Rare Radiative B decay in Standard Model and Beyond

by

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Dedicated to my beloved parents,
who valued my education above all else

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Abstract

The dissertation is based upon the theoretical study. The Standard Model is the most compact theory in Particle Physics to explain the interaction of particles at fundamental level. The Standard Model is a well testified theory via different experimentation, but still there are some open issues in the theory. To address such issues, researchers explore the theory beyond Standard Model. We study various channels of Higgs decay and computed their decay rate such as $H \to f\bar{f}$, AA, gg and $\gamma\gamma$. Further we study the radiative rare B decay $b \to s\gamma$ within Standard Model and the calculation of Wilson coefficient $C_{7\gamma}$ is performed explicitly. The new Physics usually appears in loop level. The literature explains various types of extensions to Standard Model. In this thesis "Adding Vector-like particles" one of the extensions generally $R \oplus \bar{R}$ to the Standard Model is used. These Vector-like particles have no contribution in Higgs decays at loop level, Higgs H is substituted by gauge singlet S in the di-photon resonance, by grasping the idea of di-photon resonance we apply it to $C_{7\gamma}$ Wilson coefficient in $b \to s\gamma$ to calculate penguin diagrams, in which Standard Model particles is substituted by Vector-like particles and gauge singlet S to probe new Physics.

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Chapter 1

Introduction

Since the beginning of mankind, man has always being curios to understand the basic nature of matter and everything that encompasses the universe. With the passage of time our understanding continuously developed. According to Greek philosopher earth, water, wind and fire were considered to be the four basic elements. In nineteenth century the major concept came into being by Dimitri Mendeleev who constructed periodic table of elements. Mendeleev's periodic table arguably gave birth to Particle Physics, as many scientists attempted to have a profound understanding of it. Atom was considered as the fundamental and smallest unit of matter, however with the discovery of electrons(1897), proton and neutron(1932) by J.J Thomson, Earnest Rutherford and James Chadwick debunked this idea. Therefore proton, neutron and electron were accounted as fundamental particles. Furthermore positron, muons, pions, kaons and neutrinos were discovered in the same year neutron were discovered. Later on quarks(fundamental particles) were found to be the constituents of protons and neutrons. In twentieth century experimental results of Particle Physics inspired the physicists to explore the nature of these particles and build a model that explained the interactions between them. They constructed a model known as Standard Model (SM) of Particle Physics. Quantum field theory is a theoretical frame work to explain the SM.

The SM particle contents are categorized into four genre; quarks, leptons, gauge bosons and scalar Higgs. The matter content within SM consists of fermions, these fermions are further categorized into three families of quarks and leptons. Leptons behave differently from quarks because they do not contribute to strong interactions. The force mediated particle are gauge bosons within the SM. Bosons are mediated particles in the interactions between fermions. Bosons, associated with three fundamental forces i-e electromagnetic, strong and weak boson($W^{\pm}, Z^0$). The scalar particle discovered in 2013 at Large Hadron Collider (LHC) known as Higgs, which satisfied all the characteristics of SM Higgs. The Higgs within SM is responsible to give mass to SM particle content and those gauge bosons, who obey electro-weak (EW) symmetry breaking. The SM is based upon the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. One of its properties is chirality, that quarks and leptons are divided into left and right handed fields. As theoretical approach the SM is a renormalizable and anomaly free theory.

In this dissertation we will come across two parts.

1. Calculating Higgs decays

2. Radiative B decays

In first part we analyzed and calculated Higgs decay channel i.e. Higgs decay to fermion anti-fermion and bosons at tree level Feynman diagram and to gluons and photons at loop level Feynman diagram. While calculating loop level Feynman diagrams we come across divergences and by implying certain constructed rules to remove these divergences such as Feynman Parametrization technique is used. In Feynman parametrization, it is calculated orderly to use variable shift, Wick's rotation and Regularization method. Furthermore the plot of Higgs branching ratio for various decay mode as a function of its mass has been analyzed. The coupling between Higgs boson and fundamental particles are defined by their masses. These types of interaction couplings are weak for particle such as up and down quarks and electrons and stronger for heavy particles such as W and Z bosons and top quark. Moreover, the coupling between SM Higgs and fermions are linearly proportional to the mass of fermion, while coupling between SM Higgs and bosons are proportional to the square of the mass of bosons. The coupling of Higgs boson to gluons is at leading order having one-loop Feynman diagram, in which Higgs boson couple to a virtual top quark $t\bar{t}$ pair. In case of photons, the one-loop Feynman diagram is due to virtual $t\bar{t}$ pair and also W^+W^- pair.

In second part of this dissertation Radiative B decays are used as a tool to test models of New Physics. In particular we are focused on the rare decay $b \to s\gamma$. These decay processes contribute at loop level electroweak penguin diagrams, in which the dominant particle is the top quark. Effective field theory(EFT) is used as a framework for radiative rare decays. We reproduce calculation of $C_{7\gamma}$ Wilson coefficient within SM. Wilson coefficient is solved by choosing an appropriate operator O_7 i.e. electromagnetic dipole operator. The Wilson coefficient $C_{7\gamma}$ is calculated explicitly by the matching condition of full and effective field theory, which then will be implemented to physics Beyond Standard Model (BSM). In theories BSM, there are various types of extensions to SM. We are specifically focused on adding Vector-Like particle to the SM. The motivation behind this theory is the anomaly cancellation by itself. These Vector-like particle are called Standard Vector-Like particle because they transform like SM fermions. Furthermore these Standard Vector-Like particle and Gauge singlet can replace the SM particle to calculate $C_{7\gamma}$ Wilson coefficient in $b \to s\gamma$ can probe new physics beyond SM.

Chapter 2

Standard Model

The SM [2] consists of electroweak and strong interactions. It is a successful model since the rise of 1960's and 1970's. One of the famous characteristics of the SM is the interaction of weak neutral current discovered in the "Gargamelle Neutrino experiment" in 1973 that is considered to be the first big achievement of the theory. The processes used in the experiment was $\nu_\mu/\nu_{\mu^-} + N \rightarrow \nu_\mu/\nu_{\mu^-} + hadrons$ (neutral current) and $\nu_\mu/\nu_{\mu^-} + N \rightarrow \mu^-/\mu^+ + hadrons$ (charged current). In concert with the data collected from these low energy experiments and similar processes in the 1970's, the SM was capable to predict the Vector-boson $W^\pm$ and Z masses. The first experiment in 1983 at CERN [3] directly produced the W and Z bosons. The measured mass was analyzed and it was in agreement with the SM predictions. A few years latter LEP measured the Z mass much more accurately. These experiments also probed the theory at loop level. One success story is of the top quark of SM, this quark was compulsory as the weak isospin partner of bottom quark. In 1995 the collider detector at Fermilab (CDF) was directly observed.

Despite the Successful theory of SM, it has some problems and deficiencies:

- Gravity is missing from SM.

- Hierarchy problem: why electroweak scale is so small?
- The problem of strong CP violation.
- There is lack of explanation for the quark masses according to their ranges i.e. few MeV to 100 GeV and lepton masses i.e. 0.5 MeV to 1.8 GeV.
- The SM shows the neutrinos are massless, in fact experiment shows that neutrinos have mass.

2.1 Gauge Theory

SM corresponds to a non-abelian gauge principle [4], it is a quantum field theory based upon local gauge invariance. Gauge principle provides a tool to transform Lagrangian that is invariant w.r.t global symmetry transformation of non-abelian symmetric $SU(N)$ group into a Lagrangian that consists of a local symmetry invariance. Suppose $\mathcal{L}(\psi(x), \partial_\mu\psi(x))$ is a Lagrangian, invariant under $SU(N)$ global transformation

$$\psi(x) \to U\psi(x), \qquad U^{-1} = U^\dagger. \tag{2.1}$$

But our desire to develop a theory i.e. invariant with respect to local $SU(N)$ transformation

$$\psi(x) \to U(x)\psi(x), \qquad U = e^{i\alpha^a(x)T^a} \tag{2.2}$$

The Lagrangian is now no more invariant under this local transformation. To preserve the local invariance, we introduce the covariant derivative D_μ

$$D_\mu = \partial_\mu - igA_\mu^a T^a \tag{2.3}$$

transform as

$$D_\mu\psi(x) \to (D_\mu\psi(x))' = U(x)(D_\mu\psi(x))$$

Where g is the arbitrary constant defined as coupling constant, A^a_μ defined as a vector fields/gauge fields and T^a are the corresponding generators that follow the commutation algebra

$$[T^a, T^b] = if^{abc}T^c$$

f^{abc} define as the structure constant. To restore gauge invariance, A_μ vector field transforms as

$$A^a_\mu \to A^{a'}_\mu = U(x)(A^a_\mu + \frac{i}{g}\partial_\mu)U^\dagger(x).$$

Finally, by adding the kinetic term for gauge field and introducing locally invariant term that depends on A_μ and its derivative, the field strength tensor $F^{\mu\nu}$ looks like

$$F^{\mu\nu,a} = \partial^\mu A^{\nu,a} - \partial^\nu A^{\mu,a} + gf^{abc}A^{\mu,b}A^{\nu,c}.$$

The product of $F^{\mu\nu,a}F^a_{\nu\mu}$ satisfies the structure of gauge theory and appears into the Lagrangian.

The new locally invariant Lagrangian takes the following form

$$\mathcal{L} = \mathcal{L}(\psi(x), D_\mu\psi(x)) - \frac{1}{4}F^{\mu\nu}F_{\nu\mu}. \tag{2.4}$$

The Gauge theory principle extended a global to local symmetry and it gives an information about gauge field interaction with itself via kinetic term and with matter fields via covariant derivative. As a conclusion it not only determines the symmetry but also gives an information about dynamics.

2.2 Renormalizability

The SM is a renormalizable QFT. Renormalizable QFT are theories in which divergences appearing from loop calculations can be discarded by hiding them into redefinition or a renormalization into physical parameters. The renormalized theory has some limitations to the Lagrangian of the theory. Roughly saying, the renormalizable theory can be determined by physical parameter known as mass dimension of the Lagrangian. If it is renormalizable, the constant c defined as coupling constant in the lagrangian

$$\mathcal{L} \propto cO$$

c posses a positive mass dimension, where O is an operator.

The mass dimension of Lagrangian is defined by space time dimension d i.e. four, so only the possible operator products will remain having dimension $d \leq 4$ in the Lagrangian. Operators consisting of higher mass dimension are equal to the coupling constant with $-ve$ mass dimension, in such condition the theory is non-renormalizable. For a long time it was considered that only renormalizable theory can explain nature due to their high predictive analysis. Effective field theory (EFT) give rise to new understanding to renormalization. In EFT the non-renormalizability is not discarded, operator with mass > 4 has contribution. Simply their effect are suppressed by powers of the theory at fundamental scale, which are larger then the energies i.e. experimentally achievable. We can respect a renormalizability of the theory such as SM theory, as an EFT that discards all the non-renormalizable terms.

2.3 The Standard Model Lagrangian

The SM Lagrangian [5] consists of the following main pieces

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{fermions}} + \mathcal{L}_{\text{higgs}} + \mathcal{L}_{\text{yukawa}} \tag{2.5}$$

$\mathcal{L}_{\text{gauge}}, \mathcal{L}_{\text{fermions}}, \mathcal{L}_{\text{higgs}}$ and $\mathcal{L}_{\text{yukawa}}$ terms correspond to the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, the matter contents of fermions, the Higgs sector and the coupling of Higgs with fermion of SM respectively.

2.3.1 Gauge Symmetry Group

The SM Lagrangian [6,7] is based on gauge symmetry group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. The $SU(3)_c$ color symmetry group explains the strong interaction between quarks corresponding to quantum chromodynamic (QCD) part. The $SU(2)_L \otimes U(1)_Y$ gauge group explains the Glashow-Weinberg-Salam electroweak interaction theory. The gauge terms Lagrangian is as follows

$$\mathcal{L}_{gauge} = -\frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i,\mu\nu} - \frac{1}{4}G^a_{\mu\nu}G^{a,\mu\nu} \tag{2.6}$$

The field strength tensor defined as

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ W^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \epsilon^{ijk} W^j_\mu W^k_\nu \\ G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_3 f^{abc} G^b_\mu G^c_\nu \end{aligned}$$

Where $W_\mu^i(i = 1, 2, 3)$ and $G_\mu^a(a = 1, ..., 8)$, and the corresponding covariant derivatives are

$$\begin{aligned} D_\mu &= \partial_\mu - ig_1(Y)B_\mu; \\ D_\mu &= \partial_\mu - ig_2(\frac{\tau^i}{2}W_\mu^i); \\ D_\mu &= \partial_\mu - ig_3(\frac{\lambda^a}{2}G_\mu^a); \end{aligned}$$

Table 2.1: Boson of the standard model

Boson	Tensor	Coupling constant	Physical sate	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
B_μ	$B_{\mu\nu}$	$g_1 = e$	photon, Z	(1,1,0)
W_μ^i	$W_{\mu\nu}^i$	g_2	W^+, W^-, photon	(1,3,0)
G_μ^a	$G_{\mu\nu}^a$	g_3	gluons	(8,1,0)

2.3.2 Fermionic Field in SM

Fermions are categorized in three generations. A charged lepton, neutrino and up and down type quarks belong to each generation. Furthermore, they are split into left and right handed fermions. Left handed fermions are doublet under $SU(2)_L$ while right handed are singlet under $SU(2)_L$ as shown in Table 2.2.

The fermionic field of SM explained by Dirac Lagrangian

$$\mathcal{L} = \bar{\Psi} i\gamma_\mu \partial^\mu \Psi - m\bar{\Psi}\Psi$$

as

$$\Psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix},$$

where as ψ_L and ψ_R are left and right handed Spinors respectively.

The Gell-Mann-Nishijima formula is defined as

$$Q = I_3 + \frac{Y}{2}$$

The Q, I_3 and Y denotes the charge, isospin and hypercharge respectively

The fermionic field Lagrangian is written as

$$\mathcal{L}_{fermion} = i\bar{l}_L \not{D}_L l_L + i\bar{q}_L \not{D}_Q q_L + i\bar{e}_R \not{D}_e e_R + i\bar{u}_R \not{D}_u u_R + i\bar{d}_R \not{D}_d d_R \tag{2.7}$$

Where $\not{D} = \gamma^\mu D_\mu$

$$\begin{aligned} D_l^\mu &= \partial_\mu - ig_1 Y_l B^\mu - ig_2 \sigma^i W^{i,\mu} \\ D_{q_L}^\mu &= \partial_\mu - ig_1 Y_{q_L} B^\mu - ig_2 \sigma^i W^{i,\mu} - ig_3 t^a G^{a,\mu} \\ D_e^\mu &= \partial_\mu - ig_1 Y_e B^\mu \\ D_{q_R}^\mu &= \partial_\mu - ig_1 Y_q B^\mu - ig_3 t^a G^{a,\mu} \qquad q_R = u_R, d_R \end{aligned}$$

Here $\sigma^i = \frac{\tau^i}{2}$ belongs to Pauli matrices are generator of $SU(2)$, $t^a = \frac{\lambda^a}{2}$ belongs to Gell-Mann matrices are generator of $SU(3)$.

