Cavity QED Based Tunable Delayed-Choice Quantum Eraser

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Dedicated to

My father Suba Khan
&
My mother Kalsoom Akhter.

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Abstract

A scheme for tunable delayed-choice Quantum Eraser is discussed. Whole eraser scheme revolves around the matter field interaction. The scheme is based upon dispersive, resonant and Ramsey interaction of an atom with the field, which is trapped inside the cavity. One of the intriguing thing about this scheme is the tunablity of fringes, in a delayed choice setup, which marks a question on our perception about time. The experimental feasibility is good enough to carry out this experiment with good overall success and fidelity.

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Chapter 1

Introduction

What is time? A question which looks easy, yet difficult to answer. Many views came from the early times, when philosophers started defining the time till date but no one can define time in a single sentence. There exist two most intriguing philosophies, one is Reductionism and other is Absolutism with respect to time [1]. Aristotle and many philosophers like Leibniz, had argued that time depends upon change. This argument is known as Reductionism with respect to time. Can someone praise such a view? One of the argument which is a conceptual states that: time is just a system in which things and events can be related temporally, which means the argument that time can exist without any change seems incoherent. The other main argument of Reductionism is epistemological: which says that there is no such period of moment of empty time, if by any means there is such a period of moment of empty time, we could not be able to tell anything about it like, how long it is or anything about its existence. Than there is another view which is known as Absolutism with respect to time, which came from many philosophers and scientists like, Plato and Newton. According to this view, Time is an absolute quantity, in other words there is a master clock of the universe in which every event occurs. Can someone endorse such a philosophy about time? There exists no such strong argument against Absolutism with respect to time, for example according to Sydney Shoemaker there can exist some moment of empty time [2]. But in the start of twentieth century, Einstein changed the concept of time, he proposed theory of relativity, that time is relative rather than absolute [3]. Cutting the story short about the philosophy of time, its better to ask, why there is a need for defining time? To answer such a question we discuss about an intriguing phenomenon known as Quantum Eraser.

Basis of quantum eraser lies in the principle of complementarity [4], which states that for each degree of freedom, the variables are a pair of complementary variables, and by complementary means we can not measure two variables at the same time. The problem is we can not measure the path followed by a photon(path distinguishability) in young's double-slit setup and fringe visibility together [5]. If we have which-path information, fringe pattern is lost and vice versa [6]. So to get around this problem an eyebrow-raising phenomenon of Quantum eraser was proposed by Scully and Druhl in 1982 [7]. The main goal of this Quantum eraser procedure was to erase which path information and to regain the lost fringes in double slit experiment. They proposed a slight variation of double slit experiment in which three level atoms were placed rather than slits. When field interacts with these atoms photon is emitted and forms an interference pattern on the screen but if the measurement was done on either of the atoms we will know through which atom, the photon was emitted. So to erase this, which-path information method was proposed. Our perception about time will get ambiguous when we put concept of delayed choice [8][9][10] along with quantum eraser. What will happen if we erase this path information after the detection of photon on the screen? The answer is too counter intuitive, interference pattern reappears [7][11]. After this proposal a number of scientists criticized this phenomenon [12], but many others worked on this and applied it on different optical system such as Double-Slit experiment [13] and in Mach Zehnder interferomter [14], recently Quantum Eraser based on modified Stern-Gerlach apparatus was proposed [15] . This phenomenon of erasing the information is still under debate, whether it erases the information completely, partially or does not erases any information at all [16]. In this thesis we will discuss Quantum Eraser phenomenon in cavity QED scenario [17][18].

1.1 Thesis Outline

Starting from chapter 2, we will recall our basics about the concept of complementarity, which was proposed by Niel Bohr, and to make his argument strong Heisenberg's uncertainty relation supports the concept of complementarity. Then we will discuss about an experimental setup which is known as Young's Double-Slit Experiment, the experimental setup is so simple that an undergraduate student can understand easily but the mystery which it involves has not been yet interpreted. Next we will discuss another experimental setup known as Mach-Zehnder Interferometer. Then will talk about the famous thought experiments proposed by J.A Wheeler, which are known as delayed choice experiments. In the end of 2nd chapter we will discuss about the phenomenon of Quantum Eraser, the phenomenon around which this whole thesis revolves around. In the third chapter, we will discuss about matter field interaction in detail and alot of mathematics will be discussed. At the end of third chapter we will be ready to apply Quantum Eraser procedure in a Cavity QED based setup. In fourth chapter we will discuss about a Cavity QED based tunable delayed choice Quantum Eraser, in which first a three level atom which will be in superposition of its lower two levels, do off resonant interaction with a cavity which would be in superposition too, in doing so the coherence of three level atom will lost. So to recover that coherence Quantum Eraser procedure will be applied. In the last chapter, chapter five, we will check the experimental feasibility of this proposed setup, and we will give some concluding remarks.

Chapter 2

Erasing The Past

The epigraph to this chapter contains, the counter intuitive nature of physical laws at microscopic world. The mathematical formalism of these laws can be comprehend easily even by an undergrad students. It was a common perception for a very long time, that if you try to interpret the weirdness of quantum mechanics, you were often deemed to be lost for science. Until with the development of quantum information and manipulation of single particle in a laboratory[22], that perception changed. Now rather than understanding these laws, physicists work on quantifying and defining them. A new kind of intuition or perception has been developed which allows them to guess the results before proving them experimentally. We leave this question to philosophers that, weather this intuition is different from understanding these laws?
This chapter is dedicated to describe the non-classical aspects of quantum mechanics. In the first section we will discuss the principle of complementarity. Then in the next section we will discuss about the interference phenomena in terms of complementarity . After that we will discuss an eye-brow raising phenomena known as Quantum Eraser. Then we will apply this phenomenon on different experimental setups.

2.1 Complementarity

The concept of complementarity was introduced by famous physicist Niel's Bohr [4][19], one of the founding father of Quantum Physics. In 1927 in Italy on the one hundredth anniversary of Alessandro Volta's death, that Neil's Bohr introduced the concept of complementartity. Bohr started his lecture by stating " a certain general point of view, which I hope will be helpful in order to harmonize the conflicting views taken by the scientist". Actually he was trying to present the difference between classical and quantum mechanical description of physical phenomenon.

Now consider an example, in classical physics, when two billiard balls strikes with each other, the state of the system can be determined or observed easily, or at least in principle with very small uncertainties. But on the other hand in quantum mechanical description, the state of the system cannot be observed or measured without any change in the state. The above statement can be visualize in an example. Recall that in Heisenberg's Gamma-ray microscope [20] the motion of electron was to be measured, by striking the electron by gamma-rays, but gamma-rays deflects electron from its original path.

Now consider the example, which was illustrated by Niel Bohr himself during his lecture [19]. He discussed the question on the nature of light, especially the interference phenomenon and than on the particle nature of light. The interference phenomenon can be seen in many experiments today, like in Young's Double-Slit experiment or in Mach-Zehnder interferometer, while the particle nature comes from the concept of photoelectric effect, the idea put forward by the Einstein and Compton-effect by Compton. The two different theories about the nature of light raised conflicting views between the scientists.

The question was, can these two conflicting theories be subsumed or we have to settle for the two radically different Physical phenomena? Bohr proposed that rather than reconciling these two theories, we should take into account the complementarity in these two different phenomena. In a nut shell what we can say is that, the evidence obtained from different experiments must not be comprehend as single picture but regarded as complementary.

The principle of complementarity can be defined as, for each degree of freedom, the variables are a pair of complementary variables [11]. Any object have certain pairs of complementary variables for examples, the position and momentum, the energy and time, the spin of two different axis, wave and particle like properties. If we try to find the position of the particle, we will be uncertain about the momentum of the particle, same is the case for the wave-particle nature of the light.

2.1.1 Complementarity and Heisenberg's Uncertainty Principle

Principle of complementarity and uncertainty principle are closely related [19]. The two complementary variables cannot be measured simultaneous, which puts limits to our knowledge. As an example consider the wave-particle nature of light both these variables are complementary to each other. If we try to look for the wave nature of the light, the particle behavior will be dominated by the wave behavior of light. Thus we can measure one of the two, at the same time.

Werner Heisenberg, who was an experimentalist in the early development of quantum theory, realized that this limitation to our knowledge, could be described in a different way [21]. Consider the famous young double slit experiment, in which photon passes through two slits and strikes the screen to form an interference pattern. Suppose that we measure the position of photon i.e. through which slit photon passed, while the interference pattern is akin to a measurement of its momentum.

Think of rather more simple apparatus as shown in the figure(2.1), in which there is only one slit through which light passes. If we consider the wave behavior of the light, diffraction pattern can be seen on the screen. To study the individual behavior of photons, we can decrease the intensity of light. But again we find the diffraction pattern, after a number of photons have accumulated on the screen. We know that the photon passes through the slit but we have no knowledge from where it passed. So there is an uncertainty in the particle's/photon's position which is "Δx", which is due to the width of the slit. As particle leaves the slit it will land some where on the screen, but again we won't be able to predict where it will land. If the particle has velocity in x direction than its corresponding momentum will be in x-axis. Basically the width of the diffraction pattern is the uncertainty "Δp" in our prediction in momentum.

Now if we try to make the slit smaller, the uncertainty Δx can be reduced, but the spread of the diffraction pattern increases which means that uncertainty in momentum

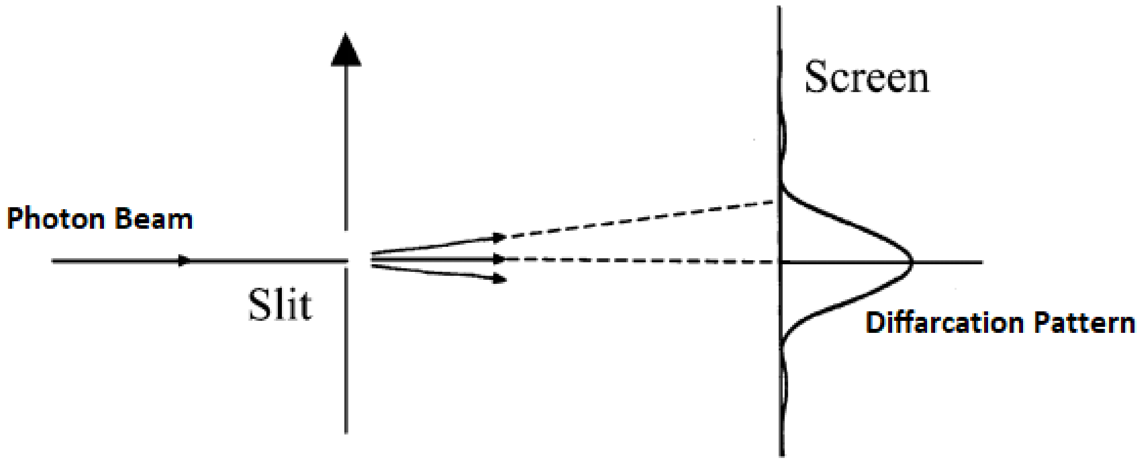

Figure 2.1: A beam of photon passing through a single slit, making diffraction pattern on the screen. The width slit is the uncertainty in the position denoted by Δx, and the spread of the diffraction pattern is the uncertainty in the momentum Δp.

Δp increases.

By doing some calculations the product of these two uncertainties comes out to be greater then the factor, $\frac{\hbar}{2}$, mathematically it can be written as,

$$\Delta x \cdot \Delta p > \frac{\hbar}{2}. \tag{2.1}$$

Where $\hbar = \frac{h}{2\pi}$, and "h" is the plank's constant. Heisenberg actually showed that, quantum mechanics requires limitation to such measurement on complementary variables. Uncertainty principle allows us to predict the results of subsequent measurements, but it also puts a fundamental limit to our knowledge. After this principle allot of experiments were proposed to measure more precisely than the uncertainty principle allows, but analysis on those experiments showed that it was impossible.

2.1.2 The Gamma-ray Microscope

To look deep into the concept of the complementarity, consider Heisenberg's Gamma-ray microscope [20]. A special microscope used for measuring the position of a particle. A particle "e" irritated with light of wavelength "λ". The uncertainty in the position of a particle "$\delta(x_e)$" using a microscope, arises due to the limitation of the resolving power, which comes from a classical relation given as,

$$\delta(x_e) \approx \frac{\lambda}{sin\varepsilon}. \tag{2.2}$$

As we can see that, uncertainty in the position of the particle can be decreased, by deceasing the wavelength of the light. Heisenberg used gamma-ray microscope for illustrating the concept of complementarity, between two complementary variables that are, position and momentum in this case.

The main idea is to decrease the indeterminacy in position by simply decreasing the wavelength of light, so in that sense gamma rays can be used. But due to decrease in wavelength the momentum of photon increases, which directly increases the uncertainty in momentum. The basis of above argument lies in the conservation of momentum, due to collision between particle and quanta of light. In the case of photon momentum depends upon the wavelength as follows,

$$p = \frac{h}{\lambda}. \tag{2.3}$$

Now consider the figure(2.2), we can say that the incoming photon scatters only if,

$$-\frac{hsin\varepsilon}{\lambda} < p_x < \frac{hsin\varepsilon}{\lambda}, \tag{2.4}$$

since we don't know for which value of momentum lies in above interval, through which photon enters the aperture, which indeed causes the uncertainty in momentum,

$$\delta p \approx \frac{2hsin\varepsilon}{\lambda}. \tag{2.5}$$

Now combining the two eq.(2.2) and (2.5), what we get is the uncertainty principle,

$$\delta(x) \cdot \delta(p) \approx 2h. \tag{2.6}$$

From above equation we can say that, the certainty in one gives rise to uncertainty in the other observable.

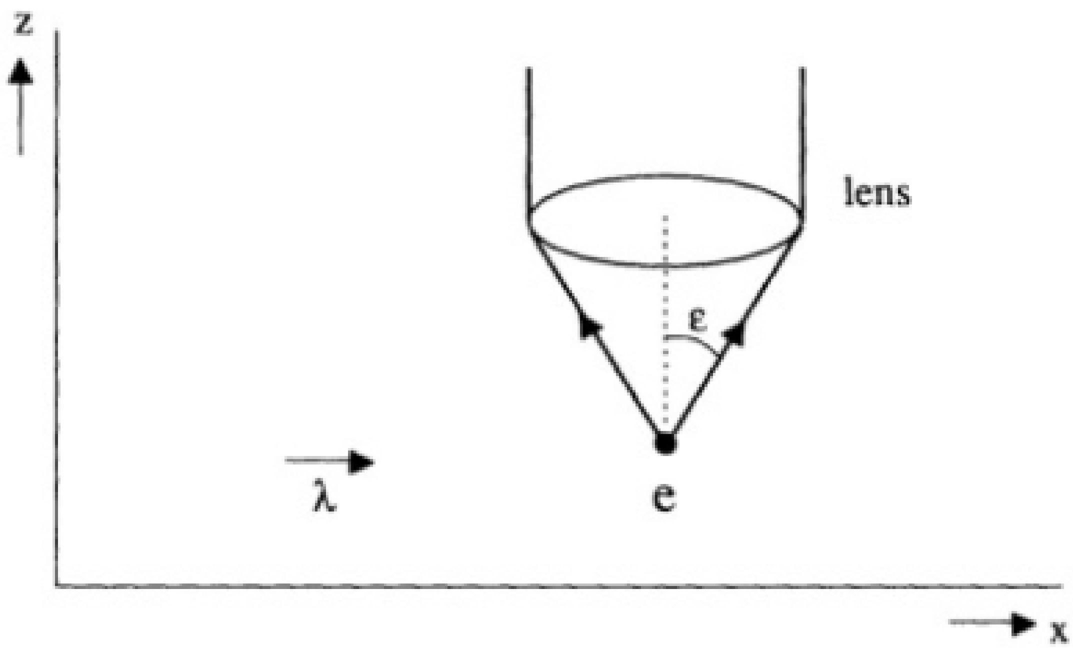

Figure 2.2: The Heisenberg's Gamma-ray Microscope

2.2 Young's Double Slit setup

In 2002, Robert Crease a philosopher and historian of science raised a question in front of physics community that, what is the most beautiful experiment in physics. More than two hundred suggestions were given. The experiments which made to the top were, Millikan oil drop experiment, Galileo's experiment on the motion of falling objects, Newton's decomposition of light with help of prism, Thomas Young's experiment on the interference of light, but the one that made to the top was slight variation of young's experiment, in a nut shell it was, young's double slit experiment applied to the interference of a single electron [28][29].

In 1803, Thomas Young an English Physicist performed and experiment in which, a black piece of paper was placed in front of hole in a window, the paper had its own hole through which sunlight was entering. Than in the path of the light beam, a card which was about 13th of an inch thick was placed at its edges, which resulted in dark

and bright fringes, which was actually the interference pattern of light [30].
In this section we shall tackle the element of mystery in its most weird form [23]. We will examine a phenomenon, which cannot be explained in a classical way, and which can be explained by quantum mechanics only. In reality young's double slit experiment only contains mystery. We can explain how it works but we cannot make this mystery go away. We will design some variations of young's double slit experiment and interpret their results in which firstly, we perform this experiment with bullets, then with waves and in the end with electrons.

2.2.1 Young's Double-slit Experiment with Bullets

In order to understand the quantum behavior of electrons, we compare their behavior in a below mentioned experimental setup, in which to see the particle behavior of electrons, we compare them with bullets.
Now consider the slight variation of young's double slit as shown in the figure(2.3), by using a source which fire bullets. Then we have a wall which has two holes in it of same size, big enough to let the bullet pass through. Behind that wall, there is another wall(mad of wood) which absorbs these bullets. On the wood wall we have movable detector which can move along x-axis. The purpose of detector is to catch the incoming bullets from the hole. The purpose to design such an experiment is to answer a question i.e. what is the probability of a particle which passed through two slits and arrive at some distance x from the center of the screen? We must include probability into it, because we don't know exactly, where that bullet can hit. A bullet which can hit at the edge of any of two holes can bounce back. The chance that bullet can arrive at the detector can be measured by counting the number of bullet it contains, in a certain time, and then taking the ratio of total number of bullets that hit the wood wall to the number of bullets in the detector. The probability is directly proportional to the number of the bullets which reaches the detector, in a certain time.

