Beam-splitting and Collective Interference of Composite Bosons

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A thesis submitted in partial fulfillment of the requirements for the Degree of **Master of Science** in **Physics**

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Dedication

My family for their Love, Endless help and Encouragement.

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Abstract

Many particles in nature are composite particles composed of elementary particles either fermions or bosons. These elementary particles shows different properties and behavior. Bosons tends to bunch together when two bosons are incident on the different ports of the beam-splitter (Hong-Ou-Mandel HOM interference) while fermions tend to anti- bunch. While exploring the formalism of composite particles specially bi-partite composite particles with Qunatum Information theory tools, we examine the nature of composite particles and prove that degree of entanglement between the constituent particles determines bosonic or fermonic behavior of composite particle. Interferometry of composite particles demonstrate the interference of composite particles. We discuss the problems of collective interference of composite bosons. First we study the case of interferometry of composite bosons whose constituent particles are interacting and then moves towards the interferometry of composite bosons whose constituent particles are non-interacting. From the results of these two cases we shows that interaction in important for the stability of composite system.

Contents

Chapter 1

Introduction

1.1 Elementary Particles

All particles that exist in nature can be classified into two categories. This classification is based on their spin difference, particles with half-integral spin are known as fermions such as Quarks, leptons, and baryons and particles with integral spin are bosons such as photons and mesons.
Fermions comply Pauli's Exclusion Principle and therefore it is not possible to co-exist in same state for two fermions. Bosons, however, have no such restriction. Another the simple way to differentiate between bosons and fermions is through Hong-Ou-Mandel interference. Two bosons come out together through the same port i.e., they bunch when they are used as the input of symmetric beam splitter. On the opposite side, two fermions turn out independently i.e., they anti-bunch.

1.2 Composite Particles

In every day life, most of the particles that we deal with are composite in nature. Therefore it is an important aspect of our reality to study different aspects of composite particles. It is the basic inspiration behind the interest to study composite particles. Particles that are made up of two or more than two sub-particles are called composite particles. Composite particles composed of fermions and bosons. Composite systems further divided into multi-partite systems and bipartite systems. Composite system composed of more than two subsystems is known as multi-partite systems while composite systems consist of only two subsystems are know as Bipartite systems. Proton is composite particle composed of one down quark and two up quarks, neutron is composed of one up and two down quarks and poin is composed of one up and one down quark. Therefore proton and neuron are the example of multi-partite systems and poin is an example of bipartite systems.

The study of composite particle is owned by the field of many-body theories[1]. An adequate amount of literature is available related to this subject but unfortunately complexity in a system arises when we increase the number of the particles of the system. For the present work, our interest is to explore that how the connection between constituent particles is responsible for several physical properties of the system. Moreover, comparison of the pure elementary particles (specifically composite boson) is part of this thesis.
When we introduce constituent particles of a system, we must have to introduce quantum correlations between them. And understanding of these non-classical correlations is mandatory to understand composite particles. The most astonishing feature of composite systems is entanglement which is heart of quantum information processing protocols.

1.2.1 Composite Particles Nature and Spin Statistics

According to spin statistics, composite particles can show fermonic nature if the number of constituent fermions are odd and bosonic nature if the number of constituent fermions are even.
As in case of proton two up quark and one down quark make odd integer spin so proton show fermoic behavior whereas in case of pion one up quark and one down quark make interger spin so pion show bosonic behaviour.

1.2.2 Composite Particles Nature and Quantum Information

The statistics of composites was recently re-considered from the perspective of quantum information. Entanglement between two constituent particles comes out to be the essential ingredient to guarantee bosonic or fermonic behavior. A pair of strongly correlated fermions have integer spin, so long as its internal structure is not probed, and is therefore expected to show bosonic behavior. For that reason, such composite systems are sometimes traditionally called composite bosons, despite the fact that the term is somewhat deceptive as not every all composite particles composed of two fermions will exhibit bosonic behaviour. Theses composite particles can exhibit a behavior ranging from fermionic to bosonic.

1.3 Non-Classical Correlation

Interference, superposition of quantum systems and tunneling are fundamental properties used to distinguish between quantum and classical systems. This is not only distinction between classical systems and quantum mechanical system. When we talk

about composite systems then there will exist correlation between constituent particles. Classical correlation means classical probabilities but quantum mechanical correlation means remote action at distance and such non-classical correlation leads us towards entanglement.
Correlation is a statistic's term. If two systems are correlated that means they are connected in such a way that we can predict a result of the second system if we know the result of the first one with some uncertainty. For example, if we have a bag filled of pieces of paper, with 00 or 11 printed on each paper having equal probability of occurrence. If we randomly pick a piece of paper and only look at one of the two numbers, then automatically we will know the other number entirely. While if we don't look at any number printed at that piece of paper, there is only fifty percent chance that we guessed correctly. That is how knowing part of the system helps to understand the other one. Above example is an example of classical correlation. The only property we were interested was a number. Let us take another example, consider we have two balls, one is blue and other is red. We give these balls to two different observers A and B with closed eyes, let say red to observer A and blue ball to observer B. The observers only know the probability of having either color but are unaware of the exact state (color) of their balls. Now, if the observer A looks at his ball and notice that his ball is red, immediately he get to know that the color of the second ball is blue. Concerning the spin of particles consider the products of the spin state of the particles 1 and 2 of the form:

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle_1 |\downarrow\rangle_2 + |\downarrow\rangle_1 |\uparrow\rangle_2). \tag{1.1}$$

When spin of the first particle is up then spin of the second particle will be down and vice-versa. Suppose we want to find the spin of a particle along x-direction, from measurement of the spin of particle 1 along S_x we can also determine the x-component of the spin of a second particle. Thus, the state of a particle is correlated and this is a classically correlated state with the probability of fifty percent outcome in this case. But what if we want to measure the spin along two directions like along x and y direction both. Suppose we have two observers Alice and Bob and the spin of a particle along x-direction is measured by Alice and that is up. Bob measures spin of that particle along y-direction and obtains spin down. Alice can never say that she has managed to measure two complementary properties simultaneously because of Bob's measurement. Imagine her surprise, then, when she tries to confirm her conclusion by measuring spin along y-direction and obtain herself spin up. Thus each of these properties is individually anti-correlated with each other. But the Bell states formed for quantum system have quantum correlation between them. This is a quantum phenomena which we cannot explain classically.
Quantum entanglement is a special kind of correlation in which connection between two systems cannot be explained by local cause.

