Normal Modes in the Ion beam-Dusty Plasma System

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Dedicated to my beloved Parents and sister(Late)

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Abstract

The existence of linear ion acoustic waves in an un-magnetized ion beam dusty plasma with Cairns distributed electrons is discussed. The ion acoustic waves are investigated in the presence of massive negatively charged dust particles, known to exist in comets and planetary rings. These dust particles have a significant effect on charge neutrality. Study shows that three normal modes exist for an electron ion plasma for large beam speed, however the presence of dust modifies the ion acoustic mode to dust ion acoustic mode. The dispersion relation derived here for low frequency electrostatic perturbations indicates the presence of three stable modes the "Dust Ion acoustic", "Fast" and "Slow" modes for large beam speed. Two stream instability in an un-magnetized multi species dusty plasma are investigated. Results show that presence of non thermal electrons increases the phase velocity of dust ion acoustic waves and an increase in ion beam velocity increases the phase velocity of Fast and Slow modes. The theoretical results may be important in laboratory ion beam driven plasmas and in space plasma where charged dust is considered as an impurity.

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Chapter 1

Introduction

1.1 Plasma

Matter can exist in three different phases namely solid, liquid and gas. The transition from one phase to another is described in terms of temperature, as the temperature increases the molecules and atoms have greater freedom of movement. In gaseous state when atoms are further heated, they have enough energy that they decompose themselves into electrons and ions. This state of gas is known as plasma.

"A plasma is a quasineutral gas of charged and neutral particles which exhibits collective behavior."

In the universe ninety-nine percent of matter is in the plasma state. This is not a precise estimate but it is rational because stellar interiors and atmospheres, nebula and a large part of interstellar hydrogen are in the state of plasma. Dust is one of the common ingredients of the plasma available in the universe, for this reason in many cases plasma coexists with the dust particles [1].

1.2 Plasma in nature

1.2.1 Solar wind

The flow of solar plasma in an ionized state and a residue of solar magnetic field that extends throughout the interplanetary space is known as the solar wind. Large difference in gas pressure between interstellar space and solar corona results in a solar wind. Regardless of the constraining behavior of solar gravity, this difference in pressure move plasma outward. Solar activity(prominent event on the sun) produces minute variations in the magnetic field of earth, which caused us to believe the existence of solar wind in 1950's. In mid 1960's, this phenomenon was observed by space probes. Measurements grabbed by spacecraft-borne devices have provided a thorough detail of solar wind over an area from inside Mercury orbit extended upto the orbit of Saturn [2].

1.2.2 Ionosphere

The planetary ionosphere requires just a neutral atmosphere and any source of ionization for the gases present in the atmosphere. The sources of ionization involve photons and energetic particles (precipitation). The process involving photons is known as photoionization and one which involves energetic particles is known as impact ionization. These photons usually originate from the sun. The energetic particles required for impact ionization can originate from the sun, galaxy (cosmic rays), magnetosphere or even from the ionosphere if a procedure for electron or local ion acceleration is operational. The energy of the ionizing particle and photons should exceed a neutral atmosphere's molecular or atomic electron's binding energy or ionization potential. All of these sources provide atmospheric ionization but the most common is solar photons. These photons with the ultraviolet and extreme ultra violet wavelength of about 10nm to 100 nm normally produce most of the planet's dayside ionosphere [3].

1.2.3 Magnetosphere

A magnetosphere is formed when solar wind interacts with the magnetic field of earth. The separation between the shocked solar wind and the magnetosphere is known as magnetopause. The magnetosphere is stretched in a direction opposite to the sun, in such a way that a tail is formed known as magnetotail. The stretching of the magnetosphere is caused due to solar winds kinetic pressure, due to which the front side of the magnetosphere is compressed while the backside is stretched. Magnetospheric plasma mainly consists of electrons, protons and He++ ions that comes from the solar wind. In addition to this, due to the terrestrial ionosphere, magnetosphere also has a fraction of He+ and O+ ions. The magnetospheric plasma is not evenly distributed but consists of different regions, each of which has a different temperature and density [4].

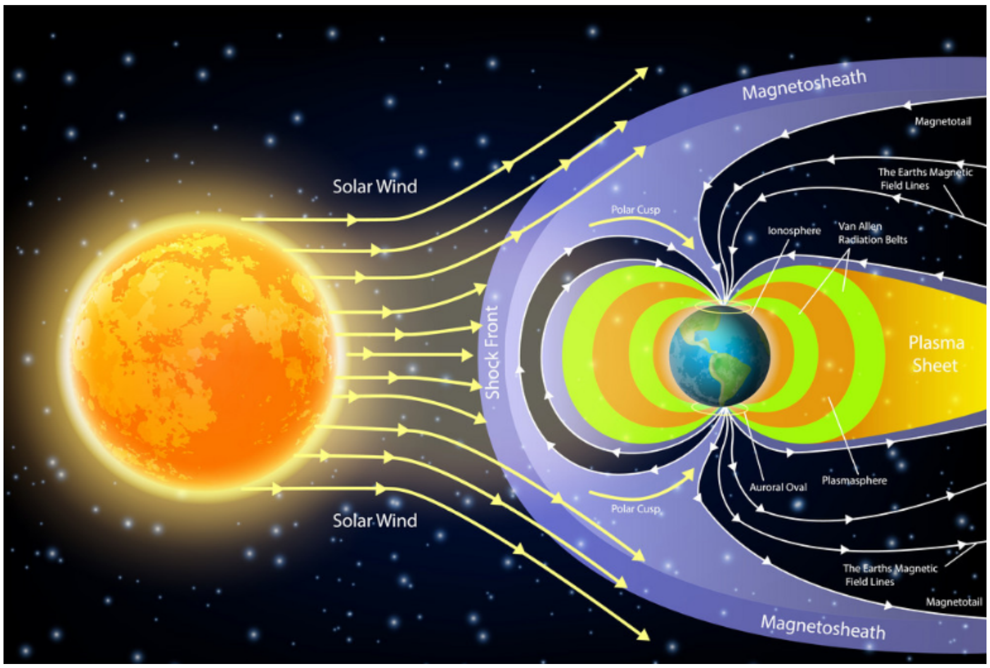

Figure 1.1: Solar wind and Magnetosphere

1.3 Characteristics of Dusty Plasma

“Dusty plasmas are defined as low-temperature partially or fully ionized electrically conducting gases whose components are charged dust grains, electrons, ions and neutral atoms.”

Sizes of dust grains are different which span over nanometers to millimeters and they are massive enough as compared to protons and electrons. Dust grains are not neutral, they are either positively or negatively charged mainly depends on the plasma exists in that environment. Dust particles can be conducting or metallic and sometimes made up of ice particulates. Plasma with the dust grains or the dust particles can be named as either 'dusty plasma' or 'dust in a plasma' which depends on the ordering of a few characteristic lengths. These lengths are plasma Debye radius(λ_D), dust grain radius(r_d), average inter-grain distance(a) and dusty plasma dimension [7].

Dust in Plasma - When $r_d << \lambda_D < a$, the charged dust particles are taken as a collection of isolated screened grains. Whenever a plasma having isolated dust grains is considered,local plasma inhomogeneities must be taken into account.

Dusty Plasma - When $r_d << a < \lambda_D$, the charged dust particles take part in collective behavior and is known as dusty plasma. Dust grains must be treated like massive charged particles just like multiply charged positive or negative ions [26].

Various methods for charging of dust grains exist. Most usual in laboratory dusty plasmas is electron bombardment, other than this radioactive decay and photo-ionization can be an important means of charging for certain astrophysical and space situations. Photo-ionization will cause the electrons to leave dust grain surface that makes dust grains positively charged. Radioactive decay rather makes dust grains have a polarity opposite to the polarity of an emitted particle in decay process e.g; dust grains will be negative if an alpha particle is emitted.

In early 1980s, Voyager images showed phenomena in Saturn rings that could not be described on just gravitational grounds. The Voyager missions discovered braids in the F-ring and spokes in the B-ring. Circumsolar dust rings, flame of a candle and noctilucent clouds in arctic troposphere, cometary comae and tails are some more examples in solar system of dusty plasma. Table 1.1 shows the contrast between electron ion and dusty plasma for different parameters [1].

Table 1.1. The basic differences between electron–ion and dusty plasmas.

Characteristics	Electron–ion plasma	Dusty plasma
Quasi-neutrality condition	$n_{e0} = Z_i n_{i0}$	$Z_d n_{d0} + n_{e0} = Z_i n_{i0}$
Massive particle charge	$q_i = Z_i e$	$\lvert q_d \rvert = Z_d e \gg q_i$
Charge dynamics	$q_i =$ constant	$\partial q_d / \partial t =$ net current
Massive particle mass	m_i	$m_d \gg m_i$
Plasma frequency	ω_{pi}	$\omega_{pd} \ll \omega_{pi}$
Debye radius	λ_{De}	$\lambda_{Di} \ll \lambda_{De}$
Particle size	uniform	dust size distribution
$\boldsymbol{E} \times \boldsymbol{B}_0$ particle drift	ion drift at low B_0	dust drift at high B_0
Linear waves	IAW, LHW, etc	DIAW, DAW, etc
Nonlinear effects	IA solitons/shocks	DA/DIA solitons/shocks
Interaction	repulsive only	attractive between grains
Crystallization	no crystallization	dust crystallization
Phase transition	no phase transition	phase transition

1.3.1 Macroscopic neutrality

Dusty plasma is macroscopically neutral like electron- ion plasma if no external disturbance exists. This tells that in equilibrium when no external forces are present, dusty plasma net resulting electric charge is zero. Hence in dusty plasma, equilibrium charge neutrality condition says

$$q_i n_{i0} = e n_{e0} - q_d n_{d0}. \tag{1.3.1}$$

where n_0 shows unperturbed number density of plasma species (i for ions, d for dust grains,e for electrons), $q_i = Z_i e$ represents ion charge, $q_d = Z_d e(-Z_d e)$ is dust particle charge when grains are positively or (negatively) charged, Z_d represents amount of charges residing on dust grain surface. $Z_d n_{do}$ could be compared to n_{io}, even for $n_{d0} << n_{i0}$ because a dust grain possesses one thousand to a range of hundred thousand elementary charges. Though in many space and laboratory plasma cases, when charging process occurs majority of electrons could stick on surface of dust grain and as a consequence, a notable depletion of electron number density in dusty plasma might take place. Therefore while considering negatively charged dust grains equation becomes

$$n_{i0} \approx Z_\mathrm{d} n_{\mathrm{d}0}. \tag{1.3.2}$$

[1, 10].

1.3.2 Debye Shielding

"The ability to shield electric field of an individual charged particle or of a surface which is at non-zero potential is known as debye shielding."

This provides a Debye radius which is shown as

$$\lambda_{\mathrm{D}} = \frac{\lambda_{\mathrm{De}}\lambda_{\mathrm{Di}}}{\sqrt{\lambda_{\mathrm{De}}^2 + \lambda_{\mathrm{Di}}^2}}. \tag{1.3.3}$$

The term λ_D is a measure of thickness of the sheath or shielding distance. For dust grains that are negatively charged $n_{e0} \ll n_{i0}$ and $T_e \geq T_i$, i.e., $\lambda_{De} \gg \lambda_{Di}$ accordingly $\lambda_D \simeq \lambda_{Di}$. This shows temperature and ions number density determines the shielding distance or the thickness of sheath of a dusty plasma. Whereas for positively charged particles and majority of ions are attached to dust grain surface, i.e., when $T_e n_{i0} \ll T_i n_{e0}$, $\lambda_{De} \ll \lambda_{Di}$ this gives $\lambda_D \simeq \lambda_{De}$. It can be concluded that temperature and electrons density determine shielding distance in a dusty plasma with positively charged dust grains [1, 7].

1.3.3 Characteristic Frequency

A dusty plasma is characterised by the stability of its macroscopic space charge neutrality just like electron-ion plasma. In case if plasma is instantaneously disturbed from equilibrium, collective particle motions take place due to resulting internal space charge field and this tends to rebuild original charge neutrality. Natural frequency of oscillations called plasma frequency characterizes the collective motions. Considering unmagnetized, uniform and cold plasma where the momentum equation, continuity equation and Poisson's equation describes the electrostatic oscillations of dust particles, electrons and ions, we get

$$\frac{\partial^2}{\partial t^2}\nabla^2\phi + 4\pi \sum_s \frac{n_{s0} q_s^2}{m_s}\nabla^2\phi = 0, \tag{1.3.4}$$

where the subscript s stands for ion, electron and dust. Integrating the above equation, we get

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}t^2} + \omega_{\mathrm{p}}^2\phi = 0, \tag{1.3.5}$$

where

$$\omega_{\mathrm{p}}^2 = \sum_s \frac{4\pi n_{s0} q_s^2}{m_s} = \sum_s \omega_{ps}^2. \tag{1.3.6}$$

1.4 Dusty Plasmas in Space

Hannes Alfven (Nobel Laureate) has always advocated existence of dust in solar nebula. In solar nebula, the coagulation of dust particles would have surely led to planetesimals from where planets and comets originate. Various physical properties of dust charges like charge, density, size, mass, depends on their surroundings and origin. There are various sources of dust particles in solar system such as man-made pollution, micrometeoroids, lunar ejecta, space debris etc. In the following few sections we will explain some significant characteristics of dust grains and their plasma environments in different domains of the solar system that is comets, Earth's atmosphere and interplanetary space [11, 12].