The result for above mentioned experimental setup can be seen in figure(2.4). The graph shows the probability P_{12} of bullets coming through both holes. We can see that

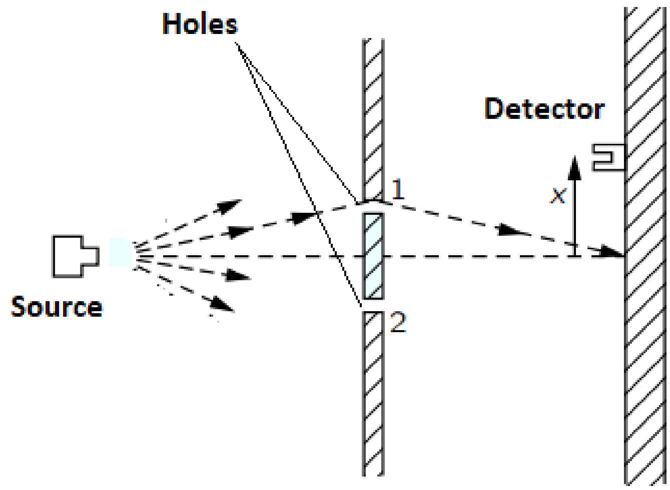

Figure 2.3: A slight variation of young's double-slit experiment for particle behavior.

probability is maximum at the middle and minimum as we go away from the center along x-axis. Now the question which arises instantly is, why it is maximum at the center of the screen? So what we can do is, we close the hole-2 so bullet can only pass through hole-1, so the probability would be P_1 and we do same for the case of P_2, which is the probability of bullet due to only hole-2. Now if we add these two probabilities we get, the probability $P_{12} = P_1 + P_2$, which was for the case when both holes were open. We must keep in mind that we don't have the interference pattern for a reason which will be explained later.

2.2.2 Young's Double-slit Experiment with waves

Now consider an experiment, to look for the results of young's double-slit setup with water waves. We can perform this experiment by generating water waves in a ripple tank and then placing a wall which has two holes/slits in it. Behind that wall there is another wall which we can call a screen and like previous experiment, we have a detector which can move on the screen along x-axis, as shown in the figure(2.5). In this case the detector measures the intensity of coming waves from the two slits.

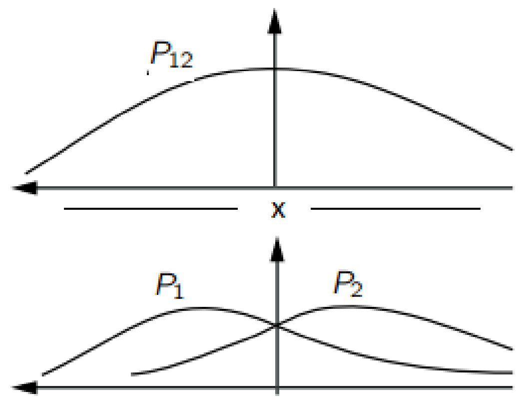

Figure 2.4: The above graph shows the probability P_{12}, which has a maxima in the center of the screen, while the second graph shows the probability P_1 or P_2 when one of the two holes was closed.

Now when we measure the intensity if wave at various position along x, the resulting curve I_{12} can be seen in the figure(2.6), which is an interference pattern. Now we wish to do as we did in the last experiment, that we close one of the slit, the corresponding intensity I_1 that comes out is, simply due to slit-1, and same for the case when we do this for slit-2, corresponding intensity comes out to be I_2 respectively.

Now as we can see that I_{12} is not the sum of I_1andI_2. This is due to the fact that at the slits diffraction takes place and two new waves are being produced. Where ever we a peak in I_{12}, we have a maxima, the waves are in phase, and where we have minima destructive interference takes place, in other words waves are out of phase.
The relationship between I_1,I_2 andI_{12}, can be explained in a following manner: The height of the water wave is actually the measure of intensity, so the wave coming from slit-1 has height $h_1 e^{\iota\omega t}$, which is in general a complex number. The intensity is mean square of the height so we can write intensity in terms of height as, $I_1 = |h_1|^2$, and it is same for the case when wave is coming from slit-2. When both slits are open

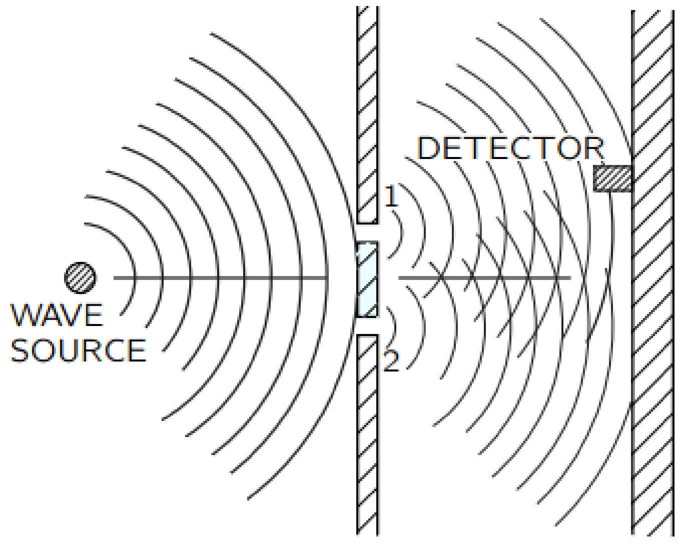

Figure 2.5: A source which is producing waves to look for the results, when we do young's double-slit experiment with waves

corresponding height of waves can be written as, $(h_1 + h_2)e^{\iota\omega t}$ and intensity I_{12} as $|h_1 + h_2|^2$.

The result that comes out is quite different from the previous experiment,

$$|h_1 + h_2|^2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2|\cos\delta, \tag{2.7}$$

$$I_{12} = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta. \tag{2.8}$$

where $\delta = e^{\iota\omega t} + e^{-\iota\omega t}$ is the phase difference. The last term is the interference term. What we can interpret from the above experiment is that, in young's double-slit experiment waves gives interference pattern.

2.2.3 Young's Double-slit experiment with electrons

Now consider the same apparatus with a slight variation, in this case we use electron gun as a source. A filament is heated when we apply voltage to it, which in return ejects electrons, then a filament is placed inside a box with a very small hole through which few electrons can pass, rest of the apparatus is same, which can be seen in the

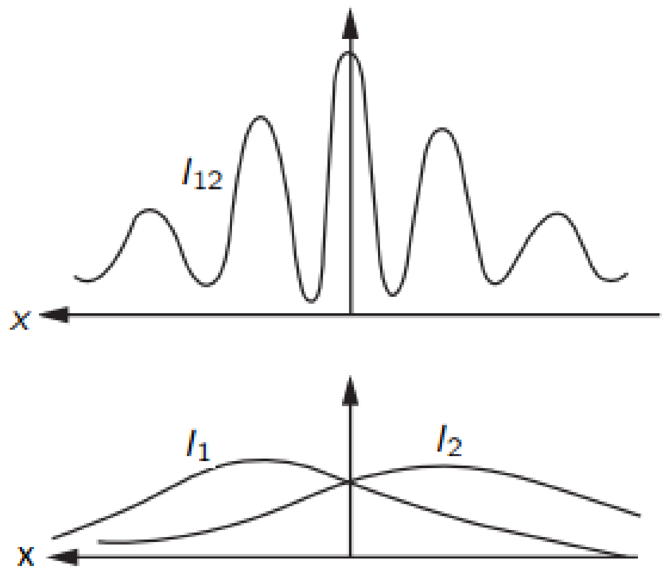

Figure 2.6: Interference pattern I_{12}, and corresponding intensities I_1 and I_2, when either of the slits were closed.

figure(2.7). The detector in this case is a geiger counter, which beeps when an electron hits it [29].

When we try to move the detector along x-axis the sound of clicks appears to be faster or slower. If we put another detector identical to the first one what we conclude is that electron hit one of them in a single interval of time, we also conclude that electrons always arrive at the screen in lumps.
To find probability where electron lands on the screen at various positions along c, we repeat the exercise as we did for the case of bullets. The probability curve is shown in the figure(2.8), which is an interference pattern.

To understand the behavior of electrons we try to analyze the curve in figure(2.8). As electrons are particles, we can make a proposition that, electron goes either through slit-1 or slit-2. The probability of electron coming through slit-1 can be marked as P_1,

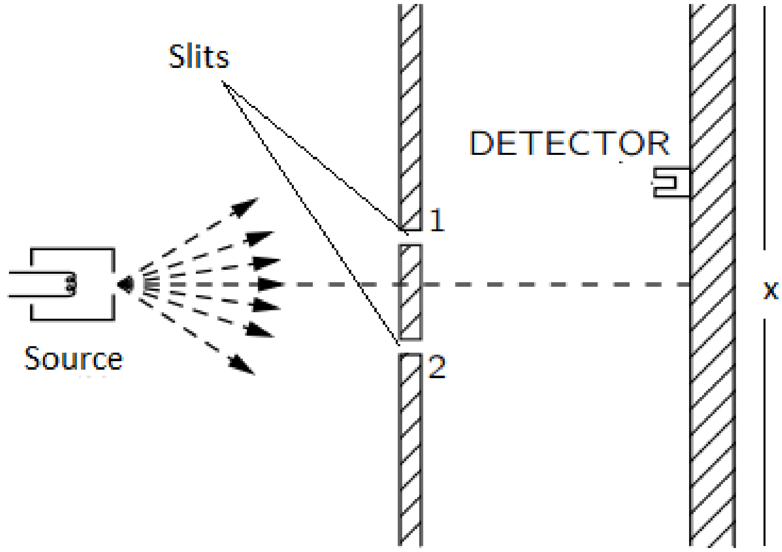

Figure 2.7: Experimental setup for interference of electrons

and for slit-2 as P_2. But comparing our experimental results with water waves and bullets experiments, we can say that for electron, $P_{12} \neq P_1 + P_2$.

Now the question is how interference pattern forms? It looks like our proposition is wrong. The weirdness comes when we try to cool the wire of filament so rate of ejection of electrons can be decreased, but if we give this experimental setup, interference pattern reappears. From this we could say that electron goes through both of the slits, but through our experimental results we concluded that electrons passes through slits in lumps.

Now young's double-slit experiment is getting mysterious, but mathematics to explain the interference pattern is very simple. It is same as we did for the case of water waves. We conclude that electron comes as particles but they are distributed over the screen like wave intensities. Before getting deep into this mysterious world, we can say undoubtedly that, our proposition is false. To support above argument we perform a simple experiment which is explained below.

We want to know the path which electron followed, to do so, we place a detector in front of slit-1 and slit-2 which gives us signal when ever electron passes through slit-

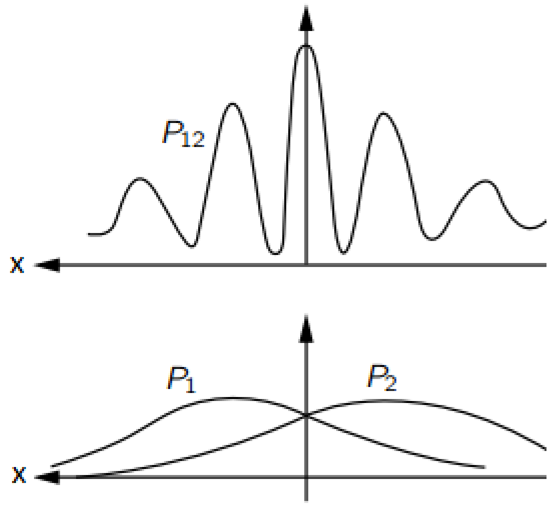

Figure 2.8: Interference pattern of electron

1 or slit-2, the apparatus can be seen in figure(2.9). When we try to measure the path followed by electron the interference pattern disappears,which can be seen in the figure(2.9) which means that photons are interacting with electrons which disturbs the motion of electrons. To get around this problem what we can do is we reduce the intensity of the detector. But now there is another problem, sometimes light flashes and sometimes do not, which means that there will be some electrons for which we know through which slit they went through and there will be some electrons for which we have no which-path information. The probability curve on the screen is interesting, we have a mixture of interference pattern and simple peaks. This is due to the reason mentioned above, which says that if we have which-path information of electrons there will be no interference pattern, and if we don't have any which-path information we have interference pattern. In the former argument, electron were behaving as particles and in the later one as waves. This is the mystery that lies in the heart of quantum mechanics.

In above mentioned experiment, it is impossible to tell through which slit electron went through with out disturbing the motion of electrons. This is the same concept

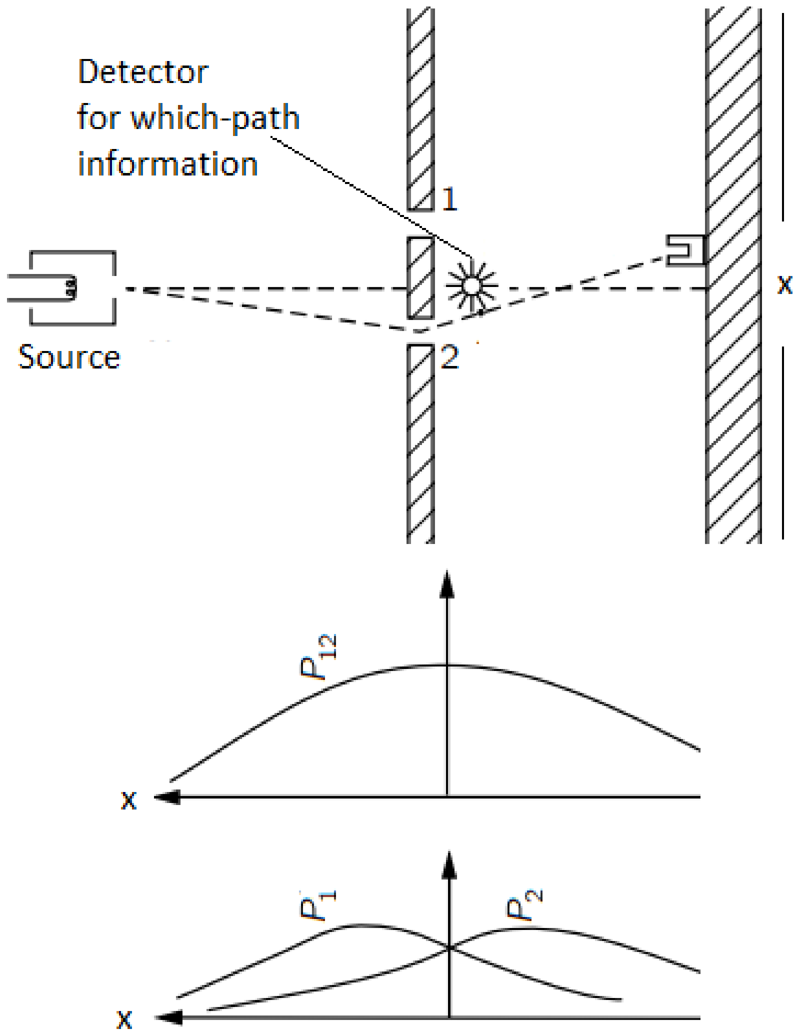

Figure 2.9: Apparatus is design to measure the path which electron followed, for that reason there is a detector in front of slits which flashes when electron passes through any of the slits.

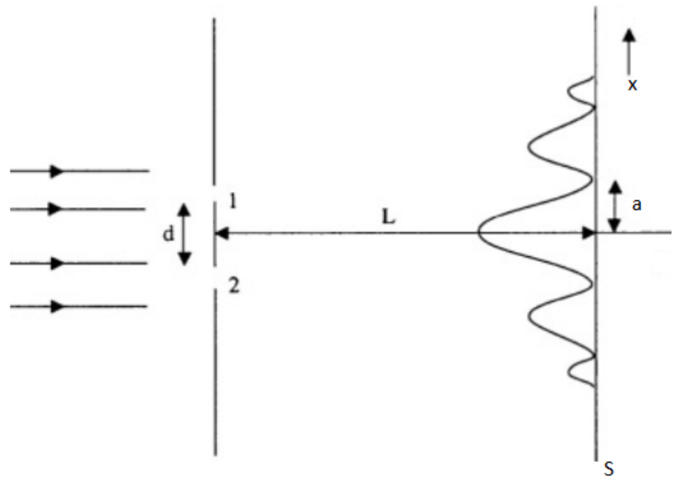

Figure 2.10: A schematic diagram of Young's Double-slit Experiment.

which was given by Heisenberg in the early age of quantum mechanics, Heisenberg proposed his uncertainty principle, which puts limits to our knowledge.

2.2.4 Young's Double-slit Experiment and Uncertainty Principle

Young's Double-slit experiment has been realized in different ways. Consider the apparatus as shown in the figure(2.10). If we are doing this experiment with electron source, we can see the interference pattern on the screen S. An electron can enter through slit-1 or slit-2 and it hits the screen at some position x. The intensity of beam at point x can be determined by the finding the probability at x, which can be written as P_x. In terms of wave functions we can write,

$$\Psi(x) = \Psi_1(x) + \Psi_2(x), \tag{2.9}$$

where $\Psi_1(x)$ and $\Psi_2(x)$, are the wave functions for electrons passing through slit 1 or 2 respectively.

