} \neq 0 \tag{1.2.8}$$

1.2.11 Compressible flows and incompressible flows

Flows in which fluid's density is variable are called compressible flows. For instance air.

Flows in which fluid's density is constant are called incompressible flows. For instance water.

1.2.12 Mach number

Mach number is a dimensionless number representing the ratio of speed of object to the speed of sound. Mathematically given as:

$$M = \frac{u}{c} \tag{1.2.9}$$

where the velocity of the object is denoted by u and speed of sound is denoted by c. Mach number is an important parameter to classify the fluid either as incompressible or compressible. Fluids having Mach number greater than 0.3 are called compressible fluids and fluids have Mach number less than 0.3 are called incompressible fluids.

1.2.13 Laminar and turbulent flow

In laminar flow the fluid particles follow a definite path that never interfere with one another. In turbulent flows the fluid particles undergoes irregular mixing and fluctuations and does not follow a definite path.

1.2.14 Newtonian fluid

A fluid that experiences shear stress that is linearly correlated to the strain rate. Mathematically defined as:

$$\tau_{xy} \propto \frac{du}{dy} \tag{1.2.10}$$

τ_{xy} denotes shear stress and du/dy denotes deformation rate respectively. Some common examples of Newtonian fluids are air and water.

1.3 Governing equations

The essentials of fluid dynamics are continuity equation, the conservation of momentum and energy equation which can be mathematically explained as follows:

1.3.1 Continuity equation

The mass conservation principle deals with the equation of continuity.

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \vec{v}) = 0 \tag{1.3.1}$$

Above mention equation represents continuity equation for compressible fluid. In case if the flow is steady we consider,

$$\frac{\partial \rho}{\partial t} = 0$$

and equation (1.3.1) will reduce to

$$\nabla.(\rho\vec{v}) = 0$$

Further if we consider density is constant then we have,

$$\vec{\nabla}.\vec{v} = 0.$$

1.3.2 Conservation of momentum

The vector equation of conservation of momentum is expressed as

$$\rho(\frac{d\vec{v}}{dt}) = -\nabla.\Pi + \rho\vec{g} \tag{1.3.2}$$

where $\frac{d\vec{v}}{dt}$ represents material derivative and right side of equation (1.3.2) represents stresses applied on the surface and body force due to gravity.
So,

$$\vec{\nabla.\Pi} = -\vec{\nabla}P + \vec{\nabla}.\tau$$

The surface forces are due to the stresses on the sides of control surface. These stresses are the sum of hydrostatic pressure and the viscous stresses (δ_{ij}) which arise from the fluid motion.

$$\Pi = -\tau$$

equation (1.3.2) becomes

$$\rho(\frac{d\vec{v}}{dt}) = -\nabla.\rho + \nabla.\delta + \rho\vec{g}.$$

1.3.3 Conservation of energy

The equation for conservation of energy is derived from the first law of thermodynamics ,whereas ,first law of thermodynamics states that,
Rate of increase of the sum of kinetic and internal energies equals the rate of energy

addition (by flow and by heat conduction) plus the rate at which fluid outside V is doing work on the fluid inside V.

$$\rho\frac{d\hat{U}}{dt} = -\vec{\nabla}.\vec{q} - \vec{\nabla}.(\Pi.\vec{v}) + (\vec{\nabla}.\Pi).\vec{v}$$

1.4 Numerical methods

In this study we convert PDEs in to ODEs to obtain the solution of the problem with the use of numerical methods. There are few numerical methods like shooting method, bvp4c and finite difference method used to convert boundary value problem into an initial value problem.
This research deals with the shooting method and bvp4c to obtain numerical approximation of the problem. Furthermore through similarity transformations PDEs are converted in to ODEs. Some basic features of these numerical methods are mentioned below.

1.4.1 Shooting method

In numerical analysis, the shooting method is a numerical method approach that works by reducing the boundary value problem into the solution of an initial value problem. Shooting technique can be used for odinary linear differential equations as well as non-linear ODEs . The solution begin to occurr at one extreme of the boundary value problem and shoot to the other extreme until the boundary condition to the other extreme approaches to its accurate value.
Let us take a second order two point BVP subjected to the boundary conditions and is expressed in the following form as

$$q'' = f(x, q, q'), \qquad q(a) = \alpha, \qquad q(b) = \beta, \tag{1.4.1}$$

where α,β are unknowns.
Following method is adapted to convert Eq. (1.4.1) into an IVP.

Let us consider the IVP

$$q'' = f(x, q, q'), \quad q(a) = \alpha, \quad q'(a) = \lambda. \tag{1.4.2}$$

In the above Eq. (1.4.2) λ is unknown so we have to find the λ because value of λ will help us calculating the value of $q(b) = \beta$. Omitting the few cases, the strategy for both linear and non-linear shooting method is identical to solve the IVPs. Non-linear problems has the similar solution as linear problems except that the base solution can not be expressed as a linear combination of each other. In case of non-linear shooting method we use an iterative technique inspite of using a simple formula so that solutions of two IVPs can be combined. We have to find the zero of the function that will represent the error i.e. the amount by which the solution to IVP fails to satisfy the boundary conditions at x=b. It can be explained in another similar statement that the amount by which $q(b, \lambda)$ misses the target value β. The error is represented by $F(\lambda)$ which is a function of the initial slope of our own choice. We obtain different errors by taking different values of λ so $F(\lambda)$ is defined as

$$F(\lambda) = y(b, \lambda) - \beta = 0. \tag{1.4.3}$$

When $q'(a) = \lambda^*$ has been calculated then the $q'(x, \lambda)$ is the wanted solution. Now zero of the error function can be found by two methods. One method is Secant method and the other one is Newton's method. We will only explain Newton's method here. For Newton's method, we will find derivative of the zero function i.e. $F(\lambda)$ to choose the value of λ^* such that Eq. (1.4.3) holds. Then

$$\lambda^* = \frac{q(b) - q(a)}{b - a} = q'(a) \tag{1.4.4}$$

$$\lambda^* = \frac{\beta - \alpha}{b - a} \tag{1.4.5}$$

We use Newton's method to approximate the solution of $q(b, \lambda) - \beta = 0$ and find a next guess λ_{i+1}.

$$\lambda_{i+1} = \lambda_i - \frac{y'(b, \lambda_i) - \beta}{y'(b, \lambda_i)} \tag{1.4.6}$$

1.4.2 bvp4c

MATLAb yield a convenient and suitable routine known as bvp4c, which is able to resolve quite sophisticated problems. The algorithm of bvp4c depends on an iteration formation to solve a scheme of non-linear equations. Bvp4c is premarily a collocation formula in which solution begins with an initial guess provided initial mesh points.

Chapter 2

SOME RESULTS OF VARIABLE VISCOSITY FOR BOUNDARY LAYER FLOW AND HEAT TRANSFER OVER AN EXPONENTIALLY STRETCHING SHEET

Chapter 2 is structured in a systematic way. Section 2.1 of this chapter is formulation of the problem. Three different cases are discussed in section 2.2. Implementation of numerical technique is in section 2.3 and section 2.4 is all about results and discussions.

2.1 Mathematical Model

We assume a steady, two dimensional, laminar flow of a Newtonian fluid past an exponentially stretching sheet.Temperature of ambient fluid is constant and is denoted by T_0 .Also the temperature of sheet is denoted by T_w. The flat surface is moving in the positive x-direction with constant speed u and y- axis is normal to it. Considering these assumptions, the governing equations following the general set up of Anderson and arseth are as follows [4]:

$$\partial_x(\rho u) + \partial_y(\rho v) = 0, \tag{2.1.1}$$

$$\rho u u_x + \rho v u_y - \partial_y(\mu u_y) = \rho u_e \frac{\partial u_e}{\partial x} \tag{2.1.2}$$

$$C_p(\rho u T_x + \rho v T_y) - \partial_y(k T_y) = 0 \tag{2.1.3}$$

where ρ is the density of the fluid and velocity in the horizontal direction is u and the velocity in the vertical direction is v, C_p represents the specific heat The temperature of the fluid is represented by T and thermal conductivity of the fluid is denoted by k.

