Engineering Entanglement in cavity QED for Quantum Networks

by

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Dedication

To

My Parents

and

Teachers

Acknowledgements:

I humbly thank Almighty **ALLAH,** The **Most** Beneficent and The **Most** Merciful, **Who** gave me the courage to complete this project. I offer my humblest thank from sense of heart to Holy Prophet "**Muhammad (P.B.U.H)",** who is forever a source of guidance for humanity.

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Abstract

Aim of the work is to engineer entanglement in cavity QED for quantum networks. We propose two schemes, first, to produce entanglement in atomic degrees of freedom using cavity QED techniques, and, second among cavities. In the first scheme, we pass two atoms, called tagged atoms, through cavity fields dispersively and entangled the atoms with cavity fields in their external degrees of freedom. Now, to transfer atom-field entanglement to atom-atom entanglement, we pass two auxiliary atoms initially in their ground state through cavity field and make them interact in non-dispersive and dispersive fashion. After interaction, the cavity field states transfer to auxiliary atoms. Hence, the external degrees of freedom of the tagged atoms are entangled with the internal degrees of freedom of the auxiliary atoms. We apply atomic detection operator on auxiliary atoms and develop multipartite entangled state. In the second scheme, we consider three different resonators, which may confine, separately, same mode or different modes of electromagnetic field. Two cavities share field entangled states with the third cavity. By applying field detection operator on the third cavity, entangled all the three cavities.

Chapter 1

Introduction

At the dawn of the 20^{th} century, experimental physics brought to light various unexplainable phenomena, such as, black body radiation, photoelectric effect and atomic structure [1], and eventually gave birth to the quantum mechanics. Later, the development of quantum mechanics saw two time periods: 1900 to1925, the first generation of quantum mechanics and, 1925 to 1928, the second generation of quantum mechanics [2].

As a critic of quantum mechanics, on 4^{th} December 1926, Einstein wrote to Max Born “Quantum mechanics is certainly imposing, but an inner voice tells me that it is not yet the real thing” [3]. His famous criticism of quantum theory came as he tried to explain quantum entanglement.

Entanglement, an important feature of quantum mechanics, is taken as a manifestation of quantum correlations. In 1935 [4] Schrödinger, the one who gave the idea of entanglement, said

“When two systems, of which we know the states of their respective representation, enter into the temporary interaction due to known forces between them and after mutual influence the systems separated again, then they no longer be described as before viz by endowing each of them with a representative of its own. I would not call that one but rather the characteristics trait of quantum mechanics”.

Einstein discussed the idea of entanglement with Rosen during 1933 Solvay conference, saying, What would you say about the following situation? “Suppose two particles moving toward each other with equal and very large momentum and interact for a very short interval of time when they pass at aknown position. Now an observer which measures the momentum of one particle, far away from the region of interaction, is able to deduce the momentum of the other particle. However, if he chooses to measure the position of first particle, he will be able to tell where the other particle is. This is correct and straight forward deduction from the principles of quantum mechanics; but is it not very paradoxical? How can the final state of the second particle be influenced by a measurement performed on the first, after all physical interactions has ceased between them” [5].

In 1935, the above idea was published in the form of a research paper by Einstein, Podolosky and Rosen [6]. The main point in statement of EPR is “If without disturbing in any way a system, we can predict with certainty the value of physical quantity, and then there exist an element of physical reality related to this quantity.” There are two fundamental assumptions in this definition, existence of element of physical reality and causality.

Causality means that two non-interacting spatially well separated objects cannot influence each other globally.

The EPR paradox seems to violate the uncertainty principle, that we can determine the position and momentum of a particle with certainty. But in fact when we measure the position (momentum) of a particle, its momentum (position) will no longer remain as before, uncertainty principle states. So after the measurement we cannot tell about the position or momentum of a particle, over which we made the measurement by measuring the position or momentum of second particle.

1.1 The version of David Bohm

David Bohm accepts the EPR concept of incompleteness and proposed hidden variable theory [7]. However his most important work in present context is to explain the EPR concept in particle spin basis instead of position-momentum basis i.e. instead of using two continuous variables, he used single discrete variable. Let us consider a source that emits electron-positron pairs. Alice and Bob are two observers who observing their spin prepared initially in the spin singlet state.

Let us consider that both choose z-axis for their measurement. When spin of the one particle is measured and is found to be along positive z-axis, then the spin of the other particle is automatically be along negative z-axis, irrespective of the distance between them. Both have $\frac{1}{2}$ probability to find $+\hbar/2$ and $-\hbar/2$. The result of Alice and Bob can never be same. Thus, the wave function for the two particles, which are prepared in spin singlet state, is

$$|\psi_s\rangle = \frac{1}{\sqrt{2}}\{|\mathrm{a}: +z, \mathrm{b}: -z\rangle - |\mathrm{a}: -z, \mathrm{b}: +z\rangle\} \quad (1.1)$$

According to quantum mechanics the spin singlet state may equally be expressed as a superposition of spin state pointing in x-direction. So they can be written in rotated eigen basis $|\pm z\rangle = \frac{1}{\sqrt{2}}(|+x\rangle \pm |-x\rangle)$.

The wave function given in equation (1.1) for the system becomes,

$$|\psi_s\rangle = \frac{1}{\sqrt{2}}\{|\mathrm{a}: +x, \mathrm{b}: -x\rangle - |\mathrm{a}: -x\rangle, \mathrm{b}: +x\rangle\}. \quad (1.2)$$

Both results of equation (1.1) and (1.2) are perfectly correlated. There exists an element of physical reality with S_{bZ} (spin component of Bob particle) that Bob can determine S_{bZ} without interacting with particle b. Since we can transfer our argument from z-axis to the x-axis, so there must be an element of physical reality related with S_{bx}. This is correct for all possible directions and bases irrespective of the distance between them [8].

1.2 Bell's inequality

Quantum mechanics predicts the probabilities for different possible measurement outcomes, whereas in the classical physics measurement outcomes are not random if we have complete knowledge of all system variables. The random measurement outcomes are possible only when we have incomplete knowledge of the system. Are there hidden variables which we cannot observe directly, but which determine the outcomes of our measurement. In 1964, John Bell provided rules which any classical hidden variable theory must obey [9]. Thus he settled a long standing question that whether the system behaves classically as EPR insisted it must be, or demonstrates quantum non locality that we call as entanglement.

According to Einstein view for quantum measurement outcomes (a, b) there exists a parameter λ, called hidden variable, that completely determines the measurement outcomes of two observers, called, Alice and Bob. Here, both Alice and Bob can make only two possible measurements (a, $\acute{a}$ for Alice and $b, \acute{b}$ for Bob) and outcomes are binary i.e. a, b ϵ (+1, -1).

For local deterministic strategy, λ_D, specifies the two measurement outcomes of Alice and two measurement outcomes of Bob. It may be any set $\lambda_D = (\mathrm{a}, \mathrm{a}', \mathrm{b}, \mathrm{b}')$. In all cases, each of these set of measured values, satisfies the expression S (λ_D) = (a + a′) b + (a − a′) b′= ± 2. Here, it is simple to note that either of the two terms (a$\pm$a′) is zero whereas the other is always equal to ± 2. After measurement, the outcomes can be averaged out in four terms $\langle \mathrm{a}, \mathrm{b}\rangle$,$\langle \mathrm{a}', \mathrm{b}\rangle$, $\langle a, \mathrm{b}'\rangle$, $\langle \mathrm{a}', \mathrm{b}'\rangle$.

For these values we can formulate it into an algebraic sum i.e.

$$\mathrm{S} = \langle \mathrm{a}, \mathrm{b} \rangle + \langle \mathrm{a}', \mathrm{b} \rangle + \langle a, \mathrm{b}' \rangle - \langle \mathrm{a}', \mathrm{b}' \rangle \quad (1.3)$$

This is the average of a quantity that has ± 2 value only and this gives us a limit. Hence for local realism criteria we get the following inequality, known as CHSH- Bell inequality given as -2$\leq S \leq$ +2 which was also tested to be correct [10]. In quantum mechanics, a measurement that results into two outcomes +1 and

-1 is defined by hermition operators with those eigen values. Therefore

$$\langle \mathrm{a}, \mathrm{b} \rangle \longrightarrow \langle A \otimes B \rangle \quad (1.4)$$

Here, A and B are two such operators. The average value of the CHSH operator given in equation (1.3) becomes

$$\mathrm{S} = [\langle A \otimes B \rangle] + [\langle \acute{A} \otimes B \rangle] + [\langle A \otimes \acute{B} \rangle] - [\langle \acute{A} \otimes \acute{B} \rangle] \quad (1.5)$$

Using the fact, $A^2 = B^2 = \acute{A}^2 = \acute{B}^2 = \mathbb{1}$, we can easily calculate $S^2 = 4\, \mathbb{1} \otimes \mathbb{1} + [A,] \otimes [B, \acute{B}]$. The maximal possible eigen value of $[A, \acute{A}]$ cannot exceed 2, because

$$\langle |[A, \acute{A}]| \rangle \leq |\langle A\acute{A} \rangle| + |\langle \acute{A}A \rangle| \quad (1.6)$$

Here, both A and $\acute{A}$ contain only +1 and -1. Hence the maximum possible eigen value of S^2 never exceeds 8. This implies that in quantum mechanics

$$|\langle S \rangle| \leq 2\sqrt{2} \quad (1.7)$$

Let us consider that we are observing the spin of two spin-1/2 particles. Here, we have

$$\mathrm{a}_1 = \sigma_{xa}, \mathrm{b}_1 = \sigma_{xb}, \mathrm{a}_2 = \sigma_{ya}, \mathrm{b}_2 = \sigma_{yb}.$$

Let the state of the particle is

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|00\rangle + \mathrm{e}^{i\pi/4}|11\rangle\}, \quad (1.8)$$

where

$$\sigma_x|0\rangle = |1\rangle, \quad \sigma_x|1\rangle = |0\rangle, \quad \sigma_y|0\rangle = i|1\rangle, \quad \sigma_y|1\rangle = -i|1\rangle.$$

Here, $|\psi\rangle$ is an entangled state. Here,$\langle \mathrm{a}, \mathrm{b} \rangle$, $\langle \mathrm{a}', \mathrm{b} \rangle$ and $\langle \mathrm{a}', \mathrm{b}' \rangle$ are equal to $\sqrt{2}/2$ and $\langle a, \mathrm{b}' \rangle$ is equal to $-\sqrt{2}/2$. Hence, S = $2\sqrt{2}$, which voilates the Bell inequality.

Actual experiments are employed to differentiate between the hidden variable theory of local realistic model and the quantum theory. Stuart Freedman and J. S. Clauser experimentally tested the inequality [11, 12] and gave the confirmation of non-local correlation of quantum mechanics which cannot be explained by any classical local model.

1.3 Entanglement

We consider two independent systems, A and B, at time t just before interaction and initially are expressed in their respective state vectors $|\psi_a(t)\rangle$ and $|\psi_b(t)\rangle$. However, their interaction with each other or with a third system (entangler) for a time, t_1, leads to state $|\psi_{ab} t_1 > t)\rangle$. The systems are entangled iff

$$|\psi_{ab}(t_1 > t)\rangle \neq |\psi_a(t)\rangle \otimes |\psi_b(t)\rangle \tag{1.9}$$

Hence, their present state is no more factorizable into original subsystems. Physically, they have now lost their identities and are non-separable in their behavior irrespective of the spatial separation between them. The strong correlation between entangled entities distinguished quantum theory from classical physics and local realism.