As we did for the case of waves, similarly we can write the probabilities as,

$$|\Psi(x)|^2 = |\Psi_1(x)|^2 + |\Psi_2(x)|^2 + \Psi_1^*(x)\Psi_2(x) + \Psi_1(x)\Psi_2^*(x), \tag{2.10}$$

$$P(x) = P_1(x) + P_2(x) + P_{int}(x). \tag{2.11}$$

Where the last two terms in eq.(2.10), are the interference terms. This is the main reason that, $P(x)$ differs from the sum of probabilities due to either of the slits along. Sometimes the above behavior of electrons can be interpreted as, electron went through both slits which is based on realist interpretation of quantum mechanics, which is completely opposite to what Bohr said. Bohr was an instrumentalist, and he thought of wave functions as a mathematical tool, according to him we must not think about what is happening between the slits and the screen.

If we want to know about the which path electron went through we must perform measurement near the slits. As long as we don't perform this measurement we get an interference pattern but as we know about the which-path information the interference pattern disappears.

For double-slit experiment uncertainty relation can be calculated through following schematic diagram in figure(2.10) [20]. There will always be uncertainty that, where the particle lands on the screen which can be denoted as δx. Now consider a distance a, form the center of first maxima to the first minima, this distance form two slits by $\lambda/2$, as L»a and d, where d is the slit separation.

$$\delta x = a = \frac{\lambda L}{2d}. \tag{2.12}$$

The uncertainty in momentum along x-axis comes out due to our limited knowledge about the electron through which slit it passed.

$$\delta P_x > \frac{Pd}{L} = \frac{h}{2a}, \tag{2.13}$$

where $P = \frac{h}{\lambda}$, and h is the plank's constant. Now combining eq.(2.12) and (2.13), gives us the uncertainty relation as follows,

$$\delta x \delta P_x \geq \frac{h}{2}. \tag{2.14}$$

The above inequality tells us that uncertainty principle has universal validity, we can support this argument by comparing the above uncertainty relation with the one that we got in Gamma-ray microscope.

2.3 Mach-Zehnder Interferometer

In 1891 Ludwig Zehnder proposed an apparatus for measuring the relative phase shift between two beams, which was then redefined by the physicist Ludwig Mach. This interferometer is highly configurable instrument. It consists of a source which produces a beam of light and than that beam passes through a beam splitter, which splits the beam into two halves, one of the beam transmits through the beam splitter and the other one is reflected by beam splitter. Than two mirrors reflects both of the beams to another beam splitter, beam goes to either detector D_1 or D_2 as shown in the figure(2.11), than on the detectors interference pattern can be seen, depending upon the path lengths or phase difference, which can be produced by some device which could be as simple as a glass slab, destructive or constructive pattern emerges [**?**]. To develop the mathematics of above system, firstly we must know, how beam splitter works, so consider a beam splitter, and a photon which passes through the beam splitter, we can describe the motion of the photon by a wave-function, that either photon follows the path-1 or path-2. The probability of photon in either of the beam cam described by two complex numbers α and β, these are the probability amplitudes so they must satisfy,

$$|\alpha|^2 + |\beta|^2 = 1. \tag{2.15}$$

We can write of a photon $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, which states that the probability of photon in upper beam is 1 and in the other beam it is 0. We can write wave function as follows,

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.16}$$

To model the beam splitter consider the diagram as shown in the figure(2.12), we have a beam coming to beam splitter from upward direction $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$, either beam reflects

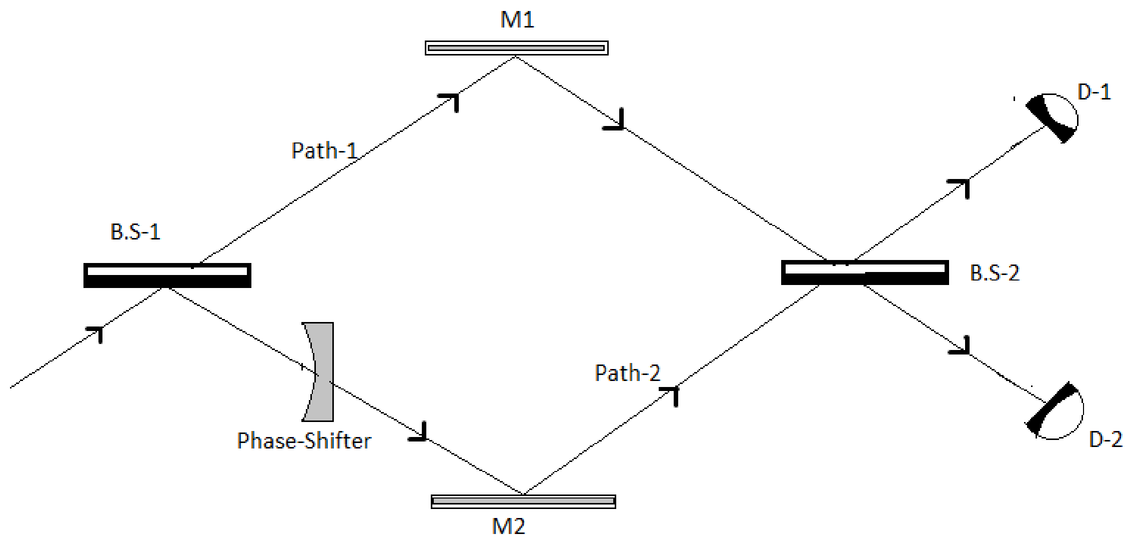

Figure 2.11: Schematic diagram of Mach-Zehnder interferometer. B.S-1 and B.S-2 are the beam splitter, M1 and M2 are the reflecting mirrors, and D-1 and D-2 are the detectors

to path-S or transmits to path-T $\begin{pmatrix} s \\ t \end{pmatrix}$, it must satisfy the probability conservation, $|s|^2 + |t|^2 = 1$,

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} s \\ t \end{pmatrix}. \tag{2.17}$$

Now consider the beam coming towards the beam splitter from downward direction $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, the beam reflects and transmits as shown in the figure(2.13),

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} u \\ v \end{pmatrix}. \tag{2.18}$$

What happens if two beams α and β passes through the beam splitter as shown in the figure(2.14), we can write it as follows,

$$\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \alpha \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \beta \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2.19}$$

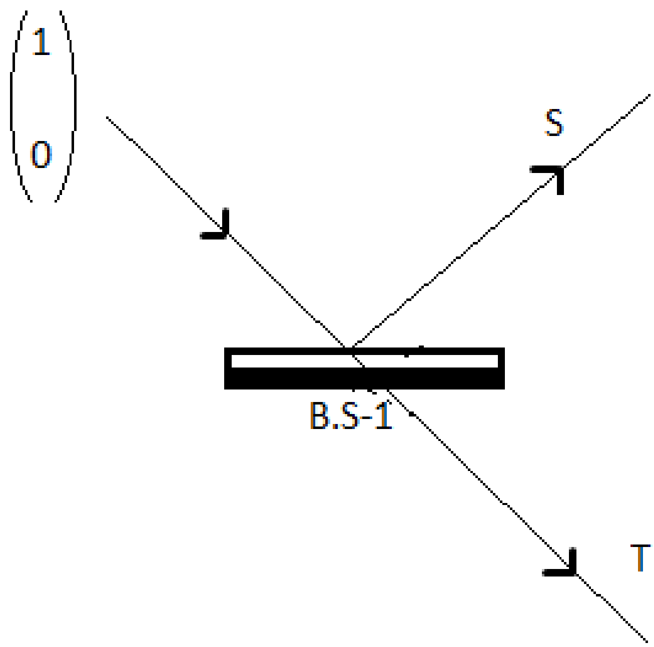

Figure 2.12: A beam which passes through the beam splitter from the upward direction.

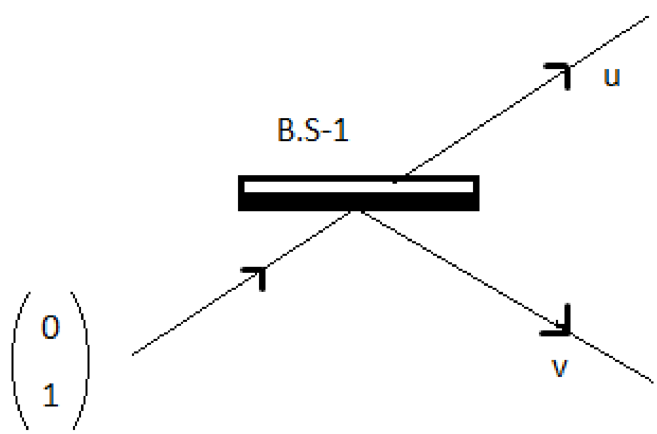

Figure 2.13: A beam coming to beam splitter from below.

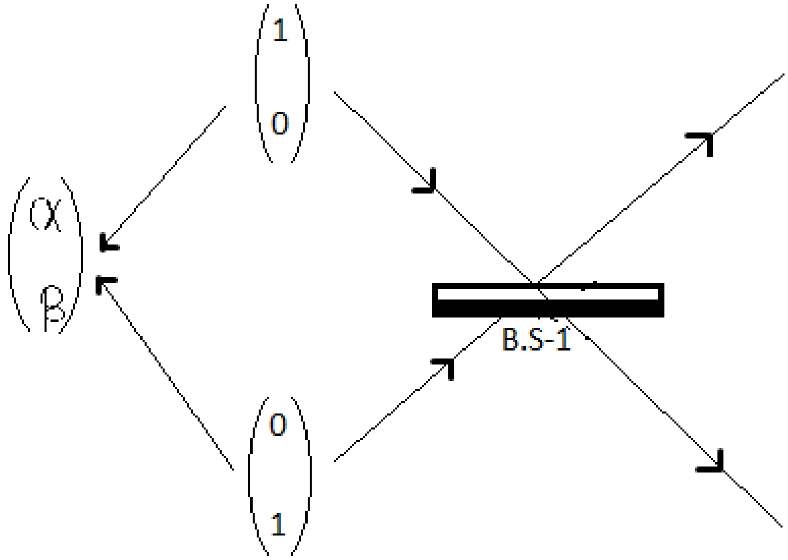

Figure 2.14: When two beams passes through a beam splitter.

and now using the conversions that we did above, we can write eq.(2.19) as,

$$\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \alpha\begin{pmatrix}s\\ t\end{pmatrix} + \beta\begin{pmatrix}u\\ v\end{pmatrix}. \tag{2.20}$$

By doing some algebraic manipulations we can write above equation as follows,

$$\begin{pmatrix}\alpha s+\beta u\\ \alpha t+\beta v\end{pmatrix} = \begin{pmatrix}s & u\\ t & v\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} \tag{2.21}$$

The term in the right of the above equation, $\begin{pmatrix}s & u\\ t & v\end{pmatrix}$, is the beam splitter. If we have a beam splitter of 50:50, that it transmits 50% of the light and reflects 50% of it, we can say that,

$$|s|^2 = |t|^2 = |u|^2 = |v|^2 = \frac{1}{2}. \tag{2.22}$$

To conserve the probability our beam splitter takes the form,

$$B.S = \begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix} \tag{2.23}$$

If we apply this beam splitter on a state $\begin{pmatrix}\alpha\\ \beta\end{pmatrix}$, the probability remains conserved, so we can say that this is a perfect 50:50 beam splitter. Now in Mach-Zehnder interferometer we have two beam splitters, which can be written as,

$$B.S_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \tag{2.24}$$

$$B.S_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{2.25}$$

Now if a beam $\begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, passes through the interferometer, the resulting output at the detectors would be,

$$B.S_2 B.S_1 \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ -\alpha \end{pmatrix} \tag{2.26}$$

If we send a single photon in state $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$, after passing through the Mach-Zehnder interferomter it would be $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Mach-Zehnder interferometer is being used as a tool to answer many fundamental questions of quantum physics, which includes quantum entanglement, quantum computation , quantum eraser and many more fundamental research topics due to its versatile configuration.

2.4 Delayed Choice Experiments

John Archibald Wheeler proposed a thought experiment which was the attempt to describe that, how quanta of light adjusts its behavior? Whether it passes through young's double-slit as a wave or as a particle, or it remain in a superposition state, neither wave nor particle until it is measured [8].

In principle in a delayed choice experiment, let the photon decide its path and just before measurement change the apparatus in such a way that its path information can be acquired. A strange thing happens, photon which choose its path as a wave behaves as a particle at the time of measurement, now the question how that photon knows that it has to behave as a particle or as a wave? One can say that, the concept of time is rather different at quantum scale, photon changes its past information because it knew what will happen in future[9].

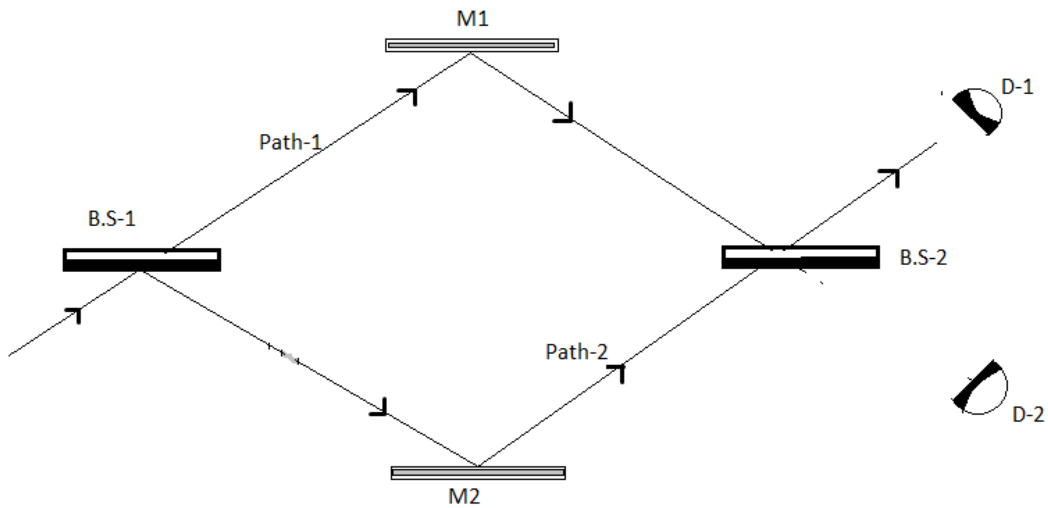

Figure 2.15: Wave behavior of photon, inside Mach-Zehnder interferometer.

Consider an experiment in which a photon moves through a Mach-Zehnder interferometer as shown in the figure(2.15). Now this photon splits at the beam splitter(B.S-1) or what we can say, it chooses its path. Either it goes through path-1 or path-2 and than after passing through beam splitter(B.S-2), it arrives at only one detector, say it strikes at D-1. No matter how many time we repeat this experiment photon arrives at only detector D-1, if no path difference is created. It shows the wave behavior of photon.

Now consider another experiment in which second beam splitter(B.S-2) is absent as shown in the figure(2.16). Photon will strike on either of the detector with equal probability and by knowing which detector it strikes we can tell which path photon was following and we get particle behavior of photon.

Now consider a thought experiment in which we let the photon decides its path in the interferometer and than we choose to change the apparatus as shown in the figure(2.17) and in figure(2.18). There are two cases.

Case-I: When we choose to take away the second beam splitter, after path was chosen.

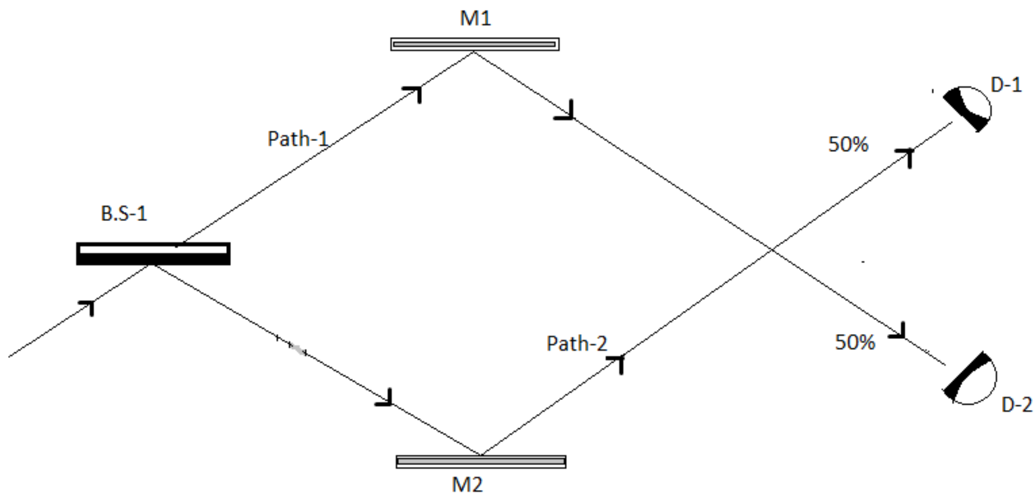

Figure 2.16: Particle behavior of photon when second beam splitter(B.S-2) is absent.

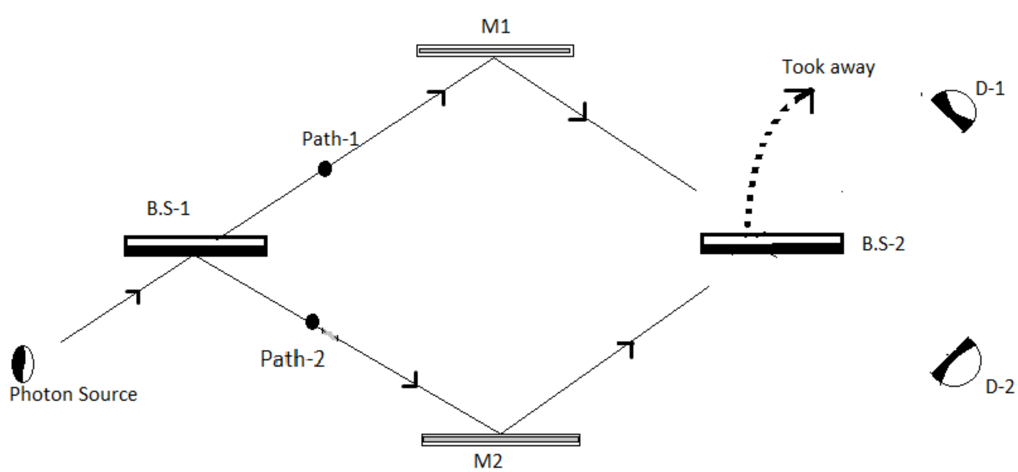

Figure 2.17: When we choose to take away the second beam splitter(B.S-2), we get the particle behavior of photon.