1.4 Quantum Entanglement

1.4.1 Historical Background

Interest in entanglement began because of the famous experiment known as Einsten-Podolsky-Rosen (EPR) paradox. In their research paper[2], Einstein et al gave an intuitive argument using the theory of quantum mechanics and theory of special relativity. They exploited the unique properties of an "EPR pair" which nowadays is known as entangled states and raised a question on the completeness of quantum mechanics. At that time, Einstein, Podolsky, and Rosen were arguing for an objective reality that was about local-realism, which quantum mechanics with its postulate of uncertainty appear to contradict. John S. Bell was the one who worked on the EPR argument further in 1960s and showed that local-realism based theories are out of scope for the correlations between measurements of entangle state predicted by quantum mechanics [3]. The inequalities derived by Bell and others were tested for entangled photons and these experiments proved the predictions of quantum mechanics [4, 5] . Although explanations given by Einstein and his fellows were not satisfactory and their conclusion is now proven to be invalid. However, it drew attention towards most important phenomena of entanglement and raised the possibility that there exist special kind of particles (entangled particles) in quantum mechanics. Even though, entangled states were identified since the beginning of quantum mechanics but recent concept of entanglement has modified and our understanding is very different from what Einstein and his fellows had in mind. Most of the present day entanglement theory is motivated by discoveries in the 1990s that use the strangeness of entanglement in various applications like in quantum teleportation [6], quantum cryptography [7] and quantum dense coding [8]. All these discoveries are experimentally demonstrated, that shows entanglement is completely quantum mechanical phenomena that have no classical replacement.

Scientists put a great attention on quantum entanglement as it is considered a vital source for quantum communication and information processing. In 1935 Einstein, Podolsky and Rosen supposed an experiment that described the incompletness of quantum mechanics theory. This experiment is called as EPR paradox. According to EPR paradox there is a hidden variable within a particle that reveals its properties during a measurement. But after many experiment it was understood that two particles can interact with each other and they cannot behave independently. Such interacting particles are called as entangled particles and this phenomenon of long distance interaction is called entanglement. Entanglement is the basic property of quantum systems that can be occur between different particles or within two or more degree of freedom of a single particle. In classical world there is no concept of entanglement between different particles that are at distance apart, but in quantum mechanics particles are entangled even if they are at distance apart.

1.4.2 Understanding the Concept of Entanglement

Quantum entanglement is a physical phenomenon in which we describe quantum state of two or more particles with reference to each other.
(1) For entangled particles, quantum state associated with each particle cannot be expressed independently but, we can define the quantum state for a whole system that contains complete information of that system. These particles are connected in such a way that action operated on any one of them influences the whole system and this connection can be explained in terms of entanglement. In terms of Schrodinger's famous paradox, entangled state can be written as:

$$|\psi\rangle_{atom-cat} = \frac{1}{\sqrt{2}}(|atom\ not\ decayed\rangle\,|cat\ alive\rangle + |atom\ decayed\rangle\,|cat\ dead\rangle). \quad (1.2)$$

(2) An entangled state is described as a state which cannot be written as a tensor product state. It can not separated into state of system A and B. Consider Alice with states $|0\rangle_A, |1\rangle_A$ and Bob with state $|0\rangle_B, |1\rangle_B$ then entangled state of Alice and Bob can be defined as:

$$|\Psi\rangle = \frac{|1\rangle_A |0\rangle_B + |0\rangle_A |1\rangle_B}{\sqrt{2}}, \quad (1.3)$$

if $|H\rangle$ is a horizontal polarization and $|V\rangle$ is a vertical polarization of a photon then entangle state in terms of polarization of photons can be written as:

$$|\Psi\rangle_{1,2} = \frac{|H_1\rangle |V_2\rangle + |V_1\rangle |H_2\rangle}{\sqrt{2}}. \quad (1.4)$$

When we perform a measurement on photon 1 and find photon to be localized in horizontal polariztion state, as a result we can immedietly guess the photon 2 will be in vertical polarization and vice versa. This is due to the superposition of quantum states which makes the outcome of measurement completely random. This means that before a measurement we cannot know about the outcome of the event. This means that if a photon 1 is collapsed in state $|V\rangle_1$ or $|H\rangle_1$ then photon 2 will automatically will collapes in $|V\rangle_2$ or $|H\rangle_2$.

1.4.3 Entanglement and Nature of Composite Particles

We relate entanglement and compositeness of a particle with each other and study the properties of a composite particle. Specifically, we study composite boson that is made up of a pair of distinguishable fermions(or bosons). With the help of entanglement, we can find out compositeness of particle and can tell how much the behavior of composite boson is close to pure boson.

1.5 Preparation of Entangled States

Spontanous Parametric Down Conversion Sources
The first experiment for polarization-entangled photon pair source was conducted with atomic-cascade decays to violate remote Bell inequality. In the beginning of 1987 entangled photons were observed from a nonlinear crystal that is pumped with an intense coherent source. So it is clear that nonlinear crystals have noval importance in most engtnaled sources.
SPDC is a strongest method leading to the emission of photon pair from nonlinear crystal. It is a process in which three waves mix together with the help of nonlinear crystal of susceptibility $\chi^{(2)}$. The phenomenon of SPDC was first intoduced in 1961 by Louisell and his co-worker .In general SPDC is a process in which a single photon decays into two lower energy daughter photons. The theory of SPDC were developed by Kleiman and modern mechanical calculations was developed by Hong and Mandel in 1985.

1.5.1 Basic principle of SPDC source

Spontaneous parametric downconversion source is based on three waves in which photon from source is passed through a nonlinear optical crystal that has an equal probabaility to be converted into daughter photons. We assumed that it is a three plane wave mixing. It has two types that are called type I and type II downconversion. In SPDC process daughter photons are characterised by their polarization from crystal axis and are called ordinary and extraordinary. In nonlinear optics the downconverted photons are called signal photons (index s) and idler photon (index i).
Polarization direction of signal and idler modes for type I downconverion is identical. It means that if the pump have a extraordinary polarization then signal and idler will be in ordinary polarization. The degenerate twin photons are emitted as cones in type II down-conversion. In laborartoray frame we refer exteraordinary photon with vertical polarization $|V\rangle$. The frequencies of three fields are expressed as $\omega_{p,s,i}$ and vectors of their fields are written as $k_{p,s,i}$. The energy and momentum conversion can be written as:

$$\omega_p = \omega_s + \omega_i, \tag{1.5}$$

$$k_p = k_s + k_i. \tag{1.6}$$

One cone contains ordinary polarized photons and other contains extra ordinary polarized photons. The opening angle of photon relies on the angle of optical axis with pump field and written as θ_p. At some value of θ_p, we get degenerate emission of photons . This will happen when two cones exactly overlap at one point. The emission of photons is in the direction of original photon. When θ_p is increased, two cones