The flow problem has the following boundary conditions:

$$(u(x,0), v(x,0), T(x,0)) = (U_w = ae^{\frac{x}{L}}, 0, T_w) \tag{2.1.4}$$
$$u \to U_e(x) = be^{\frac{x}{L}}, \quad T \to T_\infty \ as \quad y \to \infty$$

We employ the following similarity variables[4],

$$\eta = \sqrt{\frac{a}{2\nu_0 L}} e^{\frac{x}{2L}} \int_0^y \frac{\rho}{\rho_0} dy, \qquad \psi = \rho_0 \sqrt{2a\nu_0 L} e^{\frac{x}{2L}} f(\eta), \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \tag{2.1.5}$$

A stream funtion $\psi(x,y)$ is defined as: [4]

$$\rho u = \frac{\partial \psi}{\partial y}, \qquad \rho v = -\frac{\partial \psi}{\partial x}. \tag{2.1.6}$$

The continuity equation is automatically satisfied under condition (2.1.6). The velocity components u and v are given by:

$$u = a e^{\frac{x}{L}} f'(\eta), \qquad v = -\frac{\rho_0}{\rho}\sqrt{\frac{a\nu_0}{2L}} e^{\frac{x}{2L}} [f(\eta) + \eta f'(\eta)] \tag{2.1.7}$$

Using Eqs. (2.1.5), (2.1.6) and (2.1.7) into Eqs. (2.1.1), (2.1.2) and (2.1.3) we get

$$2f'^2 - ff'' = (\frac{\mu\rho}{\mu_0\rho_0} f'')' + 2\epsilon^2, \tag{2.1.8}$$

$$Pr_0 \frac{C_p}{C_{p0}} f\theta' + \left(\frac{k\rho}{k_0\rho_0}\theta'\right)' = 0, \tag{2.1.9}$$

where $Pr_0 = \mu_0 C_{p0}/k_0$ is Prandtl number, $\epsilon = \frac{b}{a}$,

Boundary conditions for similarity variable will be transformed as follows:

$$\begin{aligned} f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \\ f'(\eta) = \epsilon, \quad \theta(\eta) = 0 \quad as \;\; \eta \to \infty, \end{aligned} \tag{2.1.10}$$

the prime represents differentiation with respect to η, f' and θ represent dimensionless velocity and temperature respectively.
The skin friction coeffcient C_f and local Nusselt number Nu_x are defined as follows Mustafa [33]:

$$C_f = \frac{\tau_w}{\rho U_w^2}, \qquad Nu_x = \frac{x q_w}{T_w - T_0}, \tag{2.1.11}$$

where τ_w is the shear stress and q_w is the heat flux, and are defined as :

$$\tau_w = \mu_w \frac{\partial u}{\partial y}, \qquad q_w = -k\frac{\partial T}{\partial y}, \tag{2.1.12}$$

using equation (2.1.11) and (2.1.12) we get

$$C_f Re^{1/2} = \frac{1}{\sqrt{2L}} f''(0), \qquad Nu_x = \frac{-x\theta'(0)}{\sqrt{2L}} Re^{1/2}, \tag{2.1.13}$$

where Re denotes the local Reynolds number.

Following cases are discussed here as mentioned in Andersson and Aarsaeth[4] .

2.2 Special Cases

2.2.1 Case A: Constant Fluid Properties

Considering fluid properties to be constant, the similarity variable η is transformed to Blasius variable [2, 6].

$$\eta = \sqrt{\frac{a}{2\nu_0 L}} e^{\frac{x}{2L}} y, , \tag{2.2.1}$$

and Eqs. (2.1.8) and (2.1.9) become

$$f''' + ff'' - 2f'^2 + 2\epsilon^2 = 0 \tag{2.2.2}$$

$$\theta'' + f\theta' Pr_0 = 0. \tag{2.2.3}$$

Boundary conditions in this case will also remain the same as calculated above in Eq. (2.1.10) Ishak et al.[4].

2.2.2 Case B: Variable Fluid Properties

Andersson and Aarseth [4], Elbashbeshy and Bazid [5] and Pantokratoras[14] following the work of Pop et al. considered viscosity as a function of temperature by

considering other fluid properties to be constant.
The momentum boundary layer Eq. (2.1.8) in this case takes the form

$$2f'^2 - ff'' = (\frac{\mu}{\mu_0}f'')' + 2\epsilon^2, \tag{2.2.4}$$

and the thermal boundary layer Eq. will remain the same as in case A. For a viscous fluid, Pop et al.[6] followed Lai and Kulacki [7] and proposed following correspondence between viscosity and temperature and demonstrated $\mu(T)$ [1], [5],[7] as follows:

$$\mu(T) = \frac{\mu_{ref}}{[1 + \gamma(T - T_{ref})]} \tag{2.2.5}$$

Writing $T_{ref} \approx T_o$, then formula is rewritten as [4]

$$\mu = \frac{\mu_0}{1 - \frac{T - T_0}{\theta_{ref}(T_w - T_0)}} = \frac{\mu_0}{1 - \frac{\theta(\eta)}{\theta_{ref}}}, \tag{2.2.6}$$

here $\theta_{ref} \equiv \frac{-1}{(T_w - T_o)\gamma}$ and operating temperature difference is represented by $(T_w - T_0)$ = ΔT [1].

2.2.3 Case C: Exponential Temperature Dependency

In this case, again viscosity is taken variable similar to case B but its different realtion with temperature is used here.[1]

$$\ln(\frac{\mu}{\mu_{ref}}) = -2.10 - 4.45\frac{T_{ref}}{T} + 6.55(\frac{T_{ref}}{T})^2, \tag{2.2.7}$$

was suggested by White [1]. Here μ_{ref} = 0.00179 kg/ms and T_{ref} = 273 K.

2.3 Numerical Methods

Shooting method is employed to solve numerically the non linear odinary differential equations along with boundary conditions for different cases.The purpose of shooting method is t0 convert a boundary value problem into an initial value problem. Two computing steps are generally involved in shooting method.The first one is to find the root by Newton-Raphson method and in the second step fifth order Runge-Kutta is used to obtain the solution of an initial value problem. We compare the results of shooting method by bvp4c which is a built-in solver in MATLAB.

Numerical solutions for the case A,B and C can be calculated with the following form of governing equations[9] The momentum and energy equations for the case A are,

$$f''' + ff'' - 2f'^2 + 2\epsilon^2 = 0 \tag{2.3.1}$$

$$\theta'' + f\theta' Pr_0 = 0. \tag{2.3.2}$$

The momentum equation for variable viscosity for case B becomes,

$$f''' = -\frac{f''\theta'}{0.25-\theta} - \frac{0.25-\theta}{0.25}[ff'' - 2f'^2 + 2\epsilon^2] \tag{2.3.3}$$

The momentum equation for exponential dependence of viscosity on temperature for case C becomes,

$$f''' = (2f'^2 - ff'' - 2\epsilon^2)(\frac{\mu_o}{\mu}) - f''\theta'(T_w - T_0)(4.45\frac{T_{ref}}{T^2} - 13.1\frac{T_{ref}^2}{T^3}) \tag{2.3.4}$$

here [1]

$$\frac{\mu}{\mu_0} = \frac{\mu_{ref}}{\mu_0} exp(-2.10 - 4.45(\frac{T_{ref}}{T}) + 6.65(\frac{T_{ref}}{T})^2). \tag{2.3.5}$$

2.4 Results and Discussions

This section involves tabular and graphical representation of numerical results. Computations are performed to study the effect of variation of prandtl number Pr, velocity ratio parameter denoted by ϵ. We observe variation in velocity gradient f"(0) and temperature gradient $\theta'(0)$ by changing Prandtl number. A slight change is observed in skin friction coefficient when prandtl number is increased keeping the velocity ratio parameter constant whereas the tenperature shows the increasing behaviour as shown in table 1.While for both case B and C skin friction coefficient and temperature gradient both shows the increasing behaviour.

Table 2.1: Values of $-\frac{\partial^2 f}{\partial \eta^2}(0)$ and $-\frac{\partial \theta}{\partial \eta}(0)$ for different parameters for Case A.

		bvp4c		shooting method	
Pr	ϵ	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$
1	0.1	1.253586	0.57114134	1.25358	0.5714134
0.7	0.1	1.2535866	0.4519569	1.25358	0.451957
10	0.1	1.2535804	2.2661002	1.25358	2.26609

Table 2.2: Values of $-\frac{\partial^2 f}{\partial \eta^2}(0)$ and $-\frac{\partial \theta}{\partial \eta}(0)$ for different values of M for Case B.

		bvp4c		shooting method	
Pr	ϵ	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$
1	0.1	2.8760822	0.4394255	2.8760559	0.43942452
0.7	0.1	2.859222	0.34476157	2.85919	0.344761
10	0.1	3.2192836	1.9808132	3.21926	1.9808

Table 2.3: Values of $-\frac{\partial^2 f}{\partial \eta^2}(0)$ and $-\frac{\partial \theta}{\partial \eta}(0)$ with different values of ϵ for Case C.

		bvp4c		shooting method	
Pr	ϵ	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$
1	0.1	2.7696185	0.44790541	2.7695906	0.44790573
0.7	0.1	2.7487684	0.35107849	2.74871	0.351077
10	0.1	3.1656117	2.0029514	3.1656	2.00294

Table 2.4: **Comparison of values of wall temperature gradient $[-\frac{\partial \theta}{\partial \eta}(0)]$ with-values of Magyari and Keller[10] and Z.Abbas et al. [11] for different values of prandtl numbers in case with a=0.**

Pr	*Magyari and Keller*	Z.Abbas et al.	Present
0.5	-0.330493	-0.330493	0.330494
1	-0.549643	-0.549643	0.549645
3	-1.122188	-1.122147	1.122092
5	-1.521243	-1.521243	1.521245
8	-.1991847	-1.1991846	1.991842
10	-2.257429	-2.257424	2.257424

In order to demonstrate the effects of viscosity and thermal conductivity, three different cases have been solved. T_0 is the ambient fluid temperature which is taken as 278K for water. T_W is the surface temperature taken as 358K. Results for constant fluid properties will be compared with inversely linear viscosity variation and the exponential variation. θ_{ref} is taken as -0.25 for water as suggested by Ling and Dybbs[22]. The velocity and temperature profiles are shown in Fig. 2.1 and 2.2.