1.4.1 Interplanetary Space

" Interplanetary space is filled up with dust which is well known as interplanetary dust."

First instinct of the interplanetary dust grains came through zodiacal light. Zodiacal light happens because of dust grains dispersed all through inner solar system, with involvement of asteroid belt. The decay due to collisional fragmentation of debris originating from the comets might be the cause of these dust particles. Asteroids generate most of the interplanetary dust in the time when mutual collisions occur in asteroid belt. Dust particles present in interplanetary space has fluffy and fragile appearance. Figure 1.2 shows outside region of these interplanetary dust grains. Few of such particles are enough fragile to disintegrate into hundreds or dozens of fragments when these particles impact collector surface. These particles of dust are mostly rich enough in carbon.

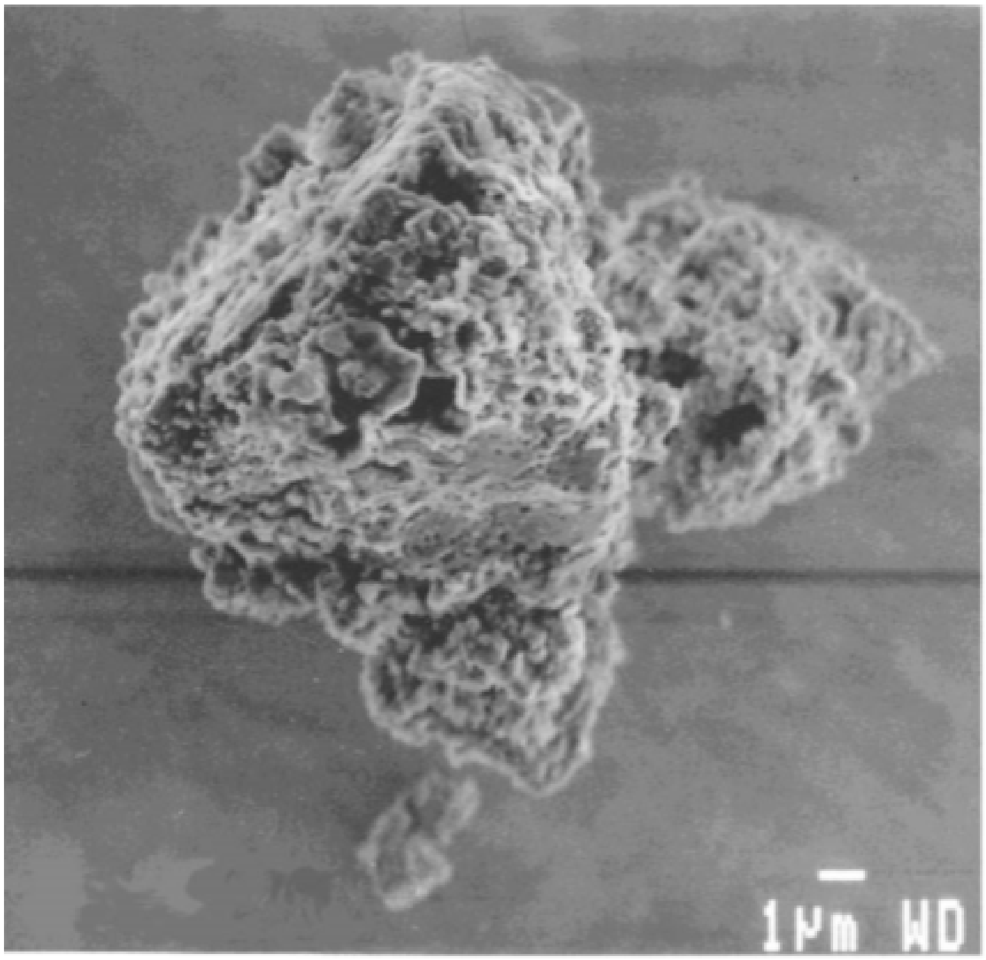

Figure 1.2: Sight of interplanetary dust particles

1.4.2 Comets

" Comets are brittle, small and asymmetrical shaped bodies that are made up of an admixture of frozen gases and non-volatile grains."

As comets advance toward Sun, they adapt extensive tails of radiant material which expands for millions of kilometers in anti-sunward direction from head. When it is far from the sun,it is understood nucleus is cold so the material within nucleus is frozen solid. When comet extends within few atomic unit of Sun, the nucleus surface starts warming and volatiles evaporate. These evaporated molecules begin to boil off and take with them little solid particles, which forms comets coma of dust and gas. The frozen nucleus can only be seen by reflected Sunlight. Nonetheless, dust reflects more Sunlight when a coma is developed and gas present in coma absorbs the ultraviolet radiation and starts to glow.

Figure 1.3: Comet Hale-Bopp showing two distinct tails

1.4.3 Planetary rings

Majority of the rings of outer giant planets (like Uranus, Neptune, Jupiter, Saturn) are basically made up of dust particles ranging from micron- to submicron-size.

Ring system of Jupiter

Voyager 1 discovered Jupiter's ring (a single picture) which was explicitly targeted to look for a faint ring system. Later on, Voyager 2 provided the complete set of images. Jupiter's ring has three vital components named as main ring, gossamer and halo ring. Orbits of two little moons Metis and Adrastea are encompassed by main ring, this might act as a source of dust which makes up most part of ring.

Ring System of Saturn

In 1610 Galileo first discovered rings of Saturn and since then it has puzzled the astronomers. These puzzles have increased since when in 1980 and 1981, Voyagers 1 and 2 provided images of ring system. These rings are named in order in which they are discovered. Particles in Saturns rings are made up of mainly ice and their sizes range from micron to meters. The images provided by Voyagers 1 and 2 showed fascinating features in Saturns ring system , these radial spokes motivated for study of dust-plasma interactions in region of planetary magnetospheres. Spokes are restricted to dense central B ring having outer edge at 1.95 R_s(R_s represents the radius of Saturn) and inner edge at 1.52 R_s. Spoke model is built on presumption that spokes have electrostatically levitated sub-micron to micron sized dust particles. A random spoke pattern is shown in figure 1.4, these spokes show typical wedge shape.

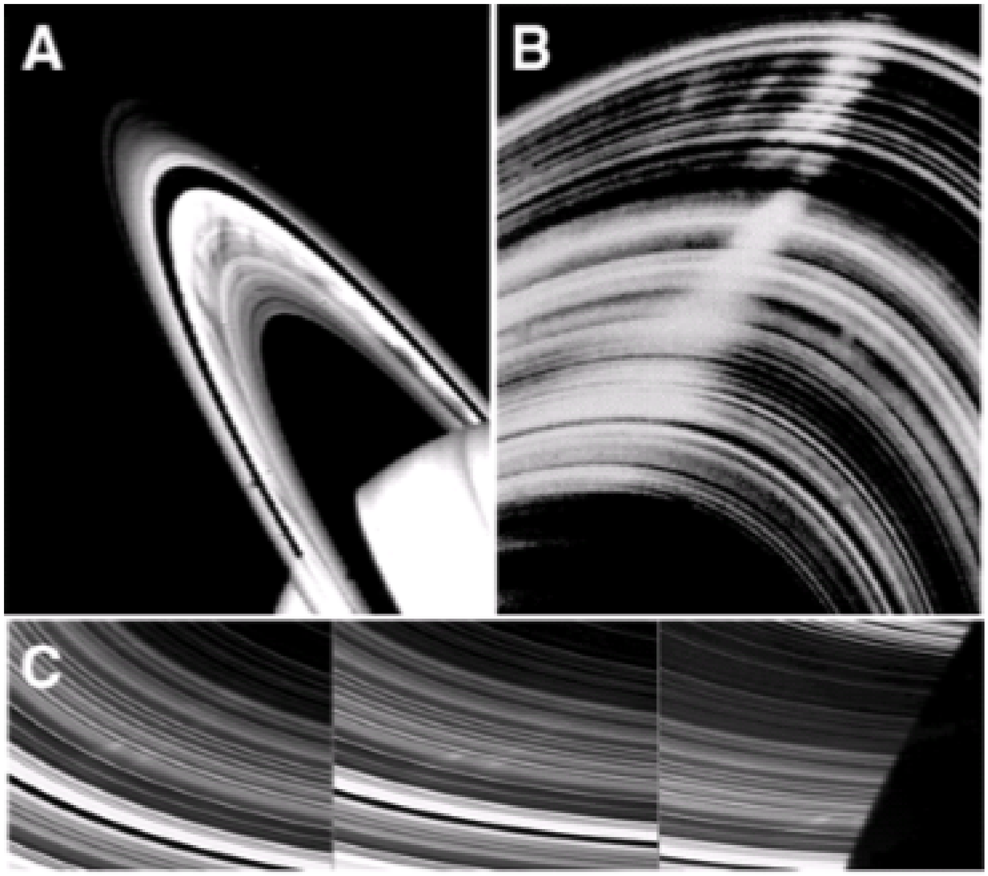

Figure 1.4: Spokes in Saturns B ring observed in 1980 by Voyager 2 (a) and (b), and in 2005 by Cassini (c). The three Cassini images cover a span of 27 min.

Ring system of Uranus

In 1977 during an inspection of a stellar occultation by a planet, Uranian ring system was discovered. The planet was encircled by five rings, this was indicated when the star blinked five times before planet and once again five more times afterward. In 1986 Voyager spacecraft took various images which provides further occultation of the ring system. A handful of additional rings were detected by Voyager cameras which showed nine prominent rings that are occupied by belts of fine dust particles.

Ring system of Neptune

Neptune has an external ring system. Instead of complete arcs, the earth based observations exhibited faint arcs. In 1989 Voyager 2 images exhibited these rings to be called complete rings having bright clumps. Out of four, one of the ring has a mysterious twisted structure. Neptune's rings are dark just like rings of Jupiter and Uranus. Small micrometer sized dust grains were detected by plasma wave instruments onboard Voyager 2. Data further disclosed that each ring plane has a severe broadband burst of noise which extends below 10Hz to above 10 KHz. Low-frequency noises in Neptune's atmosphere that has a strange orientation of magnetic fields might be due to the dust charges [13].

1.4.4 Earth's atmosphere

Most valuable part of Earth's environment where dust charge particles are seen is polar summer mesopause located somewhere between 80 and 90 km in altitude. Formation of a unique cloud called 'noctilucent cloud' (NLCs) has been a significant event observed in polar summer mesopause. These clouds were first reported in 1885 and have always known to be different from the rest of the clouds. During International Geophysical Year(1957-58)the launched rocket grenades discovered another property of polar mesopause that it was colder in summers as compared to winters. This discovery supported the opinion that NLCs were made up of ice which formed at exceptionally low temperatures surprisingly below 100 Kelvin [1, 6].

1.5 Waves in Plasma

Waves are significant as waves propagate energy from one point to another point. Information is transmitted out of plasma through waves so a spectator can infer what is happening inside the plasma. Waves can either grow or decay and can either increase the plasma energy or drive energy out of it. The number of feasible waves in plasma is larger than normal fluid because plasma has a minimum two fluids, as sound waves are the only waves that are possible in normal fluid.

1.6 Instabilities in Plasma

A Maxwellian distribution is used for particles when plasma is in thermodynamic equilibrium. When plasma deviates from the thermodynamic equilibrium condition, a free source of energy is given to the particles, which under some conditions give rise to plasma instabilities. Such a deviation can take place both in inhomogeneous plasma and homogeneous plasma systems. In a homogeneous plasma system, the deviation from thermodynamic equilibrium occurs in velocity space. One of the examples of instabilities which occur due to deviation in velocity space is the ion-acoustic instability.

- In the case of linear instability, a slight linear distortion will let the ball roll down the hill, following unstable case sets in spontaneously.

Figure 1.5: Linear Instability

- In non-linear instability the sphere is stable for a small amplitude of disturbance, but for large amplitudes, it becomes unstable.

Figure 1.6: Non-Linear Instability

Streaming Instabilities

“ Either a current is driven through plasma or a beam of energetic particles travels through plasma so that different species have drifts relative to one another.”

The instabilities such as two stream and MHD instabilities can occur in an ideal plasma where collisions play no role [9].

1.7 Theoretical Approaches to study behavior of plasma

Plasma dynamic is described by the interaction of charged particles with external magnetic and electric fields. When the charged particles move within plasma, they create a local charge concentration, which gives rise to internal electric field. Electric currents will be generated by the motion of these particles which also causes internal magnetic fields to be generated. Plasma shows different characteristics such that their densities, temperature, and degree of ionization can change. The effect of collisions and electromagnetic forces also has great importance in the study of plasma dynamics. Hence to study different behaviors of plasma, different plasma models are used.

1.7.1 Single particle approach

An applied electric or magnetic field affects the motion of particles in the plasma. To describe the motion of a single particle, when it comes under the influence of such fields, the single-particle approach is used. While describing the motion using single-particle approach the collective behavior of plasma is neglected. This approach is useful in the study of very low-density plasma e.g; while studying the energetic particles or the cosmic rays in Van Allen radiation belts.

1.7.2 Plasma as a fluid

Magnetohydrodynamics mode

The study of electrically conducting fluids is known as Magnetohydrodynamics (MHD). In this approach, plasma is taken as a single conducting fluid. It is used to describe equilibrium and large-scale stability of the magnetized plasma. A drawback of this approach is that the macroscopic properties of each species within the plasma, such as velocity, density and temperature will be lost while taking an average of these properties into account. To study plasma behavior using the Magnetohydrodynamic (MHD) model, we use the hydrodynamic equations coupled with Maxwell's equations.

