New Longitudinal Waves in Electron-Positron-Ion Quantum Plasma

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Dedication

Dedicated to My Sweet Parents, My Brother and My Loving Sisters

And

To All My Teachers

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QM

Abstract

We have taken into account the recently developed quantum kinetic model, for linear wave analysis in which quantum effects are incorporated. Two types of quantum plasma are considered here. The first type consist of electron-positron-ion degenerate Fermi gas in which we take $m_e = m_p$, here damped zero sound waves exist in contrast to undamped one in degenerate electron-ion plasma. Second type of quantum plasma is composed of electron-hole-ion plasma, where $m_e \neq m_h$. Here longitudinal quantum sound waves exist which have no analogue in quantum electron-ion plasma. The excitation of these longitudinal quantum sound waves by low density electron beam is examined. Moreover, the zero sound waves and longitudinal quantum sound waves are examined for degenerate ions along with degenerate electrons and positrons.

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Chapter 1

Introduction

Plasma physics is the study of large number of charged particles, in which the long-range coulomb force is much larger than the force due to neighboring particle. Tonks and Langmuir were first used the word plasma by in 1929. The charged particles exhibit a collective behavior i.e., motion of charged particles affect the motion of other charged particles far away via fields. It was first described by Lord Rayleigh in 1966 [3]. Plasmas are commonly found in the outer layers of the sun and stars, which are made up of matter in an ionized state. From these regions charge particles emerge out to form a wind that filled the interplanetary space [4]. The plasmas are also observed in compact objects like white dwarfs, neutron stars or black holes. Thus it was claimed that most of the matter in the universe is in the plasma state. The plasma state is defined as an electrified gas with the atoms dissociated into positive ions and negative electrons. This gas is quasi neutral i.e., neutral enough so that one can take $n_i \approx n_e \approx n$ where n is the plasma density but not as neutral so that various interesting electromagnetic effects vanishes. The terrestrial plasmas are limited to a few examples: the flash of lightning bolt, the ionosphere, the magnetosphere contains plasma in the Earth's surrounding space environment and the slight amount of ionization in a rocket exhaust [5]. Laboratory plasmas can also exist with wide range of applications in semiconductor device fabrication including reactive-ion etching, sputtering, surface cleaning and plasma-enhanced chemical vapour deposition. In laboratory, plasma can also be formed when high power lasers interact with materials.

The matter particles such as electrons, protons, neutrons are fermions which obey the laws of quantum mechanics. Therefore all plasmas are in some sense quantum, so in general plasma can be described in two categories i.e., Classical Plasma and Quantum Plasma. In classical plasma the quantum nature of the constituent particles dose not affect the macroscopic dynamics of plasma. By high temperature and low density, we can characterize such plasma. In this regime the de-Broglie wavelength ($\lambda_T = \frac{h}{\sqrt{2\pi m k_B T}}$) is so small. Therefore particles behaves point like, and wavefunctions will not overlapp. But when the density ($n_e \approx 10^{20} - 10^{30}$) of plasma increases or its temperature decreases, then we can enter in a regime where quantum mechanical laws effect the macroscopic dynamics of the system.

1.1 Quantum Plasmas

Before the concepts of quantum plasma researchers deal with the free electron gas (single particle) model in which the effect of periodic ionic lattice (The ions form a regular lattice in which the ionic bonds act in all directions, where ionic bonds are the electrostatic force of attraction between oppositely charged ions) the motions of electrons will be taken into account. Here motion of one electron is quite independent of all the other electrons and there will be no coulomb interactions. Later in the decade 1950-60, David Joseph Bohm and Pines studied the electromagnetic interactions of dense electron gas (e.g., metals). By electromagnetic interactions it means that the coulomb interactions were not ignored and here first time the free electron gas were treated as quantum plasma. Quantum plasma is characterized in-terms of high density with low temperature and here the de-Broglie wavelength is comparable to the inter particle distance, so that the overlapping of wave function occurs. However, high particle density is not the necessary criteria for quantum plasma. Because when the semiconductor plasma is cooled up to $T = 0K$ then quantum effects dominates and Fermi-Dirac statistics is applied for the particles. Besides temperature and density the intrinsic spin also contributes to quantum effects, but while dealing with classical plasma this intrinsic effect is ignored.

1.1.1 Quantum Plasma Environments

Due to its diverse application quantum plasma is gaining interest in modern technology e.g., metallic and semiconductor Nanostructures, metal clusters, spintronics, Nanotubes etc. In semiconductor plasma the electron density ($n_e \geqslant 10^{16} - 10^{18} cm^{-3}$) is much less than in metals ($n_e \approx 10^{20} - 10^{23} cm^{-3}$), but due to attenuation of components, the de-Broglie wavelength is comparable to the inter particle distance. The quantum mechanical effects such as tunneling are assumed to play a central role in the behavior of electronic component to be constructed in next upcoming years. Quantum plasma also occur in planetary interiors in compact astrophysical objects e.g, in the interior of white dwarf, magnetosphere's of neutron star etc. In interior of white dwarf stars the density is some ten orders of magnitudes larger than the ordinary solids. Due to such high number density ($n \approx 10^{30}$) a white dwarf can be as hot as fusion plasma (10^8K) but still behave quantum mechanically [1].

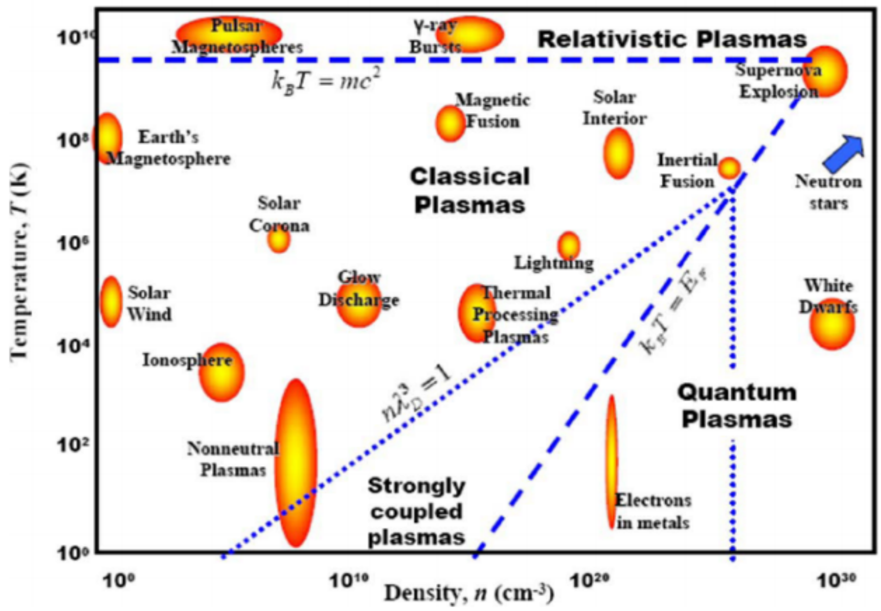

Figure 1.1: Various plasma regimes characterized by temperature and density.

1.1.2 Basic Regimes of Classical and Quantum Plasma

The de-Broglie wavelength which is the additional length scale, is introduced when quantum effects taken into account. The larger the de-Broglie wavelength the more important the quantum effect is. As for ions the de-Broglie wavelength is small therefore ions are treated classically whereas electrons and positrons are treated quantum mechanically, which form a complex system. However dimensionless analysis can reduce the complexity of the problem, that give insight to fundamental scales (time, length and velocity etc). A number of parameters that represent these fundamental scales in a classical and quantum plasma are explained here. These dimensionless parameters characterize or distinguish plasma in its different regimes that either it is classical or quantum. Moreover either it is dominated by collisional or collision less effects. The coulomb coupling parameter (g_c, g_Q), The quantum degeneracy parameter (γ) and the typical densities and temperature are the defining features of dense plasma.

1.1.3 Degeneracy Parameter (γ)

A gas is in the quantum plasma regime when the average de-Broglie wavelength is comparable to average inter-particle distance i.e., $\lambda_B \geqslant d$, where $\lambda_B = \frac{\hbar}{m v_{t\alpha}}$ ($\hbar$ is Planck constant v_{T_α} is the thermal velocity (α=e,i etc)). The electron and positron can be no longer described by considering classical point particle. The inter particle distance is approximately $d \sim (n)^{\frac{-1}{3}}$, where n being the density of particles. Mathematically the condition for quantum plasma is written as

$$n\lambda_B^3 \geq 1. \tag{1.1}$$

The Eq.(1.1) will be satisfied, when the temperature is low and corresponding density of particles is high, then the particles get close to each other. In such case the de-Broglie wave length increases so the overlapping of wave-functions occurs and quantum effects dominate.

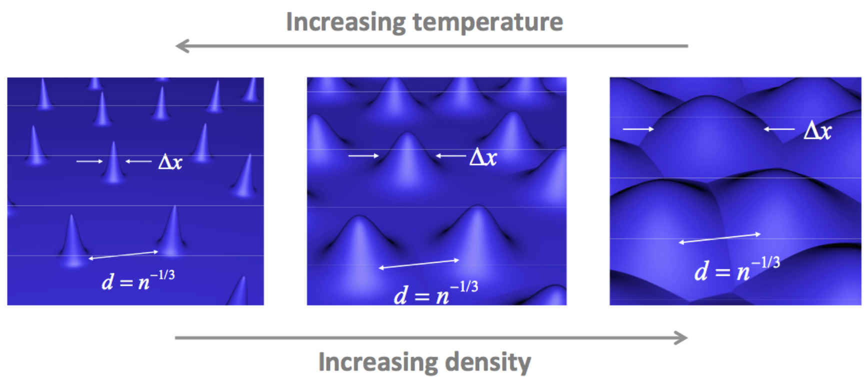

Figure 1.2: In this figure Δx shows the de-Broglie wavelength and d is the inter particle distance whose relation with density of particles is written as $d = n^{\frac{-1}{3}}$

In terms of degeneracy parameter γ (which is the ratio of Fermi energy to that of thermal energy or in terms of temperature is expressed as the ratio of Fermi temperature (T_F) and Thermal temperature(T) i.e., $\gamma = \frac{T_F}{T}$) these two regimes (Classical(non-degenerate) and Quantum(degenerate)) can be differentiated. The Fermi temperature is defined as

$$T_F = \frac{\hbar^2}{2mk_B}(3\pi^2)^{\frac{2}{3}} n^{\frac{2}{3}},$$

where k_B is the Boltzmann constant. The de-Broglie wavelength is

$$\lambda_T = \sqrt{\frac{\hbar^2}{4\pi m k_B}}.$$

Therefore T_F can be expressed as

$$T_F = \frac{T}{2}(3\pi^2)^{\frac{2}{3}}(n\lambda_T^3)^{\frac{2}{3}},$$

hence γ can be written as

$$\gamma = \frac{1}{2}((3\pi^2)^{\frac{2}{3}}(n\lambda_T^3)^{\frac{2}{3}}.$$

When $\gamma \geq 1$ than plasma is in quantum regime and when $\gamma < 1$ than the plasma is in classical regime. While moving from classical to quantum regime the plasma statistics changes from Maxwellian to Fermi Dirac distribution. The plasma degeneracy is an important effect in quantum regime and the average inter particle distance is the scale for plasma degeneracy. Beside this scale there are some other length scales which play an important role in plasma degeneracy even in classical (non-degenerate) hot plasma e.g, Landau length $\frac{Ze_\alpha^2}{k_B T_\alpha}$. When the λ_B is comparable or greater then Landau length then, quantum effects will prominent in classical regime. Mathematically,

$$\lambda_B \geq \frac{Ze_\alpha^2}{k_B T_\alpha}.$$

There is another example in which λ_B is greater then the landau length but less than the average distance of inter particle . Hence quantum interference can occur in this regime its example is in typical fusion plasma having densities $n = 10^{18} cm^{-3}$ and $T = 10^8 K$. Spinning motion of charged particles is an other example of quantum effects in classical plasma [6].

1.1.4 Coupling Parameter (g_c, g_Q)

The coupling parameter in classical and quantum regime are g_c and g_Q. Through these parameters the ideality (collision less or weakly coupled) or non-ideality (collisional or strongly coupled) of plasma can be checked, g_c is define as the ratio of average potential energy or interaction (electric) energy $\sim \frac{e^2 n^{\frac{1}{3}}}{\epsilon_0}$ and average kinetic $\sim k_B T$. This coupling parameter is a measure of the degree to which many-body interactions affects the dynamics of particles in the system. So the coupling dimensionless parameter is

$$g_c = 4\pi \frac{e^2 n^{\frac{1}{3}}}{k_B T}. \tag{1.2}$$

When g_c is small or $g_c << 1$ then it means that thermal effects are dominant and binary collisions are weak so the plasma is ideal (weakly coupled). On the other hand when it is large i.e., $g_c \geq 1$ means binary collisions are dominated and collisions are not ignored, here plasma is considered to be non-ideal (strongly coupled). In terms of Debye length

$$\lambda_D = \sqrt{\frac{k_B T}{4\pi n e^2}},$$

the coupling parameter can be expressed as

$$g_c = \frac{1}{4\pi} (\frac{1}{n\lambda_D^3})^{\frac{2}{3}}.$$

When Debye length is larger than inter particle distance then g_c is small which indicates that it is ideal (collision-less). For Quantum regime the dimensionless parameter is same as in Eq.(1.2) but the thermal energy is replaced by Fermi energy, and Debye length should be replaced by Thomas Fermi screening length λ_{Df}. Quantum coupling parameter can be expressed as

$$g_Q = \frac{1}{4\pi}(\frac{1}{n\lambda_{Df}^3})^{\frac{2}{3}}. \tag{1.3}$$

When $g_Q << 1$ it indicates degenerate collision-less regime and when $g_Q \geq 1$ then it is degenerate collisional regime. The coupling and degeneracy parameters are the

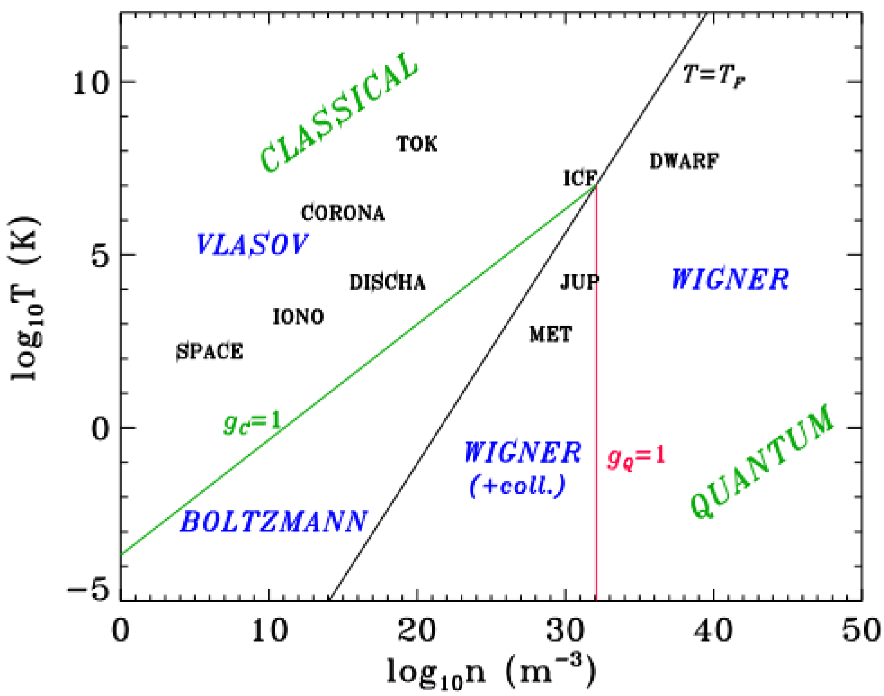

Figure 1.3: Here the Log T and Log n plane is divided into 4 regimes, two of which are classical and two are quantum mechanical region which is further divided into collisional and non collisional regime. In this figure g_c is classical coupling parameter and g_Q is quantum coupling parameter. This figure is taken from Ref[1]

functions of density and temperature. In figure (1.4) the straight line corresponds to $\gamma = g_c = g_Q = 1$, which differentiate the various plasma regime.