So

$$\begin{aligned} \mathcal{L}_{\text{fermion}} &= i\bar{l}_L \gamma^\mu \big(\partial_\mu + ig_1 B_\mu(-\frac{1}{2}) + ig_2 W_\mu^i \frac{\tau^i}{2}\big) l_L \\ &+ i\bar{q}_L \gamma^\mu \big(\partial_\mu + ig_1 B_\mu(\frac{1}{6}) + ig_2 W_\mu^i \frac{\tau^i}{2} + ig_3 G_\mu^a \frac{\lambda^a}{2}\big) q_L \\ &+ i\bar{e}_R \gamma^\mu \big(\partial_\mu + ig_1 B_\mu(-\frac{2}{2})\big) e_R \\ &+ i\bar{u}_R \gamma^\mu \big(\partial_\mu + ig_1 B_\mu(\frac{2}{3}) + ig_3 G_\mu^a \frac{\lambda^a}{2}\big) u_R \\ &+ i\bar{d}_R \gamma^\mu \big(\partial_\mu + ig_1 B_\mu(-\frac{1}{3}) + ig_3 G_\mu^a \frac{\lambda^a}{2}\big) d_R \end{aligned}$$

Table 2.2: Fermion of the standard model

Notation	I_3	Y	Q	Contents	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
l_L	$\binom{1/2}{-1/2}$	-1	$\binom{0}{-1}$	$\binom{\nu_{eL}}{e_L}\binom{\nu_{\mu L}}{\mu_L}\binom{\nu_{\tau L}}{\tau_L}$	$(1, 2, \frac{-1}{2})$
q_L	$\binom{1/2}{-1/2}$	$\frac{1}{3}$	$\binom{2/3}{-1/3}$	$\binom{u_L}{d_L}\binom{c_L}{s_L}\binom{t_L}{b_L}$	$(3, 2, \frac{1}{6})$
e_R	0	-2	-1	$e_R \quad \mu_R \quad \tau_R$	$(1, 1, 1)$
u_R	0	$\frac{4}{3}$	$\frac{2}{3}$	$u_R \quad c_R \quad t_R$	$(\bar{3}, 1, \frac{-2}{3})$
d_R	0	$\frac{-2}{3}$	$\frac{-1}{3}$	$d_R \quad s_R \quad b_R$	$(\bar{3}, 1, \frac{1}{3})$

Charged Current

According to weak interaction theory, the weak interactions only exist on left quark's and lepton's doublet.

$$\begin{aligned}\mathcal{L}_{\text{fermions}} &= i(\bar{u}_L, \bar{d}_L)\gamma_\mu(\partial^\mu - \frac{1}{2}igW_i^\mu\tau_i)\begin{pmatrix}u_L\\ d_L\end{pmatrix}\\ &= i\bar{u}_L\gamma_\mu\partial^\mu u_L + i\bar{d}_L\gamma_\mu\partial^\mu d_L - \frac{1}{2}g\bar{u}_L\gamma_\mu W^{-\mu}d_L - \frac{1}{2}g\bar{d}_L\gamma_\mu W^{+\mu}u_L\end{aligned}$$

The pauli matrices $(i = 1, 2)$ are used. $W^\pm$ gauge boson are responsible for flavor changing from up to down and down to up as well. These kind of interactions are called charge current [8].

$$\mathcal{L}_{CC} = -\frac{1}{2}g\bar{u}_L\gamma_\mu W^{-\mu}d_L - \frac{1}{2}g\bar{d}_L\gamma_\mu W^{+\mu}u_L \tag{2.8}$$

Figure(2.1) shows an example of Feynman diagram of muon decay mediated by the W boson.

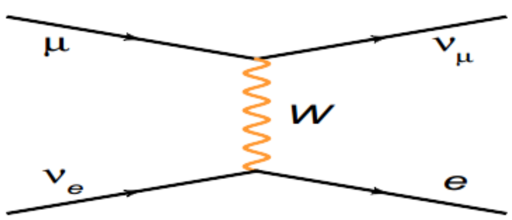

Figure 2.1: Muon decay

2.3.3 Higgs Lagrangian

The Higgs sector be explained in more detail in sec. 2.4 by introducing a new complex scalar doublet ϕ.

$$\phi = \begin{pmatrix} \phi_+ \\ \phi_0 \end{pmatrix}$$

it transform as

$$\begin{pmatrix} SU(3)_c & SU(2)_L & U(1)_Y \\ 1 & 2 & \frac{1}{2} \end{pmatrix}$$

The scalar doublet embedded in the Lagrangian as

$$\mathcal{L}_{Higgs} = |\big(\partial_\mu + ig_1 B_\mu(\frac{1}{2}) + ig_2 W^i_\mu \frac{\tau^i}{2}\big)\phi|^2 - \frac{m^2}{2}|\phi|^2 - \frac{\lambda}{4}|\phi|^4 \tag{2.9}$$

2.3.4 Higgs and Yukawa Terms

The dynamic of a spin-0 scalar field can be explained through Higgs part.

$$\mathcal{L}_{higgs} = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$$

The potential is

$$V(\phi) = m^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$

Where ϕ is a field defined as an isospin doublet

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \tag{2.10}$$

This field ϕ couples the Higgs boson with the fermion fields using Yukawa coupling. We can further expand the lagrangian by the coupling between the fermion doublets and field ϕ to introduce mass terms for the fermions. This gives rise to the new terms, known as Yukawa interactions, preserved by symmetries. The Yukawa terms Lagrangian is given as

$$\begin{aligned} \mathcal{L}_{yukawa} &= \bar{\psi}_L Y \phi \psi_R + h.c \\ \mathcal{L}_{yukawa} &= Y_u \bar{q}_L \phi u_R + Y_d \bar{q}_L \tilde{\phi} d_R + Y_L \bar{l}_L \tilde{\phi} e_R + h.c \end{aligned} \tag{2.11}$$

Q_L and L_L are defined as left handed quarks and leptons respectively.

$$l_L = P_L \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \qquad q_L = P_L \begin{pmatrix} u \\ d \end{pmatrix}$$

u_R, d_R and e_R are right handed up-type, down-type quarks and lepton respectively.

$u_R = P_R u, \qquad d_R = P_R d, \qquad e_R = P_R e$

where

$$P_L = \frac{(1-\gamma_5)}{2}, \qquad P_R = \frac{(1+\gamma_5)}{2}$$

Y_u, Y_d,and Y_L are Yukawa couplings for up-type, down-type quarks and lepton respectively. The Yukawa coupling Y_q where($q = u, d, l$) are 3×3 matrices. Local symmetry breaking can be achieved by substituting various value for ϕ field in Eq(2.10).

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \longrightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ V + h(x) \end{pmatrix} \tag{2.12}$$

The vacuum expectation value (VEV) will be explain in more detail in sec. 2.4 is not zero and expected at $\frac{V}{\sqrt{2}}$, where $h(x)$ is a perturbation around new VEV represented as the Higgs boson. The Yukawa terms in $\mathcal{L}$(2.11) will be

$$\mathcal{L}_{yukawa} = \frac{V}{\sqrt{2}}\bar{u}_L Y_u u_R + \frac{V}{\sqrt{2}}\bar{d}_L Y_d d_R + \frac{V}{\sqrt{2}}\bar{e}_L Y_L e_R + h.c \tag{2.13}$$

2.4 Spontaneous Symmetry Breaking

2.4.1 The ϕ^4 Theory

Symmetries play an important role in Physics, Noether theorem states that differential symmetry conforms to a conserved quantity. When a symmetric system having a symmetry group goes into vacuum state and if it does not remain symmetric and the vacuum expectation value is not zero any more than the symmetry is spontaneously broken.

The ϕ^4 theory is the wide-eyed example of SSB [9, 10]. The Lagrangian looks like

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{4}\lambda\phi^4 \tag{2.14}$$

The potential $V(\phi) = \frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4$, where ϕ is a real scalar field. taking minima of the potential

$$0 = \frac{dV(\phi)}{d\phi} \Longrightarrow \phi(m^2 + \lambda\phi^2) = 0$$

Two conditions for the Vacuum expectation value(VEV) arise

$<\phi>_0 = 0$ for $m^2 > 0$ which is trivial and $<\phi>_0 = \pm\sqrt{\frac{-m^2}{\lambda}} = \pm V$. For $m^2 < 0$, we have two minima for this case as shown in figure(2.2). Redefining the field having fluctuation described by $\phi(x)$. $\eta(x) = V + \eta(x)$. The new Lagrangian looks like

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}[\partial_\mu(\eta+V)]^2 - \frac{1}{2}(-\lambda V^2)(\eta+V)^2 - \frac{1}{4}\lambda(\eta+V)^4 \\ \partial_\mu V = 0 \longrightarrow \mathcal{L} &= [\frac{1}{2}(\partial_\mu\eta)^2 - \frac{1}{2}(-2m^2)\eta^2 - \frac{1}{4}\lambda\eta^4] - \lambda V\eta^3 + \frac{1}{4}\lambda\eta^4 \end{aligned} \tag{2.15}$$

The first term looks same as the original Lagrangian with a new mass $m = \sqrt{-2m^2}$. The last term is constant which is not relevant and the second last term that contains η^3 is the strong candidate for SSB since $V(\eta)$ is not symmetric any more for a transformation $\eta \rightarrow -\eta$.

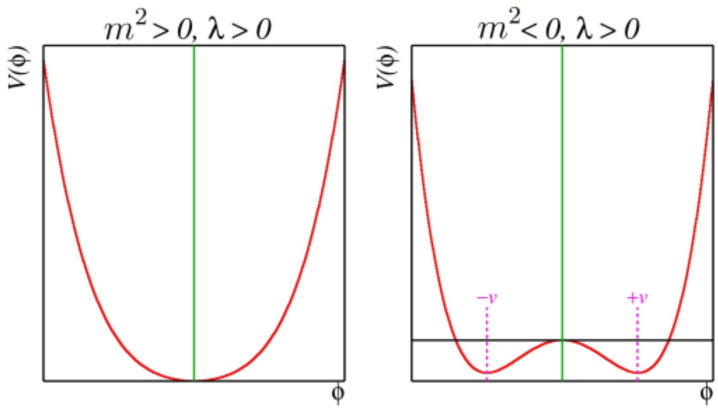

Figure 2.2: Potential of ϕ^4
theory when $m^2 > 0$ and $m^2 < 0$.

2.4.2 Goldstone's Boson(Theorem)

According to SSB, one observes the phenomena of massless scalar bosons known as Goldstone's boson theorem. To analyse this phenomena we take two fields σ and π within Lagrangian(2.14)

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu\sigma)^2 + (\partial_\mu\pi)^2 - m^2(\sigma^2+\pi^2)] - \frac{1}{4}\lambda(\sigma^2+\pi^2)^2 \tag{2.16}$$

The potential is $V(\sigma,\pi) = \frac{1}{2}m^2(\sigma^2+\pi^2) + \frac{1}{4}\lambda(\sigma^2+\pi^2)^2$. Now the Lagrangian has dis-

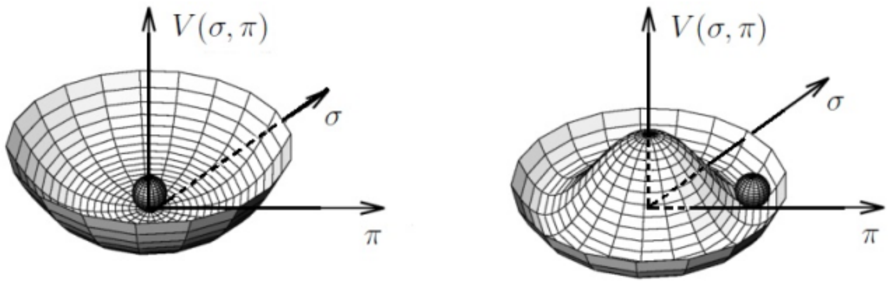

Figure 2.3: Potential of σ, π
field when $m^2 > 0$ and $m^2 < 0$ [11].

crete symmetry $(\sigma, \pi) \longrightarrow (-\sigma, \pi)$ and continuous symmetry $(\sigma, \pi) \longrightarrow R(-\sigma, \pi)$ as $R = \begin{pmatrix} \cos\gamma & -\sin\gamma \\ \sin\gamma & \cos\gamma \end{pmatrix} \in SO(2)$ Now let's check $\frac{dV}{d\pi} = 0$ and $\frac{dV}{d\sigma} = 0$ as a result
$\pi[m^2 + \lambda(\sigma^2 + \pi^2)] = 0$ and $\sigma[m^2 + \lambda(\sigma^2 + \pi^2)] = 0$
as shown in Fig. 2.3 there are two solutions $m^2 > 0$ and $m^2 < 0$ for $m^2 > 0$ is trivial $< \sigma >_0 = 0$ and $< \pi >_0 = 0$ for $m^2 < 0$ is non trivial$< \sigma^2 + \pi^2 >_0 = (\frac{-m^2}{\lambda})^{\frac{1}{2}}$. The transformation is $\pi \longrightarrow \pi$ and $\sigma \longrightarrow \eta - V$ the new Lagrangian looks like.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \eta)^2 + \frac{1}{2}(\partial_\mu \pi)^2 - \frac{1}{2}m^2\eta^2 - \lambda[\frac{1}{4}(\eta^2 + \pi^2) + V\eta\pi^2 + V\eta^3] + \frac{1}{4}\lambda V^4 \tag{2.17}$$

This looks like ϕ^4 theory in which η field gets mass $m_\eta = \sqrt{-2m^2}$. π field has no mass term. In this model the continous symmetry is broken spontaneosly. The most important consequence is π field gives a massless scalar boson known as Goldstone-boson.

2.4.3 Higgs Mechanism

Higgs mechanism [12] is an interesting phenomena that explains how to give masses to gauge bosons and fermions in the Standard Model(SM). Higgs mechanism is utilized to get rid of the Goldstone theorem. The condition holds that the Lagrangian will be invariant under local transformation.

$$\phi(x) \longrightarrow \phi\prime(x) = e^{ig\alpha(x)}\phi(x), \qquad \phi*(x) \longrightarrow \phi\prime*(x) = e^{-ig\alpha(x)}\phi(x)$$

The Lagrangian is

$$\mathcal{L} = (D_\mu\phi)^\dagger(D^\mu\phi) + m^2\phi^\dagger\phi - \lambda(\phi^\dagger\phi)^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.18}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, ϕ is a complex scalar field and A_μ is defined as massless gauge boson field, m and $\lambda > 0$ are real parameters obtained using the same method of $\mathcal{L}(2.14)$ which is already discussed in detailed. Replacing ∂_μ by Covariant derivative

$$\partial_\mu\phi \longrightarrow D_\mu\phi, \qquad \partial_\mu\phi^\dagger \longrightarrow (\mathcal{D}_\mu\phi)^\dagger$$

where,

$$\mathcal{D}_\mu = \partial_\mu + igA_\mu$$

and

$$A_\mu \longrightarrow A_\mu - \partial_\mu\alpha.$$

Considering $\alpha(x) = \frac{\eta(x)}{V}$, the gauge transform as

$$\begin{aligned} \phi &\longrightarrow \phi\prime = e^{ig\frac{\eta}{V}}\phi \\ A_\mu &\longrightarrow A_\mu - \partial_\mu\eta \end{aligned}$$

Appling these transformation the Lagrangian (2.18) is invariant. By substituting $\phi(x) = \frac{V+h(x)}{\sqrt{2}}$ in Eq(2.18), we obtain

$$\mathcal{L} = \frac{1}{2}[(\partial_\mu - igA_\mu)(V+h)(\partial^\mu + igA^\mu)(V+h)] + \frac{1}{2}m^2(V+h)^2 - \frac{1}{4}\lambda(V+h)^4 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2.19}$$

The interaction terms in the Lagrangian(2.19) are h^3, h^4, hAA and h^2AA. The quadratic terms in the Lagrangian correspond to the mass terms i-e ($\frac{g^2V^2}{2}A_\mu A^\mu$) and ($-\lambda V h^2$) that refer to the gauge boson and scalar boson mass respectively. The gauge boson A_μ eats up the Goldstone boson and gives it a mass. The complex scalar field ϕ and massless gauge boson is converted to a real scalar field and massive gauge boson respectively. The massive gauge boson(physical boson) is called a Higgs boson and the phenomena through which it gives mass to the gauge boson is known as Higgs mechanism.