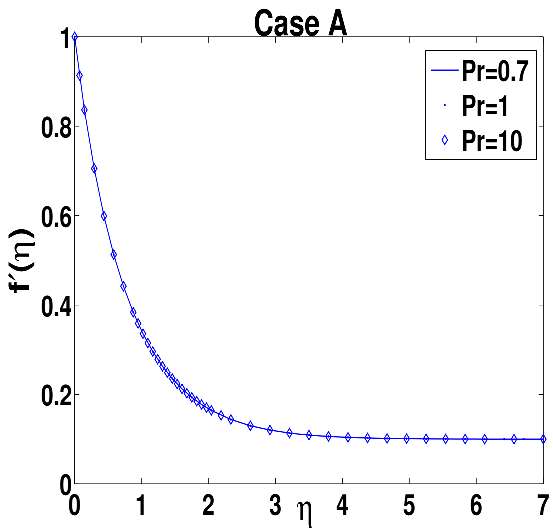


Figure 2.1: Depiction of velocity curve for case A with distinct values of Prandtl number ($\epsilon = 0.1$).

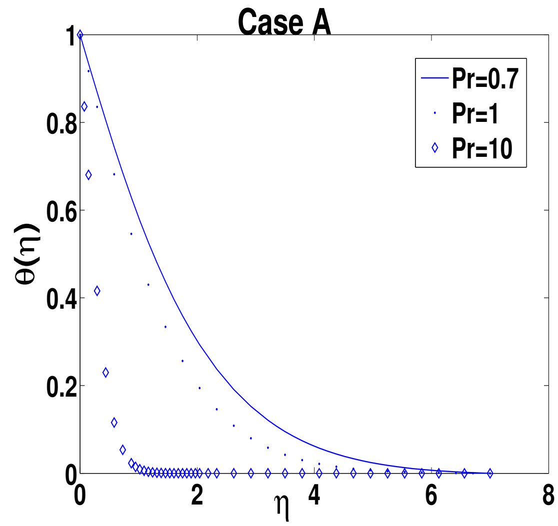


Figure 2.2: Depiction of temperature curve for case A with distinct values of Prandtl number ($\epsilon = 0.1$).

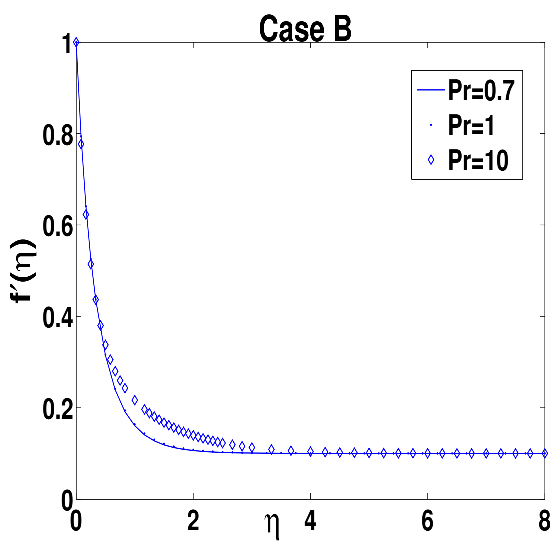


Figure 2.3: Depiction of velocity curve for case B with distinct values of Prandtl number ($\epsilon = 0.1$).

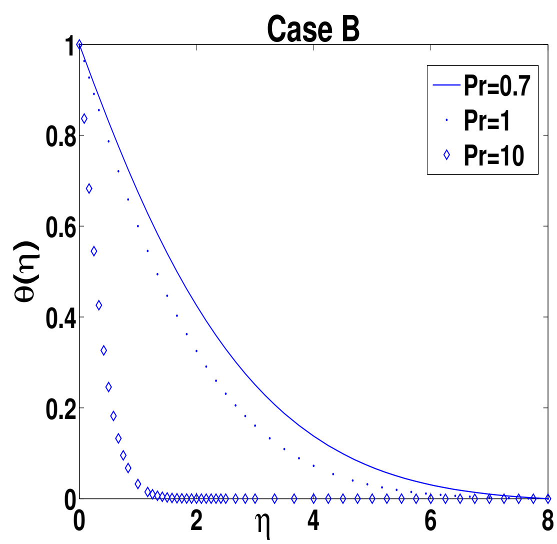


Figure 2.4: Depiction of temperature curve for case B with distinct values of Prandtl number ($\epsilon = 0.1$).

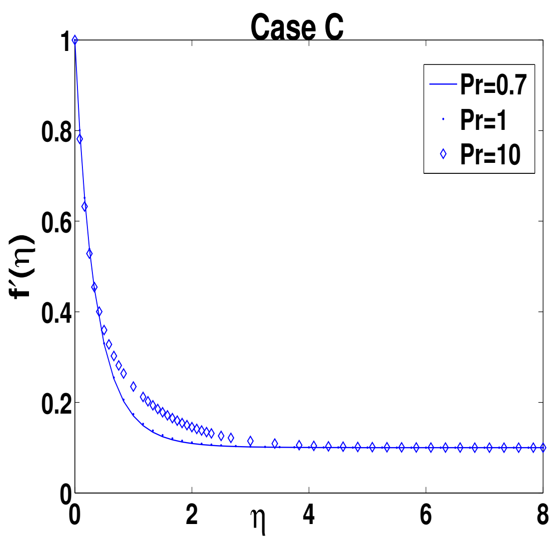


Figure 2.5: Depiction of velocity curve for case C with distinct values of Prandtl number ($\epsilon = 0.1$).

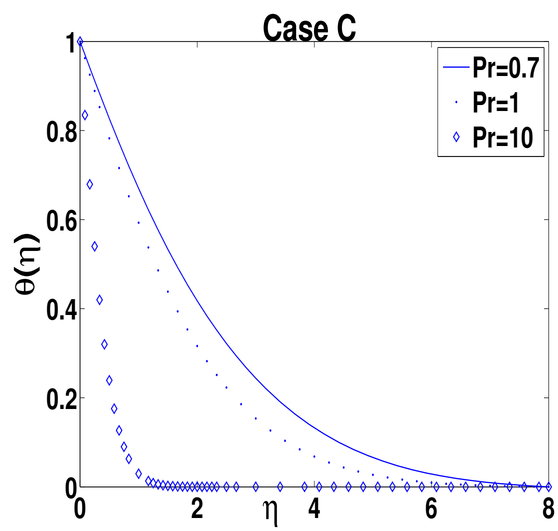


Figure 2.6: Depiction of temperature curve for case C with distinct values of Prandtl number ($\epsilon = 0.1$).

Chapter 3

Some Results of variable Viscosity for MHD Boundary Layer Flow Over an Exponentially Stretching Sheet

3.1 Mathematical Model

Let us consider two dimensional steady incompressible flow near a stagnation point located at y=0 over a stretching sheet. B_0 is the uniform magnetic field that is applied normal to the sheet.A small reynold's number is a justification to neglect the induced magnetic field for MHD flow. The flow is restricted to $y > 0$. $T_w(x)$ is the temperature of the sheet.We suppose $T = T_w(x) = T_0 + T_\infty e^{\frac{cx}{2L}}$ where T_0 is the ambient temperature and c is constant.

The basic equations governing such type of MHD flow can be written as follows [4]:

$$\partial_x(\rho u) + \partial_y(\rho v) = 0 \tag{3.1.1}$$

$$\rho(uu_x + vu_y) = \partial_y(\mu u_y) + \rho U_0 \frac{\mathrm{d}U_0}{\mathrm{d}x} + \sigma B_0^2(U_0 - u) \tag{3.1.2}$$

$$\rho C_p(uT_x + vT_y) = \partial_y(kT_y) \tag{3.1.3}$$

where ρ, μ and C_p and k represents fluid viscosity, coefficient of fluid viscosity, specific heat and variable thermal conductivity. Temperature of the respective fluid is represented by T.

The suitable boundary conditions for the velocity and temperature components for Eqs. (3.1.1)-(3.1.3) are given by;

$$\begin{gathered}(u(x,0), v(x,0), T(x,0)) = (U_w = ae^{\frac{x}{L}}, 0, T_w = T_0 + T_\infty e^{\frac{cx}{2L}}), \\ u \to U_0(x) = be^{\frac{x}{L}}, \quad T \to T_0 \; as \quad y \to \infty.\end{gathered} \tag{3.1.4}$$

Here stretching velocity of the fluid in the x-direction is $U_w(x) = ae^{\frac{x}{L}}$ at y=0 when the plate is stretched along x-axis.The temperture of the stretching sheet is T_w where a and b are constants.