1.2 Electron-Positron-Ion Quantum Plasma

The Astrophysical and laboratory plasma may have multi-components. A multi-component plasma composed of fully or partially ionized mixture of charged and neutral particle that satisfy the condition of quasi-neutrality. The four widely studied multi-component plasma systems are electron-positron-ion plasma, pair plasma, dusty plasma and multi-ion plasma.

1.2.1 Pair Plasma

There are many sources of pair production $(e^+, e-)$ in laboratory as well as in Astrophysical environment. In laboratory the pair production takes place through $light-matter$, $light-light$ interactions. When an intense laser interact with solid target (light-matter) or when two high intensity laser beams (light-light) interact with each other then pair production occurs. In light-matter interaction pair production occurs through two processes, first is BH-process (Bethe-Heitler) and second is Trident process. In BH-process the laser accelerated the electrons up to MeV energies. These accelerated electrons then interact with nuclei of target material to give bremsstrahlung photons (e.m radiation produced by deceleration of charged particles, the loss in kinetic energy of moving particles is converted into photons by satisfying the law of conservation of energy), which than interact with nuclei to produce electron-positron pairs. Moreover in Trident process the accelerated electrons directly produce pairs upon interactions with target nuclei. In light-matter interaction when laser interacts with a solid target the density of positron as high as $10^{16}/cm^3$ and pair density approaching $10^{21}/cm^3$ has been acheived [7]. These processes occur at fundamental frequency known as plasma frequency. When the frequency of laser is greater then plasma frequency $\omega > \omega_p$ than laser interaction with plasma can occur otherwise laser cannot pass through the plasma. On the other hand when counter propagating laser beam pass through dense

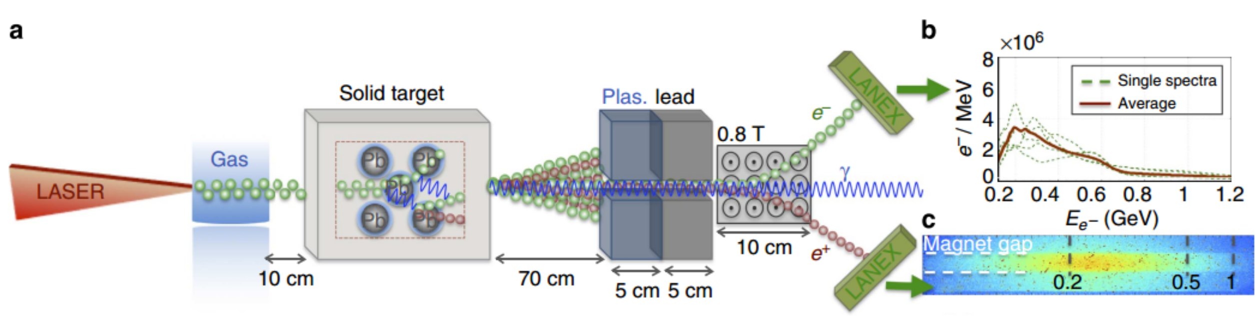

Figure 1.4: laser plasma interaction and how electron positron are produced [2].

plasma they accelerated the plasma electrons to ultra-relativistic speeds, which in turn emit photons having energy comparable to laser photons. The energy of these photons are $\sim 100MeV$, when they interact with laser photons then pairs are produced. The intensity of the laser beams is achieved up to $10^{22}Wcm^{-2}$ and the latest running setups like High Power Laser Energy Research Facility (HIPER) and Extreme Light Infrastructure(ELI) are expected to achieve the laser intensity as high as $10^{26}Wcm^{-3}$ by the end of 2020. When a γ-ray strikes a nuclei it gives rise to electron-positron pair only when the energy of these γ-ray photons is greater then the threshold energy. The threshold energy is equivalent to the rest mass energy of two electrons, i.e., $1.02MeV$. According to law of conservation of energy when the energy of photon is greater then $1.02MeV$ than it can split into two particles i.e., e^+, e^-. According to $E = mc^2$ when the positron come to rest then it may interacts with other electrons as a result photons or γ-rays are produced. In the magnetosphere of Pulsars (a magnetized neutron star emitting electromagnetic radiation) and magnetars (a neutron star with ultra-strong magnetic field) pairs are produced as these objects are highly magnetized. Due to the rotation of these highly magnetized objects electric field is induced. The component of this induced electric field which is parallel to magnetic field accelerates the electron upto ultra-relativistic speeds. These high energy particles are constrained to move in a curved magnetic field and emit high energy photons (whose energy is greater then the pair production threshold). Thus they create pairs by the interaction of two photons, when the magnetic field is strong $B > 10^{12}G$ as in Magnetars Ref[7].

1.2.2 Electron-Positron-Ion Plasma Environment

Electron-positron-ion plasma is present in interstellar medium which is the matter and radiation that exists in space between the star systems in a galaxy. This matter includes gas in ionic, atomic and molecular form as well as dust and cosmic rays. In Astrophysical objects positrons are produced when a cosmic ray nuclei interact with atom, however in laboratory plasma a short relativistically strong laser pulse interact with matter as a result epi plasma is formed due to pair production

$$\gamma + A \rightarrow A + e^+ + e^-.$$

In compact Astrophysical objects, for example in white dwarfs, magnetars, pulsars and neutron stars etc degenerate plasmas exist. A high pair creation rate is expected in compact objects due to high densities of positrons and electrons therefore here electrons and positrons are treated relativistically degenerate. However ions are also present in astrophysical environment. The existence of a fraction of ion in astrophysical plasmas have been confirmed by "Advanced Satellite for cosmology and Astrophysics (ASCA)" [8], therefore the equilibrium quasi-neutrality condition for epi plasma is

$$n_{0e} \approx n_{0p} + n_{0i}.$$

The intrinsic symmetry between electron (e^-) and positron (e^+) particles with in the e-p plasma makes there dynamics quite different from e-i plasma. In e-p plasma due

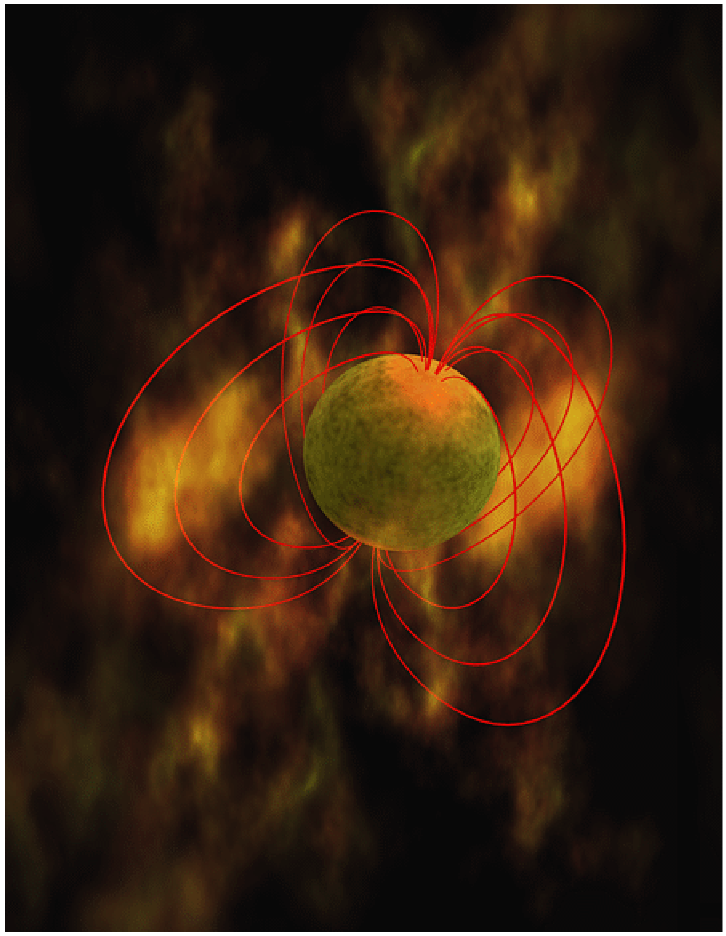

Figure 1.5: magnetic field lines of magnetar according to Artist's conception (NASA logos)

to the mass symmetry of charged particles, induced different growth rates of series of kinetic and fluid instabilities [9]. When ions are introduced in e-p plasma then there dynamic scales changes significantly, linear wave spectrum also increases and wide range of frequencies are available in three component (e-p-i) plasma, as compared to two component (e-p or e-i) plasma. Another aspect of epi plasma is that many non-linear phenomenon could happen and possibility of wave-wave interaction increases. e-p plasma is extensively studied theoretically in astrophysical context as well as in laboratory experiments in which due to the short lifetime of positron, the transport (plasma) mechanism in tokamakit can be study. By injecting of neutral positronium atom, the positron is introduced in tokamak, and it get ionized by plasma. In tokamak the confinement of particles is less than the annihilation time of positrons in plasma. [10, 11].

1.2.3 Relativistic Degenerate Plasma

Relativistic degenerate plasma exist extensively in fusion and astrophysical environments. Now a days, relativistic plasmas can also be produced by intense Laser-matter interactions [12], when the laser power is high enough to cause the plasma oscillation velocity to become highly relativistic. The relativistic factor becomes greater than 1 which give rise to variety of ultra-relativistic effects. In laboratory e-p plasma has been produced by the interaction of solid gold target with an intense picosecond laser, here the result shows that the positron density can be up to $10^{26}cm^{-3}$ [13]. The degenerate electron, positron may be non-relativistic, relativistic and ultra-relativistic depending upon the ratio of Fermi energy and rest mass energy of an electron. In relativistic degenerate plasma Fermi energy is either comparable or larger than rest mass energy of an electron i.e., $E_F >> m_0c^2$, where $E_F = \frac{(3\pi^2)^{\frac{2}{3}}\hbar^2}{2m_\alpha}n_\alpha^{\frac{2}{3}}$. Thus very high density must be reached to set relativistic degeneracy, such densities naturally exist in dwarf stars and neutron stars. The core of a white dwarf has a high average density which is typically $\sim 10^{34}cm^{-3}$, where the electrons and positrons are relativistically degenerate with weak electrostatic interaction. While the ions may consider as classical or strongly coupled. In the outer mantle with a lower density ($\sim 10^{26}cm^{-3}$) the electron may be weakly relativistic degenerate [14].

Chapter 2

Kinetic Theory

2.1 Theoretical Study of Plasma Physics

In order to describe the plasma system that consist of number of particles there are three level of description i.e., exact microscopic description, kinetic theory and the fluid or macroscopic description. The microscopic description is exact classically in the sense that if a system consist of a few thousand particles than one imagines to write down Newtons law (F=ma) for each individual particle and then solve the interacting trajectories. This description is inappropriate because numerical analysis of this system is quite complex and the system cannot be fully solved. Therefore we move towards kinetic description or kinetic theory in which statistical and probability concepts and ensemble averaging is considered. In kinetic theory the identity or precise location of particles are lost and we need complete description about the particle motion, therefore it is still considered as microscopic theory. It can be reduced further and we considered fluid theory in which macroscopic quantities such as temperature and velocity density gives the description of physical system in space and time. In fluid theory we don't need the individual particle motion to describe the physical system.

2.2 Kinetic and Fluid Description

The fluid theory is the simplest description of plasma that have been used so far but there are some phenomenon where fluid theory is inadequate to apply. The dependent variables in fluid theory are functions of only four variables i.e., x, y, z and t. Therefore in this theory the velocity distribution of each species is assumed to be Maxwellian every where and it can be described by one number i.e., the temperature. As an example when we consider two different velocity distribution the fluid theory dosen't distinguish the difference between them as long as the area under the curve is same. If the thermal velocity of charged particle is close to the phase velocity of waves than the wave particle interaction cannot be described by the fluid equations, because this interaction depends on the dynamics of the particles i.e., phase space distribution function in which the velocity and position are independent variables. Therefore we need to consider velocity distribution for each species i.e., $f = f(r, v, t)$, here f is function of seven independent variables. This treatment is known as kinetic theory and it provides the complete description of the plasma with self consistent fields [15].

2.3 Quantum Kinetic Theory

In Quantum kinetic description of plasma the particles should be treated as waves, then it is quite difficult to solve the Schroedinger's equation for the N particle wave function of the system. However it can be simplified by considering that plasma is nearly ideal i.e., we focused on the collective behavior of plasma rather then the dynamics of each and every particle. Hence the plasma can be considered as the collection of quantum particles. The dynamical equations that are used for the description of quantum system is Schroedinger's equation (for pure states) and density-matrix equation (for mixed states). There are commonly three approaches used for the mathematical modeling of quantum plasma i.e., the Wigner-Poisson, the Schroedinger-Poisson and the Dirac Maxwell. These approaches define the statistical and hydrodynamic behaviour of plasma particles at quantum scales. The quantum hydrodynamic model (QHD) have been developed to study the dynamics of transport phenomenon of charge particle in a system that interact through self-consistent electrostatic potential. Manfredi gives insight to quantum plasma in his paper "How to Model Quantum Plasma" [1] that urge plasma physicist to propose comprehensive quantum plasma models. For instance, the development of a new quantum kinetic model for Fermi particles by Tsintsadze [16] was actually incited by a previously proposed work of Kuzelev [17]. This model provides a very understandable and easy explanation of the particle dispersion effects. Moreover recently developed spin kinetic theories bring new effects in the limelight. In particular, these newly developed models of spin quantum plasmas encompass the concept of some new kinds of resonances caused by electron spin motion [18, 19, 20, 21], which cannot be found in the formerly developed theories.

2.3.1 Quantum Kinetic Equation

Let us first consider the case when we include the external magnetic field and neglecting the spin effects. The quantum Hamiltonian of charged spin- $\frac{1}{2}$ particle in an electromagnetic field is written as

$$\widehat{H} = \frac{1}{2m_\alpha}(\widehat{P} - \frac{e_\alpha}{c}A(r,t))^2 + e_\alpha\phi(r,t),$$

where $\widehat{P} = -i\hbar\nabla$, A(r,t) and $\phi(r,t)$ are the scalar and vector potentials. After Simplifying the above equation, we get

$$\widehat{H} = \frac{1}{2m_\alpha}[-\hbar^2\nabla^2 + \frac{e_\alpha^2}{c^2}A^2(r,t) + \frac{i\hbar e_\alpha}{c}(\nabla.A(r,t) + A(r,t).\nabla)] + e_\alpha\phi(r,t).$$

By using $\frac{i\hbar e_\alpha}{2m_\alpha c}[\nabla.A(r,t) + A(r,t).\nabla] = \frac{i\hbar e}{m_\alpha c}[A(r,t).\nabla]$ the above Hamiltonian becomes

$$\widehat{H} = \frac{-\hbar^2\nabla^2}{2m_\alpha} + \frac{e_\alpha^2 A^2(r,t)}{2m_\alpha c^2} + \frac{i\hbar e_\alpha}{m_\alpha c}(A(r,t).\nabla) + e_\alpha\phi(r,t). \tag{2.1}$$

Now by considering the time dependent Schrodinger equation and inserting the above hamiltonian in it, we get

$$i\hbar\frac{\partial}{\partial t}\psi_\alpha(r,t) = \widehat{H}\psi_\alpha(r,t),$$

$$i\hbar\frac{\partial}{\partial t}\psi_\alpha(r,t) = [\frac{-\hbar^2\nabla^2}{2m_\alpha} + \frac{e_\alpha^2 A^2(r,t)}{2m_\alpha c^2} + \frac{i\hbar e_\alpha}{m_\alpha c}(A(r,t).\nabla) + e_\alpha\phi(r,t)]\psi_\alpha(r,t), \tag{2.2}$$

here we use the Madelung representation [17], [22] of wave function i.e.,

$$\psi_\alpha(r,t) = R_\alpha(r,t)e^{\frac{iS_\alpha(r,t)}{\hbar}},$$

where $R_\alpha(r,t)$ and $S_\alpha(r,t)$ are real magnitude and real phase function. Now using this Madelung representation in Eq.(2.2) L.H.S can be written as

$$i\hbar\frac{\partial}{\partial t}[R_\alpha(r,t)e^{\frac{i}{\hbar}S_{\alpha(r,t)}}] = i\hbar\frac{\partial R_\alpha}{\partial t}e^{\frac{i}{\hbar}S_\alpha(r,t)} - R_\alpha(r,t)\frac{\partial S_\alpha(r,t)}{\partial .t}e^{\frac{i}{\hbar}S_{\alpha(r,t)}}.$$