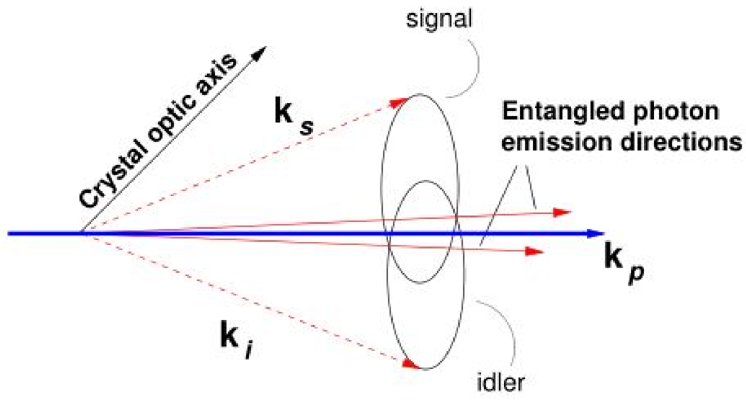

Figure 1.1: Signal and idler mode for spontaneous parametric down convervion source

come close and interact at two points having pump beam in center. Photons coming through these points are completly indistingishable excluding polarization states. So two possible indistinguisable decay paths available at these interaction and causes the emission of polarized entangled state. The emitted state can be written as:

$$|\psi\rangle = \frac{(|H_1, V_2\rangle + e^{i\phi}|H_2 V_1\rangle)}{\sqrt{2}}, \tag{1.7}$$

where ϕ is a phase angle between ordinary and extraordinary polarized light introduced by the crystal birefringence. Global phase may be ignored. In addition to realtive phase, the crystal also introduce longitudinal and transvers walk-off. In longitudinal walk-off ordinary and extraordinary have different speeds throughout and easily distingishable from each other. For transver walk-off ordinary and extraordinary waves propagate in different directions and are separeted after passing through the down conversion crystal. After the walk-offs are corrected, one of the four maximally entangled Bells states can be obtained by using half wave plate and quarter wave in the source route.

$$|\psi^{\pm}\rangle = \frac{(|H_1, V_2\rangle \pm |H_2, V_1\rangle)}{\sqrt{2}}, \tag{1.8}$$

$$|\phi^{\pm}\rangle = \frac{(|H_1, H_2\rangle \pm |V_1, V_2\rangle)}{\sqrt{2}}, \tag{1.9}$$

above four state have noval importance and have been used in many quantum information theoretical models such as quantum teleportation.

1.6 Thesis Outline

In chapter 1, we discuss introduction to elementary particles, composite particles and the nature of composite particles. In chapter 2, there is Quantum mechanics of beam splitter and Interferometry of elementary fermions and bosons. While moving next to chapter 3, a detail discussion on basic concepts that are used in formulism of composite bosons via quantum information. We view composite boson with quantum information eye and discuss role of entanglement in deciding the nature of composite bosons. In chapter 4,we discuss the cases of collective interference of composite bosons and at the end in chapter no. 5 there is summary and conclusion of my thesis.

Chapter 2

Beam-splitting and Interferometry of Elementary Particles

In this chapter we discuss interferometry of elementary particles. First we discuss classical light with traditional interferometer and then instead of classical light we replace it with quantum field.

2.1 Classical Beam-splitter

Consider a lossless plane beam splitter on which a light of amplitude E is incident. Plane beam-splitter will partially reflect and partially transmit the incident light.

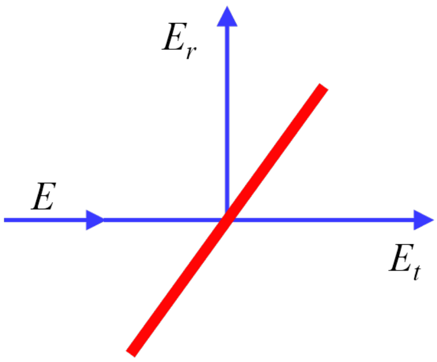

Figure 2.1: Classical Beam Splitter

The amplitudes of the reflected beam E_r and transmitted beam E_t are written as:

$$E_r = rE \quad ; \quad E_t = tE, \tag{2.1}$$

where r and t are the reflectance and transmittance respectively of the beam splitter. For 50:50 beam splitter, $|r| = |t| = 1/\sqrt{2}$.

$$|E|^2 = |E_r|^2 + |E_t|^2,$$

this follows the condition as:

$$|r|^2 + |t|^2 = 1.$$

2.2 Quantum Mechanical Beam-splitter

For quantum mechanical beam-splitter, we replace classical light amplitudes with annihilation operators $\hat{a}$, $\hat{b}$, $\hat{c}$ and $\hat{d}$ as shown in the figure.

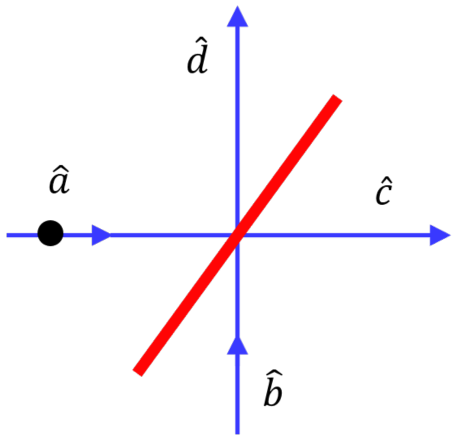

Figure 2.2: Quantum mechanical beam-splitter

$$\hat{c} = t\hat{a} + r\hat{b} \quad ; \quad \hat{d} = r\hat{a} + \hat{b}, \tag{2.2}$$

$$\begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} t_1 & r_2 \\ r_1 & t_2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}, \tag{2.3}$$

$$\begin{pmatrix} c \\ d \end{pmatrix} = U \begin{pmatrix} a \\ b \end{pmatrix}. \qquad U = \begin{pmatrix} t_1 & r_2 \\ r_1 & t_2 \end{pmatrix} \tag{2.4}$$