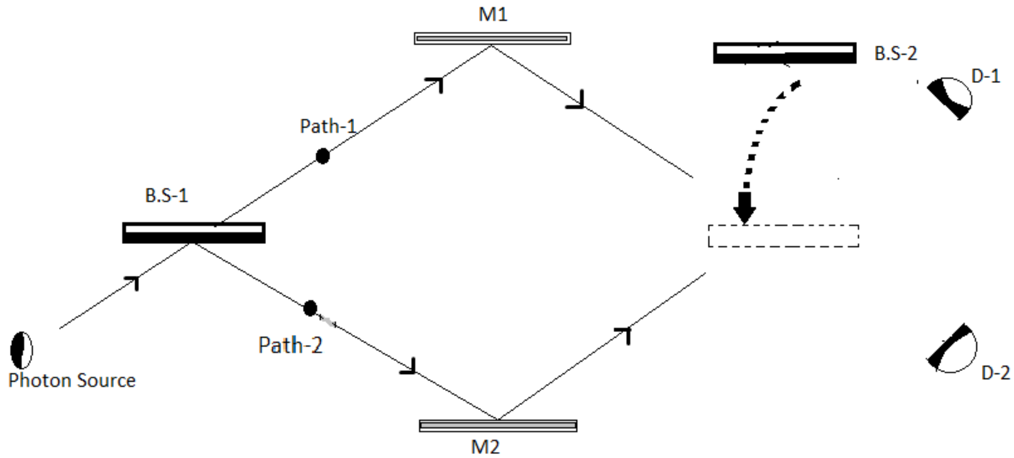

Figure 2.18: When we choose to put back the second beam splitter after the decision of photon to follow which path, we get wave behavior of photon.

Case-II: When we put back second beam splitter, after the path was chosen.

In the first case, which is shown in the figure(2.17), when photon has already chosen its path and we remove the second beam splitter, photon behaves as a particle. Although it passes through first beam splitter as wave but it is recorded as a particle.

In the second case, there is no second beam splitter, and photon is allowed to choose its path. Than we choose to put back second beam splitter as shown in the figure(2.18), results are counter intuitive. Photon which was behaving as particle at the first beam splitter, suddenly changes its behavior to a wave.
This thought experiment directly questions our perception about time. Does the photon knows what will happen in the future, so it erases its past information to behave accordingly? One can say in a wider sense that Gedanken/Delayed Choice Experiments directly illustrates the concept of complementarity.

2.5 Quantum Eraser

It all started with the concept of complementarity, which was proposed by Neil Bohr. If we try to measure the wave behavior of photon its particle behavior will disappear or vice versa. We cannot see both of these behaviors in a single experimental setup. To make things weird J.A. Wheeler's gedanken experiments question our perception about time. In attempt to answer the problem that , how can we see both of these behaviors in a single experiment, an intriguing phenomenon known as Quantum Eraser was proposed [7].
In 1982 Marlan O. Scully and Kai Druhl proposed an experiment to probe the information that is accessible to an observer. Eraser phenomenon was used to erase the past information of a quantum entity. They proposed a slight variation of young's double slit experiment. Instead of two slits they placed two three level atoms, as shown in the figure(2.19).

A radiation field E_1 which completely resonates with the atomic levels l_1 and l_2. Atom was excited by the radiation field from level l_3 to level l_1 and when it de-excites from level l_1 to l_2 it emits a β photon. The β photon makes an interference pattern of the screen, but if one measures the atomic state of the atoms, and if he found it to be in l_2, one can notify which atom emitted the β photon. With our text book wisdom and the limitation imposed by the uncertainty principle, we can say that if we have the which-path information the fringe pattern will be lost. So if we know which atom emitted the photon, the fringe pattern on the screen will be lost. Now the question is can we retrieve back those lost fringes, the answer is yes. Using an eraser phenomenon one can retrieve back those lost fringes, by erasing the information stored in the atomic states.
To erase that information allow us to involve another level in our atoms which can be named as l_2', as shown in the figure(2.20). After the interaction of field E_1 and the emission of the β photon, another field E_2 is applied on these atom which is resonating with l_2' and l_2. The radiation E_2 excites the atom from l_2 to l_2', and after de-excitation

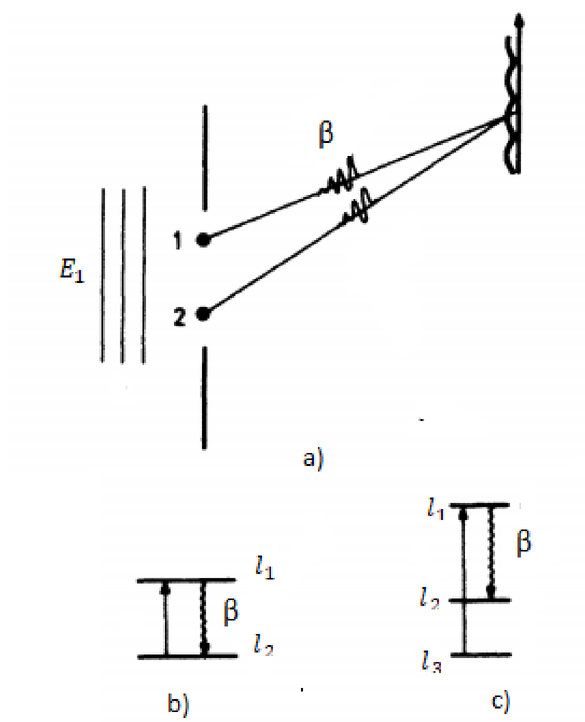

Figure 2.19: Above diagram shows a slight variation of young's double slit experiment. (a) shows two atoms having placed instead of slits and radiation E_1 excites these atoms. (b) shows the transition between atomic levels l_1 and l_2, with the emission of a β photon. (c) shows the atomic cascade decay of a three level atom with the emission of a photon.

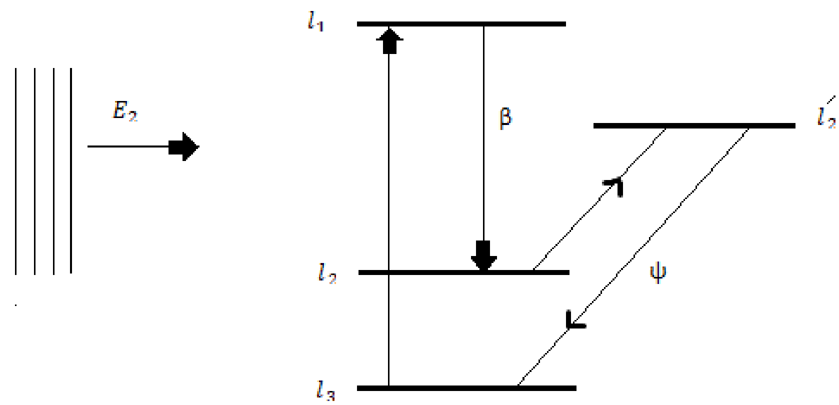

Figure 2.20: A schematic diagram showing the interaction of field which excites atom from level l_2 to l_2'
, after de-excitation ψ photon emits when it decays from l_2' to l_3.

to l_3 another photon ψ will be emitted. But one can tell through which atom that ψ photon was emitted so are still stuck in our problem i.e. fringe pattern will not appear as we thought.

So consider another experimental setup in which our atoms are placed inside an elliptical optical cavity, as shown in the figure(2.21). Our atoms are placed at the focus of this elliptical cavity and a photon detector is placed at the common focal point, which is the center of this cavity. Cavity is transparent to the radiations E_1, E_2, and β photon, but it bounds the ψ photon. This makes sense as all of these radiations have different frequency, than a ψ photon. The ψ photon is captured at the center of this cavity by the photon detector. The mathematics of this setup is very promising it shows that interference pattern is restored, as now we are unable to tell through which atom that ψ photon came, neither we can tell about the β photon. Which completes the eraser procedure, the experimental setup was forced into a state in such a way that we cannot obtain which-path information.

Later in 1999 Kim et al. and Marlan O. Scully proposed another experiment to

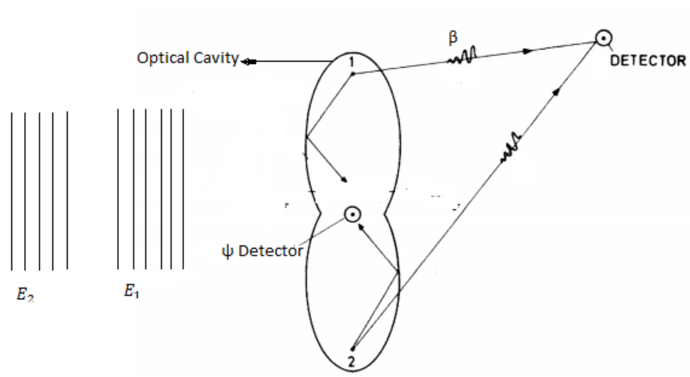

Figure 2.21: Above scheme is used to erase the information stored inside the atoms, in which our atoms were place inside an optical cavity, which is transparent to all frequencies except ψ photon.

realize the quantum eraser phenomenon [13]. Rather than atomic decay at the slits, a second order non linear crystal(BBO) was placed just after the two slits in young's double slit experiment. The BBO crystal produces two entangled photons by parametric down conversion. When a photon ϕ from a photon source passes through the slits A or B, it interacts with a BBO crystal, it produces two new photons ϕ_1 and ϕ_2. Both of the photons have half of the frequency as that of the original photon ϕ. Now one of the photon say ϕ_1 goes to the detector $D-0$ where from statistical count it produces an interference pattern, and the other photon ϕ_2 goes to a complex system of 50:50 beam splitters, mirrors and detectors, as shown in the figure(2.22).

If the photon ϕ_2 reflects through the beam splitters and goes to either of the detector $D-1$ or $D-2$, we have which-path information. Lets say detector $D-1$ records a click, we can say that the photon ϕ came through the slit A. As we have this information the fringe pattern on the detector $D-0$ is lost as fringe visibility and path distinguishability are two complementary variables.
The miracle happens when the photon ϕ_2 passes through the beam splitters, and detector $D-3$ or $D-4$, records a click, we lose the which-path information. Due to lost of this information fringe pattern at $D-0$ reappears. At the end of the experiment

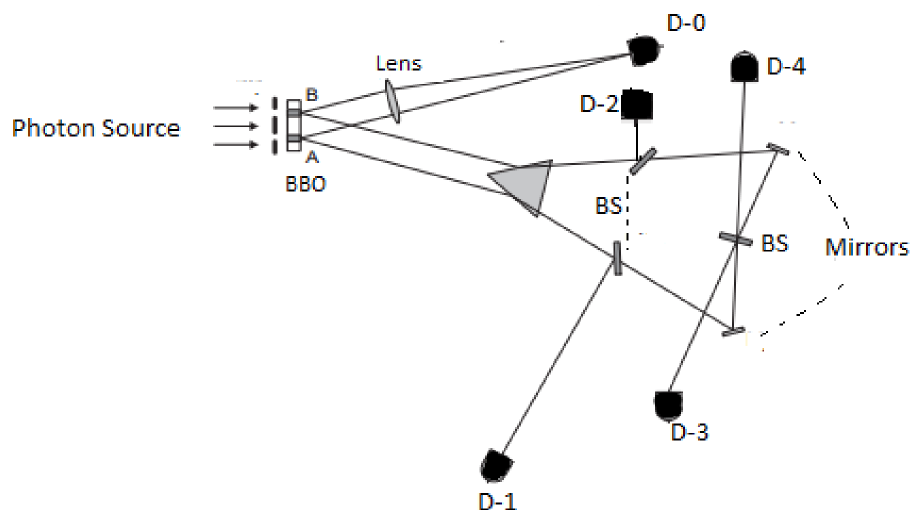

Figure 2.22: Experimental setup for the eraser procedure to recover back the interference fringes.

the results of the detector $D-0$ looks like a mess. But this can be explained if we try to separate out those photons which were detected at $D-3$ and $D-4$ and there counter parts at $D-0$ from those which were detected at $D-1$ or $D-2$, we see that those which were detected at $D-3$ or $D-4$ gives us interference pattern as which path information was lost and those which were detected at $D-1$or $D-2$ gives us particle behavior.

Strange thing happens when we delay the detection of photon ϕ_2. If detector $D-0$ already has the click due to ϕ_1 photon and ϕ_2 goes to the detector $D-1$ or $D-2$ fringe pattern will not appear, no matter how much we delay the detection of ϕ_2 photon. This raises some fundamental questions: How does the photon ϕ_1 knows that photon ϕ_2 will hit the detector $D-1$ or $D-2$, even photon ϕ_2 was faraway from detection? How does the photon $\phi-1$ knows what will happen in future and it changes its past behavior to act accordingly? Is there any meaning of Future, Past and Present at quantum scale?

2.5.1 Eraser Procedure with Mach-Zehnder Interferometer

An interesting quantum eraser phenomenon was reported in 2010 by T.L. Dimitrova and A. Weis [14]. They reported single photon quantum erasing procedure with the help of a Mach-Zehnder interferometer.It has been reported that photon which carries the which path information can be erased after it has left the interferometer. The erasing of which-path information restores the interference fringes.

Eraser procedure on waves

We start with a classical Mach-Zehnder interferometer, which is shown in the figure(2.23). We inject a classical laser field which travels through the interferometer. After passing through the first beam splitter it diverges to two paths path-1 and path -2. Mirrors introduce some phase difference to these beams. When these two complementary beams converge at second beam splitter interferecne pattern can be seen on the detectors, the intensity of the interference pattern comes out to be,

$$I_{12}(\delta\phi) = \frac{I_o}{2}(1 \pm \cos\delta\phi). \tag{2.27}$$

Where $\delta\phi$ is the phase difference introduced due to path difference created by one of the movable mirror with the help of a piezoelectric transducer.

Now we discuss the quantum erasing phenomenon on waves. Consider a special Mach-Zehnder interferometer as shown in the figure(2.23). We pump a laser field into the interferometer which is polarized at 45^0. The state of such a beam can be written in Jones vector notation as,

$$\psi = \frac{E_o}{2}\begin{pmatrix}1\\1\end{pmatrix}. \tag{2.28}$$

The first beam splitter splits the beam into two beams with equal intensities, as we have a 50:50 beam splitter, with reflection and transmission coefficients as, $r = \frac{\iota}{\sqrt{2}}$ and $t = \frac{1}{\sqrt{2}}$ respectively. Then these two beams passes through two orthogonal polarizers on there paths. We know that from our text books that two orthogonal beams do not

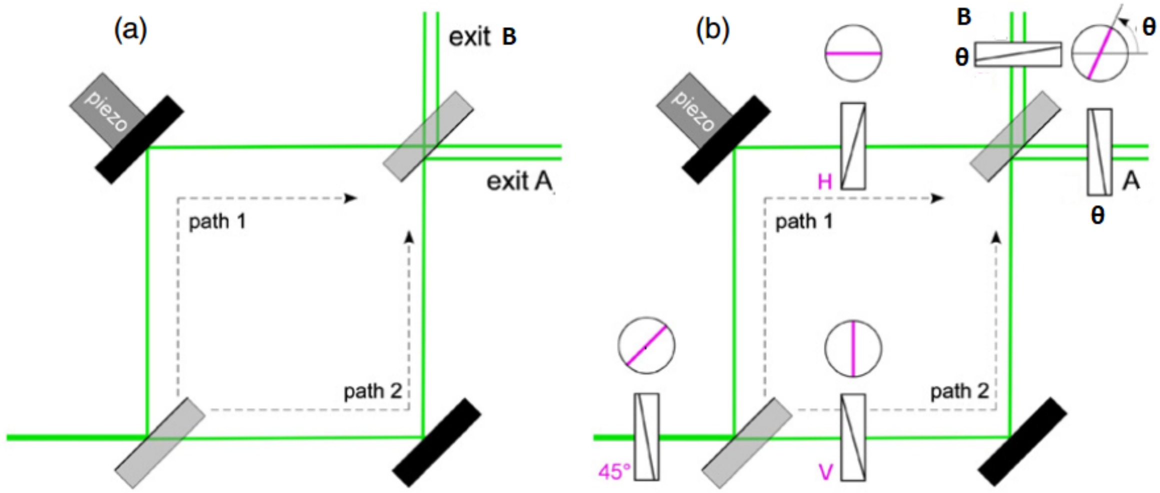

Figure 2.23: In the above figure(a) shows the simple Mach-Zehnder interferometer with adjustable path difference. Figure (b) is for the erasing procedure. Starting from the left bottom we have a polarizer of 45^o after that we have a beam splitter which splits the beams. H and V shows the horizontal and vertical polarizer. The polarizer at the end which can be oriented at any angle θ is the eraser polarizer [14].

show interference. So just before the detectors we put another polarizer which oriented at some angle θ. In matrix notation vertical and horizontal polarizers are written as,

$$M_H = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \tag{2.29}$$

$$M_V = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{2.30}$$

The erasing polarizer which is at the end of the apparatus is oriented at some angle θ. Its matrix can be written as,

$$M_\theta = R^{-1}(\theta) M_H R(\theta) = \begin{pmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{pmatrix}. \tag{2.31}$$

Where $R(\theta)$ is the rotational matrix in two dimensions, which is written as,

$$R(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{pmatrix}. \tag{2.32}$$

The wave function for the two beams just before the measurement at the B exit comes out to be,

$$\psi_{B1} = M_\theta r M_H r \psi_o e^{\iota\phi_1}, \tag{2.33}$$

$$\psi_{B2} = M_\theta t M_V t \psi_o e^{\iota\phi_2}. \tag{2.34}$$

Where ψ_{B1} and ψ_{B2} are the wave functions for the beams which came along the path-1 and path-2 with a phase difference $e^{\iota\phi_1}$ and $e^{\iota\phi_2}$ respectively. Where these phase difference can be written as,

$$\phi_1 = 2\pi\frac{L_1}{\lambda} + \Delta\phi, \tag{2.35}$$

$$\phi_2 = 2\pi\frac{L_2}{\lambda}. \tag{2.36}$$

where L_1 and L_2 are the length traveled by each beams. $\Delta\phi$ is the phase difference introduced to change in length that was produced by the help of movable mirror which was attached to a piezoelectric transducer. The intensity of beams which comes out at B can be written as,

$$I_B = |\psi_{B1} + \psi_{B2}|^2 = \frac{I_o}{8}(1 - \cos\delta\sin 2\theta). \tag{2.37}$$

We can write same for the beams which exits from side A,

$$I_A = |\psi_{A1} + \psi_{A2}|^2 = \frac{I_o}{8}(1 + \cos\delta\sin 2\theta). \tag{2.38}$$

The total intensity comes out to $I = I_B + I_A = \frac{1}{4}$. The above eq.(2.37) and eq.(2.38) clearly shows that it is the same intensity as written in the eq.(ref2.27), which directly means that our erasing procedure is successful.