After neglecting the nonlinear term the R.H.S will become

$$\begin{aligned} &= [-\frac{\hbar^2}{2m_\alpha}\nabla^2 R_\alpha(r,t) - \frac{i\hbar}{m_\alpha}\nabla R_\alpha(r,t).\nabla S_\alpha(r,t) - \frac{i\hbar}{2m_\alpha}R_\alpha(r,t).\nabla^2 S_\alpha(r,t) \\ &+ \frac{e_\alpha^2 A^2(r,t)R_\alpha(r,t)}{2m_\alpha c^2} + \frac{i\hbar A(r,t)}{m_\alpha c}.\nabla R_\alpha(r,t) - \frac{e_\alpha A(r,t)}{m_\alpha c}.\nabla S_\alpha R_\alpha(r,t) \\ &+ e_\alpha\phi(r,t)R_\alpha(r,t)]e^{\frac{i}{\hbar}S_\alpha(r,t)} \end{aligned}.$$

Hence the resulting equation becomes

$$\begin{aligned} &i\hbar\frac{\partial R_\alpha}{\partial t}e^{\frac{i}{\hbar}S_\alpha(r,t)} - R_\alpha(r,t)\frac{\partial S_\alpha(r,t)}{\partial t}e^{\frac{i}{\hbar}S_{\alpha(r,t)}} = [-\frac{\hbar^2}{2m_\alpha}\nabla^2 R_\alpha(r,t) - \frac{i\hbar}{m_\alpha}\nabla R_\alpha(r,t).\nabla S_\alpha(r,t) \\ &- \frac{i\hbar}{2m_\alpha}R_\alpha(r,t).\nabla^2 S_\alpha(r,t) + \frac{e_\alpha^2 A^2(r,t)R_\alpha(r,t)}{2m_\alpha c^2} + \frac{i\hbar A(r,t)}{m_\alpha c}.\nabla R_\alpha(r,t) \\ &- \frac{e_\alpha A(r,t)}{m_\alpha c}.\nabla S_\alpha R_\alpha(r,t) + e_\alpha\phi(r,t)R_\alpha(r,t)]e^{\frac{i}{\hbar}S_\alpha(r,t)} \end{aligned}. \tag{2.3}$$

Now we have to separate the real and imaginary parts of above equation. By comparing the imaginary part of above equation the resulting equation is continuity equation i.e.,

$$\frac{\partial R_\alpha^2}{\partial t} + \nabla . \frac{R_\alpha^2 P_\alpha}{m_\alpha} = 0$$

and the real parts becomes

$$\begin{aligned} &-\frac{\hbar^2}{2m_\alpha}\nabla^2 R_\alpha(r,t) + \frac{e_\alpha^2}{2m_\alpha c^2}A^2(r,t)R_\alpha(r,t) - \frac{e_\alpha}{m_\alpha c}A(r,t).\nabla S_\alpha(r,t)R_\alpha(r,t) \\ &+ e_\alpha\phi(r,t)R_\alpha(r,t) = -R_\alpha(r,t)\frac{\partial S_\alpha(r,t)}{\partial t}, \end{aligned} \tag{2.4}$$

dividing both side by $-R_\alpha(r,t)$

$$\frac{\partial S_\alpha(r,t)}{\partial t} = \frac{\hbar^2}{2m_\alpha}(\frac{\nabla^2 R_\alpha(r,t)}{R_\alpha(r,t)}) - \frac{e_\alpha^2 \nabla A^2(r,t)}{2m_\alpha c^2} + \frac{e_\alpha}{m_\alpha c}A(r,t).\nabla S_\alpha(r,t) - e_\alpha\phi(r,t).$$

Now taking the gradient of above equation and also neglecting the second term on R.H.S because this term is so small as compared to other terms

$$\frac{\partial \nabla S_\alpha(r,t)}{\partial t} = \frac{\hbar^2}{2m_\alpha}\nabla(\frac{\nabla^2 R_\alpha(r,t)}{R_\alpha(r,t)}) + \frac{e_\alpha}{m_\alpha c}\nabla(A(r,t).\nabla S_\alpha(r,t)) - e_\alpha\nabla\phi(r,t), \tag{2.5}$$

where $p_\alpha = m_\alpha v_\alpha - \frac{e_\alpha}{c}A(r,t)$, $m_\alpha v = \nabla S_\alpha(r,t)$. Therefore

$$\nabla S_\alpha = p_\alpha + \frac{e_\alpha}{c}A(r,t),$$

time derivative of above equation becomes

$$\frac{\partial(\nabla S_\alpha(r,t))}{\partial t} = \frac{dp_\alpha}{dt} + \frac{e_\alpha}{c}[\frac{\partial A(r,t)}{\partial t} + (v.\nabla)A(r,t)]. \tag{2.6}$$

As the total derivative of vector potential is

$$\frac{dA(r,t)}{dt} = [\frac{\partial A(r,t)}{\partial t} + (v.\nabla)A(r,t)].$$

Now using Eq.(2.6)in (2.5), we get

$$\frac{dp_\alpha}{dt} + \frac{e_\alpha}{c}[\frac{\partial A(r,t)}{\partial t} + (v.\nabla)A(r,t)] = \frac{\hbar^2}{2m_\alpha}\nabla(\frac{\nabla^2 R_\alpha(r,t)}{R_\alpha(r,t)}) + \frac{e_\alpha}{c}\nabla(A(r,t).v) - e_\alpha\nabla\phi(r,t). \tag{2.7}$$

The identity that we consider here is

$$v_\alpha \times (\nabla \times A(r,t)) = \nabla(v_\alpha.A(r,t)) - (v_\alpha.\nabla)A(r,t)$$

by rearranging the terms, we get

$$\nabla(v.A(r,t)) = v_\alpha \times (\nabla \times A(r,t)) + (v.\nabla)A(r,t).$$

The dot product is commutative so we write above equation as

$$\nabla(A(r,t).v_\alpha) = v_\alpha \times (\nabla \times A(r,t)) + (v_\alpha.\nabla)A(r,t).$$

Now using the above identity in Eq.(2.7) yields

$$\frac{dp_\alpha}{dt} = \frac{\hbar^2}{2m_\alpha}\nabla(\frac{\nabla^2 R_\alpha(r,t)}{R_\alpha(r,t)}) + \frac{e_\alpha}{c}(v_\alpha \times (\nabla \times A(r,t)) + (v_\alpha.\nabla)A(r,t) - e_\alpha\nabla\phi(r,t)$$
$$- \frac{e_\alpha}{c}[\frac{\partial A(r,t)}{\partial t} + (v_\alpha.\nabla)A(r,t)]. \tag{2.8}$$

In electromagnetic field the modified electric and magnetic fields are

$$E = -\nabla\phi(r,t) - \frac{\partial A(r,t)}{c\partial t}$$

and

$$H = \nabla \times A(r,t).$$

Using these modified electric and magnetic fields in above equation, we get

$$\frac{dp_\alpha}{dt} = \frac{\hbar^2}{2m_\alpha}\nabla(\frac{\nabla^2 R_\alpha(r,t)}{R_\alpha(r,t)}) + e_\alpha[E + \frac{v_\alpha \times H}{c}]. \tag{2.9}$$

The Quantum Boltzmann equation which is also called landau's equation is

$$\frac{\partial f_\alpha(r,p,t)}{\partial t} + (v_\alpha.\nabla)f_\alpha(r,p,t) + \frac{dp_\alpha}{dt}\frac{\partial f_\alpha(r,p,t)}{\partial t} = c(f_\alpha),$$

here collisional effects are negligible so R.H.S of above equation is zero therefore

$$\frac{\partial f_\alpha(r,p,t)}{\partial t} + (v_\alpha.\nabla)f_\alpha(r,p,t) + \frac{dp_\alpha}{dt}\frac{\partial f_\alpha(r,p,t)}{\partial t} = 0. \tag{2.10}$$

As Fermionic particles satisfy the above equation therefore using Eq.(2.9) in (2.10) the resulting equation is quantum kinetic equation which includes the dispersion effects of particles

$$\frac{\partial f_\alpha(r,p,t)}{\partial t} + (v_\alpha.\nabla)f_\alpha(r,p,t) + \frac{\hbar^2}{2m_\alpha}\nabla(\frac{\nabla^2 R_\alpha(r,t)}{R_\alpha(r,t)}) + e_\alpha[E + \frac{v_\alpha \times H}{c}]\frac{\partial f_\alpha(r,p,t)}{\partial t} = 0. \tag{2.11}$$

2.4 Mathematical Approaches in Quantum Plasma

When we move towards low temperature i.e., in quantum regime numerous new issues emerge, so to express the new effects in quantum plasma we have the following many body approaches or models.

2.4.1 Schroedinger-Poisson System

In quantum plasma (dense) within a small volume, there are large number of particles . Its inappropriate to solve Schroedinger equation for the trajectories of each individual particle. So by generalization from single body to N-body dynamics we can solve the problem. In N-body dynamics we neglect the higher correlations between the two bodies for simplification. Therefore the wave functions for N particle i.e., $\psi(x_1, x_2, ...x_N, t)$ can be written as i.e.,

$$\psi(x_1, x_2, ..., t) = \psi_1(x_1, t), \psi_2(x_2, t).....\psi_N)(x_N, t).$$

This leads to N independent Schroedinger equations

$$i\hbar\frac{\partial\psi_k(x,t)}{\partial t} = \frac{\hbar^2}{2m}\Delta\psi_N(x,t) + eV(x,t)\psi_k(x,t),$$

where $k = 1, 2, ...N$ and V(x,t) is the electrostatic potential (self consistent), given by the Poisson's equation

$$\Delta V(x,t) = 4\pi e(\sum_{k+1}^{N} p_k|\psi_k(x,t)|^2 - n_0),$$

where $\psi_k(x,t)$ is the wave function of N pure states. The probability occupation p_i of different quantum states for Fermi particles is given by Fermi Dirac statistics i.e.,

$$p_k = \frac{1}{e^{\frac{(E-E_F)}{k_B T}} - 1}$$

by definition $\sum_{k=1}^{N} p_k = 1$. In Schroedinger-Poisson model, the quantum mechanical equation of motion and long-range self consistent interactions are main aspects of quantum plasma. This model is consider to be the quantum analogue of Vlasov Poisson model, because majority of suppositions of two models are the same. For example, neglected collisions, electrostatic potential is considered and single particle dynamics is utilized.

2.4.2 Wigner-Poisson Approach

In detailing of quantum mechanics, in order to find the probability density in position space x we take square of the wave functions, $\rho(x) = |\psi(x)|^2$, similar expression exist in momentum space. So its convenient to express wave function in phase space that have position and momentum coordinates. In 1932, Wigner recommended a phase space distribution of quantum mechanics by methods of joint probability distribution. Main objective of Wigner is to establish a relation between quantum and classical mechanics by expressing Boltzmann factor, as the functions of both position and momentum. Wigner distribution for mixed quantum states $\psi_N(x,t)$ each classified by occupation probability p_i [23] is

$$W(x,v,t) = \frac{m}{2\pi\hbar} \sum_{k=1}^{N} p_i \int_{-\inf}^{+\inf} d\lambda \psi_i^*(x + \frac{\lambda}{2}, t)\psi_i(x - \frac{\lambda}{2}, t) e^{\frac{\iota m v \lambda}{\hbar}},$$

where the normalization condition is $\sum_{k=1}^{N} p_k = 1$. The density is defined as

$$n(x,t) = \int_{-\inf}^{+\inf} W(x,v,t) dV.$$

2.4.3 Quantum Hydrodynamic Approach

QHD model or quantum fluid model is generalization of classical fluid model, here the conservation laws of particles, energy and momentum are used to express transport equations. Comparatively QHD is the simplest description that uses collective dynamics rather than S-P and W-P(phase space dynamics). By making use of average quantities both S-P and W-P lead to QHD equations.

Chapter 3

New Longitudinal Waves in Electron Positron Ion Quantum Plasma

3.1 Introduction

In this chapter we study longitudinal waves in epi quantum plasma. For this purpose two different quantum plasmas are considered. One comprised of Fermi gas which is composed of electron, positron and ion (i.e., $m_e = m_p$), and the other consist of electron hole and ions (in this case mass of hole is greater than the mass of electron here the mass difference arises due to interaction between particles). In these two quantum plasmas, we assume that ions are stationary and they provide a uniform fixed background. Semiconductors which contains light electrons and heavy positive holes (charge carriers), can be degenerate ($n_e \geq 10^{16} - 10^{18} cm^{-3}$) with effective mass of electron $m_e^* \approx (0.01 - 0.1)m_e$ and it occurs at temperature $T < 10^2 K$ [24]. The wavelength of the particles is larger than the inter particle distance r_0 at low temperature in degenerate solid state, and in quantum Fermi liquid . Pauli exclusion principle and screening of coulomb interaction between the particles is responsible for such long mean free path. Also in solid state and liquid quantum plasma the effective masses of holes and negative electrons are different from free electrons due to interaction between particles.

3.2 Linear Analysis of Quantum Kinetic Equation

In this section we consider three component (electron positron-ion) collisionless plasma at low temperature i.e., $T \approx 0K$ and than taken into account the linear Landau damping. For longitudinal waves $E(r,t) = -\nabla\phi_\alpha$ and here we take B=0 and the amplitude function in term of density can be expressed as $R_\alpha(r,t) = \sqrt{n_\alpha(r,t)}$ so

Eq.(2.11) can be written as

$$\frac{\partial f_\alpha(r,p,t)}{\partial t}+(v_\alpha.\nabla)f_\alpha(r,p,t)-e_\alpha\nabla\phi_\alpha\frac{\partial f_\alpha(r,p,t)}{\partial t}+\frac{\hbar^2}{2m_\alpha}\nabla(\frac{\nabla^2\sqrt{n_\alpha(r,t)}}{\sqrt{n_\alpha(r,t)}})\frac{\partial f_\alpha(r,p,t)}{\partial p}=0. \tag{3.1}$$

Now by linearizing the above equation with respect to small perturbations in the field i.e., $f_\alpha = f_{\alpha 0}+\delta f_\alpha$, $\phi_\alpha=\phi_{\alpha 0}+\delta\phi_\alpha$,$n_\alpha = n_{\alpha 0}+\delta n_\alpha$, we get

$$\frac{\partial(\delta f_{\alpha 0})}{\partial t}+(v.\nabla f_{\alpha 0})-e_\alpha\nabla\delta\phi_\alpha\frac{\partial f_{\alpha 0}}{\partial p}+\frac{\hbar^2}{2m_\alpha}\nabla.\frac{\nabla^2\delta n_\alpha}{(n_{\alpha 0}+\delta n_\alpha)}(\frac{\partial f_{\alpha 0}}{\partial p}+\frac{\partial\delta f_{\alpha 0}}{\partial p})=0. \tag{3.2}$$

In the linear analysis we consider $\frac{n_{\alpha 0}}{\delta n_\alpha}>1$, so the above equation becomes

$$\frac{\partial(\delta f_\alpha)}{\partial t}+(v.\nabla f_\alpha)-e_\alpha\nabla\delta\phi_\alpha\frac{\partial f_{\alpha 0}}{\partial p}+\frac{\hbar^2}{4m_\alpha}\nabla.\frac{\nabla^2\delta n_\alpha}{n_{\alpha 0}}\frac{\partial f_{\alpha 0}}{\partial p}=0. \tag{3.3}$$

