Atomic Scattering From Cavity Field And Entanglement To Distant Cavities

by

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Dedicated to

My Parents and my siblings for their Love,
Endless support &
Encouragement.

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Starting with the name of Allah (S.W.T), the most Merciful, the most beneficient. All praise is for Allah (S.W.T), we seek his guidance and ask for His forgiveness. I am thankful to Allah (S.W.T), who gave me courage, guidance and the love to complete this research. Without His help this could never be possible. I cannot forget the ideal man on the earth and the most respectful personality for whom Allah (S.W.T) created the whole universe, Prophet Hazrat Muhammad (P.B.U.H).
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Abstract

We have reviewed atomic scattering from cavity field and its application to entangle distant cavities. In particular we are studying the entanglement generation between atomic momenta and photon number state of the cavity. This is then extended to employ entanglement swapping and generate entanglement between distant cavities. The principle of the model is based upon the scattering of atoms and fulfilling the conditions of Bragg scattering. This leads to the entanglement generation between atomic momenta of atoms and the cavities if cavities are in the superposition state of 0 and 1 photons. Further, the Einstein-Podolsky-Rosen(EPR) pairs are made by passing these atomic momenta through the beam splitters where the cavity fields act as beam splitter for atomic momenta states. The beam splitters acts as EPR state analyzer which enables entanglement swapping and entangles the distant cavities.

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Chapter 1

Introduction

Quantum theory is the theoretical basis of modern physics and the nature and behavior of matter and energy on the atomic and subatomic level is explained by quantum theory. Quantum theory is based on the principle that matter and energy have dual nature i.e, they have both light nature and wave nature. Quantum theory explains many phenomena like black-body radiation, photoelectric effect and uncertainty principle which are unexplained by classical mechanics. Quantum theory emerged from the idea of Max Planck which assumed that the light is made up units of energy. Then, Albert Einstein gave the idea that energy in light waves is made up of energy packets which are called quanta.

Quantum teleportation is an application of quantum mechanics, which transfers the quantum state from one point to another without sending the original state. This becomes possible due to the entanglement. In quantum information entanglement plays a vital role. Entanglement is a correlation between two or more systems which is non-classical. Quantum entanglement has many applications in the field of quantum computation and quantum information such as teleportation, quantum dense coding and cryptography. Quantum computers are also based on the idea of entanglement. Quantum entanglement is used in entanglement swapping. Entanglement swapping is a procedure of entangling systems which have never interacted. Entanglement swapping is already studied but it is not applied to entangle the atoms at large distances.

The interaction of an atom and field can be studied in different ways in which a two level atom interacts with the field. Cavity QED techniques are used for the interaction of the atom and field. In particular atomic momenta can be used as a tool to entangle distant cavities by entanglement swapping. In Sec. 1.1, we will tell about the qubit and in Sec. 1.2, we will tell about the quantum states. Entanglement is being discussed in Sec. 1.3. Entanglement swapping is discussed in Sec. 1.4 and entanglement swapping experiment with photons is explained in Sec. 1.5 and with ions is discussed in Sec. 1.6. Then the atom and photon interferometry is explained in Sec 1.7 and Sec. 1.8 respectively.

1.1 Qubit

In classical mechanics, the basic unit of information is called bit. It exits in the form of '0' or '1', which tells either the state is 'true' or 'false'. While in quantum mechanics it is called qubit. It is in the superposition state of '0' and '1' while in classical mechanics bit is not in superposition state.
The superposition state of bit is written as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle,$$

where, α and β are the probabilities. The sum of the probabilities is equal to 1.

$$|\alpha|^2 + |\beta|^2 = 1.$$

1.2 Quantum State

Quantum state is the superposition of all the states in Hilbert space. It can be either pure or mixed. Quantum state is a vector in Hilbert space which contains all the information about the system. State is expressed in bra-ket notation which was given by Dirac. State is written as $|\psi\rangle$ the ket notation and $\langle\psi|$ in bra notation. Here ψ represents the state. The state of a system is usually expressed as the sum of its probability amplitudes given as:

$$|\psi\rangle = \Sigma_n C_n |\psi\rangle,$$

where, C_n is the probability amplitude.

1.3 Entanglement

Entanglement is an important quantum mechanical resource which plays an important role in the field of quantum computation and quantum information. Entanglement was first studied by Schrodinger. Further it was questioned by Einstein, Podolsky and Rosen. Entanglement is created when two particles interact in such a way that by doing measurement on one particle we can get the information about the other particle. If the measurement on one particle affects the state of the other, then the particles are said to be entangled. But Einstein said that this interaction is not possible as it is non-local. Non-locality means that the effect of a quantum system should be at a distance. Einstein called it a "spooky action at a distance". An entangled state can never be described as a separable state or in the form of a tensor product. Entangled state can be fully defined only by considering all its particles. One or two particles cannot describe the state fully.

1.3.1 Mathematical Definition for Pure States

A pure state is a coherent superposition of two or more states. A state $|\psi\rangle_k$ is pure if its density matrix ρ has following properties:

- It is hermitian, that is, $\rho^\dagger = \rho$.
- Its trace is equal to one. Trace(ρ^2) = 1.
- $\rho^2 = \rho$. [1]

For pure states, if $|\psi\rangle_A$ and $|\psi\rangle_B$ are states of subsystem of a system $|\psi\rangle_{AB}$ then $|\psi\rangle_{AB}$ is an entangled state of system if its states cannot be written as tensor product of state of two sub-systems

$$|\psi\rangle_{AB} \neq |\psi\rangle_A \otimes |\psi\rangle_B,$$

The states $|\psi\rangle_A$ and $|\psi\rangle_B$ are thus non-separable and are called entangled.

1.3.2 Mathematical Definition for Mixed States

For mixed states, the density matrix ρ_{AB} of a composite system is separable if and only if

$$\rho_{AB} = \Sigma_k \; p_k \rho_A \otimes \rho_B.$$

For the sets of density matrices $\{\rho_A\}$ and $\{\rho_B\}$ and for probabilities $\Sigma_k \; p_k = 1$, $\forall k$, $\Sigma_k \; p_k \geq 0$. The separable state is also known as unentangled [2].

1.4 Entanglement Swapping

Entanglement swapping is a technique which is very closely related to the teleportation. Entanglement swapping is also called teleporting entanglement. The idea of entanglement swapping was given by Zukowski, Zeilinger, Horne and Ekert [3].
Before entanglement swapping, it was thought that the two particles can only get entangled if they come from the same source or they must have interacted in the past to get entangled. But after entanglement swapping technique, it was discovered that the particles can get entangled even if they did not have any interaction in the past.
Alice and Bob both have entangled pairs. They send their particles to Charlie. Then Charlie perform BSM(Bell state measurement) on the two particles which entangles the Alice and Bob particles.
We can describe entanglement swapping in such a way, that Alice and Bob share an entangled pair of particles and Charlie and David. Bob teleports his pair of particle to Charlie, so, as a result of teleportation, David's particle is in the same state in which Alice's was at the beginning. Now, Alice and David's particles get entangled without any interaction [3].
Entanglement swapping is done experimentally in the past. Many experiments have done on entanglement swapping of which one of them is the entanglement swapping of photons that have never interacted [4].

1.5 Entanglement Swapping of Photons that Have Never Interacted in the Past

To obtain entanglement, the state of two particles is projected on an entangled state. For this projective measurement we do not require a direct interaction between the two particles. When the two pair of particles will get entangled, then by doing a EPR state measurement on partner particles will collapse the other partner particles into an entangled state.

For the entanglement swapping, we consider two EPR sources, each of them emitting a pair of entangled particles simultaneously. We have assumed that there are polarized entangled photons in the state.

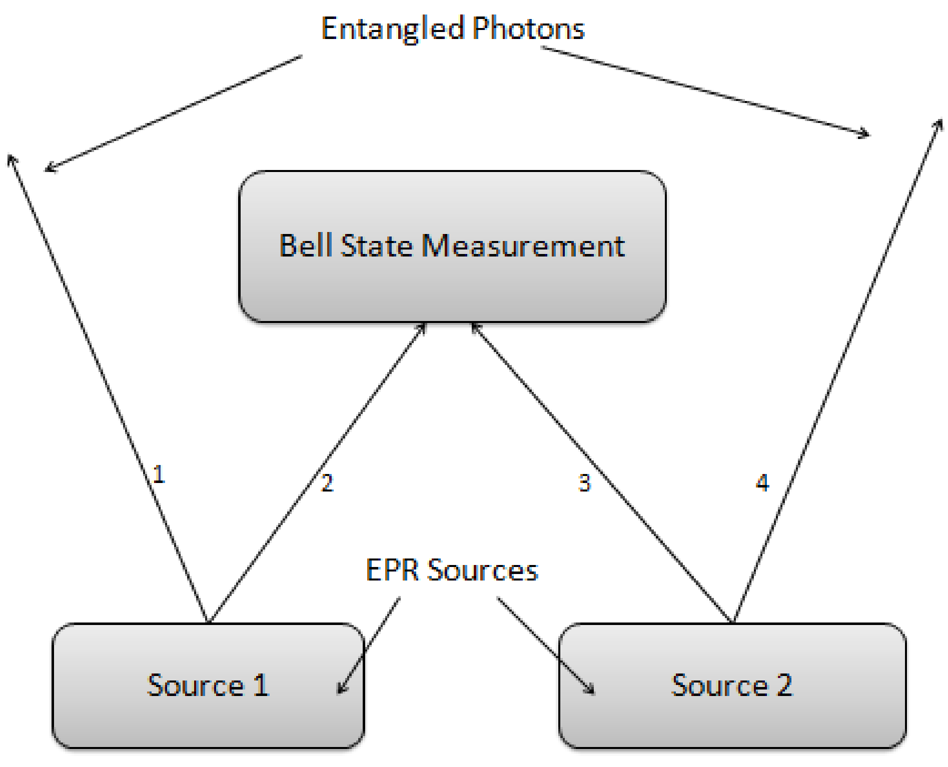

Figure 1.1: Principle of entanglement swapping

In Fig. 1.1 two EPR sources are shown that produce two entangled pair of photons which are pair 1, 2 and 3, 4. One of the photon of each of the pair is projected to Bell-state measurement. As a result of this, the other two photons 3 and 4 are projected into an entangled state.

$$|\psi\rangle_{1234} = \frac{1}{2}(|H\rangle_1|V\rangle_2 - |V\rangle_1|H\rangle_2) \times (|H\rangle_3|V\rangle_4 - |V\rangle_3|H\rangle_4),$$

Here, $|H\rangle$ and $|V\rangle$ indicates the horizontal and vertical polarization of photons. The total state describes the entanglement of photons 1 and 2 (3 and 4) in an antisymmetric polarization. Although, there is no entanglement between any photons of 1 and 2 with any photon of 3 and 4, but by performing a joint EPR state measurement on the photons 2 and 3, the photon 2 and photon 3 are projected on one of the the four EPR states, which form a complete basis for combined photons state 2 and 3.

$$|\psi^{\pm}\rangle_{23} = \frac{1}{\sqrt{2}}(|H\rangle_2|V\rangle_3 \pm |V\rangle_2|H\rangle_3),$$

$$|\phi^{\pm}\rangle_{23} = \frac{1}{\sqrt{2}}(|H\rangle_2|H\rangle_3 \pm |V\rangle_2|V\rangle_3),$$

This measurement projects photons 1 and 4 onto a Bell state which is a different Bell state from the photons 2 and 3. For the initial state of four photons, it has seen that the photons 1 and 4 are collapsed to the state identical to the state of photons 1 and 2. So the photon state will be:

$$|\psi\rangle_{1234} = \frac{1}{2}(|\psi^+\rangle_{14}|\psi^+\rangle_{23} + |\psi^-\rangle_{14}|\psi^-\rangle_{23} + |\phi^+\rangle_{14}|\phi^+\rangle_{23} + |\phi^-\rangle_{14}|\phi^-\rangle_{23}),$$

In all the cases the emerging photons 1 and 4 are entangled although they have never interacted in the past [4]. The state of photons 1 and 2 is destroyed as they are measured.