Entanglement can be generated when we interact an atom with a cavity field [13]. We interact a two level atom, which is initially in an excited state $|\mathrm{e}\rangle$, with a cavity field which is in the vacuum state $|0\rangle$. After the interaction equal to $\frac{\pi}{2}$ Rabi pulse, there is 50% probability that the atom remains in the excited state $|\mathrm{e}\rangle$ or comes to ground state $|g\rangle$. Hence, when the atom remains in excited state $|\mathrm{e}\rangle$, the cavity remains in vacuum state $|0\rangle$, but, if the atom comes to ground state $|g\rangle$ then the cavity is in one photon state $|1\rangle$. The interaction generates the atom-field entangled state, as

$$|\ \psi\ \rangle = \frac{1}{\sqrt{2}}\{|g, 1\rangle + |\mathrm{e}, 0\rangle\ \} \tag{1.10}$$

There are many types of entangled states such as Bell states, NOON state, GHZ state, W state and Cluster state. Bell states are the maximally entangled bi-partite state of the form

$$|\psi^{+}\rangle = \frac{1}{\sqrt{2}}\{|0_a\rangle\otimes|0_b\rangle + |1_a\rangle\otimes|1_b\rangle\}, \tag{1.11}$$

$$|\psi^{-}\rangle = \frac{1}{\sqrt{2}}\{|0_a\rangle\otimes|0_b\rangle - |1_a\rangle\otimes|1_b\rangle\}, \tag{1.12}$$

$$|\varphi^{+}\rangle = \frac{1}{\sqrt{2}}\{|0_a\rangle\otimes|1_b\rangle + |1_a\rangle\otimes|0_b\rangle\}, \tag{1.13}$$

$$|\varphi^{-}\rangle = \frac{1}{\sqrt{2}}\{|0_a\rangle\otimes|1_b\rangle - |1_a\rangle\otimes|0_b\rangle\}, \tag{1.14}$$

The Bell states are orthonormal and normalized, so provide a complete set of bases.

A NOON state is many partite entangled state of the form

$$|\psi_{NOON}\rangle = \frac{1}{\sqrt{2}}\{|N_a, 0_b\rangle + e^{iN\theta}|0_a, N_b\rangle\}. \tag{1.15}$$

This represents a superposition of N-particles in mode “a” and zero particle in mode “b” and vice versa. The highly non-classical entangled NOON state are used in quantum lithography and metrology, where its important application is in producing interference fringes with a resolution of λ/ N. Here, N represents number of entangled particles. Greenberger-Horn-Zeilinger state is a certain type of entangled state which involves at least three subsystems of the following form.

$$|\mathrm{GHZ}\rangle = {}^{1}/_{\sqrt{2}}\{|0\rangle^{\otimes M} + |1\rangle^{\otimes M}\}, \tag{1.16}$$

Where, $M > 2$.

For three partite system $|\mathrm{GHZ}\rangle = \frac{1}{\sqrt{2}}\{|000\rangle + |111\rangle\}$. (1.17)

An interesting point is that when we trace one of the three systems of GHZ state we get a non-entangled mixed state.

The W state is three partite quantum entangled state of the following form

$$|\mathrm{W}\rangle = \frac{1}{\sqrt{3}}\{|001\rangle + |010\rangle + \ |100\rangle\ \}. \tag{1.18}$$

The W-state is different from GHZ due to an interesting property, that if measurement is performed on one of the three qubits, remaining system of two qubits exhibit entanglement in the reduced Hilbert space. The W state is an ideal resource for communication due to highly nonclassicality as compared to GHZ state.

1.4 Entanglement Applications

1.4.1 Quantum cryptography

After the discovery of the non-locality of the quantum entangled states, scientists started thinking on its applications and the first one of them is quantum cryptography by Stephen Wiesner [13]. He did his work in the 70s but published much later in1983. Many classical cryptographic systems were proposed at different times but everyone has its own problem. Wiesner gave quantum mechanical demonstration of cryptography. Charles Bennet and Gilles Brassard, extended this idea practically and published a paper in which they proposed a quantum key distribution protocol (QED), called BB84 protocol [14]. Charless and Gills along with F. Bessette, L. Salvail and J. Smolin demonstrated it experimentally in lab where the parties are separated by a distance of 32cm and transfer rate is 10 bits per second [15]. In 2002, the task was achieved over a distance of 20km [16]. In 2003, Andrew Shields and his co-worker, proposed a prototype QKD system which is able to transfer information over a distance of 122km using optical fiber at a rate of 2kb/s [17], there are now commercially available. Artur Ekert is the first one who proposed the quantum cryptography based on entanglement [18]. This technique was published in an improved version and experimentally demonstrated, in1998, upto a distance of 10km [19] and later extended by same authors using fiber optic to a 50Km distance [20]. Anton Zeilinger, using photon polarization entanglement, demonstrated internet Bank transfer over 1.45km distance and later for 7.8km using free space optical link.

1.4.2 Quantum teleportation

Another important discovery is quantum teleportation proposed by Bennett in 1993 [21]. In teleportation process, we have a qubit which can be transported exactly from one location to another, without the qubit being transmitted through the intervening space. After the discovery of teleportation by Bennet, many theoretical proposals for atomic and field states teleportation were proposed [22, 23]. Theoretical and experimental work has been done on quantum teleportation in both independent state and squeeze state teleportation. Classical communication is used with the existence of entanglement. We follow the following steps

1) An EPR pair is generated and distributed to two separate locations between the two observers Alice and Bob.
2) Alice performs Bell`s state measurement on her part of entangled state and the unknown state that is to be teleported.
3) Through classical channel, Alice sends two bits of classical information to Bob.
4) Bob uses unitary operation in order to get the qubits which Alice wish to transport [24].

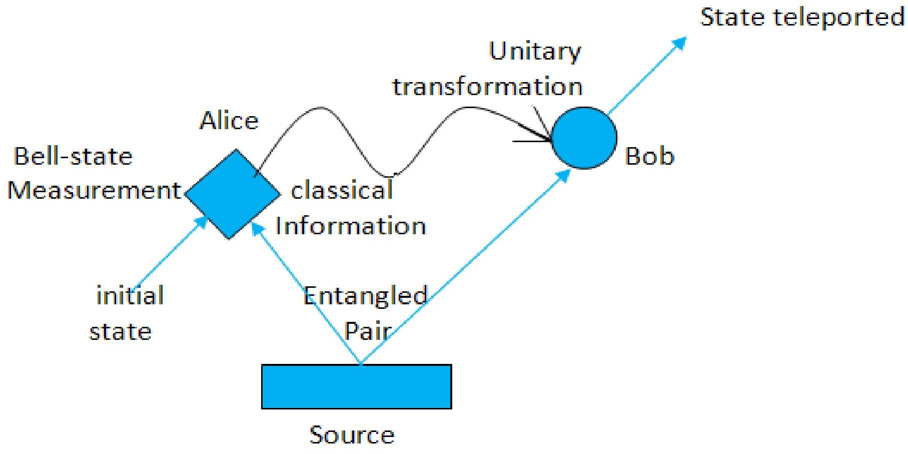

Fig 1.1: We show the teleportation process between two observers Alice and Bob. The source shares an entangled pair between Alice and Bob. Alice performs unitary operation on the unknown set which wants transmit and sent her result Bob. He apply unitary operation to get the desired result.

Let us consider, Alice wish to transmit a qubit in an unknown state $|\varphi_c\rangle$ to Bob, i.e.

$$|\varphi_c\rangle = a|0_c\rangle + b|1_c\rangle \tag{1.19}$$

Let's, Alice and Bob share a maximally entangled pair in the state

$$|\varphi_{AB}\rangle = \frac{1}{\sqrt{2}}\{|0_A 0_B\rangle + |1_A 1_B\rangle\} \tag{1.20}$$

Hence the combined state of the three qubits given in equation (i) and (ii), one the teleported and other the entangled, is

$$|\varphi_{ABC}\rangle = \frac{1}{\sqrt{2}}\{a|0_A 0_B 0_C\rangle + a|1_A 1_B 0_C\rangle + b|0_A 0_B 1_C\rangle + b|1_A 1_B 1_C\rangle\} \tag{1.21}$$

Now we apply CNOT gate and Hadamard gate operations, which have the following operations

$$\text{CNOT } |a, b\rangle = |a, a \oplus b\rangle \tag{1.22}$$

Here, first qubit is controlling and second one is target qubit. This gate flips the second bit if the first bit is 1 and acts trivially when the first bit is zero. The Hadamard gate has following operation

$$\mathrm{H}|0\rangle = \frac{1}{\sqrt{2}}\{|0\rangle+|1\rangle\} \tag{1.23}$$

$$\mathrm{H}|1\rangle = \frac{1}{\sqrt{2}}\{|0\rangle-|1\rangle\} \tag{1.24}$$

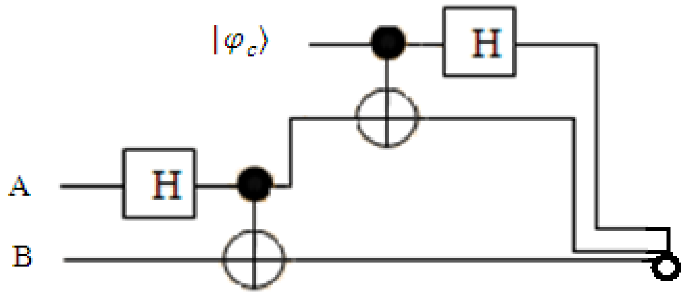

Fig 1.2: We show the operations of the CNOT gate and Hadamard gate on qubits and their sequence (left to right) for teleportation process. Here, A and B are two qubits of entangled state and $|\varphi_c\rangle$ is the state to be teleported.

Hence, after the application of these operations given in equation (1.22), (1.23) and (1.24) the final state expressed by equation (1.21) of the combined system becomes

$$|\varphi\rangle = \frac{1}{2}\{|0_A 0_C\rangle\}\{\mathrm{a}|0\rangle + \mathrm{b}|1\rangle\}+\{|1_A 0_C\rangle\}\{\mathrm{a}|0\rangle - \mathrm{b}|1\rangle\}$$

$$+\{|0_A 1_C\rangle\}\{\mathrm{b}|0\rangle + \mathrm{a}|1\rangle\} + \{|1_A 1_C\rangle\}\{-\mathrm{b}|0\rangle + \mathrm{a}|1\rangle\} \tag{1.25}$$

$$= \{||\varphi^+\rangle\}\{\mathrm{a}|0\rangle + \mathrm{b}|1\rangle\}+\{|\varphi^-\rangle\}\{\mathrm{a}|0\rangle - \mathrm{b}|1\rangle\}$$

$$+\{|\psi^+\rangle\}\{\mathrm{b}|0\rangle + \mathrm{a}|1\rangle\} + \{|\psi^-\rangle\}\{-\mathrm{b}|0\rangle + \mathrm{a}|1\rangle\} \tag{1.26}$$

Alice performs measurement, which collapse the four possibilities given in equation (1.26) into one and yield two classical bits. These two bits are sent to

Bob, which, he uses to know about which operators (X, Y, Z, I) is applied to his qubits in order to place it in state given in equation (1.19).

1.4.3 Quantum dense coding

It is a process of exchange of information by transferring a single qubit in place of two classical bits, with existence of entanglement [25]. It was proposed by C. H. Bunnett and S. J. Wiesner in 1992 [26]. The dense coding was experimentally demonstrated and mostly discussed for the discrete quantum variable [27]. However quantum dense coding for continuous variable has many applications in quantum communication and in development of quantum information process [28].