The Poisson equation for longitudinal waves can be written as

$$-\nabla^2\phi_\alpha = 4\pi\Sigma_\alpha e_\alpha n_\alpha. \tag{3.4}$$

By linearizing the above equation, we get

$$\nabla^2(\delta\phi_\alpha)=-4\pi\Sigma_\alpha e_\alpha\delta n_\alpha.$$

The density of particles is expressed as

$$\delta n_\alpha = 2\int\frac{dp}{(2\pi\hbar)^3}\delta f_\alpha,$$

where 2 is due to the spin of electron i.e., (up and down spin). As here all the perturbed quantities vary like $\exp i(k.v-\omega t)$ so above Eq.(3.4) and (3.5) becomes

$$-i\omega\delta f_\alpha+i(v.k)\delta f_\alpha-ie_\alpha\delta\phi_\alpha k.\frac{\partial f_{\alpha 0}}{\partial p}+i\frac{\hbar^2k^2}{4m_\alpha}\frac{\delta n_\alpha}{n_{\alpha 0}}k.\frac{\partial f_{\alpha 0}}{\partial p}=0, \tag{3.5}$$

$$k^2\delta\phi_\alpha = 4\pi\Sigma_\alpha e_\alpha\delta n_\alpha. \tag{3.6}$$

By simplifying Eq.(3.5), we get

$$\delta f_\alpha = e_\alpha\delta\phi_\alpha\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{(k.v-\omega)}+\frac{\hbar^2k^2}{4m_\alpha}\frac{\delta n_\alpha}{n_{\alpha 0}}\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{k.v-\omega}. \tag{3.7}$$

Now integrate both sides of above equation over the volume element $2\int\frac{dp}{(2\pi\hbar)^3}$ i.e.,

$$2\int\frac{dp}{(2\pi\hbar)^3}\delta f_\alpha = e_\alpha\delta\phi_\alpha\int\frac{2}{(2\pi\hbar)^3}\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{(k.v-\omega)}dp+\frac{\hbar^2k^2}{4m_\alpha}\frac{\delta n_\alpha}{n_{\alpha 0}}\int\frac{2}{(2\pi\hbar)^3}\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{k.v-\omega}dp. \tag{3.8}$$

As $\delta n_\alpha = 2\int \frac{dp}{(2\pi\hbar)^3}\delta f_\alpha$ so the above equation becomes

$$\delta n_\alpha = e_\alpha \delta\phi_\alpha \int \frac{2}{(2\pi\hbar)^3}\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{(k.v-\omega)}dp + \frac{\hbar^2 k^2}{4m_\alpha}\frac{\delta n_\alpha}{n_{\alpha 0}}\int \frac{2}{(2\pi\hbar)^3}\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{k.v-\omega}dp, \tag{3.9}$$

we can write above equation as,

$$\delta n_\alpha = \frac{e_\alpha \delta\phi_\alpha \int \frac{2}{(2\pi\hbar)^3}\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{(k.v-\omega)}dp}{1 - \frac{\hbar^2 k^2}{4m_\alpha n_{\alpha 0}}\int \frac{2}{(2\pi\hbar)^3}\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{k.v-\omega}dp}. \tag{3.10}$$

Let

$$I = \int \frac{2}{(2\pi\hbar)^3}\frac{k.\frac{\partial f_{\alpha 0}}{\partial p}}{k.v-\omega}dp \tag{3.11}$$

then the above equation becomes

$$\delta n_\alpha = \frac{e_\alpha \delta\phi_\alpha I}{1 - \frac{\hbar^2 k^2}{4m_\alpha n_{\alpha 0}}I}, \tag{3.12}$$

where $n_{\alpha 0}$ represents the equilibrium density of the plasma species. We assume that at nearly T=0K positrons and electrons are completely degenerate, and their respective equilibrium density functions for plasma particles are considered as the step function

$$f_{\alpha 0} = \delta(E_{F\alpha} - E_\alpha),$$

where $f_{\alpha 0}$ is equal to 1 when $E_{F\alpha} \geq E_\alpha$ otherwise it is zero, here $E_{F\alpha}$ is the Fermi energy ($E_{F\alpha} = \frac{V_{F\alpha}^2}{2}$) and E_α is the thermal energy of electron and positron and

$$\frac{\partial f_{\alpha 0}}{\partial p} = -v\delta(E_\alpha - E_{F\alpha}).$$

In spherical coordinates $d^3p = p^2 sin\theta d\theta dp d\phi$. The corresponding components that we consider here are $p_z = p_\parallel$ $k = (0,0,k)$, $v_z = vcos\theta$. Then the Eq.(3.11) becomes

$$I = -4\pi k \int_0^\infty p^2 \frac{\partial f_{\alpha 0}}{\partial p}dp \int_0^\pi \frac{sin\theta cos\theta}{\omega - kv_\alpha cos\theta}d\theta.$$

After performing the θ-integration, we get

$$\int_0^\pi \frac{sin\theta cos\theta}{\omega - kv_\alpha cos\theta}d\theta = \frac{-2}{kv_\alpha}[1 - \frac{\omega}{2kv_\alpha}\ln\frac{(\omega + kv_\alpha)}{(\omega - kv_\alpha)}].$$

Therefore I integral takes the form

$$I = 8\pi \int_0^\infty p^2 \frac{\partial f_{\alpha 0}}{\partial p}dp[\frac{1}{v_\alpha}(1 - \frac{\omega}{2kv_\alpha}\ln\frac{(\omega + kv_\alpha)}{(\omega - kv_\alpha)})].$$

Here we assumed that when $T \rightarrow 0$ than the Fermi distribution is a step function therefore $T_F >> T$ so that $E_F >> E$, where T is the thermal temperature, T_F is the Fermi degeneracy temperature. Which means that all the energy states below the Fermi level are filled and all those above are vacant. As $E^2 = \frac{p^2}{2m_\alpha}$ so by using this we move from momentum to energy distribution where

$$\frac{\partial f_{\alpha 0}}{\partial E} = -\frac{2}{(2\pi\hbar)^3}\delta(E - E_{F\alpha}).$$

Now using this in above integral, we get

$$I = -\frac{8\pi}{(2\pi\hbar)^3}\int_{m_0c^2}^{\infty}(2m_\alpha E^2)\delta(E - E_{F\alpha})dE[\frac{1}{v_\alpha}(1 - \frac{\omega}{2kv_\alpha}\ln\frac{(\omega + kv_\alpha)}{(\omega - kv_\alpha)})].$$

When $E = E_{F\alpha}$ then $p = p_{F\alpha}$ and $v = v_{F\alpha}$, then the above integral becomes

$$I = -\frac{p_{F\alpha}^2}{\pi^2\hbar^3 v_{F\alpha}}[1 - \frac{\omega}{2kv_{F\alpha}}\ln\frac{(\omega + kv_{F\alpha})}{(\omega - kv_{F\alpha})}].$$

Multiply and divide R.H.S of above equation by $3p_{F\alpha}$. As $n_{0\alpha} = \frac{p_{F\alpha}^3}{3\pi^2\hbar^3}$ employing this in above

$$I = -\frac{3n_{0\alpha}}{m_\alpha v_{F\alpha}^2}[1 - \frac{\omega}{2kv_{F\alpha}}\ln\frac{(\omega + kv_{F\alpha})}{(\omega - kv_{F\alpha})}]. \tag{3.13}$$

Using Eq.(3.13) in (3.12)

$$\delta n_\alpha = \frac{e_\alpha\delta\phi_\alpha[-\frac{3n_{0\alpha}}{m_\alpha v_{F\alpha}^2}(1 - \frac{\omega}{2kv_{F\alpha}}\ln\frac{(\omega+kv_{F\alpha})}{(\omega-kv_{F\alpha})})]}{1 - \frac{\hbar^2k^2}{4m_\alpha n_{0\alpha}}[-\frac{3n_{0\alpha}}{m_\alpha v_{F\alpha}^2}(1 - \frac{\omega}{2kv_{F\alpha}}\ln\frac{(\omega+kv_{F\alpha})}{(\omega-kv_{F\alpha})})]}. \tag{3.14}$$

By using δn_α in Eq,(3.7) and rearranging the terms, we get

$$1 + \sum_\alpha \frac{3\omega_\alpha^2}{\Gamma_\alpha k^2 v_{F\alpha}^2}[1 - \frac{\omega}{2kv_{F\alpha}}\ln\frac{(\omega + kv_{F\alpha})}{(\omega - kv_{F\alpha})}] = 0, \tag{3.15}$$

where $\omega_{p\alpha}^2 = \frac{e_\alpha^2 n_{0\alpha}}{m_\alpha\epsilon_0}$ is the plasma frequency of the particles and

$$\Gamma_\alpha = 1 + \frac{3\hbar^2k^2}{4m_\alpha^2 v_{F\alpha}^2}[1 - \frac{\omega}{2kv_{F\alpha}}\ln\frac{(\omega + kv_{F\alpha})}{(\omega - kv_{F\alpha})}]$$

is the Madelung term which arises due to the particle dispersion effect in quantum regime. Eq.(3.15) is the generalized dispersion relation of longitudinal waves in quantum plasma.

3.3 Zero Sound Waves

In neutral Fermi liquid two modes of vibration exist, first one is low frequency mode ($\omega\tau << 1$) where τ is the mean free path time of the quasi particle, $\tau \sim \frac{1}{T^2}$. In this mode wavelength is long as compared to mean free path of the quasi particle which corresponds to ordinary sound also known as first sound. The propagation of these waves takes place through density variation therefore regions of compression and rare fraction are formed. Now when the temperature is reduced ($\omega\tau \sim 1$) than ordinary sound attenuates. Landau suggested that at high frequencies and low temperature for which $\omega\tau >> 1$, a new type of sound exist which is called zero sound or second sound. Here necessary criteria is that the wavelength of the wave is much less than the Thomas Fermi screening length or the mean free path of the quasi particles, collisions is not the criteria for the propagation of wave nor to establish thermodynamic equilibrium. Thus zero sound is non-equilibrium type of wave propagation which involves periodic deformation of Fermi surface (i.e., a variation in the distribution function). These waves are also called temperature wave which doesn't involve density variation. Landau[25] study the weak interaction between He^3 atoms at sufficiently low temperature and comes with the conclusion that in degenerate electron-ion plasma undamped zero sound ($\omega \simeq kv_{Fe}$) can propagate almost in an ideal Fermi gas and electron oscillations do not damp until $\frac{\omega}{k} \to v_{Fe}$ [6]. In degenerate electron-ion plasma in the collision-less approximation dissipation is totally absent. As $\omega >> kv_{Fe}$, $\omega >> kv_{Fi}$ so the dispersion relation are purely real in this frequency range, where as in degenerate electron positron-ion plasma the situation changes drastically due to the existence of positron and damping also exist.

3.4 Positron Zero Sound Waves in epi Quantum Plasma

The generalized dispersion relation of longitudinal waves are

$$1 + \sum_{\alpha} \frac{3\omega_{p\alpha}^2}{\Gamma_\alpha k^2 v_{F\alpha}^2}[1 - \frac{\omega}{2kv_{F\alpha}} \ln \frac{(\omega + kv_{F\alpha})}{(\omega - kv_{F\alpha})}] = 0. \quad (3.16)$$

First we consider it without the Madelung term here $\alpha = e, p$

$$1 + \frac{3\omega_{pp}^2}{k^2 v_{Fp}^2}[1 - \frac{\omega}{2kv_{Fp}} \ln \frac{(\omega + kv_{Fp})}{(\omega - kv_{Fp})}] + \frac{3\omega_{pe}^2}{k^2 v_{Fe}^2}[1 - \frac{\omega}{2kv_{Fe}} \ln \frac{(\omega + kv_{Fe})}{(\omega - kv_{Fe})}] = 0. \quad (3.17)$$

When we choose the frequency range $kv_{Fe} > \omega \sim kv_{Fp}$, we take $m_e = m_p$ so that

$$\ln(\omega - kv_{Fe}) = \ln|\omega - kv_{Fe}| - \iota\pi\delta(\omega - kv_{Fe}),$$

then the above equation becomes

$$1+\frac{1}{k^2\lambda_p^2}[1-\frac{\omega}{2kv_{Fp}}\ln|\frac{\omega+kv_{Fp}}{\omega-kv_{Fp}}|]+\frac{1}{k^2}\lambda_e^2[1-\frac{\omega}{2kv_{Fe}}\ln|\frac{\omega+kv_{Fe}}{\omega-kv_{Fe}}|-\frac{i\pi}{2kv_{Fe}}\delta(\omega-kv_{Fe})]=0, \tag{3.18}$$

where $\delta(\omega-kv_{Fe})=1$ when $\omega<kv_{Fe}$ otherwise it is zero and $\lambda_e=\frac{v_{Fe}}{\sqrt{3}\omega_{pe}}$, $\lambda_p=\frac{v_{Fp}}{\sqrt{3}\omega_{pp}}$ are the electron and positron Fermi screening length. Now in order to simplify the above equation multiplying $2k^2\lambda_p^2$ on both side and than by rearranging the terms, we get

$$-\frac{\omega}{kv_{Fp}}\ln|\frac{\omega+kv_{Fp}}{\omega-kv_{F}p}|-\frac{\lambda_p^2}{\lambda_e^2}\frac{\omega}{kv_{Fe}}\ln||\frac{\omega+kv_{Fe}}{\omega-kv_{Fe}}|=-2k^2\lambda_p^2-\frac{2\lambda_e^2+2\lambda_p^2}{\lambda_e^2}-\frac{i\pi\omega\lambda_p^2}{kv_{Fe}\lambda_e^2}. \tag{3.19}$$

As $kv_{Fe}>kv_{Fp}$ so $\lambda_e^2>>\lambda_p^2$ due to this the second term on L.H.S is small in comparison to first term and can be neglected. After using the condition $\omega\sim kv_{Fp}$ and $\rho^2=\frac{\lambda_e^2\lambda_p^2}{(\lambda_e^2+\lambda_p^2)}$ the above equation becomes

$$\ln(\frac{\omega}{2kv_{Fp}}-\frac{1}{2})=-2[k^2\lambda_p^2+\frac{\lambda_p^2}{\rho^2}]-\frac{i\pi\omega'}{kv_{Fe}}\frac{\lambda_p^2}{\lambda_e^2}. \tag{3.20}$$

Taking the exponential on both sides and by rearranging the terms the dispersion relation of zero sound waves without Madelung term can be obtained

$$\omega=kv_{Fp}[1+2exp[-2(k^2\lambda_p^2+\frac{\lambda_p^2}{\rho^2})-\frac{i\pi\omega'}{kv_{Fe}}\frac{\lambda_p^2}{\lambda_e^2}]]. \tag{3.21}$$

As $\frac{\lambda_p^2}{\lambda_e^2}=(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}$ so the above dispersion relation can be written as

$$\omega=kv_{Fp}[1+2exp[-2(1+k^2\lambda_p^2+(\frac{n_0e}{n_{0p}})^{\frac{1}{3}})-\frac{i\pi\omega'}{kv_{Fe}}(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}]]. \tag{3.22}$$

Now we have to separate the real and imaginary part of frequencies. For this let

$$\theta=\frac{\pi\omega'}{kv_{Fe}}(\frac{n_{oe}}{n_{0p}})^{\frac{1}{3}},$$

then the above equation is written as

$$\omega=kv_{Fp}[1+2exp[-2(1+k^2\lambda_p^2+(\frac{n_0e}{n_{0p}})^{\frac{1}{3}})]exp[-i\theta]. \tag{3.23}$$

By using the de moivre's theorem $[exp(-i\theta)=cos\theta-isin\theta]$ as $(\frac{v_{Fe}}{v_{Fp}}=(\frac{n_{oe}}{n_{0p}})^{\frac{1}{3}}$ so $\theta=\frac{\pi\omega'}{kv_{Fp}})$ the above equation becomes

$$\omega=kv_{Fp}[1+2exp[-2(1+k^2\lambda_p^2+(\frac{n_0e}{n_{0p}})^{\frac{1}{3}})][cos(\frac{\pi\omega'}{kv_{Fp}})-isin(\frac{\pi\omega'}{kv_{Fp}})]. \tag{3.24}$$

By applying the approximation $\omega' \sim kv_{Fp}$ the imaginary term vanishes therefore we can't neglect a small exponential term also. When $\theta \approx 0$ than $cos\theta \approx 1$ but $sin\theta \approx \theta$, here we assumed that $exp[-2(1 + k^2\lambda_p^2 + (\frac{n_0e}{n_{0p}})^{\frac{1}{3}})] << 1$. Therefore we first find the ω_r and then use this in imaginary part of frequency in place of ω' such that the approximation becomes $\omega_r \sim \omega' \sim kv_{Fp}$. For real part $\omega' \sim kv_{Fp}$ so ω_r becomes

$$\omega_r = kv_{Fp}[1 - exp[-2(1 + k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}})]]. \tag{3.25}$$