The following similarity variables are introduced in this problem [4],

$$\eta = \sqrt{\frac{a}{2\nu_0 L}} e^{\frac{x}{2L}} \int_0^y \frac{\rho}{\rho_0} dy, \qquad \psi = \rho_0 \sqrt{2a\nu_0} L^{\frac{x}{2L}} f(\eta), \qquad \theta(\eta) = \frac{T - T_0}{T_w - T_0}, \tag{3.1.5}$$

where ψ is the stream function [4]. Velocities and the stream function has the following relationship [6]

$$\rho u = \frac{\partial \psi}{\partial y}, \qquad \rho v = -\frac{\partial \psi}{\partial x}. \tag{3.1.6}$$

x and y components of velocity are produced as

$$u = ae^{\frac{x}{L}} f'(\eta), \qquad v = -\frac{\rho_0}{\rho}\sqrt{\frac{a\nu_0}{2L}} e^{\frac{x}{L}} [f(\eta) + \eta f'(\eta)] \tag{3.1.7}$$

u_x and u_y are calculated as follows

$$\frac{\partial u}{\partial x} = u_x = \frac{a}{L} e^{\frac{x}{L}} f'(\eta) + \frac{a}{2L} e^{\frac{x}{L}} \eta f''(\eta) \tag{3.1.8}$$

$$\frac{\partial u}{\partial y} = u_y = ae^{\frac{x}{L}} f''(\eta) \sqrt{\frac{a}{2\nu_0 L}} e^{\frac{x}{2L}} \frac{\rho}{\rho_0} \tag{3.1.9}$$

Multiply $\frac{\partial u}{\partial x}$ by ρu and $\frac{\partial u}{\partial y}$ by ρv we get,

$$\rho u \frac{\partial u}{\partial x} = \frac{\rho a^2}{L} e^{\frac{2x}{L}} f'^2(\eta) + \frac{\rho a^2}{2L} e^{\frac{2x}{L}} \eta f'(\eta) f''(\eta) \tag{3.1.10}$$

and

$$\rho v \frac{\partial u}{\partial y} = -\frac{\rho a^2}{2L} e^{\frac{2x}{L}} f(\eta) f''(\eta) - \frac{\rho a^2}{2L} e^{\frac{2x}{L}} \eta f'(\eta) f''(\eta) \tag{3.1.11}$$

Adding above two equations we get,

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \quad \frac{\rho a^2}{L} e^{\frac{2x}{L}} f'^2(\eta) - \frac{1}{2} f(\eta) f''(\eta) \tag{3.1.12}$$

Next we multiply μ by $\frac{\partial u}{\partial y}$ and calculating a derivative with respect to y, equation changes to

$$\partial_y(\mu \frac{\partial u}{\partial y}) = \partial_y(\mu a e^{\frac{x}{L}} f''(\eta) \sqrt{\frac{a}{2\nu_0 L}} e^{\frac{x}{2L}} \frac{\rho}{\rho_0}) \tag{3.1.13}$$

$$\partial_y(\mu \frac{\partial u}{\partial y}) = \frac{a^2}{2\nu_0 L} e^{\frac{2x}{L}} \frac{\rho}{\rho_0} [\mu \frac{\rho}{\rho_0} f''(\eta)^{'}] \tag{3.1.14}$$

Subsituting all above equations in Eq (3.1.2) transform them into following BVP

$$\left(\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) + \rho U_0 \frac{\mathrm{d}U_0}{\mathrm{d}x} + \sigma B_0^2 (U_0 - u), \tag{3.1.15}$$

$$(\frac{\rho a^2}{L}e^{\frac{2x}{L}}f'^2(\eta)-\frac{1}{2}f(\eta)f''(\eta)) = (\frac{a^2}{2\nu_0 L}e^{\frac{2x}{L}}\frac{\rho}{\rho_0}[\mu\frac{\rho}{\rho_0}f''(\eta)^{'}])+\rho(be^{\frac{x}{L}})(\frac{b}{L}e^{\frac{x}{L}})+\sigma B_0^2((be^{\frac{x}{L}})-(ae^{\frac{x}{L}}f'(\eta))), \tag{3.1.16}$$

Multipling whole equation by $\frac{L}{\rho a^2 e^{\frac{2x}{L}}}$ we get,

$$f'^2 - \frac{1}{2}ff'' = \frac{1}{2}[\frac{\mu\rho}{\mu_0\rho_0}f'']' + \frac{b^2}{a^2} + \frac{\sigma B_0^2 L}{\rho a e^{\frac{x}{L}}}(\frac{b}{a} - f'(\eta)) \tag{3.1.17}$$

$$f'^2 - \frac{1}{2}ff'' = \frac{1}{2}[\frac{\mu\rho}{\mu_0\rho_0}f'']' + \lambda^2 + \frac{\sigma B_0^2 L}{\rho a e^{\frac{x}{L}}}(\lambda - f'(\eta)) \tag{3.1.18}$$

$$2f'^2 - ff'' = [\frac{\mu\rho}{\mu_0\rho_0}f'']' + 2\lambda^2 + \frac{2\sigma B_0^2 L}{\rho a e^{\frac{x}{L}}}(\lambda - f'(\eta)) \tag{3.1.19}$$

$$2f'^2 - ff'' = [\frac{\mu\rho}{\mu_0\rho_0}f'']' + 2\lambda^2 + M(\lambda - f'(\eta)) = 0 \tag{3.1.20}$$

M is a magnetic parameter equal to $\frac{2\sigma B_0^2 L}{\rho a e^{\frac{x}{L}}}$ and λ a velocity ratio parameter $\frac{b}{a}$,

Now differentiating T with respect to x and y, As

$$T = T_0 + \theta(T_W - T_0) \tag{3.1.21}$$

From boundary condition we have,

$$T = T_w(x) = T_0 + T_\infty e^{\frac{cx}{2L}} \tag{3.1.22}$$

so,

$$T_w(x) - T_0 = T_\infty e^{\frac{cx}{2L}} \tag{3.1.23}$$

so eq 1.1.21 will become

$$T = T_0 + \theta(T_\infty e^{\frac{cx}{2L}}) \tag{3.1.24}$$

Now differentiating T with respect to x and y ,

$$\frac{\partial T}{\partial x} = \theta' T_\infty e^{\frac{cx}{2L}} \sqrt{\frac{a}{2\nu_0 L}} \frac{e^{\frac{x}{2L}}}{2L} \int \frac{\rho}{\rho_0} dy + \theta T_\infty e^{\frac{cx}{2L}} \frac{c}{2L} \tag{3.1.25}$$

$$\frac{\partial T}{\partial y} = \theta' T_\infty e^{\frac{cx}{2L}} \sqrt{\frac{a}{2\nu_0 L}} e^{\frac{x}{2L}} \frac{\rho}{\rho_0} \tag{3.1.26}$$

$$\frac{\partial}{\partial y}(\kappa \frac{\partial T}{\partial y}) = \frac{a\rho e^{\frac{x}{L}}}{2\mu_0 L}(T_w - T_0)(\kappa\theta' \frac{\rho}{\rho_0})' \tag{3.1.27}$$

Putting all these above in energy equation leads to,

$$Pr_0 \frac{C_p}{C_{p0}}(cf'\theta - f\theta') = \left(\frac{k}{k_0}\theta' \frac{\rho}{\rho_0}\right)', \tag{3.1.28}$$

where Pr_0 is a Prandtl number equal to $\mu_0 C_{p0}/k_0$ and $\nu_0 = \frac{\mu_0}{\rho_0}$

The boundary conditions are transformed in to the following form :

$$\begin{aligned} f'(0) = 1, \quad \theta(0) = 1, \quad , f(0) = 0 \\ f'(\eta) = \lambda, \quad \theta(\eta) = 0 \quad as \;\; \eta \to \infty, \end{aligned} \tag{3.1.29}$$

the prime is a differentiation with respect to η where f' and θ denote velocity and temperature and both are dimensionless.

3.2 Special Cases

3.2.1 Case A: Constant Fluid Properties

Considering all fluid properties constant in this case will transform Eq 1.1.20 and 1.1.28 to

$$f''' + 2\lambda^2 + M(\lambda - f') - 2f'^2 + ff'' = 0 \tag{3.2.1}$$

$$\theta'' + Pr_0(cf'\theta - f\theta') = 0. \tag{3.2.2}$$

Above two equations are subject to same boundary conditions (1.1.29) Eq. (10) Ishak et al.

3.2.2 Case B: Variable Fluid Properties

Viscosity and thermal conductivity both are taken variable in this case.