All the four Bell`s maximally entangled states can be generated by operation on single qubit. Let us consider Alice and Bob share maximally entangled state i.e.

$$|\psi_{AB}^{+}\rangle = \frac{1}{\sqrt{2}}\{|0_A 0_B\rangle + |1_A 1_B\rangle\}. \tag{1.27}$$

Then Alice, using the four unitary operations (X, Y, Z, I), can generate any of the Bell`s maximally entangled state, as

$$\mathrm{I}|\psi_{AB}^{+}\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle + |1_A 1_B\rangle) = |\psi_{AB}^{+}\rangle, \tag{1.28}$$

$$\mathrm{X}|\psi_{AB}^{+}\rangle = \frac{1}{\sqrt{2}}(|1_A 0_B\rangle + |0_A 1_B\rangle) = |\varphi_{AB}^{+}\rangle, \tag{1.29}$$

$$\mathrm{Y}|\psi_{AB}^{+}\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle - |1_A 1_B\rangle) = |\psi_{AB}^{-}\rangle, \tag{1.30}$$

$$\mathrm{Z}|\psi_{AB}^{+}\rangle = \frac{1}{\sqrt{2}}(|1_A 0_B\rangle - |0_A 1_B\rangle) = |\varphi_{AB}^{-}\rangle. \tag{1.31}$$

Alice then sends her qubits to Bob, who operates CNOT gate operation and measuring the target bits, to distinguish between $|00\rangle \pm |11\rangle$ and $|10\rangle \pm |01\rangle$. Bob operates Hadamard gate to distinguish between the sign in the superposition as

$$|\psi^{+}\rangle \rightarrow \text{CNOT} \rightarrow \frac{1}{\sqrt{2}}(|00\rangle + |10\rangle) \rightarrow \text{H} \rightarrow |00\rangle, \tag{1.32}$$

$$|\varphi^{+}\rangle \rightarrow \text{CNOT} \rightarrow \frac{1}{\sqrt{2}}(|11\rangle + |01\rangle) \rightarrow \text{H} \rightarrow |01\rangle, \tag{1.33}$$

$$|\varphi^{-}\rangle \rightarrow \text{CNOT} \rightarrow \frac{1}{\sqrt{2}}(|11\rangle - |10\rangle) \rightarrow \text{H} \rightarrow |10\rangle, \tag{1.34}$$

$$|\psi^{-}\rangle \rightarrow \text{CNOT} \rightarrow \frac{1}{\sqrt{2}}(|01\rangle - |11\rangle) \rightarrow \text{H} \rightarrow |11\rangle. \tag{1.35}$$

Hence, by Bell`s measurement Bob decodes her massage and gets two classical bits that she wants to transfer. Dense coding permits secure communication.

1.4.4 Entanglement swapping

The meaning of entanglement swapping is to transfer entanglement from one pair to another by means of suitable local operations. In entanglement swapping [29], we have two pairs of entangled states, (1 and 2), (3 and 4), by local measurements on pair, 2 and 3, entangled the other two, 1 and 4. The entanglement swapping protocol have been demonstrated experimentally with different systems [30]. In entanglement swapping we entangled to the system by operation without any interaction between these systems.

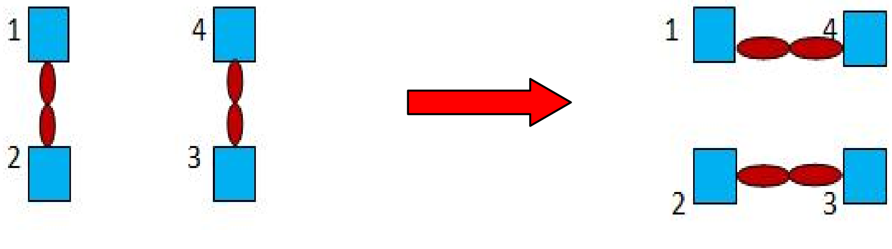

Fig 1.3: We show entanglement swapping; initially two pairs (1&2) and (3&4) are entangled and by local measurement entanglement swap to pairs 1&4 and 2&3.

Let us consider two entangled states $|\varphi_{12}^{+}\rangle$ and $|\varphi_{34}^{+}\rangle$, such that

$$|\varphi_{12}^{+}\rangle|\varphi_{34}^{+}\rangle=\left\{\frac{1}{\sqrt{2}}(|0_1 0_2\rangle+|1_1 1_2\rangle)\right\}\left\{\frac{1}{\sqrt{2}}(|0_3 0_4\rangle+|1_3 1_4\rangle)\right\}, \tag{1.36}$$

$$=\frac{1}{2}\{|0000\rangle_{1234}+|0011\rangle_{1234}+|1100\rangle_{1234}+|1111\rangle_{1234}\}, \tag{1.37}$$

$$=\frac{1}{2}\{|0000\rangle_{1423}+|0101\rangle_{1423}+|1010\rangle_{1423}+|1111\}\rangle_{1423}\}. \tag{1.38}$$

Now, we perform Bell`s state measurement on qubits 2 and 3 of equation (iii), we get

$$|\varphi\rangle_{1423} = 1/2\,\{|\varphi_{14}^{+}\rangle|\varphi_{23}^{+}\rangle\}+1/2\{|\varphi_{14}^{-}\rangle|\varphi_{23}^{-}\rangle\}$$

$$+1/2\{|\psi_{14}^{+}\rangle|\psi_{23}^{+}\rangle\}+1/2\{|\psi_{14}^{-}\rangle|\psi_{23}^{-}\rangle\}. \tag{1.39}$$

Thus, by Bell`s state measurement on 2 and 3, we entangled 1 and 4, although they never interact. Entanglement swapping shows that direct interaction is not necessary to generate entanglement.

1.8 Outline

Our aim is to engineer multipartite entangled state in cavity QED for quantum networks. In the thesis, first, we develop two partite entangled state of atomic external degree of freedom with cavity fields using Bragg diffraction. Later, we develop atom-atom and field-field entangled states for quantum networks.

In the second chapter, we give a brief introduction to Bragg diffraction. We explain the scheme proposed for non-dispersive interaction of atoms with cavity field and develop the effective Hamiltonian for this interaction. We explain the scheme for the development of maximally entangled Bell states in the atomic external degrees of freedom. In addition, we explain field-field entanglement.

In the third chapter, we explain the schemes proposed for engineering of multipartite entangled states. In the first scheme, we entangled atoms with cavity fields in external degrees of freedom. Later, we interact auxiliary atoms with cavity fields to transfer cavity fields state to the internal state of the auxiliary atoms. In the last step we use atom detector which detect the auxiliary atoms in their excited state and develop entangled state. In the second scheme, we have three cavities system A, B and C, where the third cavity C has two modes, C_1 and C_2. We develop entanglement of two cavities, A and B, in two modes of cavity C which are distinguishable. By using field detection operator on third cavity we develop entanglement among the three cavity fields. In the end, we give conclusion and experimental parameters for our proposed schemes.

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Chapter 2

Engineering Bi-partite Entanglement

In this chapter, we explain two schemes to develop Bi-partite entangled states. These are basic models for our proposed schemes for entanglement extended to many parties. In the first scheme, we explain the entanglement between cavity fields and atomic external degrees of freedom using Bragg diffraction [1], whereas in the second scheme, we develop entanglement between the cavities field states [2, 3].

2.1 Entanglement via atomic Bragg's scattering

W. H. Bragg and W. L. Bragg first time demonstrated the diffraction of X-rays from crystal [12]. Bragg's explain the coherent and incoherent scattering from crystal plane. When the phase shift of scattered wave is integral multiple of 2π the waves interfere constructively. Energy and momentum are conserved during Bragg's scattering. In atomic Bragg scattering due to energy conservation, atoms get a momentum kick and deflect at various angles due to light induced force. We take large detuning between field frequency and any atomic transition frequency. The scattering of the atom takes place in the Bragg regime and in the Raman Nath regime.

2.1.1 The model

We interact two super-cooled two-level atoms with cavity field which is in the superposition state of zero and one photon. The direction is chosen such that electric field is polarized along y-axis and propagates along x-axis. The atom interacts with the cavity field at an angle, θ, to the normal having initial momentum, P_i. The momentum of atom has two components, one along the field, P_0 and second normal to the field P_z. The momentum component, P_z, is very large and we treat it classically. The longitudinal component of momentum changes and atom leaves the cavity at different angles whose probability distribution gives us information about photon number of the field.

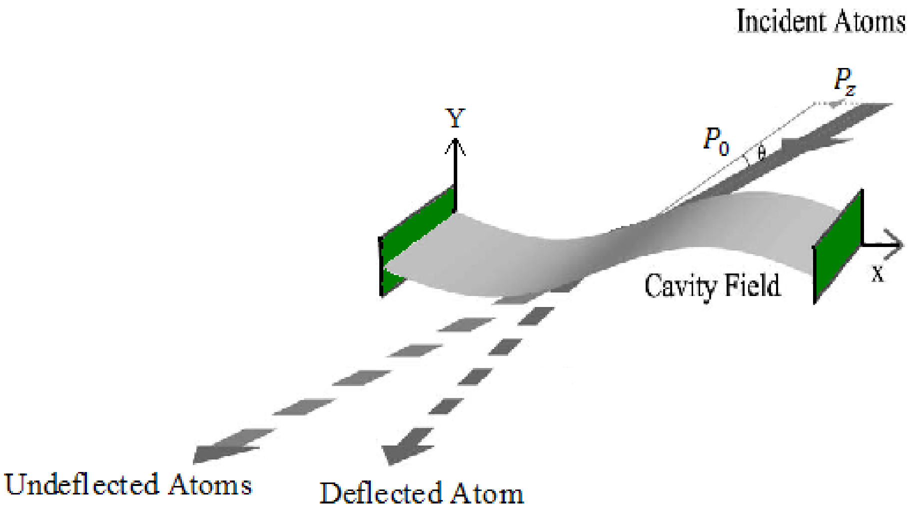

Fig. 2.1: We show that highly detuned atomic beam is scattered by quantized cavity field. The component $\boldsymbol{P_z}$ is very large as compared to component$\boldsymbol{P_0}$.

2.1.2 Basic requirements

The spontaneous emission causes the emission of photons in any arbitrary direction. Hence to avoid spontaneous emission, we take large detuning between atomic and field frequencies. The photon number in the field remains same during interaction as in large detuning only virtual transition take place. This implies that atom undergoes even number of Rabi oscillations. The atom is taken initially in ground state and after interaction it does not leave the cavity in excited state.

2.1.3 Basic phenomena

When atom interacts with cavity field a transfer of recoil momentum, Δp=ħk, takes place between atom and field. The field profile in the cavity is $\varepsilon_0 = \frac{1}{2}\varepsilon(e^{ikx} + e^{-ikx})$. Since the field has standing wave profile, the atom sees two light waves moving in opposite direction, one in its velocity direction and other opposite to it. Suppose atom absorbs a photon from the wave moving in its direction, it gets a recoil momentum +ħk to conserve momentum. Now in de-excitation it can emit the photon in either of the two waves. If it emits the photon in the wave moving in same direction, it gets a recoil momentum –ħk to conserve momentum. So the net change in momentum is zero in this case i.e. Δp=0. However if it emits photon in opposite wave then gets a recoil momentum +ħk, then, Δp=2ħk. This is the maximum momentum transfer to atom in one Rabi cycle. Hence the net momentum change of the atom is given as

$$p_l = p_0 + \ell\hbar k. \tag{2.1}$$

Here, ℓ is an even integer. Thus the atoms can scatter into any momentum components separated by momenta 2ħk.