Single Photon Quantum Erasing

The erasing procedure on a single photon is more intriguing, as it is directly related to the question that through which path photon went through [14]. So let us consider a system in which electron went through a Mach-Zehnder interferometer, the corresponding state vector can be written as,

$$|\psi\rangle = r\,|1\rangle\, e^{\iota\delta\phi} + t\,|2\rangle\,, \tag{2.39}$$

where $|1\rangle$ and $|2\rangle$ refers to the path, and $\delta\phi$ is the phase difference. After bit of algebra, the probability of a photon to be detected at either A or B comes out to be,

$$P_{12} = \frac{1}{2}(1 \pm \cos\delta\phi). \tag{2.40}$$

Now consider that same system that was used for the classical laser light. The state of the photon after passing through a polarizer which is oriented at 45^o can be written as,

$$|\psi_o\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle)\,|in\rangle\,. \tag{2.41}$$

Where $|in\rangle$ denotes the direction of incoming photon, $|H\rangle$ and $|V\rangle$ are the horizontal and vertical states which currently have equal probability. Beam splitter acts as projection operator, which is written as $BS1 = r\,|1\rangle\,\langle in| + t\,|2\rangle\,\langle in|$, on the above state. The new state of the photon after the beam splitter comes out to be,

$$|\psi_{BS1}\rangle = \frac{1}{\sqrt{2}}(r\,|1\rangle\,|H + t\,|2\rangle\,|V\rangle\rangle + r\,|1\rangle\,|V\rangle + t\,|2\rangle\,|H\rangle). \tag{2.42}$$

Than the photon reflects through the mirrors which introduced spatial phase difference, than it passes through the two orthogonal ploarizers. The state of the photon after vertical and horizontal polarizer comes out to be,

$$|\psi_{HV}\rangle = \frac{1}{\sqrt{2}}(r\,|1\rangle\,|H\rangle\,e^{\iota\delta\phi} + t\,|2\rangle\,|V\rangle). \tag{2.43}$$

After passing of photon through the second beam splitter(BS2) the state of the photon comes to be,

$$|\psi_{BS2}\rangle = \frac{1}{\sqrt{2}}(r^2\,|H\rangle\,|B\rangle\,e^{\iota\delta\phi} + tr\,|H\rangle\,|A\rangle\,e^{\iota\delta\phi} + rt\,|V\rangle\,|A\rangle + t^2\,|B\rangle). \tag{2.44}$$

Now we have the eraser polarizer which acts on the above state but before that just for the simplification, we apply a projector in the direction of B. After the projection operator $P_B = |B\rangle\,\langle B|$, the state becomes,

$$|\psi_B\rangle = \frac{1}{2\sqrt{2}}(-\,|H\rangle\,e^{\iota\delta\phi} + |V\rangle)\,|B\rangle\,. \tag{2.45}$$

The eraser now only allows the photon to pass which is only in $|B\rangle$ direction and as our polarizer can be oriented at any angle θ, the action of this polarizer comes out to be,

$$|\psi_{AE}\rangle = \frac{1}{2\sqrt{2}}[-\cos^2\theta\,|H\rangle\, e^{\iota\delta\phi} + \sin^2\theta\,|V\rangle + \sin\theta\cos\theta(|H\rangle - |V\rangle\, ee^{\iota\delta\phi})]\,|B\rangle \quad (2.46)$$

Where $|\psi_{AE}\rangle$ is the state of the system after the photon has passed through the eraser polarizer. The probability of the photon to be at the exit B can be calculated as,

$$P_B = \langle\psi_{AF}|\psi_{AF}\rangle\,. \quad (2.47)$$

which comes out to be,

$$P_B = \frac{1}{8}(1 - \cos\delta\phi\sin 2\theta). \quad (2.48)$$

The above equation is same as we calculated for the case of waves. Similarly we can find the probability of photon to be at the exit A .

This chapter leaves us with just one question: What is Time? May be Human beings lives in a perpetual present, which is completely sealed off from the past and moving relentlessly into the Future.

Chapter 3

Atom Field Interaction

This chapter is dedicated to the mathematical frame work of atom field interaction, and the equations governing these interactions. We will start with the quantization of field and than we will derive different Hamiltonian for different scenarios i.e. when field is classical and atom is quantized, and than for quantized field and quantized atom as well in quantum mechanial picture of atom field interaction. In the end equation for effective Hamiltonian will be derived which will be used later in our system.

3.1 Field Quantization

We start with quantizing the field in free space. For this purpose we take Maxwell's equations in free space [25]. These equation relates electric field E and the magnetic field H, along with displacement vector D and magnetic field intensity B. Maxwell's equation are as follows,

$$\nabla \times H = \frac{\partial D}{\partial t}, \tag{3.1}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}, \tag{3.2}$$

$$\nabla \cdot B = 0, \tag{3.3}$$

$$\nabla \cdot D = 0. \tag{3.4}$$

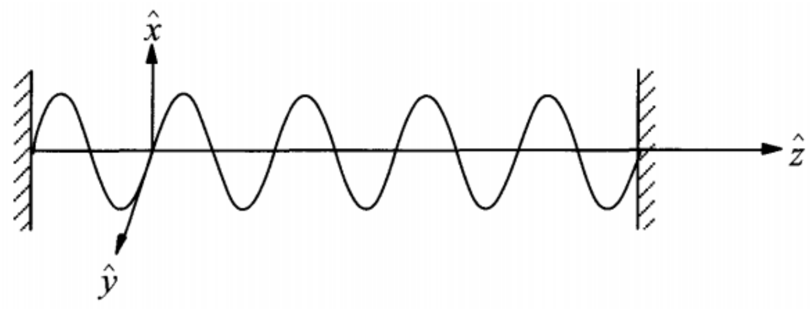

Figure 3.1: Electromagnetic field trapped inside a cavity, which is polarized along x-axis and oscillating along z-axis[25].

Where B and D can be written as, $B = -\mu_o H$ and $D = \epsilon_o E$ respectively. The ϵ_o is the permittivity and μ_o is the permeability of free space.Taking the curl of eq.(3.2),

$$\nabla \times (\nabla \times E) = \nabla \times (\frac{-\partial B}{\partial t}). \tag{3.5}$$

By using the property,

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E, \tag{3.6}$$

the equation 3.5 takes the form,

$$\nabla^2 E - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} E = 0. \tag{3.7}$$

Where using $\nabla \cdot E = 0$ in equation (3.6), as we are considering source free region. The equation (3.7) is also known as the wave equation. Now for the quantization of the field, we consider a field which is trapped inside a cavity of length L as shown in the figure, where as field is polarized along x-axis and oscillating along the z-axis as shown in figure(3.1)[25]. The solution of equation (3.7) can be written as,

$$E_x(z,t) = Aq(t)sin(kz), \tag{3.8}$$

where q is the mode amplitude with dimensions of length, $k = \dfrac{\pi}{L}$ is the wave number, and $A = \left(\dfrac{2\nu^2 m}{V\epsilon_o}\right)^{\frac{1}{2}}$ with $\nu = \dfrac{\pi c}{L}$ being the eigen frequency, $V = AL$ where

A is the area of the optical resonator/cavity and V is the volume of the cavity. The constant m has the units of mass which is used to include analogy with harmonic oscillator. We can write same solution of wave equation for magnetic field as well which would be,

$$H_y = A\frac{\dot{q}\epsilon_o}{k}cos(kz). \tag{3.9}$$

The classical Hamiltonian for the field is written as,

$$H = \int_V d\tau(\epsilon_o E_x^2 + \mu_o H_y^2), \tag{3.10}$$

where we took the integration over the volume of the cavity. Now putting eq.(3.8) and (3.9) in above eq. (3.10) which after some algebraic manipulation takes the form,

$$H = \frac{1}{2}(m\nu^2 q^2 + \frac{p^2}{m}), \tag{3.11}$$

where $p = m\dot{q}$ is canonical momentum of the mode. The eq.(3.11) is the Hamiltonian of the radiation field as sum of independent energies. To quantize the field we consider p and q as operators which must follows,

$$[q_i, p_{i'}] = \iota\hbar\delta_{ii'}, \tag{3.12}$$

$$[q_i, q_{i'}] = [p_i, p_{i'}] = 0, \tag{3.13}$$

where $i = 1, 2, 3, ...,$ tells about the number of modes present in the cavity if there is only one mode in the cavity than we take $i = 1$. Now writing q and p in terms of lowering and raising operators.

$$ae^{-\iota\nu t} = \frac{1}{\sqrt{2m\hbar\nu}}(m\nu q + \iota p), \tag{3.14}$$

$$a^{\dagger}e^{\iota\nu t} = \frac{1}{\sqrt{2m\hbar\nu}}(m\nu q - \iota p), \tag{3.15}$$

by solving the commutator of eq.(2.12) we get,

$$H = \hbar\nu\left(\frac{1}{2} + a^{\dagger}a\right). \tag{3.16}$$

The above equation is the Hamiltonian of the field inside the resonating cavity. Now we can write electric field and magnetic field in terms of lowering and raising operators, the equations (3.8) and (3.9) would become,

$$E_x(z,t) = \left(\frac{\hbar\nu}{V\epsilon_o}\right)^{\frac{1}{2}} (ae^{-\iota\nu t} + a^\dagger e^{\iota\nu t}) sin(kz), \tag{3.17}$$

$$H_y(z,t) = -\iota\epsilon_o \left(\frac{\hbar\nu}{\epsilon_o V}\right)^{\frac{1}{2}} (ae^{-\iota\nu t} - a^\dagger e^{\iota\nu t}) cos(kz). \tag{3.18}$$

Now our field is quantized as we can see in equations mentioned above.

3.2 Atom Field Interaction

The simple most problem involving the atom field interaction is the interaction of two level atom with the single mode field. In order to do so we must know about the total energy of the system. So I will derive Hamiltonian of the atom and and than the total Hamiltonian of the system means combined Hamiltonian of the atom and the field, later which will be used to describe the interaction of an atom with some field. So we start our discussion by a brief introduction about the a general Hamiltonian and than we will talk about the Hamiltonian of atom field interaction. Basically Hamiltonian is the total energy of the system, which is the sum of kinetic energy and potential energy of the system. In quantum mechanics we take Hamiltonian to be as operator which comes from its classical analogue, which when applied on some eigen state gives all possible energies. Because of its usage in explaining the dynamics of time evolved state, it is very important in many formulation of quantum mechanics. Mathematically we can write it as,

$$H = \hat{T} + \hat{V}. \tag{3.19}$$

Where $\hat{t}$ and $\hat{V}$ are the operators of kinetic and potential energies, which can be written as follows,

$$\hat{T} = \frac{\hat{p^2}}{2m}, \tag{3.20}$$

$$\hat{p} = -\iota\hbar\nabla, \tag{3.21}$$

$$\hat{V} = V(r,t). \tag{3.22}$$

Where $\hat{p}$ is the momentum operator, m is the mass of the particle. Now the eq. (3.19) becomes,

$$H = \frac{\hat{p^2}}{2m} + \hat{V}(r,t). \tag{3.23}$$

Now consider we have some quantum system in the state $|\Psi(t)\rangle$ at some time t,

$$H|\Psi(t)\rangle = \iota\hbar\frac{\partial}{\partial t}|\Psi(t)\rangle. \tag{3.24}$$

The above equation is the famous time dependent Schrodinger equation. For state initially at $t = 0$, we can use unitary operator to find the time evolved state,

$$|\Psi(t)\rangle = exp\left(\frac{-\iota Ht}{\hbar}\right)|\Psi(0)\rangle. \tag{3.25}$$

Where $\hat{U} = exp\left(\frac{-\iota Ht}{\hbar}\right)$ is the unitary operator for a closed quantum system.

3.2.1 Semi-Classical Picture

Now consider a charged particle interacting with electromagnetic field having scalar potential $U(x,t)$ and vector potential $A(x,t)$. The Hamiltonian of such a system can be written as,

$$H = \frac{1}{2m}[\hat{p} - eA(x,t)]^2 + eU(x,t) + V(x). \tag{3.26}$$

The above equation is known as minimal coupling Hamiltonian. Where $\hat{p}$ is the canonical momentum operator and $V(x)$ is the electrostatic potential of the nucleus with the electron. This Hamiltonian can be reduced to simple form by using dipole approximation. In dipole approximation the electron and nucleus feels the same electromagnetic field by vector potential $A(x_o + x, t)$. This vector potential under dipole $kx << 1$ approximation can be written as,

$$A(x_o + x, t) = A(t)e^{[\iota k.(x_o + x)]}, \tag{3.27}$$

$$A(x_o + x, t) = A(t)e^{(\iota k.x_o)(1 + \iota k.x + ...)}, \tag{3.28}$$

$$A(x_o + x, t) = A(t)e^{\iota k.x_o}. \tag{3.29}$$

$$\tag{3.30}$$

Now using the Hamiltonian of eq. (3.26) in Schrodinger equation and using the above approximation we arrive at,

$$\left[-\frac{\hbar^2}{2m}[\nabla - \frac{\iota e}{\hbar}A(x_o,t)]^2 + V(x)\right]\psi(x,t) = \iota\hbar\frac{\partial}{\partial t}\psi(x,t), \tag{3.31}$$

here we again consider that we are in domain of radiation guage so,

$$U(x,t) = 0, \tag{3.32}$$

$$\nabla \cdot A = 0. \tag{3.33}$$

Now by using change of variable and introducing a new wave function,

$$\psi(x,t) = exp\left[\frac{\iota e}{\hbar}A(x_o,t).x\right]\phi(x,t). \tag{3.34}$$

Now using eq.(3.31) and (3.34) we get,

$$\iota\hbar\left[\frac{\iota e}{\hbar}\dot{A}\cdot x\phi(x,t) + \dot{\phi}(x,t)\right]exp\left(\frac{\iota e}{\hbar}A\cdot x\right) = exp\left(\frac{\iota e}{\hbar}A\cdot x\right)\left[\frac{\hat{p}^2}{2m} + V(x)\right]\phi(x,t), \tag{3.35}$$

and after some algebraic manipulations and some rearrangements the above equation becomes,

$$\iota\hbar\phi(\dot{x},t) = [H_o - ex\cdot E(x_o,t)]\phi(x,t), \tag{3.36}$$

where I have used $E = \dot{A}$ and $H_o = \frac{\hat{p}^2}{2m} + V(x)$ is the unperturbed Hamiltonian, here in our case,

$$H = H_o + H_I. \tag{3.37}$$

Where H_o is the free part of the Hamiltonian which doesn't take part in the interaction while H_I is the interaction part,

$$H_I = -ex.E(x_o,t). \tag{3.38}$$

The above Hamiltonian is important as it tells us about the atom's interaction with classical field, field which is not quantized.