1.6 Long Distance Entanglement with Ions and Photons

For entanglement, photons are excellent in a way that they preserve their coherence over large distances and they propagate very fast. For entanglement swapping and

purification, it is of our interest that we have systems which can be stored and between these systems we can realize the quantum gates easily. The systems we are using should have the coherence time larger than the propagation time of photons over larger distances. So to fulfill these requirements ions are the best choice.
In this scheme, by doing the joint detection on two photons the distant trapped ions are entangled, each photon comes from an ion. A pair of ions is shared between Alice and Bob. Entanglement is created between this pair of ions. Alice and Bob both have an ion which have λ energy levels. The excited state $|e\rangle$ can decay into two meta stable states $|x_1\rangle$ and $|x_2\rangle$ and in this decay it emits a photon which have two orthogonal polarization modes d1 and d2. For simplicity we have assumed that the probability of transitions from $e \to x_1$ and $e \to x_2$ are same.
Both ions are excited to the state $|e\rangle_A\ |e\rangle_B$. A photon is emitted by each ion and the state becomes:

$$\frac{1}{2}\big[|x_1\rangle_A a_1^\dagger + |x_2\rangle_A a_2^\dagger\big]\big[|x_1\rangle_B b_1^\dagger + |x_2\rangle_B b_2^\dagger\big]|0\rangle, . \qquad \text{(a)}$$

which shows that each ions is maximally entangled with the emitted photon by itself. Photons from A and B propagate to intermediate location where partial EPR state analysis is performed as shown in Fig. 3.1.
When the two photons will be detected in $|\psi_\pm\rangle = \frac{1}{\sqrt{2}}(a_1^\dagger b_2^\dagger \pm a_2^\dagger b_1^\dagger)|0\rangle$ state then the two distant ions will also be projected to the state $|\psi_\pm\rangle$ corresponding to $\frac{1}{\sqrt{2}}(|s_1\rangle_A|s_2\rangle_B \pm |s_2\rangle_A|s_1\rangle_B)$. While the remaining $|\phi_\pm\rangle$ cannot be distinguished [5].

1.7 Atom Interferometry

An atom interferometer is based on exploiting the wave character of atoms. To observe the interference, the atomic beam is subjected to a periodic scattering realized with massive gratings or optical lattices. Depending on the relative phase between the different paths, the atoms interfere constructively or destructively, and an interference pattern can be observed on a distant screen. The atom continues on either of two spatially separate paths, the interferometer arms. When the paths are recombined, the probability that the atom is found depends upon the phase

difference between them, which determines whether the two waves will add or cancel. This phase is shifted by the atoms coupling to electromagnetic fields, gravity, inertial forces, and other influences.
Interferometers using atoms rather than light can measure acceleration and rotation to high precision. Because atoms are slower than light, atom interferometers have the potential to reach greater inertial sensitivity than optical interferometers. The atoms inside an atom interferometer are controlled by beam splitters and mirrors. The first beam splitter that an incoming matter wave encounters separates the wave into two different paths. The accumulation of phase along the two paths leads to interference at the last beam splitter, whose two output channels produce complementary probability amplitudes for detecting atoms.

1.8 Photon Interferometry

In photon interferometers, we use a light source that strikes the beam-splitter BS1 and after passing through the BS1 it diverts into two parts as shown in Fig. 1.2. Beam gets reflected and other get transmitted through the beam-splitter. Reflected beam is named as R and transmitted one is named as T. Both are then projected to the mirrors. After passing through the mirrors they are deflected to the second beam-splitter BS2. After which, one ray is again transmitted and other is reflected. So, we get TR+RT at detector D1 and TT+RR at detector D2. As shown in Fig. 1.2. This procedure tells us about the particle nature of light.

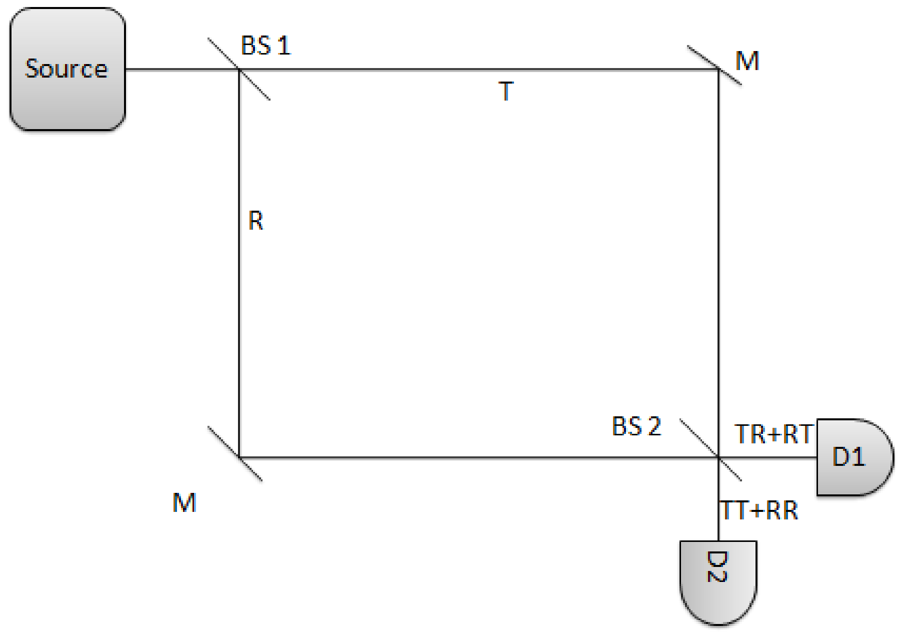

Figure 1.2: Photon Interferometer

1.9 Atom-Field Interaction

Atom can interact with the field in many ways. It can be scattered or it can be absorbed and emitted from the field in Rabi cycles. The atom can get scattered in either Bragg regime or Raman-Nath regime.

In Bragg regime, atoms are incident in ground state at some angle to the standing wave cavity field. Here the momentum component along the field is considerable. The injection rate is taken so small that only one atom can enter the cavity at one time. Then the atom passes through the field and get absorbed and emitted in Rabi cycles. Rabi cycle is a cycle in which when a two-level atom is illuminated by a coherent beam of light, then it absorbs photon and emits them during stimulated emission. Energy is conserved which allows only two possible directions for the scattering atom. Through this process the momentum of the atom gets kick of $2\hbar k$ or $0\hbar k$.

In Raman-Nath regime, atoms enter the field at almost normal to the field. Here the momentum component along the field is very small as it becomes ignorable. So the momentum transfer is also very small. It is an inelastic scattering in which kinetic energy is not conserved. This regime can be achieved when we take very small interaction time. When the interaction time is very small then due to energy time uncertainty principle $\Delta E \Delta t \geq \hbar/2$, energy becomes very large and atom can get scatter in many possible directions.

Entanglement generation is very important step for entanglement swapping between distant cavities. For entanglement generation between the atoms and cavities, the cavities are taken in Bragg regime. When atoms pass through the cavities they get scattered in two possible directions. These two scattered atoms from each cavity, pass through the beam splitters and then BSM entangles the atoms and cavities by their atomic momenta. Here the beam splitter used is also a cavity which is in superposition state of zero and one photon.

1.10 Thesis Outline

In this thesis, we give a review on scattering of atoms from cavity field and how this process can be used to entangle distant cavities using entanglement swapping.

In Chapter 2, we describe the full mathematical and physical description of our system. We explain our system that how it is being used to entangle the cavities. We explain the Bragg regime and conditions to achieve it. Then we explain the Bragg scattering in full detail and then development of Hamiltonian. From this Hamiltonian we develop the equations of motion for solving the system. Then we solve them to obtain the solution of these equations.

In chapter 3, we explain the entanglement generation between the atoms to their atomic momenta by using the cavities in Bragg regime. Then we explain the entanglement of these atoms to the cavities. Then by doing the Bell-state measurement we explain the entanglement of two distant cavities. After that, the detectors detect the state of the atom.

In chapter 4, we conclude our thesis.

Chapter 2

Fundamental Process of Atomic Scattering from Cavity Field

In this chapter, we tell about the atom-field interaction from the cavity field. We send atoms with some initial momentum $|P_0\rangle$ in the cavity. The variation in the momentum of the atoms after passing through the cavity are studied. Further, these atomic momenta are used to entangle the cavities.
In Sec. 2.1, we explain the model used for the entangling of cavities and its physical description. In Sec. 2.2, we tell the requirements to meet the conditions of Bragg regime. In Sec. 2.3, we explain the basic phenomenon occurring inside the cavity. In Sec. 2.4, we explain the Bragg scattering in detail and its types.

2.1 Model

We have two cavities in which, we inject a beam of two level atom such that there is only one atom in each cavity at a time by keeping the injection rate very small. We keep the atoms at very large detuning which means that the frequency of the field is kept much larger than the transition frequency. We take one-dimensional quantized field for simplicity. We inject the atom at some angle to the cavity with some initial momentum. We take the component of momentum along the cavity to be smaller than the perpendicular component of the cavity. When atom interacts with the optical field the longitudinal component gets a shift and get scattered at different angles, keeping the energy and momentum conserved.

By energy conservation, we achieve the Bragg regime in which an atom gets deflected in only two possible directions. The interaction of these two atoms from the two cavities entangles the atoms and hence, their momentums are entangled. Then, we pass these atoms through the beam splitters to detect them.

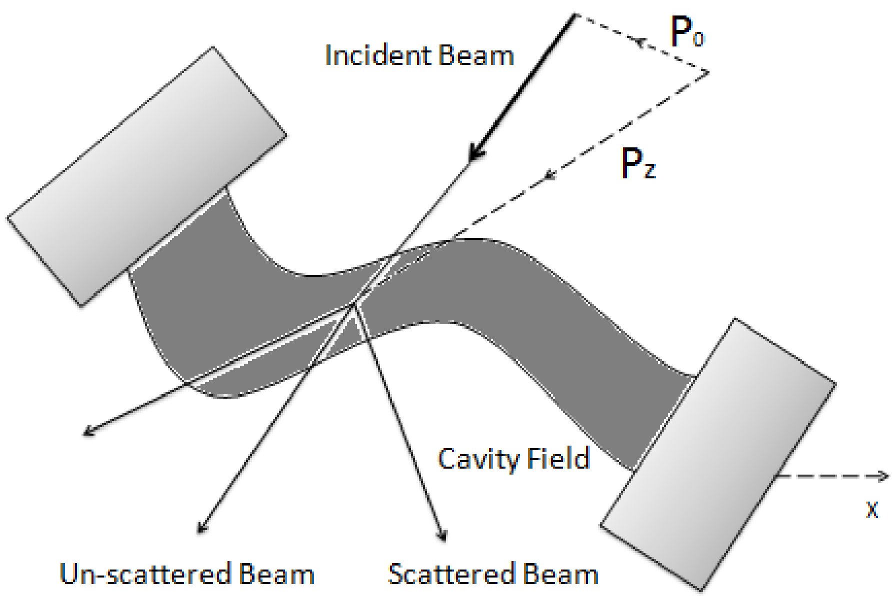

Figure 2.1: Suggested Experimental Setup: Highly detuned atomic beam is scattered by a quantized cavity field by ϵ_x at various angles. The momentum component P_0 is very small compared to the normal component P_z.

2.2 Requirements

Our purpose is to entangle distant cavities by taking the cavities in Bragg regime, to achieve Bragg Regime and to entangle only one atom at a time we take following measures.

(i) The atoms are taken in initially ground state so that when an atom undergoes even number of oscillations it deposit no photon in the field. Thus the photon numbers in the field remains conserved.