Multi fluid model

In this model, each species of the plasma is treated as a separate fluid element. The advantage of the multi-fluid model over MHD is that different behavior of different species within the plasma can be taken into account. For example at the same spatial point within the plasma, different plasma components can have different velocity, temperature, and pressure.

1.7.3 Kinetic theory of plasma

While using the fluid model approach the information regarding the velocity distribution of the particle is lost, as the fluid variables are a function of position and time only. The physical properties of the plasma that are dependent on this microscopic detail can only be discovered by an illustration in six-dimensional (r,v) space so starting with the distribution function, $f(\mathrm{r}, \mathrm{v}, t)$, rather than starting with the density of particles $n(\mathrm{r}, t)$ at position r and time t, evolution of the distribution function is explained by the kinetic theory.

$$\int f(\mathrm{r}, \mathrm{v}, t) dv = n(\mathrm{r}, t)$$

1.8 Important Aspects of Dusty Plasma

The branch of physics known as dusty plasma has gained immense significance in recent times, not just in the academic perspective but also in many new aspects such as semiconductor technology, crystal physics, fusion devices, modern astrophysics, biophysics, plasma chemistry etc. Some of them are briefly discussed in the following sections.

1.8.1 Space science and astrophysics

In October 1980, Voyager 1 sped past Saturn and delivered pictures of strange dark spokes extending around B ring . This provided with the first hint that the space plasma particles charged the dust particles which could explain some of the heavenly features. Hill and Mendis(1981), Morfill and Goertz(1983) proposed that the spokes in the Saturn ring might be charged dust sculpted by electrostatic forces. Mendis and Hill gave the opinion that the electrification of dust might be due to electrons hurled into the Saturn's ring through processes such as aurora occurring near Saturn. On the other hand, Morfill and Goertz ascribed the electrification to the burst of plasma which generated as micrometeoroids bombarded the boulder of ring. As shown in figure 1.4, the two of the above described phenomenas would produce spoke like regions of electrification where electrostatic repulsion between boulders and dust particles would give rise to trails of dust grains.

1.8.2 Semiconductor industry

Most of the funds are spent on space and fusion plasma research but fortunately in the recent past a new implementation of plasma physics has gained immense importance, the implementation of low temperature partially ionized plasmas(also known as dusty plasma) in the making of material processing and chips. In 1985 Roth et al performed the work on silane discharges and presented the phenomena for the first time that dust problems are interlinked with plasma technologies. In 1989 Selwyn et al provided the evidence by working on sputtering tools or microelectronics etching.

1.8.3 Fusion research

For International Thermonuclear Experimental Reactor and upcoming fusion reactors, dust is definitely a significant safety issue. A huge fraction of hydrogen ($>0.2H/C$) might be present in dust particles which might lead to important tritium inventories. In vacuum or coolant leak, these scattered dust particles can be chemically reactive and they may immediately react with water vapor or oxygen. Migration of dust particles is also an important aspect. Due to monotonous condensation and evaporation and thermophoretic forces the dust particles may gather at cold areas of the device. They might fill gaps and block spacings which are basically introduced for the engineering purposes. Issue of dust in tokamaks is gaining interest these days. The dust particles obtained in tokamaks TEXTOR,ALCATOR TFTR, DIII-D and C-MOD, etc., ranging from $\sim$ 100nm to $\sim$ 100μm.

1.9 Thesis Outline

In chapter 2 we deal with the mathematical formulation of fluid equations and the Maxwell equation. The phenomena of two stream instability for an electron ion plasma is described graphically and mathematically. In the end this chapter illustrates the theoretical and mathematical description of ion acoustic waves, dust acoustic waves and dust ion acoustic waves predicted by Shukla and Silin in 1992.

In chapter 3, we derive the dispersion relation for dust ion acoustic waves. First we use the cairns distribution function for electrons to find the electron density. Further we do sinusoidal perturbation of momentum,continuity and adiabatic pressure equation derived in chapter 2 for ions and ion beam. These equations are complemented by Poisson equation for electrostatic potential associated with dust ion-acoustic perturbation. This chapter graphically compares the Maxwellian and Cairns distribution for different values of non thermal parameter α.

In chapter 4, we find out the graphical representation of two stream instability between ion and ion beam in a dusty plasma. It graphically analyses the effect of non thermal electrons on dust ion acoustic wave. In the end of the chapter we graphically discuss the effect of ion beam parameters on fast, slow and dust ion acoustic modes.

Chapter 2

Fluid description of Plasma

2.1 Waves in Two Fluid Plasma

Generally, waves in plasma are electromagnetic. Maxwell equations deal with magnetic and electric field of a wave.

$$\nabla \cdot \mathbf{E}(\mathbf{x}, t) = \rho_q(\mathbf{x}, t)/\epsilon_0, \tag{2.1.1}$$

$$\nabla \cdot \mathbf{B}(\mathbf{x}, t) = 0, \tag{2.1.2}$$

$$\nabla \times \mathbf{E}(\mathbf{x}, t) = -\frac{\partial \mathbf{B}(\mathbf{x}, t)}{\partial t}, \tag{2.1.3}$$

$$\nabla \times \mathbf{B}(\mathbf{x}, t) = \mu_0 \mathbf{j}(\mathbf{x}, t) + \mu_0 \epsilon_0 \frac{\partial}{\partial t} \mathbf{E}(\mathbf{x}, t). \tag{2.1.4}$$

The charge and current density in the above equation are the quantities that associate wave fields to the response of plasma. For determining the current density its important to understand the impact of wave fields on plasma. Lorentz force law can be used to derive current density when plasma is cold in a way that all species in plasma have thermal velocities that can be ignored. Lorentz law states that how a single particle motion is affected by magnetic field and electric field. Every particle of species in cold plasma at certain time and position will react in absolutely the same way and macrophysical quantities like current density can be derived just by summing all of the particles in a volume element. The other extreme is using kinetic treatment, collisonless Boltzmann (Vlasov) equation is used to analyze the effect on distribution function of particles by wave fields. In the middle of the above two extremes magnetohydrodynamic equations can be used which are obtained by utilizing Vlasov equation's moments or equations for two fluid plasma can be used. The advantage of using magnetohydrodynamic equations is simplicity; fluid equations has time and three spatial dimensions, relatively the Vlasov theory phase space is seven dimensional. Significant effect like Landau damping (originated by resonance with the particles that are moving at phase speed of the wave) cannot be obtained by fluid equation. The respective equations for two fluid plasma are:

$$\frac{\partial n_s}{\partial t} + \boldsymbol{\nabla}.(n_s u_s) = 0, \tag{2.1.5}$$

$$\frac{\partial u_s}{\partial t} + u_s.\boldsymbol{\nabla} u_s - \frac{q_s}{m_s}(E + u_s \times B) + \frac{\boldsymbol{\nabla} p_s}{n_s m_s} = \frac{F'}{n_s m_s}, \tag{2.1.6}$$

$$j = \sum_s n_s q_s u_s, \tag{2.1.7}$$

$$p_q = \sum_s n_s q_s. \tag{2.1.8}$$

In the above equations all variables are dependent on x and t where s denotes ions and electrons. To provide a whole detail of plasma, two equations must be added. In these equations the fluid pressure is related to fluid variables n_s and u_s . The fluid equations system is closed by describing fluid pressure (polytropic law)

$$p_s = constant \times (n_s)^{\gamma_s} = n_s T_s. \tag{2.1.9}$$

γ_s is specific heat ratio. Different waves arise in space plasma. Waves are either named by region in which they exist, like auroral kilometric, plasma spheric hiss, auroral hiss radiation. Waves can either be named according to the frequency range in which they appear when it is close to plasma's natural frequency e.g., ion cyclotron waves, upper hybrid resonance noise,electron cyclotron waves and lower hybrid waves. Waves that carry no magnetic field are named as electrostatic waves e.g., electrostatic ion cyclotron wave [5, 8].

2.2 Fluid equation of motion

The fundamental conservation laws of momentum, mass and energy in a fluid govern average properties of particles. The equations of fluid are hydrodynamic equations if electromagnetic field effects are not taken into account. But the electric field, current and magnetic field are definitely significant in plasmas therefore magnetohydrodynamic equations are introduced.

2.2.1 Momentum equation

For a single particle, the equation of motion is given as:

$$m\frac{d\mathbf{v}}{dt} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}), \tag{2.2.1}$$

Considering no collisions and thermal motions and multiplying above equation by n(density) which gives fluid equation.

$$mn\frac{d\mathbf{u}}{dt} = qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}). \tag{2.2.2}$$

Considering one dimensional x space, taking any characteristic of a fluid represented by H(x,t). In a frame moving with fluid the variation in H with respect to time is represented as a sum of terms.

$$\frac{d\mathbf{H}(x,t)}{dt} = \frac{\partial \mathbf{H}}{\partial t} + \frac{\partial \mathbf{H}}{\partial x}\frac{dx}{dt} = \frac{\partial \mathbf{H}}{\partial t} + u_x\frac{\partial \mathbf{H}}{\partial x},$$

The change of H in fluid at a fixed point is represented by ist term, and second term shows variation in H as the spectator moves along fluid in a region where H is unlike. This can be written in three dimensions as

$$\frac{d\mathbf{H}}{dt} = \frac{\partial \mathbf{H}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{H}. \tag{2.2.3}$$

This is normally known as convective derivative. While considering plasma H is taken as fluid velocity u.

$$mn\left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u}\right] = qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}). \tag{2.2.4}$$

$\frac{\partial u}{\partial t}$ represents time derivative keeping the frame fixed.

Taking thermal motions in consideration, its compulsory to add pressure force on right side of equation (2.2.4). Pressure term arises because of random motion of particles in and out of an element of fluid [9].

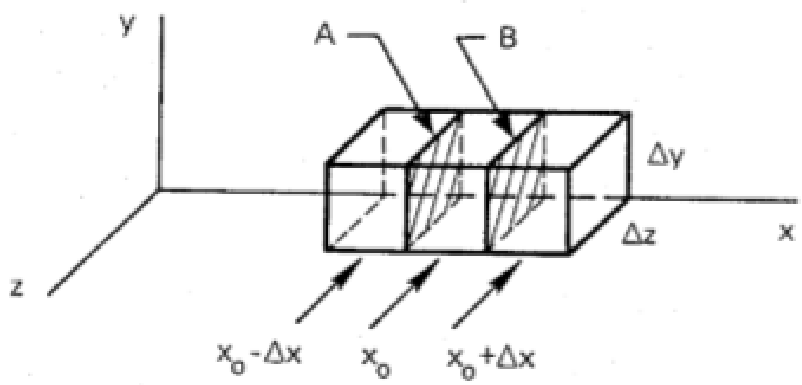

Figure 2.1:

In figure 2.1 for convenience considering only the x direction motion of fluid element through A and B face, that is centered at $(x, \frac{\Delta y}{2}, \frac{\Delta z}{2})$. Number of particles that are passing per second through face A, with velocity v_x is

$$\Delta n_v v_x \Delta y \Delta z. \tag{2.2.5}$$

At x_0 the fluid element's total change of momentum is

$$\frac{\partial}{\partial t}\left(nmu_x\right)\Delta x\Delta y\Delta z = -m\frac{\partial}{\partial x}\left(n\bar{v}_x^2\right)\Delta x\Delta y\Delta z, \tag{2.2.6}$$

For 1-D Maxwellian distribution

$$\frac{1}{2}m\bar{v}_{xx}^2 = \frac{1}{2}KT,$$

Equation (2.2.6) now becomes

$$\frac{\partial(nmu_x)}{\partial t} = -m\frac{\partial}{\partial x}[n(\overline{u_x^2}) + 2\overline{uv_{xr}} + \overline{v_{xr}^2}] = -m\frac{\partial}{\partial x}[n(u_x^2 + \frac{KT}{m})]. \tag{2.2.7}$$

The mass conservation equation is

$$\frac{\partial n}{\partial t} + \frac{\partial (nu_x)}{\partial x} = 0,$$

This equation allows us to modify the equation (2.1.6) where two of the term goes to zero because of mass conservation condition. Pressure is defined as P=nKT.

$$mn(\frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x}) = -\frac{\partial p}{\partial x},$$

The above equation is the pressure gradient force. Considering it in three dimensions and adding the electromagnetic forces the above equation becomes,

$$mn \left[\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right] = qn(\mathbf{E} + \mathbf{u} \times \mathbf{B}) - \nabla p. \tag{2.2.8}$$

2.3 Continuity equation

In a volume V, the absolute particles N can change if there exists a net flux of particles over the surface S encompassing that volume. Divergence theorem gives us (nu is the flux density of the particle).

$$\frac{\partial N}{\partial t} = \int_V \frac{\partial n}{\partial t} dV = -\oint (nu.dS) = -\int_V (\boldsymbol{\nabla}.nu) dV, \tag{2.3.1}$$

The above integrals should be equal for it to be true for any volume V.

$$\frac{\partial n}{\partial t} + \boldsymbol{\nabla}.(nu) = 0. \tag{2.3.2}$$

Any additional terms can be included on the right hand side [9].