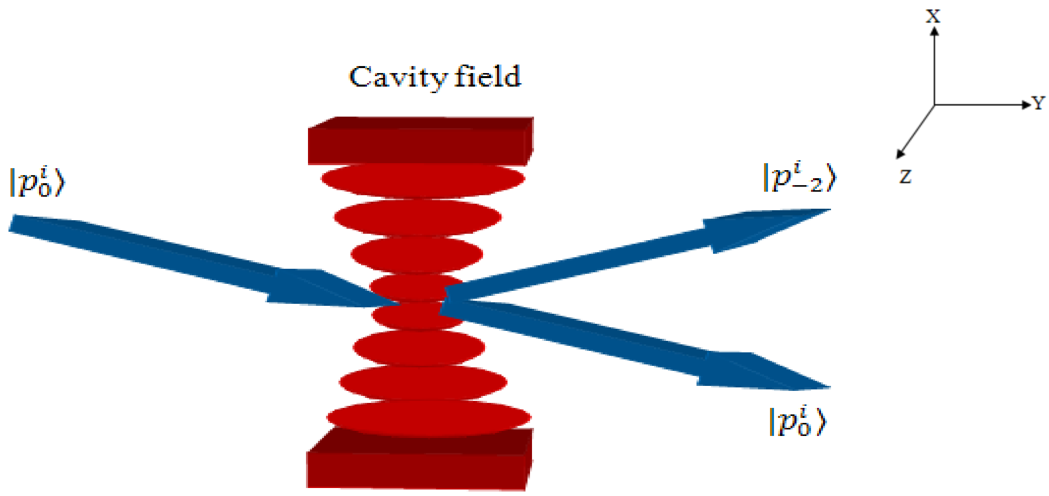

Fig 2.2: We show the dispersive interaction of atom with cavity field in the superposition state. For zero photon state their momentums remain same and for one photon state it gets a momentum kick.

The atomic scattering has two regimes depending on strength of recoil force, Bragg regime and Ramen-Nath regime. The Bragg regime is long interaction time regime [4, 5, 6] and is achieved when recoil energy is larger than energy of interaction [7, 8, 9]. The Ramen-Nath regime is complementary to Bragg regime, having short interaction time [10] and recoil energy is smaller than interaction energy [11].

2.1.4 Atomic scattering in Bragg Regime

The mechanical action of light on material particles is well known physical phenomenon. Bernhardt, B. W. Shore and R. J. Cook explain the atomic theory of Bragg diffraction from crystal [5, 6] and demonstrated experimentally [13, 14]. In 1988 Peter J. Martin et al. [15] demonstrated the Bragg diffraction of sodium atom. As the momentum splitting is coherent in

Bragg diffraction, so atomic mirror, beam splitters and atomic interferometer can be constructed [16, 17, 18, 19]. In optical Bragg scattering, light waves interacts with crystal and scatter from atomic plane of crystal. The Braggs scattering have two conditions

i) During the interaction, energy is conserved.

ii) The reflected waves interfere constructively only if it satisfies the condition,

$$2d\sin\theta = n\lambda. \tag{2.2}$$

Here, d is atomic spacing, λ is wavelength of incident light, n is order of Bragg diffraction and θ is the angle made by the incident beam with normal. From first condition, that is, energy conservation

$$\frac{P_{in}^2}{2M} = \frac{P_{out}^2}{2M}, \tag{2.3}$$

$$\frac{l(l+l_0)\hbar^2 k^2}{2M} = 0. \tag{2.4}$$

So either, l = 0, means that atoms goes undeflected or, $l = -l_0$, means that the momentum component along the propagation direction, of deflected atom, is reversed.

The second Bragg condition allows momentum transfer, during the interaction, is only for discrete values of the initial atomic momentum [8, 9, 20, 21].

$$P_0 = \frac{l_0}{2}\hbar k. \tag{2.5}$$

Where, l_0 = 2,4 and 6 etc. gives the first, second and third order Bragg scattering [6]. The order of the Bragg scattering is changed by changing the angle θ.

2.1.5 Effective Hamiltonian

In order to write the total Hamiltonian governing the interaction of the atom with the field, we treat the atom and the field quantum mechanically. The Hamiltonian for the quantized field is

$$H_F = \hbar\nu\,(\hat{a}^\dagger \hat{a} + \tfrac{1}{2}). \qquad (2.6)$$

where $\hat{a}^\dagger$ and $\hat{a}$ are field creation and annihilation operators. We take two level atom with ground state, $|b\rangle$, with energy E_b and excited state, $|a\rangle$, with energy E_a. The atomic Hamiltonian is

$$H_A = E_a\,|a\rangle\langle a| + E_b\,|b\rangle\langle b|, \qquad (2.7)$$

$$= \tfrac{1}{2}(E_a - E_b)\,(|a\rangle\langle a| - |b\rangle\langle b|) + \tfrac{1}{2}(E_a + E_b)\,(|a\rangle\langle a| + |b\rangle\langle b|).$$

Introducing the operators, $\sigma_Z = |a\rangle\langle a| - |b\rangle\langle b|$ the inversion operator, $\sigma_+ = |a\rangle\langle b|$($\sigma_- = |b\rangle\langle a|$) atomic raising (lowering) operator. Using $E_a - E_b = \hbar\omega$ and ignoring the constant energy term $\frac{1}{2}(E_a + E_b)$, also take $|a\rangle\langle a| + |b\rangle\langle b| = I$. We get

$$H_A = \frac{1}{2}\hbar\omega\sigma_Z. \qquad (2.8)$$

The atom-field interaction Hamiltonian is

$$H_i = -e\boldsymbol{X}_e.\mathbf{E} \qquad (2.9)$$

Here, $e\boldsymbol{X}_e$ is the dipole moment with $\boldsymbol{X}_e$ is the position vector of charge e and **E** is electric field intensity. The direction is chosen such that field is polarized along y-axis and is propagated along x-axis. So the electric field intensity is given as

$$\mathbf{E} = \mathcal{E}\,(\boldsymbol{X}_e + \mathbf{X})\,(\hat{a} + \hat{a}^\dagger)\cos(kx). \qquad (2.10)$$

Here, $\mathcal{E}$ is field amplitude, cos (kx) is field profile in cavity along x-axis. We consider dipole approximation, i.e. the field profile is same throughout the dimension of atom and take $\mathcal{E}(\boldsymbol{X}_e+\mathbf{X}) = \mathcal{E}(\boldsymbol{X})$. The interaction Hamiltonian under the dipole approximation is

$$H_i = -e\boldsymbol{X}_e\mathcal{E}(\mathbf{X})\ (\hat{a} + \hat{a}^\dagger \cos(kx)\ . \qquad (2.11)$$

Putting the identity, $|a\rangle\langle a|+|b\rangle\langle b| = I$, we get

$$H_i = -e(|a\rangle\langle a| + |b\rangle\langle b|)\boldsymbol{X}_e(|a\rangle\langle a| + |b\rangle\langle b|)\mathcal{E}(\mathbf{X})(a + a^\dagger)\cos(kx),$$

$$= -e(\wp_{ab}\hat{\sigma}_+ + \wp_{ba}\hat{\sigma}_-)\mathcal{E}(\mathbf{X})(\hat{a} + \hat{a}^\dagger)\cos(kx). \qquad (2.12)$$

With $\wp_{ab} = \wp_{ba}^* = e\langle a|X_e|b\rangle$, is the electric dipole moment matrix element. We define the Rabi frequency as $g = -{}^{\varepsilon\wp_{ab}}/_{\hbar}$, we get

$$H_i = \hbar(g\hat{\sigma}_+ + g^*\hat{\sigma}_-)(\hat{a} + \hat{a}^\dagger)\cos(kx). \qquad (2.13)$$

By rotating wave approximation, we drop the energy non-conservative term i.e. $a^\dagger\hat{\sigma}_+$ and $a\hat{\sigma}_-$**.** The total Hamiltonian for the system is

$$H = \frac{\hat{p}^2}{2M} + \hbar\nu\hat{a}^\dagger\hat{a} + \frac{\hbar\omega}{2}\sigma_Z + \hbar\ (g\hat{\sigma}_+\hat{a} + g^*\hat{\sigma}_-\hat{a}^\dagger)\cos(kx). \qquad (2.14)$$

Here, M is mass of atom. In case of large detuning between field frequency and atomic transition frequency, the derivative of the probability amplitude of the wave function with respect to time and position become vanishingly small. Drop the zero energy term, the effective interaction picture Hamiltonian under adiabatic approximation in which the wavelength and atomic size are the comparable and the field profile through the atom remain constant [22] is

$$H_{eff} = \frac{\hat{p}^2}{2M} - \frac{\hbar|g|^2}{2\Delta}\hat{n}\hat{\sigma}_-\hat{\sigma}_+(1+\cos 2kx). \qquad (2.15)$$

Here, $\Delta = \nu - \omega$ is detuning between the field frequency and atomic transition frequency and $\hat{n}$ is the field operator.

2.1.6 Generation of entangled state

We interact two super cooled two level atoms with cavity field dispersively, where cavity is in superposition state of zero and one photon. The wave function for the system under the above Hamiltonian, in product form, is

$$|\psi_{at.f}(t)\rangle = \frac{1}{\sqrt{2(2m+1)}}\sum_{l=-m}^{m}\sum_{l'=-m}^{m}[\, c_{0,p_l}^{(1)}(t)c_{0,p_{l'}}^{(2)}(\mathrm{t})|p_l^{(1)},p_{l'}^{(2)},0\rangle+$$

$$c_{1,p_l}^{(1)}(t)c_{1,p_{l'}}^{(2)}(\mathrm{t})|p_l^{(1)},p_{l'}^{(2)},1\rangle]. \quad (2.16)$$

c_{n,p_l}^{j}(t), is the probability amplitude for atom j=1, 2 at any time t, with momentum, p_l and, $p_{l'}$ in the presence of the field. The atoms evolution during interaction is given by Schrödinger equation

$$i\,\hbar\frac{\partial}{\partial t}|\psi_{at.f}(\mathrm{t})\rangle = H_{eff}\,|\psi_{at.f}(\mathrm{t})\rangle. \quad (2.17)$$

We find set of coupled rate equation for probability amplitude, $c_{n,p_l}^{(j)}$, viz

$$i\frac{\partial}{\partial t}c_{n,p_l}^{(j)}(\mathrm{t}) = -\omega_{rec}\,(\ell+\ell_0)c_{n,p_l}^{(j)}(\mathrm{t})\text{-}\,\frac{\chi^n}{2}\{c_{n,p_{l+i\hbar k}}^{(j)}(\mathrm{t})\text{-}\,c_{p_{n,l-2\hbar k}}^{(j)}(\mathrm{t})\}, \quad (2.18)$$

where $\omega_{rec} = \frac{\hbar k^2}{2m}$ is the recoil frequency of the atom and $\chi^n = \frac{|g|^2 n}{2\Delta}$ is the effective Rabi frequency. In Braggs scattering, the recoil frequency is much larger than effective Rabi frequency i.e. $\omega_{rec}>>>\chi^n$ [7, 10]. In this region conservation of energy provides ℓ= 0 and ℓ $=\ell_0$, which show that atom have two possible momentum propagation after scattering one is $p_0=\frac{l_0}{2}\hbar$k and other is $p_{-l_0}=\frac{-l_0}{2}\hbar$k [20, 21, 22]. Thus, we get set of coupled rate equation for Braggs region under these condition

$$i\frac{\partial}{\partial t}c_{n,p_0}^{(j)}(\mathrm{t}) = -\,\frac{\chi^n}{2}\{c_{n,p_2}^{(j)}(\mathrm{t})+\,c_{n,p_{-2}}^{(j)}(\mathrm{t})\}, \quad (2.19)$$

$$i\frac{\partial}{\partial t}c_{n,p_{-2}}^{(j)}(\mathrm{t}) = \omega_{rec}\,(-2)(-2+\ell_0)c_{n,p_{-2}}^{(j)}\text{-}\,\frac{\chi^n}{2}\{c_{n,p_0}^{(j)}+\,c_{n,p_{-4}}^{(j)}\}, \quad (2.20)$$

$$i\frac{\partial}{\partial t}c^{(j)}_{n,p_{-4}}(t) = \omega_{rec}\,(-4)(-4+\ell_0)c^{(j)}_{n,p_{-4}} - \frac{\chi^n}{2}\{c^{(j)}_{n,p_{-2}} + c^{(j)}_{n,p_{-6}}\}, \qquad (2.21)$$