Now if $r_1 = r_2$; $t_1 = t_2$ then,

$$U = \begin{pmatrix} \cos(\theta) & \exp(i\phi)\sin(\theta) \\ -\sin(\theta)\exp(-i\phi) & \cos(\theta) \end{pmatrix}, \tag{2.5}$$

$$c = a(\theta) = a\cos(\theta) + ib\sin(\theta) \quad ; \quad d = b(\theta) = b\cos(\theta) + ia\sin(\theta), \tag{2.6}$$

By taking derivative of these equations,

$$\frac{\mathrm{d}a(\theta)}{\mathrm{d}\theta} = ib(\theta) \;\; ; \;\; \frac{\mathrm{d}b(\theta)}{\mathrm{d}\theta} = a(\theta), \tag{2.7}$$

Initial conditions are $a(0) = a \;\; and \;\; b(0) = b$.
To evaluate the Hamiltonian, we use Heisenberg equation.

$$\frac{\mathrm{d}a(\theta)}{\mathrm{d}\theta} = -\frac{i}{\hbar}\left[a(\theta)\;,\;H\right], \tag{2.8}$$

$$ib(\theta) = -\frac{i}{\hbar}\left[a(\theta)\;,\;H\right], \tag{2.9}$$

$$i(b\cos(\theta) + ia\sin(\theta)) = -\frac{i}{\hbar}\left[a(\theta)\;,\;H\right]. \tag{2.10}$$

First we have to solve the following commutation relation,

$$[a(\theta)\;,\;H] = a(\theta)H - Ha(\theta), \tag{2.11}$$

now we have to select such form of Hamiltonian which satisfy the above Heisenberg equation and is given as:

$$H = -\hbar(a^\dagger b + ab^\dagger). \tag{2.12}$$

2.3 Fock Space

Fock Space is important because we use it to study many particles system, as well as the system where number of particles may not be conserved. For example, in non-ideal optical cavity there is possibility of leakage of photons. Another example is excited atoms, these atoms emit a photon. Formalism that allows us to describe such type of systems in a good way is explained below. This formalism is actually build by using concept of creation and annihilation operators, we use to describe harmonic oscillator in quantum mechanics. Let us consider a system having single particle. First we describe this basic system and later we can easily generalize it for more than one particle. As we are observing quantum mechanically, we cannot forget importance of vacuum state. In a system, there must be state that represents zero particle. We denote vacuum state by $|0\rangle$ and its inner product with itself $\langle 0|0\rangle$ equal to one. Now just like harmonic oscillator, we define the creation and annihilation operators as $\hat{a}_n^\dagger$ and $\hat{a}_m$ respectively. It is important to keep in mind that here harmonic oscillator is not involved, its just that we are defining operators in *ad hoc*. Now the state of single particle is $|1\rangle = \hat{a}^\dagger|0\rangle$, thus creation operator is adding one particle in system and annihilation operator just removes a particle from the system.

$$\langle 0|\hat{a}\hat{a}^\dagger|0\rangle = \langle 0|0\rangle = 1.$$

2.3.1 Fermions

The properties of annihilation and creation operators for fermions are:

$$\hat{a}_n^\dagger|0\rangle = |\varphi_n\rangle, \tag{2.13}$$

$$\hat{a}_m^\dagger|\varphi_n\rangle = \hat{a}_m^\dagger\hat{a}_n^\dagger|0\rangle = |\varphi_m\varphi_n\rangle = -|\varphi_n\varphi_m\rangle, \tag{2.14}$$

$$\hat{a}_n|\varphi_n\rangle = |0\rangle, \tag{2.15}$$

$$\hat{a}_n|0\rangle = 0. \tag{2.16}$$

Here n is quantum number of fermion. From Eq. (2.14), we can define that vector states are anti-symmetric with interchange of any two fermions.
Anti-commutation relations for fermions are:

$$\{\hat{a}_n^\dagger, \hat{a}_m^\dagger\} = \{\hat{a}_n, \hat{a}_m\} = 0, \{\hat{a}_n, \hat{a}_m^\dagger\} = \delta_{nm}, \tag{2.17}$$

$$\{\hat{a}_n^\dagger, \hat{b}_m^\dagger\} = \{\hat{a}_n, \hat{b}_m\} = \{\hat{a}_n^\dagger, \hat{b}_m\} = \{\hat{a}_n, \hat{b}_m^\dagger\} = 0. \tag{2.18}$$

where $\hat{a}$ and $\hat{b}$ are operators belong to two distinguishable fermions.

2.3.2 Bosons

Following are the properties that creation and annihilation operator for bosons have:

$$\hat{c}_\beta^\dagger = |0, 0, 0...., m_\beta = 0, ...\rangle = |\varphi_\beta\rangle = |0, 0, ..., m_\beta = 1, 0, ...\rangle, \tag{2.19}$$

$$\hat{c}_\beta|m_1, m_2, ..., m_\beta = 0, ...\rangle = 0, \tag{2.20}$$

$$\hat{c}_\beta^\dagger|m_1, m_2, ..., m_\beta, ...\rangle = \sqrt{(m_\beta + 1)}|m_1, m_2, ..., m_\beta + 1, ...\rangle, \tag{2.21}$$

$$\hat{c}_\beta|m_1, m_2, ..., m_\beta, ...\rangle = \sqrt{(m_\beta)}|m_1, m_2, ..., m_\beta - 1, ...\rangle. \tag{2.22}$$

Bosons follow the commutation relation,

$$[\hat{c}_n, \hat{c}_m^\dagger] = \delta_{nm}. \tag{2.23}$$

For all bosons to be in same mode then above relations become:

$$\hat{c}^\dagger|0\rangle = 0, \tag{2.24}$$

$$\hat{c}|0\rangle = 0, \tag{2.25}$$

$$\hat{c}^\dagger|m\rangle = \sqrt{(m + 1)}|m + 1\rangle, \tag{2.26}$$

$$\hat{c}|m\rangle = \sqrt{m}|m - 1\rangle, \tag{2.27}$$

$$[\hat{c}, \hat{c}^\dagger] = 1. \tag{2.28}$$

2.4 Basic Statistics of Single and Two Photon States at Beam-Splitter

2.4.1 Single Particle and Beam-splitter

Consider a 50:50 beam-splitter, a single particle is taken as an input on its one port [9]. This particle has a chance of 50% to come out on either output port (c) or (d). The input state of beam-splitter undergo a transformation as:

$$|a\rangle = \frac{1}{\sqrt{2}}(|c\rangle + i\,|d\rangle). \tag{2.29}$$

For simplicity, we assume beam splitter is symmetrical, this symmetry implies that there is a phase shift of $\pi/2$ in the reflected part [10].