2.4 Adiabatic Pressure Equation

The energy evolution equation is given as

$$\frac{N}{2}\frac{\mathrm{d}P_\sigma}{\mathrm{d}t} + \frac{2+N}{2} P \nabla \cdot \mathbf{u}_\sigma = -\nabla \cdot \mathbf{Q}_\sigma + \mathbf{R}_{\sigma\alpha} \cdot \mathbf{u}_\sigma - \left(\frac{\partial W}{\partial t} \right)_{E\sigma\alpha}. \tag{2.4.1}$$

First term on right side shows heat flux, second term represents frictional heating of species σ caused by frictional drag where last term on the right side shows the collision rate.

Adiabatic limit:

The collisional and heat flux terms are so small that they can be neglected as compared to left hand side quantities; this happens when $V_{ph} \gg v_{r\sigma}$. " Adiabatic is a Greek word which means impassable which is used here to denote that no amount of heat is flowing" i.e., the volume under consideration is thermally isolated from the outside world.

$$\frac{N}{2}\frac{\mathrm{d}P_\sigma}{\mathrm{d}t} + \frac{2+N}{2}P\nabla \cdot \mathbf{u}_\sigma = 0, \tag{2.4.2}$$

The energy equation can be simplified in adiabatic limit by rearranging continuity equation (2.3.2)

$$\nabla \cdot \mathbf{u}_\sigma = -\frac{1}{n_\sigma}\frac{\mathrm{d}n_\sigma}{\mathrm{d}t}, \tag{2.4.3}$$

and now substituting this expression into the left-hand side of Eq. (2.4.2) to obtain

$$\frac{1}{P_\sigma}\frac{\mathrm{d}P_\sigma}{\mathrm{d}t} = \frac{\gamma}{n_\sigma}\frac{\mathrm{d}n_\sigma}{\mathrm{d}t}, \tag{2.4.4}$$

where $\gamma = \frac{N+2}{N}$, and the operator d/dt is the convective derivative i.e,

$$\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathbf{u}_\sigma \cdot \nabla.$$

which indicates the temporal rate of change seen by an observer who is moving with mean fluid velocity u_σ of species σ [8].

2.5 Equation Of State

An additional equation is required which relates pressure P to "n"(number density). This is commonly known as "the equation of state."

$$P = Cn^\gamma, \tag{2.5.1}$$

where C is a constant and γ is the ratio of specific heat at constant volume to constant pressure i.e, $\frac{C_p}{C_v}$. Taking gradient of the above formula

$$\boldsymbol{\nabla} P = C\gamma n^{\gamma-1}\boldsymbol{\nabla} n = \frac{\gamma P \boldsymbol{\nabla} n}{n}, \tag{2.5.2}$$

Therefore

$$\frac{\boldsymbol{\nabla} P}{P} = \frac{\gamma \boldsymbol{\nabla} n}{n}. \tag{2.5.3}$$

- Considering isothermal compression $\gamma = 1$

$$\boldsymbol{\nabla} P = KT\boldsymbol{\nabla} n. \tag{2.5.4}$$

- KT changes for adiabatic compression condition, this gives γ value greater than one.

$$\gamma = \frac{2+N}{N}. \tag{2.5.5}$$

  "N" shows the degree of freedom.

The equation of state demands that heat flow should be small enough to be neglected, in other words, thermal conductivity should be low.

2.6 Poisson's Equation

From Poissons equation,

$$\nabla \cdot E = \frac{\rho}{\epsilon_o}, \tag{2.6.1}$$

where $\rho = \rho_e + \rho_i, \rho_e = -en_e$ and $\rho_i = Zen_i$, For electrostatic case $E = -\nabla\phi$. By putting values we get

$$-\nabla^2\phi = \frac{e\left(Zn_i - n_e\right)}{\epsilon_o},$$

By using Guassian units

$$\nabla^2\phi = 4\pi e\left(n_e - Zn_i\right). \tag{2.6.2}$$

2.7 Two Stream Instability

Consider a uniform plasma that has stationary ions and v_0 is the velocity of electrons in comparison with the ions in such a way that the spectator is in a frame that is in motion with the flow of ions. Considering the case of cold plasma where $KT_e = KT_i = 0$ and $B_0 = 0$. The equations of motion in their linearized form are given as:

$$Mn_0 \frac{\partial \mathbf{v}_{i1}}{\partial t} = en_0 \mathbf{E}_1, \tag{2.7.1}$$

$$mn_0 \left[\frac{\partial \mathbf{v}_1}{\partial t} + (\mathbf{v}_0 \cdot \nabla) \mathbf{v}_1 \right] = -en_0 \mathbf{E}_1. \tag{2.7.2}$$

As we have taken $v_{i0} = 0$, so the term $(\mathbf{v}_0 \cdot \nabla) \mathbf{v}_1$ doesn't appear in (2.7.1). In equation (2.7.2), the quantity $(\mathbf{v}_{e1} \cdot \nabla) \mathbf{v}_0$ doesn't exist because v_0 has been assumed uniform. Looking into the electrostatic waves of the following type:

$$\mathbf{E}_1 = Ee^{i(kx - \omega t)} \hat{\mathbf{x}}, \tag{2.7.3}$$

Here the direction of k and v_0 is along $\hat{\mathbf{x}}$. Solving equations (2.7.1) and (2.7.2) gives

$$-i\omega M n_0 v_{i1} = en_0 \mathbf{E}_1 \Rightarrow \quad v_{i1} = \frac{ie}{M\omega} E\hat{\mathbf{x}}. \tag{2.7.4}$$

$$mn_0 (-i\omega + ikv_0) v_{e1} = -en_0 \mathbf{E}_1 \Rightarrow \quad v_{e1} = -\frac{ie}{m} \frac{E\hat{\mathbf{x}}}{\omega - kv_0}. \tag{2.7.5}$$

Omitting the subscript x as the velocities v_{j1} are in the x direction. The ion continuity equation gives

$$\frac{\partial n_{i1}}{\partial t} + n_0 \nabla \cdot \mathbf{v}_{i1} = 0, \tag{2.7.6}$$

Applying sinusoidal perturbation we get

$$n_{i1} = \frac{k}{\omega} n_0 v_{i1},$$

Substituting the value of v_{i1} from equation (2.7.4)

$$n_{i1} = \frac{ien_0 k}{M\omega^2} E. \tag{2.7.7}$$

As $\nabla n_0 = \mathbf{v}_{0i} = 0$, so the remaining terms in $\nabla \cdot (n\mathbf{v}_i)$ will be zero. The electron continuity equation can be written as,

$$\frac{\partial n_{e1}}{\partial t} + n_0 \nabla \cdot \mathbf{v}_{e1} + (\mathrm{v}_0 \cdot \nabla)\, n_{e1} = 0, \tag{2.7.8}$$

Applying sinusoidal perturbation and substituting the value of v_{e1} from equation (2.7.5) we get

$$n_{e1} = \frac{kn_0}{\omega - kv_0} v_{e1} = -\frac{iekn_0}{m\,(\omega - kv_0)^2} E. \tag{2.7.9}$$

We are using Poisson's equation rather than plasma approximation because unstable waves have plasma oscillation in a high frequency range.

$$\epsilon_0 \nabla \cdot \mathbf{E}_1 = e\,(n_{i1} - n_{e1})\,, \tag{2.7.10}$$

Using the value of n_{i1} from equation (2.7.7) and n_{e1} from equation (2.7.9) we get

$$ik\epsilon_0 E = e\,(ien_0 kE) \left[\frac{1}{M\omega^2} + \frac{1}{m\,(\omega - kv_0)^2} \right], \tag{2.7.11}$$

Finding the dispersion relation by dividing the above equation by $ik\epsilon_0 E$

$$1 = \omega_p^2 \left[\frac{m/M}{\omega^2} + \frac{1}{(\omega - kv_0)^2} \right], \tag{2.7.12}$$

There will be four solutions as it is a quartic equation and due to real coefficients, the four solutions can be arranged in two complex conjugate pairs. The positive imaginary part gives exponentially growing wave, the negative imaginary part gives a damped wave. As roots appear in conjugate pairs, there will be one wave which will be always unstable. We can write the above equation as

$$1 = \frac{m/M}{x^2} + \frac{1}{(x-y)^2} = F(x,y) \tag{2.7.13}$$

where $x = \frac{\omega}{\omega_p}$ and $y = \frac{kv_0}{\omega_p}$. F(x,y) is plotted as a function of x for any prescribed value of y.

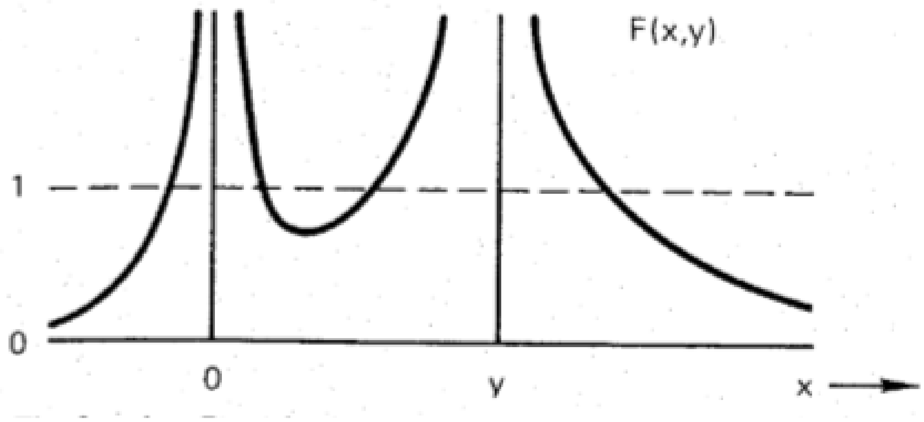

Figure 2.2: Function F(x,y) in two stream instability in a case when plasma is stable

F(x,y) has singularities in figure(2.2) at x=y and x=0. This line intersects with curve F(x,y)=1 thus giving values of x which satisfy the dispersion relation. There are four real roots as figure(2.2) indicates four intersections.

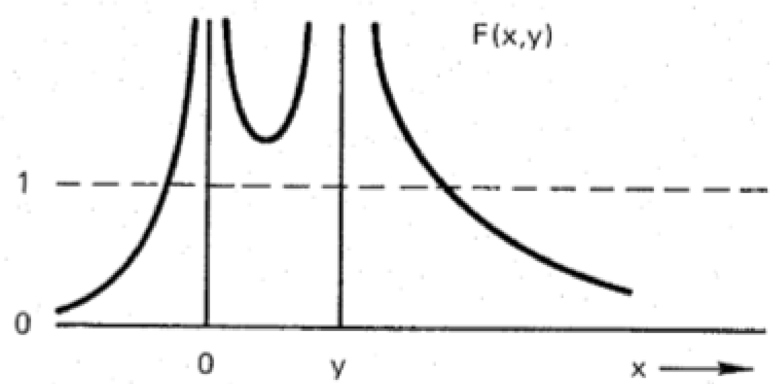

Figure 2.3: Function F(x,y) in two stream instability in case when plasma is unstable

For smaller values of y, the graph looks like the one in figure (2.3). This one has only two real roots because it has two intersection points. The remaining two roots should be complex, out of which one definitely corresponds to an unstable wave. Smaller values of kv_0 correspond to un-stable plasma. For any value of v_0 oscillations in the range of long wavelength leads to unstable plasma. Equation (2.7.14) gives a maximum rate of growth when $\frac{m}{M}$<<1 [15, 27, 42].

$$Im(\frac{\omega}{\omega_p}) \approx (\frac{m}{M})^{1/3}. \tag{2.7.14}$$

2.8 Acoustic modes

Acoustic modes are electrostatic modes.

“ Electrostatic waves start from a charge imbalance in an initially quasi-neutral fluid element. This charge imbalance accelerates the electrons and ions in the neighbourhood, resulting in charges accelerating back and forth. These oscillations are defined as electrostatic waves as they only involve the electric field. The oscillating magnetic field is zero.”

2.8.1 Ion Acoustic mode

Ion waves are low pressure and low-frequency waves that occur in plasma. The dispersion relation of ion waves is similar to that of sound waves hence they are called the Ion acoustic waves (IAW). “ Ion acoustic waves are longitudinal oscillations of ions and electrons in a plasma just like sound waves travelling in a neutral gas.” The momentum equation in the absence of the magnetic field for ions is

$$Mn\left[\frac{\partial v_i}{\partial t} + (v_i.\Delta)\, v_i\right] = -en\Delta\phi - \gamma_i KT_i \Delta n, \tag{2.8.1}$$

Applying sinusoidal perturbation

$$-i\omega M n_0 v_{i_1} = -en_0 ik\phi_1 - \gamma_i KT_i ik n_{i_1}. \tag{2.8.2}$$

For electrons

$$n_e = n = n_0 \exp\left(\frac{e\phi_1}{KT_e}\right) = n_0\left(1 + \frac{e\phi_1}{KT_e} + \dots\right),$$
$$n_e = n_0 + n_{e1},$$
$$n_{e_1} = n_e - n_0 = n_0\frac{e\phi_1}{KT_e},$$

For low frequency oscillations we can use the quasi-neutrality condition i.e. $n_{i1} = n_{e1}$. Therefore

$$n_{i_1} = n_0 \frac{e\phi_1}{KT_e}.$$

From equation of continuity of ions

$$i\omega n_{i_1} = n_0 ikv_{i_1}. \tag{2.8.3}$$

Substituting above values in equation (2.8.2)

$$\frac{\omega}{k} = \left(\frac{KT_e + \gamma_i KT_i}{M} \right)^{\frac{1}{2}} = v_s. \tag{2.8.4}$$

The above equation gives dispersion relation for ion-acoustic waves, where v_s is the ion-acoustic speed. For $KTi \approx 0$, the ion-acoustic waves still exist and the ion-acoustic velocity is given by

$$v_s = \frac{KT_e}{M}. \tag{2.8.5}$$

This assumption is not true for higher frequencies, closer to ω_{pi}, because the electron and ion motion becomes uncorrelated. In the case of high frequency oscillation we will use the Poissons equation instead of the quasi-neutrality condition.

$$\nabla^2 \phi = \frac{e}{\epsilon_0} \left(n_{i_1} - n_{e_1} \right), \tag{2.8.6}$$

$$\epsilon_0 k^2 \phi = n_{i_1} - n_{e_1}, \tag{2.8.7}$$

where

$$n_{e_1} = n_0 \frac{e\phi_1}{KT_e},$$

$$n_{i_1} = \frac{k}{\omega} n_0 v_{i_1}.$$

Substituting the value of v_{i1} and n_{i_1}, n_{e_1} in equation (2.8.7) , we get

$$\frac{\omega}{k} = \left(\frac{KT_e}{M} \frac{1}{1 + k^2 \lambda_D^2} + \frac{\gamma_i KT_i}{M} \right)^{\frac{1}{2}}. \tag{2.8.8}$$

Unlike sound waves present in an ordinary gas these waves are extremely damped unless the ion temperature is much less than electron temperature and the collision frequency is much less than wave frequency [31].