3.2.2 Quantum Mechanical Picture

In the last section we explained the interaction of an atom with classical field, where field was not quantized. Now in quantum mechanical picture of atom field interaction we will take quantized field that we derived in this chapter before. So to explain the interaction of such a system where atom and field both are quantized Hamiltonian can be written as follows [**?**],

$$H = H_A + H_F + H_I. \tag{3.39}$$

Where H_F is the Hamiltonian of the quantized field written as,

$$H_F = \hbar\nu\left(a^\dagger a + \frac{1}{2}\right). \tag{3.40}$$

Where H_A is Hamiltonian of a two level atom,

$$H_A = E_a \left|a\right\rangle \left\langle a\right| + E_b \left|b\right\rangle \left\langle b\right|, \tag{3.41}$$

The above Hamiltonian of two level atom can be derived by using the completeness relation $\left|a\right\rangle \left\langle a\right| + \left|b\right\rangle \left\langle b\right| = 1$ and using property $\left\langle i|j\right\rangle = \delta_{ij}$ where $\left|a\right\rangle$ and $\left|b\right\rangle$ are the ground and first excited levels of the atom with corresponding energies E_a and E_b respectively. Now using completeness relation and electric field operator $\hat{E} = \epsilon\varepsilon(a+a^\dagger)$ in eq. (3.38).We can express H_A and ex in terms of atomic transition operators $\sigma_{ab} = \left|a\right\rangle \left\langle b\right|$, now H_A can be written as,

$$H_A = \sum_i E_i \sigma_{ii}, \tag{3.42}$$

$$ex = \sum_{i,j} e \left|i\right\rangle \left\langle i\right| x \left|j\right\rangle \left\langle j\right| = \sum_{i,j} p_{ij}\sigma_{ij}, \tag{3.43}$$

where $p_{ij} = e \left\langle i\right| x \left|j\right\rangle$ is the electric dipole transition matrix element and $\varepsilon = \left(\frac{\hbar\nu}{2\epsilon_o V}\right)^{\frac{1}{2}}$ has the units of electric field. Now eq. (3.38) would take the form,

$$H_I = \hbar\mu[\left|a\right\rangle \left\langle b\right| + \left|b\right\rangle \left\langle a\right|](a + a^\dagger). \tag{3.44}$$

Now using eq. (3.40,3.42,3.44) in eq. (3.39), the total Hamiltonian would become,

$$H = \hbar\nu(a^\dagger a) + \sum_i E_i \sigma ii + \hbar \sum_{i,j} \mu^{ij}\sigma_{ij}(a + a^\dagger). \tag{3.45}$$

Where $\mu^{ij} = -\frac{p_{ij} \cdot \epsilon\varepsilon}{\hbar}$ is known as coupling constant which tells us about how strong is the interaction of atom and field, and it depends upon the strength of the field and the atomic dipole and angle between dipole and the field. We have to note that I have ignored the ground level energy because it does not take part in the interaction, and the atomic dipole matrix element is $p_{ab} = p_{ba}$. Now Hamiltonian can be written as,

$$H = \hbar\nu(a^\dagger a) + E_a\sigma_{aa} + E_b\sigma bb + \hbar\mu(\sigma_{ab} + \sigma_{ba})(a + a^\dagger). \tag{3.46}$$

Now doing some algebraic manipulations where using $E_a - E_b = \hbar\omega$ and $\sigma_{aa} + \sigma_{bb} = 1$ and writing the atomic part in the Hamiltonian as,

$$E_a\sigma_{aa} + E_b\sigma_{bb} = \frac{1}{2}\hbar\omega(\sigma_{aa} - \sigma_{bb}) + \frac{1}{2}(E_a + E_b). \tag{3.47}$$

Ignoring the constant energy terms $\frac{E_a+E_b}{2}$, for convenience we change the notation as follows,

$$\sigma_z = |a\rangle\langle a| - |b\rangle\langle b|, \tag{3.48}$$

$$\sigma_+ = |a\rangle\langle b|, \tag{3.49}$$

$$\sigma_- = |b\rangle\langle a|. \tag{3.50}$$

After using above equations and doing some rearrangements, Hamiltonian will take the form,

$$H = \hbar\nu a^\dagger a + \frac{1}{2}\hbar\omega\sigma_z + \hbar\mu(\sigma_+ + \sigma_-)(a + a^\dagger) \tag{3.51}$$

In the above equation the terms $a^\dagger\sigma_-$ describes the process where atom dexcites to lower level resulting in the emission of a photon and the term $a\sigma_+$ tells the opposite. But the other two terms $a\sigma_-$ and $a^\dagger\sigma_+$ tells the loss in energy while atom dexcites to lower state which violates the law of conservation of energy, so we ignore these terms in our Hamiltonian and it will takes the form as follows,

$$H = \hbar\nu a^\dagger a + \frac{1}{2}\hbar\omega\sigma_z + \hbar\mu(\sigma_+ a + a^\dagger\sigma_-). \tag{3.52}$$

3.2.3 Interaction Picture

In the last section we derived the Hamiltonian for the interaction of two level atom with quantized field having frequency ν, referring to eq. (3.52),

$$H_o = \hbar\nu a^\dagger a + \frac{1}{2}\hbar\omega\sigma_z, \tag{3.53}$$

$$H_I = \hbar\mu(\sigma_+ a + a^\dagger\sigma_-), \tag{3.54}$$

$$H = H_o + H_I. \tag{3.55}$$

The equation (3.53) describes the Hamiltonian in the dipole and rotating wave approximation(RWA), RWA is basically the approximation in which we neglect the faster oscillating terms, which means we neglect the terms which includes $\nu + \omega$, where ν is the frequency of the field and ω is the transition frequency of atom. It is better to work out mathematics in interaction picture, we will derive an equation which tells us basically which part of the Hamiltonian takes part in the interaction. For that reason we write interaction Hamiltonian as,

$$\hat{H}(t) = e^{\left(\frac{\iota H_o t}{\hbar}\right)} H_I e^{\left(-\frac{\iota H_o t}{\hbar}\right)}. \tag{3.56}$$

Now using Baker's formula which is given as,

$$e^{\alpha A} B e^{-\alpha A} = B + \alpha[A, B] + \frac{\alpha^2}{2!}[A, [A, B]] + ... \tag{3.57}$$

from eq.(3.56) we have,

$$\hat{H}(t) = \hbar\mu(\sigma_+ a e^{\iota\Delta t} + a^\dagger\sigma_- e^{-\iota\Delta t}). \tag{3.58}$$

The above equation is the interaction Hamiltonian known as Jaynes-Cummings-Paul Model Hamiltonian, where $\Delta = \nu - \omega$ is the detuning which arises when both frequencies, frequency of field and transition frequency are not resonant. For the case when atomic levels transition frequency completely resonates with oscillating frequency of field, detuning goes to zero $\Delta = 0$.

3.2.4 Effective Hamiltonian

In the last section we derived the Hamiltonian in interaction picture. The equation can be used for the cases when there is detuning. So we consider system of a two level atom which is interacting off-resonantly with the field.The time evolution of off-resonant interaction Hamiltonian,

$$\hat{H}(t) = \hbar\mu(\sigma_+ a e^{\iota\Delta t} + a^\dagger \sigma_- e^{-\iota\Delta t}), \tag{3.59}$$

can be approximated by the Effective Hamiltonian [27], which is writtena as,

$$H_{eff} = -\frac{\hbar\mu^2}{\Delta}\left[\sigma_z a^\dagger a + \frac{1}{2}(\sigma_z + 1)\right]. \tag{3.60}$$

WHere Δ is the detuning, μ is the coupling constant, σ_z is the atomic inversion operator, and $a^\dagger$ and a are the lowering and raising operators respectively. Above eq.(3.60) can be derived in the limit of large detuning and using the time evolution operator,

$$\hat{U}(t) = e^{\frac{\iota H_I t}{\hbar}}, \tag{3.61}$$

$$\hat{U}(t) = 1 - \frac{\iota}{\hbar}\int_0^t dt'\hat{H}(t') - \frac{1}{\hbar^2}\int_0^t dt'\hat{H}(t')\int_0^{t'} dt''\hat{H}(t''). \tag{3.62}$$

Where we only keep upto second order. Now we calculate the time evolution operator for off-resonant interaction. We first evaluate the integral,

$$\int_0^t dt'\hat{H}(t') = \frac{\hbar\mu}{\iota\Delta}\left[\sigma_- a^\dagger(e^{\iota\Delta t} - 1) - \sigma_+ a(e^{-\iota\Delta t} - 1)\right]. \tag{3.63}$$

Now using the above mentioned result to obtain second order contribution, which comes out ot be,

$$\begin{aligned}
&\int_0^t dt'\hat{H}(t')\int_0^{t''}\hat{H}(t'') \\
&\quad = \frac{\hbar^2\mu^2}{\iota\Delta}\int_0^t dt'\left(\sigma_- a^\dagger e^{\iota\Delta t'} + \sigma_+ a e^{-\iota\Delta t'}\right)\left[\sigma_- a^\dagger(e^{\iota\Delta t'} - 1) - \sigma_+ a(e^{\iota\Delta t'} - 1)\right] \\
&\quad = \frac{\hbar^2\mu^2}{\iota\Delta}\int_0^t dt'[\sigma_-^2 a^{\dagger 2}(e^{2\iota\Delta t'} - e^{\iota\Delta t'}) + \sigma_-\sigma{+}aa^\dagger \\
&\quad - \sigma_+^2 a^2(e^{-2\iota\Delta t'} - e^{-iota\Delta t'}) - \sigma_+\sigma aa^\dagger(e^{-\iota\Delta t'} - 1)].
\end{aligned} \tag{3.64}$$

The above equation can be simplified using, $\sigma_- = |b\rangle\langle a|b\rangle\langle a| = 0$ and so is $\sigma_+^2 = 0$, which becomes after some rearrangements,

$$\int_0^t dt' \hat{H}(t') \int_0^{t''} \hat{H}(t'') = \frac{\hbar^2\mu^2}{\iota\Delta} \int_0^t dt' \left[\sigma_-\sigma_+ a^\dagger a(e^{\iota\Delta t'} - 1) - \sigma_+\sigma_- aa^\dagger (e^{-\iota\Delta t'})\right]. \quad (3.65)$$

When we do the remaining integration and use $[a, a^\dagger] = 1$, and $\sigma_+\sigma_- - \sigma_-\sigma_+ = |a\rangle\langle b|b\rangle\langle a| - |b\rangle\langle a|a\rangle\langle b| = |a\rangle\langle a| - |b\rangle\langle b| = \sigma_z$ and $\sigma_+\sigma_- = |a\rangle\langle b|b\rangle|a\rangle = |a\rangle\langle a| = \frac{1}{2}(\sigma_z + 1)$,

$$\int_0^t dt' \hat{H}(t') \int_0^{t''} \hat{H}(t'') = -\iota^2 \frac{\hbar^2\mu^2}{\Delta} \left[(\sigma_+\sigma_- - \sigma_-\sigma_+) aa^\dagger + \sigma_+\sigma_-\right] t, \quad (3.66)$$

$$\int_0^t dt' \hat{H}(t') \int_0^{t''} \hat{H}(t'') = -\iota^2 \frac{\hbar^2\mu^2}{\Delta} \left[\sigma_z a^\dagger a + \frac{1}{2}(\sigma_z + 1)\right] t. \quad (3.67)$$

Now as we have solved the whole integration we are in position to rewrite our unitary operator $\hat{U}(t)$ in eq.(3.61) as,

$$\hat{U}(t) = 1 - \frac{\iota}{\hbar}\left(-\frac{\hbar\mu^2}{\Delta}\left[\sigma_z a^\dagger a + \frac{1}{2}(\sigma_z + 1)\right] t\right). \quad (3.68)$$

Where we have ignored the first order contribution as it is constant oscillating term in time, and we use the second order contribution as it is linear in time, so I can rewrite my unitary operator in a simple way as,

$$\begin{aligned} \hat{U}(t) &= 1 - \frac{\iota}{\hbar} H_{eff} t, \\ &= exp\left[\frac{-\iota H_{eff} t}{\hbar}\right]. \end{aligned} \quad (3.69)$$

Where H_{eff} is the required result,

$$H_{eff} = -\frac{\hbar\mu^2}{\Delta}\left[\sigma_z a^\dagger a + \frac{1}{2}(\sigma_z + 1)\right]. \quad (3.70)$$

which will be used later, when we will talk about case in which an atom interacts off-resonantly with a single mode of field trapped inside a cavity.

Chapter 4

Cavity Based Tunable Delayed Choice Quantum Eraser

In this chapter a different scheme of quantum eraser will be explained, where atom will do different interactions with the field in cavity [17]. First a three level atom will interact off-resonantly with the field, we tag a three level atom with cavity and cavity will be in superposition state. This interaction of atom and field will be governed by effective Hamiltonian derived in the previous chapter. Than three level atom will be exposed to Ramsey field. The Ramsey fringes will not form due to tagging of atom with the cavity field. In order to recover these fringes or to recover the lost information eraser phenomenon will used in which a two level atom interacts with the same cavity. Solving mathematical equations leads to entanglement between three level and two level atom. Than two level atom again interacts with Ramsey field. By adjusting the interaction time of two level atom with Ramsey field we will be able to manipulate interference fringes and path distinguishability.

4.1 Tagging of Three Level Atom With Cavity

In the tagging process system consists of three parts as shown in the figure 4.1[17]. First we have a three level atom which is in its superposition state written as $\frac{(|a\rangle+|b\rangle)}{\sqrt{2}}$ where $|a\rangle$ and $|b\rangle$ are ground and first excited levels and the third level is named as $|c\rangle$. The reason that we took atom to be in superposition is it exhibits interference

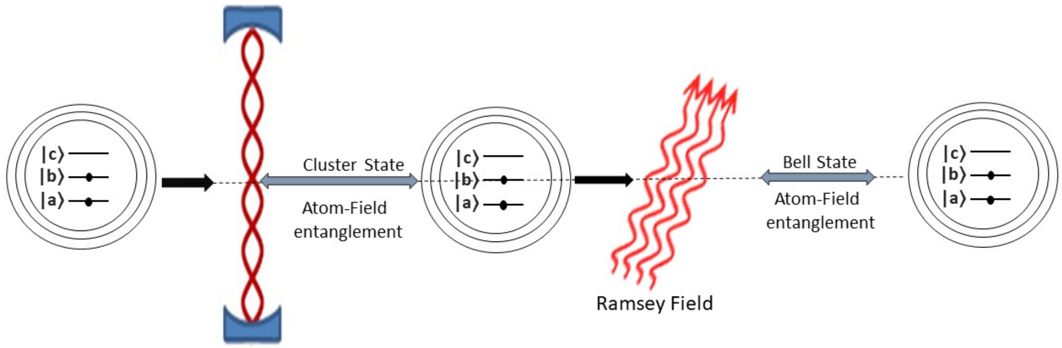

Figure 4.1: Figure shows the tagging process of three level atom with the cavity. Starting from left we have initial state $|\Psi_{t=0}\rangle$, than After the off resonant interaction for interaction time $t = \frac{\pi\delta}{\mu^2}$ state will be $\left|\Psi_{t=\frac{\pi\delta}{\mu^2}}\right\rangle$, and in the end we have the bell state $|\Psi_T\rangle = \frac{1}{\sqrt{2}}(|0_c, a\rangle + |1_c, b\rangle)$ [17].

pattern when interacts with a Ramsey field. The second one is cavity which is also in superposition state written as $\frac{(|0_c\rangle+|1_c\rangle)}{\sqrt{2}}$.

Cavity field interacts dispersively with upper two levels of the atom $|b\rangle$ and $|c\rangle$. This interaction will be governed by the effective Hamiltonian given as,

$$H_{eff} = \frac{\hbar\mu^2}{\delta}[\hat{a}\hat{a^\dagger}\,|c\rangle\,\langle c| - \hat{a^\dagger}\hat{a}\,|b\rangle\,\langle b|]. \tag{4.1}$$

Where μ is the coupling constant and δ is the difference between the transition frequency of levels and the frequency of the field inside the cavity, where as $\hat{a}$ and $\hat{a^\dagger}$ are the creation and annihilation operators which discribes the dynamics of the field. Before the interaction between atom and the cavity field initial state of the system can be written as,

$$\left|\Psi_{(t=0)}\right\rangle = \frac{1}{2}(|a, 0_c\rangle + |a, 1_c\rangle + |b, 0_c\rangle + |b, 1_c\rangle). \tag{4.2}$$

The time evolution of such a state is governed by Schrodinger wave equation $\iota\hbar\frac{\partial}{\partial(t)}|\Psi_t\rangle = H_{eff}|\Psi\rangle$, the time evolved state can be proposed as,

$$|\Psi_t\rangle = C_a^{0_c}|a, 0_c\rangle + C_a^{1_c}|a, 1_c\rangle + C_b^{0_c}|b, 0_c\rangle + C_b^{1_c}|b, 1_c\rangle + C_c^{0_c}|c, 0_c\rangle + C_a^{2_c}|a, 2_c\rangle . \quad (4.3)$$

Now by applying the effective Hamiltonian on this proposed state, on the right side of the Schrodinger wave equation we are left with,

$$H_{eff}|\Psi_t\rangle = \frac{\hbar\mu^2}{\delta}[C_c^{0_c}|c, 0_c\rangle - C_b^{1_c}|b, 1_c\rangle]. \quad (4.4)$$

Our Schrodinger equation takes the form,

$$\begin{aligned} \dot{C_a^{0_c}}|a, 0_c\rangle + \dot{C_a^{1_c}}|a, 1_c\rangle + \dot{C_b^{0_c}}|b, 0_c\rangle + \dot{C_a^{1_c}}|a, 1_c\rangle + \dot{C_c^{0_c}}|c, 0_c\rangle + \dot{C_a^{2_c}}|a, 2_c\rangle \\ = \frac{\iota\mu^2}{\delta}[C_c^{0_c}|c, 0_c\rangle - C_b^{1_c}|b, 1_c\rangle]. \end{aligned} \quad (4.5)$$

By solving a bit of more algebra we get the following set of equations,

$$\dot{C_c^{0_c}} = \frac{\iota\mu^2}{\delta}C_c^{0_c} \quad , \quad \dot{C_b^{1_c}} = \frac{\iota\mu^2}{\delta}C_b^{1_c} \quad (4.6)$$

$$\dot{C_a^{0_c}} = 0 \quad , \quad \dot{C_a^{1_c}} = 0 \quad , \quad \dot{C_b^{0_c}} = 0 \quad , \quad \dot{C_a^{2_c}} = 0. \quad (4.7)$$

Now by using the initial conditions $C_a^{0_c}(t=0) = C_a^{1_c}(t=0) = C_b^{0_c}(t=0) = C_b^{1_c}(t=0) = \frac{1}{2}$ and $C_c^{0_c}(t=0) = C_a^{2_c}(t=0) = 0$, our time evolved state comes out to be,

$$|\Psi_t\rangle = \frac{1}{2}[|a, 0_c\rangle + |a, 1_c\rangle + |b, 0_c\rangle + e^{\frac{-\iota\mu^2 t}{\delta}}|b, 1_c\rangle]. \quad (4.8)$$

For the interaction time $t = \frac{\pi\delta}{\mu^2}$ the above state will be,

$$\left|\Psi(t = \frac{\pi\delta}{\mu^2})\right\rangle = \frac{1}{2}[|a, 0_c\rangle + |a, 1_c\rangle + |b, 0_c\rangle - |b, 1_c\rangle]. \quad (4.9)$$

For the third and the last part in the tagging process atom interacts with the classical Ramsey field. The interaction time is set in accordance to the symmetric Hadamard transformation i.e. $|a\rangle \rightarrow \frac{(|a\rangle + |b\rangle)}{\sqrt{2}}$ and $|b\rangle \rightarrow \frac{(|a\rangle - |b\rangle)}{\sqrt{2}}$, by putting these values in eq.(4.9), the final state will be,

$$|\Psi_T\rangle = \frac{1}{\sqrt{2}}(|0_c, a\rangle + |1_c, b\rangle). \quad (4.10)$$

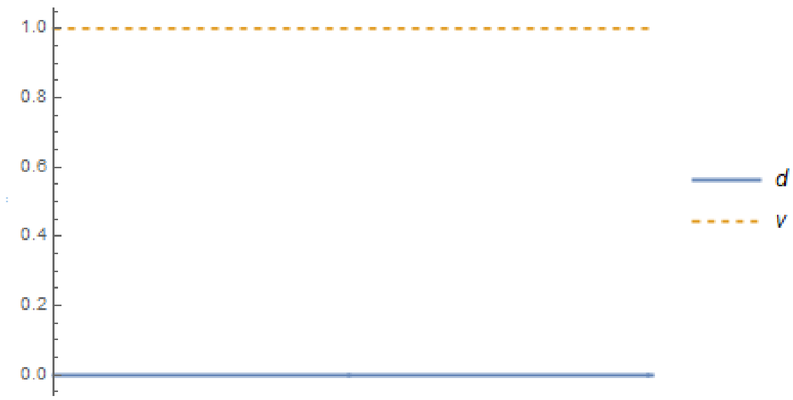

Figure 4.2: Figure illustrates that coherence of three level atom was lost due to its interaction with cavity field.