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$$i\frac{\partial}{\partial t}c^{(j)}_{n,p_{-l_0+4}} = \omega_{rec}\,(-\ell_0+4)(4)c^{(j)}_{n,p_{-l_0+4}} - \frac{\chi^n}{2}\{c^{(j)}_{n,p_{-l_0+6}} - c^{(j)}_{n,p_{-l_0+2}}\}, \quad (2.22)$$

$$i\frac{\partial}{\partial t}c^{(j)}_{n,p_{-l_0+2}} = \omega_{rec}\,(-\ell_0+2)(2)c^{(j)}_{n,p_{-l_0+2}} - \frac{\chi^n}{2}\{c^{(j)}_{n,p_{-l_0+4}} + c^{(j)}_{n,p_{-l_0}}\}, \quad (2.23)$$

$$i\frac{\partial}{\partial t}c^{(j)}_{n,p_{-l_0}} = -\frac{\chi^n}{2}\{c^{(j)}_{n,p_{-l_0+2}} + c^{(j)}_{n,p_{-l_0-2}}\}. \qquad (2.24)$$

In the above set of equations, for $\ell = 0$ and $\ell = -\ell_0$, the diagonal term vanish. Under adiabatic condition i.e. large detuning case in which no transition occur we can ignore the time derivative of probability amplitude. Only retaining the lowest order term of $\frac{\chi^n}{2}$ in co-efficient and putting back, we get two coupled equation for $(l_0 > 2)$ as

$$i\frac{\partial}{\partial t}C^{(j)}_{n,p_{l_0}} = A_n C^{(j)}_{n,p_{l_0}}(t) - \frac{1}{2}B_n C^{(j)}_{n,p_{-l_0}}(t), \qquad (2.25)$$

$$i\frac{\partial}{\partial t}C^{(j)}_{n,p_{-l_0}} = A_n C^{(j)}_{n,p_{-l_0}}(t) - \frac{1}{2}B_n C^{(j)}_{n,p_0}(t),$$

where

$$A_n = \frac{\chi^{n}/2}{\omega_{rec}\;(l_0-2)(2)}\,.$$

$$|B_n| = \frac{\chi^{n\frac{1}{2}}}{(2\omega_{rec})^{\frac{l_0}{2}-1}\{(l_0-2)(l_0-4)\ldots\ldots 4.2\}^2}.$$

After solving these equations, we get

$$C^{(j)}_{n,p_{\pm l_0}}(t) = e^{-iA_n t}\{C^{(j)}_{n,\pm p_{l_0}}(0)\cos(\tfrac{1}{2}B_n t) + iC^{(j)}_{n,\mp P_{l_0}}(0)\sin(\tfrac{1}{2}B_n t)\}. \qquad (2.26)$$

To generate entanglement between the external degrees of freedom of atoms, 1 and 2, we prepare them in their linear momentum state $|p^1_{+l_0}\rangle$ and $|p^2_{-l_0}\rangle$. The initial conditions on atomic probability amplitude are

$$C^{(1)}_{n,p_{+l_0}}(0) = C^{(2)}_{n,p_{-l_0}}(0) = 1, \text{ and } C^{(1)}_{n,p_{-l_0}}(0) = C^{(2)}_{n,p_{+l_0}}(0) = 0.$$

Putting these values, we get

$$C^{(1)}_{n,p_{+l_0}}(\mathrm{t}) = C^{(2)}_{n,p_{-l_0}}(\mathrm{t}) = e^{-iA_n t}\cos(\tfrac{1}{2}B_n t). \tag{2.27}$$

$$C^{(1)}_{n,p_{-l_0}}(\mathrm{t}) = C^{(2)}_{n,p_{+l_0}}(\mathrm{t}) = i\, e^{-iA_n t}\sin(\tfrac{1}{2}B_n t). \tag{2.28}$$

The combined state of two atoms in their external degrees of freedom and cavity field states is

$$|\psi_{at,f}(\mathrm{t})\rangle = \frac{1}{\sqrt{2}}\left[|p^{(1)}_l, p^{(2)}_{l^`}, 0\rangle + \textstyle\sum_{l=+l_0,-l_0}\sum_{l'=+l_0,-l_0} C^{(1)}_{1,p_{l_0}}\, C^{(2)}_{1,p_{l'}}\, |p^{(1)}_l, p^{(2)}_{l'^`}, 1\rangle\right]. \tag{2.29}$$

This gives a three-partite entangled state. Both $A_n and\ |B_n|$ vanish for n = 0. Hence, for an interaction time t = sπ/B_1, s is an odd integer, leads as to atom-field entangled state. So

$$|\psi_{at,f}(\mathrm{t})\rangle = \frac{1}{\sqrt{2}}\left[|p^{(1)}_{+l_0}, p^{(2)}_{-l_0}, 0\rangle - e^{-i\varphi}|p^{(1)}_{-l_0}, p^{(2)}_{+l_0}, 1\rangle\right], \tag{2.30}$$

where the phase, φ = 2s$\pi A_1/B_1$, depends on order of Bragg scattering. The second atom interacts for a time t = 2rπ/B_1, where r is an even integer. So φ = 2(s+r)$\pi A_1/B_1$. The second atom is initially in its ground state and is in resonance with optical cavity field. The time of interaction correspond to half of the Rabi cycle. The state vector becomes

$$|\psi_{at,f}(\mathrm{t})\rangle = \frac{1}{\sqrt{2}}\left[|p^{(1)}_{+l_0}, p^{(2)}_{-l_0}, \mathrm{b}\rangle - e^{-i\varphi}|p^{(1)}_{-l_0}, p^{(2)}_{+l_0}, a\rangle\right]. \tag{2.31}$$

After passing the atom through a $\frac{\pi}{2}$ Ramsey pulse [28], leads to superposition $|b\rangle = \frac{1}{\sqrt{2}}[|b\rangle+|a\rangle]$ and $|a\rangle = \frac{1}{\sqrt{2}}[|b\rangle-|a\rangle]$. For probe atom in their ground state $|b\rangle$, we get the following Bell basis

$$|\psi_{at}(t)\rangle = \frac{1}{\sqrt{2}}\left[|p^{(1)}_{+l_0}, p^{(2)}_{-l_0}\rangle - e^{-i\varphi}|p^{(1)}_{-l_0}, p^{(2)}_{+l_0}\rangle\right]. \tag{2.32}$$

And for atom in their excited state $|a\rangle$, we get other bell basis

$$|\psi_{at}(t)\rangle = \frac{1}{\sqrt{2}}\left[|p^{(1)}_{+l_0}, p^{(2)}_{-l_0}\rangle + e^{-i\varphi}|p^{(1)}_{-l_0}, p^{(2)}_{+l_0}\rangle\right]. \tag{2.33}$$

Hence, controlling the interaction time, we get the other two Bell bases given below

$$|\psi_{at}(t)\rangle = \frac{1}{\sqrt{2}}\left[|p^{(1)}_{+l_0}, p^{(2)}_{+l_0}\rangle - e^{-i\varphi}|p^{(1)}_{-l_0}, p^{(2)}_{-l_0}\rangle\right]. \tag{2.34}$$

$$|\psi_{at}(t)\rangle = \frac{1}{\sqrt{2}}\left[|p^{(1)}_{+l_0}, p^{(2)}_{+l_0}\rangle + e^{-i\varphi}|p^{(1)}_{-l_0}, p^{(2)}_{-l_0}\rangle\right]. \tag{2.35}$$

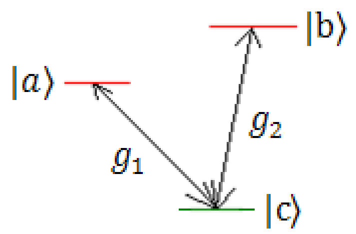
This theoretical suggestion can be realized experimentally by following the setup of references [20, 23]. In this process a Rubidium atom interact with cavity field in superposition state for 30ns. Later, pass another Rubidium atom for interaction time of 60ns. The life time of atom is 300ns. The life time of cavity is 600ns, which is greater than total experimental time.

2.2 Engineering of entanglement between cavities

Here, we discuss proposed scheme to engineer EPR-Bell state [2, 3] between different modes of electromagnetic field. The interaction of three-level atoms with cavity having two modes is desirable to investigate for two photon resonance transition [2].

2.2.1 The model

We use a three level atom in V-configuration; the transition between upper two levels is forbidden and interacts with cavity in vacuum state. The three levels of atoms are $|a\rangle$, $|\mathrm{b}\rangle$ and $|\mathrm{c}\rangle$ with their energies E_a, E_b and E_c respectively. The upper two levels are prepared in superposition with the help of Ramsey field before it enters into the cavity. We interact this atom with cavity having two modes A and B with velocity ω_A and ω_B.



The cavity have two modes A and B which are in resonance with transition frequencies i.e. $\omega_A = \frac{E_A - E_C}{\hbar}$ and $\omega_B = \frac{E_B - E_C}{\hbar}$. The time of interaction is equal to π Rabi cycle.