Acoustic modes in the presence of dust

In unmagnetized, uniform and collisionless dusty plasmas there are two classes of acoustic modes with a weak Coulomb coupling between dust charges. They are dust acoustic (DA) and dust ion-acoustic (DIA) waves. Three stable modes propagate in the beam plasma for large beam speeds. In the absence of a beam and for long wavelength limit the stable ion-acoustic mode exists. Static dust grains modify the Ion acoustic mode into Dust Ion Acoustic Mode. The propagation velocity increases as the beam velocity is increased from zero and ion acoustic mode is Doppler shifted into:

Slow beam Mode (lower phase velocity).

Fast beam Mode (higher phase velocity).

[17, 19, 20, 28, 37].

2.8.2 Dust acoustic waves (DAW)

Dust acoustic waves have been predicted in 1990 by Rao et al in a collisionless dusty plasma having multi species which are ions, electrons and dust grains that are negatively charged. Dust acoustic waves phase velocity in comparison to ion and electron thermal speed is given as:

$$\sqrt{\frac{T_{\mathrm{d}}}{m_{\mathrm{d}}}} \ll v_{\mathrm{ph}} \ll \sqrt{\frac{T_{\mathrm{i}}}{m_{\mathrm{i}}}}. \tag{2.8.9}$$

The pressure gradient is balanced by electric force, giving Boltzmann ion and electron number density perturbations n_{j1} that are.

$$n_{\mathrm{el}} \approx n_{\mathrm{e0}} \frac{e\phi}{k_{\mathrm{B}} T_{\mathrm{e}}}, \tag{2.8.10}$$

$$n_{\mathrm{i1}} \approx -n_{\mathrm{i0}} \frac{e\phi}{k_{\mathrm{B}} T_{\mathrm{i}}}. \tag{2.8.11}$$

The dust number density perturbation is obtained from the dust continuity equation and the dust momentum equation. All these equations are closed by Poisson's equation.

The dispersion relation is given as:

$$\omega^2 = 3k^2V_{\mathrm{Td}}^2 + \frac{k^2C_{\mathrm{D}}^2}{1+k^2\lambda_{\mathrm{D}}^2}, \tag{2.8.12}$$

where $C_D = \omega_{pd}\lambda_D$ is the DA speed. Since $\omega >> kV_{Td}$, we can deduce the DA wave frequency from the above equation,

$$\omega = \frac{kC_{\mathrm{D}}}{(1+k^2\lambda_{\mathrm{D}}^2)^{1/2}}, \tag{2.8.13}$$

which in the long-wavelength limit reduces to

$$\omega = kZ_{\mathrm{d0}}\left(\frac{n_{\mathrm{d0}}}{n_{\mathrm{io}}}\right)^{1/2}\left(\frac{k_{\mathrm{B}}T_{\mathrm{i}}}{m_{\mathrm{d}}}\right)^{1/2}\left[1+\frac{T_{\mathrm{i}}}{T_{\mathrm{e}}}\left(1-\frac{Z_{\mathrm{d0}}n_{\mathrm{d0}}}{n_{\mathrm{i0}}}\right)\right]^{-1/2}. \tag{2.8.14}$$

where dust grains are negatively charged. Last equation discloses that restoring force originates from pressures of inertialess ions and electrons, whereas dust mass gives inertia to support the waves. Frequency of dust acoustic waves is quite smaller than dust plasma frequency.

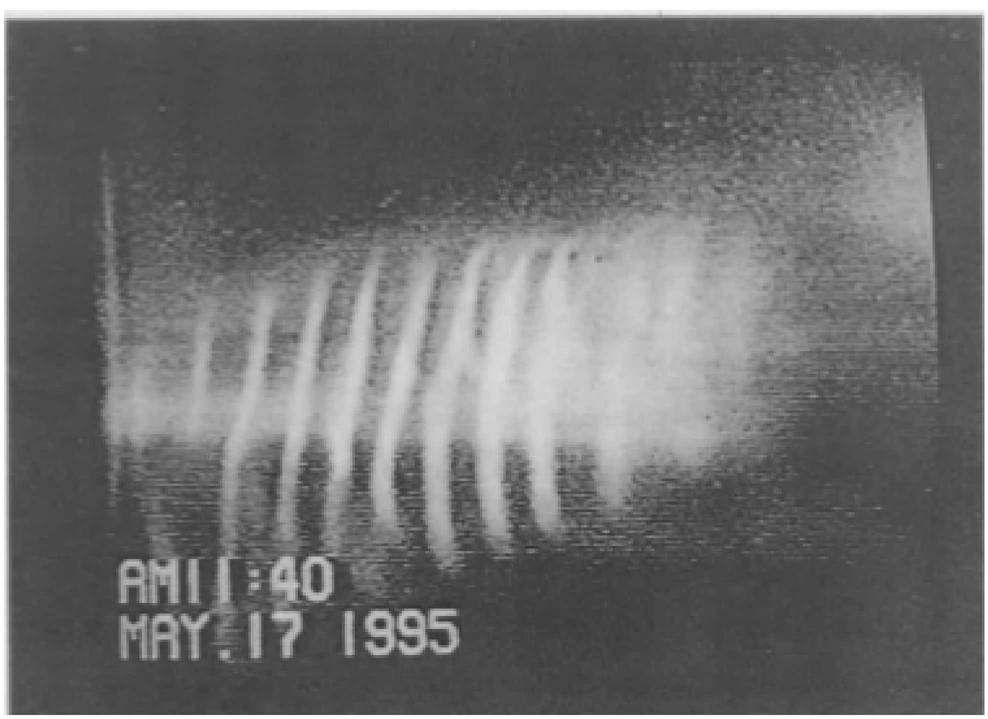

Figure 2.4: Experimental Verification of DAW

2.8.3 Dust ion acoustic waves (DIA)

In 1992, Shukla and Silin predicted DIA waves [23]. Phase velocity of the DIA waves in comparison to ion and electron thermal speed is given as

$$\sqrt{\frac{T_i}{m_i}} \ll v_{\mathrm{ph}} \ll \sqrt{\frac{T_{\mathrm{e}}}{m_{\mathrm{e}}}}. \tag{2.8.15}$$

Equation (2.8.10) gives the electron number density perturbation. The ion number density perturbation n_{i1} is obtained from ion continuity equation,

$$\frac{\partial n_{\mathrm{il}}}{\partial t} + n_{\mathrm{i0}} \nabla \cdot v_{\mathrm{i}} = 0. \tag{2.8.16}$$

and ion momentum equation is

$$\frac{\partial v_i}{\partial t} = -\frac{e}{m_i}\nabla\phi - \frac{3k_{\mathrm{B}}T_i}{m_i n_{i0}}\nabla n_{i1}. \tag{2.8.17}$$

the dispersion relation is

$$\omega^2 = \frac{k^2 C_{\mathrm{S}}^2}{1 + k^2\lambda_{\mathrm{De}}^2}, \tag{2.8.18}$$

where $C_S = \omega_{pi}\lambda_{De} = \sqrt{\frac{n_{i0}}{n_{e0}}}c_s$ and $c_s = \sqrt{\frac{k_B T_e}{m_i}}c_s$. In the long wavelength limit above equation reduces to

$$\omega = k\left(\frac{n_{\mathrm{i0}}}{n_{\mathrm{c0}}}\right)^{1/2} c_{\mathrm{s}}. \tag{2.8.19}$$

$n_{i0} > n_{e0}$ for negatively charged dust grains therefore the phase velocity ($V_p = \omega/k$) of the DIA waves in a dusty plasma is greater than c_s. [15, 16, 33].

Chapter 3

Methodology

In this chapter the dispersion relation for dust ion acoustic waves is studied by deriving its expression which results due to two stream instability of ions and ion beam and non thermal distribution of electrons. Due to Cairns distribution a factor of β is generated in the dispersion relation which does not appear in case of thermal electrons where Maxwellian distribution is used. The dispersion relation for DIA waves is derived by using a system of equations which are momentum equation, continuity equation and adiabatic pressure equation for ion and ion beams. Poisson equation closes the system of equations for electrostatic potential by substituting value of electron density calculated by Cairns distribution in the first step.

The model consists of dust particles that are negatively charged and stationary, non thermal electrons that follow the Cairns distribution. Furthermore, the model consists of ion beams and inertial warm ions of equal mass. The dust charges undergo no charge fluctuations as they are stationary and so for that reason, they will only alter the equilibrium charge neutrality, which is given as

$$\sum q_j N_{j0} = 0, \text{ i.e.,} \quad N_{i0} + N_{b0} = N_{e0} + Z_d N_{d0}. \tag{3.0.1}$$

Here $N_{j0}q_j$ represents the equilibrium charge density for the j^{th} species. ($j = i$ in case of ions, $j = b$ in case of ion beam and for electrons $j = e$). For dust particles, the equilibrium density is given by $N_{d0}(-Z_d e)$ where Z_d is size of dust charge [18].

3.1 Distribution function

Maxwellian Distribution

"The function describing the instantaneous density of particles in phase-space is known as distribution function ." It is indicated by $f(x, v, t)$. For plasma in thermal equilibrium, Maxwellian distribution is the most predictable distribution [35].

$$f(u) = A \exp\left(-\frac{1}{2}mu^2/KT\right). \tag{3.1.1}$$

In the above equation, K denotes the Boltzmann's constant, $\frac{mu^2}{2}$ shows the kinetic energy, fdu represents the number of particles per m^3 having velocity in the range of u and u+du.

$$K = 1.38 \times 10^{-29} \mathrm{J}/^{\circ}\mathrm{K}. \tag{3.1.2}$$

Number of particles per m^3 is given as:

$$n = \int_{-\infty}^{\infty} f(u)du. \tag{3.1.3}$$

The constant A is related to density as:

$$A = n\left(\frac{m}{2\pi KT}\right)^{1/2}. \tag{3.1.4}$$

In this distribution particle's average kinetic energy is

$$E_{\text{av}} = \frac{\int_{-\infty}^{\infty} \frac{1}{2}mu^2 f(u)du}{\int_{-\infty}^{\infty} f(u)du}. \tag{3.1.5}$$

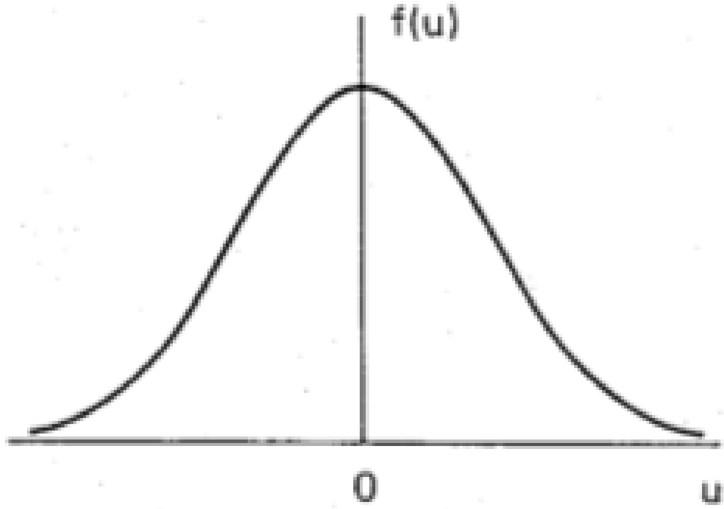

Figure 3.1: A Maxwellian velocity distribution

Cairns Distribution

Non-thermal plasmas are found naturally in the Uranus, magnetospheres and ionosphere's of Earth, Saturn, Mercury and in the solar wind [21, 23]. When a specie is non-thermal Cairns distribution is used for a system having large number of fast particles. Cairns distribution shows a high energy tail overlying on a Maxwellian distribution as shown in figure 3.2. The non thermality of electrons might change the essence of ion sound solitary structures, positive and negative density perturbations of solitons can occur [22, 14, 17]. The Cairns distribution is given as

$$F_e(v) = \frac{N_{e0}}{(2\pi V_{te}^2)^{1/2}} \frac{\left[1 + \alpha\,(v/V_{te})^4\right]}{(1+3\alpha)} \exp\left[\frac{-\,(v/V_{te})^2}{2}\right]. \tag{3.1.6}$$

where v is the electron speed and N_{e0} is the equilibrium electron number density, α is constant and V_{te} is electron thermal velocity. In the presence of electric field, the cairns distribution function given in (3.1.6) becomes

$$F_e(v) = \frac{N_{e0}}{(2\pi V_{te}^2)^{1/2}} \frac{[1+\alpha(\frac{v^2}{V_{te}^2} - \frac{2e\phi}{KT})^2]}{(1+3\alpha)} \exp\left[\frac{-(v/V_{te})^2}{2} + \frac{e\phi}{KT}\right].$$