Consider momentum boundary layer equation in this case,

$$(\frac{\mu\rho}{\mu_0\rho_0}f'')' + 2\lambda^2 + M(\lambda - f'(\eta)) - 2f'^2 + ff'' = 0. \tag{3.2.3}$$

For a viscous fluid, Bachok et al.[21] followed Ling and Dybbs [22] and assumed that the viscosity varies inversely with temperature. Lai and Kulacki [20] suggested $\mu(T)$ [4], [20],[22] given by the following relation

$$\mu(T) = \frac{\mu_{ref}}{[1 + \gamma(T - T_{ref})]} \tag{3.2.4}$$

Here γ dependent on a reference temperature T_{ref}, is a fluid property. Writing $T_{ref} \approx T_o$, then formula (14) is rewritten as [4]

$$\mu = \frac{\mu_0}{1 - \frac{T-T_0}{\theta_{ref}(T_w-T_0)}} = \frac{\mu_0}{1 - \frac{\theta(\eta)}{\theta_{ref}}}, \tag{3.2.5}$$

Putting value of the $\frac{\mu}{\mu_0}$ from the above relation in momentum boundary layer equation,

$$[\frac{1}{1 - \frac{\theta(\eta)}{\theta_{ref}}} f'']' + 2\lambda^2 + M(\lambda - f'(\eta)) - 2f'^2 + ff'' = 0 \tag{3.2.6}$$

$$[\frac{\frac{\theta'(\eta)}{\theta_{ref}} f''}{(\frac{\theta_{ref}-\theta(\eta)}{\theta_{ref}})^2} + \frac{1}{\frac{\theta_{ref}-\theta(\eta)}{\theta_{ref}}} f'''] + 2\lambda^2 + M(\lambda - f'(\eta)) - 2f'^2 + ff'' = 0 \tag{3.2.7}$$

After simplification we get,

$$f''' = -\frac{\theta'(\eta)}{(\theta_{ref} - \theta(\eta))} f'' + \frac{(\theta_{ref} - \theta(\eta))}{\theta_{ref}} (-2\lambda^2 - M(\lambda - f'(\eta)) + 2f'^2 - ff'') \tag{3.2.8}$$

Now consider energy equation,

$$Pr_0 \frac{C_p}{C_{p0}} (cf'\theta - f\theta') = \left(\frac{k}{k_0} \theta' \frac{\rho}{\rho_0} \right)', \tag{3.2.9}$$

As κ is a variable so,

$$\kappa = \kappa_0(1 + \lambda\theta) \tag{3.2.10}$$

$$\frac{\kappa}{\kappa_0} = (1 + \lambda \frac{T - T_0}{T_w - T_0}) \tag{3.2.11}$$

$$\frac{\kappa}{\kappa_0} = (1 + \lambda\theta) \tag{3.2.12}$$

Considering all other fluid properties to be constant energy equation will become,

$$Pr_0(cf'\theta - f\theta') = \left(\frac{k}{k_0}\theta'\right)', \tag{3.2.13}$$

$$Pr_0(cf'\theta - f\theta') = ((1 + \lambda\theta)\theta')', \tag{3.2.14}$$

$$Pr_0(cf'\theta - f\theta') = (1 + \lambda\theta)\theta'' + \lambda\theta'^2, \tag{3.2.15}$$

where $\theta_{ref} \equiv \frac{-1}{(T_w - T_o)\gamma}$ and operating temperature difference is denoted by $(T_w - T_0)$ = ΔT [4].

3.2.3 Case C: Exponential Temperature Dependency

Viscosity and thermal conductivity are again considered variable in this case,due to which energy equation remain the same but the relation for temperature dependent viscosity takes another form [4]

$$\log_e\left(\frac{\mu}{\mu_{reference}}\right) = c_1 + c_2\frac{T_{reference}}{T} + c_3\left(\frac{T_{reference}}{T}\right)^2, \tag{3.2.16}$$

here $c_1 = -2.10$, $c_2 = -4.45$, $c_3 = 6.55$, $\mu_{reference} = 0.00179$ kg/ms and $T_{reference} = 273$ K.

3.3 Results and Discussions

In the current section, numerical results are presented in tabular and graphical forms. Shooting method is employed to solve the system of non-linear ODEs numerically. Numerical results obtained from this method are then compared with bvp4c. Computations are carried out to investigate the effect of variation of Prandtl number

Pr, magnetic parameter M, ϵ denoting velocity ratio parameter, temperature index parameter c.

Variation of different parameters shows variation in velocity gradient $-f''(0)$ and temperature gradient $-\theta'(0)$ and given in tables (3.1-3.3). Increasing prandtl number shows an increase in temperature gradient whereas skin coefficient is decreasing. When magnetic parameter is increased, an increasing effect on skin friction coefficient is observed while temperature gradient decreases for the case A and B. Comparison of three different cases have been showed below.

Table 3.1: Numerical output of -$\frac{\partial^2 f}{\partial \eta^2}(0)$ and $-\frac{\partial \theta}{\partial \eta}(0)$ for Case A.

				bvp4c		Shooting Method	
Pr	M	ϵ	c	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$
0.7	0.1	0.1	1	1.285658	0.7755519	1.285651	0.7755509
1	-	-	-	1.285684	0.9713938	1.285651	0.9713867
3	-	-	-	1.285666	1.874469	1.285651	1.874469
7	-	-	-	1.285653	3.015463	1.285651	3.015476
10	-	-	-	1.285652	3.661816	1.285652	3.661843
0.7	0.2	-	-	1.316948	0.7698457	1.316924	0.7698421
-	0.3	-	-	1.347466	0.7643757	1.347458	0.7643752
-	0.4	-	-	1.377331	0.7591334	1.377304	0.7593398
-	0.5	-	-	1.406535	0.7540962	1.406506	0.7540921
10	0.5	0.5	-0.5	0.9479515	1.379364	0.9479514	1.379354
-	-	-	0	0.9479518	2.347711	0.9479518	2.347687
-	-	-	1	0.9479533	3.758663	0.9479514	3.758704
-	-	-	2	0.9479518	4.831548	0.9479514	4.831617
-	-	-	-1.5	0.9479516	2.569593	0.9479514	2.569553
10	0.1	0.1	1	1.285651	3.661843	1.285651	3.661843
-	-	0.2	-	1.221606	3.681157	1.221604	3.681183
-	-	0.3	-	1.133234	3.706352	1.133233	3.706377
-	-	0.4	-	1.023311	3.735983	1.023461	3.7735959
-	-	0.5	-	0.8938668	3.768995	0.8938667	3.769038

Table 3.2: Numerical output of $-\frac{\partial^2 f}{\partial \eta^2}(0)$ and $-\frac{\partial \theta}{\partial \eta}(0)$ for Case B.

				bvp4c		shooting method	
M	Pr	ϵ	c	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$
0.1	1	0.1	1	2.9848125	0.71666998	2.984761	0.7166663
0.2	-	-	-	3.0544435	0.70954247	3.054388	0.7095464
0.3	-	-	-	3.1223643	0.70288331	3.122316	0.7028884
0.4	-	-	-	3.1887519	0.6966408	3.188675	0.6966449
0.5	-	-	-	3.2536403	0.69076563	3.0545405	0.70954847

Table 3.3: Numerical output of $-\frac{\partial^2 f}{\partial \eta^2}(0)$ and $-\frac{\partial \theta}{\partial \eta}(0)$ for Case C.

				bvp4c		shooting method	
ϵ	c	M	Pr	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$
0.1	1	5	1	3.7524107	0.64851597	3.757235	0.6628378
0.2	-	-	-	3.480537	0.6994253	3.480876	0.7007433
0.3	-	-	-	3.162261	0.729269	3.162306	0.7295696
0.4	-	-	-	2.8049452	0.74943324	2.804935	0.7495075
0.5	-	-	-	2.4125211	0.76416864	2.412513	0.7641879

Table 3.4: Numerical output of $-\frac{\partial^2 f}{\partial \eta^2}(0)$ and $-\frac{\partial \theta}{\partial \eta}(0)$ (M= ϵ=1/10 and c=1)

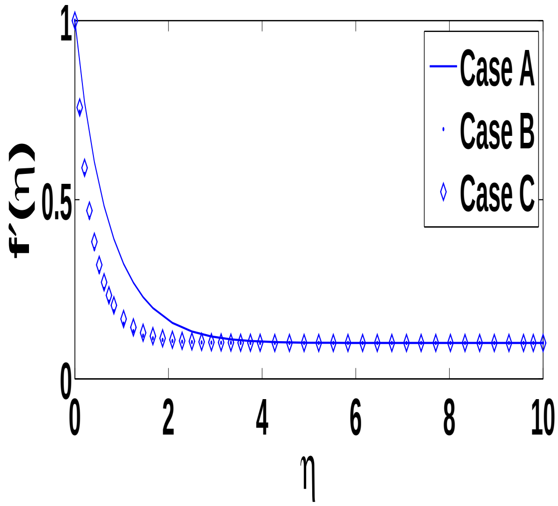
			bvp4c		shooting method	
Cases	ϵ	Pr	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$	$-\frac{\partial^2 f}{\partial \eta^2}(0)$	$-\frac{\partial \theta}{\partial \eta}(0)$
	0.1	0.7				
A			1.285658	0.7755519	1.285651	0.7755509
B			2.9599614	0.56290812	2.9599228	0.56290653
C			2.8069739	0.57720581	2.8069414	0.57720408
	0.1	1				
A			1.285684	0.9713938	1.285651	0.9713867
B			2.9848125	0.71666998	2.984761	0.7166663
C			2.8382754	0.73472495	2.8382053	0.73474384
	0.1	10				
A			1.285652	3.661816	1.285652	3.661843
B			3.4370474	3.0775832	3.4371001	3.0774977
C			3.3590037	3.1109399	3.3590036	3.110923

Table 3.5: **Comparison of values of wall temperature gradient** $[-\frac{\partial \theta}{\partial \eta}(0)]$ with-values of Magyari and Keller[10] for different values of prandtl numbers in case with c=0.