2.2.2 Creation of field entanglement

The atom we use is three-level in which the upper two levels are in superposition. Initially two cavity modes are vacuum. The initial state of the system is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}[|a\rangle + e^{i\varphi}|b\rangle]|0_A, 0_B\rangle. \tag{2.36}$$

The interaction Hamiltonian for the system under the dipole and rotating wave approximation can be written as

$$\mathrm{H} = \hbar g_1 \{a|a\rangle\langle c| + a^\dagger|\mathrm{c}\rangle\langle a|\} + \hbar g_2 \{\mathrm{b}|\mathrm{b}\rangle\langle c| + \mathrm{b}^\dagger|\mathrm{c}\rangle\langle \mathrm{b}|\}, \tag{2.37}$$

where, g_1 and g_2, are vacuum Rabi frequencies and $a^\dagger, \mathrm{b}^\dagger$ and a, b are cavity fields creation and annihilation operators. After interaction the atom-field state vector can be written as

$|\psi_{(A,B)}(t)\rangle = c_{a,0,0}|a,0,0\rangle + c_{b,0,0}|\mathrm{b},0,0\rangle + c_{c,1,0}|c,1,0\rangle + c_{c,0,1}|c,0,1\rangle.$ (2.38)

$c_{a,m,n}$, $c_{b,m,n}$, and, $c_{c,m,n}$, are probability amplitudes of atom in state $|a\rangle$, $|\mathrm{b}\rangle$, and, $|c\rangle$, while, m and n, are number of photons in modes A and B respectively. By Schrödinger equation, the rate equations for the probability amplitudes is

$$\frac{d}{dt} c_{a,0,0} = -i g_1 c_{c,1,0} \tag{2.39}$$

$$\frac{d}{dt} c_{c,1,0} = -i g_1 c_{a,0,0} \tag{2.40}$$

$$\frac{d}{dt} c_{b,0,0} = -i g_2 c_{c,0,1} \tag{2.41}$$

$$\frac{d}{dt} c_{c,0,1} = -i g_2 c_{b,0,0} \tag{2.42}$$

Solving equations (2.39) and (2.40), we get

$$\frac{d^2}{dt^2} c_{a,0,0} + \hbar g_1^2 c_{a,0,0} = 0. \tag{2.43}$$

Similarly, we can find the other probability amplitudes by solving the above rate equations. Their solution gives the atom-field entangled state as

$$|\psi_{(A,B)}(t)\rangle = \frac{1}{\sqrt{2}}[\cos(g_1 t)\,|a,0{,}0\rangle - i\sin(g_1 t)\,|c,1{,}0\rangle + e^{i\varphi}\cos(g_2 t)\,|\mathrm{b},0{,}0\rangle$$
$$-ie^{i\varphi}\sin(g_2 t)\,|c,0{,}1\rangle]. \tag{2.44}$$

After interaction, the atom is detected in ground state and takes equal probability amplitude to produce maximal entangled state i.e.

$$\sin(g_1 t) = \sin(g_2 t).$$

The interaction time can be calculated from probability amplitude given in equation (2.44). For an interaction time m$\pi/2g_1$ and n$\pi/2g_2$ (where m and n are odd integers) with modes A and B, the probability amplitude for detecting the atom in ground state is maximum. After interaction the atom develops an entangled state between two cavity field modes as

$$| \psi (\mathrm{A, B}) \rangle = \frac{-i}{\sqrt{2}}[|0_A, 1_B\rangle + e^{i\varphi}|1_A, 0_B\rangle]. \tag{2.45}$$

By adjusting the interaction time of atom with cavity field, we get another Bell state

$$| \psi (\mathrm{A, B}) \rangle = \frac{-i}{\sqrt{2}}[|0_A, 1_B\rangle - e^{i\varphi}|1_A, 0_B\rangle]. \tag{2.46}$$

To generate the other two bases we will not consider superposition in $|a\rangle$ and $|\mathrm{b}\rangle$. The transition from state $|a\rangle$ to $|c\rangle$ is in resonance with mode A and from $|\mathrm{b}\rangle$ to $|c\rangle$ is in resonance with mode B. By adjusting the interaction time such that atoms experiences $\frac{\pi}{2}$ pulse, hence there is equal probability for atom to remain in their ground state or in their excited state. After interaction, we get

$$| \psi (\mathrm{A, B}) \rangle = \frac{1}{\sqrt{2}}[|a, 0_A\rangle + |c, 1_A\rangle]\otimes|0_B\rangle. \tag{2.47}$$

Apply laser field to excite the atom from state $|c\rangle$ to state $|\mathrm{b}\rangle$ and have no effect if atom is in excited state $|a\rangle$. After interaction, the final state is

$$| \psi (\mathrm{A, B}) \rangle = \frac{1}{\sqrt{2}}\Big[|0_A, 0_B\rangle \pm |1_A, 1_B\rangle\Big], \tag{2.48}$$

which are the Bell maximally field entangled states. Now to engineer GHZ state, repeat this process using different exciting state. Thus we can develop GHZ entangled state

$$| \psi (\mathrm{AB.....N}) \rangle = \frac{1}{\sqrt{2}}[|0_A 0_B \ldots \ldots . . 0_N\rangle + |1_A 1_B \ldots \ldots . . 1_N\rangle]. \tag{2.49}$$

For the proposed scheme, we consider slow Rubidium atoms with life time of few milliseconds. The atoms are prepared in higher Rydberg states, and pass through high Q-superconducting cavity with a speed of 400m/s. The interaction time of atom is of order of few tens of microseconds, which is much smaller than cavity life time [2].

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Chapter 3

Engineering Entanglement for quantum networks

Entanglement provides a successful way to develop quantum channels and is, therefore, important in quantum communication, quantum cryptography, and quantum computation. The bipartite entanglement has been successfully generated between two electromagnetic cavities [1], multimode's of single cavity [2] and in internal [3, 4, 5, 6] as well as in external degrees of freedom of atoms [7] using Bragg diffraction regime. The Bragg regime cavity QED techniques have many interesting applications in quantum information, entanglement engineering [8, 9], teleportation [10] and in the development of quantum logic gates [11]. The entanglement in multipartite states, such as Noon state, W state, Cluster state and Graph state have also been reported [12, 13, 14]. The experimental advancements in generating quantum correlations in multipartite systems have enabled researchers to develop quantum networks based on cavity QED techniques [15] and colored Laser [16]. The atom-cavity system forms universal nodes capable of sending, receiving, storing and releasing photonic quantum information. The transfer of the quantum state and creation of the maximally entangled state are have also being demonstrated [17].

3.1 Introduction

In our proposed scheme, we suggested two techniques to generate multipartite entanglement using cavity QED. In the first technique, we develop entanglement in atomic degrees of freedom and in the second technique, we engineer entanglement among cavities. In the first scheme, we have two type of atoms tagged atoms and auxiliary atoms. We interact these atoms with two cavities in non-dispersive and dispersive fashion with cavity which is in superposition state. First, we dispersively interact two-level tagged atoms with the cavity fields, which entangled the tagged atoms in their external degrees of freedom with cavity field states. Later, we used two auxiliary atoms which interact in non-dispersive and dispersive fashion with the cavity fields. The states of the cavity fields transfer to auxiliary atoms states and we can erase the cavities from our system. Hence by auxiliary atoms interaction we transfer the atom-field entanglement to atom-atom entanglement. Now by atomic detection operator on the auxiliary atoms, we develop entanglement of the entangled states, among the tagged atom's external degrees of freedom and the auxiliary atom's internal states. In second scheme, we have three cavities system A, B and C. Cavity C has two modes C_1 and C_2 which are distinguishable. The two cavities, A and B, are in entangled state with the third cavity, C, in two modes, C_1 and C_2. This entanglement is engineered by non-dispersive interaction of two-level atom in their excited state. Now we interacts an atoms in λ configuration with cavity in ground state. The lower two-levels are in superposition state. After interaction the atoms is detected in their excited state. Hence we develop entanglement of the entangled states among the cavities.

3.2 Engineering entanglement of the entangled state in atomic degrees of freedom

3.2.1 The Model

Our proposed scheme for generation of multipartite entanglement in atomic external degrees of freedom is based on cavity QED techniques. We used two sets of atoms, one to engineering momentum entangled state with cavity fields called tagged atom and the other called auxiliary atoms which used to erase cavity information [13]. The tagged atoms are two level atoms, initially prepared in ground state $|g\rangle$, with transverse momentum $|p_0\rangle$, interacting dispersively with cavity fields which is in superposition of vacuum and one photon state. For simplicity, we take only the first order Bragg's diffraction regime. So the atomic momentum state is tagged with cavity field state through entanglement correlation. Auxiliary atoms are again two-level atoms, initially in ground state, undergo prescribe non-dispersive and dispersive interaction with cavity fields to erase the cavity information respectively.

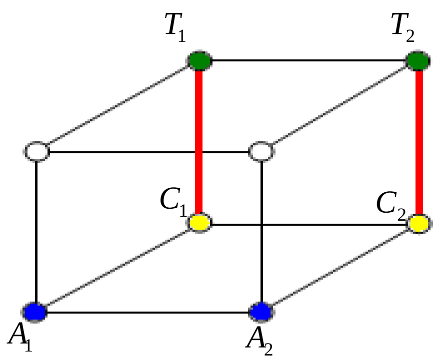

Fig 3.1 We show our basic model. The green edges show tagged atoms, Yellow show cavities, Blue corner show auxiliary atoms and red line show entanglement.

3.2.1 Tagging of atom in momentum space

The tagging procedure of atom in momentum space with cavity field, initially prepared in superposition of zero and one photon state i.e. $(|0\rangle+|1\rangle)/\sqrt{2}$ [18, 19, 20], is based on dispersive Bragg diffraction. When atom enters the cavity, its momentum does not change if the cavity is in state $|0\rangle$ and get a momentum kick when cavity is in state $|1\rangle$. The atom after its interaction for predetermine time, exit the cavity in its ground state with two equally probable discrete momentum states, $|p_0\rangle$, and, $|p_{-2}\rangle$ [7, 10, 21, 22]. The longitudinal component of momentum is quite large, whereas the transverse component of the momentum spread is negligible and treated quantum mechanically [21, 22]. Hence, due to two possible momentum split the cavity fields become entangled with the tagged atoms in their external degrees of freedom.

We consider the atom in its ground state $|g\rangle$, with momentum $|p_0\rangle$, interacting with cavity field in superposition state. The initial state vector for the system before interaction is

$$|\psi(0)\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\otimes|g,\ p_0\rangle. \tag{3.1}$$

The interaction Hamiltonian under dipole and rotating wave [23] approximation is

$$H = \hat{p}_x^2/2M + \hbar\delta\sigma_Z/2 + \hbar\mu\cos(kx)\,[\sigma_{eg}\hat{c} + \hat{c}^\dagger\sigma_{ge}]. \tag{3.2}$$

Here, $\hat{x}$ ($\hat{p}_x$) is the position (momentum) of center of mass of atom along x-axis, $\sigma_{eg}=|e\rangle\langle g|$ ($\sigma_{ge}=|g\rangle\langle e|$) is atomic raising (lowering) operator, $\sigma_Z=|e\rangle\langle e|-|g\rangle\langle g|$ is inversion operator, $\hat{c}$ ($\hat{c}^\dagger$) is field annihilation (creation) operator, δ is atom-field detuning, μ is vacuum Rabi frequency.

The state vector for the Bragg atom-field interaction at arbitrary time t, is

$$|\psi(t)\rangle = e^{-i\left(\frac{p_0^2}{2M}-\frac{\delta}{2}\right)}\sum_{\xi=-\infty}^{\infty}\{A_{0,g}^{p_\xi}(t)\,|0, g, p_\xi\rangle + A_{1,g}^{p_\xi}(t)\,|1, g, p_\xi\rangle$$

$$+ A_{0,e}^{p_\xi}(t)\,|0, e, p_\xi\rangle\}. \tag{3.3}$$

Here, ξ is summation for transverse atomic momentum during interaction, $A_{j,k}^{p_\xi}$ is probability amplitude with j = 0, 1 where j is the field state, k = e, g is atomic internal state and $|p_\xi\rangle$ is atom transverse momentum. For mathematical convenience we have introduced global phase factor. When detuning δ, is very large the spontaneous emission probability reduces and gives the persistence of atomic wave packet coherence when traveling.

Schrödinger equation for the system after interaction in the adiabatic approximation, for a time of interaction t = $\frac{2\pi\delta}{\mu^2}$ [17, 18, 19, 20, 21, 34, 35, 36] gives the following atom-field entangled state

$$|\,\psi\rangle = \frac{1}{\sqrt{2}}(|0,p_0\rangle + |1,p_{-2}\rangle)\,|g\rangle. \tag{3.4}$$

Thus, due to cavity field superposition state we get two possible momentum states for the atom which develops an entangled state between tagged atoms and cavity fields.