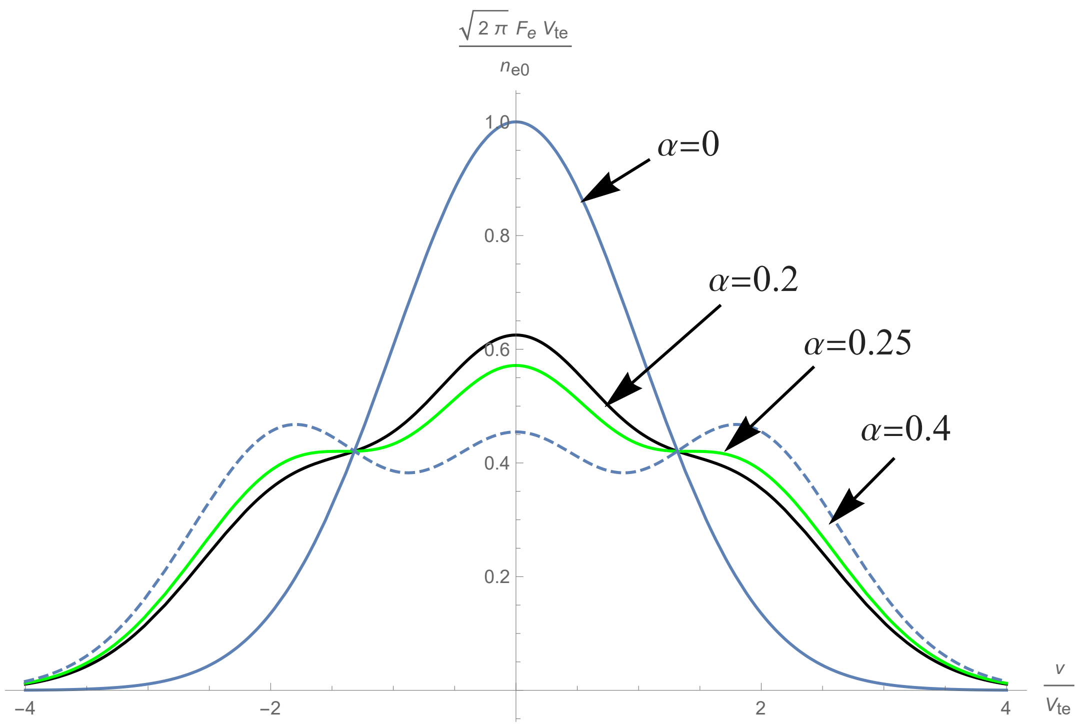

Figure 3.2: Plot of $\sqrt{2\pi}F_eV_{te}/N_{e0}$ against v/V_{te} for increasing values of α.

As it can be seen in Figure 3.2 which is a plot of cairns distribution for different values of α. For $\alpha = 0$, the cairns distribution function is reduced to Maxwellian distribution. For $\alpha = 0.2$, the cairns distribution is shown in black colour, the tail of graph increases which indicates that there are large number of fast particles present in the system. Similarly as the value of α increases there is a sharp increase in its tail [36, 41]. Integrating the distribution function

$$N_e(\varphi) = \int_{-\infty}^{\infty} F_e(v)\mathrm{d}v,$$

$$N_e(\phi) = \frac{N_{e0}}{2\pi V_{te}(1+3\alpha)} \exp\left(\frac{e\phi}{KT}\right) \int_{-\infty}^{+\infty} ([1+\alpha\frac{v^4}{V_{te}^4}+4\alpha[\frac{e\phi}{KT}]^2-4\alpha\frac{e\phi}{KT}[\frac{v}{V_{te}}]^2] \exp\left[\frac{-(v/V_{te})^2}{2}\right])dv,$$

$$N_e(\phi) = \frac{N_{e0}}{2\pi V_{te}(1+3\alpha)} \exp\left(\frac{e\phi}{KT}\right)[\sqrt{2\pi}V_{te}+4\alpha(\frac{e\phi}{KT})^2\sqrt{2\pi}V_{te}+\frac{\alpha}{V_{te}^4}3\sqrt{2\pi}V_{te}^5-\frac{4\alpha}{V_{te}^2}(\frac{e\phi}{KT})\sqrt{2\pi}V_{te}^3],$$

Simplifying gives

$$N_e(\phi) = N_{e0}exp(\frac{e\phi}{KT})[\frac{1}{1+3\alpha} + \frac{4\alpha}{1+3\alpha}(\frac{e\phi}{KT})^2 + \frac{3\alpha}{1+3\alpha} - \frac{4\alpha}{1+3\alpha}(\frac{e\phi}{KT})],$$

In the above equation, let $\frac{4\alpha}{1+3\alpha} = \beta$

$$= N_{e0}\left[1 - \beta\left(\frac{e\phi}{k_B T_e}\right) + \beta\left(\frac{e\phi}{k_B T_e}\right)^2\right] \exp\left[\frac{e\phi}{k_B T_e}\right],$$

Let $\frac{e\phi}{k_B T_e} << 1$, using Maclaurin series for $e^x = 1 + x$ and neglecting the higher order terms, we get

$$= N_{e0}[1 - \beta(\frac{e\phi}{k_B T_e})](1 + \frac{e\phi}{k_B T_e}),$$

$$N_e(\phi_1) = N_{e0}\left\{1 + (1-\beta)\left[\frac{e\phi_1}{k_B T_e}\right]\right\}. \tag{3.1.7}$$

3.2 Fluid Model

The fluid equations govern the dynamics of ion beam and inertial and warm ions. The electrostatic oscillations due to internal space charge field of ions, electrons are represented by continuity equation. The equation in un-normalized quantities is given as:

$$\frac{\partial N_j}{\partial t'} + \frac{\partial}{\partial x'}(N_j V_j) = 0. \tag{3.2.1}$$

The momentum equation in un-normalized quantities is given as:

$$\frac{\partial V_j}{\partial t'} + V_j\frac{\partial V_j}{\partial x'} + \frac{q_j}{m_j}\frac{\partial \phi}{\partial x'} + \frac{1}{N_j m_j}\frac{\partial P_j}{\partial x'} = 0. \tag{3.2.2}$$

Adiabatic pressure equation is given as:

$$\frac{\partial P_j}{\partial t'} + V_j\frac{\partial P_j}{\partial x'} + \gamma P_j\frac{\partial V_j}{\partial x'} = 0. \tag{3.2.3}$$

Poisson's equation closes the system of equations (3.2.1),(3.2.2),(3.2.3).

$$\varepsilon_0\frac{\partial^2 \varphi}{\partial x'^2} = e\left(N_e + Z_d N_{d0} - N_i - N_b\right). \tag{3.2.4}$$

The perturbations in a system give rise to waves. Like a sound wave arises because of the perturbations caused by pressure change and these pressure perturbations are carried by sound waves. To evaluate the reaction of a system, the system is taken in equilibrium initially and small perturbations are taken into account. The fluid equations described above are nonlinear but restricting to disturbances of small amplitude, one can find differential equations that are linear. If the velocity of plasma is perturbed the static equilibrium is disturbed and thus this perturbation gives rise to perturbation of mass density and fluid pressure. Linearizing all three fluid equations by taking the perturbation in pressure, density, velocity such as

$$N_j = N_{j0}+N_{j1}, \quad V_j = V_{j0}+V_{j1} \quad \text{and} \quad P_j = P_{j0}+P_{j1} \text{ with } \quad V_{j0} \rightarrow (V_{i0}, V_{b0}) \equiv (0, V_{b0}) \,.$$

and eliminating the subscript j for our convenience. Neglecting the terms having higher powers as linear theory is valid. Fourier transformation is used after linearization as the unperturbed system is static and uniform. Sinusoidal perturbation of the continuity equation given in (3.2.1) gives :

$$\frac{\partial N_1}{\partial t'} + \frac{\partial}{\partial x'}\left(N_0 V_1 + N_1 V_0\right) = 0,$$

$$V_1 = (\frac{\omega}{k} - V_0)\frac{N_1}{N_0}. \tag{3.2.5}$$

Sinusoidal perturbation of the momentum equation(3.2.2)

$$\frac{\partial V_1}{\partial t'} + V_0\frac{\partial V_1}{\partial x'} + \frac{q}{m}\frac{\partial \phi_1}{\partial x'} + \frac{1}{(N_0+N_1)m}\frac{\partial (P_0+P_1)}{\partial x'} = 0,$$

$$\phi_1 = \frac{m}{q}[(\frac{\omega}{k} - V_0)]V_1 - \frac{1}{ikm(N_0+N_1)}\frac{(P_0+P_1)}{\partial x'}],$$

Substituting the value of V_1 from (3.2.5)

$$\phi_1 = \frac{m}{q}[(\frac{\omega}{k} - V_0)^2\frac{N_1}{N_0} - \frac{1}{ikm(N_0+N_1)}\frac{(P_0+P_1)}{\partial x'}]. \tag{3.2.6}$$

Sinusoidal perturbation of the adiabatic pressure equation (3.2.3)

$$\frac{\partial (P_0+P_1)}{\partial t'} + (V_0+V_1)\frac{\partial (P_0+P_1)}{\partial x'} + \gamma(P_0+P_1)\frac{\partial (V_0+V_1)}{\partial x'} = 0,$$

$$P_1 = \frac{\gamma P_0 N_1}{N_0}. \tag{3.2.7}$$

Putting the value of P_1 in equation (3.2.6)

$$\phi_1 = \frac{m}{e}[(\frac{\omega}{k} - V_0)^2 \frac{N_1}{N_0} - \frac{\gamma N_1 P_0}{m(1+\frac{N_1}{N_0})(N_0)^2}],$$

Substituting $P_0 = N_0 K_B T_j$ and multiplying, dividing by T_e

$$\phi_1 = \frac{m}{e}[(\frac{\omega}{k} - V_0)^2 - \frac{\gamma K_B T_j T_e}{m(1+\frac{N_1}{N_0})T_e}]\frac{N_1}{N_0},$$

Substituting the values $C_s^2 = \frac{K_B T_e}{m}$ and $\sigma = \frac{T_j}{T_e}$ and approximating $1 + \frac{N_1}{N_0} \approx 1$ since $\frac{N_1}{N_0} << 1$,

$$\phi_1 = \frac{m}{e}[(\frac{\omega}{k} - V_0)^2 - \gamma\sigma C_s^2]\frac{N_1}{N_0}. \tag{3.2.8}$$

Sinusoidal perturbation of the Poisson's equation (3.2.4) and substituting the charge neutrality equation gives

$$\varepsilon_0 k^2 \phi_1 = e(N_{e1} - N_{j1}),$$

Putting the value of electron density $N_e(\phi_1) = N_{e0}\left\{(1-\beta)\left[\frac{e\phi_1}{K_B T_e}\right]\right\}$,

$$\varepsilon_0 k^2 \phi_1 = \frac{e^2 \phi_1 N_0 (1-\beta)}{K_B T_e} - eN_1,$$

Putting the value of N_1 as obtained from equation (3.2.6) and taking $\lambda_{Deff}^2 = \frac{\varepsilon_0 T_e K_B}{e^2 N_{i0}}$, $\mu_e = \frac{N_{e0}}{N_{i0}}$, $C_s^2 = \frac{K_B T_e}{m}$ [22]. Taking the summation on the right-side as $j = i, b$

$$k^2 \lambda_{Deff}^2 + \mu_e(1-\beta) = \Sigma_{j=i,b}[\frac{C_s^2 \frac{N_{j0}}{N_{i0}}}{(\frac{\omega}{k} - V_{j0})^2 - \gamma\sigma C_s^2}],$$

k is normalized with λ_{Deff}^{-1} , ω with ω_{pi}. Let $u_{b0} = \frac{V_{b0}}{C_s}$ and by applying the summation we get the final expression for $\gamma = 3$ as:

$$k^2 + \mu_e(1-\beta) = \frac{1}{\frac{\omega^2}{k^2} - 3\sigma_i} + \frac{\mu_b}{((\omega/k) - u_{b0})^2 - 3\sigma_b}. \tag{3.2.9}$$

$\mu_b = \frac{N_{b0}}{N_{i0}}$. The first term in the denominator $(\omega/k) \to \sqrt{3\sigma_i}$ gives the dust ion acoustic mode, the second term $(\omega/k - u_{bo}) \to \sqrt{3\sigma_b}$ gives Fast mode and $(\omega/k - u_{bo}) \to -\sqrt{3\sigma_b}$ corresponds to Slow mode.