Pr	*Magyari and Keller [10]*	Present Results
0.5	-0.594338	0.5943396
1	-0.954782	0.9547853
3	-1.869075	1.869071
5	-2.500135	2.500116
8	-3.242129	3.242075
10	-3.660379	3.660302

Three different cases are represented together to demonstrate the influence of temperature dependent viscosity. T_w is the surface temperature taken as 358K. T_0 =278K is the temperature of the ambient fluid. Figures 3.1 and 3.2 are produced to show velocity and temperature results for various cases (A,B,C). In case C, reduction in temperature profile can easily be seen in comparison with the remaining two cases, i.e. A and B. Figs. 3.3-3.7 shows the velocity and temperature profiles when magnetic parameter is varied. The effect of variation of magnetic parameter can be clearly observed. The presence of a magnetic field to an electrically conducting fluid give rise to a resistive force called the Lorentz force. The motion of the fluid become slow because of this force. Lorentz force enhances when magnetic parameter is enhanced, due to which fluid motion decreases and momentum boundary layer thickness decrease. Thickness of thermal boundary layer increases with an increase of magnetic parameter. While in all other cases thickness of thermal boundary layer decreases. Owing to an increase in c the width of thermal boundary layer reduces which is depicted in curves 3.8-3.10.

Width of thermal boundary layer decreases because of an increase in the prandtl number whereas momentum boundary layer remains unchanged Temperature Profile reduces with the decrease in Prandtl number which is shown in Fig. 3.12 while variation of Prandtl number causes no change in velocity profile. Infuence of change of parameter ϵ is shown in Figs. 3.15-3.18, for the two cases i.e. A and C.



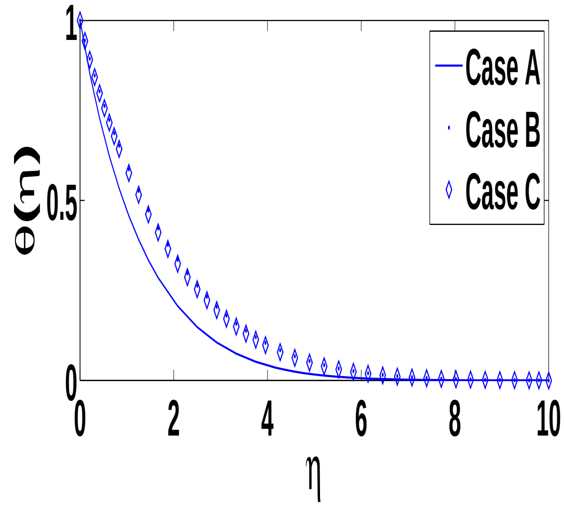
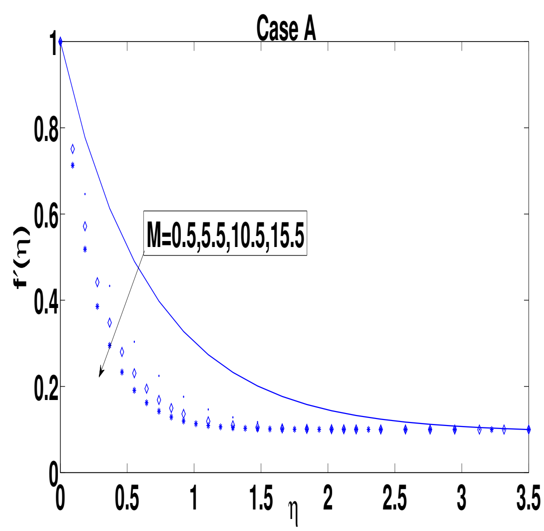


Figure 3.1: Depiction of velocity curve for distinct cases (Pr= 0.7, c = 1 and m= ϵ = 0.1).

Figure 3.2: Depiction of temperature curve for distinct cases (Pr= 0.7, c = 1 and m= ϵ = 0.1).



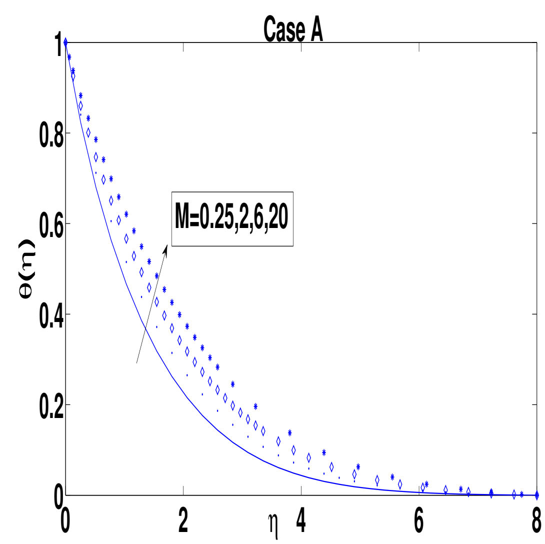


Figure 3.3: Depiction of velocity curve for distinct values of Magnetic parameter m (m=0.5,5.5,10.5,15.5) with $c = 1$, $\epsilon = 0.1$ and Pr=0.7.

Figure 3.4: Depiction of temperature curve for distinct values of Magnetic parameter m (m=0.25,2,6,20) with $c = 1$, $\epsilon = 0.1$ and Pr=0.7.

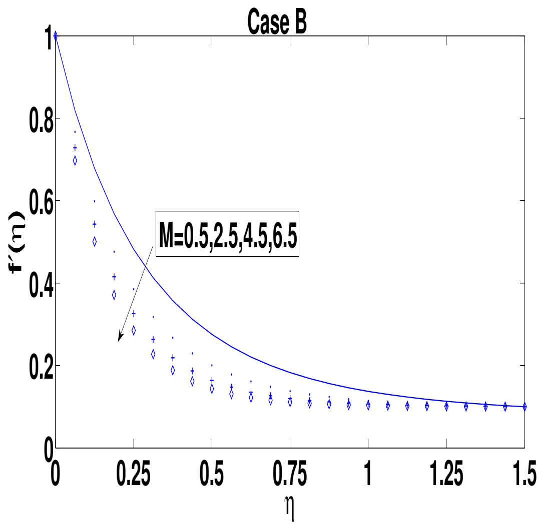


Figure 3.5: Depiction of velocity curve for distinct values of Magnetic parameter m with $c = 1$, $\epsilon = 0.1$ and Pr=0.7.

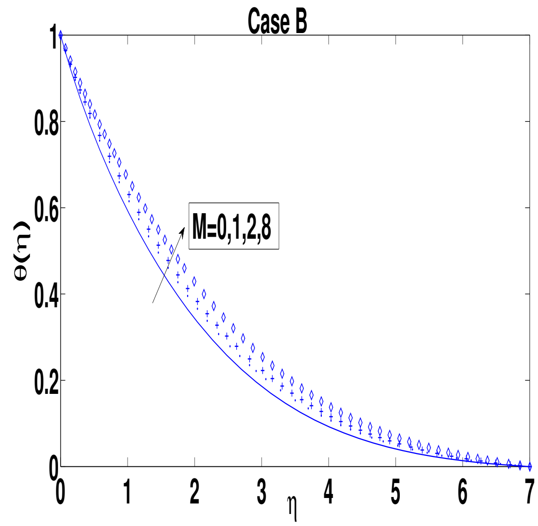


Figure 3.6: Depiction of temperature curve for distinct values of m (ϵ =0.1 with $c = 1$and Pr=o.7).

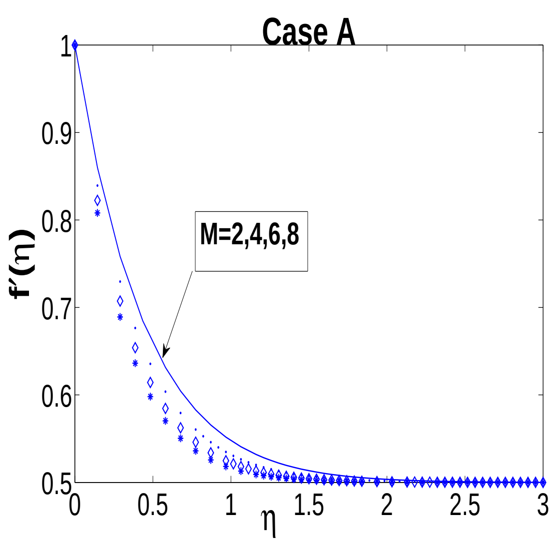


Figure 3.7: Depiction of velocity curve for distinct values of m ($c = 1$, $\epsilon = 0.5$ and Pr=10).