(ii) Atom must leave the cavity in ground state, thus taking away no photons with it. This is achieved by emission of photon followed by absorption of photon.

(iii) Emission must not be spontaneous as it leads to emission of photon in arbitrary direction.

We keep the field frequency much larger than the transition frequency. As a result, atoms experience it as large detuning, such that

$$\Delta^2 \gg \gamma^2 + g^2,$$

$\Delta = \nu - \omega$ denotes the detuning, ν is the field frequency, ω is the transition frequency of the atom, γ is the spontaneous emission decay rate and g is the Rabi frequency. Rabi frequency tells us about the oscillation of atoms between two levels. It also tells the atomic population fluctuation between the levels of the field.

$$g_{m,n} = \frac{\overrightarrow{d_{m,n}}.\overrightarrow{E_0}}{\hbar},$$

Here, $d_{m,n}$ is the dipole moment of transition from initial state m to final state n and $\overrightarrow{E_0} = \epsilon E_0$ is the vector electric field amplitude including the polarization.

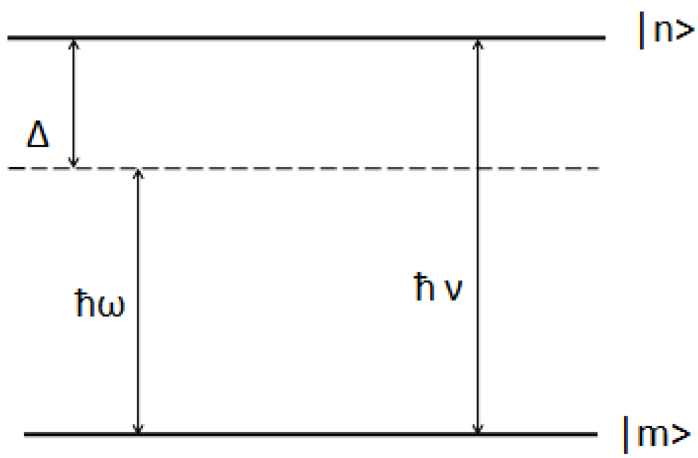

Figure 2.2: Energy Level Diagram: $|n\rangle$ is the excited level and $|m\rangle$ is the ground level. Δ is the detuning between two levels.

2.3 Basic Phenomenon

When an atom passes through the cavity it interacts with the optical field, and during interaction it emits and absorbs photons. Whenever a photon of frequency $\nu = ck$, where c is the velocity of light and k is the wave number $k = 2\pi/\lambda$, is absorbed or emitted a transfer of recoil momentum $\Delta P = \hbar k$ takes place between atom and field. As we have taken the normal component of momentum of the atom to be very large so it will be treated classically and we will take the momentum component along the cavity in quantum treatment.
The profile of the standing wave cavity field is given by

$$\varepsilon_0 = \frac{1}{2}\varepsilon(e^{ikx} + e^{-ikx}). \tag{2.3.1}$$

When the atom enters the cavity it experiences standing waves of field there. Waves are moving opposite to each other with a phase difference of 180^o. If atom absorbs the photon in its own direction and gets a transition from lower to upper level then to conserve momentum it gets a recoil momentum of $+\hbar$k. Then by emitting a photon in either direction it transits from upper to lower level. If it is stimulated to emit the photon in its own direction then it gets a recoil momentum of -$\hbar k$. Then there is no momentum transfer and the net momentum in this case will be $\Delta P = 0$ i.e the photon is restored in the cavity. If the atom is stimulated to emit the photon in opposite direction then it will get a recoil momentum of $+\hbar k$ and in this case there will be a momentum kick of $\hbar k$. Thus when atom returns to ground state the net momentum will be $\Delta P = 2\hbar k$. This is the greatest possible momentum kick in one Rabi cycle which is due to the counter propagating waves.
As we know that the recoil momentum is transferred in the form of $+2\hbar k$, thus the momentum of exiting atom from the cavity will be

$$P_l = P_0 + l\hbar k, \tag{2.3.2}$$

where, l is an even integer.
Thus atoms can scatter in any possible momentum component spaced by momenta $2\hbar k$. The strength of recoil force on an atom, divides the atomic scattering into two regimes; Raman-Nath Regime and Bragg Regime. Our interest is in Bragg regime. Bragg Regime is possible when the recoil energy is greater than the interaction energy of an atom between its energy levels.

2.4 Bragg Scattering

Bragg scattering of x-rays from the crystal planes was shown by W . H . Bragg and W . L . Bragg in 1912, for which they got Noble prize in 1915. Bragg scattering from neutrons was first discovered in 1946 from where the neutron interferometry got started [6].

2.4.1 Optical Bragg Scattering

Scattering of the atom through standing wave cavity field in Bragg regime can be best understood if we consider the optical Bragg scattering.

In optical Bragg scattering, the light waves incident on a crystal plane, scatter from the crystal planes of atoms. This means the crystal planes are matter gratings from which the light waves are reflecting. This Bragg scattering requires two conditions to be fulfilled.

(i) Energy should be conserved during interaction. This requires that the light waves should reflect at the same angle at which they are incident on the crystal planes.

(ii) Reflected waves from the adjacent planes interfere constructively only if the incident angle satisfies the condition

$$2d\cos\theta = n\lambda,$$

where, θ is the angle that the incident wave makes with normal to the lattice plane, d is the lattice spacing and λ is the wavelength of the incident light. As shown in figure, only at these values of θ, the phase difference between two reflected light waves is equal to an integral multiple of wavelength and Bragg's reflected waves superimpose constructively. Here n is the order of Bragg scattering which tells us the number of wavelengths that fit in the distance of $2dcos\theta$.

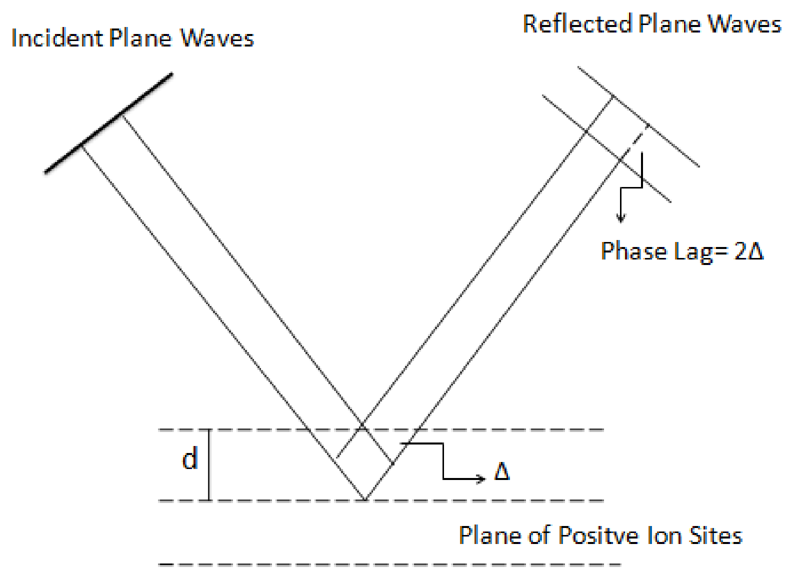

Figure 2.3: Optical Bragg Scattering: (a) Scattering of plane waves from lattice sites.

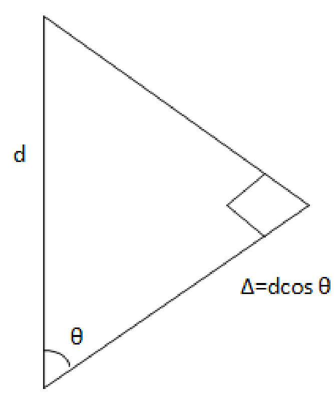

Figure 2.4: (b) Optical Bragg Scattering

2.4.2 Atomic Bragg Scattering

Bragg scattering of atoms from standing light waves was first discovered by Martin in 1988. In case of atomic Bragg scattering from the standing wave cavity field, wave particle duality is the guiding principle. Previously we discussed the scattering of light waves from the crystal planes, now we will discuss the atomic Bragg scattering, by considering the scattering of matter de-Broglie matter waves from an optical lattice, which has the lattice spacing or periodicity as

$$d_{\text{light}} = \frac{\lambda_{\text{light}}}{2}, \tag{2.4.1}$$

The first condition of conservation of energy of Bragg reflection implies that

$$\frac{P_{\text{in}}^2}{2M} = \frac{P_{\text{out}}^2}{2M}, \tag{2.4.2}$$

$$\frac{l(l+l_0)}{2M}\hbar^2 k^2 = 0, \tag{2.4.3}$$

either l = 0, or l = -l_0

Thus either the atom goes undeflected or deflected such that the momentum component along the direction is reversed i.e, it is deflected at the same angle at which it was incident.
Incident angle satisfies the Bragg condition for constructive interference as

$$2d_{\text{light}} \cos\theta = n\lambda_{dB},$$

where, n is the order of Bragg scattering. This Bragg condition allows momentum transfer only for discrete initial values of atomic momenta i.e,

$$P_0 = \frac{l_0}{2}\hbar k, \tag{2.4.4}$$

where, l_0 is an even integer and l_0 =2, 4 and 6 etc corresponds to first, second and third order Bragg scattering respectively. It is clear, that by changing the angle and longitudinal component of momentum, we can change the order of Bragg scattering. We are using first order Bragg scattering for simplicity.

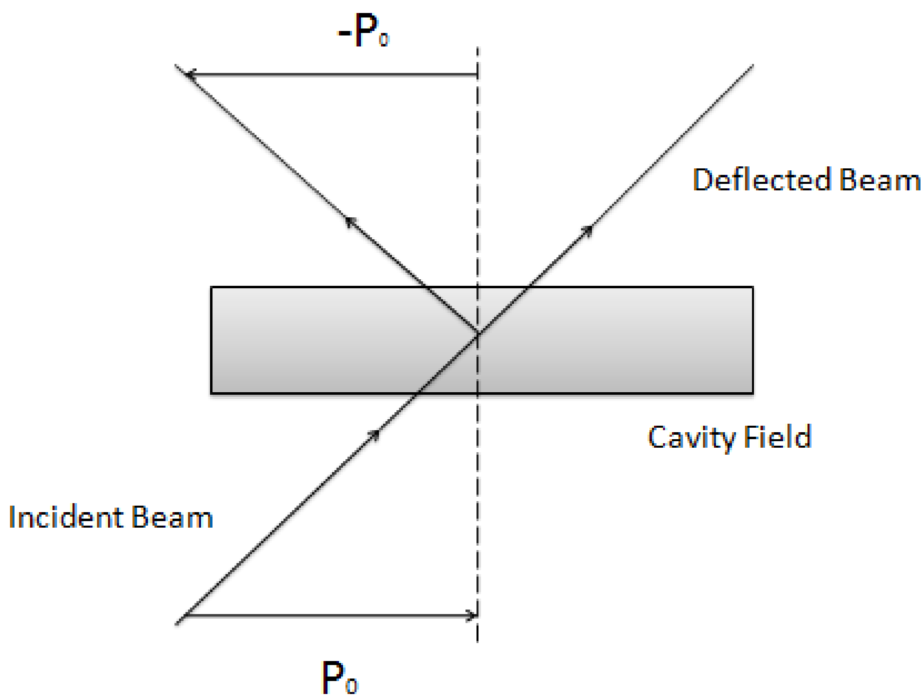

Figure 2.5: Scattering in Bragg Regime: Conservation of energy allows only two possible directions of deflection. Either the momentum component goes un deflected as P_0 or deflects at $-P_0$

2.5 Development of Hamiltonian

In order to calculate the Hamiltonian we take both, the field and probing atom, quantum mechanically. The atom is considered to be of mass M and is completely described by its centre of mass momentum P and position r and its internal states. The energy of free quantized field in terms of creation operator $a^\dagger$ and annihilation operator a is given as

$$H_F = \hbar\nu(a^\dagger a + \frac{1}{2}). \tag{2.5.1}$$

The internal states of two-level atom are labelled as, the ground state $|m\rangle$ with eigen energy E_m and the excited state as $|n\rangle$ with eigen energy E_n.