Chapter 4

Results and Discussion

Ion beam has generated three longitudinal electrostatic modes: fast, slow and ion-acoustic mode. Right-hand side of dispersion relation derived in equation (3.2.14) is a quartic equation (fourth power) thus it has four roots.

$$k^2 + f_e(1-\beta) = \frac{1}{\frac{\omega^2}{k^2} - 3\sigma_i} + \frac{f_b}{((\omega/k) - u_{b0})^2 - 3\sigma_b}. \tag{4.0.1}$$

The first term on right-hand side of dispersion relation diverges at $(\frac{\omega}{k})^2 = 3\sigma_i$ and the second term diverges at $(\frac{\omega}{k} - u_{bo})^2 = 3\sigma_b$. The four asymptotes obtained are $\omega_1 = -\sqrt{3\sigma_i k^2}$, $\omega_2 = \sqrt{3\sigma_i k^2}$, $\omega_3 = u_{bo}k + \sqrt{3\sigma_b k^2}$ and $\omega_4 = u_{bo}k - \sqrt{3\sigma_b k^2}$. If we plot ω vs $F(\omega)$ we will get a graph shown in figure 4.1.

ω_1, ω_2,ω_3,ω_4 are vertical asymptotes in figure 4.1. These asymptotes correspond to the zeros of the denominator of the dispersion relation (4.0.1) where ω_1 is a negative asymptote. Two stream instability phenomena between ion beam and ion is shown in the plot of ω vs $F(\omega)$. Between ω_2 and ω_3 a straight horizontal line exists (indicated in orange colour) at a minimum value $F(\omega_m) \simeq 2.6758$, above this line the wave is unstable. The upward parabola has its minimum value at $\simeq 2.6758$ which shows the onset of linear instability. The downward parabola between ω_1 and ω_2 shows one of the maximum value and the other maximum value exists between ω_3-ω_4. The coefficents of equation (4.0.1) are real therefore it gives two complex roots that are complex conjugates of each other. Comparing this two stream instability plot with the generic case of electron ion two stream instability where the source of instability was electron beam as described in the chapter 2, there is an increase in the growth rate of wave. As one knows that dust particles significantly increases the waves growth rate and damping of wave becomes difficult.

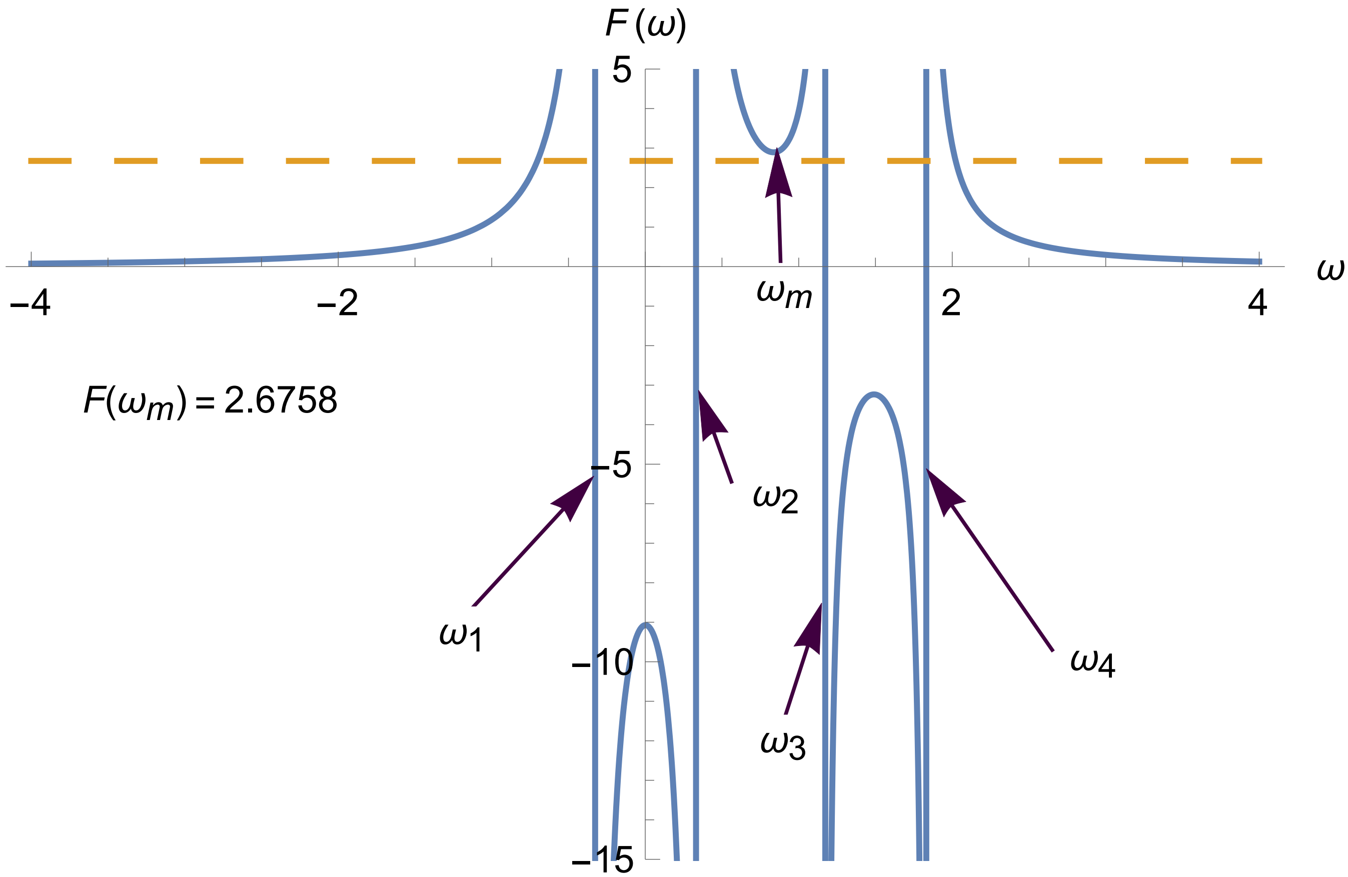

Figure 4.1: Plot of dispersion relation

Considering limiting case , μ_b=0, waves propagate for $V_p = \omega/k$

$$V_p = \sqrt{3\sigma_i + \frac{1}{k^2 + \mu_e(1-\beta)}}, \tag{4.0.2}$$

$$V_p = \sqrt{3\sigma_i + \frac{1}{k^2 + \mu_e(1-\frac{4\alpha}{1+3\alpha})}}. \tag{4.0.3}$$

The equation shows that phase speed of a DIA wave significantly increases because of non-thermal electrons.

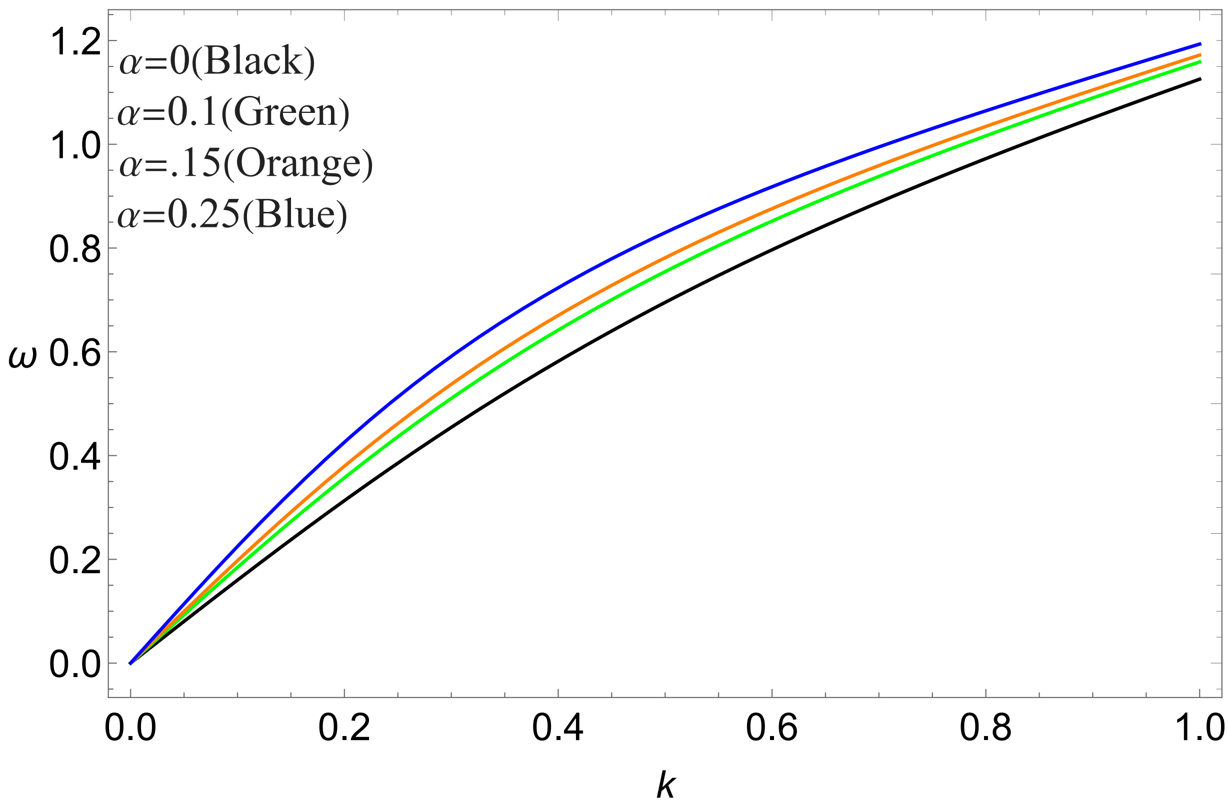

Figure 4.2: Plot of dispersion relation for $\mu_b = 0$

Figure 4.2 shows varying phase velocity (indicated by different colours) for increasing values of α which is the non thermal parameter that describes the number of fast electrons in our system. As it can be seen as the value of non thermal parameter increases from zero to 1 and then to 2 and 2.5 the phase velocity increases as indicated by mathematical relation (4.0.2). Absence of non thermal electrons ($\beta = 0$) gives

$$V_p = \sqrt{3\sigma_i + \frac{1}{k^2 + \mu_e}}, \tag{4.0.4}$$

which exists in long-wavelength limit i.e., $k \ll 1$ as

$$V_p = \sqrt{3\sigma_i + 1/\mu_e}, \tag{4.0.5}$$

Considering cold ions, σ_i

$$V_p = \sqrt{1/\mu_e}. \tag{4.0.6}$$

This is in exact accordance with the phase speed obtained for cold dust.

The graphical study of the dispersion relation is carried out by contour plotting ω vs k for three wave modes F mode, S and DIA mode which propagate along beam direction. Figure 4.3 shows three modes for varying values of u_{b0}. It is observed in the figure that as ion beam speed is increased from 4 (represented by Green colour) to 5 (represented by black colour) the phase speed of both Fast and Slow mode is increased whereas the dust ion-acoustic mode has no effect. This happens because ion beam is the source of generation of fast and slow modes whereas dust ion acoustic mode appears for zero beam velocity.

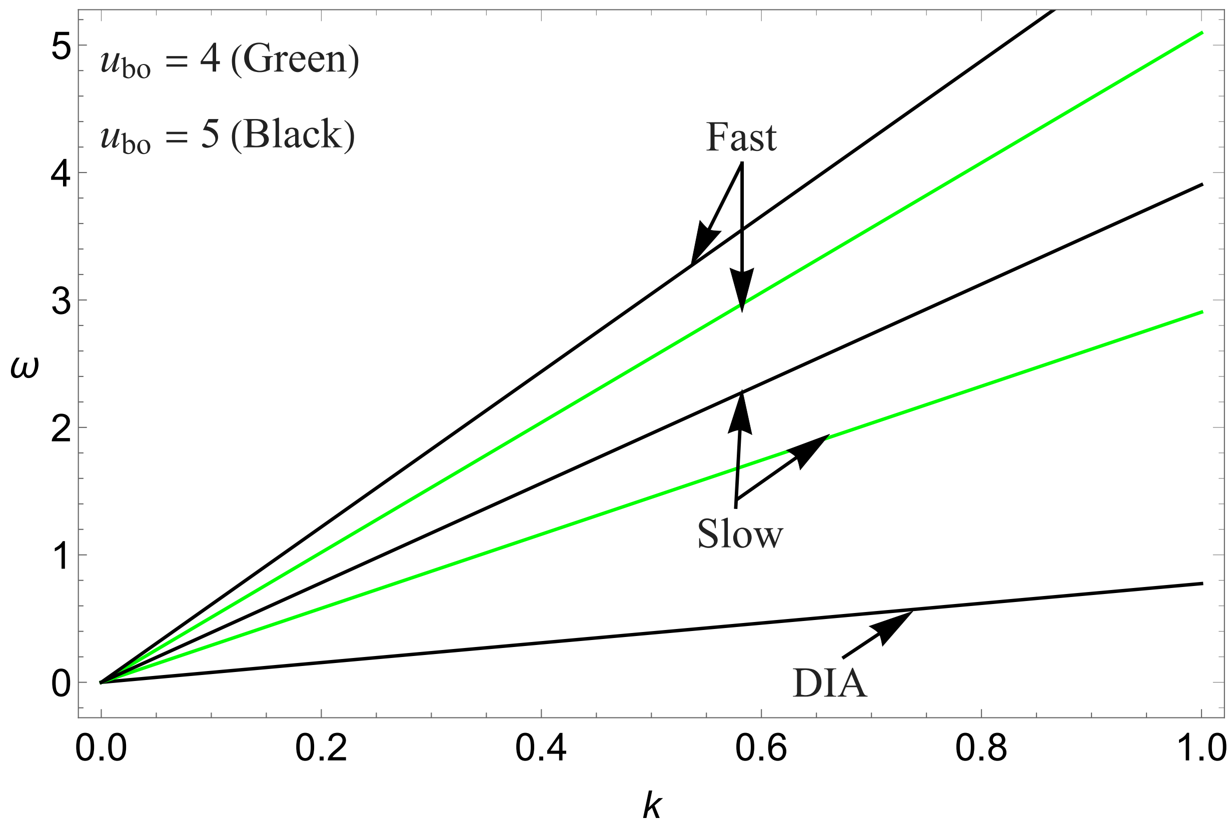

Figure 4.3: Dispersion relation against wavenumber k for $\sigma_b = 0.4$,μ_b=0.9,σ_i=0.2 with values of $u_{b0} = 4$, 5.