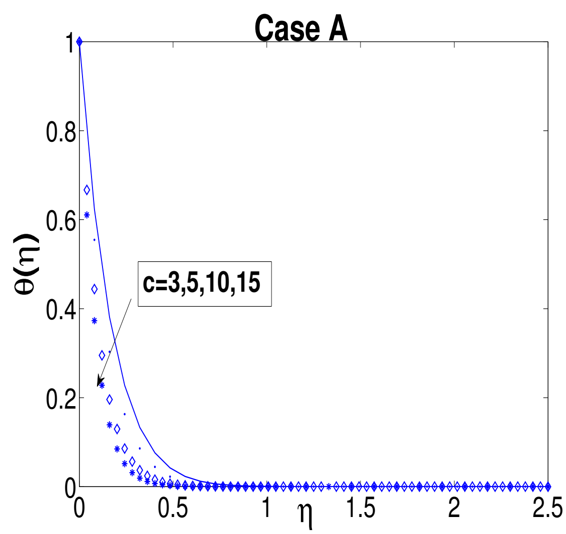


Figure 3.8: Depiction of temperature curve for distinct values of temperature index parameter c (c=3,5,10,15) and ϵ =0.1 and Pr=10.

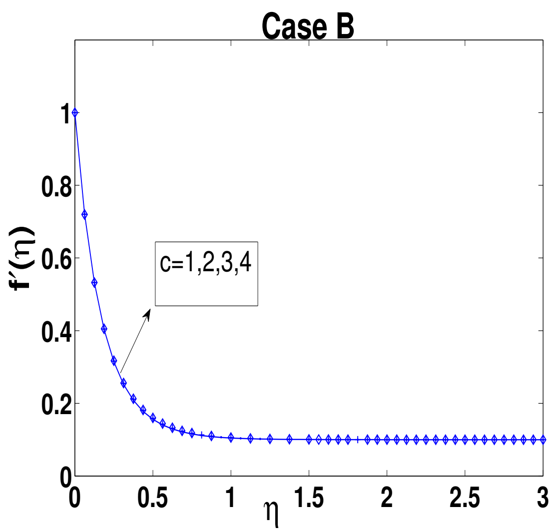


Figure 3.9: Depiction of velocity curve for distinct values of n (c = 1, m=5, Pr=1 and $\epsilon = 0.1$).

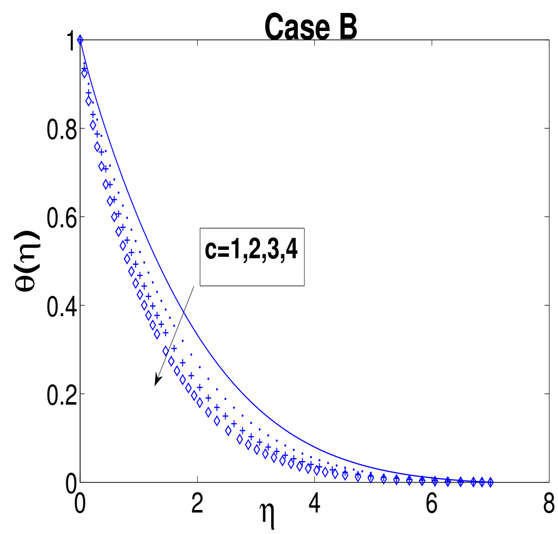


Figure 3.10: Depiction of temperature curve for distinct values of n with m=5, c = 1, $\epsilon = 0.1$ and Pr=1.

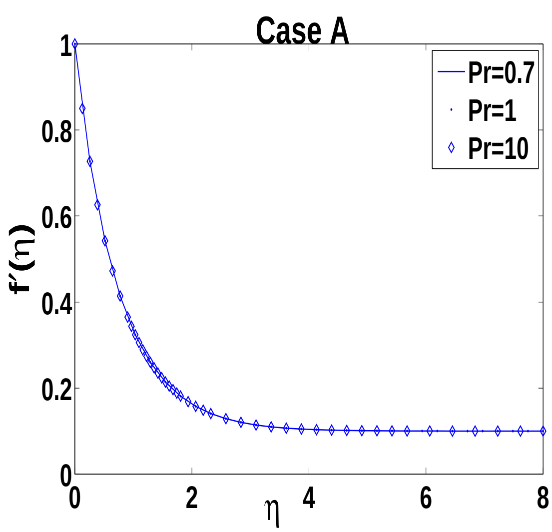


Figure 3.11: Depiction of velocity curve for distinct values of Prandtl with m=0.5.

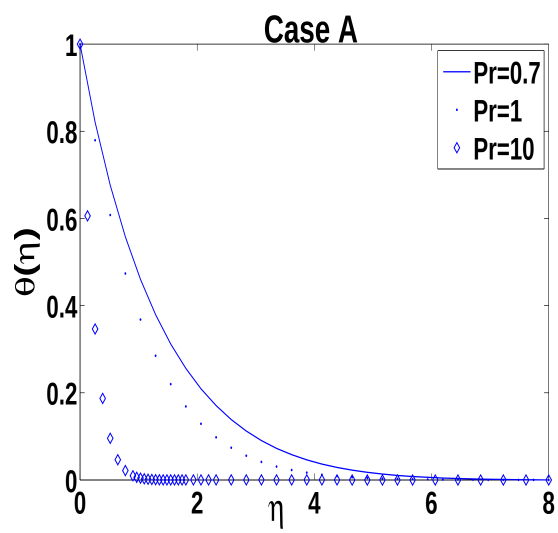


Figure 3.12: Depiction of temperature curve for distinct values of Prandtl with m=0.5.

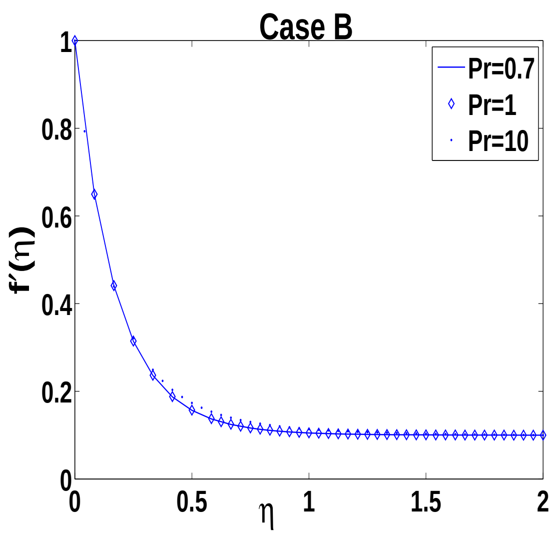


Figure 3.13: Effect on velocity curve by varying Prandtl number with m=5.

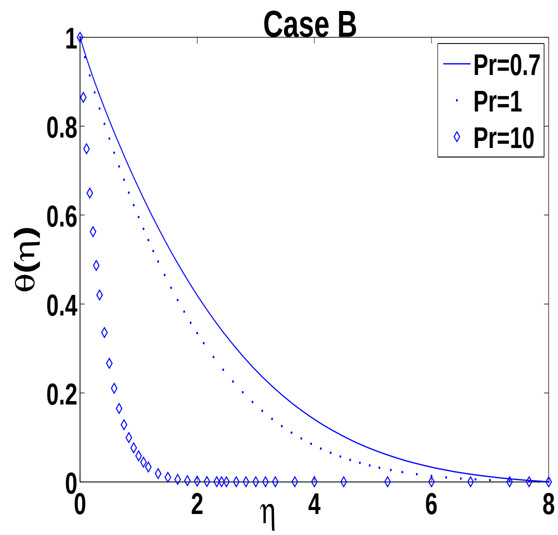


Figure 3.14: Effect on temperature curve by varying Prandtl number with m=5.