3.2.2 Interaction of auxiliary atoms

We interact two auxiliary atoms non-dispersively and dispersively with cavity fields which are in entangled state with tagged atoms to transform atoms-fields entanglement to atoms-atoms entanglement. The auxiliary atoms are taken initially in ground state, $|g_1\rangle$ and $|g_2\rangle$. The initial state of our system before interaction is

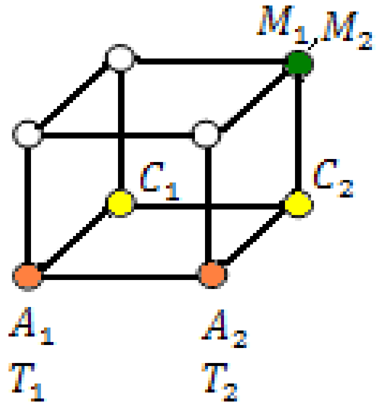

Fig 3.2: We show two tagged atoms, T_1 and T_2, each in initial entangled state with cavities C_1 and C_2 respectively. Two auxiliary atoms, A_1 and A_2, in their ground state $|g_1\rangle$ and $|g_2\rangle$ at orange edge (lower, front), the yellow (lower, back) edge show cavities (C_1 and C_2) and the green edge (upper, back) show measurement operators, M_1 and M_2.

$$| \psi(t_1)\rangle = \frac{1}{\sqrt{2}}[(\left|0_1, p_0^{(1)}\right\rangle + \left|1_1, p_{-2}^{(1)}\right\rangle) \otimes \left|g^{(1)}\right\rangle]\otimes\frac{1}{\sqrt{2}}[(\left|0_2, p_0^{(2)}\right\rangle$$

$$+ |1_2, p_{-2}^{(2)}\rangle)\otimes|g^{(2)}\rangle]. \tag{3.5}$$

Now in the next step, we interact the first auxiliary atom with first cavity non-dispersively followed by the Hamiltonian, H = ħ $\mu_r(\hat{\sigma}_{eg}\hat{c} + \hat{\sigma}_{ge}\hat{c}^{\dagger})$ under dipole and rotating wave approximation. Here, μ_r is atom-field coupling constant, $\hat{\sigma}_{ge}$ $(\hat{\sigma}_{eg})$ are atomic raising (lowering) operators. The state of the system, given in equation (3.5), after interaction is

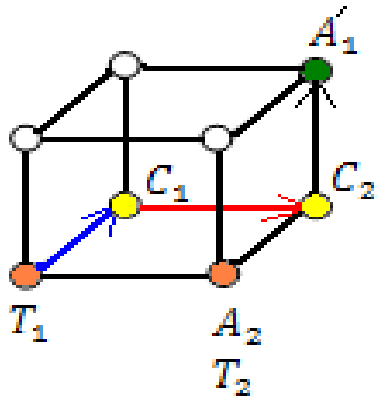

Fig 3.3: We show the Interaction of first auxiliary atom resonantly with cavity C_1 and dispersively with cavity C_2. By this interaction we transfer the state of first cavity to the first auxiliary atom.

$$|\,\psi(t_1)\rangle = \frac{1}{2}\{[|0_1, g^{(1)}, p_0^{(1)}\rangle + \cos(\mu_r t_1)\,|1_1, g^{(1)}, p_{-2}^{(1)}\rangle -$$

$$i\sin\mu_r\,|0_1, e^{(1)}, p_{-2}^{(1)}\rangle] \otimes (|0_2, p_0^{(2)}\rangle + |1_2, p_{-2}^{(2)}\rangle) \otimes |g^{(2)}\rangle\}. \tag{3.6}$$

The first cavity comes to vacuum state for an interaction time, $t_1 = \pi/2\mu_r$, of a π-Rabi cycle. As the state of the first cavity transfer to auxiliary atom-1, so we trace the first cavity. Hence, the state vector of the system expressed by equation (3.6) becomes

$$|\,\psi_1\rangle = \frac{1}{\sqrt{2}}\left(\left|g^{(1)}, p_0^{(1)}\right\rangle - i\left|e^{(1)}, p_{-2}^{(1)}\right\rangle\right) \otimes \left(\frac{1}{\sqrt{2}}|0_2, p_0^{(2)}\rangle + |1_2, p_{-2}^{(2)}\rangle\right) \otimes |g^{(2)}\rangle. \tag{3.7}$$

In the second step, auxiliary atom-1 interact dispersively with second cavity govern by Hamiltonian given by $H_d = \hbar\lambda\,(\hat{c}\hat{c}^{\dagger}|e\rangle\langle e| - \hat{c}^{\dagger}\hat{c}|g\rangle\langle g|)$. Here, $\lambda = \mu_d^2/\Delta$ is the effective Rabi frequency, Δ, atom-field detuning, and, μ_d^2, is the atom-field coupling constant [21]. This dispersive interaction with the second cavity develops the following atom-field entangled state

$$|\psi(t_3)\rangle = \frac{1}{2}\{|g^{(1)}, 0_2, p_0^{(1)}, p_0^{(2)}\rangle - e^{i\lambda t_2}|g^{(1)}, 1_2, p_0^{(1)}, p_{-2}^{(2)}\rangle$$
$$+ie^{-i\lambda t_2}|e^{(1)}, 0_2, p_{-2}^{(1)}, p_0^{(2)}\rangle - ie^{-2i\lambda t_2}|e^{(1)}, 1_2, p_{-2}^{(1)}, p_{-2}^{(2)}\rangle\}\otimes|g^{(2)}\rangle. \quad (3.8)$$

When auxiliary atom-1 leave the second cavity, we interact resonantly auxiliary atom-2 with second cavity, under the resonance interaction Hamiltonian, H = ħ $\mu_r(\hat{\sigma}_{ge}\hat{c} + \hat{\sigma}_{eg}\hat{c}^{\dagger})$. The state of the system for an interaction time, $t_3 = \pi/2\mu_r$, which equal to π-Rabi cycle is

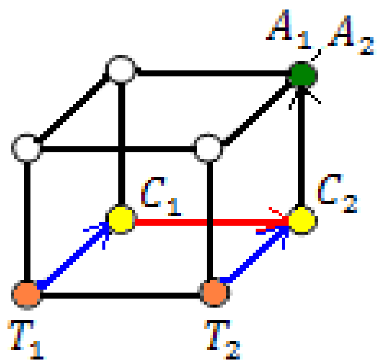

Fig 3.4: We show the resonant interaction of second auxiliary atom with cavity C_2. By this interaction we transfer the state of the second cavity to the second auxiliary atom. Hence we transfer the atom-field entanglement to atom-atom entanglement.

$$|\psi_2\rangle = \frac{1}{2}\left\{|g^{(1)}, g^{(2)}, p_0^{(1)}, p_0^{(2)}\rangle - ie^{i\lambda t_2}|g^{(1)}, e^{(2)}, p_0^{(1)}, p_{-2}^{(2)}\rangle\right.$$
$$\left. - ie^{-i\lambda t_2}\left|e^{(1)}, g^{(2)}, p_{-2}^{(1)}, p_0^{(2)}\right\rangle - e^{-2i\lambda t_2}\left|e^{(1)}, e^{(2)}, p_{-2}^{(1)}, p_{-2}^{(2)}\right\rangle\right\}. \quad (3.9)$$

3.2.3 Engineering entanglement of atomic entangled state

We apply the atomic detection operator needs to detect the atom in excited state. For this purpose we apply an ionization pulse to the atoms which ionize them when the atom is in excited state and have no effect if atom is in ground state. After ionizing the atom we use a detector which detects them. When detector detect no atoms it mean that both are in ground state we get non-entangled state $\left|p_0^{(1)}, p_0^{(2)}\right\rangle$. When both are detected in excited then again we have non-entangled state $\left|p_{-2}^{(1)}, p_{-2}^{(2)}\right\rangle$. But when detector detect only single atom then their either auxiliary atom-1 is in excited state or auxiliary-atom2 are in excited state. So applying the operator, $\frac{1}{\sqrt{2}}\{|e^{(1)}\rangle\langle e^{(1)}|+|e^{(2)}\rangle\langle e^{(2)}|$ and rearranging the equation (3.9) gives

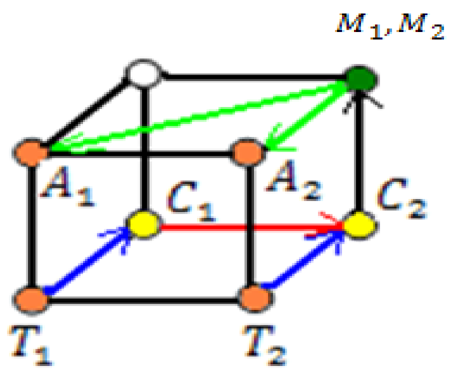

Fig 3.5: We show the state detection operation. we entangled auxiliary atoms in their internal degrees of freedom and tagged atom external degrees of freedom.

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\{-\mathrm{i}e^{-\mathrm{i}\lambda t_2}|g^{(2)}, p_{-2}^{(1)}, p_0^{(2)}\rangle|e^{(1)}\rangle - e^{-2\mathrm{i}\lambda t_2}|e^{(2)}, p_{-2}^{(1)}, p_{-2}^{(2)}\rangle|e^{(1)}\rangle$$

$$-\mathrm{i}e^{\mathrm{i}\lambda t_2}|g^{(1)}, p_0^{(1)}, p_{-2}^{(2)}\rangle|e^{(2)}\rangle - e^{-2\mathrm{i}\lambda t_2}|e^{(1)}, p_{-2}^{(1)}, p_{-2}^{(2)}\rangle|e^{(2)}\rangle\}, \quad (3.10)$$

$$= \frac{1}{\sqrt{2}}\{\left(-\mathrm{i}e^{-\mathrm{i}\lambda t_2}|g^{(2)}, p_0^{(2)}\rangle - e^{-2\mathrm{i}\lambda t_2}|e^{(2)}, p_{-2}^{(2)}\rangle\right)|e^{(1)}, p_{-2}^{(1)}\rangle$$

$$-\left(\mathrm{i}e^{\mathrm{i}\lambda t_2}|g^{(1)}, p_0^{(1)}\rangle - e^{-2\mathrm{i}\lambda t_2}|e^{(1)}, p_{-2}^{(1)}\rangle\right)|e^{(2)}, p_{-2}^{(2)}\rangle\}. \quad (3.11)$$

Here, each probability amplitude given in equation (3.11) depends on time. To get equal probability amplitude we choose time, $t_2 = \frac{\pi}{2\lambda}$, so

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}\{(-|g^{(2)}, p_0^{(2)}\rangle + |e^{(2)}, p_{-2}^{(2)}\rangle)|e^{(1)}, p_{-2}^{(1)}\rangle$$

$$+(|g^{(1)}, p_0^{(1)}\rangle + |e^{(1)}, p_{-2}^{(1)}\rangle)|e^{(2)}, p_{-2}^{(2)}\rangle\}. \qquad (3.12)$$

The resultant entangled state is in atomic internal and external degrees of freedom.

3.3 Engineering entanglement among cavities field

3.3.1 The Model

We consider a three partite system of Alice, Bob and Charles, such that, Alice-Charles and Bob-Charles are in one photon entangled states. The cavity C have two modes C_1 and C_2, which are distinguishable. To develop Alice-Charles and Bob-Charles entanglement we interacts two-level atom in their excited state. We interact this with cavity A and mode C_1 which entangled them. Similarly we interacts another atom in their excited state with Cavity B and mode C_2 and develop an entangled state between them.