Here we can see in above equation that the original coherence of atom has been lost and to visualize this we can use Englert-Greenberger relation to find the visibility and path distinguishability and if we do so we can see as shown in the figure(4.2) that path distinguishability goes to zero. Atom is coupled with the cavity field, to recover this lost coherence an eraser process will be applied which we will discuss in the next section.

4.2 Delayed-Choice Eraser Procedure

In the tagging process the coherence of three level atom was lost. To recover that coherence Quantum Eraser process will be used to recover coherence. In erasing process we consider a two level atom as shown in the figure(4.3) [17], having ground and excited levels denoted as $|g\rangle$ and $|e\rangle$ respectively.

This two level atom interacts with the same cavity field which interacted with three level atom before. Initially the atom is in ground state and the initial state of the system can be written as,

$$|\Psi_{ep}\rangle = \frac{1}{\sqrt{2}}(|0_c, a\rangle + |1_c, b\rangle) \otimes |g\rangle \,. \tag{4.11}$$

Atom interacts with cavity field resonantly governed by the Hamiltonian given below,

$$\nu = \hbar\mu_{eg}[\hat{a} \, |e\rangle \, \langle g| + \hat{a^{\dagger}} \, |g\rangle \, \langle e|]. \tag{4.12}$$

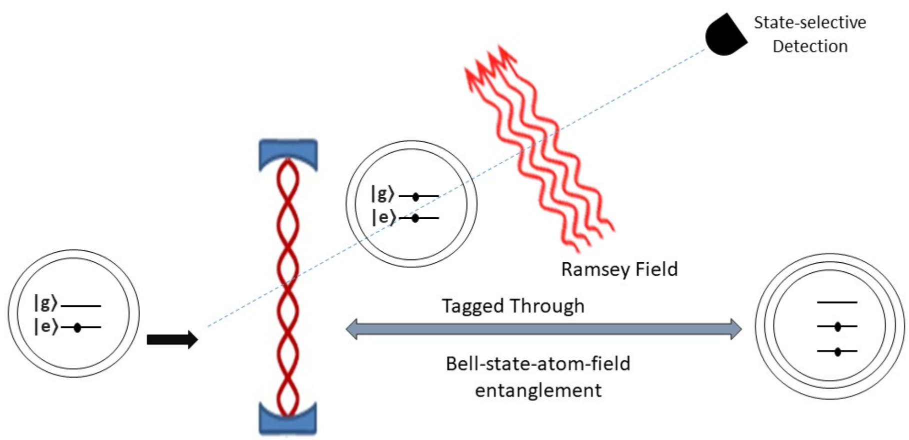

Figure 4.3: Interaction of two level atom with the same cavity field and then with the Ramsey field. Figure illustrates that two level atom is coupled with three level atom after its interaction with cavity field [17]

As it is completely resonant interaction $\delta = 0$. Proposed state of the system can be written as follows,

$$|\Psi_t\rangle = C^{0_c}_{a,g}|0_c, a, g\rangle + C^{1_c}_{b,g}|1_c, b, g\rangle + C^{0_c}_{b,e}|0_c, e, b\rangle . \tag{4.13}$$

For the time evolved state we use Schrodinger equation $\iota\hbar\frac{\partial}{\partial(t)}|\Psi_t\rangle = \nu|\Psi_t\rangle$, which after some simple algebra and by projecting these states $\langle 0_c, e, b|$ and $\langle 1_c, g, b|$ we are left with following coupled equations,

$$\iota\hbar\dot{C}^{0_c}_{b,e} = \hbar\mu_{eg}C^{1_c}_{b,g} \tag{4.14}$$

$$\iota\hbar\dot{C}^{1_c}_{b,g} = \hbar\mu_{eg}C^{0_c}_{b,e} \tag{4.15}$$

To solve these coupled equations for the values of probability amplitudes, we will use matrices method to find the values. For that purpose consider eq.(4.14) and eq. (4.15) where the initial conditions are given as $C^{0_c}_{a,g}(t=0) = C^{1_c}_{b,g} = \frac{1}{\sqrt{2}}$ and $C^{0_c}_{b,e} = 0$, we can write these equation in matrices form as follows,

$$X' = \begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = \begin{bmatrix} 0 & -\iota\mu_{eg} \\ -\iota\mu_{eg} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Where as $x_1 = C^{0_c}_{b,e}$ and $x_2 = C^{1_c}_{b,g}$. To find $C^{1_c}_{b,e}$ and $C^{0_c}_{b,e}$ we will find the eigenvalues of the above matrix, so for that reason we consider the characteristic equation $A - \lambda I = 0$,

$$|A - \lambda I| = \begin{vmatrix} -\lambda & -\iota\mu_{eg} \\ -\iota\mu_{eg} & -\lambda \end{vmatrix} = 0.$$

$$\lambda^2 - (-\iota\mu_{eg})^2 = 0. \tag{4.16}$$

From the eq.(4.16) we get our eigenvalues as follows,

$$\lambda_+ = \iota\mu_{eg}, \lambda_- = -\iota\mu_{eg}. \tag{4.17}$$

Solution to the equation X' can be calculated by using the following equation,

$$X(t) = C_1 v_1 e^{\lambda_+ t} + C_2 v_2 e^{\lambda_- t}. \tag{4.18}$$

Where,

$$X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Where as v_1 and v_2 are the eigenvectors corresponding to eigenvalues λ_+ and λ_- which came out to be,

$$v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Now for finding the C_1 and C_2 we take $t = 0$, for that case using the initial condition $x_1 = C^{1_c}_{b,e} = 0$ and $x_2 = C^{0_c}_{b,g} = \frac{1}{\sqrt{2}}$, we put all these values in eq.(4.18), and we get,

$$C_1 + C_2 = 0, -C_1 + C_2 = \frac{1}{\sqrt{2}} \tag{4.19}$$

Now solving for $C - 1$ and C_2, we get $C_1 = -\frac{1}{2\sqrt{2}}$ and $C_2 = \frac{1}{2\sqrt{2}}$, now we use these values to get our probability amplitudes, we put back these values in eq.(4.18) and we get,

$$x_1 = C^{0_c}_{b,e} = -\frac{1}{\sqrt{2}}\iota sin(\mu_e g) \tag{4.20}$$

and,

$$x_2 = C^{1_c}_{b,g} = \frac{1}{\sqrt{2}}cos(\mu_e g). \tag{4.21}$$

Now we have the probability amplitudes of our proposed state so we can put back values in eq.(4.13) and our state becomes,

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}[|0_c, g, a\rangle + cos(\mu_{eg}t)\,|1_c, g, b\rangle - \iota sin(\mu_{eg}t)\,|0_c, e, b\rangle] \tag{4.22}$$

Now for the interaction time $t = \frac{\pi}{2\mu_{eg}}$ the information that was lost inside the cavity is transferred to this two level atom leaving the cavity in vacuum state $|0_c\rangle$, now our state of the system after the interaction of two level atom with cavity comes out to be,

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}[|g, a\rangle - \iota\,|e, b\rangle]. \tag{4.23}$$

From the above eq.(4.23) we can see that our two level atom is now coupled with three level atom.

4.2.1 Interaction with Ramsey Field

Now this two level atom which is coupled with three level atom interacts with a classical field also known as Ramsey Field, as three level atom did. This interaction follows the Hamiltonian as given bellow,

$$H = \frac{\hbar\Omega_R}{2}[e^{-\iota\phi}\left|e\right\rangle\left\langle g\right| + e^{\iota\phi}\left|g\right\rangle\left\langle e\right|], \tag{4.24}$$

where $\Omega_R = \dfrac{|p_{eg}|\epsilon}{\hbar}$ is the Rabi frequency, $p_{ge} = q\left\langle g\right|\hat{x}\left|e\right\rangle$ is the dipole matrix element, and ϕ denotes the phase between dipole and the field. The initial state of the system becomes,

$$\left|\Psi(t=0)\right\rangle = \frac{1}{\sqrt{2}}[\left|g,a\right\rangle - \iota\left|e,b\right\rangle], \tag{4.25}$$

from the above equation our proposed state or time evolved proposed state can be written as,

$$\left|\Psi(t)\right\rangle = [C_{g,a}\left|g,a\right\rangle + C_{e,a}\left|e,a\right\rangle - \iota C_{e,b}\left|e,b\right\rangle - \iota C_{g,b}\left|g,b\right\rangle]. \tag{4.26}$$

Now as we have done before for finding the time evolved state, we consider Schrodinger equation once again. When we apply the Hamiltonian on our proposed state we are left with,

$$H\left|\Psi\right\rangle = \frac{\hbar\Omega_R}{2}[e^{-\iota\phi}C_{g,a}\left|e,a\right\rangle + \iota e^{-\iota\phi}C_{e,b} + e^{\iota\phi}C_{e,a}\left|g,a\right\rangle - \iota e^{\iota\phi}C_{e,b}\left|g,b\right\rangle] \tag{4.27}$$

$$\iota\hbar[\dot{C_{g,a}}\left|g,a\right\rangle + \dot{C_{e,a}}\left|e,a\right\rangle - \iota\dot{C_{e,b}} - \iota\dot{C_{g,b}}\left|g,b\right\rangle] = H\left|\Psi\right\rangle \tag{4.28}$$

Now taking projection of $\left|g,a\right\rangle$,$\left|e,a\right\rangle$,$\left|e,b\right\rangle$, and$\left|g,b\right\rangle$ on equations (4.27) and (4.28), we will be left with,

$$\dot{C_{g,a}} = -\frac{\iota\Omega_R}{2}e^{\iota\phi}C_{e,a}, \tag{4.29}$$

$$\dot{C_{e,a}} = -\frac{\iota\Omega_R}{2}e^{-\iota\phi}C_{g,a}, \tag{4.30}$$

$$\dot{C_{e,b}} = -\frac{\iota\Omega_R}{2}e^{-\iota\phi}C_{g,b}, \tag{4.31}$$

$$\dot{C_{g,b}} = -\frac{\iota\Omega_R}{2}e^{\iota\phi}C_{e,b}. \tag{4.32}$$

Now we have four coupled differential equations for the dynamics of our system. To find the probability amplitudes we will decouple these equations using Laplace transformation method under the initial conditions, $C_{g,a}(t=0)=\frac{1}{\sqrt{2}}$,$C_{e,b}(t=0)=\frac{1}{\sqrt{2}}$,$C_{e,a}(t=0)=0$ and $C_{g,b}(t=0)=0$. We first consider the equations 4.29 and 4.30, these equations can be written in Laplace transformation method as follows,

$$x^{'}+\frac{\iota\Omega_R}{2}e^{\iota\phi}y=0, \tag{4.33}$$

and

$$y^{'}+\frac{\iota\Omega_R}{2}e^{-\iota\phi}x=0, \tag{4.34}$$

where $x^{'}=\dot{C_{g,a}}$ and $y^{'}=\dot{C_{e,a}}$,

$$[sL(x)-x(0)]+\frac{\iota\Omega_R}{2}e^{\iota\phi}L(y)=0, \tag{4.35}$$

$$[sL(y)-y(0)]+\frac{\iota\Omega_R}{2}e^{-\iota\phi}L(x)=0. \tag{4.36}$$

Now using the initial values $x(0)=C_{g,a}(0)=\frac{1}{\sqrt{2}}$ and $y(0)=C_{e,a}=0$ and multiplying the equations with s and $\frac{\iota\Omega_R}{2}e^{\iota\phi}$, after bit of algebra we will be left with,

$$\frac{1}{\sqrt{2}}s+s^2L(x)+\frac{\Omega_R^2}{4}L(x)=0. \tag{4.37}$$

Now using the Laplace inverse transform on the equation,

$$L(x)=\frac{s}{\sqrt{2}}\left[\frac{1}{s^2+\frac{\Omega_R^2}{4}}\right], \tag{4.38}$$

we get,

$$x=C_{g,a}=\frac{1}{\sqrt{2}}cos\left(\frac{\Omega_R t}{2}\right). \tag{4.39}$$

Using above equation we can find the value of our other probability amplitude or by performing these steps again we can find $y=C_{e,a}$, which comes out to be,

$$y=C_{e,a}=\frac{-\iota}{\sqrt{2}}sin\left(\frac{\Omega_R t}{2}e^{-\iota\phi}\right). \tag{4.40}$$

We can perform this whole procedure for the other two coupled differential eq.(4.31) and (4.32), the probability amplitudes comes out to be,

$$C_{e,b} = \frac{1}{\sqrt{2}} cos\left(\frac{\Omega_R t}{2}\right), \tag{4.41}$$

and,

$$C_{g,b} = \frac{-\iota}{\sqrt{2}} e^{\iota\phi} sin\left(\frac{\Omega_R t}{2}\right). \tag{4.42}$$

Finally we have all the probability amplitudes of our time evolved state, using equations (4.39),(4.40),(4.41), and (4.42) in eq.(4.26) we get our final state of the system,

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}\left[\left(cos(\frac{\Omega_R t}{2})|a\rangle - e^{\iota\phi} sin(\frac{\Omega_R t}{2})|b\rangle\right) \otimes |g\rangle - \iota e^{-\iota\phi}\left(sin(\frac{\Omega_R t}{2})|a\rangle + e^{\iota\phi} cos(\frac{\Omega_R t}{2})|b\rangle\right)\right. \tag{4.43}$$

4.2.2 Delayed-Choice Tunability

From the eq.(4.43) we can see that the coherence of three level atom has been recovered after performing delayed choice eraser procedure. We can check this by using Englert-Greenberger relation once again. The path distinguishability that was lost in the tagging process (referring to Figure 4.2) can not only be recovered but can also be made tunable by adjusting the interaction time of two level atom with Ramsey field much latter in time. Using Enlert-Greenberger relation we can calculate fringe visibility "V" and path distinguisability "D" of eq.(4.43). To calculate "V" and "D", we know that,

$$D = \left|\frac{|C_A|^2 - |C_B|^2}{|C_A|^2 + |C_B|^2}\right|, \tag{4.44}$$

and,

$$V = 2\frac{|C_A.C_B^*|}{|C_A|^2 + |C_B|^2}. \tag{4.45}$$

From eq.(4.43), firstly for "D" C_A and C_B comes out to be,

$$C_A = \frac{1}{\sqrt{2}} cos\left(\frac{\Omega_R t}{2}\right), C_B = -\frac{1}{\sqrt{2}} e^{\iota\phi} sin\left(\frac{\Omega_R t}{2}\right). \tag{4.46}$$

Using eq.(4.45) and (4.46) we get "D" as follows,

$$D = \left||cos\left(\frac{\Omega_R t}{2}\right)|^2 - |-e^{\iota\phi} sin\left(\frac{\Omega_R t}{2}\right)|^2\right|. \tag{4.47}$$

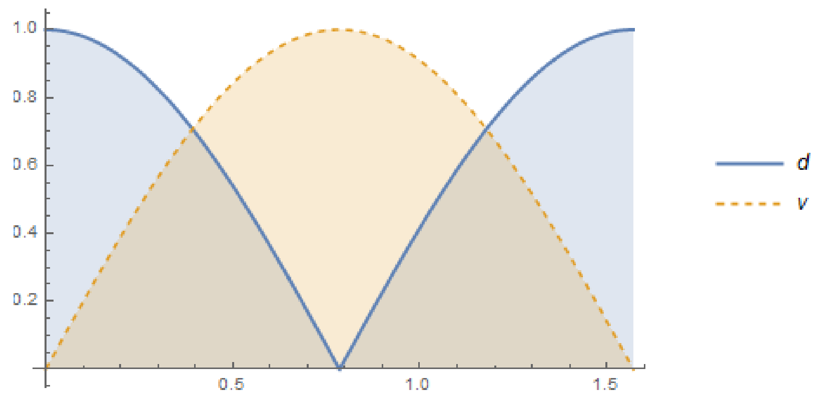

Figure 4.4: Distinguishability "D" and Visibility "V" are plotted for the interaction time upto π pulse

Similarly we can calculate the visibility "V" by using C_A and C_B given below,

$$C_A = -\frac{\iota}{\sqrt{2}} sin\left(\frac{\Omega_R t}{2}\right), C_B = -\frac{\iota}{\sqrt{2}} cos\left(\frac{\Omega_R t}{2}\right) e^{\iota\phi}. \tag{4.48}$$

Now using eq.(4.48) the visibility comes out to be,

$$V = 2\left|e^{\iota\phi} sin\left(\frac{\Omega_R t}{2}\right) cos(\frac{\Omega_R t}{2})\right|. \tag{4.49}$$

Considering the phase difference "ϕ" to be zero, and plotting visibility and distinguishability at different interaction time we can see as shown in the figure(4.4) that path distinguishability or coherence of the three level atom has been recovered

In comparison to the figure(4.2), we can easily see in figure(4.4) that coherence has been recovered. This coherence is recovered after it was lost due to the interaction of three level atom with the cavity, by post selecting the interaction time "t" of two level atom with Ramsey field. This forces us to think about the concept of time in Quantum world. This completes our Delayed-Choice Quantum Eraser procedure.