The imaginary part of frequency becomes

$$\omega_i = -2kv_{Fp}exp[-2(1+k^2\lambda_p^2+(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}})]sin(\pi - 2\pi exp[-2(1+k^2\lambda_p^2+(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}})]). \tag{3.26}$$

Let $\theta = 2\pi exp[-2(1 + k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}})]$, as $sin(\pi - \theta) = sin\theta$ and when $\theta << 1$ then $sin\theta \approx \theta$. Therefore the imaginary part of frequency is modified as

$$\omega_i = -4\pi kv_{Fp}exp[-4(1 + k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}})]. \tag{3.27}$$

Now we have to find the dispersion relation of zero sound waves with Madelung term. For this we consider again the general dispersion relation Eq.(3.16) for electron and positron.

$$1 + \frac{3\omega_{pp}^2}{k^2v_{Fp}^2}\frac{1}{\Gamma_p}[1 - \frac{\omega}{2kv_{Fp}}\ln\frac{(\omega + kv_{Fp})}{(\omega - kv_{Fp})}] + 1 + \frac{3\omega_{pe}^2}{k^2v_{Fe}^2}\frac{1}{\Gamma_e}[1 - \frac{\omega}{2kv_{Fe}}\ln\frac{(\omega + kv_{Fe})}{(\omega - kv_{Fe})}] = 0. \tag{3.28}$$

As

$$\Gamma_e = 1 + \frac{3\hbar^2k^2}{4m_e^2v_{Fe}^2}[1 - \frac{\omega}{2kv_{Fe}}\ln\frac{\omega + kv_{Fe}}{\omega - kv_{Fe}}],$$

where $\omega_q^2 = \frac{\hbar k^2}{4m_e^2}$ is quantum oscillation frequency so

$$\Gamma_e = 1 + \frac{3\omega_q^2}{k^2v_{Fe}^2}[1 - \frac{\omega}{2kv_{Fe}}\ln\frac{\omega + kv_{Fe}}{\omega - kv_{Fe}}].$$

By rearranging the terms and noting that $\omega_{pq}^2 = \omega_{pe}^2 = \omega_q^2$ the above equation becomes

$$\frac{v_{Fe}^2k^2}{3\omega_q^2}(\Gamma_e - 1) = [1 - \frac{\omega}{2kv_{Fe}}\ln\frac{\omega + kv_{Fe}}{\omega - kv_{Fe}}], \tag{3.29}$$

$$\frac{v_{Fp}^2k^2}{3\omega_q^2}(\Gamma_p - 1) = [1 - \frac{\omega}{2kv_{Fp}}\ln\frac{\omega + kv_{Fp}}{\omega - kv_{Fp}}]. \tag{3.30}$$

Now by using Eq.(3.29) and (3.30) in Eq.(3.28), we get

$$1 + \frac{\omega_{pe}^2}{\omega_q^2}(\frac{\Gamma_e - 1}{\Gamma_e}) + \frac{\omega_{pp}^2}{\omega_q^2}(\frac{\Gamma_p - 1}{\Gamma_p}) = 0. \quad (3.31)$$

Multiplying both side by $\omega_q^2 \Gamma_p$

$$\omega_q^2 \Gamma_p + \omega_{pe}^2 \Gamma_p (1 - \frac{1}{\Gamma_e}) + \Gamma_p \omega_{pp}^2 = \omega_{pp}^2,$$

hence Γ_p becomes

$$\Gamma_p = (\frac{\omega_{pp}^2}{\omega_q^2 + \omega_{pe}^2 (1 - \frac{1}{\Gamma_e}) + \omega_{pp}^2}). \quad (3.32)$$

As

$$\Gamma_p = 1 + \frac{3\hbar^2 k^2}{4m_p^2 v_{Fp}^2}[1 - \frac{\omega}{2kv_{Fp}} \ln \frac{\omega + kv_{Fp}}{\omega - kv_{Fp}}],$$

let $\gamma_{qp}^2 = \frac{v_{Fp}^2}{3\omega_q^2}, \gamma_p = \omega - kv_{Fp}$ whereas quantum oscillation frequency is $\omega_q^2 = \frac{\hbar^2 k^4}{4m_p^2}$ therefore

$$\Gamma_p = 1 + \frac{1}{k^2 \gamma_{qp}^2}[1 - \frac{\omega}{2kv_{Fp}} \ln \frac{\omega + kv_{Fp}}{\gamma_p}].$$

When $\omega \to kv_{Fp}$ then

$$\Gamma_p = 1 + \frac{1}{k^2 \gamma_{qp}^2}[1 - \ln \frac{2kv_{Fp}}{\gamma_p}]. \quad (3.33)$$

Now comparing Eq.(3.32) and (3.33) gives

$$1 + \frac{1}{k^2 \gamma_{qp}^2}[1 - \frac{\omega}{2kv_{Fp}} \ln \frac{2kv_{Fp}}{\gamma_p}] = \frac{\omega_{pp}^2}{\omega_{qp}^2 + \omega_{pe}^2 (1 - \frac{1}{\Gamma_e}) + \omega_{pp}^2},$$

multiplying both sides by $2k^2 \gamma_{qp}^2$, we get

$$\ln(\frac{\gamma_p}{2kv_{Fp}}) = \frac{-2(1 + k^2 \gamma_{qp}^2)[\omega_{pe}^2 (\frac{\Gamma_e - 1}{\Gamma_e}) + \omega_{qp}^2] - 2\omega_{pp}^2}{\omega_{pp}^2 + \omega_{pe}^2 (\frac{\Gamma_e - 1}{\Gamma_e}) + \omega_{qp}^2}. \quad (3.34)$$

As

$$\frac{\Gamma_e - 1}{\Gamma_e} = \frac{1 + \frac{3\omega_q^2}{k^2 v_{Fe}^2}[1 - \frac{\omega}{2kv_{Fe}} \ln \frac{\omega + kv_{Fe}}{\omega - kv_{Fe}}] - 1}{1 + \frac{3\omega_q^2}{k^2 v_{Fe}^2}[1 - \frac{\omega}{2kv_{Fe}} \ln \frac{\omega + kv_{Fe}}{\omega - kv_{Fe}}]},$$

when $\omega << kv_{Fe}$ then $\ln(\omega - kv_{Fe}) = \ln|\omega - kv_{Fe}| - i\pi(\omega - kv_{Fe})$ hence

$$\frac{\Gamma_e - 1}{\Gamma_e} = \frac{\frac{3\omega_q^2}{k^2 v_{Fe}^2}[1 + \frac{i\pi\omega}{2kv_{Fe}}]}{1 + \frac{3\omega_q^2}{k^2 v_{Fe}^2}[1 + \frac{i\pi\omega}{2kv_{Fe}}]}.$$

Let $k^2 v_{Fe}^2 >> \omega_q^2$ so drop this term in the denominator because this term is small in comparison to 1

$$\frac{\Gamma_e - 1}{\Gamma_e} = \frac{3\omega_q^2}{k^2 v_{Fe}^2}[1 + \frac{i\pi\omega}{2kv_{Fe}}]. \tag{3.35}$$

By using Eq.(3.35) in (3.34), we get

$$\ln(\frac{\gamma_p}{2kv_{Fp}}) = -2[1 + \frac{\frac{v_{Fe}^2}{v_{Fe}^2}\omega_{pe}^2 + \frac{v_{Fe}^2}{v_{Fe}^2}\omega_{pe}^2 \frac{i\pi\omega}{2kv_{Fe}} + k^2 v_{Fp}^2}{\omega_{pp}^2 + \omega_{pe}^2 \frac{3\omega_q^2}{k^2 v_{Fe}^2} + \omega_{pe}^2 \frac{3\omega_q^2}{k^2 v_{Fe}^2}\frac{i\pi\omega}{2kv_{Fe}} + \omega_{qp}^2}].$$

Multiplying and divide the second term of above equation by ω_{pp}^2 and as $\omega_{pp}^2 >> \omega_{qp}^2$ so neglect the two terms in the denominator and $\frac{v_{FP}^2}{v_{Fe}^2}\frac{\omega_{Pe}^2}{\omega_{pp}^2} = (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}$. So using this in above equation it becomes

$$\ln(\frac{\gamma_p}{2kv_{Fp}}) = -2[1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}} + \frac{i\pi\omega}{2kv_{Fe}}(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1 + \frac{3\omega_q^2}{k^2 v_{Fe}^2}(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}].$$

Let $a = \frac{3\omega_q^2}{k^2 v_{Fe}^2}(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}$ using this in above equation

$$\ln(\frac{\gamma_p}{2kv_{Fp}}) = -2[1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}} + \frac{\iota\pi\omega}{2kv_{Fe}}(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1 + a}],$$

simplifying the above equation

$$\ln(\frac{\gamma_p}{2kv_{Fp}}) = [-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1 + a}) - \frac{i\pi\omega}{kv_{Fe}}\frac{(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1 + a}],$$

taking exponential on both side, we get

$$\omega = kv_{Fp}[1 + 2exp[-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1 + a}) - \frac{i\pi\omega}{kv_{Fe}}\frac{(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1 + a}]]. \tag{3.36}$$

This is the required dispersion relation of zero sound waves with Madelung term . Now we have to separate its real and imaginary parts by using de moivre's theorem

$exp(i\theta) = cos\theta - \iota sin\theta$ here $\theta = \frac{\frac{\pi\omega}{kv_{Fe}}(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1+a}$, $\frac{v_{Fe}}{v_{Fp}} = (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}$ and using the approximation $\omega \sim kv_{Fp}$, we get

$$\omega = kv_{Fp} + 2kv_{Fp}exp[-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1+a})][cos(\frac{\pi}{1+a}) - isin(\frac{\pi}{1+a})].$$

As $\omega = \omega_r - \iota\omega_i$ so putting this in above equation and than comparing its real and imaginary parts of frequency i.e.,

$$\omega_r + i\omega_i = kv_{Fp} + 2kv_{Fp}exp[-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1+a})][cos(\frac{\pi}{1+a}) - isin(\frac{\pi}{1+a})].$$

Hence

$$\omega_r = kv_{Fp}[1 + 2kv_{Fp}exp[-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1+a})]cos(\frac{\pi}{1+a})], \tag{3.37}$$

$$\omega_i = -2kv_{FP}exp[-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1+a})]sin(\frac{\pi}{1+a}). \tag{3.38}$$

Zero sound waves with or without Madelung term in degenerate electron positron-ion plasma provides a new spectrum that incorporates the damping rates of positron oscillation, here a is called the quantum parameter. Which can take any value but only those values significantly effect the real and imaginary frequency at which the wavenumber is approximately close to inverse of λ_{Fp}. When $a \leq 1$ then it will have no significant effect on real frequency. As an example let we set $a \simeq 1$ and $k^2\lambda_p^2 \sim (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}$ that satisfy the condition of semiconductor plasma. Hence Eq.(3.39) and (3.40) becomes

$$\omega_r \simeq kv_{Fp}$$

$$\omega_i = -2kv_{Fe}exp(-2[1 + \frac{k^2\lambda_p + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{2}]).$$

In degenerate semiconductor plasma the crystal lattice vibrations occurs at approximately $\omega \approx kv_{Fp}$. A packet of these waves can travel throughout the crystal with a definite energy and momentum. In quantum mechanical terms these waves can be treated as particle, called a phonon. In other words these crystal lattice vibrations are also called phonon waves. So above equation shows that these are phonon waves and they are purely quantum in nature. The zero sound waves in degenerate electron-ion plasma exist only in the lime $k^2\lambda_p^2 >> 1$. But here it exist even in the limit $k^2\lambda_p^2 \sim 1$ because of the existence of $(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}$ in the exponential part of above equations [26].

3.5 Logitudinal Sound Waves

In semiconductor plasma the electrons and holes interact with coulombic field, if same magnitude of electric field is applied to electrons in both vacuum and inside the crystal. They will accelerate at a different rate from each other due to the existence of different potential inside the crystal. So the electrons and holes inside the crystal will have a different mass than that of free electron in vacuum. This altered mass is called an effective mass. In degenerate semiconductor plasma, the complete description of

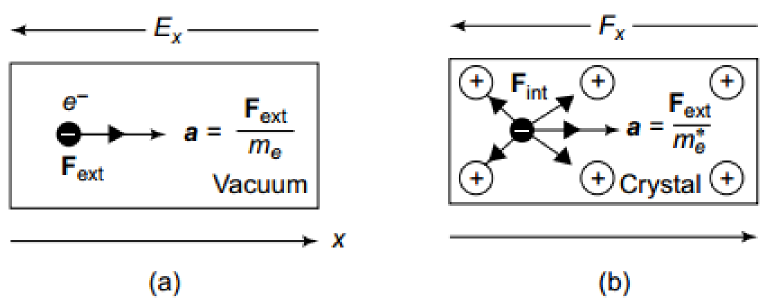

Figure 3.1: An external force F_{ext} is applied to an electron (a) in vacuum (b) in a solid

electrons are based on their wave functions. The electron wave function will be of the form $\exp(\pm k.r)$ where k is the wave vector. When external electric field is applied it will accelerate the electron with acceleration $a = \frac{qE}{\hbar^2}\frac{d^2E}{dk^2}$ here $m_{eff} = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$ in order to interpret the acceleration equation as $F = ma$ [27]. In semiconductors we can categorized the electron-hole-ion plasma as an epi-plasma by taking the mass of hole smaller or larger than positron. However in this section we consider the special case in which the mass of positron is considered greater then effective mass of electrons. Effective mass difference arises due to the interaction and momentum transfer between the particles. The effective mass of electrons is not always less than the effective mass of holes, it depends on the type of semiconductor. For Silicon and GaAs $\frac{m_e}{m_p} < 1$

Table 3.1: Electron and Hole effective masses m_e, m_p where m_0 is the rest mass of an electron

Effective mass	Si	Ge	GaAs	InAs	AlAs
m_e	$0.26m_0$	$0.12m_0$	$0.068m_0$	$0.023m_0$	$2.0m_0$
m_p	$0.39m_0$	$0.30m_0$	$0.50m_0$	$0.30m_0$	$0.3m_0$

and its the basic criteria for the existence of these waves in the frequency regime $kv_{Fe} > \omega > kv_{Fp}$, where as in other semiconductor like AlAs $\frac{m_e}{m_p} > 1$ holds. In previous section, the new type of zero sound waves are found where as here in electron-hole-ion plasma a new longitudinal quantum waves are found out, which have nothing to do in quantum electron-ion plasma. In the next section the low density mono-energetic straight electron beam will examine by using the excitation of these waves.

3.5.1 Longitudinal Sound Waves without Madelung Term

In positron sound waves we choose the frequency range $kv_{Fe} > \omega > kv_{Fp}$, here $m_p > m_e$ as in semiconductor plasma in which mass of hole is greater than mass of electron. For this frequency range the Madelung terms are

$$\Gamma_e = 1 + \frac{3\omega_{qe}^2}{k^2 v_{Fe}^2}(1 + \frac{\iota\pi\omega}{2kv_{Fe}}),$$

and

$$\Gamma_p = 1 - \frac{\omega_{pq}^2}{\omega^2}[1 + \frac{3}{5}(\frac{kv_{Fp}}{\omega})^2].$$

At first we consider the dispersion relation of Eq.(3.16) without the Madelung term (Γ_α) i.e.,

$$1 + \frac{3\omega_{pe}^2}{k^2 v_{Fe}^2}[1 + \frac{\iota\pi\omega}{2kv_{Fe}}] - \frac{\omega_{pp}^2}{\omega^2}[1 - \frac{3}{5}(\frac{kv_{Fp}}{\omega})^2] = 0. \tag{3.39}$$

where $\omega^2 >> k^2 v_{Fp}^2$, so above equation becomes

$$1 + \frac{3\omega_{pe}^2}{k^2 v_{Fe}^2}[1 + \frac{i\pi\omega}{2kv_{Fe}}] - \frac{\omega_{pp}^2}{\omega^2} = 0. \tag{3.40}$$

By separating the real and imaginary part, we get

$$\omega_r^2(1 + \alpha) - 3\alpha\beta\omega_i\omega_r^2 = \omega_{pp}^2, \tag{3.41}$$

$$2i\omega_i\omega_r(1 + \alpha) + i\alpha\beta\omega_r^3 = 0. \tag{3.42}$$

where $\alpha = \frac{3\omega_{pe}^2}{k^2 v_{Fe}^2}$, $\beta = \frac{2}{kv_{Fe}}$. In long wavelength limit i.e., $k^2 v_{Fe}^2 << \omega_{pe}^2$, the real and imaginary part of frequency becomes

$$\omega_r = \frac{1}{\sqrt{3}}(\frac{m_e}{m_p}\frac{n_{0p}}{n_{0e}})^{\frac{1}{2}} kv_{Fe}, \tag{3.43}$$

$$\omega_i = -\frac{\pi}{12}(\frac{m_e}{m_p}\frac{n_{0p}}{n_{0e}})kv_{Fe}. \tag{3.44}$$

As $\frac{m_e}{m_p}\frac{n_{0p}}{n_{0e}} << 1$ so the imaginary part is smaller than the real part. We also note that this spectrum (3.43) and damping rate (3.44) are novel and they described the propagation of new waves with slow damping and are called positron sound waves.