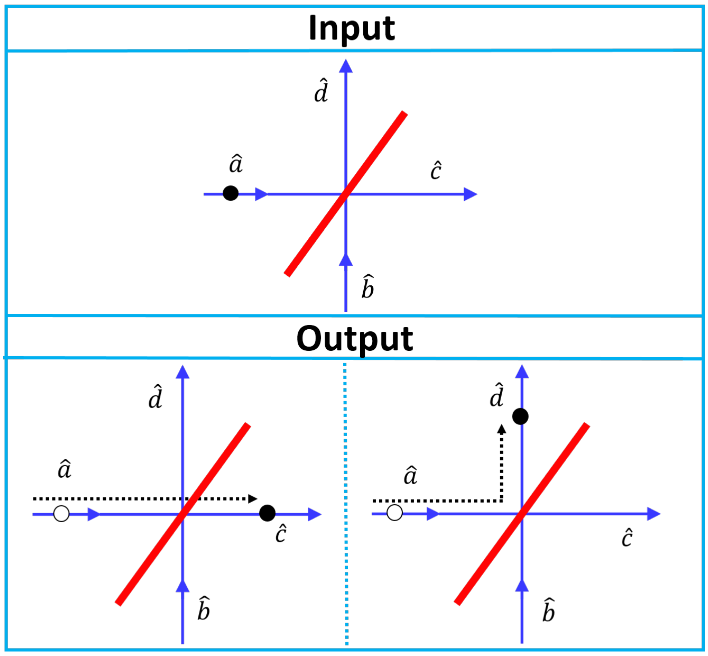

Figure 2.3: The input particle is either reflected or transmitted

2.4.2 Two Particles on Same Port of Beam-splitter

Let two particles as an input on the same port of the beam-splitter. In this case we have various possibilities. The initial state is defined as:

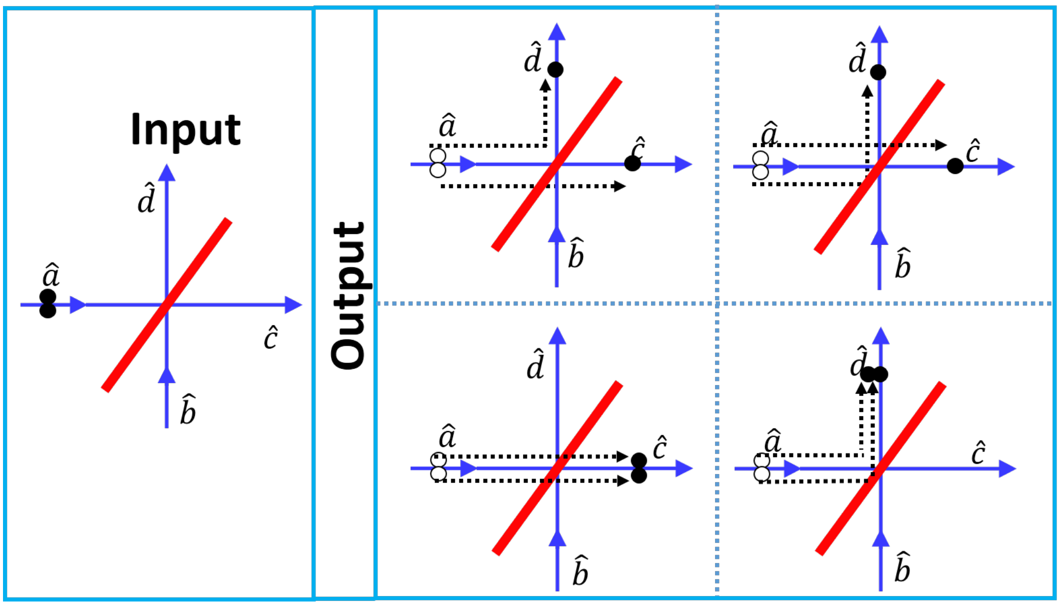

Figure 2.4: Two particles on same port of beam splitter

$$|\psi_{in}\rangle = |a\rangle_1 |a\rangle_2 ,$$

where $|a\rangle_1$and $|a\rangle_2$ means that particle 1 and particle 2 are incident on port (a).

$$|a\rangle_1 |a\rangle_2 = \frac{1}{2}(|c\rangle_1 + i\,|d\rangle_1)(i\,|c\rangle_2 + |d\rangle_2),$$

$$|a\rangle_1 |a\rangle_2 = \frac{1}{2}(|c\rangle_1 |c\rangle_2 + i\,|c\rangle_1 |d\rangle_2 + i\,|d\rangle_1 |c\rangle_2 - |d\rangle_1 |d\rangle_2). \tag{2.30}$$

The two particles behave as independent just as classical particles do. Therefore, following are the probabilities

Probability for both particles on port c = 25%
Probability for both particles on port d = 25%
Probability for one particle on c and one on d = 50%

2.4.3 Two Particles on Different Ports of Beam-splitter

Consider two particles which are indistinguishable, here we have no informaion whether particle 1 or particle 2 is incident on beam port (a) or (b) [11]. So the state is in superposition of $|a\rangle_1 |b\rangle_2$ and $|a\rangle_1 |b\rangle_2$.
In case of the two particles are bosons, The state of system is written as:

$$|\psi\rangle_{boson} = \frac{1}{\sqrt{2}}(|a\rangle_1 |b\rangle_2 + |a\rangle_1 |b\rangle_2). \tag{2.31}$$

In case of the two particles are fermions, The state of system is written as:

$$|\psi\rangle_{fermions} = \frac{1}{\sqrt{2}}(|a\rangle_1 |b\rangle_2 - |a\rangle_1 |b\rangle_2). \tag{2.32}$$

The output state in case of two fermions on each port of the beam splitter is given as [12]:

$$|\psi\rangle_{fermions} = \frac{1}{\sqrt{2}}(|c\rangle_1 |d\rangle_2 - |d\rangle_1 |c\rangle_2), \tag{2.33}$$

this shows that the two fermions will come out on different output ports.
The output state in case of two bosons on each port of the beam splitter is given by [12]:

$$|\psi\rangle_{bosons} = \frac{i}{\sqrt{2}}(|c\rangle_1 |c\rangle_2 + |d\rangle_1 |d\rangle_2), \tag{2.34}$$

this shows that the two bosons will come together either on (c) or (d) port. The experimental verification of this case is studied in 1987 [13].