Chapter 5

Analysis and Conculsion

An idea of Delayed-Choice Quantum Eraser has been discussed in a cavity QED scenario. Cavity QED is a state of the art tool, and proves to be fruitful whenever we discuss about matter field interaction. The option of adjustable fringe visibility and path distinguishability nicely unveil the mysteries of quantum mechanics. For an experimental setup that was discussed in chapter four, High Quality Cavities are now available with lifetime in the range of seconds [34], as cavity can sustain field in them for seconds, cavity decay is not a constraint anymore for successful experimental results.

5.1 Experimental Feasibility

It has been reported recently by Haroche et al., the feasibility of interaction of streams of thousands of atoms with the cavity field before decoherence related to the cavity decay [17][32]. So in the above context the experimental setup discussed here seems straight forward. But there are few factors which can disturb the outcomes of this experiment that are success probability and fidelity. The factors which disturbs the merit of results are coupling dispersion, cavity anisotropies and random velocities taken from velocity distribution. These factors can be integrated into interaction time errors operationally, by dealing carefully with the most dominant factor, the velocity distribution of atoms.

5.1.1 Success Probability

The Success Probability of the erasing procedure depends upon the resonant interaction of the two level atom with the cavity field. Consider the expression,

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}[|0_c, g, a\rangle + cos(\mu_{eg}t)\,|1_c, g, b\rangle - \iota sin(\mu_{eg}t)\,|0_c, e, b\rangle]. \tag{5.1}$$

Now consider the interaction time error Δt in above scenario, where Δt has very small value. The eq.(5.1) becomes,

$$|\Psi(t+\Delta t)\rangle = \frac{1}{\sqrt{2}}[|0_c, g, a\rangle + cos(\mu_{eg}(t+\Delta t))\,|1_c, g, b\rangle - \iota sin(\mu_{eg}(t+\Delta t))\,|0_c, e, b\rangle], \tag{5.2}$$

for $t = \frac{\pi}{2}$, the eq.(5.2) becomes,

$$|\Psi(t+\Delta t)\rangle = \frac{1}{\sqrt{2}}[|0_c, g, a\rangle - sin(\mu_{eg}\Delta t)\,|1_c, g, b\rangle - \iota cos(\mu_{eg}\Delta t)\,|0_c, e, b\rangle], \tag{5.3}$$

Compare eq.(5.3) with eq.(4.23), what we can say that second term in eq.(5.3) is unwanted, the success of our experimental setup depends upon the two states $|0_c, g, a\rangle$ and $|0_c, e, b\rangle$, so the success probability comes out to,

$$P_{sucess} = 1 - \frac{1}{2}sin^2(\mu_{eg}\Delta t). \tag{5.4}$$

Where Δt being the interaction time error.

5.1.2 Fidelity

The fidelity of the erasing procedure depends upon resonant and Ramsey interaction of the two level atom. Generally fidelity is given as $F = |\langle\Psi_{ideal}|\Psi_{exp}\rangle|^2$, where as $|\Psi_{ideal}\rangle$ is the expression without any interaction errors, and was calculated as in eq.(4.43). To calculate $|\Psi_{exp}\rangle$, consider the eq.(5.3), and ignoring the second term as it puts limit to the success of this experiment, the expression becomes,

$$|\Psi(\Delta t)\rangle = \frac{1}{\sqrt{2}}[|0_c, g, a\rangle - \iota cos(\mu_{eg}\Delta t)\,|0_c, e, b\rangle], \tag{5.5}$$

Now in state $|\Psi(\Delta t)\rangle$ we can trace out the cavity, as cavity is off now, and we let the two level atom interact with Ramsey field. For that purpose recall the Hamiltonian in chapter four which was,

$$H = \frac{\hbar\Omega_R}{2}[e^{-\iota\phi}|e\rangle\langle g| + e^{\iota\phi}|g\rangle\langle e|]. \quad (5.6)$$

The proposed state comes out to be,

$$|\Psi(t_R)\rangle = [C_{g,a}|g,a\rangle + C_{e,a}|e,a\rangle + C_{e,b}|e,b\rangle + C_{g,b}|g,b\rangle], \quad (5.7)$$

under the initial conditions, $C_{g,a}(t=0) = \frac{1}{\sqrt{2}}$,$C_{e,b}(t=0) = \frac{1}{\sqrt{2}}(-\iota cos(\mu_{eg}\Delta t))$, $C_{e,a}(t=0) = 0$ and $C_{g,b}(t=0) = 0$. Once again using the Schrodinger equation and applying Hamiltonian on eq.(5.7), we are left with four coupled differential equation written as follows,

$$\dot{C_{g,a}} = -\frac{\iota\Omega_R}{2}e^{\iota\phi}C_{e,a}, \quad (5.8)$$

$$\dot{C_{e,a}} = -\frac{\iota\Omega_R}{2}e^{-\iota\phi}C_{g,a}, \quad (5.9)$$

$$\dot{C_{e,b}} = -\frac{\iota\Omega_R}{2}e^{-\iota\phi}C_{g,b}, \quad (5.10)$$

$$\dot{C_{g,b}} = -\frac{\iota\Omega_R}{2}e^{\iota\phi}C_{e,b}. \quad (5.11)$$

Using the Laplace transform method to solve these coupled differential equation and invoking the interaction time error and the phase error into the expression leads to,

$$|\Psi(t_R+\Delta t_R)\rangle = \frac{1}{\sqrt{2}}[cos(\frac{\Omega_R}{2}(t_R+\Delta t_R))|g,a\rangle - \iota e^{-\iota(\phi+\Delta\phi)}sin(\frac{\Omega_R}{2}(t_R+\Delta t_R))|e,a\rangle$$
$$+\iota cos(\mu_{eg}\Delta t)cos(\frac{\Omega_R}{2}(t_R+\Delta t_R))|e,b\rangle - cos(\mu_{eg}\Delta t)e^{\iota(\phi+\Delta\phi)}sin(\frac{\Omega_R}{2}(t_R+\Delta t_R))|g,b\rangle] \quad (5.12)$$

The eq.(5.12) is the $|\Psi_{exp}\rangle$, which includes the interaction time errors, which is the desired expression to calculate fidelity. Now consider that, the two level atom is detected in its excited state, the eq.(4.43) and eq.(5.12) reduces to,

$$\langle\Psi_{ideal}| = \frac{1}{\sqrt{2}}(e^{\iota\phi}sin(\frac{\Omega_R}{2}t_R))\langle a| + cos(\frac{\Omega_R}{2}t_R)\langle b|, \quad (5.13)$$

$$|\Psi_{exp}\rangle = \frac{1}{\sqrt{2}}[e^{-\iota(\phi+\Delta\phi)}sin(\frac{\Omega_R}{2}(t_R+\Delta t_R))|a\rangle + cos(\mu_{eg}\Delta t)cos(\frac{\Omega_R}{2}(t_R+\Delta t_R))|b\rangle]. \quad (5.14)$$

Now normalized fidelity comes out be,

$$F = \left| \left[e^{-\iota\Delta\phi} sin(\frac{\Omega_R}{2} t_R) sin(\frac{\Omega_R}{2}(t_R + \Delta t_R)) + cos(\mu_{eg\Delta t}) cos(\frac{\Omega_R}{2} t_R) cos(\frac{\Omega_R}{2}(t_R + \Delta t_R)) \right] \right|^2 . \tag{5.15}$$

Where as $\Delta\phi$,Δt and Δt_R are the upper bounded variables in their respective units. For the interaction time $t_R = \frac{\pi}{2}$, fidelity has been simulated against frequency of fidelity, for thousand runs, for different interaction time errors in figures(5.1,5.2,5.3,5.4), where we can see that as interaction time errors decreases fidelity of the experimental setup increases. Fidelity goes to unity, when interaction time errors are limited to $o.1 \times 10^{-6}$, this is the minimum interaction time error required to carry out successful experiment. If we increase the interaction time error by just little a factor i.e. $\Delta t_R = 1 \times 10^{-5}$s, the fidelity decreases drastically which can be seen in figure(5.5). Haroche et al. used an average atomic velocity of 503 m/s with uncertainty spread of just 2 m/s [17][32][33], which corresponds to interaction time error in the range of just 10^{-7}s which is a very small value in itself. But for number of interactions using thermally accelerated atoms makes the scenario worse. To accumulate such problem using cold atomic beams yielded from magneto-optical trap results in a sharp beam with negligible velocity spread which in turns results in to near ideal success probability and fidelity[17][31].

5.2 Conclusion

In a nut shell the Tunable Delayed-Choice Quantum Eraser as discussed here unveils the counter-intuitive and mystifying features of Quantum Mechanics in a prominent way and can also be used to test the interpretation of theory. The most counter-intuitive feature that arises from the phenomenon of Quantum Eraser is that, it questions our perception about time. The concept of time is different when we deal with classical or quantum systems. So there is a need for redefining Time. The results of this experimental setup shows that concept of complementarity is one of the key principle of quantum mechanics, one cannot talk about this theory without considering this important principle. The experimental setup as discussed in this thesis shows that when we the which-path information, the coherence of three level atom has beem lost.

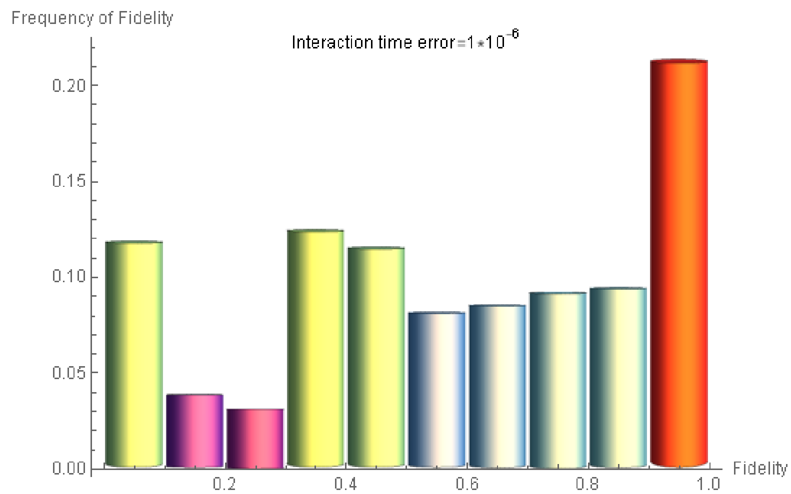

Figure 5.1: The above graph is between frequency of fidelity and fidelity for thousands runs, when fidelity goes to 1, it means experimental results are upto the merit. This graph has been plotted for fidelity at $t_R = \frac{\pi}{2}$, where Δt_R and $\Delta\phi$ are the upper bounded random variables. For interaction error time $\Delta t_R = 1 \times 10^{-6}$ there are chances of zero fidelity as well which means that, at this interaction time error our results do not meet our expectations.

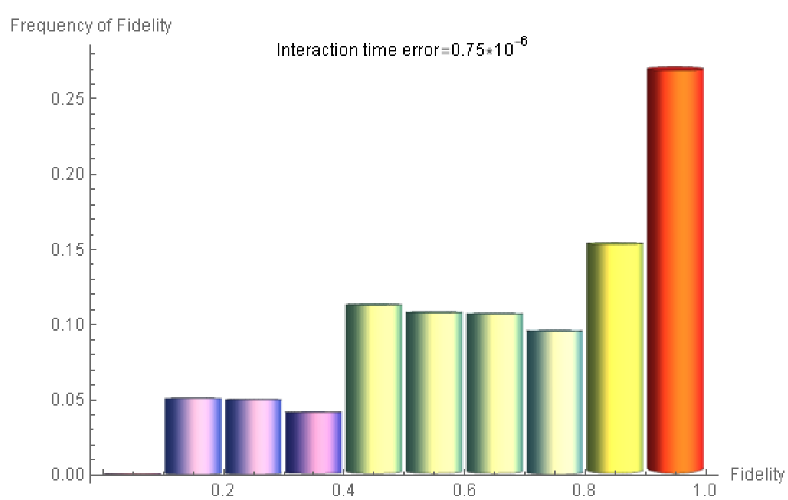

Figure 5.2: Compared to the figure(5.1)
,as the interaction time is $\Delta t_R = 0.75 \times 10^{-6}$, there are more chances of getting fidelity 1.

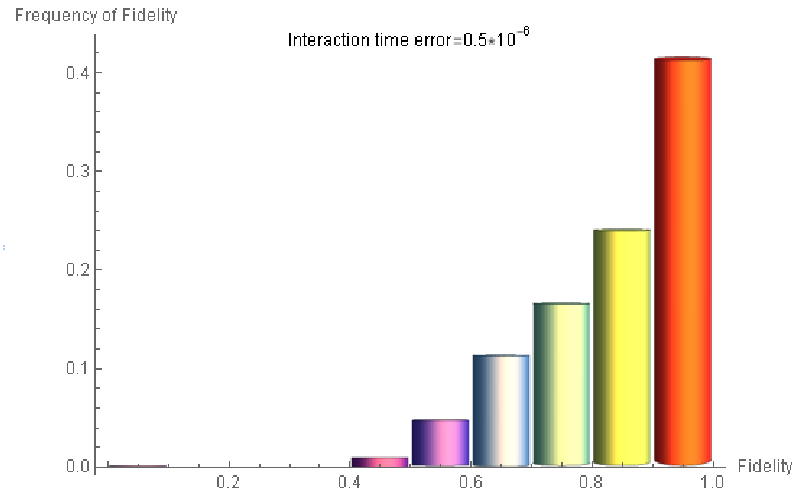

Figure 5.3: As we limit the interaction time error below $\Delta t_R = 0.5 \times 10^{-6}$ the experimental results looks more fruitful.

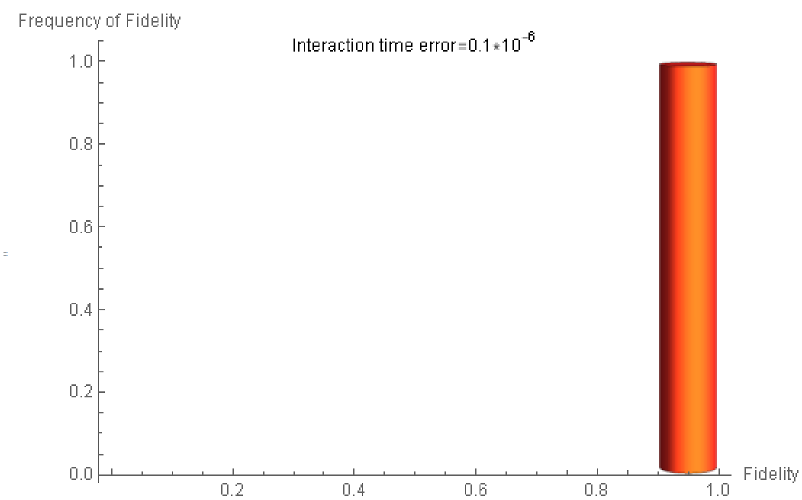

Figure 5.4: The frequency of fidelity goes to 1 as interaction time error is below 0.1×10^{-6}. Which means in the above mentioned experimental setup, experiment can be taken out efficiently when ever interaction time error is less than 0.1×10^{-6}.

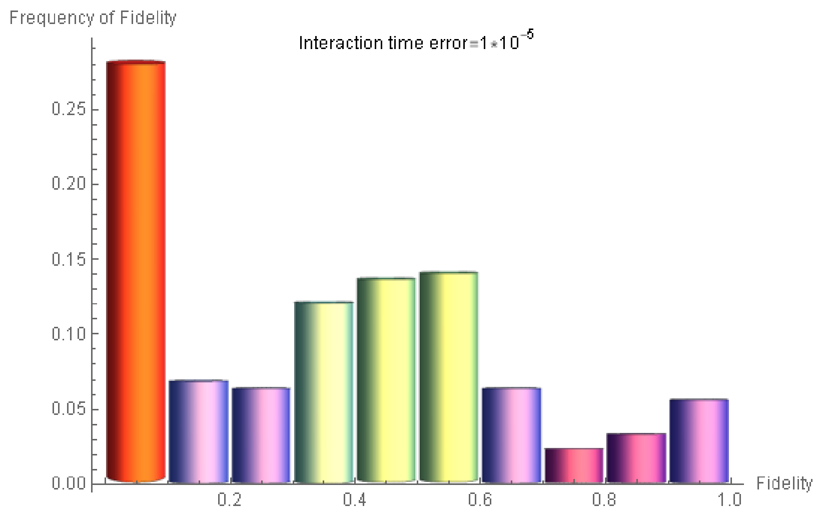

Figure 5.5: The fidelity decreases drastically, when interaction time error increases from the minimum, required to carryout experiment successfully.

Another thing that came up is that there is no experimental difference in real time and Delayed-Choice eraser, no matter when we try to retrieve the coherence of three level atom it can be recovered [13][16](but we have to take into account the factor of cavity decay). One cannot tell through which slit photon came through until a measurement is done, and by doing the measurement the interference pattern, which hints out that reduction of state should be treated as a mental process concerning information rather than a physical process [17][35]. The scheme presented here can be efficiently taken out in a Cavity QED research scenario.

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