Figure 4.4 shows a little variation in fast and slow mode and no variation in dust ion acoustic mode by changing the ion beam temperature ratio. The ion beam temperature ratio is increased from 0.4 (Green colour) to 0.9 (Black colour). The phase speed of the fast mode increased by increasing σ_b. Phase speed of the slow mode decreased as the σ_b increased but the effect on phase speed by increasing ion beam speed in figure 4.3 was greater. This happens because ion beam speed is a source of energy which causes instability so it has a greater effect on phase speeds of fast and slow mode [29].

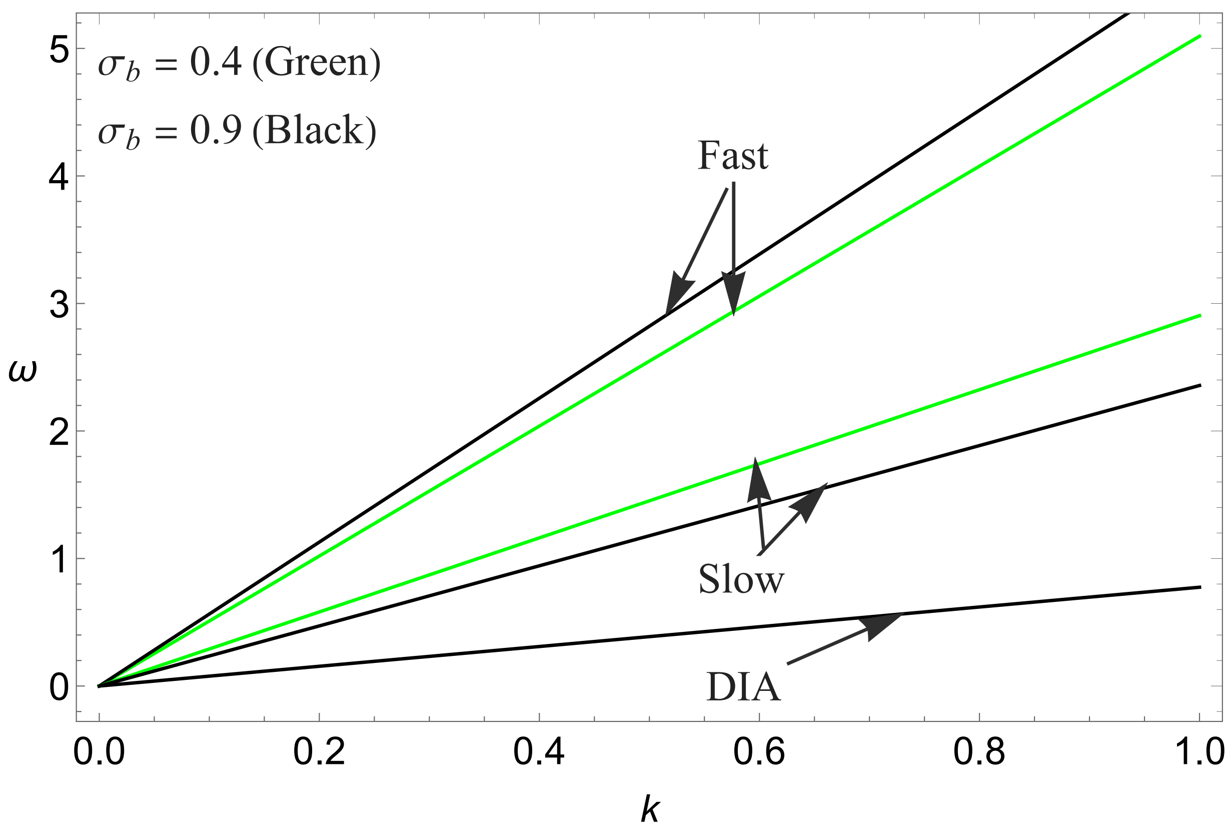

Figure 4.4: Dispersion relation against wavenumber k for $u_{b0} = 4$,μ_b=0.9,σ_i=0.2 with values of $\sigma_b = 0.4$,0.9.

Figure 4.5 describes the effect of ion beam density ratio on three modes. Ion beam density ratio is increased from 0.4 (Green colour) to 0.9 (Black colour). It can be seen that none of them is effected by ion beam density ratio. This happens because while considering the fluid model for our dispersion relation there is no critical value for f_b whereas kinetic model has a critical value for f_b and above this value DIA mode is shifted into Fast mode. Below this critical value the DIA mode propagates without shifting to fast mode.

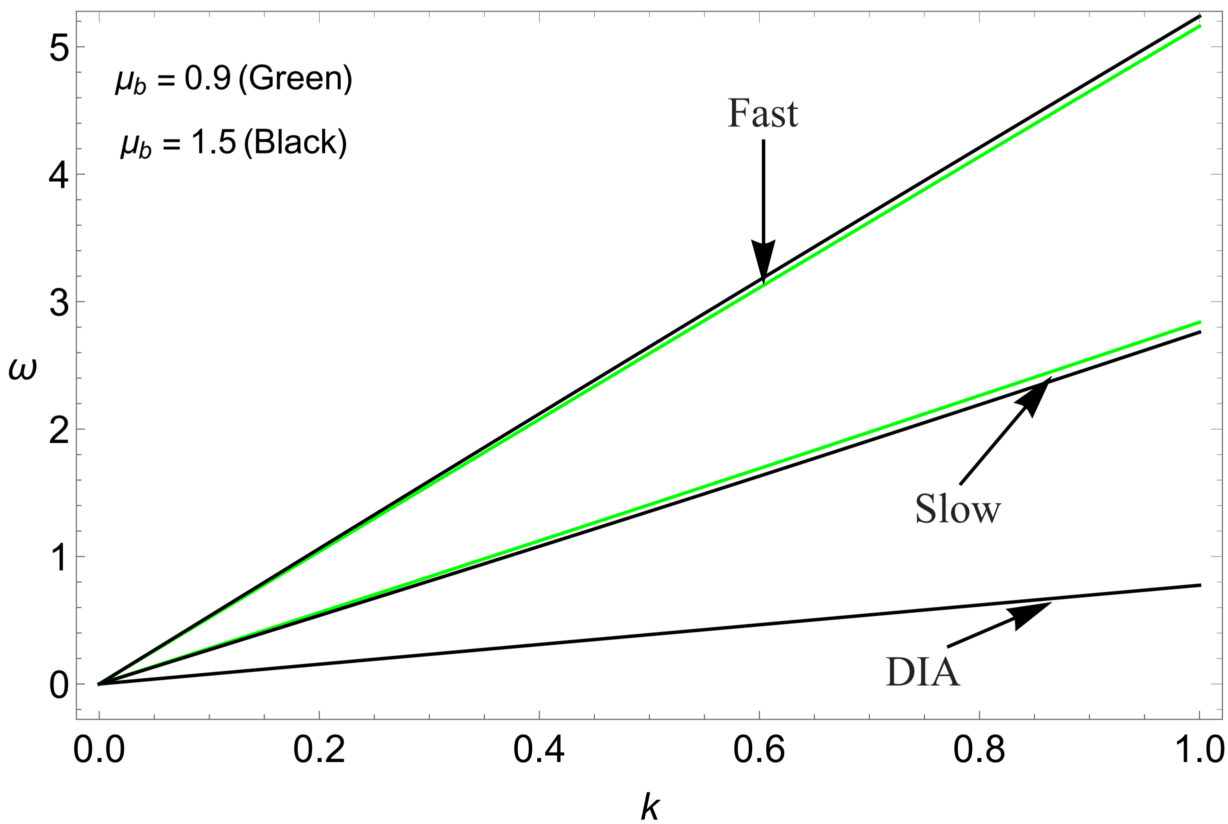

Figure 4.5: Dispersion relation against wavenumber k for $\sigma_b = 0.4$, u_{bo}= 4,σ_i=0.2 with values of $\mu_b = 0.9$,1.5

Figure 4.6 shows that by increasing ion temperature ratio the only mode effected is dust ion-acoustic mode. The ion temperature ratio is increased from 0.2 (Green) to 0.8 (Black). Dust ion-acoustic waves phase speed increases as value of ion temperature ratio increases. This happens because the dust ion acoustic mode is generated because of presence of ions whereas fast and slow mode does not depend on presence of ions.

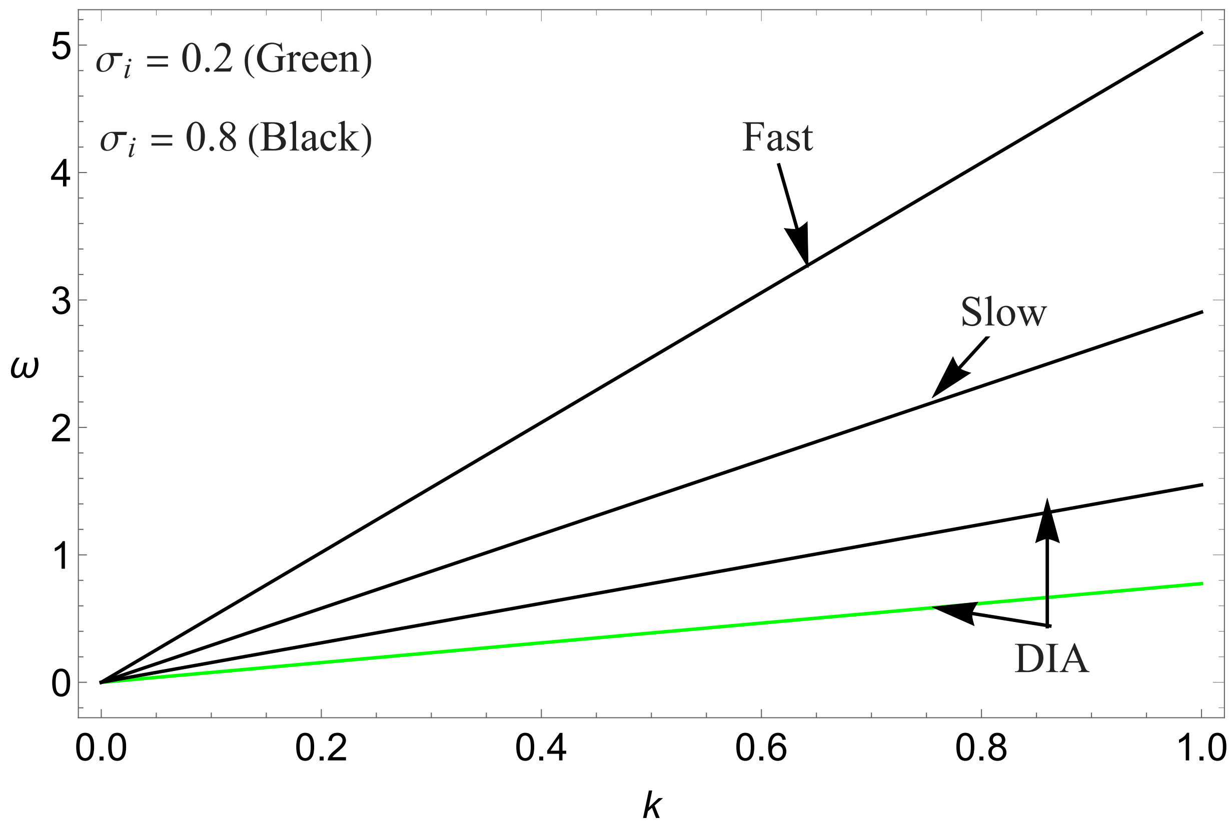

Figure 4.6: Dispersion relation against wavenumber k for $\mu_b = 0.9$,$\sigma_b = 0.4$, $u_{bo} = 4$ with values of $\sigma_i = 0.2$, 0.8.

4.1 Conclusion

The corresponding dispersion relation yields three longitudinal electrostatic modes which propagates for larger beam velocities. These three modes are dust ion acoustic,fast and slow modes that propagate along beam direction. Increasing the non thermal parameter value increased the phase velocity of DIA wave. The two stream instability in dusty plasma between ion and the energy source ion beam occurs when $(\omega/k) \rightarrow \sqrt{3\sigma_i}$ or when $(\omega/k - u_{b0}) \rightarrow \pm\sqrt{3\sigma_b}$. The theoretical results shows that the phase speed of both fast and slow mode is increased by increasing ion beam speed whereas the DIA-mode has no effect. A little variation in fast and slow mode and no variation in dust ion acoustic mode occurs by changing the ion beam temperature ratio. By varying ion beam density ratio none of the modes is effected while only DIA-mode is effected by varying ion beam temperature ratio.

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