2.5 CKM matrix and Fermion masses

The masses of gauge boson $W^\pm$ and Z gets through the SSB of the gauge group $SU(2)_L \otimes U(1)_Y$. How can one generate the missing masses of fermions? we desperately required a term that couple the fermions with Higgs doublet. They must be gauge invariant and renormalizable. These terms are called Yukawa terms in the Lagrangian. The Lagrangian for the charge lepton corresponding to first generation is

$$\mathcal{L}^{Leptons}_{Yukawa,1} = -Y_e \bar{e}' \phi^\dagger \begin{pmatrix} e \\ \nu_e \end{pmatrix}'_L + h.c. \tag{2.20}$$

For the three generations, the Lagrangian is written in the generalized form as

$$\mathcal{L}_{Yukawa}^{Leptons} = -(\bar{e}'_R \quad \bar{\mu}'_R \quad \bar{\tau})Y_l \begin{pmatrix} \phi^\dagger \begin{pmatrix} e \\ \nu_e \end{pmatrix}'_L \\ \phi^\dagger \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}'_L \\ \phi^\dagger \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L \end{pmatrix} + h.c. \tag{2.21}$$

According to Eq.(2.12) after giving VEV the $\mathcal{L}_{Yukawa}^{Leptons}$ splits into two parts. One part explains the interaction of leptons with physical Higgs and other part is explained by

$$\mathcal{L}_{Mass}^{Leptons} = -(\bar{e}'_R \quad \bar{\mu}'_R \quad \bar{\tau})M_l \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}'_L \tag{2.22}$$

where,

$$M_l = \frac{V}{\sqrt{2}} Y_l$$

In principle M_l is an arbitrary 3 complex matrix and cannot be named as mass matrix. However the charge lepton fields are possible to transform in such fashion that M_l is defined as diagonal matrix with positive real or zero number elements. The Lagrangian derived by applying this type of transformation to all of its term will latter be expressed as the mass eigenstate of the leptons. The new Lagrangian of charge current carries flavor mixing term. All lepton fields now taking place are mass eigenstate, for distiction we use without prime notation.

To analyze the quarks masses d, s and b are the down-type quark masses, the Yukawa

Lagrangian is same as the one in Eq.(2.21) with Y_q^d Yukawa matrix. The up-type quark is a bit different, we replace ϕ with $i\sigma_2\phi^*$ as the $SU(2)_L$ doublet. Where σ_2 is the pauli matrix

$$\mathcal{L}_{Yukawa}^{U-quarks} = -(\bar{u}'_R \quad \bar{c}'_R \quad \bar{t}'_R) Y_q^u \begin{pmatrix} i\sigma_2\phi^* \begin{pmatrix} u \\ d \end{pmatrix}_L' \\ i\sigma_2\phi^* \begin{pmatrix} c \\ s \end{pmatrix}_L' \\ i\sigma_2\phi^* \begin{pmatrix} t \\ b \end{pmatrix}_L \end{pmatrix} + h.c. \tag{2.23}$$

The Yukawa matrices are diagonalized by using the unitary transformation of the quark fields explicity it is given as below

$$\begin{pmatrix} u \\ c \\ t \end{pmatrix}_L' = V_u \begin{pmatrix} u \\ c \\ t \end{pmatrix}_L \quad , \quad \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R' = U_u \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R$$
$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L' = V_d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \quad , \quad \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R' = U_d \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R . \tag{2.24}$$

Where V_u, U_u, V_d, U_d belong to $U(3)$. In the lepton sector only one set exists like these matrices, which diagonalizes the yukawa matrices and that is the reason behind the Lagrangian having different mass eigenstates from the weak eigenstate. The quarks generation mix with each other defined by the CKM matrix known as Cabibbo-Kobayashi-Maskawa matrix.

$$V_{CKM} = V_u^\dagger V_d.$$

We can introduce these quarks coupling terms with $W^{\pm}$ bosons

$$\mathcal{L}_{CC}^{Quarks} = -\frac{e}{2\sin\theta_W}(W_\mu^+ J^{\mu,-} + W_\mu^- J^{\mu,+}) \tag{2.25}$$

where,

$$J^{\mu,-} = (\bar{u} \quad \bar{c} \quad \bar{t})_L V_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L \tag{2.26}$$

In $\mathcal{L}(2.8)$ for the Charge current will become

$$\begin{aligned} \mathcal{L}_{CC} &= -\frac{1}{2} g \bar{u}_L \gamma_\mu W^{-\mu} d_L - \frac{1}{2} g \bar{d}_L \gamma_\mu W^{+\mu} u_L \\ &= -\frac{1}{2} g \bar{u}_L V_L^{u\dagger} V_L^d \gamma_\mu W^{-\mu} d_L - \frac{1}{2} g \bar{d}_L V_L^{d\dagger} V_L^u \gamma_\mu W^{+\mu} u_L \end{aligned} \tag{2.27}$$

The $V_L^{u\dagger} V_L^d$ mattrix product consisting of off-diagonal terms causes the transition of coupling of quarks from one doublet to the other doublets involving weak transition and charged current. This phenomena is called quark mixing and d'_L defined for down type quarks consists of mixed quark mass states.

$$d'_L = \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_L^{u\dagger} V_L^d d_L = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{2.28}$$

For instance the first element d' is the superposition mass state of d, s and b that depends on V_{ud}, V_{us} and V_{ub}. The $V_L^{u\dagger} V_L^d$ matrix product is called Cabibbo-Kobayashi-Maskawa (CKM) matrix [13]. The components are calculated by experimental analysis [14].

$$\begin{pmatrix} |V_{ud}| \approx 0.974 & |V_{us}| \approx 0.25 & |V_{ub}| \approx 0.003 \\ |V_{cd}| \approx 0.225 & |V_{cs}| \approx 0.973 & |V_{cb}| \approx 0.04 \\ |V_{td}| \approx 0.009 & |V_{ts}| \approx 0.040 & |V_{tb}| \approx 0.999 \end{pmatrix} \tag{2.29}$$

2.5.1 Standard Parametrization

The CKM matrix describe in the standard parametrization [14]

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \tag{2.30}$$

$$= \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}s_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.31}$$

Where $c_{lm} = cos\theta_{lm}$, $s_{lm} = sin\theta_{lm} \quad (l, m = 1, 2, 3)$ and δ is the phase with the range $0 \leq \delta \leq 2\pi$. The four independent parameter are $s_{12} = |V_{us}|$, $s_{13} = |V_{ub}|$, $s_{23} = |V_{cb}|$ and δ. s_{12}, s_{13} and s_{23} are obtained by tree level decays mediated through transitions $s \longrightarrow u$, $b \longrightarrow u$ and $b \longrightarrow c$ respectively. The phase is obtained from loop level sensitive to $|V_{td}|$ through CP violating transition. The standard parametrization is suitable for numerical calculations.

Chapter 3

Higgs Decay Modes

To analyze the Higgs decay, first of all we have to know the complete information about its coupling at tree level with the massive particles of SM. In this chapter Higgs decay at tree level has been calculated that means to calculate fermion anti-fermion and weak boson channels and at loop level massless final state arises bosons such as gluons and photons [1].

3.1 Coupling of Higgs Boson

The coupling of Higgs is associated with the masses of fermions and bosons. The Higgs boson will preferably decay into heaviest particle allowed by phase space. The Higgs boson couples with gauge boson are $A = W, Z$ and the heavy fermions are $f = \tau, \mu, t, c, b$. Their masses [1] values are

$$\begin{aligned} m_W = 80.42\, GeV, \quad m_Z = 91.18\, GeV, \quad m_\tau = 1.777\, GeV, \\ m_\mu = 0.106\, GeV, \quad m_t = 178 \pm 4.3\, GeV, \\ m_b = 4.88 \pm 0.07\, GeV, \quad m_c = 1.64 \pm 0.07\, GeV. \end{aligned} \tag{3.1}$$

The Higgs boson interaction with gauge boson HAA calculated from Lagrangian(2.19) by considering quadratic term

$$g_{HAA} = \delta_v \frac{m_v^2}{v} \tag{3.2}$$

where $\delta_v = 2$ for W and $\delta_v = 1$ for Z boson

The Higgs boson couple to fermion is given by [15]

$$g_{Hf\bar{f}} = \frac{\sqrt{2}m_f}{v} \tag{3.3}$$

where $v = (\sqrt{2}G_F)^{\frac{-1}{2}} \simeq 246$ GeV

3.2 Higgs Decay to fermion antifermion

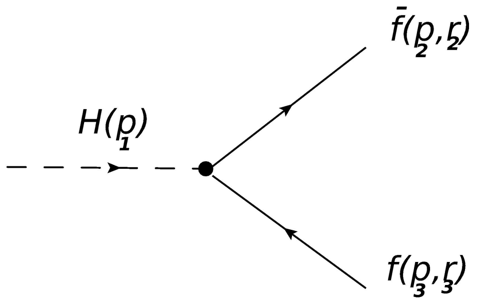

Figure 3.1: Higgs decay to fermion and anti-fermion

Higgs H particle coupling is directly proportional to particle of fermion mass and also the branching ratio is directly proportional to m_f^2. The partial decay width for $H \to f\bar{f}$ at tree level computation is as below

$H(p_1) \to f(p_2)f(\bar{p_3})$

The Transition Amplitude is

$$
\begin{aligned}
-i\mathcal{M}_{H\to f\bar{f}} &= \frac{im_f}{v}\bar{U}_{r2}V_{r3} \\
i\mathcal{M}^{\dagger}_{H\to f\bar{f}} &= \frac{(-i)m_f}{v}\bar{V}_{r3}U_{r2} \\
\sum_{r2,r3}|\mathcal{M}^2_{H\to f\bar{f}}| &= \sum_{r2,r3}\mathcal{M}\mathcal{M}^{\dagger} = \frac{(m_f)^2}{v^2}\sum_{r2,r3}\bar{V}_{r3}U_{r2}\bar{U}_{r2}V_{r3}
\end{aligned} \tag{3.4}
$$

After some algebraic steps we come across to the result

$$
\begin{aligned}
\sum_{r2,r3}|\mathcal{M}^2_{H\to f\bar{f}}| &= \frac{4m_f^2}{v^2}\frac{M_H^2}{2}(1-4\frac{m_f^2}{M_H^2}) \\
&= \frac{2m_f^2}{v^2}M_H^2(1-4\frac{m_f^2}{M_H^2})
\end{aligned} \tag{3.5}
$$

The decay width is defined by the formula

$$
\Gamma(H\to f\bar{f}) = \sum|\mathcal{M}_{H\to f\bar{f}}|^2 \quad \frac{1}{8\pi M_H^2}|\vec{p}| \tag{3.6}
$$

p is computed and plugged in(3.6) gives the final form

$$
\begin{aligned}
\Gamma(H\to f\bar{f}) &= \frac{2m_f^2}{v^2}M_H^2(1-4\frac{m_f^2}{M_H^2})\frac{1}{8\pi M_H^2}\frac{M_H}{2}(1-\frac{4m_f^2}{M_H^2})^{\frac{1}{2}} \\
&= \frac{M_H}{8\pi} \quad \frac{m_f^2}{V^2}(1-\frac{4m_f^2}{M_H^2})^{\frac{3}{2}}
\end{aligned} \tag{3.7}
$$

Defining the velocity of the final fermions

$$
\begin{aligned}
B_f = (1-\frac{4m_f^2}{M_H^2})^{\frac{1}{2}} \\
\Gamma(H\to f\bar{f}) = \frac{M_H}{8\pi} \quad \frac{m_f^2}{V^2} \quad B_f^3
\end{aligned} \tag{3.8}
$$

3.3 Higgs decay to Weak Boson

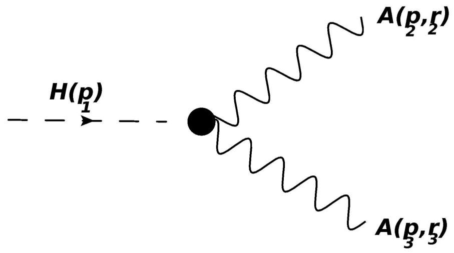

Figure 3.2: Higgs decay to vector bosons A

The Higgs decay to weak boson is defined in a general notation A, where $A = W, Z$. After calculation we plug in A with W and Z, where ZZ decay width has extra $\frac{1}{2}$ factor due to identical particles

The transition amplitude looks like

$$\begin{aligned} -i\mathcal{M} &= \epsilon_\mu(\vec{p}_2,\vec{r}_2)(\frac{2M_A^2}{V})g^{\mu\nu}\epsilon_\nu(\vec{p}_3,\vec{r}_3) \\ i\mathcal{M}^\dagger &= \epsilon^*_{\mu'}(\vec{p}_2,\vec{r}_2)(\frac{2M_A^2}{V})g^{\mu'\nu'}\epsilon^*_{\nu'}(\vec{p}_3,\vec{r}_3) \\ \sum_{r_i}|M_{H\to AA}|^2 &= \frac{4M_A^4}{V^2}g^{\mu\nu}g^{\mu'\nu'}\sum_{pol}\epsilon_\mu(\vec{p}_2,\vec{r}_2)\epsilon^*_{\mu'}(\vec{p}_2,\vec{r}_2)\sum_{pol}\epsilon_\nu(\vec{p}_3,\vec{r}_3)\epsilon^*_{\nu'}(\vec{p}_3,\vec{r}_3) \end{aligned} \tag{3.9}$$

After few steps we get

$$\sum_{r_i}|M_{H\to AA}|^2 = \frac{\cancel{4M_A^4}}{V^2}\left(\frac{M_H^2}{\cancel{4M_A^4}}\right)\left(1 - \frac{4M_A^2}{M_H^2} + \frac{12M_A^4}{M_H^4}\right) \tag{3.10}$$

For Decay as we know that

$$\Gamma(H \to AA) = \sum |M_{H\to AA}|^2 \frac{|\vec{p}|}{8\pi M_H^2}\ , \tag{3.11}$$

Where p is computed from relativistic relation $E_A^2 = p^2 + M_A^2$

$$\begin{aligned} \vec{p} &= \left(\frac{M_H^4}{4} - M_A^2\right)^{\frac{1}{2}} \\ |\vec{p}| &= \frac{M_H}{2}\left(1 - \frac{4M_A^2}{M_H^2}\right)^{\frac{1}{2}} \end{aligned} \tag{3.12}$$

Putting (3.10) and (3.12)into (3.11) as a Consequence

$$\begin{aligned} \Gamma(H \to AA) &= \frac{M_H^4}{V^2}\left(1 - \frac{4M_A^2}{M_H^2} + \frac{12M_A^4}{M_H^4}\right)\frac{1}{8\pi M_H^2}\left(\frac{M_H}{2}(1 - \frac{4M_A^2}{M_H^2})^{\frac{1}{2}}\right) \\ &= \frac{M_H^3}{16\pi V^2}\left(1 - \frac{4M_A^2}{M_H^2}\right)^{\frac{1}{2}}\left(1 - \frac{4M_A^2}{M_H^2} + \frac{12M_A^4}{M_H^4}\right) \end{aligned}$$

$$x \equiv \frac{M_A^2}{M_H^2}$$

$$\Gamma(H \to AA) = \frac{M_H^3}{16\pi V^2}(\sqrt{1-4x})(1 - 4x + 12x^2) \tag{3.13}$$

3.4 Higgs Decay to Gluons

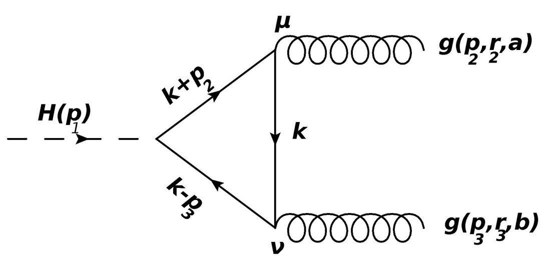

Figure 3.3: Higgs decay to gluons First diagram

$H(p_1) \to g(p_2)g(p_3)$

Higgs decay to gluons is a loop process. The decay rate is considered to be low as compared to tree level but it is not true exactly because due to heavy top quark mass this process generates high decay rate, therefore it must be taken into account.