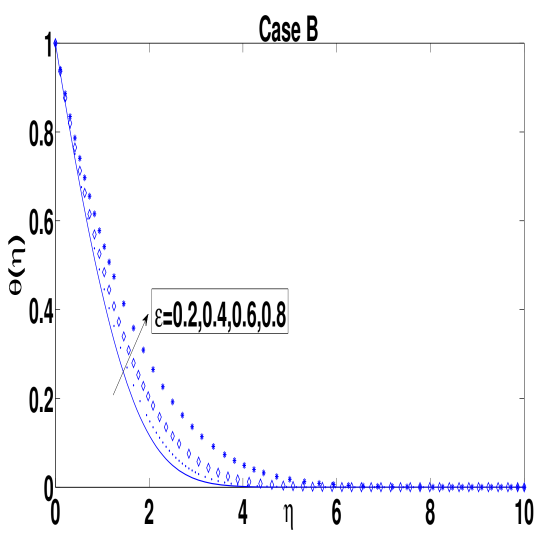


Figure 3.15: Difference in velocity curve depicting Case C for distinct values of ϵ with m=0.1, Pr=0.7

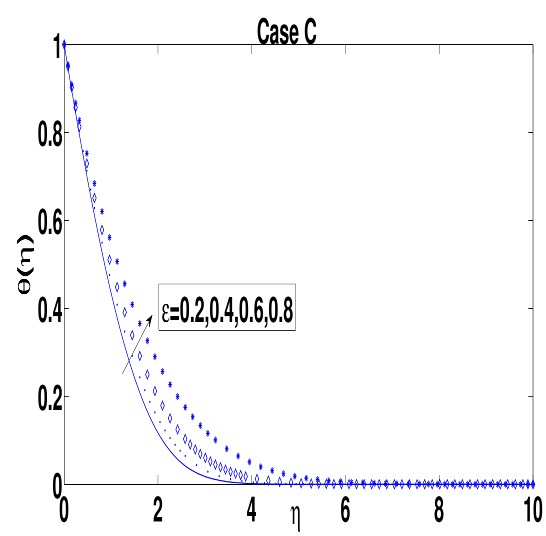


Figure 3.16: Difference in temperature curve depicting Case C for distinct values of ϵ (m=0.1, Pr=0.7)

Chapter 4

Conclusion

In this chapter, all the results of previous chapters are concluded concisely. In this dissertation, we have studied MHD flow with variable fluid properties over an exponentially stretching sheet. Variable viscosity and variable thermal conductivity are mainly focused in this research while taking other fluid properties as constant. Non-linear PDEs are converted into non-linear ODEs by employing similarity transformations. We have examined different cases while taking variable thermal conductivity along with the exponential case of temperature dependency of the fluid in order to observe the MHD flow and investigation of heat transfer over a stretching sheet. The effects of various paramters such as magnetic parameter M, Prandtl number Pr, velocity ratio paramter ϵ and temperture index parameter c on MHD flow and heat transfer are studied. Numerical results for skin friction and local Nusselt number are calcultaed and presented in tables. Velocity and temperature curves are presented graphically and comparison of numerical calculations with previous literature has also been done.

Bibliography

[1] B. C. Sakiadis, Boundary-layer behaviour on continuous solid surfaces. The Boundary-layer flow on a continuous at surface, *J. A ppl. Math.*, **7** (1961) 221-225.

[2] H. Blasius, Grenzschichten in flussigkeiten mit kleiner reibung, Z. Angew. Math. Phys., (1908) 1-37.

[3] L. J. Crane, Flow past a stretching plane, Z. Angew. Math. Phys., **21** (1970) 645-647.

[4] H. Andersson and J. Aarseth, Sakiadis flow with variable fuid properties revisited, *Int. J. Eng. Sci.*, **45** (2007) 554-561.

[5] E. M. A Elbashbeshay and M. A. A. Bazid, The effect of temperature-dependent viscosity on heat transfer over a continuous moving surface, *J. Phys. D: Appl. Phys.*, **33** (2000) 2716-2721.

[6] A. Ishak, K. Jafar, R. Nazar, I. Pop, MHD stagnation point flow towards a stretching sheet, Phys A: Stat. Mech. Appl. (2009)**388**(17) 3377-3383.

[7] S. Mukhopadhyay, G. C. Layek and Sk. A. Samad, Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity, *Int. J. Heat Mass Transfer*, **48** (2005) 4460-4466.

[8] Nemat Dalir, Mohammad Dehsara and S. Salman Nourazar, Entropy analysis for magnetohydrodynamic flow and heat transfer of a Jeffrey nanofluid over a stretching sheet, *Int. J. Eng. Sci.* **79** (2015) 351-362.

[9] F. M. Ali, R. Nazar, N. M. Arifin and I. Pop, MHD mixed convection boundary layer flow toward a stagnation point on a vertical surface with induced magnetic field. *J. Heat Transfer* **133** (2011) Article ID: 022502-16.

[10] T. M. Agbaje, S. Mondal, Z.G. Makukula, S.S. Motsa and P. Sibanda, A new numerical approach to MHD stagnation point flow and heat transfer towards a stretching sheet, *Ain Shams Eng. J.* (2015).

[11] A. Chakrabarti and A.S. Gupta, Hydromagnetic flow and heat transfer over a stretching sheet, *Quart. Appl. Math.*, **37** (1979) 73-78.

[12] T. C. Chiam, Magnetohydrodynamic heat transfer over a non-isothermal stretching sheet, *Acta Mech.*, **122** (1997) 169-179.

[13] M. A. A. Mahmoud, Thermal radiation effects on MHD flow of a micropolar fluid over a stretching surface with variable thermal conductivity, *Physica. A.* **375** (2007) 401-410.

[14] T. Hayat, S. Asad, M. Mustafa, A. Alsaedi, MHD stagnation-point flow of Jeffrey fluid over a convectively heated stretching sheet, *Comput. Fluids*, **108** (Feb. 15, 2015) 179-185.

[15] M. H. Cobble, Magnetohydrodynamic flow with a pressure gradient and fluid injection, *J. Eng. Math.* **11** (1977) 249-256.

[16] K. A. Helmy, Solution of the boundary layer equation for a power law fluid in magneto-hydrodynamics, *Acta. Mech.*, **102** (1994) 25-37.

[17] S. P. Anjali Devi and M. Thiyagarajan, Steady nonlinear hydromagnetic flow and heat transfer over a stretching surface of variable temperature, *Heat Mass Transfer*, **42** (2006) 671-677.

[18] N. Afzal, Heat transfer from a stretching surface, *Int. J. Heat Mass Transfer*, **36**(1993) 1128-1131.

[19] O. D. Makinde, W. A. Khan, and J. R. Culham, MHD variable viscosity reacting flow over a convectively heated plate in a porous medium with thermophoresis and radiative heat transfer, *Int. J. Heat Mass Transfer*, **93** (2016) 595-604.

[20] F. C. Lai and F. A. Kulacki, The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, *Int. J. Heat Mass Transfer* **33**,1990 1028-1031.

[21] N. Bachok, A. Ishak and I. Pop, Boundary layer flow and heat transfer with variable fuid properties on a moving at plate in a parallel free stream, *J. Appl. Math.* **2012** (2012), Article ID 372623, 10 pages.

[22] J. X Ling and A. Dybbs, Forced convection over a flat plate submersed in a porous medium:variable viscosity case, *Am. Soc. Mech. Eng.* **114**, (1987) 87-123.

[23] Xu, H. "Series solutions of unsteady three-dimensional MHD flow and heat transfer in the boundary layer over an impulsively stretching plate", European Journal of Mechanics / B Fluids, 200701/02

[24] Nadeem, S., and Noor Muhammad. "Impact of stratification and Cattaneo-Christov heat flux in the flow saturated with porous medium", Journal of Molecular Liquids, 2016.

[25] Bachok, Norfifah, Anuar Ishak, and Ioan Pop."Boundary Layer Flow and Heat Transfer with Variable Fluid Properties on a Moving Flat Plate in a Parallel Free Stream", Journal of Applied Mathematics, 2012."

[26] K. Swain, S.K. Parida, G.C. Dash, MHD Heat and Mass Transfer on Stretching Sheet with Variable Fluid Properties in Porous Medium, **86** (2018) 706-726

[27] N.S.M. Adnan, N.M. Arifin, Stability analysis of boundary layer flow and heat transfer over a permeable exponentially shrinking sheet in the presence of thermal radiation and partial slip, *J. Phys. Conf. Ser. Pap.* (2017)

[28] S. Mukhopadhyay, MHD boundary layer flow and heat transfer over an exponentially stretching sheet embedded in a thermally stratified medium, *Alex Eng J.* (2013) **52** 259-265.

[29] K. V. Prasad, K. Vajravelu and P. S. Datti, The effects of variable fluid properties on the hydro-magnetic flow and heat transfer over a non-linearly stretching sheet, *Int. J. Thermal Sci*, **49** (2010) 603-610.

[30] A. Ishak, MHD boundary layer flow due to an exponentially stretching sheet with radiation effect, *Sains Malays.* **40** (2011) 391-395.

[31] M. Mustafa, M. A. Farooq, T. Hayat, A. Alsaedi, Numerical and Series Solutions for Stagnation-Point Flow of Nanofluid over an Exponentially Stretching Sheet, *PLoS ONE* (2013) **8**

[32] M. S. Abel, M. M. Nandeppanavar, V. Basanagouda, Effects of Variable Viscosity, Buoyancy and Variable Thermal Conductivity on Mixed Convection Heat Transfer Due to an Exponentially Stretching Surface with Magnetic Field, *Proceedings of the National Academy of Sciences, India Section A: Physical Sciences* (2017) **87** 247256.

[33] M. Mustafa, Viscoelastic flow and heat transfer over a non-linearly stretching sheet: OHAM solution, *J. Appl. Fluid Mech.,* **9** (2016) 1321-1328.