As we are dealing with acoustic waves so its better to express the waves in terms of positron sound velocity i.e., $v_s = \sqrt{\frac{m_e}{3m_p}} v_{Fe}$

$$\omega_r = (\frac{n_{0p}}{n_{0e}})^{\frac{1}{2}} k v_s \tag{3.45}$$

$$\omega_i = -\frac{\pi}{4\sqrt{3}} (\frac{n_{0p}}{n_{0e}})(\frac{m_e}{m_p})^{\frac{1}{2}} k v_s. \tag{3.46}$$

Now we consider Eq.(3.410 and (3.42) for short wavelength limit i.e.,$k^2 v_{Fe}^2 >> \omega_{pe}^2$

$$\omega_r^2 = \omega_{pp}^2 \tag{3.47}$$

$$\omega_i = -\frac{3\pi \omega_{pp}^2 \omega_{pe}^2}{(4kv_{Fe})(k^2 v_{Fe}^2)}. \tag{3.48}$$

In term of Positron sound velocity above equation becomes

$$\omega_r^2 = \omega_{pp}^2 \tag{3.49}$$

$$\omega_i = -\frac{\pi}{4} \frac{\omega_{pp}^2 \omega_{pe}^2 m_e \sqrt{m_e}}{k^3 v_s^3 m_p \sqrt{m_p}}. \tag{3.50}$$

These are new longitudinal waves in epi quantum plasma which are absent in Fermi electron-positron-ion gas where $m_p = m_e$. The necessary condition for these waves i.e., $\frac{m_p}{m_e} > (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}$ must be satisfied.

3.5.2 Longitudinal Sound Waves with Madelung Term

Now we consider Eq.(3.16) to incorporate the Madelung term in positron sound waves i.e.,

$$1 + \frac{\omega_{pe}^2}{\omega_{qp}^2} (\frac{\Gamma_e - 1}{\Gamma_e}) + \frac{\omega_{pp}^2}{\omega_{qp}^2} (\frac{\Gamma_p - 1}{\Gamma_p}) = 0. \tag{3.51}$$

For $kv_{Fe} > \omega > kv_{Fp}$

$$\frac{\Gamma_e - 1}{\Gamma_e} = \frac{3\omega_{qe}^2}{k^2 v_{Fe}^2} (1 + \frac{i\pi\omega}{2kv_{Fe}}), \tag{3.52}$$

$$\frac{\Gamma_p - 1}{\Gamma_p} = -\frac{\frac{\omega_{qp}^2}{\omega^2}}{1 - \frac{\omega_{qp}^2}{\omega^2}}. \tag{3.53}$$

Putting Eq.(3.52) and (3.53) in (3.51), we get the resulting equation

$$\omega^2 - \omega_{qp}^2 + (\frac{3\omega_{pe}^2}{k^2 v_{Fe}^2} \omega^2 - \frac{3\omega_{pe}^2}{k^2 v_{Fe}^2} \omega_{qp}^2)(1 + \frac{i\pi\omega}{2kv_{Fe}}) = \omega_{pp}^2. \tag{3.54}$$

The real and imaginary part of above equation becomes

$$\omega_r^2 = \omega_{qp}^2 + \frac{\omega_{pp}^2}{(1 + \frac{3\omega_{pe}^2}{k^2 v_{Fe}^2})}. \tag{3.55}$$

$$\omega_i = -\frac{\frac{\pi}{2kv_{Fe}}\omega_r^2}{2(\frac{1+\frac{3\omega_{pe}^2}{k^2 v_{Fe}^2}}{\frac{3\omega_{pe}^2}{k^2 v_{Fe}^2}})}. \tag{3.56}$$

For long wavelength limit or $k^2\lambda_e^2 << 1$ then the real and imaginary parts of frequencies are

$$\omega_r^2 = \omega_{qp}^2 + (\frac{m_e}{m_P}\frac{n_{0p}}{n_{0e}})(\frac{k^2 v_{Fe}^2}{3}), \tag{3.57}$$

$$\omega_i = -\frac{\pi}{12}(\frac{m_e}{m_P}\frac{n_{0p}}{n_{0e}})kv_{Fe}. \tag{3.58}$$

For short wavelength limit or $k^2\lambda_e^2 >> 1$ the real and imaginary part of frequencies becomes

$$\omega_r^2 = \omega_{qp}^2 + \omega_{pp}^2, \tag{3.59}$$

$$\omega_i = -\frac{3\pi}{4kv_{Fe}}\frac{\omega_{pp}^2\omega_{pe}^2}{k^2 v_{Fe}^2}. \tag{3.60}$$

The quantum correction term or Madelung term appear for both long and short wavelength, in real frequencies but the damping rates are unaffected by this term.

3.6 Beam-Plasma Interaction

In Beam-Plasma interaction, a beam of electron is injected into a degenerate EPI plasma. The density of electron beam n_b is much less than the plasma density so we assume that the beam (electron) obeys the Maxwellian distribution. Starting from Eq.(3.1) and proceeding with the same procedure for electron and positron(hole), whereas ions are treated as non-degenerate which provide a uniform fixed background. So dispersion relation which includes the electron beam contributions is,

$$1 + \delta\epsilon_e + \delta\epsilon_p + \delta\epsilon_b = 0.$$

where

$$\delta\epsilon_e = 1 + \frac{3\omega_{pe}^2}{\Gamma_e k^2 v_{Fe}^2}[1 - \frac{\omega}{2kv_{Fe}} \ln\frac{(\omega + kv_{Fe})}{(\omega - kv_{Fe})}] = 0$$

$$\delta\epsilon_p = 1 + \frac{3\omega_{pp}^2}{\Gamma_e k^2 v_{Fp}^2}[1 - \frac{\omega}{2kv_{Fp}} \ln\frac{(\omega + kv_{Fp})}{(\omega - kv_{Fp})}] = 0.$$

Now for beam part we consider Eq.(3.5) and use it in Eq.(3.6) it takes the form

$$1+\frac{4\pi e^2}{k^2}[\frac{I_1}{1+\frac{\hbar^2k^2I_1}{4m_en_0}}]=0. \tag{3.61}$$

As in 1D case

$$I_1=k\int\frac{\frac{\partial f_{p_x}}{\partial p_x}}{\omega-kv_x}dp_x,$$

where the distribution function is

$$f(p_x)=\frac{n_0}{(2\pi m_eT_e)^{\frac{1}{2}}}exp(-\frac{p_x^2}{2m_eT_e}).$$

Using this in I_1 and by setting appropriate substitution $\frac{p_x^2}{2m_eT_e}=s^2$, $p_x=\sqrt{2m_eT_e}ds$, $p_x=\sqrt{2m_eT_e}ds$. The integral I_1 becomes

$$I_1=\frac{n_0}{\sqrt{\pi}T_e}\int\frac{se^{-s^2}}{(s-x^{'})}ds. \tag{3.62}$$

Now by inserting Eq.(3.62) in (3.61), we get

$$1+\frac{\frac{4\pi e^2}{k^2}\frac{n_0}{T_e\sqrt{\pi}}\int\frac{se^{-s^2}}{(s-x^{'})}ds}{1+\frac{\omega_{qb}^2}{k^2v_{tb}^2}\frac{1}{\sqrt{\pi}}\int\frac{se^{-s^2}}{(s-x^{'})}ds}=0. \tag{3.63}$$

By taking the derivative of plasma dispersion function[28]

$$\frac{1}{\sqrt{x}}\int\frac{e^{-s^2}}{(s-x^{'})}ds=Z(x^{'}),$$

which gives

$$Z^{'}(x^{'})=\frac{1}{\sqrt{x}}\int\frac{se^{-s^2}}{(s-x^{'})^2}ds.$$

Integrating the above equation by parts, we get

$$\int\frac{se^{-s^2}}{(s-x^{'})}ds=\sqrt{\pi}(1+x^{'}Z(x^{'})).$$

Now we have to insert above equation in (3.63) i.e.,

$$1+\frac{\frac{\omega_{pb}^2}{k^2v_{th}^2}(1+x^{'}Z(x^{'}))}{1+\frac{\omega_{qb}^2}{k^2v_{th}^2}(1+x^{'}Z(x^{'}))}=0. \tag{3.64}$$

The dispersion function has the following identities

$$Z(x^{'}) \approx -2x^{'}(1 - \frac{2x^{'2}}{3} + \frac{4x^{'4}}{15}.....) + i\pi e^{-x^{'2}} \qquad |x^{'}| << 1 \tag{3.65}$$

$$Z(x^{'}) \approx -\frac{1}{x^{'}}(1 + \frac{1}{2x^{'2}} + \frac{3}{4x^{'4}} + ...) + i\sqrt{\pi}e^{-x^{'2}} \qquad |x^{'}| >> 1. \tag{3.66}$$

So by using these expansion the dispersion relation for beam part becomes

$$\delta\epsilon_b = 1 + \frac{\frac{\omega_{pb}^2}{k^2 v_{th}^2}(1 - I_+(x))}{1 + \frac{\omega_{qb}^2}{k^2 v_{th}^2}(1 - I_+(x))} = 0. \tag{3.67}$$

As

$$\Gamma_b = 1 + \frac{\omega_{qb}^2}{k^2 v_{tb}^2}(1 - I_+(x)),$$

where $I_+(x) = xe^{-\frac{x^2}{2}} \int d\tau e^{\frac{\tau^2}{2}}$ is the modified form of plasma dispersion function[29]. Here $x = \frac{\omega - k.u}{kv_{tb}}$ or $x = \frac{\omega^{'}}{kv_{tb}}$ where u is the velocity of electron beam and $\omega - k.u$ is due to Doppler effect. When $x >> 1$ or when $\omega^{'} >> kv_{tb}$ the asymptotic value of modified plasma dispersion function is

$$I_+(x) = 1 + \frac{1}{x^2} + \frac{3}{x^4} + ... - i\sqrt{\frac{\pi}{2}}xe^{-\frac{-x^2}{2}}.$$

Here we consider the terms upto second order, therefore $I_+(x) = 1 + \frac{1}{x^2}$ if x»1 and $I_+(x) \approx -\iota\sqrt{\frac{\pi}{2}}x$ if x«1 or $\omega^{'} << kv_{tb}$. When we consider the frequency range $kv_{Fe} > \omega > kv_{Fp}$ and $x >> 1$ or $\omega^{'} >> kv_{tb}$ than from previous section

$$\delta\epsilon_e = 1 + \frac{3\omega_{pe}^2}{k^2 v_{Fe}^2}[1 + \frac{i\pi\omega}{2kv_{Fe}}],$$

$$\delta\epsilon_p = -\frac{\omega_{pp}^2}{\omega^2 - \omega_{qp}^2}.$$

The beam part becomes

$$\delta\epsilon_b = -\frac{\omega_{pb}^2}{(\omega - k.u)^2 - \omega_{qb}^2},$$

and the dispersion relation becomes

$$1 + \frac{3\omega_{pe}^2}{k^2 v_{Fe}^2}[1 + \frac{i\pi\omega}{2kv_{Fe}}] - \frac{\omega_{pp}^2}{\omega^2 - \omega_{qp}^2} - \frac{\omega_{pb}^2}{(\omega - k.u)^2 - \omega_{qb}^2}. \tag{3.68}$$

Considering the instability in long wavelength limit i.e., $k^2v_{Fe}^2 << \omega_{pe}^2$ and frequency range $kv_{Fe} > \omega > kv_{Fp}$, in electron and positron part $\omega = \omega_r + \gamma$ and in beam part we use $\omega = \omega_r = k.u + \omega_{qb} + \gamma$. Hence the above dispersion relation becomes

$$1 + \frac{3\omega_{pe}^2}{k^2v_{Fe}^2} - \frac{\omega_{pp}^2}{\omega_r^2 + 2\omega_r\gamma} - \frac{\omega_{pb}^2}{2\omega_{qb}\gamma} = 0.$$

Rationalize the positron part for simplification i.e.,

$$1 + \frac{3\omega_{pe}^2}{k^2v_{Fe}^2} - \frac{\omega_{pp}^2}{(\omega_r^2 - \omega_{qp}^2)}(1 - \frac{2\omega_r\gamma}{(\omega_r^2 - \omega_{qp}^2)}) - \frac{\omega_{pb}^2}{2\omega_{qb}\gamma} = 0,$$

where $\gamma = \imath\omega_i$. The imaginary part of above equation is

$$\omega_i = \frac{1}{2}(\frac{m_p}{mb}\frac{n_{0b}}{n_{0p}})^{\frac{1}{2}}\frac{(\omega_r^2 - \omega_{qp}^2)}{\sqrt{\omega_r\omega_{qb}}}. \tag{3.69}$$

By inserting ω_r^2 from Eq.(3.57) than above equation become

$$\omega_i = \frac{1}{2}\frac{\sqrt{n_{0p}n_{0b}}}{n_{0e}}\frac{m_e}{\sqrt{m_bm_p}}\frac{k^2v_{Fe}^2}{3\sqrt{\omega_r\omega_{qp}}}. \tag{3.70}$$

For the same frequency range $kv_{Fe} > \omega > kv_{Fp}$ but $\omega' << kv_{tb}$ and for $k^2v_{Fe}^2 >> \omega_{Fe}^2$ than Eq.(3.68) casts in the form

$$1 + \frac{3\omega_{pe}^2}{k^2v_{Fe}^2}[1 + \frac{i\pi\omega}{2kv_{Fe}}] - \frac{\omega_{pp}^2}{\omega^2 - \omega_{qp}^2} + \frac{\omega_{pb}^2}{k^2v_{tb}^2}(1 + i\sqrt{\frac{\pi}{2}}\frac{\omega'}{kv_{tb}}) = 0. \tag{3.71}$$

Here we use $\omega = \omega_r + i\omega_i$ then the above equation becomes

$$1 + \frac{3\omega_{pe}^2}{k^2v_{Fe}^2}[1 + \frac{i\pi\omega_r}{2kv_{Fe}}] - \frac{\omega_{pp}^2}{(\omega_r^2 - \omega_{qp}^2)}(1 - \frac{2\omega_r i\omega_i}{(\omega_r^2 - \omega_{qp}^2)}) + \frac{\omega_{pb}^2}{k^2v_{tb}^2}(1 + i\sqrt{\frac{\pi}{2}}\frac{i(\omega_r - k.u)}{kv_{tb}}) = 0.$$

Now by comparing the imaginary terms on both side, which leads to following result

$$\omega_i = -\frac{3\pi}{4}(\frac{m_p}{m_e}\frac{n_{0e}}{n_{0p}})\frac{(\omega_r^2 - \omega_{qp}^2)^2}{k^3v_{Fe}^3} - \frac{1}{2}\sqrt{\frac{\pi}{2}}(\frac{m_p}{m_e}\frac{n_{oe}}{n_{op}})\frac{(\omega_r - k.u)}{k^3v_{tb}^3}(\omega_r^2 - \omega_{qp}^2)^2. \tag{3.72}$$

For instability $\omega_i > 0$ which means that we must have

$$-\frac{3\pi}{4}(\frac{m_p}{m_e}\frac{n_{0e}}{n_{0p}})\frac{(\omega_r^2 - \omega_{qp}^2)^2}{k^3v_{Fe}^3} - \frac{1}{2}\sqrt{\frac{\pi}{2}}(\frac{m_p}{m_b}\frac{n_{ob}}{n_{op}})\frac{(\omega_r - k.u)}{\omega_r k^3v_{tb}^3}(\omega_r^2 - \omega_{qp}^2)^2 > 0$$

$$-3\frac{\pi}{2}(\frac{n_{0e}}{m_e}) > \sqrt{\frac{\pi}{2}}(\frac{n_{0b}}{m_b})\frac{\omega_r - k.u}{\omega_r}\frac{v_{Fe}^3}{v_{tb}^3},$$

rearranging the terms

$$k.u > \omega_r(1 + 3\sqrt{\frac{\pi}{2}}\frac{m_b}{m_e}\frac{n_{0b}}{n_{0e}}\frac{v_{tb}^3}{v_{Fe}^3})$$

which is the required condition of kinetic instability.