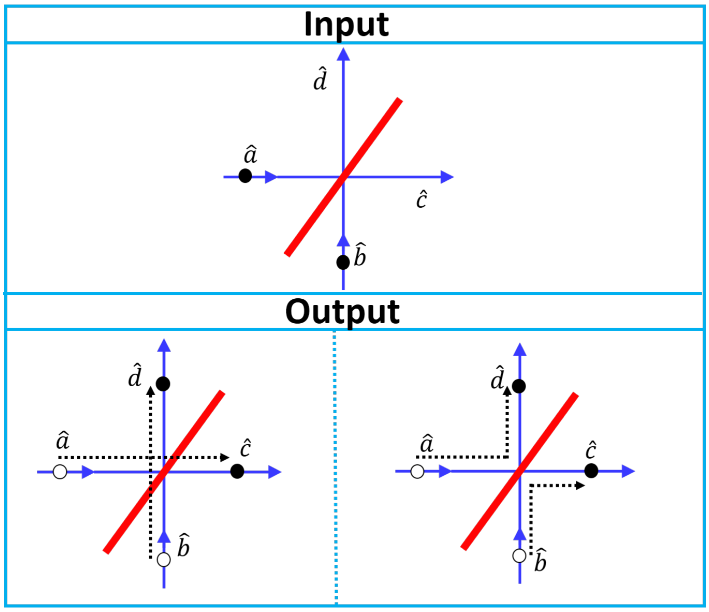

Figure 2.5: If the two particles are bosons, they found on same output ports.

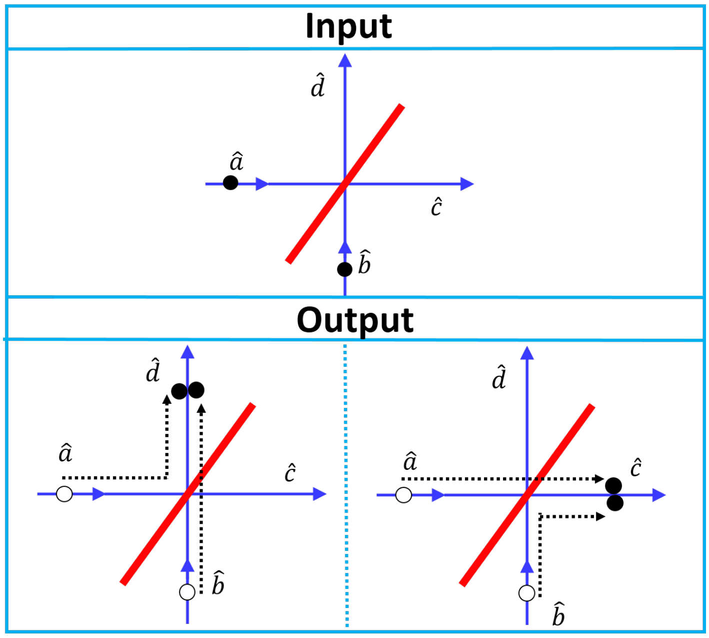

Figure 2.6: If the two particles are fermions, they found on different output ports.

Chapter 3

Bipartite Entanglement and Stability of Composite Bosons

In this chapter, we will discuss composite particle particularly a bipartite boson using the quantum information approach. We will use the tools of quantum information to develop the formalism that helps to understand the internal structure of composite boson. We discuss the coboson made up of pair of fermions in detail, however, we also discuss briefly coboson comprised of pair of bosons. In this chapter we will see that we can know information all about the composite behavior of composite boson with the help of entanglement. We found that measurement of the degree of entanglement between the sub-particles explains the deviation of the composite character of composite particle from a pure bosonic character. In other words, it explains how closely composite boson is behaving like pure boson. This phenomena entails some interesting ideas about the constituent particles that these particles are somehow bound by quantum entanglement. For the discussion of a composite particle and its behavior in a bipartite system the mechanical binding forces are actually not necessary because these forces usually help us only as physical means when we try to apply the quantum correlations. Below in this chapter the underlying role of entanglement will be discussed using concept of second quantization, on the bases of properties of ladder operators associated with composite particles. In quantum mechanics, the state of a system is expressed in complex vector space by a state vector. The state vector is either as a ket vector $|\alpha\rangle$ or a bra vector $\langle\beta|$. This way of writing state vectors in Quantum Mechanics is called Dirac Notation. Suppose we prepare a state $|\psi\rangle$ with Hilbert space H and expand it in a series of basis states $|\phi_i\rangle$ as:

$$|\psi\rangle = \sum_i a_i |\phi_i\rangle , \tag{3.1}$$

where a_i are the amplitudes for each basis state.

3.1 Fundamental Concepts of Quantum Information

Quantum mechanics allows a quantum system to be in superposition state. Superposition is the property of a quantum system to be in two states at the same time. However, when we observe a system, the system has to decide where to be and we can only see it in one of those two states. Quantum information is the effort to both understand and use the properties of the quantum world. We use the concepts of quantum information to understand the internal structure of composite system and its properties. In this section, we describe the basic concepts and mathematical methods that will help us to understand a later subject. We discuss the state of a composite quantum system, specifically, bipartite quantum system and construct the formalism to express the states of the composite quantum system in terms of states of subsystems. We also explain the concept of entanglement and relate it with Schmidt decomposition.

3.2 Preliminary Definitions

3.2.1 Composite Systems

A composite quantum system can decompose naturally into its subsystems, where every subsystem is proper quantum system. Usually, we distinguish the subsystems from each other on the basis of the distance between them which must be larger than the individual subsystem's size. For example, hydrogen atom is composite in nature since it consists of an electron and proton. Another common example of a composite system is the string of ions in which every ion acts as a subsystem [14].

Multipartite Compoiste System

In quantum mechanics, we often associate a Hilbert space H, with a physical system. If we have some system that is made up of two or more than two subsystems (known as multipartite system), Hilbert space H associated with that system is given by tensor product of all Hilbert spaces of subsystems.

$$H = H_1 \otimes H_2 \otimes H_3 \otimes H_4... \tag{3.2}$$

Bipartite Composite System

In this section, we comprehensively take a look at the bipartite coboson from the perspective of quantum information. We take a look at the fermionic and bosonic algebra for ideal fermions and bosons and modify it for the composite boson.