The transition amplitude is

$$i\mathcal{M} = \frac{im_f}{v}\int\frac{d^4k}{(2\pi)^4}Tr[\frac{i((\not k+\not p_2)+m)}{(k+p_2)^2-m^2}(ig\gamma^\mu t^a)\frac{i(\not k+m)}{k^2-m^2}(ig\gamma^\nu t^b)\frac{i(\not k-\not p_3+m)}{(k-p_3)^2-m^2}] \times\epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3} \quad (3.14)$$

Trace calculation

$$Tr[((\not k+\not p_2)+m)\gamma^\mu(\not k+m)\gamma^\nu(\not k-\not p_3+m)] \quad (3.15)$$

using property of trace

$$\begin{aligned} Tr[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma] &= 4(g^{\mu\nu}g^{\rho\sigma}-g^{\mu\rho}g^{\nu\sigma}+g^{\mu\sigma}g^{\nu\rho}) \\ Tr[\gamma^\mu\gamma^\nu] &= 4g^{\mu\nu} \\ Tr[odd\gamma] &= 0 \end{aligned}$$

Result of Trace

$$\begin{aligned} &= 4m[4k^\mu k^\nu+2p_2^\mu k^\nu-2p_3^\nu k^\mu+p_2^\nu p_3^\mu-p_2^\mu p_3^\nu+g^{\mu\nu}(m^2-p_2\cdot p_3-k^2)] \\ Tr[t^at^b] &= \frac{1}{2}\delta_{ab} \end{aligned}$$

Calculating the Denominator by using Feynman parameterization

$$\frac{1}{A_1A_2....A_n} = \int_0^1 dx_1dx_2....dx_n \quad \frac{\delta(\sum x_i-1)(n-1)!}{[x_1A_1+.....+x_nA_n]^n} \quad (3.16)$$

$$\frac{1}{ABC} = \int_0^1 dxdydz \quad \frac{\delta(x+y+z-1)2!}{[xA+yB+zC]^3} \quad (3.17)$$

$$Denominator = [((k+p_2)^2-m^2)(k^2-m^2)((k-p_3)^2-m^2)]$$

Defining

$$\begin{aligned} A &= (k+p_2)^2 - m^2 \\ B &= (k-p_3)^2 - m^2 \\ C &= k^2 - m^2 \end{aligned}$$

by pluging these value in (3.17) as a result

$$= xp_2^2 + 2kp_2x + yp_3^2 - 2kp_3y + k^2 - m^2 \tag{3.18}$$

To get rid of these linear terms we shift momentum variables.

$$\ell = k + p_2x - p_3y \qquad \Rightarrow k = \ell - p_2x + p_3y$$

Replacing k as a result (3.18)

$$D = \ell^2 + 2p_2p_3xy - m^2 \quad = \ell^2 - \Delta$$

where

$$\Delta = -2p_2p_3xy + m^2$$

The Equation (3.14) looks like

$$\mathcal{M} = \frac{-im_f g^2}{v} \int \frac{d^4\ell}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} dy \quad \frac{4m \cdot 2N^{\mu\nu}}{[\ell^2 - \Delta]^3} \epsilon^a_{\mu,r_2} \epsilon^b_{\nu,r_3} \tag{3.19}$$

Evaluating the $N^{\mu\nu}$ by applying the shift of momentum variables

$$\ell = k + p_2x - p_3y \qquad \Rightarrow k = \ell - p_2x + p_3y$$

The new $N^{\mu\nu}$ looks like

$$\begin{aligned} N^{\mu\nu} &= 4\ell^\mu\ell^\nu - g^{\mu\nu}\ell^2 + p_2^\mu p_2^\nu(-2x+4x^2) + p_3^\mu p_3^\nu(-2y+4y^2) + p_2^\mu p_3^\nu(-4xy+2y-1+2x) \\ &\quad + p_2^\nu p_3^\mu(1-4yx) + g^{\mu\nu}(m^2 - p_2p_3 + 2p_2p_3xy) \end{aligned} \tag{3.20}$$

The condition for "onshell gluons " is $\epsilon_\mu p^\mu = 0$

Applying $\epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}$ to (3.20)

$$\begin{aligned} \epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}N^{\mu\nu} &= 4\epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}\ell^\mu\ell^\nu - \epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}g^{\mu\nu}\ell^2 + \epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}p_2^\mu p_2^\nu(-2x+4x^2) + \epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}p_3^\mu p_3^\nu(-2y+4y^2) \\ &\quad + \epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}p_2^\mu p_3^\nu(-4xy+2y-1+2x) + \epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}p_2^\nu p_3^\mu(1-4yx) \\ &\quad + \epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}g^{\mu\nu}(m^2 - p_2p_3 + 2p_2p_3xy) \end{aligned}$$

Plugging in (3.19)

$$\mathcal{M} = \frac{-im_f g^2}{v}\int \frac{d^4\ell}{(2\pi)^4}\int_0^1 dx \int_0^{1-x} dy \quad \frac{4m \cdot 2N^{\mu\nu}}{[\ell^2-\Delta]^3}\epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3} \tag{3.21}$$

Now

$$I \equiv \int \frac{d^4\ell}{(2\pi)^4}\int_0^1 dx \int_0^{1-x} dy \frac{N^{\mu\nu}}{[\ell^2-\Delta]^3}$$

Integral solving into two parts

$$I = \int_0^1 dx \int_0^{1-x} dy(I_1 + I_2) \tag{3.22}$$

$$I_1 \equiv \int \frac{d^4\ell}{(2\pi)^4}\frac{4\ell^\mu\ell^\nu - g^{\mu\nu}\ell^2}{[\ell^2-\Delta]^3} \tag{3.23}$$

put $C = p_2^\nu p_3^\mu(1-4yx) + g^{\mu\nu}(m^2 - p_2p_3 + 2p_2p_3xy)$

$$I_2 \equiv \int \frac{d^4\ell}{(2\pi)^4}\frac{C}{[\ell^2-\Delta]^3} \tag{3.24}$$

$$I_1 = \int \frac{d^4\ell}{(2\pi)^4} \frac{4\ell^\mu \ell^\nu - g^{\mu\nu}\ell^2}{[\ell^2 - \Delta]^3} \tag{3.25}$$

$$\ell^\mu \ell^\nu \to \frac{1}{d}\ell^2 g^{\mu\nu}$$

$$= \int \frac{d^4\ell}{(2\pi)^4} \frac{4\ell^2 h^{\mu\nu}}{d} - g^{\mu\nu}\ell^2$$

$$= \int \frac{d^4\ell}{(2\pi)^4} (\frac{4}{d} - 1)\ell^2 g^{\mu\nu}$$

In numerator we have two terms $4\ell^\mu\ell^\nu - g^{\mu\nu}\ell^2$ having divergence. these terms can be simplified by dimensional regularization methods.

$$\int \frac{d^d\ell \quad \ell^2}{(2\pi)^d(\ell^2 - \Delta)^n} = \frac{(-1)^{n-1}i}{(4\pi)^{\frac{d}{2}}} \frac{d}{2} \frac{\Gamma((n - \frac{d}{2}) - 1)}{\Gamma(n)} (\frac{1}{\Delta})^{n-\frac{d}{2}-1}$$

$$\int \frac{d^d\ell}{(2\pi)^d} (\frac{4}{d} - 1) \frac{1}{(\ell^2 - \Delta)^n} \ell^2 g^{\mu\nu} = \frac{(-1)^{n-1}i}{(4\pi)^{\frac{d}{2}}} (\frac{4}{d} - 1) \frac{d}{2} \frac{\Gamma((n - \frac{d}{2}) - 1)}{\Gamma(n)} (\frac{1}{\Delta})^{n-\frac{d}{2}-1}$$

for $d = 4 - 2\epsilon$, and $n = 3$

$$= \frac{i\epsilon}{(4\pi)^2} g^{\mu\nu} \frac{\Gamma(\epsilon)}{\Gamma(3)} (\frac{4\pi}{\Delta})^\epsilon \tag{3.26}$$

As we know that $\Gamma(n) = (n - 1)!$

$\Gamma(3) = 2$

$\Gamma(x) = \frac{1}{x} - \gamma + O(x)$

$\Gamma(\epsilon) = \frac{1}{\epsilon} - \gamma + O(\epsilon)$

plugging in (3.26)

$$\begin{aligned}I_1 &= \frac{i\epsilon}{(4\pi)^2}g^{\mu\nu}(\frac{4\pi}{\Delta})^{\epsilon}\frac{1}{2}(\frac{1}{\epsilon}-\gamma+O(\epsilon)) \\ &= \frac{i}{(4\pi)^2}g^{\mu\nu}(\frac{4\pi}{\Delta})^{\epsilon}\frac{1}{2}\frac{\cancel{\epsilon}}{\cancel{\epsilon}}-\frac{\gamma\epsilon i g^{\mu\nu}}{(4\pi)^2}(\frac{4\pi}{\Delta})^{\epsilon}+O(\epsilon) \\ \epsilon \longrightarrow 0 & \\ &= \frac{i}{16\pi^2}\frac{g^{\mu\nu}}{2}-0 \\ &= \frac{ig^{\mu\nu}}{32\pi^2}\end{aligned}$$

Now let's compute I_2

$$I_2=\int\frac{d^4\ell}{(2\pi)^4}\frac{C}{[\ell^2-\Delta]^3}$$

From Peskin Appendix [10]

$$\int\frac{d^d\ell}{(2\pi)^d(\ell^2-\Delta)^n}=\frac{(-1)^n i}{(4\pi)^{\frac{d}{2}}}\frac{\Gamma(n-\frac{d}{2})}{\Gamma(n)}(\frac{1}{\Delta})^{n-\frac{d}{2}}$$

As $d=4$,$n=3$

$$\begin{aligned}I_2 &= \frac{-iC}{32\pi^2\Delta} \\ I_1+I_2 &= \frac{ig^{\mu\nu}}{32\pi^2}-\frac{iC}{32\pi^2\Delta} \\ &= \frac{ig^{\mu\nu}\Delta-iC}{32\pi^2\Delta}\end{aligned} \tag{3.27}$$

put $C=p_2^{\nu}p_3^{\mu}(1-4yx)+g^{\mu\nu}(m^2-p_2p_3+2p_2p_3xy)$

and $\Delta=-2p_2p_3xy+m^2$

$$I_1+I_2 = \frac{i}{32\pi^2}(\frac{(p_2^{\nu}p_3^{\mu}-g^{\mu\nu}p_2p_3)(1-4xy)}{-\Delta}) \tag{3.28}$$

So equation (3.22)⇒

$$I = \frac{i}{32\pi^2}\int_0^1 dx \int_0^{1-x} dy \left(\frac{(p_2^\nu p_3^\mu - g^{\mu\nu}p_2p_3)(1-4xy)}{-\Delta}\right)$$

Now $H = \int_0^1 dx \int_0^{1-x} dy \frac{(1-4xy)}{-\Delta}$

$$I^{\nu\mu} = \frac{i}{32\pi^2}H(p_2^\nu p_3^\mu - g^{\mu\nu}p_2p_3) \tag{3.29}$$

Equation (3.19)⇒ for first diagram

$$M = (-i)\frac{m}{v}g^2(4m)(\frac{1}{2}\delta_{ab})2\epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}I^{\nu\mu} \tag{3.30}$$

Second Diagram:

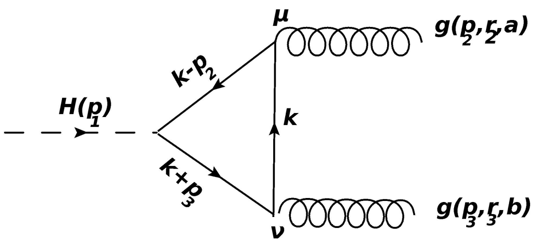

Figure 3.4: Higgs decay to gluons second diagram contribution

$$\mathcal{M} = \frac{(-i)g^2m}{v}\epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}(\frac{1}{2}\delta_{ab})\int\frac{d^4k}{(2\pi)^4}\frac{Tr[((\not{k}-\not{p}_2)+m)\gamma^\mu(\not{k}+m)\gamma^\nu((\not{k}+\not{p}_3)+m)]}{[(k-p_2)^2-m^2][k^2-m^2][(k+p_3)^2-m^2]} \tag{3.31}$$

The final result for the trace is

$$= 4m[4k^\mu k^\nu + 2p_2^\mu k^\nu - 2k^\mu p_3^\nu + g^{\mu\nu}(m^2 - p_2p_3 - k^2) + p_2^\nu p_3^\mu - p_2^\mu p_3^\nu] \tag{3.32}$$

As calculated separately the second diagram by using the Feynman parametrization giving the same result as for the first diagram for $I^{\mu\nu}$

$I^{\mu\nu} = \int \frac{d^4\ell}{(2\pi)^4}\int_0^1 dx \int_0^{1-x} dy \frac{N^{\mu\nu}}{[\ell^2-\Delta]^3}$

So it means that the amplitude for both the diagrams are equivalent

$\mathcal{M}$(first diagram)=$\mathcal{M}$(second diagram)

The total Amplitude is

$\mathcal{M}$=$\mathcal{M}$(first diagram)+$\mathcal{M}$(second diagram)

$\mathcal{M}$=$\mathcal{M}_1$+$\mathcal{M}_2$

$\Rightarrow \mathcal{M}_1$=$\mathcal{M}_2$

$\mathcal{M}$=2$\mathcal{M}_1$

$|\mathcal{M}|^2 = 4\mathcal{M}_1\mathcal{M}_1^\dagger$

$$\begin{aligned}
\mathcal{M} &= (-i)g^2(4m)(\frac{1}{2}\delta_{ab})\epsilon^a_{\mu,r_2}\epsilon^b_{\nu,r_3}I^{\nu\mu}\cdot 2 \\
\mathcal{M}^\dagger &= i\frac{m}{v}g^2(4m)(\frac{1}{2}\delta_{ab})\epsilon^{a*}_{\mu,r_2}\epsilon^{b*}_{\nu,r_3}I^{\nu\mu*}\cdot 2 \\
\sum_{r_2,r_3}|\mathcal{M}_{H\to gg}|^2 &= 4\sum_{r_2,r_3,a,b}(\frac{m}{v})^2(2g^2m)^2\delta_{ab}\delta_{ab}\epsilon^a_{\mu,r_2}\epsilon^{a*}_{\tau,r_2}\epsilon^b_{\nu,r_3}\epsilon^{b*}_{\rho,r_3}I^{\nu\mu}I^{\tau\rho*} \qquad (3.33)
\end{aligned}$$

The sum over spins and colors are

$$\begin{aligned}
\sum_{a,b}\delta_{ab}\delta_{ab} &= 8 \\
\sum_{r_2,r_3}\epsilon^{a*}_{\tau,r_2}\epsilon^a_{\mu,r_2}\epsilon^{b*}_{\rho,r_3}\epsilon^b_{\nu,r_3} &= g_{\tau\mu}g_{\rho\nu} \\
g_{\tau\mu}g_{\rho\nu}I^{\nu\mu}I^{\tau\rho*} &= I^{\nu\mu}I^*_{\nu\mu}
\end{aligned}$$

$$\sum_{r_2,r_3}|\mathcal{M}_{H\to gg}|^2 = 4\cdot 4g^4(\frac{m^2}{v^2})m^2 8\cdot 4I^{\nu\mu}I^*_{\nu\mu} \qquad (3.34)$$

Now computing,

$$\begin{aligned}
I^{\nu\mu}I^*_{\nu\mu} &= (\frac{i}{32\pi^2})H(p_2^\nu p_3^\mu - g^{\mu\nu}p_2p_3)\times(\frac{-i}{32\pi^2})H(p_{2\nu}p_{3\mu} - g_{\mu\nu}p_2p_3) \\
&= \frac{1}{512\pi^4}H^2(p_2\cdot p_3)^2 \qquad (3.35)
\end{aligned}$$

Plugging (3.34) into (3.35)

$$\sum |\mathcal{M}_{H\to gg}|^2 = \frac{512}{512} g^4 \frac{m^4}{v^2\pi^4} H^2 (p_2\cdot p_3)^2$$
$$= \frac{g^4 m^4 H^2 (p_2\cdot p_3)^2}{v^2\pi^4} \tag{3.36}$$