The atomic Hamiltonian H_A is given by

$$H_A = E_n|n\rangle\ \langle n| + E_m|m\rangle\ \langle m|,$$

It can be written as,

$$H_A = \frac{1}{2}(E_n - E_m)(|n\rangle\ \langle n| - |m\rangle\ \langle m|) + \frac{1}{2}(E_n + E_m)(|n\rangle\ \langle n| + |m\rangle\ \langle m|). \quad (2.5.2)$$

Since we are considering two level atom, we can treat its internal states in close analogy with spin 1/2 particles. Therefore, we adopt the pauli matrices formalism and introduce operators σ_z, σ_+ and σ_- as

$$\sigma_z = |n\rangle\ \langle n| - |m\rangle\ \langle m|,$$

$$\sigma_+ = |n\rangle\ \langle m|,$$

$$\sigma_- = |m\rangle\ \langle n|,$$

where, σ_+, σ_- act as atomic raising and lowering operators, respectively.
Now, using $(E_n - E_m) = \hbar\omega$ and using the above defined operator σ_z, we get

$$H_A = \frac{1}{2}\hbar\omega\sigma_z, \quad (2.5.3)$$

where, we have taken $|n\rangle\ \langle n| + |m\rangle\ \langle m| = 1$ and $(E_n + E_m)$ is constant additive term.

Atom Field Interaction

When an atom of dipole moment -ex_e interacts with an electric field E, during passage through the cavity then the interaction Hamiltonian will be:

$$H_i = -ex_e.E,$$

where, x_e is the polarization vector of electron having charge e. As we have assumed that the electric field is $y-polarized$ and it is propagating in x-direction, so electric field is given by

$$E = \varepsilon(x + x_e)\cos kx(a + a\dagger)e_x,$$

where, ε is the amplitude of the field, $\cos kx$ gives us the profile of the field and e_x is the unit vector along x-axis.
Since we are considering a point like particle, therefore we assume that the profile of the field remains same throughout the dimension of the atom.
Thus using dipole approximation we can write interaction Hamiltonian as

$$H_i = -ex_e\varepsilon(x)\cos kx(a + a^\dagger).$$

By using the identity $|n\rangle\ \langle n| + |m\rangle\ \langle m| = 1$, we get

$$H_i = -e(|n\rangle\ \langle n| + |m\rangle\ \langle m|)r_e(|n\rangle\ \langle n| + |m\rangle\ \langle m|)\varepsilon(r)\cos kx(a + a^\dagger).$$

$$H_i = -(\wp_{nm}\sigma_+ + \wp_{nm}\sigma_-)\varepsilon(r)\cos kx(a + a^\dagger),$$

where, $\wp_{nm} = \wp_{nm}* = e\langle n|r_e|m\rangle$ is the matrix element of the electric dipole moment.
We define Rabi frequency as $g = -\wp_{nm}\varepsilon/\hbar$, and we get

$$H_i = \hbar\cos kx(g\sigma_+ + g*\sigma_-)(a + a^\dagger).$$

Now we can apply the rotating wave approximation by dropping the terms in which energy is not conserved. These terms are $a^\dagger\sigma_+$ and $a\sigma_-$ where the first term shows that the gain of energy while emitting a photon and the 2nd term shows the loss of energy while absorbing a photon which is not feasible. So H_i takes the following form

$$H_i = \hbar\cos kx(g\sigma_+ a + g*a^\dagger\sigma_-). \tag{2.5.4}$$

During this interaction, atom absorbs a photon and goes to a higher state $(\sigma_+ a)$ term and then emits a photon and returns to lower state $(a^\dagger\sigma_-)$ term. This means that atom undergoes Rabi oscillation with frequency g.

2.6 Total Hamiltonian

We can express the complete Hamiltonian by adding field Hamiltonian, atomic Hamiltonian and interaction Hamiltonian, which is given by

$$H = \frac{P^2}{2M} + \hbar\nu a^\dagger a + \frac{1}{2}\hbar\omega\sigma_z + \hbar\cos kx(g\sigma_+ a + g*a^\dagger\sigma_-), \tag{2.6.1}$$

here, the first term is added on account for the centre of mass momentum of the atom.

As we have taken the component normal to the cavity P_y to be very large, therefore it will be treated classically and the component along the cavity will be treated quantum mechanically. So, we can replace P^2 by P_x^2 and the total Hamiltonian becomes,

$$H = \frac{P_x^2}{2M} + \hbar\nu a^\dagger a + \hbar\omega\sigma_z + \hbar\cos kx(g\sigma_+ a + g * a^\dagger\sigma_-). \tag{2.6.2}$$

2.7 Interaction Picture Hamiltonian

Total Hamiltonian can be converted to *interaction picture* Hamiltonian by using the following transformation

$$V = \exp(iH_0t/\hbar)H_1\exp(iH_0t/\hbar), \tag{2.7.1}$$

where, H_0 is the unperturbed and H_1 is the interaction part of the Hamiltonian, H. We can separate H_0 and H_1 from the total Hamiltonian as

$$H_0 = \hbar\nu a^\dagger a + \frac{1}{2}\hbar\omega\sigma_z. \tag{2.7.2}$$

$$H_1 = \frac{P_x^2}{2M} + \hbar\cos kx(g\sigma_+ a + g * a^\dagger\sigma_-). \tag{2.7.3}$$

Thus interaction picture Hamiltonian is given as,

$$H_1 = \frac{P_x^2}{2M} + (\frac{\hbar g}{2}e^{i\Delta t}\sigma_+ a + \frac{\hbar g^*}{2}e^{-i\Delta t}a^\dagger\sigma_-)\cos kx. \tag{2.7.4}$$

2.8 Equations of Motion

In order to calculate the momentum distribution of the exiting atoms we require equations of motion of the probability amplitudes of atoms in momentum space. We can take atom-field wave function as

$$|\psi(t)\rangle = \sum_j\sum_l(C_{P_l}^{n,j}|n,j,P_l\rangle + C_{P_l}^{m,j}|m,j,P_l\rangle), \tag{2.8.1}$$

here, $C_{P_l}^{i,j}$ is the probability amplitude of the atom in state $i = n, m$ with probability amplitude P_l and cavity is having j photons in it. We have considered discrete momentum space.

We can develop the rate equations for the probability amplitude by using the Schrodinger equation

$$i\hbar\frac{\partial|\psi(t)\rangle}{\partial t} = V|\psi(t)\rangle. \tag{2.8.2}$$

Under consideration of large detuning, we assume that atom does not exist in excited state. Due to this reason we take the temporal and spatial change in probability amplitude of excited state to be zero. This is known as **secular approximation** or **adiabatic approximation**. This implies that

$$P_x^2 C_{P_l}^{n,j} = -\hbar^2\frac{\partial^2 C_{P_l}^{n,j}}{\partial x^2} = 0, \tag{2.8.3}$$

and

$$\frac{\partial C_{P_l}^{n,j}}{\partial t} = 0. \tag{2.8.4}$$

As, we know $\langle n|m\rangle = 0$ and $\langle m|n\rangle = 0$ and from equation(2.8.2), we get equations of motion for probability amplitude as:

$$i\hbar\frac{\partial C_{P_l}^{(n,j)}(t)}{\partial t} = \frac{\hbar g*}{2}\sqrt{j+1}e^{-i\triangle t}\big(C_{P_l+\hbar k}^{(m,j+1)}(t) + C_{P_l-\hbar k}^{(m,j+1)}(t)\big), \tag{2.8.5}$$

$$i\hbar\frac{\partial C_{P_l}^{(m,j)}(t)}{\partial t} = \frac{\hbar g}{2}\sqrt{j}e^{-i\triangle t}\big(C_{P_l+\hbar k}^{(n,j-1)}(t) + C_{P_l-\hbar k}^{(n,j-1)}(t) + \frac{P_l^2}{2M}C_{P_l}^{(m,j)}(t)\big), \tag{2.8.6}$$

Now, we take

$$C_0(t) = e^{i\triangle t}C_{P_l}^{n,j}(t), \tag{2.8.7}$$

and using secular approximation on equation(2.8.5) we get,

$$i\hbar\frac{\partial C_0(t)}{\partial t} = \frac{\hbar g*}{2}\sqrt{j+1}\big(C_{P_{l+\hbar k}}^{(m,j+1)}(t) + C_{P_{l-\hbar k}}^{(m,j+1)}(t)\big) - \hbar\triangle C_0(t), \tag{2.8.8}$$

We get,

$$C_0(t) = -\frac{g*}{2\triangle}\sqrt{j+1}\big(C_{P_{l+\hbar k}}^{(m,j+1)}(t) + C_{P_{l-\hbar k}}^{(m,j+1)}(t)\big). \tag{2.8.9}$$

Compare the value of C_0 to equation(2.8.7), and we get the equation for $C_{P_l}^{(n,j)}(t)$:

$$C_{P_l}^{n,j}(t)e^{i\triangle t} = -\frac{g*}{2\triangle}\sqrt{j+1}\big(C_{P_{l+\hbar k}}^{(m,j+1)}(t) + C_{P_{l-\hbar k}}^{(m,j+1)}(t)\big). \tag{2.8.10}$$

Now, replace $j \longrightarrow j-1$ and $P_l \longrightarrow P_{l+\hbar k}$,

$$C_{P_{l+\hbar k}}^{n,j-1}(t)e^{i\triangle t} = -\frac{g*}{2\triangle}\sqrt{j}\big(C_{P_{l+2\hbar k}}^{(m,j)}(t) + C_{P_l}^{(m,j)}(t)\big), \tag{2.8.11}$$

and $j \longrightarrow j-1$ and $P_l \longrightarrow P_{l-\hbar k}$,

$$C_{P_{l-\hbar k}}^{n,j-1}(t)e^{i\triangle t} = -\frac{g*}{2\triangle}\sqrt{j}\big(C_{P_l}^{(m,j)}(t) + C_{P_{l-2\hbar k}}^{(m,j)}(t)\big). \tag{2.8.12}$$

Now, put equation(2.8.11) and (2.8.12) in equation(2.8.6),

$$i\hbar\frac{\partial C_{P_l}^{(m,j)}(t)}{\partial t} = -\frac{\hbar|g|^2 j}{2\triangle}\Big(\frac{C_{P_{l+2\hbar k}}^{(m,j)}(t) + C_{P_{l-2\hbar k}}^{(m,j)}(t)}{2} + C_{P_l}^{(m,j)}(t)\Big) + \frac{P_l^2}{2M}C_{P_l}^{(m,j)}(t). \tag{2.8.13}$$

Re-arranging,

$$i\hbar\frac{\partial C_{P_l}^{(m,j)}(t)}{\partial t} = \Big(-\frac{\hbar|g|^2 j}{2\triangle} + \frac{P_l^2}{2M}\Big)C_{P_l}^{(m,j)}(t) - \frac{\hbar|g|^2 j}{4\triangle}\big(C_{P_{l+2\hbar k}}^{(m,j)}(t) + C_{P_{l-2\hbar k}}^{(m,j)}(t)\big). \tag{2.8.14}$$

We can ignore the first term which is constant additive term and putting the value of P_l from equation(2.3.2), we get

$$i\hbar\frac{\partial C_{P_l}^{(m,j)}(t)}{\partial t} = \frac{\hbar k}{2M}l(l+l_0)C_{P_l}^{(m,j)}(t) - \frac{\hbar|g|^2 j}{4\triangle}\big(C_{P_{l+2\hbar k}}^{(m,j)}(t) + C_{P_{l-2\hbar k}}^{(m,j)}(t)\big). \tag{2.8.15}$$