Let us consider a composite bipartite particle C in Hilbert space H comprised of two sub-particles. Let we denote sub-particles with A and B. Hilbert space H_A be the Hilbert space of subsystem associated with particle A and H_B be the Hilbert space of other subsystem related with particle B. Therefore the Hilbert space (H) of composite particle is expressed as the tensor product of Hilbert spaces of sub-particles.

$$H = H_A \otimes H_B. \tag{3.3}$$

Both constituent particles are distinguishable and can be either bosons or fermions. Collectively, both particles behave as coboson. We will find that how much nature of coboson is deviating from ideal boson. The state vector of composite bipartite sysem can be written in form as;

$$|\psi\rangle_C = \sum_{i,j=0}^{\infty} \alpha_{ij} |i\rangle_A \otimes |j\rangle_B, \tag{3.4}$$

where $|i\rangle_A$ and $|j\rangle_B$ are the state vectors of the subsystem.

3.2.2 Pure States

A pure state is the state where we have exact information about the system. Pure state $|\psi\rangle$ is given by Density operator as:

$$\rho = |\psi\rangle \langle\psi| . \tag{3.5}$$

Density operator for pure state always follows the following conditions as given by:

1. $\rho^2 = \rho$
2. $Tr(\rho^2) = 1$

3.2.3 Mixed States

Mix state is defined as a linear combination of different pure states. In real life experiments, instead of single quantum system often there are collection of quantum systems (ensemble). Also each member of ensemble can be found in more than one quantum states. Thus most of the time we encounter mix states instead of pure states.

For mixed states, the density matrix ρ_{AB} of a composite system is separable if and only if

$$\rho_{AB} = \sum_k P_k \rho_A \rho_B. \tag{3.6}$$

3.2.4 Separable States

Consider a composite system consists of two distinct subsystems. Let a pure bipartite quantum state $|\phi\rangle \in H_1 \otimes H_2$; it is called separable state if and only if it can be written as a tensor product of pure states of subsystem, such that

$$|\phi\rangle_{sep} = |\phi_1\rangle \otimes |\phi_2\rangle \,. \tag{3.7}$$

Example

$$|\phi\rangle = \frac{1}{\sqrt{2}}[|0\rangle_1 |0\rangle_2 + |0\rangle_1 |1\rangle_2],$$

$$|\phi\rangle = \frac{1}{\sqrt{2}}[|0\rangle_1][|0\rangle_2 + |1\rangle_2],$$

$$|\phi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle \,.$$

The results of measurements on pure separable states are not correlated i.e measurement taken on one subsystem do not change the possible outcomes of other subsystem. The reduced density matrices are give by the partial trace over the first and second subsystem

$$\rho_1 = Tr_2(\rho) \quad ; \quad \rho_2 = Tr_1(\rho)$$

where ρ is the whole density matrix and is defined by $\rho = |\phi\rangle \langle\phi|$.
The density matrix of separable state can also be written as the tensor product of reduced density matrices of the subsystem as given by:

$$\rho = \rho_1 \otimes \rho_2. \tag{3.8}$$

3.2.5 Entangled States

Entangled state of a bipartite system cannot be expressed as a tensor product of states of subsystem given as:

$$|\phi\rangle \neq |\phi_1\rangle \otimes |\phi_2\rangle \,. \tag{3.9}$$

Example: Examples of entangle bipartite states is Bell state,

$$|\phi\rangle = \frac{1}{\sqrt{2}}[|0\rangle_1 |0\rangle_2 + |1\rangle_1 |1\rangle_2].$$

So this type of state cannot be written like Eq.3.9. Information contain by the entangled state is not described completely in terms of the states of subsystem. The results of measurement on entangled states are correlated. Also for an entangled state, one can find the density matrix as:

$$\rho = |\phi\rangle \langle\phi| \neq \rho_1 \otimes \rho_2. \tag{3.10}$$

3.2.6 Density Operator

So far, in quantum mechanics we have dealt with the system which are completely described by state vector and in such representation state vector contain all the information about the system. In quantum mechanics there is also an alternative and more general approach analogous to the state vector approach known as density operator or density matrix approach. This is more convenient way to thinking for some commonly encountered scenarios in quantum mechanics[15].

For a given ensemble $[|\phi_i\rangle, p_i]$ of N pure states $|\phi_i\rangle$ with probability p_i, the density operator ρ is given by:

$$\rho = \sum_{i=1}^{N} p_i |\phi_i\rangle \langle\phi_i|, \tag{3.11}$$

where,

$$\sum_{i=1}^{N} p_i = 1.$$

Density operator must satisfies the following conditions:

1. $\rho = \rho^\dagger$ density operator must be Hermitain.
2. $Tr(\rho) = 1$.
3. For a state vector $|u\rangle$, $\langle u| \rho |u\rangle >= 1$, means ρ is a positive operator.

3.3 Density Matrix Formalism

"Density matrix" is a very powerful formalism in which we describe quantum state by its density matrix. It is an alternative formalism to describe a quantum state by Dirac notations (bra-ket notation). Density operator is an average operator and basically useful in describing statistical mixture. It is denoted by ρ.

In the case of pure states where the state of a system is definite, ρ can be constructed by the outer product of state $|\psi\rangle$. To see this, let we consider some operator $\hat{Q}$, we find the average value (expectation value) of this operator. The expectation value $\langle\hat{Q}\rangle$ can be written as:

$$\langle\hat{Q}\rangle = \langle\psi|Q|\psi\rangle.$$

Expanding $|\psi\rangle$ in its orthonormal states, we get the form as

$$\langle\hat{Q}\rangle = (\gamma_1^*\langle u_1| + \gamma_2^*\langle u_2| + \gamma_3^*\langle u_3|.........\gamma_m^*\langle u_m|)Q(\gamma_1|u_1\rangle + \gamma_2|u_2\rangle + \gamma_3|u_3\rangle.........\gamma_m|u_m\rangle)$$

$$= \sum_{k,l=1}^{n} \gamma_k^*\gamma_l\langle u_k|Q|u_l\rangle$$

$$= \sum_{k,l=1}^{n} \gamma_k^* \gamma_l Q_{k,l}. \tag{3.12}$$

As the expansion coefficient can be written as:

$$\gamma_m = \langle u_n | \psi \rangle,$$

and the complex conjugate is

$$\gamma_m^* = \langle \psi | u_n \rangle,$$

that means $\gamma_k^* \gamma_l = \langle u_l (|\psi\rangle\langle\psi|) u_k \rangle$. Thus the average value of operator $\hat{Q}$ is

$$\langle \hat{Q} \rangle = \sum_{k,l=1}^{n} \gamma_k^* \gamma_l Q_{kl} \sum_{k,l=1}^{n} \langle u_l | \psi \rangle \langle \psi | u_k \rangle Q_{kl}.$$