Now computing the integral H

$$H = \int_0^1 dx \int_0^{1-x} dy \frac{(1-4xy)}{-\Delta}$$
$$= \frac{1}{2p_2p_3} \int_0^1 dx \int_0^{1-x} dy \frac{(1-4xy)}{xy - \frac{m^2}{2p_2p_3}}$$
$$= \frac{1}{2p_2p_3} B(n) \tag{3.37}$$

$B(n) \equiv \int_0^1 dx \int_0^{1-x} dy \frac{(1-4xy)}{xy-\frac{m^2}{2p_2p_3}}$, $n \equiv \frac{m^2}{2p_2p_3}$

$$H = \frac{n}{m^2} B(n)$$
$$H^2 = \frac{n^2}{m^4} B^2(n) \tag{3.38}$$

Plugging into (3.36)

$$\sum |\mathcal{M}_{H\to gg}|^2 = \frac{g^4 m^4 (p_2\cdot p_3)^2}{v^2\pi^4} \frac{n^2}{m^4} |B(n)|^2 \tag{3.39}$$

Defining Kinematics (Center of mass frame).

$$p_1^\mu = (M_H, 0), \qquad p_2^\mu = (p, \vec{p}), \qquad p_3^\mu = (p, -\vec{p})$$

As a result

$$p_2 \cdot p_3 = \frac{1}{2} M_H^2 \tag{3.40}$$

plugging into (3.39)

$$\begin{aligned}\sum|\mathcal{M}_{H\to gg}|^2 &= \frac{g^4\cancel{m^4}(\frac{1}{2}M_H^2)^2}{v^2\pi^4}\frac{n^2}{\cancel{m^4}}|B(n)|^2 \\ &= \frac{g^4}{4v^2\pi^4}M_H^2 n^2|B(n)|^2\end{aligned}$$

$$\alpha = \frac{g^2}{4\pi} \qquad \Longrightarrow \alpha^2 = \frac{g^4}{16\pi^2}$$

$$= \frac{4\alpha^2}{\pi^2 v^2}M_H^4 n^2|B(n)|^2 \tag{3.41}$$

The formula for decay rate is

$$\Gamma(H\to gg) = \sum|\mathcal{M}_{H\to gg}|^2\frac{1}{8\pi M_H^2}|\vec{p}| \tag{3.42}$$

$|\vec{p}| = \frac{1}{2}M_H$

$$\begin{aligned}\Gamma(H\to gg) &= \frac{4\alpha^2}{\pi^2v^2}M_H^4 n^2|B(n)|^2\frac{1}{8\pi M_H^2}(\frac{1}{2}M_H) \\ &= \frac{2\alpha^2}{\pi^2v^2}\frac{M_H^3}{8\pi}n^2|B(n)|^2\end{aligned} \tag{3.43}$$

For identical particle decay we multiply by symmetry factor $\frac{1}{2}$

$$\Gamma(H\to gg) = \frac{\alpha_s^2}{\pi^2v^2}\frac{M_H^3}{8\pi}n^2|B(n)|^2 \tag{3.44}$$

$I^2 = n^2|B(n)|^2$

$\frac{1}{9}|I(\frac{1}{n})|^2 = \left(\frac{1}{3}B(\frac{m^2}{M_H^2})\right)^2 \qquad , \qquad v = \frac{2m_w}{g}$, $g = \frac{e}{sin\theta_w}$

$v = \frac{2m_w sin\theta_w}{e} \Longrightarrow v^2 = \frac{4m_w^2 sin^2\theta_w}{e^2}$

Putting these value in equation (3.44).

$$\Gamma(H \to gg) = (\frac{\alpha M_H}{8sin^2\theta_w})\frac{M_H^2}{m_w^2}\frac{\alpha_s^2}{9\pi^2}|\sum I(\frac{M_H^2}{m^2})|^2 \tag{3.45}$$

3.5 Higgs decay to Di-Photon

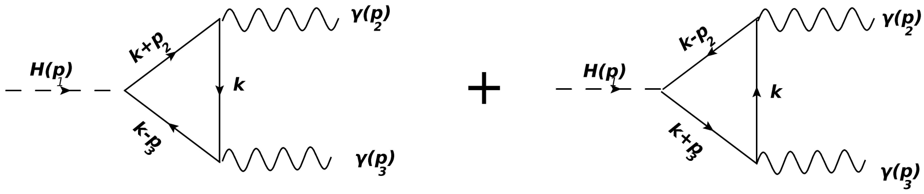

Figure 3.5: Higgs decay to photons

One of the Golden Channels for Higgs decay is "Higgs decay to Di-photon" as shown in Fig. 3.5 searched at CERN(LHC). This process has been examined many year ago [16]. As we know that photons are massless, therefore Higgs decay can not be calculated directly at tree level, the reason Higgs decay to di-photon is calculated at loop level. In the loop the top quark and W boson can contribute. The amplitude is calculated with the top quark loop. All of the calculation for the amplitude is same as done for gluons in sec. 3.4. Gluons are color charge so this factor is excluded. Only include a factor of electric charge Q_f with factor $N_c(f)$, substituting e with g_s in α term and sum over all charged fermions.
The final expression of decay looks like

$$\Gamma_{(H\to\gamma\gamma)} \quad = \quad \left(\frac{\alpha M_H}{8sin^2\theta_w}\right)\frac{M_H^2}{m_w}\frac{\alpha_s^2}{18\pi^2}|\sum_f \mathcal{Q}_f^2 N_c(f)|^2$$

3.6 Branching Ratio and Total Decay Width

Partial decay widths of quarks and leptons are compared. From Eq(3.8) $\Gamma(H \to f\bar{f})$ decay width depends upon mass term m_f^2 and $\Gamma(H \to A\bar{A})$ in Eq(3.13) depends upon mass term

m_A^3. As a consequence, the decay width of vector boson pairs is seen larger than fermion anti-fermion as shown in Fig(3.6), giving a general summary of all branching ratios.

The Fig(3.6) shows the branching ratio for Higgs boson relies on M_H(Higgs mass). The

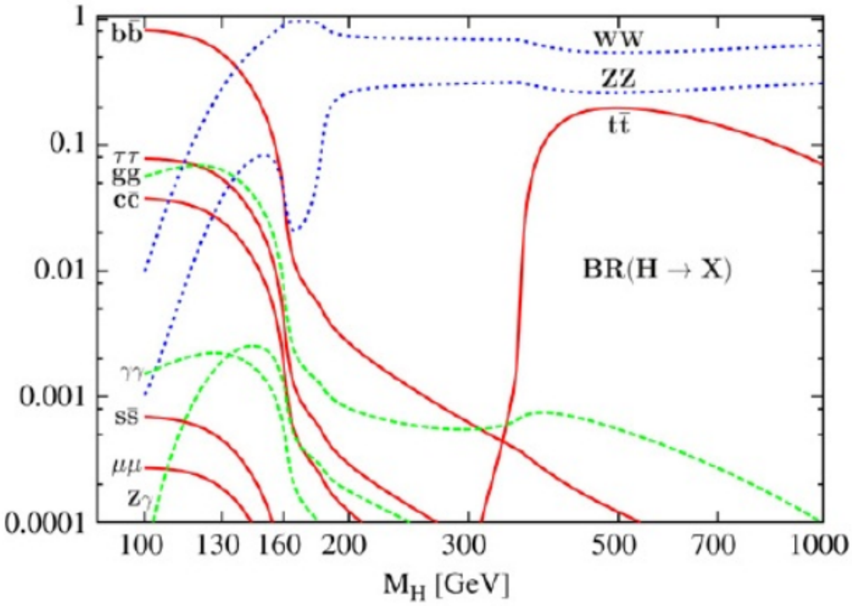

Figure 3.6: Higgs Boson Branching Ratio [1]

Higgs mass is divided into three parts as observed: a low mass, an intermediate mass and a high mass which have range $M_H < 130\,GeV$, $130\,GeV < M_H < 200\,GeV$ and $M_H > 200\,GeV$ respectively. The decay process $b\bar{b}$ pair is the most significant process for light Higgs boson having branching ratio of 75%. Subsequently $H \to \tau^+\tau^-$ and $H \to c\bar{c}$ with branching ratio $\sim 6\%\, and\, 2-3\%$ respectively. For a mass $M_H \sim 120\,GeV$ the $H \to gg$ is significant with BR$\sim$ 7%. The $H \to \gamma\gamma$ with branching ratio is only a few per mille but this channel has a significant contribution towards the Higgs search. The di-photon decay shows a clear signal in detector. $H \to Z\gamma\gamma$, $\mu\mu$ and $s\bar{s}$ are less than few per mille, which are somewhat relevent. In the low mass range the decay $H \to WW$, ZZ increases but both processes consist of at least one virtual vector boson, therefore the energy is below threshold and maximum energy of these decay is not attainable. The Higgs decay $H \to WW$ dominates when the energy is approaching to the threshold energy while the WW both becomes real. Specially in the mass range of $160\,GeV < M_H < 180\,GeV$ the WW real bosons are produced at this threshold energy with 100% approximate branching ratio. In the above mass range the $H \to b\bar{b}$ is the only non negligible decay process having a less contribution with BR $\sim 50\%$ at the starting of this mass range and drop to few percent at the end. In high mass range, $M_H > 200\,GeV$ the decay process are almost the Higgs decay into massive gauge bosons. The $H \to WW$ with BR$\sim 60\%$ and $H \to ZZ$ with BR$\sim 30\%$. The $H \to t\bar{t}$ channel opens at the $M_H \sim 300\,MeV$ but it is not significant before the threshold $M_H \sim 350\,Gev$(top quark threshold) with BR$\sim 10\%$ and it goes on decreasing for higher Higgs mass. Generally the Higgs have intensions to couple with heavier particles, so it is observable that below the WW threshold $M_H < 160\,GeV$, the $b\bar{b}$ is more attractive decay and for above $M_H > 160\,GeV$, the decay process of WW and ZZ are best-liked.

The total decay width Γ_H is shown in Fig(3.7). When the Higgs mass increases the total

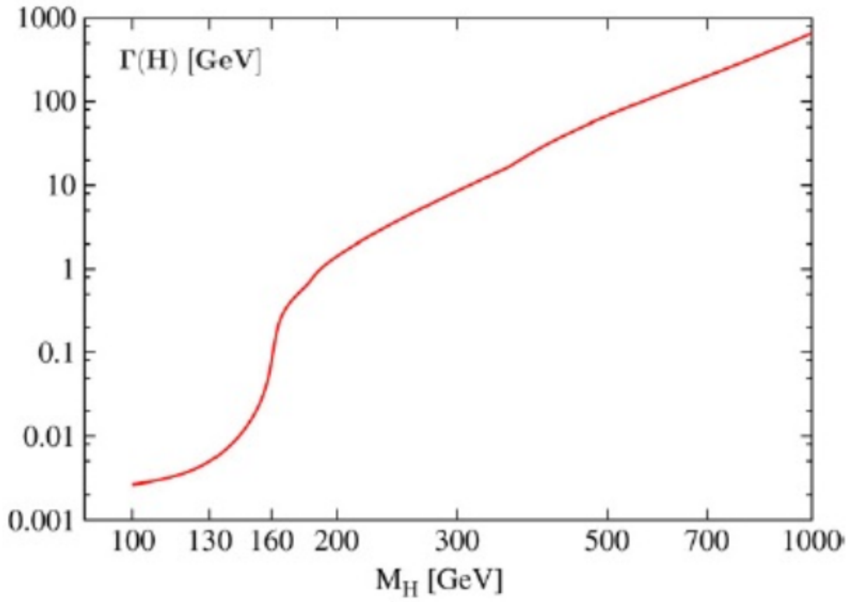

Figure 3.7: Higgs Boson Total decay width [1]

decay width also increases. It is narrow in low mass range, the $\Gamma_H < 1\,MeV$ and wide in intermediate mass range for $M_H \geq 500\,GeV$, Γ_H is almost equal to its mass, that is very uncommon. As in the case of W or Z boson in the detector we have a clear signal for Higgs mass particle, which is much higher than the decay width [17].

Chapter 4

Rare Radiative decay in the Standard Model ($b \to s\gamma$)

4.1 Theoretical frame work of B decay

In this chapter we will further explain the concept of Wilson coefficient. We will describe the effective field theory and basics of weak decays formalism [18]. and more about the calculation of $C_{7\gamma}$ Wilson coefficient.

4.1.1 Effective Field Theory

Effective Field Theory (EFT) [19,20] is derived from Quantum Field Theory (QFT), which is a technique to deal with multi-scale problems. Consider that QFT consists of characteristic energy scale K and lets assume that we are interested at some lower scale physics where $E \ll K$. To setup an EFT we take a cutoff μ below K scale and integrate out the heavy degree of freedom which means that to remove the heavier particles w.r.t the cutoff scale. The EFT contains light degree of freedom only and consider as limited case (low energy) of full theory. The effective Lagrangian is

$$\mathcal{L}_{eff} = \sum_{n\geq 0} C_n(\mu) O_n \tag{4.1}$$

The Lagrangian is an infinite sum over the operators O_n, where $C_n(\mu)$ is coupling constant known as Wilson coefficients. So one can ask about the predictability of this theory. The answer to above question is by substituting the coupling constant $C_n(\mu)$ with dimensionless constant c_{i_n}. So the new form of Lagrangian is

$$\mathcal{L}_{eff} = \mathcal{L}^0 + \sum_{n>0} \sum_{c_{i_n}} \frac{c_{i_n}}{K^n} O_{i_n} \tag{4.2}$$

The higher dimension of operator are suppressed with the increasing power of K. The lowest dimensional operators is more important due to which one can truncate the series and only the finite couplings and number of operators will remain.

4.1.2 Operator Product Expansion

We can illustrate the phenomena of operator product expansion (OPE) by simple example of weak decay of $c \longrightarrow su\bar{d}$ shown in Fig(4.1) The amplitude of the decay is

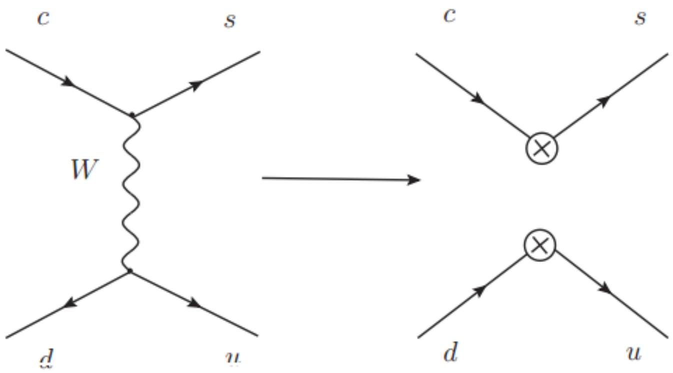

Figure 4.1: Left shows full theory and at Right the effective theory in $c \longrightarrow su\bar{d}$

$$\begin{aligned}
\mathcal{M}_{Full} &= \frac{g_2^2}{8}V_{cs}^*V_{ud}[\bar{u}_s(p_s)\gamma_\mu(1-\gamma_5)u_c(p_c)]\frac{g^{\mu\nu}}{k^2-m_w^2}[\bar{u}_u(p_u)\gamma_\nu(1-\gamma_5)u_d(p_d)] \\
\mathcal{M}_{Full} &= \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}[\bar{u}_s(p_s)\gamma_\mu(1-\gamma_5)u_c(p_c)]\frac{m_w^2}{k^2-m_w^2}[\bar{u}_u(p_u)\gamma^\mu(1-\gamma_5)u_d(p_d)] \quad (4.3)
\end{aligned}$$

G_F is Fermi constant

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8m_w^2} \quad (4.4)$$

Expanding the amplitude to $O(\frac{k^2}{m_w^2})$

$$\mathcal{M}_{Full} = -\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}[\bar{u}_s(p_s)\gamma_\mu(1-\gamma_5)u_c(p_c)][\bar{u}_u(p_u)\gamma^\mu(1-\gamma_5)u_d(p_d)] + O(\frac{k^2}{m_w^2}) \quad (4.5)$$

Where k is the momentum transferred due to W propagator and its value is small as compared to m_w. We can neglect the terms $O(\frac{k^2}{m_w^2})$ without any hesitation from Eq(4.5). Now the full amplitude will be approximately equal to

$$\mathcal{M}_{Full} \approx -\frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}[\bar{u}_s(p_s)\gamma_\mu(1-\gamma_5)u_c(p_c)][\bar{u}_u(p_u)\gamma^\mu(1-\gamma_5)u_d(p_d)] \quad (4.6)$$

The same result is obtained by the effective Hamiltonian

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}V_{cs}^*V_{ud}[\bar{s}\gamma_\mu(1-\gamma_5)c][\bar{u}\gamma^\mu(1-\gamma_5)d] + higher\ Dim\ operator \quad (4.7)$$

This corresponds to the low energy scale, where the heavier particles momenta is integrated out and the higher dimension operator represented by the terms of order $O(\frac{k^2}{m_w^2})$. The OPE idea is grasped through above example.