Now, we can see that the atom is oscillating due to large detuning having Rabi frequency $\frac{|g|^2 j}{2\Delta}$, so, effective Hamiltonian can be written as:

$$H_{\text{eff}} = -\frac{\hbar|g|^2}{2\Delta}j\sigma_-\sigma_+\Big(\frac{e^{-i2kx} + e^{+i2kx}}{2} + 1\Big) + \frac{P_x^2}{2M}, \tag{2.8.16}$$

$$H_{\text{eff}} = -\frac{\hbar|g|^2}{2\Delta}j\sigma_-\sigma_+(\cos 2kx + 1) + \frac{P_x^2}{2M}. \tag{2.8.17}$$

Now, we need to derive the coupled equations for $\pm l_0$. So for this purpose, we take values of l_0 from $l=0$ to $l=-l_0$ and get coupled equations:

$$i\frac{\partial C_{P_0}^{m,j}}{\partial t} = -\frac{|g|^2 j}{4\Delta}\big(\underline{C_{P_2}^{m,j}} + C_{P_{-2}}^{m,j}\big), \tag{2.8.18}$$

$$i\frac{\partial C^{m,j}_{P_{-2}}}{\partial t} = \frac{\hbar k^2}{2M}(-2)(-2+l_0)C^{m,j}_{P_{-2}} - \frac{|g|^2 j}{4\Delta}(C^{m,j}_{P_0} + C^{m,j}_{P_{-4}}), \tag{2.8.19}$$

$$i\frac{\partial C^{m,j}_{P_{-4}}}{\partial t} = \frac{\hbar k^2}{2M}(-4)(-4+l_0)C^{m,j}_{P_{-4}} - \frac{|g|^2 j}{4\Delta}(C^{m,j}_{P_{-2}} + C^{m,j}_{P_{-6}}), \tag{2.8.20}$$

$$\vdots$$

$$i\frac{\partial C^{m,j}_{P_{-l_0+4}}}{\partial t} = \frac{\hbar k^2}{2M}(4)(-l_0+4)C^{m,j}_{P_{-l_0+4}} - \frac{|g|^2 j}{4\Delta}(C^{m,j}_{P_{-l_0+6}} + C^{m,j}_{P_{-l_0+4}}), \tag{2.8.21}$$

$$i\frac{\partial C^{m,j}_{P_{-l_0+2}}}{\partial t} = \frac{\hbar k^2}{2M}(2)(-l_0+2)C^{m,j}_{P_{-l_0+2}} - \frac{|g|^2 j}{4\Delta}(C^{m,j}_{P_{-l_0+4}} + C^{m,j}_{P_{-l_0}}), \tag{2.8.22}$$

$$i\frac{\partial C^{m,j}_{P_{-l_0}}}{\partial t} = -\frac{|g|^2 j}{4\Delta}(C^{m,j}_{P_{-l_0+2}} + \underline{C^{m,j}_{P_{-l_0-2}}}), \tag{2.8.23}$$

here, diagonal terms vanish at $l = 0$ and $l = -2l_0$. The non-diagonal terms dominate in the limit when $\frac{\hbar k^2}{2M}$ is much much larger than $\frac{|g|^2 j}{4\Delta}$. We see that probability is oscillating between two terms $l = 0$ and $l = -l_0$ and outside this probability range, probability is very low. So, we can ignore all the underlined terms. Keeping only the lowest coefficients of $\frac{|g|^2 j}{4\Delta}$, and back substituting the values we get the coupled equations for $l_0 > 1$:

$$i\frac{\partial C^{m,j}_{P_0}}{\partial t} = -\frac{(|g|^2 j/4\Delta)}{\frac{\hbar k^2}{2M}(l_0-2)(2)}C^{m,j}_{P_0} + \frac{(-1)^{l_0/2}(|g|^2 j/4\Delta)^{l_0/2}}{(\frac{\hbar k^2}{2M})^{l_0/2-1}[(l_0-2)(l_0-4)...4.2]^2}C^{m,j}_{P_{-l_0}}, \tag{2.8.24}$$

$$i\frac{\partial C^{m,j}_{P_{-l_0}}}{\partial t} = -\frac{(|g|^2 j/4\Delta)}{\frac{\hbar k^2}{2M}(l_0-2)(2)}C^{m,j}_{P_{-l_0}} + \frac{(-1)^{l_0/2}(|g|^2 j/4\Delta)^{l_0/2}}{(\frac{\hbar k^2}{2M})^{l_0/2-1}[(l_0-2)(l_0-4)...4.2]^2}C^{m,j}_{P_0}, \tag{2.8.25}$$

These two equations are:

$$i\frac{\partial C^{m,j}_{P_0}}{\partial t} = AC^{m,j}_{P_0} - \frac{1}{2}BC^{m,j}_{P_{-l_0}}, \tag{2.8.26}$$

and

$$i\frac{\partial C^{m,j}_{P_{-l_0}}}{\partial t} = AC^{m,j}_{P_{-l_0}} - \frac{1}{2}BC^{m,j}_{P_0}, \tag{2.8.27}$$

These equations are decoupled by Laplace transformation by re-arranging above equations, we get:

$$i\dot{C}^{m,j}_{P_0} + iAC^{m,j}_{P_0} - \frac{i}{2}BC^{m,j}_{P_{-l_0}} = 0, \tag{2.8.28}$$

$$i\dot{C}^{m,j}_{P_{-l_0}} + iAC^{m,j}_{P_{-l_0}} - \frac{i}{2}BC^{m,j}_{P_0} = 0. \tag{2.8.29}$$

Now applying Laplace transformation on equation(2.8.28) and equation(2.8.29)

$$s\bar{C}_{P_0}(s) - C_{P_0}(0) + iA\bar{C}_{P_0}(s) - \frac{i}{2}B\bar{C}_{P_{-l_0}}(s) = 0,$$

$$i\bar{C}_{P_{-l_0}}(s) - C_{P_{-l_0}}(0) + iA\bar{C}_{P_{-l_0}}(s) - \frac{i}{2}B\bar{C}_{P_0}(s) = 0$$

Re-arranging above equations, we get

$$\bar{C}_{P_0}(s) = \frac{iB}{2}\frac{1}{s+iA}\bar{C}_{P_{-l_0}}(s) + \frac{1}{s+iA}C_{P_0}(0), \tag{2.8.30}$$

$$\bar{C}_{P_{-l_0}}(s) = \frac{iB}{2}\frac{1}{s+iA}\bar{C}_{P_0}(s) + \frac{1}{s+iA}C_{P_{-l_0}}(0). \tag{2.8.31}$$

Now, put the value of $\bar{C}_{P_{-l_0}}(s)$ from equation(2.8.31) in equation(2.8.30) and re-arranging equation we get:

$$\bar{C}_{P_0}(s) = \frac{iB}{2}\frac{1}{(s+iA)^2 + B^2/4}C_{P_{-l_0}}(0) + \frac{s+iA}{(s+iA)^2 + B^2/4}C_{P_0}(0). \tag{2.8.32}$$

Applying inverse laplace transform

$$C_{P_0}(t) = e^{-iAt}[\cos(1/2Bt)C_{P_0}(0) + i\sin(1/2Bt)C_{P_{-l_0}}(0)]. \tag{2.8.33}$$

Now, putting value of $\bar{C}_{P_0}(s)$ from equation(2.8.30) in equation(2.8.31) we get:

$$\bar{C}_{P_{-l_0}}(s) = \frac{iB}{2}\frac{1}{(s+iA)}\left(\frac{1}{1+\frac{B^2/4}{(s+iA)^2}}\right)C_{P_0}(0) + \frac{1}{(s+iA)}\left(\frac{1}{1+\frac{B^2/4}{(s+iA)^2}}\right)C_{P_{-l_0}}(0). \tag{2.8.34}$$

Now, applying inverse laplace transform

$$C_{P_{-l_0}}(t) = e^{-iAt}[\cos(1/2Bt)C_{P_{-l_0}}(0) + i\sin(1/2Bt)C_{P_0}(0)]. \tag{2.8.35}$$

Equation(2.8.33) and equation(2.8.35) can be written as:

$$C^{(m)}_{j,+l_0} = e^{-iA_jt}[C^{(m)}_{j,+l_0}(0)\cos(\frac{1}{2}B_jt) + iC^{(m)}_{j,-l_0}(0)\sin(\frac{1}{2}B_jt)], \tag{2.8.36}$$

$$C^{(m)}_{j,-l_0} = e^{-iA_jt}[C^{(m)}_{j,-l_0}(0)\cos(\frac{1}{2}B_jt) + iC^{(m)}_{j,+l_0}(0)\sin(\frac{1}{2}B_jt)], \tag{2.8.37}$$

where,

$$A_j \equiv \begin{cases} -\frac{(|g|^2 j/4\Delta)^2}{\omega_{rec}(l_0-2)(2)} & For l_0 \neq 2, \\ 0 & l_0 = 2, \end{cases} \tag{2.8.38}$$

and

$$B_j \equiv \begin{cases} -\frac{(|g|^2 j/2\Delta)^{l_0/2}}{2\omega_{rec}^{(l_0/2-1)}[(l_0-2)(l_0-4)...(4.2)]} & For l_0 \neq 2 \\ |g|^2 j/2\Delta & l_0 = 2. \end{cases} \tag{2.8.39}$$

So, probability of exiting atom flips with a cosine function[7].

Chapter 3

Entangling Distant Cavities By Entanglement Swapping

In this chapter, we study the entanglement of cavities and atoms through the beam splitter action and the formation of EPR pairs. The detection of these four EPR states will throw these EPR states into one of the corresponding four Bell states.
In Sec. 3.1, Bell states are discussed. Sec. 3.2, explains the Bell theorem. In Sec. 3.3, Bell state measurement and in Sec. 3.4, teleportation is discussed. In Sec. 3.5, entanglement of cavities is discussed. In Sec. 3.6, we apply the Bell-state measurement upon the atoms exiting from these entangling cavities. When the Bell-state measurement is done, then the cavities that are non-interacting are also entangled. In Sec. 3.7, we tell how these exiting atoms from the beam-splitters are detected at the detectors. In the detection process, we get one of the four Bell states at the detectors.

3.1 Bell States

There are four Bell states which are maximally entangled. These are also known as EPR pairs. These are given as

$$|\phi^{+}\rangle = \frac{1}{\sqrt{2}}|00\rangle + |11\rangle$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}|00\rangle - |11\rangle$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}|01\rangle + |10\rangle,$$

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}|01\rangle - |10\rangle,$$

The most important property that entangled state has is the correlation, such that measurement on the two qubits are correlated.