Chapter 4

Positron Zero Sound Waves in epi-Quantum Plasma

4.1 Zero Positron Sound Waves

The quantum effects of lighter species (electrons, positrons and so on) are prominent due to their smaller mass than those of heavior (ion) species, which may behave classically or quantum mechanically depending upon the degeneracy parameter i.e., $n\lambda_B^3$, which should be greater than unity for quantum effects to be significant. Moreover, it is predicted that the main constituents of compact astrophysical objects such as white dwarfs are degenerate electrons, positrons, light nuclei (He, Hydrogen or Carbon) and heavy nuclei (ferrous or molybdenum). When these stars contract to very high densities, matter in their interiors will cool and become degenerate under certain conditions [30]. Therefore in these environments we may treat ions as degenerate along with electrons and positrons, here the densities of ions, electrons and positrons are approximately $n_{i0} = 1.1 \times 10^{29} cm^{-3}$, $n_{e0} = 9.1 \times 10^{29} cm^{-3}$, $n_{p0} = 1.5 \times n_{e0}$. Hence in this section we consider the propagation of electrostatic perturbations in four component degenerate plasma containing degenerate inertial ions having both positive and negative ions, degenerate electrons, positrons and static heavy positively charged ions [30]. The quasi-neutrality condition at equilibrium is $n_{e0} + Z_i n_{i0} \approx Z_h n_{h0} + Z_p n_{p0}$, where Z_s is the charge per ion (s=i,h,p). Moreover, we assumed that $m_i > m_p = m_e$ and $T_{Fi} < T_{Fp} < T_{Fe}$ due to difference in there equilibrium number densities. The general dispersion relation of longitudinal waves in degenerate plasma is

$$1 + \sum_{\alpha} \frac{3\omega_{p\alpha}^2}{\Gamma_\alpha k^2 v_{F\alpha}^2}[1 - \frac{\omega}{2kv_{F\alpha}} ln\frac{(\omega + kv_{F\alpha})}{(\omega - kv_{F\alpha})}] = 0, \tag{4.1}$$

where $\alpha = e, p, i$. At first we consider it without Madelung term, choosing the intermediate range of phase velocities $\omega >> kv_{Fi}$, $kv_{Fp} \sim \omega < kv_{Fe}$ and noting that

$\delta(\omega - kv_{Fe}) = 1$ when $kv_{Fe} > \omega$ otherwise it is zero. Using these approximations in the above equation

$$-\frac{\omega}{kv_{Fp}} ln|\frac{\omega + kv_{Fp}}{\omega - kv_{F}p}| - \frac{\lambda_p^2}{\lambda_e^2}\frac{\omega}{kv_{Fe}} ln||\frac{\omega + kv_{Fe}}{\omega - kv_{Fe}}| - 2k^2\lambda_p^2(\frac{\omega_{pi}^2}{\omega^2}) = -2k^2\lambda_p^2 - \frac{2\lambda_e^2 + 2\lambda_p^2}{\lambda_e^2} - \frac{i\pi\omega\lambda_p^2}{kv_{Fe}\lambda_e^2}. \quad (4.2)$$

As $v_{Fe}^2 >> v_{Fp}^2$ so the second term on L.H.S is small as compare to first term so it can be neglected. Above equation can be simplified by using the approximation $\omega \sim kv_{Fp}$ i.e.,

$$-ln|\frac{2kv_{Fp}}{\omega - kv_{F}p}| = 2(\frac{\omega_{pi}^2}{\omega_{pp}^2}) - 2k^2\lambda_p^2 - \frac{2\lambda_e^2 + 2\lambda_p^2}{\lambda_e^2} - \frac{i\pi\omega\lambda_p^2}{kv_{Fe}\lambda_e^2}. \quad (4.3)$$

As $\frac{\omega_{pi}^2}{\omega_{pp}^2} = \frac{Z_i^2}{Z_p^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}$ and $\frac{\lambda_p^2}{\lambda_e^2} = (\frac{n_{0e}}{n_{0p}})\frac{T_{Fp}}{T_{Fe}} = \alpha$ using this in above equation

$$\omega = kv_{Fp}[1 + 2exp[-2(1 + k^2\lambda_p^2 - \frac{1}{3}(\frac{Z_i^2}{Z_p^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}) + \alpha) - \frac{i\pi\omega'}{kv_{Fe}}\alpha]]. \quad (4.4)$$

The real and imaginary part of above dispersion relation becomes

$$\omega_r = kv_{Fp}[1 - exp[-2(1 + k^2\lambda_p^2 - \frac{1}{3}(\frac{Z_i^2}{Z_p^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}) + \alpha)]], \quad (4.5)$$

$$\omega_i = -4\pi kv_{Fp}exp[-4(1 + k^2\lambda_p^2 - \frac{1}{3}(\frac{Z_i^2}{Z_p^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}) + \alpha)]. \quad (4.6)$$

Now we consider the dispersion relation (4.1) with Madelung term and using the same technique that we used in chapter 3 i.e.,

$$1 + \frac{\omega_{pi}^2}{\omega_{qi}^2}(\frac{\Gamma_i - 1}{\Gamma_i}) + \frac{\omega_{pe}^2}{\omega_q^2}(\frac{\Gamma_e - 1}{\Gamma_e}) + \frac{\omega_{pp}^2}{\omega_q^2}(\frac{\Gamma_p - 1}{\Gamma_p}) = 0. \quad (4.7)$$

Multiplying both side by $\omega_q^2\Gamma_p$ and rearranging the terms, we get

$$\Gamma_p = (\frac{\omega_{pp}^2}{\omega_q^2 + \frac{\omega_{qp}^2}{\omega_{qi}^2}\omega_{pi}^2(1 - \frac{1}{\Gamma_i}) + \omega_{pe}^2\frac{\omega_{qp}^2}{\omega_{qe}^2}(1 - \frac{1}{\Gamma_e}) + \omega_{pp}^2}). \quad (4.8)$$

As

$$\Gamma_p = 1 + \frac{1}{k^2\gamma_{qp}^2}[1 - ln\frac{2kv_{Fp}}{\gamma_p}], \quad (4.9)$$

$$\frac{\Gamma_e - 1}{\Gamma_e} = \frac{3\omega_{qe}^2}{k^2v_{Fe}^2}[1 + \frac{i\pi\omega}{2kv_{Fe}}], \quad (4.10)$$

$$\frac{\Gamma_i - 1}{\Gamma_i} = -\frac{\omega_{qp}^2}{\omega_2}. \tag{4.11}$$

Using Eq.(4.9), (4.10) and (4.11) in Eq.(4.8), we get the required dispersion relation

$$\omega = kv_{Fp}[1 + 2exp[-2(1 + \frac{k^2\lambda_p^2 - \frac{1}{3}(\frac{Z_i^2}{Z_p^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}) + \alpha}{1+a}) - \frac{i\pi\omega}{kv_{Fe}}\frac{\alpha}{1+a}]]. \tag{4.12}$$

The real and imaginary part of above dispersion relation becomes

$$\omega_r = kv_{Fp}[1 + 2kv_{Fp}exp[-2(1 + \frac{k^2\lambda_p^2 - \frac{1}{3}(\frac{Z_i^2}{Z_p^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}) + \alpha}{1+a})]cos(\frac{\pi}{1+a})], \tag{4.13}$$

$$\omega_i = -2kv_{Fp}exp[-2(1 + \frac{k^2\lambda_p^2 - \frac{1}{3}(\frac{Z_i^2}{Z_p^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}) + \alpha}{1+a})]sin(\frac{\pi}{1+a}), \tag{4.14}$$

here

$$a = \frac{\omega_{qp}^2}{k^2v_{Fp}^2}[\alpha - \frac{Z_i^2}{Z_P^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}].$$

4.2 Positron Sound Waves

Here we consider degenerate plasma by treating ions, electrons and positrons as degenerate. Noting that here we consider the effective masses, as in semiconductor plasma and we assume that the effective mass of hole is greater than mass of electron i.e., $m_p > m_e$, moreover we assumed that mass of ion become approximately equal to effective mass of hole i.e., $m_i = m_p$ and choosing the frequency range $kv_{Fi} < \omega < kv_{Fe}$ and $\omega > kv_{Fp}$. First we consider the case without Madelung term. The general dispersion relation are

$$1 + \sum_\alpha \frac{3\omega_{p\alpha}^2}{k^2v_{F\alpha}^2}[1 - \frac{\omega}{2kv_{F\alpha}}ln\frac{(\omega + kv_{F\alpha})}{(\omega - kv_{F\alpha})}] = 0, \tag{4.15}$$

here $\alpha = e, i, p$. Hence the above dispersion relation is

$$\omega^2(1+\alpha) + i\alpha\beta\omega^3 = \omega_{pi}^2 + \omega_{pp}^2.$$

The the real and imaginary part of above equation becomes

$$\omega_r^2 = \frac{\omega_{pi}^2}{(1 + \frac{3\omega_{pe}^2}{k^2v_{Fe}^2})} + \frac{\omega_{pp}^2}{(1 + \frac{3\omega_{pe}^2}{k^2v_{Fe}^2})}, \tag{4.16}$$

$$\omega_i = -\frac{\beta\omega_r^2}{2(\frac{1+\alpha}{\alpha})}. \tag{4.17}$$

When $k^2 v_{Fe}^2 >> \omega_{pe}^2$ or we can say in the limit of short wavelength, above real and imaginary parts become

$$\omega_r^2 = \omega_{pp}^2 + \omega_{pi}^2, \tag{4.18}$$

$$\omega_i = -\frac{3\pi\omega_{pe}^2}{(4kv_{Fe})(k^2 v_{Fe}^2)}(\omega_{pp}^2 + \omega_{pi}^2). \tag{4.19}$$

In term of positron acoustic velocity it becomes

$$\omega_i = -\frac{\pi}{2}\frac{\omega_{pp}^2\omega_{pe}^2 m_e\sqrt{m_e}}{k^3 v_s^3 m_p\sqrt{m_p}}(1 + \frac{Z_i^2}{Z_p^2}\frac{n_{0i}}{n_{0p}}). \tag{4.20}$$

When $k^2 v_{Fe}^2 << \omega_{pe}^2$ or in the limit of long wavelength Eq.(4.35) and (4.36) becomes

$$\omega_r = \frac{1}{\sqrt{3}}(\frac{m_e}{m_p}\frac{n_{0p}}{n_{0e}} + \frac{m_e}{m_p}\frac{Z_i^2}{Z_e^2}\frac{n_{0i}}{n_{0e}})^{\frac{1}{2}} kv_{Fe}, \tag{4.21}$$

$$\omega_i = -\frac{\pi}{12}(\frac{m_e}{m_p}\frac{n_{0p}}{n_{0e}} + \frac{m_e}{m_p}\frac{Z_i^2}{Z_e^2}\frac{n_{0i}}{n_{0e}})kv_{Fe}. \tag{4.22}$$

In term of positron acoustic velocity it becomes

$$\omega_i = -\frac{\pi}{4\sqrt{3}}(\frac{n_{0i}}{n_{0p}} + \frac{Z_i^2}{Z_e^2}\frac{n_{0p}}{n_{0e}})(\frac{m_e}{m_p})^{\frac{1}{2}} kv_s. \tag{4.23}$$

Now we consider Madelung term and choosing the same frequency range by repeating the same procedure the real part of the dispersion relation for long and short wavelength becomes

$$\omega_r^2 = \omega_{qp}^2 + (\frac{n_{0i}}{n_{0p}} + \frac{m_e}{m_p}\frac{n_{0p}}{n_{0e}})(\frac{k^2 v_{Fe}^2}{3}), \tag{4.24}$$

$$\omega_r^2 = \omega_{qp}^2 + \omega_{pp}^2 + \omega_{pe}^2. \tag{4.25}$$

The damping rates are unaffected in the presence of Madelung term.

Chapter 5

Results and Discussion

5.1 Zero Sound Waves in Electron-Positron-ion Quantum plasma

The real and imaginary part of frequencies with Madelung term are

$$\omega_r = kv_{Fp}[1 + 2exp[-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1+a})]cos(\frac{\pi}{1+a})], \tag{5.1}$$

$$\begin{aligned}\omega_i = -2kv_{FP}exp[-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1+a})][sin(\frac{\pi}{1+a}) + (\frac{2\pi}{1+a}) \\ exp[-2(1 + \frac{k^2\lambda_p^2 + (\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}}}{1+a})]cos^2(\frac{\pi}{1+a})],\end{aligned} \tag{5.2}$$

here a is called the quantum parameter, which can take any value but only those values significantly affect the real and imaginary frequency at which $k\lambda_{Fp} \sim 1$ which implies $\lambda_{Fp} \approx \lambda_B$ and it is shown in figure(5.1) and (5.2). When a increases then due to this frequency shift is created and as a result these waves shifted upward. When we move towards more short wavelength limit then waves merge together this can be shown in figure(5.2). Quantum effects are associated with de-Broglie wavelength larger the de-Broglie wavelength is larger the quantum effect. In long wavelength limit zero sound waves do not exist, because these waves are high frequency waves.

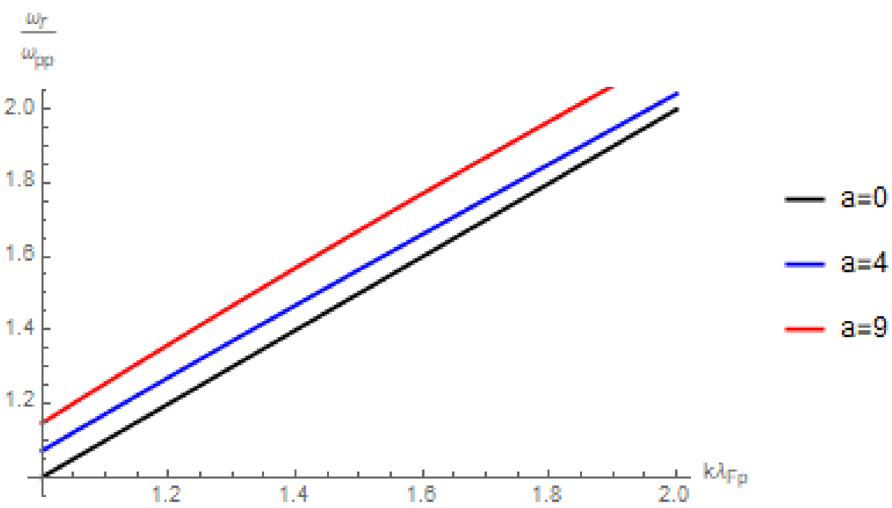

Figure 5.1: Plot of Eq.(5.1) for arbitrary value of a. Here we consider the fixed ratio $(\frac{n_{0e}}{n_{0p}})^{\frac{1}{3}} = 6$

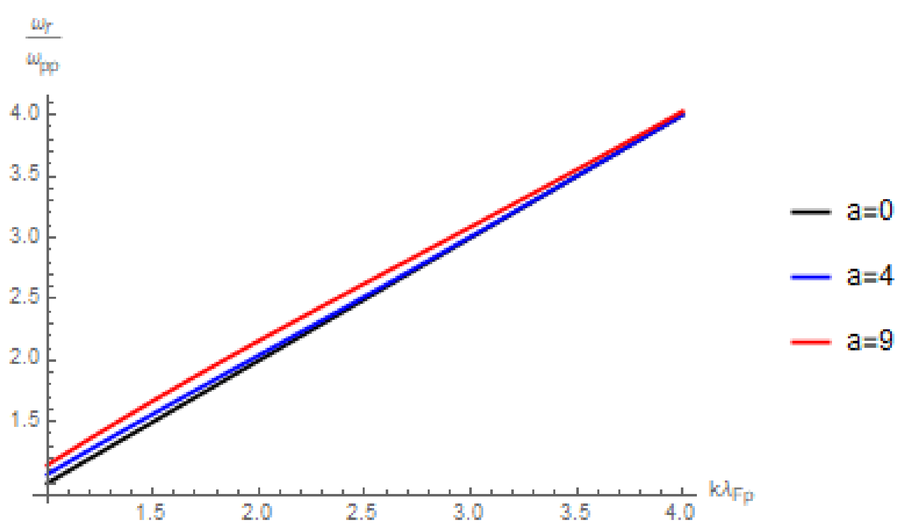

Figure 5.2: Plot of Eq.(5.1) for arbitrary value of a, when we move towards more short wavelength limit.