4.1.3 Effective Hamiltonian

The Effective Hamiltonian is the basic ingredient to discuss the weak decays

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) O_i(\mu) \tag{4.8}$$

Where O_i is a complete set of six dimensional local operators. The V_{CKM} matrix elements and C_i Wilson coefficient explain the strength of the operators. Both the Wilson coefficients and local operator are dependent on the cutoff scale μ. Above the cutoff scale μ we have high energy effects while at below cutoff scale μ we have low energy effect. All of the high energy scale effects are absorbed into the Wilson coefficients and the low energy effects are absorbed into local operators. Now the physics is separated into two regime i-e the low and high energy regime. This is the significant property of the operator product expansion.

4.1.4 Wilson Coefficients

Wilson coefficient can be solved by choosing an appropriate operator basis that corresponds to a set of operators. In this way the effective Hamiltonian is defined as the linear combination of these operators. From the matching condition $\mathcal{M}_{full} = \mathcal{M}_{eff}$ amplitude we get the Wilson coefficient $C_i(\mu)$

$$\mathcal{M}_{full} = \mathcal{M}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) < O_i(\mu) > \tag{4.9}$$

$< O_i(\mu) >$ bracket denoted matrix element to the relevant operator $O_i(\mu)$. This is called the matching condition of the full theory with effective theory. The full theory deals with the particles having dynamical degree of freedom while in effective theory we integrate out the heavy degree of freedom. For larger μ scale we can use perturbation theory for matching and Wilson coefficient depend upon the mass of the integrated out particles. From our previous example the effective theory matching condition is

$$\mathcal{M}_{full} = \mathcal{M}_{eff} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) < O_i(\mu) >$$

where

$$O = [\bar{s}\gamma_\mu(1-\gamma_5)c][\bar{u}\gamma^\mu(1-\gamma_5)d]$$

We observe the Wilson coefficient $C_i(\mu) = 1$ in the example. We did not take Quantum Chromodynamic (QCD) effects in considerations because of the complication arising due to a second operator, which has a complicated color structure. Therefore we would not include QCD effect in thesis.

4.2 Flavor Changing Neutral Currents

Flavor changing neutral currents (FCNCs) are prohibited at tree level in the SM. For example the b quarks has no direct coupling with s and d quarks. From Eq(2.27) we see for electromagnetic current the flavor changing charged currents in the SM. FCNCs at loop level can be mediated through W bosons. Fig(4.2) shows box and penguin diagrams which permit flavor changing neutral transitions. We will focus our discussion on the penguin diagrams throughout thesis. One of the most important characteristics of flavor physics is FCNCs. They allow the measurement of CKM matrix and are very sensitive to Physics beyond standard model. In the SM FCNCs are suppressed. GIM mechanism is a tool to ensure the rationale of the above discussions.

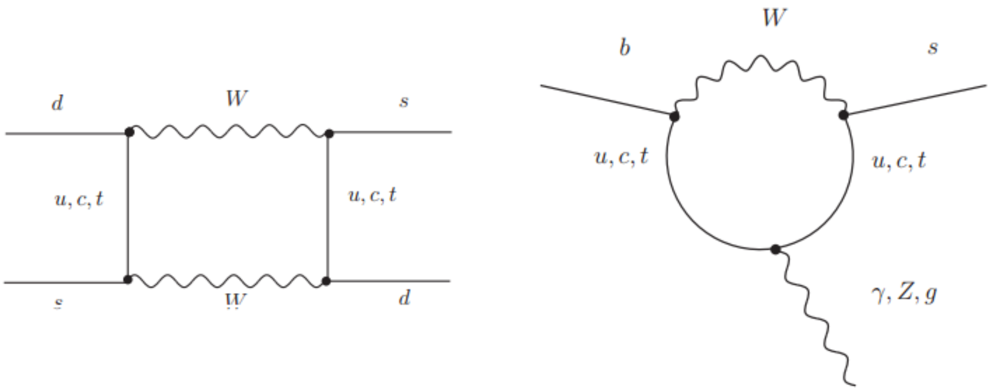

Figure 4.2: Left shows Box and at Right Penguin is located

4.3 GIM Mechanism

S.L Glashow, J.Iliopolous, L.Maiani introduced the GIM mechanism [21] in 1970. Due to this discovery they introduced fourth quark and the charm quark, which were not known at that time. The GIM mechanism can be studied through the diagrams in Fig(4.2), where in $b \longrightarrow s\gamma$ the whole amplitude is the sum of the diagram consisting of u, c and t in the loop.

$$\mathcal{M} = \mathcal{M}(m_u^2)V_{ub}V_{us}^* + \mathcal{M}(m_c^2)V_{cb}V_{cs}^* + \mathcal{M}(m_t^2)V_{tb}V_{ts}^* \tag{4.10}$$

The unitarity condition of CKM matrix is

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0 \tag{4.11}$$

If the masses of the quarks are $m_u = m_c = m_t$ then the amplitude have to be zero and still at loop level FCNCs will be forbidden. But the quarks have different masses and particularly $m_t \gg m_u, m_c$. Therefore the amplitude will be proportional to $ln(\frac{m_t^2}{m_w^2})$. The top quark is extremely massive, therefore the loop diagram is not strongly suppressed. Hence it is used to test the SM or in search of new physics beyond SM.

4.4 B decays Formalism

Rare B decays formalism is described in the low energy regime of effective field theory, where the particles with heavy degree of freedom are integrated out. In this scenario we have $W^{\pm}$ boson and the top quark t. We only deal with the dimension six operators and s quark mass has been neglected due to the strong suppression by higher dimensional operators.
The effective Hamiltonian can be written at the scale $\mu = O(m_b)$ for the decay $b \longrightarrow s\gamma$ [22].

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} [\sum_{i=1}^{6} C_i(\mu_b) O_i + C_{7\gamma}(\mu_b) O_{7\gamma} + C_{8\gamma}(\mu_b) O_{8\gamma}] \tag{4.12}$$

O_i denotes the relevant local operators. $C_i(\mu_b)$ is defined as the Wilson coefficient, which absorb the W and top quark mass completely. From Eq(4.11), the unitarity condition as we know that

$$\begin{aligned} V_{cb}V_{cs}^* &= -V_{ub}V_{us}^* - V_{tb}V_{ts}^* \\ &= -V_{tb}V_{ts}^* (\frac{V_{ub}V_{us}^*}{V_{tb}V_{ts}^*} + 1) \\ &= -V_{tb}V_{ts}^* + O(\mu^2) \end{aligned}$$

In the loop u, c and t quarks appear but due to the condition μ^2, $V_{cb}V_{cs}^*$ and $V_{ub}V_{us}^*$ is suppressed relative to $V_{tb}V_{ts}^*$, therefore u and c are neglected.

The operators are defined as:

Current-Current Operator

$$O_1 = (\bar{s}_\beta c_\beta)_{V-A}(\bar{c}_\alpha b_\alpha)_{V-A}$$
$$O_2 = (\bar{s}_\beta c_\alpha)_{V-A}(\bar{c}_\alpha b_\beta)_{V-A}$$

QCD Penguin Operator

$$O_3 = (\bar{s}_\beta b_\beta)_{V-A}\sum_q(\bar{q}_\alpha q_\alpha)_{V-A}$$
$$O_4 = (\bar{s}_\beta b_\alpha)_{V-A}\sum_q(\bar{q}_\alpha q_\beta)_{V-A}$$
$$O_5 = (\bar{s}_\beta b_\beta)_{V-A}\sum_q(\bar{q}_\alpha q_\alpha)_{V+A}$$
$$O_6 = (\bar{s}_\beta b_\alpha)_{V-A}\sum_q(\bar{q}_\alpha q_\beta)_{V+A}$$

where $q = u, d, s, c, b$

Magnetic Dipole Operator

$$O_{7\gamma} = -\frac{em_b}{8\pi^2}\bar{s}_\alpha\sigma_{\mu\nu}(1+\gamma^5)b_\alpha F^{\mu\nu}$$
$$O_{8\gamma} = -\frac{g_3 m_b}{8\pi^2}\bar{s}_\alpha\sigma_{\mu\nu}(1+\gamma^5)T^\alpha_{\alpha\beta}b_\alpha G^{a,\mu\nu}$$

Where $\alpha, \beta =$ Color indices, e and g_3 define as electromagnetic and strong coupling. $F^{\mu\nu}$ and $G^{a,\mu\nu}$ denote field strength tensor. $(\bar{q}_\alpha q_\beta)_{V\pm A} = \bar{q}_\alpha\gamma^\mu(1+\gamma^5)q_\beta$ used as a short hand notation.

Fig(4.3) shows the penguin diagrams that follow the Feynman rule, in which the magnetic dipole operators come into being by insertion of mass on the external line which indicate b quark line for QED and QCD. Where $m_s << m_b$, therefore we neglect the s quark mass

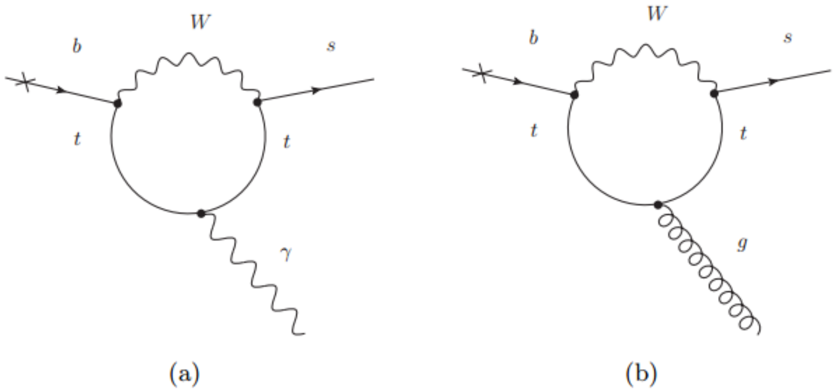

Figure 4.3: (a)Photon.(b)Gluon

insertion and s quark has no contribution.

4.5 Electromagnetic Dipole Operator

$O_{7\gamma}$ is the electromagnetic dipole operator. Without QCD correction $O_{7\gamma}$ operator is responsible for the $b \longrightarrow s\gamma$ decay. The relevant wilson coefficient for $O_{7\gamma}$ operator is $C_{7\gamma}$. When we match the amplitude of full theory with the effective theory at the scale $\mu_W = O(m_w)$, then we are able to calculate $C_{7\gamma}$. The diagram contributing at order of one loop is shown in Fig(4.4). We get the Wilson coefficient $C_{7\gamma}$ when we solve the diagram having photon contribution. Feynman Gauge is used, where it requires the contribution of diagrams involving exchange of Goldstone bosons as shown in Fig(4.5) In calculation we define masses of the light particles as zero. Only b quark mass is included till linear order. The term $m_b^2 = p_b^2 = 0$. The detailed calculation of the diagrams are discussed in sec. 4.6. The Wilson coefficient $C_{7\gamma}$ is obtained by the matching condition.

$$\mathcal{M}_{Full} = \mathcal{M}_{eff} = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} C_{7\gamma} < O_{7\gamma} > \tag{4.13}$$

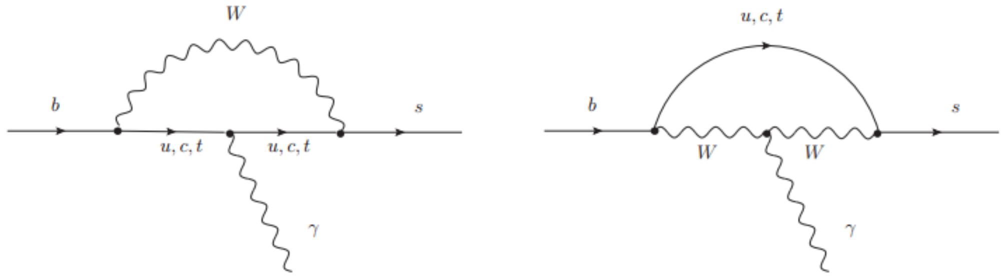

Figure 4.4: $b \longrightarrow s\gamma$ decay at one loop

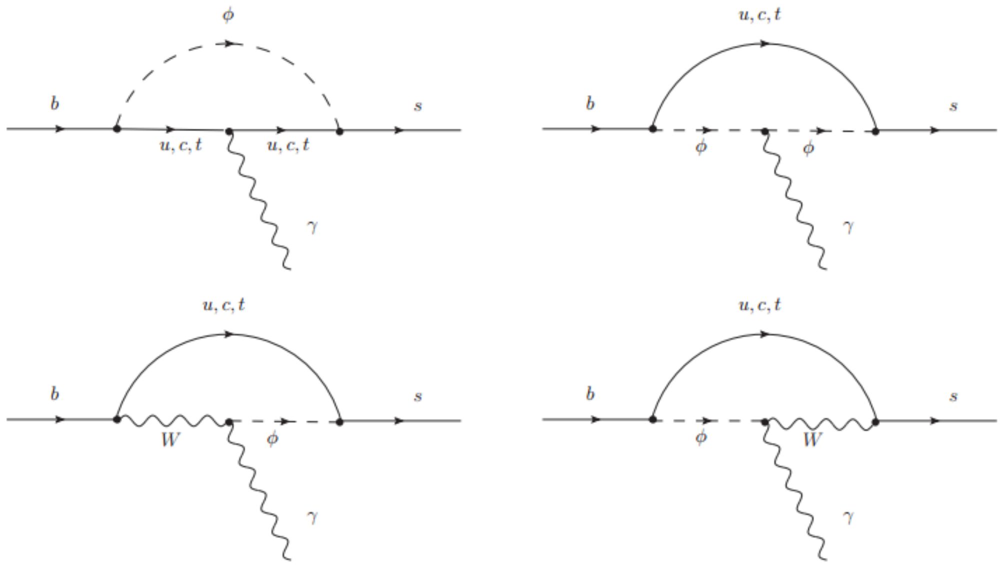

Figure 4.5: $b \longrightarrow s\gamma$ decay at one loop contributing Goldstone bosons

At last we get the result for $C_{7\gamma}$ in agreement with [23]

$$C_{7\gamma}(\mu_W) = -\frac{X}{2}\left[\frac{\frac{3}{2}X^2 + \frac{5}{12}X - \frac{7}{12}}{(X-1)^3} - \frac{(\frac{3}{2}X^2 - X)lnX}{(X-1)^4}\right] \tag{4.14}$$

4.6 Calculation of $C_{7\gamma}$ in the Standard Model

For on-shell i.e. $q^2 = 0$, $p_s^2 = m_s^2$, $p_b^2 = m_b^2$ since $m_s << m_b$, so s quark mass=0, b quark mass considered only to linear order i-e $m_b^2 = 0$.