3.2 Bell Theorem

For two observables a and b which have average values given by

$$\langle ab\rangle = \langle a'b\rangle = \langle ab'\rangle = \frac{1}{\sqrt{2}},$$

$$\langle a'b'\rangle = -\frac{1}{\sqrt{2}},$$

Bell-inequality is given as

$$\langle ab\rangle + \langle a'b\rangle + \langle ab'\rangle - \langle a'b'\rangle = 2\sqrt{2}, \tag{3.2.1}$$

Bell state violates the Bell-inequality maximally. For example, for Bell state $|\psi_-\rangle$, we can show that the Bell inequality is violated. For this purpose, we take four observables a, a', b and b'. These observables can take the following values:

$$a = \overrightarrow{\sigma}_A.\hat{a}., a' = \overrightarrow{\sigma}_A.\hat{a}'.$$

$$b = \overrightarrow{\sigma}_B.\hat{b}., b' = \overrightarrow{\sigma}_B.\hat{b}'.$$

They have eigen values ± 1. Now, for $|\psi_-\rangle$ state

$$\langle ab\rangle = \langle\psi_-|ab|\psi_-\rangle,$$

Applying operators $\overrightarrow{\sigma}_x^B$ and $\overrightarrow{\sigma}_x^A$ on $|\psi_-\rangle$

$$|\psi_-\rangle = \frac{1}{\sqrt{2}}[|0_A 1_B\rangle - |1_A 0_B\rangle],$$

$$\overrightarrow{\sigma}_x^B \frac{1}{\sqrt{2}}[|0_A 1_B\rangle - |1_A 0_B\rangle] = \frac{1}{\sqrt{2}}[|00\rangle - |11\rangle],$$

$$\overrightarrow{\sigma}_x^A \frac{1}{\sqrt{2}}[|0_A 1_B\rangle - |1_A 0_B\rangle] = \frac{1}{\sqrt{2}}[|11\rangle - |00\rangle],$$

Operating $\overrightarrow{\sigma}_x^B$ on $|\psi_-\rangle$ is as if we are applying $\overrightarrow{-\sigma}_x^A$. So,

$$\langle ab\rangle = \langle\psi_-|(\overrightarrow{\sigma}^A.a)(\overrightarrow{\sigma}^B.b)|\psi_-\rangle$$

$$= \langle\psi_-|\Sigma_i \overrightarrow{\sigma}_i^A.\hat{a}_i \Sigma_j \overrightarrow{\sigma}_j^B.\hat{b_j}|\psi_-\rangle, \tag{3.2.2}$$

$$= \Sigma_i \Sigma_j a_i b_j \langle\psi_- \overrightarrow{\sigma}_i^A \overrightarrow{\sigma}_j^B|\psi_-\rangle$$

Putting the value of $\overrightarrow{\sigma}^A$

$$\langle ab\rangle = -\Sigma_i \Sigma_j a_i b_j \langle\psi_- \overrightarrow{\sigma}_i^A \overrightarrow{\sigma}_j^A|\psi_-\rangle,$$

$$\langle ab\rangle = -\Sigma_i \Sigma_j a_i b_j Tr_A(\overrightarrow{\sigma}_i^A \overrightarrow{\sigma}_j^A \rho_A)$$

From,

$$\rho_A = \frac{1}{2}[|0\rangle\langle 0| + |1\rangle\langle 1|] = \frac{1}{2}I,$$

So,

$$\langle ab\rangle = -\frac{1}{2}\Sigma_i \Sigma_j a_i b_j Tr_A(\overrightarrow{\sigma}_i^A \overrightarrow{\sigma}_j^A),$$

$$= -\Sigma_i \Sigma_j a_i b_j \delta_{ij} = -\Sigma_j a_j b_j = -\hat{a}.\hat{b},$$

$$= \cos(\theta_1 - \theta_2) = -\cos\theta_{ab},$$

$$\langle C\rangle = \langle ab\rangle + \langle a'b\rangle + \langle ab'\rangle - \langle a'b'\rangle,$$

The angle between the four observables a, a', b and b' is $\pi/4$. So the equation becomes

$$\langle C\rangle = -\cos\frac{\pi}{4} - \cos\frac{\pi}{4} - \cos\frac{\pi}{4} + \cos\frac{3\pi}{4},$$

$$= -4/\sqrt{2},$$

Now, taking the mod

$$|\langle C\rangle| = 2\sqrt{2} \leq 2,$$

So, the Bell inequality is violated by Bell states.
The entangled states which have the reduced density matrix equal to $1/2I(identity)$ are called maximally-entangled states. Bell states are maximally-entangled states. There are also states which are entangled but not maximally entangled. If we have the state $|\psi\rangle_{AB} = \frac{1}{\sqrt{3}}(|11\rangle + |12\rangle + |21\rangle)$, then

$$\rho_{AB} = \frac{1}{3}(|11\rangle\langle 11| + |11\rangle\langle 12| + |11\rangle\langle 21| + |12\rangle\langle 11| \\ + |12\rangle\langle 12| + |12\rangle\langle 21| + |21\rangle\langle 11| + |21\rangle\langle 12| + |21\rangle\langle 21|)$$

Then we find that,

$$\rho_A = \frac{2}{3}|1\rangle\langle 1| + \frac{1}{3})|1\rangle\langle 2| + \frac{1}{3})|2\rangle\langle 1| + \frac{1}{3})|2\rangle\langle 2|.$$

This state is entangled but not maximally entangled, the density matrix is not equal to $1/2I$.

3.3 Bell-State Measurement

Bell-state measurement is defined to be the projection of two qubits on maximally-entangled Bell states. It is main operation in many quantum processes like teleportation and entanglement swapping. There are many types of Bell-state measurements.

Here we discuss the linear optical method of Bell-state measurement. In this process, we are able to distinguish between two symmetric Bell states $|\psi^{+}\rangle$ and $|\psi^{-}\rangle$ but we cannot distinguish between the anti-symmetric Bell states $|\phi^{+}\rangle$ and $|\phi^{-}\rangle$ through the detectors. That is why it is called $50-50$ beam splitter action.
In this technique, two photons are incident (as shown in Fig below) upon $50-50$ beam splitter and they are detected at the corresponding (single-photon) detectors [8].

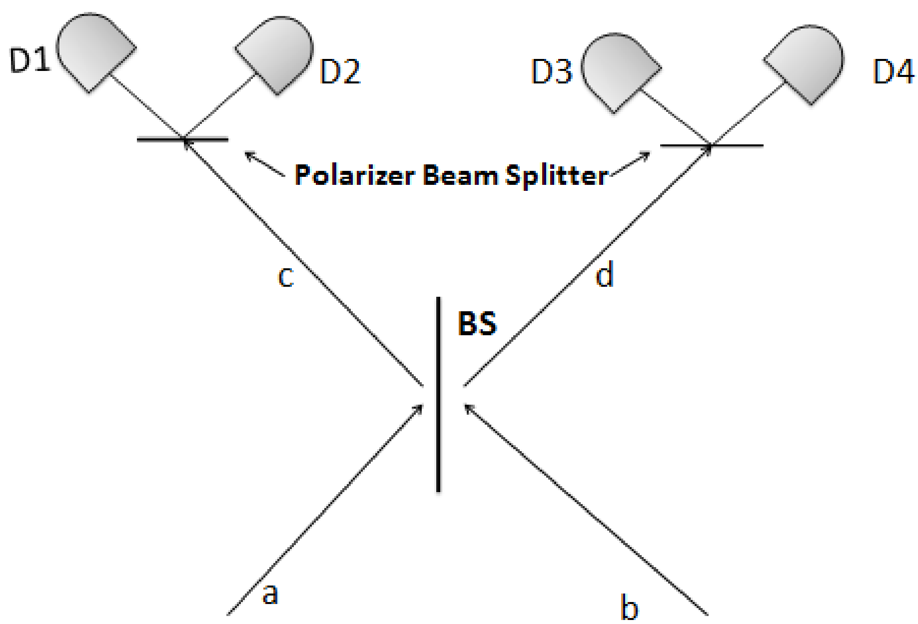

Figure 3.1: Partial Bell-state analyzer

In Fig. 3.1 one photon comes from mode a and other comes from mode b. Only if the two photons are in the antisymmetric Bell state $|\psi_{-}\rangle$, there will be one photon in each output mode of the first beam splitter (BS), c and d. Therefore a coincidence detection between D1 and D3 or D2 and D4 identifies this state. D4 identifies this state. If the photons are in the state $|\psi_{+}\rangle$, they both go into c or both into d and are then split by the subsequent polarizing beam splitters (PBS), because they have orthogonal polarizations. Therefore coincidences between D1 and D2 or between D3 and D4 signify a state $|\psi_{+}\rangle$. For the two other Bell states $|\psi_{-}\rangle$ and $|\phi_{+}\rangle$ both photons go to the same detector.
Beam splitter action is defined as

$$a_1^{\dagger} = \frac{1}{\sqrt{2}}(a_1^{\dagger} + ia_2^{\dagger}),$$

where a and $a^{\dagger}$ are the ladder operators.

$$a_1^{\dagger} = \frac{1}{\sqrt{2}}(ia_1^{\dagger} + a_2^{\dagger}),$$

$a^{\dagger}$ replaced by $h^{\dagger}$ and $v^{\dagger}$ for horizontal and vertical polarization respectively. Under the above transformation, the four Bell-states evolve as

$$|\psi^{+}\rangle = \frac{1}{\sqrt{2}}[h_1^{\dagger}v_2^{\dagger} + h_2^{\dagger}v_1^{\dagger}]|0\rangle \rightarrow \frac{i}{\sqrt{2}}[h_1^{\dagger}v_2^{\dagger} + h_2^{\dagger}v_1^{\dagger}]|0\rangle,$$

$$|\psi^{-}\rangle = \frac{1}{\sqrt{2}}[h_1^{\dagger}v_2^{\dagger} - h_2^{\dagger}v_1^{\dagger}]|0\rangle \rightarrow \frac{i}{\sqrt{2}}[h_1^{\dagger}v_2^{\dagger} - h_2^{\dagger}v_1^{\dagger}]|0\rangle,$$

$$|\phi^{+}\rangle = \frac{1}{\sqrt{2}}[h_1^{\dagger}h_2^{\dagger} + v_1^{\dagger}v_2^{\dagger}]|0\rangle \rightarrow \frac{1}{\sqrt{2}}[h_1^{\dagger 2} + h_2^{\dagger 2} + v_1^{\dagger 2} + v_2^{\dagger 2}]|0\rangle,$$

$$|\phi^{-}\rangle = \frac{1}{\sqrt{2}}[h_1^{\dagger}h_2^{\dagger} - v_1^{\dagger}v_2^{\dagger}]|0\rangle \rightarrow \frac{1}{\sqrt{2}}[h_1^{\dagger 2} + h_2^{\dagger 2} - v_1^{\dagger 2} - v_2^{\dagger 2}]|0\rangle,$$

where, 1, 2 stands for the 1st and 2nd qubit respectively.
For $|\psi^{+}\rangle$ and $|\psi^{-}\rangle$, we get two different results at the output but for $|\phi^{+}\rangle$ and $|\phi^{-}\rangle$ we get the same results but with a phase difference of $\pi/2$. For $|\psi^{+}\rangle$ both photons reach at the same detector but with different polarizations while for $|\psi^{-}\rangle$

one photon reaches at each detector. For $|\phi^+\rangle$ and $|\phi^-\rangle$, both photons reaches at the same detectors with same polarizations. This Bell-state measurement can distinguish between $|\psi^+\rangle$ and $|\psi^-\rangle$ while for $|\phi^+\rangle$ or $|\phi^-\rangle$ it gives degenerate results i.e. we get the results with 50 % probability [8].

3.4 Entanglement of Cavities

We now give the procedure to generate entanglement between two distant cavities. This is done by entanglement swapping procedure explained in Sec. 1.5. Here entanglement is swapped from that between two pairs of atomic momentum state and cavity filed state to that between cavities by making EPR state measurement on the two atomic momentum states.

We can write the wave function for two- atom field pairs as

$$|\psi(t)\rangle = [\frac{1}{\sqrt{2}}(|0_1, P_{l_0}^{(1)}\rangle + ie^{-i\phi}|1_1, P_{-l_0}^{(1)}\rangle)] \otimes [\frac{1}{\sqrt{2}}(|0_2, P_{l_0}^{(2)}\rangle + ie^{-i\phi}|1_2, P_{-l_0}^{(2)}\rangle)], \quad (3.4.1)$$

where, $\phi = r\pi A_1/B_1$. Here A_1 and B_1 are the constants.