Now we consider imaginary part of these waves, as imaginary part of frequency is negative which indicate damping of these waves, here damping exist due to wave-particle interaction. Whereas damping is significant when $k\lambda_{Fp} \sim 1$ this is shown in figure(5.3).

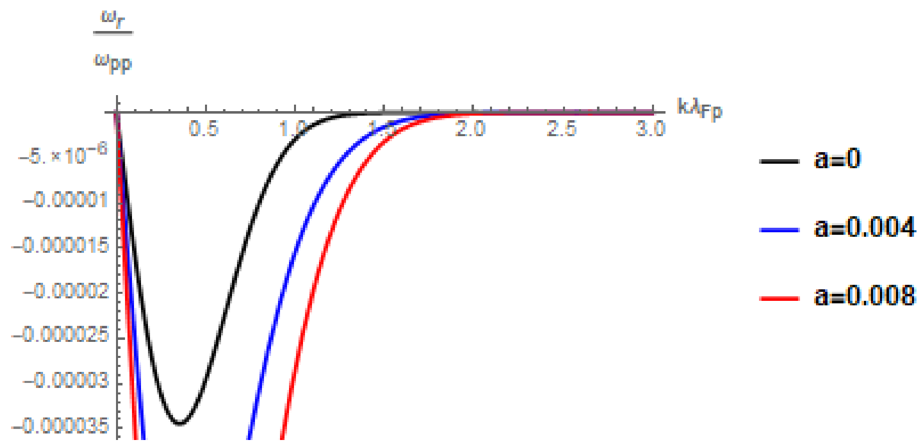

Figure 5.3: Plot of Eq.(5.2) for arbitrary value of a.

5.2 Positron Zero sound Waves when we assumed ions as Degenerate

The real and imaginary part of positron zero sound waves when we assumed that ions are also degenerate along with electrons and positrons with frequency approximation $\omega > kv_{Fi}, kv_{Fe} > \omega \sim kv_{Fp}$.

$$\omega_r = kv_{Fp}[1 + 2kv_{Fp}exp[-2(1 + \frac{k^2\lambda_p^2 - \frac{1}{3}\gamma + \alpha}{1+a})]cos(\frac{\pi}{1+a})], \tag{5.3}$$

$$\omega_i = -2kv_{Fp}exp[-2(1 + \frac{k^2\lambda_{Fp}^2 - \frac{1}{3}\gamma + \alpha}{1+a})][sin(\frac{\pi}{1+a}) + (\frac{2\pi}{1+a}) \\ exp[-2(1 + \frac{k^2\lambda_{Fp}^2 - \frac{1}{3}\gamma + \alpha}{1+a})]cos^2(\frac{\pi}{1+a})] \tag{5.4}$$

,

here $a = \frac{\omega_{qp}^2}{k^2 v_{Fp}^2}[\gamma - \alpha]$ and $\alpha = \frac{\lambda_p^2}{\lambda_e^2} = (\frac{n_{0e}}{n_{0p}})\frac{T_{Fp}}{T_{Fe}}$ and $\gamma = \frac{Z_i^2}{Z_P^2}\frac{m_p}{m_i}\frac{n_{0i}}{n_{0p}}$. When we treat ions as degenerate and taking $m_i > m_p = m_e$ so that $T_{Fi} < T_{Fp} < T_{Fe}$ for light nuclei like hydrogen the zero sound waves propagate in the same way in both short and long wavelength limit. But for heavy nuclei like Helium these waves significantly differ. When there is no quantum effect i.e., a=0, then ions, electrons and positrons move gradually at low perturbed frequency, as the perturbed frequency increases then ion

inertia becomes important, here only electrons and positrons move gradually.
When quantum effects are taken into account then initially ions respond but the quantum effects associated with ions are very small therefore the graph appear to seem merge initially. When we move towards high perturb frequency then these effects prominently exist due to electrons and positrons, here quantum effects appear only when $k\lambda_{Fp} \approx \lambda_B$. Therefore when we move towards more short wavelength then these effects vanishes like in previous case. Now we consider imaginary part of these waves, the trend of the waves is same like in previous case, here damping exist because of wave-particle interaction and is significant when $k\lambda_{Fp} \sim 1$, But damping rate is high because of the presence of ions.

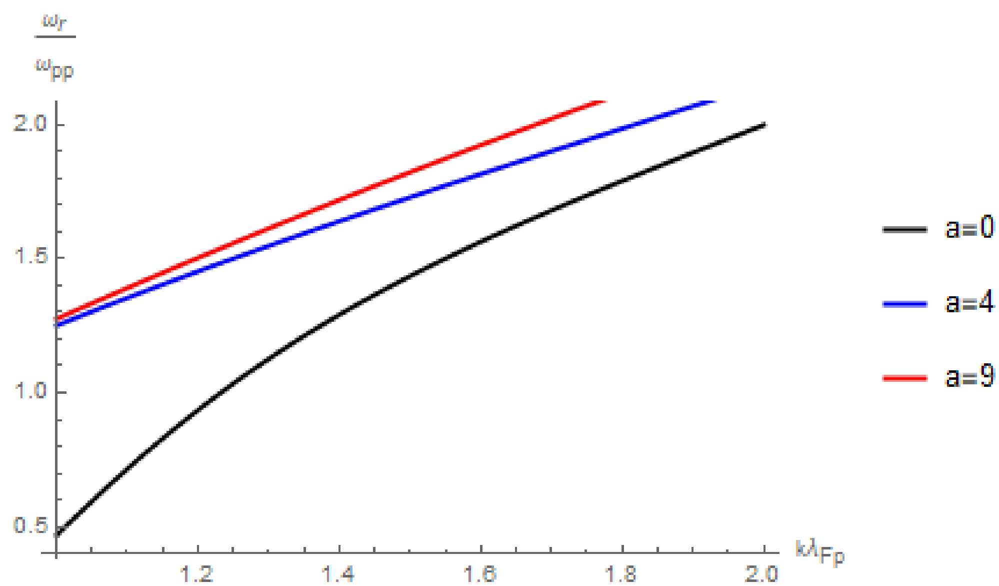

Figure 5.4: Plot of Eq.(5.3) for arbitrary value of a.

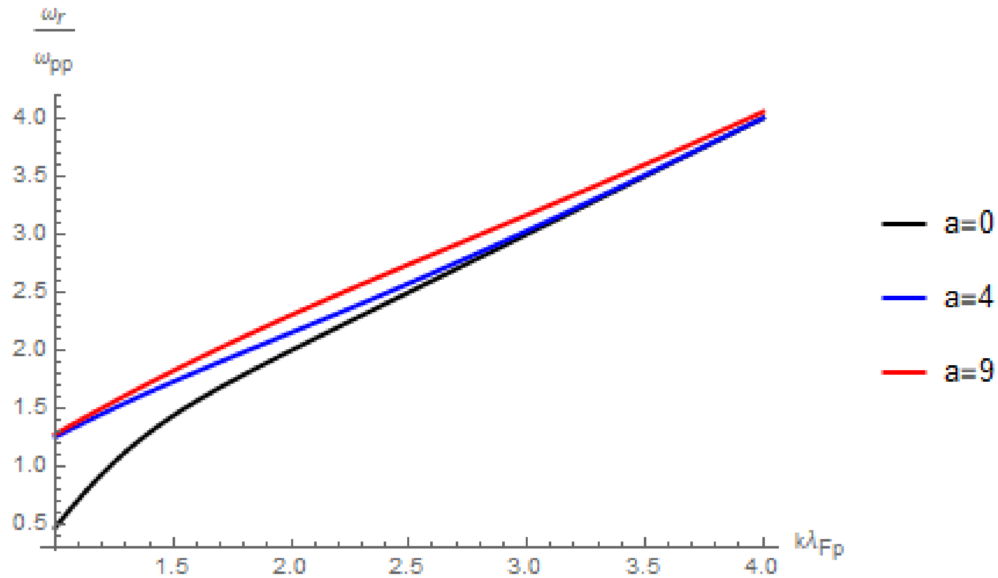

Figure 5.5: Plot of Eq.(5.3) for arbitrary value of a.

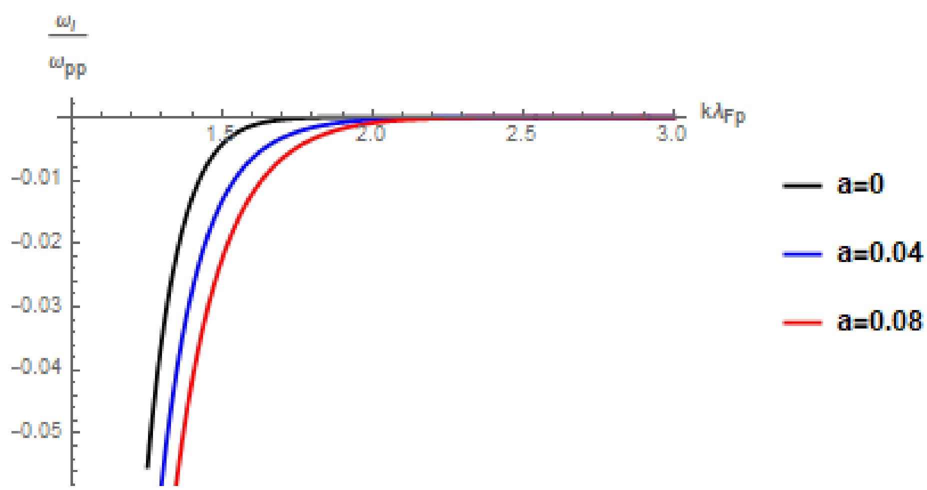

Figure 5.6: Plot of Eq.(5.4) for arbitrary value of a.

5.3 Special Cases with $m_h > m_e$

The imaginary frequency for long wavelength limit ($k^2\lambda_e^2 << 1$) is

$$\omega_i = -\frac{\pi}{4\sqrt{3}}(\frac{n_{0p}}{n_{0e}})(\frac{m_e}{m_p})^{\frac{1}{2}}kv_s. \tag{5.5}$$

The imaginary frequency for short wavelength limit ($k^2\lambda_e^2 >> 1$) is

$$\omega_i = -\frac{\pi}{4}\frac{\omega_{pp}^2\omega_{pe}^2 m_e\sqrt{m_e}}{k^3 v_s^3 m_p\sqrt{m_p}} \tag{5.6}$$

The arbitrary values of $(\frac{m_e}{m_p}) = 0.1$ and $(\frac{n_{0p}}{n_{0e}}) = 0.01$ are taken so that they satisfy the condition $\frac{m_e}{m_p} > (\frac{n_{0p}}{n_{0e}})^{\frac{1}{3}}$. Note that the mass ratio of electron to holes depends on the type of material which is used in semiconductor e.g., Silicon and GaAs, $\frac{m_e}{m_p} < 1$ holds, while opposite in AlAs i.e, $\frac{m_e}{m_p} > 1$. Fig(5.3) and Fig(5.4) shows that the damping for short wavelength waves is quite larger than the long wavelength waves. Low damping rates give rise to long-lived oscillations. So in quasi-neutral medium, these positron or longitudinal sound waves should propagate. For long wavelength limit this spectrum is valid i.e, $k^2\lambda_\alpha^2 << 1$, whereas in short wavelength limit both electrons and holes move at different frequency due to difference in their masses as a result these waves are highly damped.

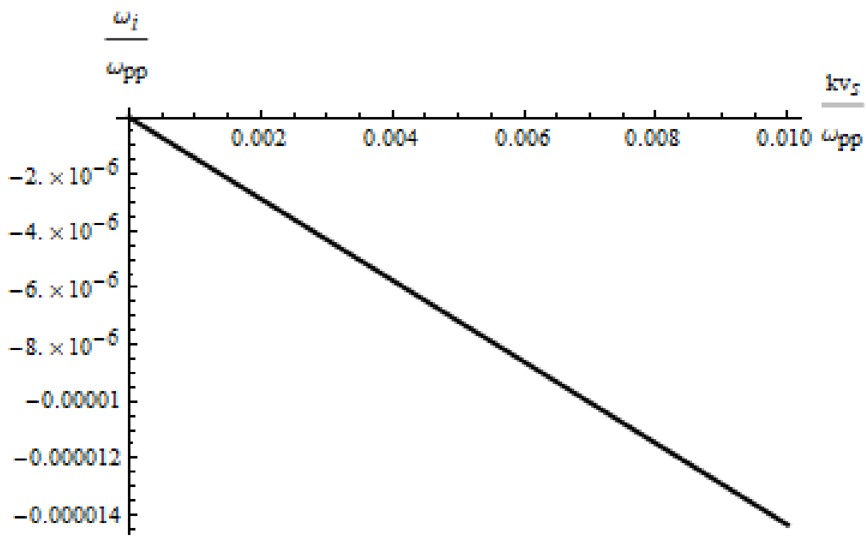

Figure 5.7: Plot of Eq.(5.5) for arbitrary values of $(\frac{m_e}{m_p})$ and $(\frac{n_{0p}}{n_{0e}})$ in long wavelength limit.

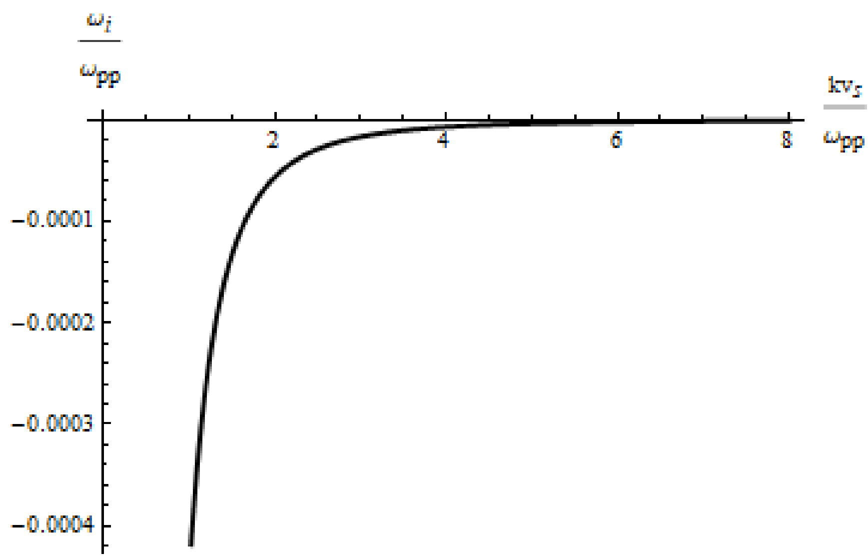

Figure 5.8: Plot of Eq.(5.6) for arbitrary values of $(\frac{m_e}{m_p})$ and $(\frac{n_{0p}}{n_{0e}})$ in short wavelength limit.

5.4 Conclusion

Considering the propagation of small longitudinal perturbations in two types of quantum plasmas i.e., electron-positron-ion and electron-hole-ion, by using Schrodinger-Poisson model we derive a general quantum dispersion relation. Considering this for some cases, in EPI plasma, we revealed a new type of damped (zero sound waves). Moreover, in electron-hole-ion plasma we found a new longitudinal quantum waves. Later the excitation of these longitudinal quantum waves are examined, and here we found the instability condition for the generation of these waves. Further we consider longitudinal quantum waves and examined by treating fully degenerate e-p-i plasma. These investigations may be useful to describe the complex phenomenon in astrophysical objects, and may have many applications in modern technology.[31]

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