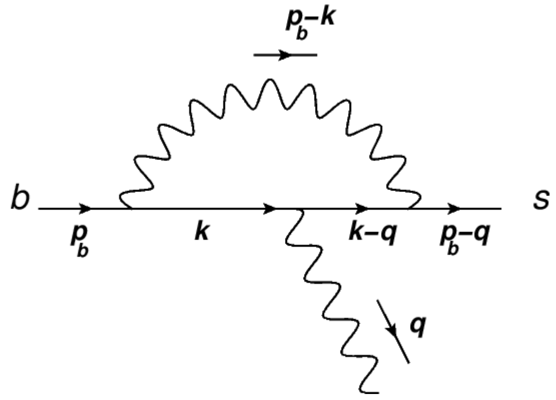

Figure 4.6: Calculation of $C_{7\gamma}$ for $b \to s\gamma$ in SM

Defined $u_s(p_s) \equiv s$, $u_b(p_b) \equiv b$.

$$\begin{aligned} i\mathcal{M} &= e\int dk V_{ts}^* \frac{g_w}{\sqrt{2}} \bar{s} \frac{\gamma^\mu(1-\gamma^5)}{2} \frac{\not{k} - \not{q} + m_t}{[(k-q)^2 - m_t^2]} \gamma^\beta \frac{\not{k} + m_t}{k^2 - m_t^2} V_{tb} \frac{g_w}{\sqrt{2}} \frac{\gamma_\mu(1-\gamma^5)}{2} b \frac{1}{[(k-p_b)^2 - m_w^2]} \cdot \epsilon(q) \\ i\mathcal{M} &= e\frac{g_w^2}{8} V_{ts}^* V_{tb} \int dk \frac{[\bar{s}\gamma^\mu(1-\gamma^5)(\not{k} - \not{q} + m_t)\gamma^\beta(\not{k} + m_t)\gamma_\mu(1-\gamma^5)b]}{[(k-q)^2 - m_t^2][k^2 - m_t^2][(k-p_b)^2 - m_w^2]} \cdot \epsilon(q) \end{aligned} \tag{4.15}$$

Denominator

For denominator we use Feynman parametrization technique.

$$\frac{1}{[(k-q)^2 - m_t^2][k^2 - m_t^2][(k-p_b)^2 - m_w^2]}$$

The general form up to 3 propagator are

$$\frac{1}{ABC} = \int_0^1 \frac{dxdydz\delta(x+y+z-1)2!}{[xA + yB + zC]^3} \tag{4.16}$$

Where,

$A \equiv (k-q)^2 - m_t^2$, $B \equiv k^2 - m_t^2$, $C \equiv (k-p_b)^2 - m_w^2$

$x = xy$, $y = 1-x$, $z = x(1-y)$.

$$xA + yB + zC = xy[(k-q)^2 - m_t^2] + (1-x)(k^2 - m_t^2) + x(1-y)[(k-p_b)^2 - m_w^2]$$

After some algebraic calculation and Feynman parametrization procedure we get

$$\int_0^1 \frac{dxdy}{xy[(k-q)^2 - m_t^2] + (1-x)(k^2 - m_t^2) + x(1-y)[(k-p_b)^2 - m_w^2]} = \int \frac{dxdy}{[\ell^2 - \Delta]^3}$$

Where $k = \ell + qxy + xp_b - xyp_b$ and $\Delta = m_t^2(1 - x + xy) + x(1-y)m_w^2$.

So,

$$i\mathcal{M} = 2\int_0^1 dy \int_0^1 xdx \int d\ell \frac{Num}{[\ell^2 - \Delta]^3} \tag{4.17}$$

Now calculating Num.

$$\begin{aligned}
\text{Num} &= \gamma^\mu(1-\gamma^5)(\not{k} - \not{q} + m_t)\gamma^\beta(\not{k} + m_t)\gamma_\mu(1-\gamma^5) \\
&= 2\gamma^\mu(\not{k} - \not{q})\gamma^\beta \not{k}\gamma_\mu(1-\gamma^5) \\
&= 2\gamma^\mu(\not{q}xy + x\not{p}_b - \not{q})\gamma^\beta(\not{q}xy + x\not{p}_b - xy\not{p}_b)\gamma_\mu(1-\gamma^5) \\
&= 2\gamma^\mu[-(1-xy)\not{q} + x(1-y)\not{p}_b]\gamma^\beta[xy\not{q} + x(1-y)\not{p}_b]\gamma_\mu(1-\gamma^5) \qquad (4.18)
\end{aligned}$$

From eqution (4.18) we get four different terms to calculate.

$$\begin{aligned}
\gamma^\mu\not{q}\gamma^\beta\not{q}\gamma_\mu(1-\gamma^5) &= -4\not{q}q^\beta(1-\gamma^5) \\
\gamma^\mu\not{q}\gamma^\beta\not{p}_b\gamma_\mu(1-\gamma^5) &= -4\not{q}p_b^\beta(1-\gamma^5) + 2\not{q}m_b\gamma^\beta(1+\gamma^5) \\
\gamma^\mu\not{p}_b\gamma^\beta\not{q}\gamma_\mu(1-\gamma^5) &= -4m_b q^\beta(1+\gamma^5) + 2m_b\not{q}\gamma^\beta(1+\gamma^5) \\
\gamma\not{p}_b\gamma^\beta\not{p}_b\gamma_\mu(1-\gamma^5) &= -4m_b p_b^\beta(1+\gamma^5)
\end{aligned}$$

In several steps we obtain a numerator by using Dirac equation $\not{p}_b(1-\gamma^5)b = m_b(1+\gamma^5)b$ and $\epsilon(q).q^\beta = 0$, $\epsilon(q)$ is a photon polarization , hence we can neglect the terms proportional to q^β.

$$\begin{aligned}\text{Num} &= 8x(1-xy)(1-y)\not{q}p_b^\beta(1-\gamma^5) - 4m_b x(1-xy)(1-y)\not{q}\gamma^\beta(1+\gamma^5) \\ &\quad +4x^2y(1-y)m_b\gamma^\beta\not{q}(1+\gamma^5) - 8x^2(1-y)^2m_b p_b^\beta(1+\gamma^5). \end{aligned} \tag{4.19}$$

Applying the Gordon identity

$$\bar{s}(\frac{p_b^\mu + p_s^\mu}{2})(1-\gamma^5)b = \frac{1}{2}m_b\bar{s}\gamma^\mu(1+\gamma^5)b + \frac{i}{2}\bar{s}\sigma^{\mu\nu}q_\nu(1-\gamma^5)b,$$

and below relations in (4.19)

$$\begin{aligned}\not{q}\gamma^\beta &= \frac{1}{2}(\{\gamma^\beta,\not{q}\} - [\gamma^\beta,\not{q}]) \simeq -\frac{1}{2}[\gamma^\beta,\not{q}], \\ \gamma^\beta\not{q} &= \frac{1}{2}([\gamma^\beta,\not{q}] + \{\gamma^\beta,\not{q}\}) \simeq \frac{1}{2}[\gamma^\beta,\not{q}].\end{aligned}$$

The simplified form of numerator is

$$\text{Num} = [2x^2y(1-y) + 2x^2(1-y)^2]m_b[\gamma^\beta,\not{q}](1+\gamma^5). \tag{4.20}$$

The Amplitude looks like

$$\mathcal{M} = -iV_{ts}^*V_{tb}\frac{g_w^2}{8}Qe2\int_0^1 dy\int_0^1 xdx\int dl\frac{Num}{[\ell^2-\Delta]^3} \tag{4.21}$$

performing integration, the momentum integration $\int \frac{1}{[\ell^2-\Delta]^3}dl = -\frac{i}{(4\pi)^2 2\Delta}$

$$\begin{aligned}\mathcal{M} &= -iV_{ts}^*V_{tb}\frac{g_w^2}{8}Qe\not{2}\int_0^1 dy\int_0^1 xdx(\frac{-iNum}{(4\pi)^2\not{2}\Delta}) \\ &= -\frac{1}{(4\pi)^2}V_{ts}^*V_{tb}\frac{g_w^2}{8}Qem_b[\gamma^\beta,\not{q}](1+\gamma^5)\underbrace{\int_0^1 dy\int_0^1 xdx(\frac{[2x^2y(1-y)+2x^2(1-y)^2]}{\Delta})}_{I}\end{aligned} \tag{4.22}$$

Calculating I

$$\begin{aligned} I &= \int_0^1 dy \int_0^1 dxx \frac{[2x^2y(1-y)+2x^2(1-y)^2]}{\Delta} \\ &= \int_0^1 dy \int_0^1 dxx \frac{[2x^2y(1-y)(\not{y}+1-\not{y})]}{\Delta} \end{aligned}$$

Where $\Delta = m_t^2(1-x+xy)+x(1-y)m_w^2$.

$$I = \int_0^1 dy \int_0^1 dx \frac{2x^3(1-y)}{\Delta}$$

Using mathematica for solving I we get

$$I = \frac{5X^3 - 9X + 6Xln(X)(1-2X)+4}{6m_w^2(X-1)^4} \tag{4.23}$$

Where, $X = \frac{m_t^2}{m_w^2}$

Plugging I in (4.22)

$$\mathcal{M}_{Full} = -\frac{1}{(4\pi)^2} V_{ts}^* V_{tb} \frac{g_w^2}{8} Qe \Big(\frac{5X^3 - 9X + 6Xln(X)(1-2X)+4}{6m_w^2(X-1)^4} \Big) m_b[\gamma^\beta, \not{q}](1+\gamma^5) \tag{4.24}$$

From effective theory

$$\mathcal{M}_{eff} = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} C_{7\gamma} < O_{7\gamma} > \tag{4.25}$$

$$O_{7\gamma} = -\frac{em_b}{8\pi^2} \bar{s} \sigma^{\mu\nu}(1+\gamma^5) b F_{\mu\nu}$$

$$\begin{aligned} \sigma^{\mu\nu} F_{\mu\nu} \longrightarrow -2i\sigma^{\mu\nu} q_\nu &= -\not{2}\frac{i^2}{\not{2}}[\gamma^\mu, \gamma^\nu] q_\nu \\ &= [\gamma^\mu, \not{q}] \end{aligned}$$

$$O_{7\gamma} = -\frac{em_b}{8\pi^2} \bar{s} [\gamma^\mu, \not{q}](1+\gamma^5) b.$$

$$
\begin{aligned}
\mathcal{M}_{eff} &= \frac{G_F}{\sqrt{2}}\Big(\frac{-em_b}{8\pi^2}\Big)V_{ts}^{*}V_{tb}C_{7\gamma}\bar{s}[\gamma^{\mu},\not{q}](1+\gamma^5)b \\
\frac{G_F}{\sqrt{2}} &= \frac{g_w^2}{8m_w^2} \\
\mathcal{M}_{eff} &= -\frac{g_w^2}{8m_w^2}\frac{em_b}{8\pi^2}V_{ts}^{*}V_{tb}C_{7\gamma}\bar{s}[\gamma^{\mu},\not{q}](1+\gamma^5)b \qquad (4.26)
\end{aligned}
$$

As comparison of $\mathcal{M}_{Full} = \mathcal{M}_{eff}$ demands

$$
C_{7\gamma} = \frac{5X^3 - 9X + 6Xln(X)(1-2X) + 4}{12(X-1)^4} \qquad (4.27)
$$

The other diagrams are similarly calculated by applying these steps , Eq(4.14) is the sum of these calculated individual diagrams shown in Fig(4.4) and (4.5).

Chapter 5

Beyond Standard Model

5.1 Chiral Anomaly

Anomaly cancellation is a key characteristic of SM. It breaks the symmetry by including quantum effects, anomaly cancellation is important for quantum consistency that is required for any gauge theory. SM,that is based on gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$,is anomaly free. To renormalize the theory , the conservation law must hold for vector and axial vector currents. The chiral transformation $\psi \to e^{i\alpha(x)\gamma^5}\psi$ holds for Dirac lagrangian

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu \psi - m\bar{\psi}\psi. \tag{5.1}$$

As a consequence

$$\partial_\mu(\bar{\psi}\gamma^\mu\gamma^5\psi) = 2im\bar{\psi}\gamma^5\psi \tag{5.2}$$

$\bar{\psi}\gamma^\mu\gamma^5\psi$ defined as J_5^μ axial current which is conserved for massless limit and fermions are massless before electroweak (EW) symmetry breaking . In these massless limits the triangular loop diagram, as shown in Fig. 5.1 , having one axial and two vector currents at vertices leads to anomaly.

$$\partial_\mu J_5^\mu = (\frac{g}{4\pi})^2 \epsilon^{\mu\nu\tau\kappa} F_{\mu\nu} F_{\tau\kappa}. \tag{5.3}$$

where g is coupling constant and $F_{\mu\nu}$ is field strength tensor.For an EW theory renormalizability, it is essential to be anomaly free.

Lagrangian can be written for the gauge coupling bosons and fermions.

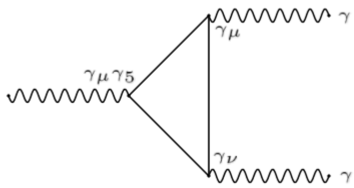

Figure 5.1: Triangular anomaly diagram consists of one axial vector and two vector currents, individually located at each vertex

$$\mathcal{L}_{int} = i\bar{\psi}_L \gamma^\mu (\partial_\mu + igT_a^L W_{a,\mu})\psi_L + i\bar{\psi}_R \gamma^\mu (\partial_\mu + igT_a^R W_{a,\mu})\psi_R. \tag{5.4}$$

The lagrangian(5.4) gives the result for current $j^\mu = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \delta\phi$ as follows

$$j_\mu^a = \bar{\psi}\gamma_\mu \Big(\frac{1-\gamma^5}{2}\Big) T_a^L \psi + \bar{\psi}\gamma_\mu \Big(\frac{1+\gamma^5}{2}\Big) T_a^R \psi \tag{5.5}$$

T_a^L and T_b^R are the hermitian matrices that obey the lie algebraic rules.

$[T_a^L, T_b^L] = if_{abc}T_c^L, \quad [T_a^R, T_b^R] = if_{abc}T_c^R$

We define unitary transformation such that

$$\begin{aligned} \psi &\rightarrow U\psi \qquad \text{Where as } U = e^{i\theta T_R}. \\ \psi^* &\rightarrow U^*\psi \qquad \text{Where as } U^* = e^{i\theta(-T_R)} = e^{i\theta T_{\bar{R}}}. \end{aligned}$$

Where as from above condition

$$T_{\bar{R}} = -T_R^* \tag{5.6}$$

Where $T_{\bar{R}} = U^{-1}T_R^a U$ if $U = I \rightarrow$ real representation, $U \neq I \rightarrow$ pseudoreal real representation.

$$[T_R^a, T_R^b] = if^{abc}T_R^C$$

$$[-T_R^{a*}, -T_R^{b*}] = if^{abc}(-T_R^{c*})$$

$$[T_{\bar{R}}^a, T_{\bar{R}}^b] = if^{abc}(T_{\bar{R}}^c)$$

$$A(R)d^{abc} = \frac{1}{2}Tr[T_R^a\{T_R^b, T_R^c\}] \tag{5.7}$$

$$A(\bar{R})d^{abc} = \frac{1}{2}Tr[T_{\bar{R}}^a\{T_{\bar{R}}^b, T_{\bar{R}}^c\}] \tag{5.8}$$

Now taking complex conjugate of Eq(5.7)

$$-A(R)d^{abc} = \frac{1}{2}Tr[-T_R^{a*}\{-T_R^{b*}, -T_R^{c*}\}] \tag{5.9}$$

$$A(\bar{R})d^{abc} = \frac{1}{2}Tr[T_{\bar{R}}^a\{T_{\bar{R}}^b, T_{\bar{R}}^c\}] \tag{5.10}$$

From Eq(5.9) and (5.10) we conclude that

$$A(\bar{R}) = -A(R) \tag{5.11}$$

To check the SM anomaly, we simply consider the one generation and other two generations are considered in the similar way. The possible anomalies of SM structure by using Table(2.2) contents are

- $\underline{SU(3)_c \otimes SU(3)_c \otimes SU(3)_c}$:

$$\begin{aligned} & 2A(3) + A(\bar{3}) + A(\bar{3}) \\ = & \ 2A(3) + 2A(\bar{3}) = 2[A(3) - A(3)] \\ = & \ 0 \end{aligned}$$

- $\underline{SU(2)_L \otimes SU(2)_L \otimes SU(2)_L}$:

$$3A(2) + A(2) = 4A(2)$$
$$\text{due to real representation} \quad A(2) = 0$$

- $\underline{U(1)_Y \otimes U(1)_Y \otimes U(1)_Y}$:

The $U(1)$ generators are just numbers, we just add the cube of hypercharge of particles

$$\begin{aligned} & 3 \times 2(\frac{1}{6})^3 + 3(\frac{-2}{3})^3 + 3(\frac{1}{3})^3 + 2(\frac{-1}{2})^3 + 1(1)^3 \\ & = \frac{-3}{4} + \frac{3}{4} = 0 \end{aligned}$$

- $\underline{SU(3)_c \otimes SU(3)_c \otimes SU(2)_L}$:

These are canceled out because of traceless property

$$\frac{1}{2}Tr(\tau_a[\lambda_a, \lambda_b]) = \frac{1}{2}Tr[\lambda_a, \lambda_b]Tr[\tau_a] = 0 \qquad \because Tr[\tau_a] = 0$$

similarly $SU(2)_L \otimes SU(2)_L \otimes SU(3)_c$ also discarded.