Atoms in their external degrees of freedom get entangled with their respective cavity fields. This entanglement is created when the atoms with some initial momentum P_{l_0} pass through the cavities and get deflected after passing through the cavities in two possible directions having atomic momenta P_{l_0} and P_{-l_0}. Then the combined wave function can be written as:

$$|\psi(t)\rangle = \frac{1}{2}(|0_1, 0_2, P_{l_0}^{(1)}, P_{l_0}^{(2)}\rangle + ie^{-i\phi}|0_1, 1_2, P_{l_0}^{(1)}, P_{-l_0}^{(2)}\rangle + ie^{-i\phi}|1_1, 0_2, P_{-l_0}^{(1)}, P_{l_0}^{(2)}\rangle - ie^{-2i\phi}|1_1, 1_2, P_{-l_0}^{(1)}, P_{-l_0}^{(2)}\rangle). \quad (3.4.2)$$

We want to project the state into one of the four Bell states. For this purpose, we add and subtract some terms. Adding $\frac{1}{2}(|0_1, 0_2, P_{l_0}^{(1)}, P_{l_0}^{(2)}\rangle + ie^{-2i\phi}|1_1, 1_2, P_{-l_0}^{(1)}, P_{-l_0}^{(2)}\rangle)$, and $\frac{1}{2}(ie^{-i\phi}|0_1, 1_2, P^{(1)}_{l_0}, P^{(2)}_{-l_0}\rangle + ie^{-i\phi}|1_1, 0_2, P^{(1)}_{-l_0}, P^{(2)}_{l_0}\rangle)$, we get

$$
\begin{aligned}
|\psi(t)\rangle = \frac{1}{2}(&|0_1, 0_2, P^{(1)}_{l_0}, P^{(2)}_{l_0}\rangle + ie^{-i\phi}|0_1, 1_2, P^{(1)}_{l_0}, P^{(2)}_{-l_0}\rangle \\
&+ ie^{-i\phi}|1_1, 0_2, P^{(1)}_{-l_0}, P^{(2)}_{l_0}\rangle - ie^{-2i\phi}|1_1, 1_2, P^{(1)}_{-l_0}, P^{(2)}_{-l_0}\rangle \\
&+ |0_1, 0_2, P^{(1)}_{l_0}, P^{(2)}_{l_0}\rangle + ie^{-2i\phi}|1_1, 1_2, P^{(1)}_{-l_0}, P^{(2)}_{-l_0}\rangle \\
&+ ie^{-i\phi}|0_1, 1_2, P^{(1)}_{l_0}, P^{(2)}_{-l_0}\rangle + ie^{-i\phi}|1_1, 0_2, P^{(1)}_{-l_0}, P^{(2)}_{l_0}\rangle). \quad (3.4.3)
\end{aligned}
$$

Now, subtracting $\frac{3}{4}(|0_1, 0_2, P^{(1)}_{l_0}, P^{(2)}_{l_0}\rangle + ie^{-2i\phi}|1_1, 1_2, P^{(1)}_{-l_0}, P^{(2)}_{-l_0}\rangle)$ and
$\frac{3}{4}(ie^{-i\phi}|0_1, 1_2, P^{(1)}_{l_0}, P^{(2)}_{-l_0}\rangle + ie^{-i\phi}|1_1, 0_2, P^{(1)}_{-l_0}, P^{(2)}_{l_0}\rangle)$,
we get:

$$
\begin{aligned}
|\psi(t)\rangle = &\frac{1}{4}(P^{(1)}_{l_0}, P^{(2)}_{l_0}\rangle + e^{-i2\phi}|P^{(1)}_{-l_0}, P^{(2)}_{-l_0}\rangle)(|00\rangle - |11\rangle) \\
&+ \frac{1}{4}(P^{(1)}_{l_0}, P^{(2)}_{l_0}\rangle - e^{-i2\phi}|P^{(1)}_{-l_0}, P^{(2)}_{-l_0}\rangle)(|00\rangle + |11\rangle) \\
&+ \frac{1}{4}ie^{-i\phi}(|P^{(1)}_{l_0}, P^{(2)}_{-l_0}\rangle + P^{(1)}_{-l_0}, P^{(2)}_{l_0}\rangle)(|01\rangle + |10\rangle) \\
&+ \frac{1}{4}ie^{-i\phi}(|P^{(1)}_{l_0}, P^{(2)}_{-l_0}\rangle - |P^{(1)}_{-l_0}, P^{(2)}_{l_0}\rangle)(|01\rangle - |10\rangle). \quad (3.4.4)
\end{aligned}
$$

Thus, we have entangled the cavities and atoms in separate four EPR pairs, and four states are also entangled with each other. By measuring the atoms in one of the four EPR states projects two cavities in corresponding Bell state.
Here, in Fig. 3.2 C1 and C2 stands for the cavity 1 and cavity 2. Atoms with initial momentum $|P^{(i)}_{l_0}\rangle$ interacts with the cavity which is already in superposition photon state of zero and one. The interaction time is adjusted such that when the cavity is in zero superposition state atom does not get deflected and has the same momentum $|P^{(i)}_{l_0}\rangle$ as was the initial momentum. For one photon state, atom deflects and has momentum state $|P^{(i)}_{-l_0}\rangle$.

3.5 EPR State Measurement

Our main purpose is to create entanglement between two distant cavities. For this purpose, we do EPR state measurement on atomic momentum states corresponding with the cavities. By doing EPR state measurement on atomic momentum states,

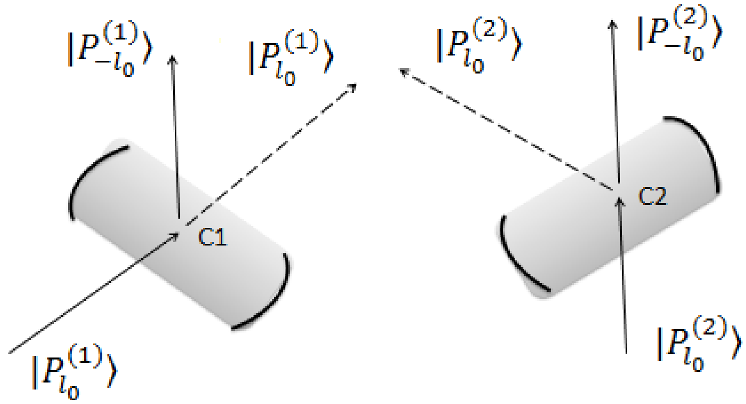

Figure 3.2: Showing dispersive interaction of atoms with the cavity field.

the field states are collapsed into one of the EPR states.

When two level atom in its ground state interacts off-resonantly with the cavity (which is in the superposition state $(|0\rangle + |1\rangle)/\sqrt{2}$) the atomic momentum states associated with the atom are entangled to the cavities. The superposition in cavity is created by passing the two-level atom in excited state for half of the Rabi cycle. Due to this interaction the atomic state is transferred to the cavity and the cavity comes into the superposition state of zero and one photon. After interacting with the cavity, the atom exits in one of the two equally probable discrete momentum states $|P_{l_0}\rangle$ and $|P_{-l_0}\rangle$. The resultant state is the entangled state of atom and cavity. After passing through the cavity these momentum states are passed through the beam splitters.

For EPR state measurement, we have two beam splitters, BS1 and BS2. The two beam splitters are prepared in superposition of zero and one photon as $(|0\rangle+|1\rangle)/\sqrt{2}$. The four incoming components from the cavities pass through the beam splitters. $|P^1_{l_0}\rangle$ and $|P^2_{l_0}\rangle$ pass through the beam splitter BS2 and $|P^1_{-l_0}\rangle$ and $|P^2_{-l_0}\rangle$ pass through the beam splitter BS1 as shown in Fig. 3.4. They can be deflected to desired cavities by using mirrors. The dispersive interaction of atom with the cavity beam splitter for an interaction time $t = 2\pi\delta'/|g'|^2$, transfers atomic momentum states into superposition, where, δ' is the atom-beam splitter field detuning and g' is the vacuum rabi-frequency [9].

Atoms and the beam splitter cavities are adjusted such that there is first order Bragg-scattering of atoms. Beam splitter action is performed as follows:

$$|P_{l_0}^{(1)}\rangle \longrightarrow |P_{l_0}^{(1)}\rangle + i|P_{l_0}^{(2)}\rangle,$$

$$|P_{l_0}^{(2)}\rangle \longrightarrow i|P_{l_0}^{(1)}\rangle + |P_{l_0}^{(2)}\rangle,$$

$$|P_{-l_0}^{(1)}\rangle \longrightarrow |P_{-l_0}^{(1)}\rangle + i|P_{-l_0}^{(2)}\rangle,$$

$$|P_{-l_0}^{(2)}\rangle \longrightarrow i|P_{-l_0}^{(1)}\rangle + |P_{-l_0}^{(2)}\rangle.$$

When the beam splitter action is performed, the first factor of equation(3.4.4) goes like this:

$$|P_{l_0}^{(1)}, P_{l_0}^{(2)}\rangle + e^{-i2\phi}|P_{-l_0}^{(1)}, P_{-l_0}^{(2)}\rangle \longrightarrow (|P_{l_0}^{(1)}\rangle + i|P_{l_0}^{(2)}\rangle, |P_{-l_0}^{(1)}\rangle + i|P_{-l_0}^{(2)}\rangle),$$

$$+e^{-i2\phi}(|P_{-l_0}^{(1)}\rangle + i|P_{-l_0}^{(2)}\rangle, i|P_{-l_0}^{(1)}\rangle + |P_{-l_0}^{(2)}\rangle),$$

$$= i(|P_{l_0}^{(1)}, P_{l_0}^{(1)}\rangle + |P_{l_0}^{(2)}, P_{l_0}^{(2)}\rangle + e^{-i2\phi}|P_{-l_0}^{(1)}, P_{-l_0}^{(1)}\rangle + e^{-i2\phi}|P_{-l_0}^{(2)}, P_{-l_0}^{(2)}\rangle). \quad (3.5.1)$$

Similarly, the first factor of second term after action of beam splitter transforms as:

$$P_{l_0}^{(1)}, P_{l_0}^{(2)}\rangle - e^{-i2\phi}|P_{-l_0}^{(1)}, P_{-l_0}^{(2)}\rangle \longrightarrow (|P_{l_0}^{(1)}\rangle + i|P_{l_0}^{(2)}\rangle, i|P_{l_0}^{(1)}\rangle + |P_{l_0}^{(2)}\rangle), \quad (3.5.2)$$

$$+e^{-i2\phi}(|P_{-l_0}^{(1)}\rangle + i|P_{-l_0}^{(2)}\rangle, i|P_{-l_0}^{(1)}\rangle + |P_{-l_0}^{(2)}\rangle),$$

$$= i(|P_{l_0}^{(1)}, P_{l_0}^{(1)}\rangle + |P_{l_0}^{(2)}, P_{-l_0}^{(2)}\rangle - e^{-i2\phi}|P_{-l_0}^{(1)}, P_{-l_0}^{(1)}\rangle - e^{-i2\phi}|P_{-l_0}^{(2)}, P_{-l_0}^{(2)}\rangle). \quad (3.5.3)$$

Then the third term of equation(3.4.4) is:

$$|P_{l_0}^{(1)}, P_{-l_0}^{(2)}\rangle + P_{-l_0}^{(1)}, P_{l_0}^{(2)}\rangle \longrightarrow (|P_{l_0}^{(1)}\rangle + i|P_{l_0}^{(2)}\rangle, i|P_{-l_0}^{(1)}\rangle + |P_{-l_0}^{(2)}\rangle),$$

$$+(i|P^{(1)}_{l_0}\rangle + |P^{(2)}_{l_0}\rangle, |P^{(1)}_{-l_0}\rangle + i|P^{(2)}_{-l_0}\rangle),$$

$$= 2i(|P^{(1)}_{l_0}, P^{(1)}_{-l_0}\rangle - |P^{(2)}_{l_0}, P^{(2)}_{-l_0}\rangle. \qquad (3.5.4)$$

And then the fourth term transforms as

$$|P^{(1)}_{l_0}, P^{(2)}_{-l_0}\rangle - |P^{(1)}_{-l_0}, P^{(2)}_{l_0}\rangle \longrightarrow (|P^{(1)}_{l_0}\rangle + i|P^{(2)}_{l_0}\rangle, i|P^{(1)}_{-l_0}\rangle + |P^{(2)}_{-l_0}\rangle),$$

$$-(|P^{(1)}_{-l_0}\rangle + i|P^{(2)}_{-l_0}\rangle, i|P^{(1)}_{l_0}\rangle + |P^{(2)}_{l_0}\rangle),$$

$$= 2(|P^{(1)}_{l_0}, P^{(2)}_{-l_0}\rangle - |P^{(1)}_{-l_0}, P^{(2)}_{l_0}\rangle). \qquad (3.5.5)$$

The interaction time of atoms with the cavities acting as a beam splitter can be controlled by using velocity selector.