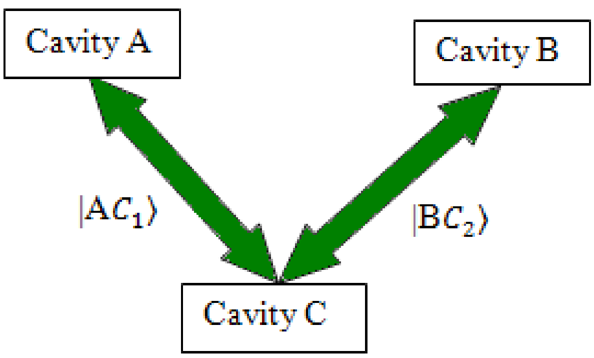

Fig. 3.6: We show the initial entangled states of the cavity A and mode C_1, and, cavity B and mode C_2.

3.3.2 Cavity field entanglement

In order to generate this entanglement, we let a two-level atom interact in its excited state $|e\rangle$, with a transition frequency ν_1 with cavity field initially in its vacuum state. The atom is initially in resonance with cavity field mode C_1, and after interaction with cavity fields A and C, is measured in its ground state$|g\rangle$. Therefore, it contributes a photon in either of the two cavities and leads to maximal entangled state [1], given as

$$|\mathrm{A}C_1\rangle = \frac{1}{\sqrt{2}}\left[|1_{\mathrm{A}}0_{\mathrm{C}_1}\rangle + |0_{\mathrm{A}}1_{\mathrm{C}_1}\rangle\right]. \tag{3.13}$$

We follow the same procedure, to develop entanglement between the cavity fields, B and C, we use second atom with transition frequency, ν_2, between the two levels and in resonance with cavity mode C_2. This gives us another maximum entangled state as

$$|\mathrm{B}C_2\rangle = \frac{1}{\sqrt{2}}\left[|1_{\mathrm{B}}0_{\mathrm{C}_2}\rangle + |0_{\mathrm{B}}1_{\mathrm{C}_2}\rangle\right]. \tag{3.14}$$

The state vector for the whole system can be written as

$$|\mathrm{A}BC_1C_2\rangle=\frac{1}{2}\left[|1_A1_B0_{C_1}0_{C_2}\rangle + |0_A0_B1_{C_1}1_{C_2}\rangle + |0_A1_B1_{C_1}0_{C_2}\rangle + |1_A0_B0_{C_1}1_{C_2}\rangle\right]. \tag{3.15}$$

Eq. (3.15) have four terms in which first term show no photon of either of two modes of cavity C, second term show that each mode of cavity C have one photon, third term show one photon in C_1 with no photon in C_2, and the last term show that no photon in C_1 with one photon in C_2. All term have same probability of occurrence.

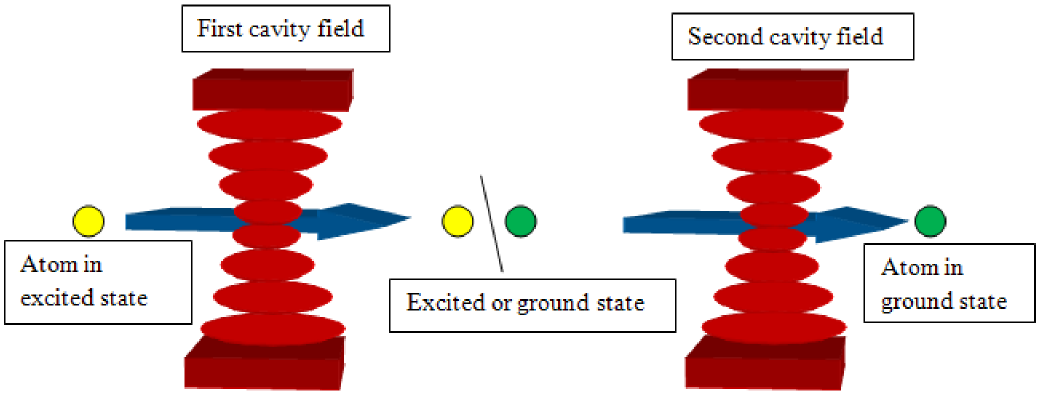

Fig 3.7: We show the resonance interaction of atom initially in their excited state with cavity fields. Finally the atom is detected in their ground state. After interaction with first cavity the atom have equal probability to remain in ground state or excited state. When passes through second cavity through resonance interaction atom is detected in ground. So by this way the two cavities get entangled.

3.3.3 Engineering entanglement of field entangled state

To develop entangled state among the entangled cavity fields we apply field detection operator on cavity C, when the two mode of cavity C are taken distinguishable. The initial state of the system is expressed by Eq. (3.15).

To engineer entanglement of the entangled states, Charles used three level atoms in Λ configuration. The atom is initially prepared in superposition of lower levels $|1\rangle$ and $|2\rangle$. The atom transition from level $|1\rangle$ to $|3\rangle$ is in resonance with mode C_1 and $|2\rangle$ to $|3\rangle$ is in resonance with C_2. After its interaction, Charles measures the atom in excited state. The detection process, $\frac{1}{\sqrt{2}}\,\{|1_{C_1}\rangle\langle 1_{C_1}| + |1_{C_2}\rangle\langle 1_{C_2}|\}\,|\mathrm{AB}C_1C_2\rangle$, leads to multipartite entangled state, expressed as

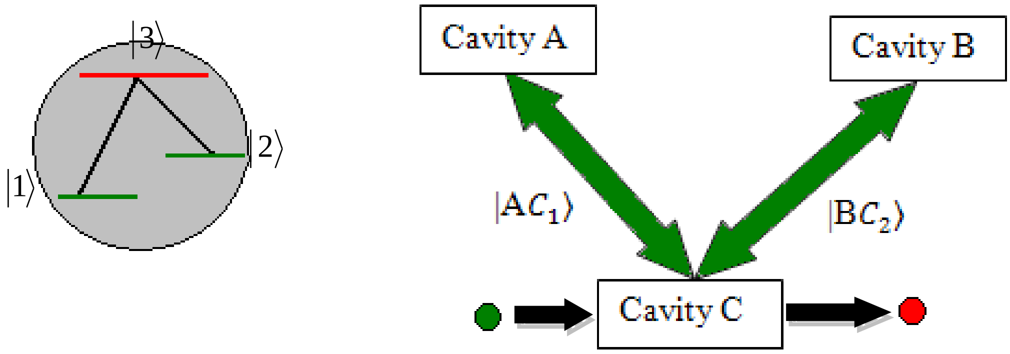

Fig 3.8: Scheme diagram for entanglement of entangled state. We prepare a three-level atom with superposition of lower two levels with help of Ramsey field. Pass this atom through cavity C having two modes.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\{|1_{C_1}\rangle\langle 1_{C_1}| + |1_{C_2}\rangle\langle 1_{C_2}|\}|\mathrm{AB}C_1C_2\rangle,$$

$$= \frac{1}{2}[|0_A 1_{C_1}\rangle|\mathrm{B}C_2\rangle + |\mathrm{A}c_1\rangle|1_{C_2}0_B\rangle],$$

$$= \frac{1}{2}[|0_A 1_{C_1}\rangle|\varphi\rangle + |\varphi\rangle|1_{C_2}0_B\rangle]. \qquad (3.16)$$

Where, $|\varphi\rangle$ is already entangled state given in equation (3.13) and (3.14).

Here, we develop entanglement of entangled state among atomic degrees of freedom and among fields. We also generalize the idea for atomic degrees of freedom to n-partite which use for quantum networks.

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CHAPTER 4:

Results and Conclusions

For multipartite entanglement in atomic degrees of freedom, we use two tagged atoms which, at first, make entangled states with two cavities. Later, we pass two auxiliary atoms in ground state through cavity fields to transform cavity field states to atoms. In the last, by atomic detection operator, if atom is detected in excited states and we get entanglement of the entangled state in atomic degrees of freedom. In our proposed scheme for optical cavity system, we have three party systems Alice, Bob and Charles. Here, Charles is separately entangled with Alice and Bob in different optical modes. By apply field detection operator at third party Charles, we entangled the already entangled states. The generation of this state requires that we detect only one photon in the indistinguishable modes.

4.1 Generalization and experimental parameters

We can easily generalize our work to N parties and the procedure leads to quantum networking. We use two sets of N-atoms, one to engineer momentum state entanglement with cavity field, called tagged atoms and the other to erase cavity information, called auxiliary atoms. First, we interact each tagged atoms with fields which gives an atom-field entangled state. Now interact first auxiliary atom non-dispersively with cavity C_1 and then dispersively with cavity C_2. We follow the same procedure for second auxiliary atom with cavity C_2 and C_3, and so on. The nth auxiliary atom interacts non-dispersively with cavity C_n, and hence the state of all cavities transfers to the auxiliary atoms. By applying detection operator on auxiliary atoms we can develop a network among N-party.

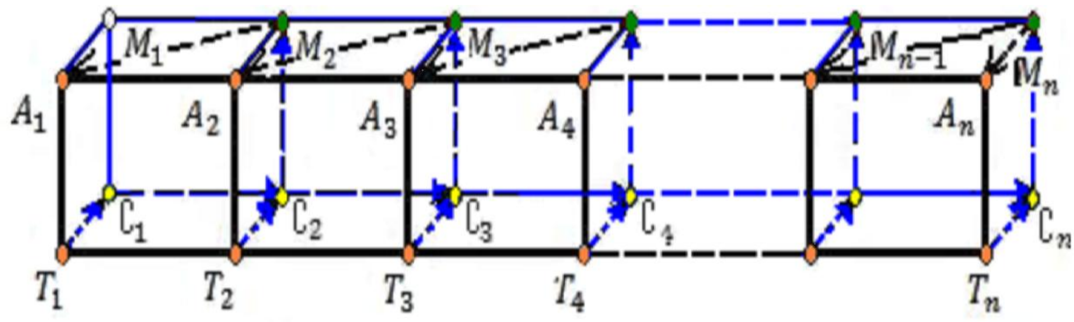

Fig 4.1: We show generalization of our work to N-parties in atomic degree of freedom. Here, each atom interacts resonantly with first cavity and dispersively with second cavity except the last one which interact resonantly with last cavity only. After interaction fields state transfer to auxiliary atoms state and by applying detection operator we can develop entangled state.

The atom-cavity system form universal nodes capable of exchange of quantum information. The transfer of the atomic quantum state and creation of the entanglement between two nodes, separated in independent laboratories at a distance of 21m and are connected by optical fiber link of length 60m, are also demonstrated experimentally [1]. To generate cavity QED entangled state we use high Q-cavities of few centimeters in length. The atoms used are of two atomic levels as circular Rydberg level [2] which has large radioactive decay time and is strongly coupled to microwave. The interaction time of atom moving with velocity of 400m/s is of order of microsecond. The cavity life time is of few milliseconds, so that, atom does not undergo radioactive decay when passing through cavity.

For the entanglement in atomic degrees of freedom, we use high Q-cavities having a life time of up to a few seconds which is greater than atom-field interaction time [3]. Microwave regime is used with the atom-field interaction is of microsecond [4]. The momentum deflection of atoms is very small so we use long arm interferometer to get sufficient separation. We can use off-resonant Braggs scattering of 4He atoms (λ= 1083nm) up to sixteen order diffraction [5]. The measurement process needs to detect the atom in the excited state. For this purpose we apply a field which ionizes the atoms when in their excited states and has no affect when in their ground states. This ionized atom passed through the field where it is deflected and detected [6].

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