- $\underline{SU(3)_c \otimes U(1)_Y \otimes U(1)_Y \text{ and } SU(2)_L \otimes U(1)_Y \otimes U(1)_Y}$:

They also follow the above same condition and get cancelled i.e. $Tr[\lambda_a] = 0$.

- $\underline{SU(3)_c \otimes SU(3)_c \otimes U(1)_Y}$:

The contribution of particles to this anomaly $SU(3)$ and $U(1)$ have to obey the following condition

$$
\begin{aligned}
Tr[\{T^a, T^b\}T^c] = \frac{1}{2}\delta abT^c = \frac{1}{2}\sum Y_i \\
= \frac{1}{2}[(\frac{1}{6})(2) + (1)(\frac{-2}{3}) + (1)(\frac{1}{3})] \\
= \frac{1}{2}[\frac{1}{3} - \frac{2}{3} + \frac{1}{3}] = 0
\end{aligned}
$$

It is observed from the above conditions that the anomaly gets cancelled out in the SM accidently, hence we come to the conclusion that SM is anomaly free.

5.2 Extension to SM

The literature explains various types of Extensions to SM. For example Super Symmetry that explain the symmetry between fermions and bosons, Minimal Super Symmetric Model (MSSM),Grand Unified Theory (GUT), Extra dimensions, String theory etc. Although there is no experimental verification for such type of theories, yet there are possibilities to build new model by theorist. Adding Vector-like particle [24] generally $R \oplus \bar{R}$ to the SM is also one of the Extensions to SM. The motivation behind this theory is the anomaly cancellation by itself followed by the following condition.

$$
\begin{aligned}
A(\bar{R}) &= -A(R) \\
A(R \oplus \bar{R}) &= A(R) \oplus A(\bar{R}) \\
&= A(R) - A(R) = 0
\end{aligned}
$$

These particles can be called more specifically as Standard Vector-like Particles. We call it standard because these Vector-like Particles transform in the same way as SM fermions. Standard Vector-like Particles and their contents are described in Table(5.1)

Table 5.1: Standard Vector-like Particles

Notation	$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$
$Q \oplus \bar{Q}$	$(3, 2, \frac{1}{6}) \oplus (\bar{3}, 2, \frac{-1}{6})$
$U \oplus \bar{U}$	$(3, 1, \frac{2}{3}) \oplus (\bar{3}, 1, \frac{-2}{3})$
$D \oplus \bar{D}$	$(3, 1, \frac{-1}{3}) \oplus (\bar{3}, 1, \frac{1}{3})$
$L \oplus \bar{L}$	$(1, 2, \frac{1}{2}) \oplus (1, 2, \frac{-1}{2})$
$E \oplus \bar{E}$	$(1, 1, 1) \oplus (1, 1, -1)$
G	$(8, 1, 0)$
W	$(1, 3, 0)$

Adjoint representation condition

$$(T_a^A)_{bc} \equiv -if_{abc}$$

$$(T_a^{\bar{A}})_{bc} = (T_a^A)^*_{bc} = -if_{abc} = (T_a^A)_{bc}$$

Those particles having adjoint representation are not the Vector-like particles due to the reason they have no contribution to the anomalies. Hence in the Table(5.1) shows that G and W do not demand to be Vector-like particle because they are associated to $SU(3)$ and $SU(2)$ adjoint representations respectively.

We have already calculated the various Higgs decay in chap. 3, by implying the grasped idea to di-photon resonance as shown in Fig. 5.2. The direct coupling of gauge singlet S with

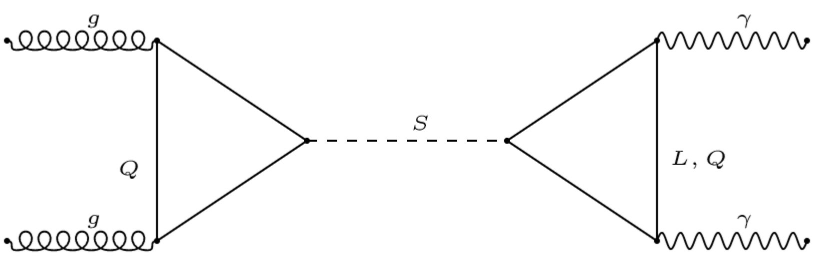

Figure 5.2: Production and di-photon decay at loop level containg gauge singlet S

SM quarks is not possible to produce S from gluon fusion. After that S can be decayed into gg(gluons) and $\gamma\gamma$(photons) through loop diagrams as shown in Fig. 5.2 via standard Vector-like particles. When LHC announces the di-photon excesses, the $gg \to S \to \gamma\gamma$ signal has been investigated and analyzed. The S production dominated by gluon fusion, can be delineated by decay widths: $S \to gg$ and $S \to \gamma\gamma$

$$\sigma(gg \to S \to \gamma\gamma) = \frac{C_{gg}}{s m_S \Gamma_{total}} \Gamma(S \to gg)\Gamma(S \to \gamma\gamma) \tag{5.12}$$

Where C_{gg} is the integral that describes the parton distribution functions of gluons, which is numerically defined as [25]: $C_{gg} = 2137$ at $\sqrt{s} = 13\,TeV$ and $C_{gg} = 174$ at $\sqrt{s} = 7\,TeV$. Where m_s and Γ_{total} are defined as mass and total decay width of S respectively. The S decay to gluons and photons are defined [26].

$$\Gamma(S \to gg) = \frac{\alpha_s^2 m_S^3}{128\pi^3} | \sum_{a=1}^{N_F} \frac{y_F^{aa}}{m_F^a} A_{\frac{1}{2}}(\tau_a) + \sum_{b=1}^{N_F} \frac{y_F^{bb}}{m_F^b} A_{\frac{1}{2}}(\tau_b)|^2 \tag{5.13}$$

$$\Gamma(S \to \gamma\gamma) = \frac{\alpha^2 m_S^3}{256\pi^3} | \sum_{a=1}^{N_F} \frac{y_F^{aa}}{m_F^a} N_c (\frac{2}{3})^2 A_{\frac{1}{2}}(\tau_a) + \sum_{b=1}^{N_F} \frac{y_F^{bb}}{m_F^b} N_c (-\frac{1}{3})^2 A_{\frac{1}{2}}(\tau_b)|^2 \tag{5.14}$$

τ_i defined as $\tau_i = \frac{m_S^2}{4(m_F^i)^2}$, where $i = a, b$. $A_{\frac{1}{2}}$ is a loop function and defined as

$$A_{\frac{1}{2}}(\tau) = \frac{2[\tau + (\tau - 1) f(x)]}{\tau^2}$$

Where $f(x)$ function reads as

$$f(x) = \begin{cases} \arcsin^2 \sqrt{x}, & \text{for } x \leq 0. \\ -\frac{1}{4}[\log \frac{1+\sqrt{1-\frac{1}{x}}}{1-\sqrt{1-\frac{1}{x}}} - i\pi]^2, & \text{for } x > 1. \end{cases} \tag{5.15}$$

We replace the Higgs H with gauge singlet S. The reason is H, S and Vector-like particles Q, $\bar{Q}$ transform under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ as $(1, 2, \frac{1}{2})$, $(1, 1, 0)$ and $(3, 2, \frac{1}{6})$, $(\bar{3}, 2, \frac{-1}{6})$ respectively. Let's check the consistency of the theory as shown below.

$H\bar{Q}Q$ transform under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

$$SU(3) \; : \; 1 \times \bar{3} \times 3 = 1 \oplus ...$$
$$SU(2) \; : \; 2 \times 2 \times 2 \neq 1 \oplus ...$$
$$U(1) \; : \; \frac{1}{2} - \frac{1}{6} + \frac{1}{6} \neq 0$$

$S\bar{Q}Q$ transform under $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$

$$SU(3) \; : \; 1 \times \bar{3} \times 3 = 1 \oplus ...$$
$$SU(2) \; : \; 1 \times 2 \times 2 = 1 \oplus ...$$
$$U(1) \; : \; 0 - \frac{1}{6} + \frac{1}{6} = 0$$

The following condition must hold to respect the theory to be consistent for all possibilities of gauge singlet S, Higgs H, all Vector-like and SM particles. Whereas vector-like and SM particles are denoted by ψ.

- $H\psi\bar{\psi}$ transform under $SU(3)_c$

$1 \times 3 \times \bar{3}$ ✓

$1 \times 1 \times \bar{1}$ ✓

$1 \times 1 \times 3$ ×

$1 \times 1 \times \bar{3}$ ×

- <u>$S\psi\bar{\psi}$ transform under $SU(3)_c$</u>

$1 \times 3 \times \bar{3}$	✓
$1 \times 1 \times \bar{1}$	✓
$1 \times 1 \times 3$	×
$1 \times 1 \times \bar{3}$	×

- <u>$H\psi\bar{\psi}$ transform under $SU(2)_L$</u>

$2 \times 2 \times 2$	×
$2 \times 1 \times 1$	×
$2 \times 1 \times 2$	✓

- <u>$S\psi\bar{\psi}$ transform under $SU(2)_L$</u>

$1 \times 2 \times 2$	✓
$1 \times 1 \times 1$	✓
$1 \times 1 \times 2$	×

- <u>$H\psi\bar{\psi}$ and $S\psi\bar{\psi}$ transform under $U(1)_Y$</u>

$\sum Y_i = 0$	✓
$\sum Y_i \neq 0$	×

As from the above discussion it is clear that H does not couple with Vector-like particles through yukawa coupling therefore the gauge singlet S is replaced and it couples through yukawa coupling. This may work for new physics beyond standard model.

From Fig. 5.3 the lagrangian [27] can be written as

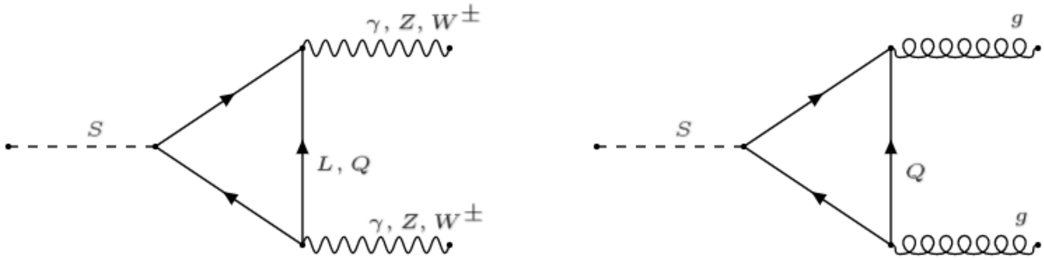

Figure 5.3: S couples to SM gauge bosons involving a Vector-like fermions in loop

$$\mathcal{L}_{BSM} = \mathcal{L}_{SM} + YSF\bar{F} + \frac{m_s^2}{2}|S|^2 + \frac{\lambda}{4!}S^4 + m_F\bar{F}F + kinetic\ term. \tag{5.16}$$

As $F \equiv Q, U, D, L, E$ Vector-like particles

The motivation given by Vector-like theory can be applied further on flavor physics sector. The Wilson coefficient $C_{7\gamma}$ in the SM is calculated in sec. 4.6 that can be calculated by applying Vector-like particle with gauge singlet instead of SM particles. Our future goal is to calculate the diagram as shown in Fig. 5.4 by using the motivation of above discussion. In these types of penguin diagrams as shown in Fig. 5.4 that are involved in calculation of

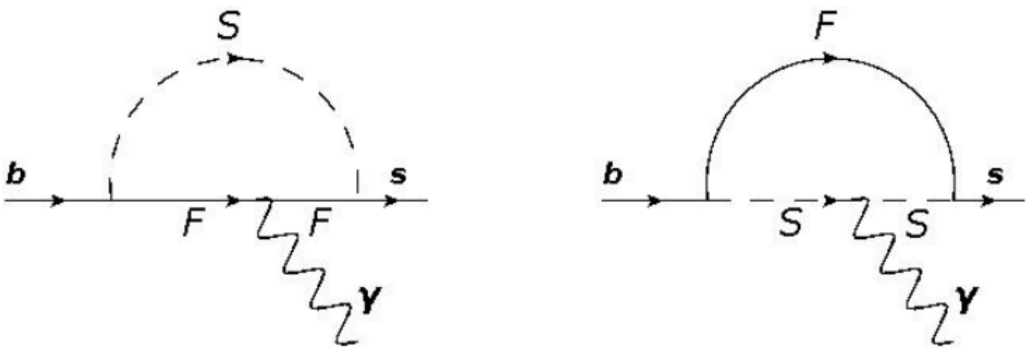

Figure 5.4: Loop diagrams consisting of gauge singlet S and Vector-like fermions F

$C_{7\gamma}$, we replace W with S and instead of SM fermion we want to use Vector-like fermions.

Conclusion

The construction of SM Lagrangian was analyzed by using the tool i.e. gauge theory and renormalizability. SM structure has been reviewed briefly and put forward the concept of Higgs mechanism to realize masses in gauge theory. Despite plentiful successes of the SM, still Higgs boson was not discoverd. For the existence of Higgs boson it is mandatory to analyze its production and decay modes. There are theoretical and experimental constrains on the Higgs mass range. In this dissertation we explicitly calculated the decay rate at tree level such as $H \to f\bar{f}, AA$ where $A = W^{\pm}, Z$ and at loop level such as $H \to gg$ and $\gamma\gamma$ and analyzed its branching ratio. The loop level calculation exhibits divergences and to remove these divergences we have used the technique of dimensional regularization, Feynman parametrization etc. Furthermore, we introduced the intuition towards effective field theory, which is the precise tool in the description of weak decays described by effective Hamiltonian that can be written as the summation of local operators with weight factor of Wilson coefficients, which can be determined by using the matching condition of effective to full theory. We also studied magnetic dipole operator which governs the decay $b \to s\gamma$ and computed explicitly the corresponding Wilson coefficient $C_{7\gamma}$. When LHC announced the di-photon excesses, the $gg \to S \to \gamma\gamma$ signal was investigated and analyzed. The S production dominated by gluon fusion, can be delineated by decay widths: $S \to gg$ and $S \to \gamma\gamma$. In which the gauge singlet S replaced the Higgs H. We applied the above motivation to flavor

Physics. As we know that SM is verified by experimental data, but there are still some problems with SM, to solve these problems one goes beyond SM by using various types of extensions. We used "adding Vector-like particles", that is one of the extensions to SM. Our future perspective is to calculate $C_{7\gamma}$ Wilson coefficient penguin loop diagrams by replacing the SM particles with Vector-like particles and gauge singlet.

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