Velocity selector is a device that is used to separate the particles according to the charges upon them. Only the particles which have the correct speed are deflected through the velocity selector while others remain be un-deflected. A velocity selector is a region in which there is a uniform electric and magnetic field. These two fields are mutually perpendicular and are also perpendicular to the velocity of the incoming particles.

Force exerted on a charged particle by electric field is given by:

$$F = qE$$

The magnitude of the force exerted by the magnetic field for the perpendicular velocity is given by:

$$F = qvB.$$

The net force is zero when two forces are equal and opposite, and particle passes undeflected through the region. As magnetic force is dependent on speed, the particles having speed less or more than the applied magnetic force will be deflected in one direction or the other [10].

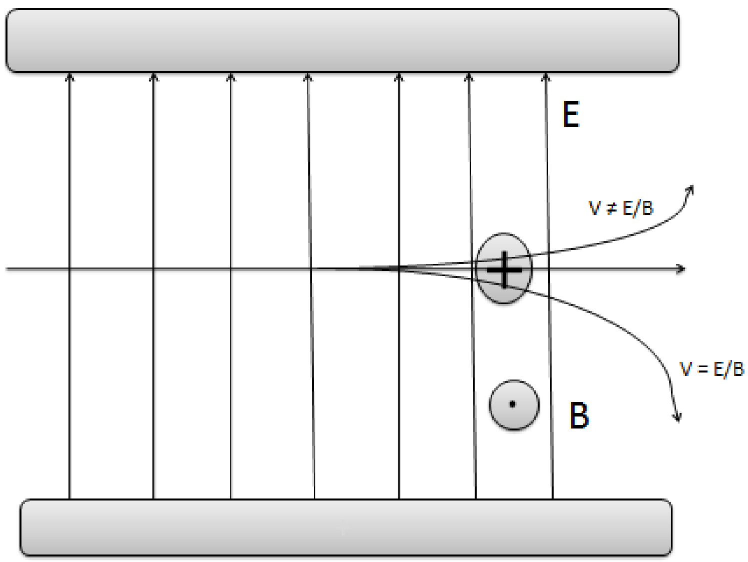

Figure 3.3: Velocity Selector: E is the electric field and B is the magnetic field. + is the in-ward direction of field.

3.6 Detection

For the detection purpose, we use four detectors D1, D2, D3 and D4 to detect the direction of atomic momentum components. These detectors are placed in spatial paths of atoms in different directions of propagation of atoms, which then corresponds to different momenta [11].

Whenever a detector clicks, it detects an atom in that direction and the atomic momentum associated with that atom. Detectors can be used to detect the fast moving atoms like Rydberg atoms ^{85}Rb. For that purpose, cavity is used in two linear orthogonal polarizations H and V. A weak magnetic field is set parallel to the incident polarization. When Rb atoms pass through the cavity then during π transitions the cavity mode is transversed to the atoms. The excited atoms can come in ground state in two possibilities. Either the excited atom can come into ground state by emitting the same polarization light H during stimulated emission and preserving the ground state number $\delta m = 0$, or by emitting light during spontaneous emission in circular polarization, where the ground number is changed to $\delta m = \pm 1$ and is collected by the polarization V. The presence of atom is identified by the detection of light [12].

For different atomic momenta different detectors give clicks. When D_1 detector detects a click then it detects atomic momentum $|P^{(2)}_{-l_0}\rangle$. When D_2 detector give a click then it detects $|P^{(1)}_{-l_0}\rangle$. Similarly, when D_3 gives a click it detects $|P^{(2)}_{l_0}\rangle$ and D_4 gives click for $|P^{(1)}_{l_0}\rangle$ atomic direction as shown in Fig. 3.4.

Below is the Fig. 3.4 which shows the EPR state measurement of atomic momenta. In which atoms are passed through the beam splitters BS1 and BS2 and then these are detected at the detectors D1, D2, D3 and D4. The undeflected atoms pass through the beam splitter BS1 while the deflected atoms pass through the BS2. Beam splitters are cavities prepared in superposition state of zero and one photon. These cavities get entangled when their atomic momenta states are detected.

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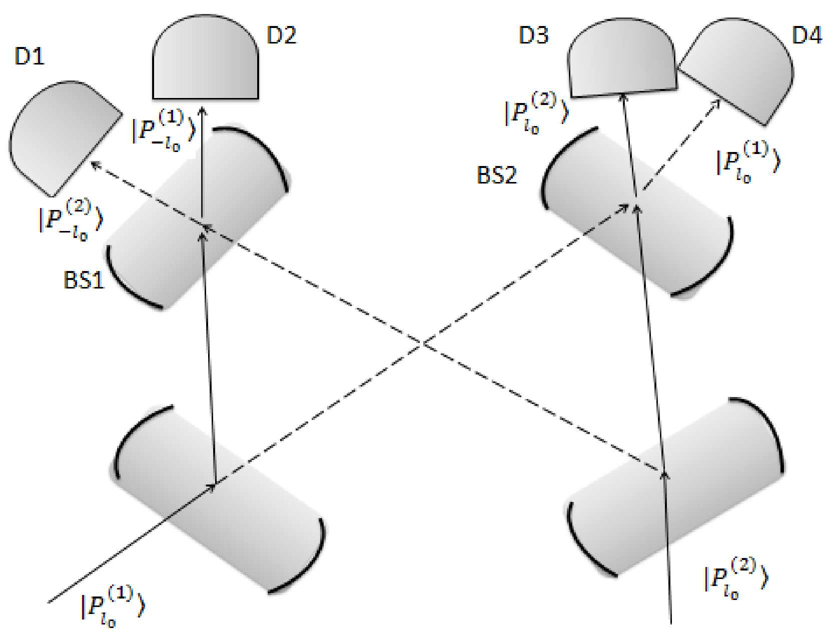

Figure 3.4: EPR state measurement

Now, combining equation(3.4.4) with equation(3.5.1-3.5.4) we get the following table for detector clicks.

Cavities State	Detectors Click
$\vert\phi^+\rangle = \frac{1}{\sqrt{2}}\vert 00\rangle + \vert 11\rangle$	Two clicks at one of the four detectors
$\vert\phi^-\rangle = \frac{1}{\sqrt{2}}\vert 00\rangle - \vert 11\rangle$	Two clicks at one of the four detectors
$\vert\psi^+\rangle = \frac{1}{\sqrt{2}}\vert 01\rangle + \vert 10\rangle$	Either we get a click at D1 and D4 or at D2 and D3
$\vert\psi^-\rangle = \frac{1}{\sqrt{2}}\vert 01\rangle - \vert 10\rangle$	Either we get a click at D2 and D4 or at D1 and D3

Table 3.1: Detector clicks of two entangled cavities

By getting different clicks on each detector, we can distinguish between four states $|\phi^+\rangle$, $|\phi^-\rangle$, $|\psi^+\rangle$ and $|\psi^-\rangle$. After the detection, the cavities are in the Bell state. With this procedure, we can distinguish only between $|\phi^+\rangle$ and $|\phi^-\rangle$, while $|\psi^+\rangle$ and $|\psi^-\rangle$ cannot be distinguished as happens in linear optical Bell-state measurement[13]. The scheme of entangling cavities through beam splitter possesses strong non-locality. There is a procedure in which high fidelity can be achieved by using microwave cavity QED which have lifetime up to seconds. In this scheme, light atoms such as He is used,which has a mass of 6.64×10^{-27} Kg and emission line at wave length $l = 543.5nm$. Chosen temperature is 230 K. The recoil energy is $\hbar k^2/2M = 1.06$MHz. For detuning 6.28 GHz the effective Rabi frequency is approximately 208 KHz. The interaction time with one cavity is approximately 8.3 ms. The total interaction time for 20 atoms comes as 166 μs. Under first order Bragg diffraction, 15-20 helium atoms are passed through the cavity and high fidelity is achieved as in given [14].

Chapter 4

Conclusion

We have reviewed atom field interaction. This review work also includes the method of entanglement for two non-interacting cavities at a distance. For this process, we have seen the entangling of the cavity fields with the atomic momenta in their external degrees of freedom. Then we have reviewed the EPR state measurement to entangle these cavities by passing them through the beam splitters, which are also the cavities in superposition state of zero and one. Then these atoms are detected at the detectors and give clicks corresponding to the state of atom.

The atom-field interaction from the cavity field in Bragg regime is thoroughly studied. The behavior of atoms is analyzed by sending them in ground state into the cavity. Then from equations of motion we reviewed the momentum variation of atoms after passing through the field.

We have studied the process of entanglement of atoms and the cavities. The atoms and cavities are first entangled, then the EPR pairs thus formed are passed through the EPR state analyzer. This leads to entanglement swapping and entangles the two cavities. This process can be further used for the entanglement of N distant cavities.

Bibliography

[1] G. Benenti, G. Casati, G. Strini, *Principles of Quantum Computation and Information*, Vol.II; World Scientific Publishing Co. Pte. Ltd, (2007).

[2] D. Esteve, J. M. Raimond, J. Dalibard, *Quantum Entanglement And Information Processing*, Vol.LXXIX; Elsevier Science Publisher, (2004).

[3] A. Whitaker, *The New Quantum Age: From Bell's Theorem to Quantum Computation and Teleportation*, (OUP Oxford) (2012).

[4] Pan, J. W., Bouwmeester, D. Weinfurter, H. Zeilinger, *Experimental entanglement swapping: Entangling photons that never interacted.*, Phys. Rev. Lett. **80**, 3891 (1998).

[5] Christoph Simon, William T. M. Irvine, *Robust Long-Distance Entanglement and a Loophole-Free Bell Test with Ions and Photons*, Phys, Rev. **91**, 110405 (2003).

[6] Peter J. Martin, Bruce G. Oldaker, Andrew H. Miklich, David E. Pitchard, *Bragg scattering of atoms from standing light wave*, Phys. Rev. **60**, 515 (1988).

[7] A. Khalique, F. Saif, *Engineering entanglement between external degrees of freedom of atoms via Bragg scattering*, Phys. Rev. A **314**, 2003.

[8] W. P. Grice, *Arbitrarily complete Bell-state measurement using only linear optical elements*, Phys. Rev. A **84**, 042331 (2011).

[9] R. Islam, A. Khosa, F. Saif, J. Bergou, *Generation of atomic momentum cluster and graph states via cavity QED*, Quantum Inf. Proc., **1**, (2012).

[10] W. Wien, W. Karl, *100 years of ion beams: Willy Wien's canal rays*, Brazilian Journal of Physics. **29**, (3) (1999)

[11] R. ul Islam, A. H. Khosa, F. Saif, *Generation of Bell, NOON and Wstates via atom interferometry*, J. Phys. B: Atom. Mol. Opt. Phys. **41**, 035505 (2008).

[12] M. L. Terraciano, R. Oson Knell, D. G. Norris, J. Jing, A. Fernandez, L. A. Orozco, *Photon burst detection of single atoms in an optical cavity*, Nat. Phys. Rev. **5**, 480 (2009).

[13] S. U, Haq, A. Khalique, *Long distance cavity entanglement by entanglement swapping using atomic momenta*, Optics Communications **334**, (2015).

[14] A. H. Khosa, M. Ikram, M. S. Zubairy, *Measurement of entangled states via atomic beam deflection in Braggs regime*, Phys. Rev. A **70**, 052312 (2004).