We call this outer product $|\psi\rangle\langle\psi|$, a density operator ρ. And the average value of operator with respect to state $|\psi\rangle$ is

$$\langle \hat{Q} \rangle = \sum_{k,l=1}^{n} \langle u_l | \rho | u_k \rangle Q_{kl}.$$

In terms of trace, we can write it as

$$\langle \hat{Q} \rangle = Tr\left(\rho Q\right).$$

For pure states, we see that

$$\rho^2 = (|\psi\rangle\langle\psi|)(|\psi\rangle\langle\psi|) = (|\psi\rangle\langle\psi|) = \rho.$$

As $Tr(\rho) = 1$, that means

$$Tr(\rho^2) = 1.$$

Thus in terms of density operator, if trace of a square of density operator is equal to one then state of a system is pure.
For mixed states, let suppose there are N number of possible states. For state $|\psi_n\rangle$, density operator can be written as $\rho_n = |\psi_n\rangle\langle\psi_n|$. Probability that system in ensemble has been prepared in state $|\psi_n\rangle$ is p_n. Then the density operator for ensemble is

$$\rho = \sum_{n=1}^{N} p_n |\psi_n\rangle | \langle \psi_n |.$$

We can characterize that given state of system is mixed state if trace of square of density operator $Tr(\rho^2) < 1$.

3.3.1 Reduced Density Operator for a Bipartite System

Let ρ is a density operator of a bipartite system consists of two subsystems A and B in Hilbert space H , then the reduced density operator ρ^A (ρ^B) is defined as:

$$\rho^A = Tr_B(\rho) \qquad ; \qquad \rho^A = Tr_A(\rho), \tag{3.13}$$

where $Tr_B(\rho)$ and $Tr_A(\rho)$ are called partial trace. The idea behind partial trace is to obtain a density operator for one of the subsystem alone.
Density operators have very important application in characterization of composite systems and working with the states of their subsystems.

3.4 Criteria to Distinguish Separable and Entangled States

The above criteria to distinguish separable and entangled states seems very simple on first sight. But if we check different states, we found that in some cases checking separability of state gets complicated. As for pure states, we have defined the criteria of separability which is the existence of the decomposition of a state into product states, or for mixed states by a convex sum of tensor products. Therefore, when we look at the given state to check the separability, we have to find such decomposition. Once a decomposition is found, it gets clear that the state is separable. But in case of failure to find decomposition, there are two possible reasons: either the state is actually separable but reasonable decomposition could not be identified, or the state is entangled so there is no decomposition. Due to this reason, there is a need for a standard but straightforward criterion to distinguish separable and entangled states which do not require an explicit search. For pure states, there are criteria which differentiate separable and entangled states unambiguously, but in the case of mixed states, this criterion is applicable only for a low dimensional system. For higher dimensional systems, this criterion can give only partial information. Here we will discuss a simple case of pure states as our primary work is basically on bipartite pure states.

3.4.1 Schmidt Decomposition

Schmidt decomposition can help us to find out whether the state of a system is separable or entangled [16]. We will discuss that how useful is the Schmidt decomposition for the measurement of bipartite entanglement for pure states [17].
Qubit System: A quantum system with two dimensional (d=2) linear independent states is called Qubit system. This type of system is represented by a two dimensional hilbert space H_2. e.g spin $\frac{1}{2}$ system and bell states. For pure bipartite systems, there

is a very important concept known as schmidt decomposition which is only valid for bipartite systems.
Let system has pure state $|\psi\rangle$ in the Hilbert space H which is given by the direct product of the Hilbert spaces of subsystems.

$$H = H_A \otimes H_B,$$

where H_A and H_B are the Hilbert spaces of the subsystems having two local basis $| \, i\rangle_A$ and $| \, j\rangle_B$ respectively. In terms of above mentioned basis, the state $|\psi\rangle$ of a system can be expressed as:

$$|\psi\rangle = \sum_{i,j} \alpha_{ij}(|i\rangle_A \otimes |j\rangle_B), \tag{3.14}$$

where α_{ij} is the expansion coefficient, which represents the overlap of a state of system with the basis vectors,

$$\begin{aligned} \alpha_{i,j} &= \langle i_A| \otimes \langle j_B|\psi\rangle, \\ &= \langle i_A|\psi\rangle\langle j_B|\psi\rangle. \end{aligned} \tag{3.15}$$

Now , let us write matrix formed by expansion coefficient α_{ij} as some $d_A \times d_B$ matrix C where d_A and d_B are equal to dimensions of Hilbert space H_A and H_B respectively.

$$[C]_{i,j} = \alpha_{i,j}. \tag{3.16}$$

As every matrix has singular value decomposition (SVD), with the help SVD we will solve this matrix to find Schmidt eigenvalues.

Singular Value Decomposition(SVD)

SVD is the factorization technique of any $m \times n$ matrix A into three matrices UDV^T. Where U and V are the orthogonal matrices of size $m \times m$ and $n \times n$, respectively. While D is diagonal matrix of size $m \times n$ and these diagonal entries are called singular values of matrix A.
To understand physically, for any vector, when matrix A apply on a vector, it rotates the vector and also stretch it. In case of circle (two dimensional case of sphere), when the matrix A apply on sphere, it rotates the circle and also stretch it, so that it becomes ellipse.
Here let we denote orthogonal vectors of circle by v_1 and v_2, while major and minor axis of ellipse are denoted by u_1 and u_2, respectively. So, when matrix A applies to vector v_1, it gives

$$Av_1 = \sigma_1 u_1,$$

Similarly, when matrix A applies to vector v_2, it gives

$$Av_2 = \sigma_2 u_2,$$

where σ_1 and σ_2 are stretching factors. In case of N-dimensional sphere (hyper sphere), after the operation we get a hyper ellipse.

$$Av_j = \sigma_j u_j,$$

.

We can see Eq. is like a eigen value problem. In the matrix form, we can write it as $[A]\begin{bmatrix} v_1 & v_2 & . & . & v_n \end{bmatrix} = \begin{bmatrix} u_1 & u_2 & . & . & u_n \end{bmatrix}\begin{bmatrix} \sigma_1 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & . & 0 & 0 \\ 0 & 0 & 0 & . & 0 \\ 0 & 0 & 0 & 0 & \sigma_n \end{bmatrix}$, here A is a matrix of order $m \times n$. Generally, it can be written as, $AV = UD$, where D is a diagonal matrix while V and U are the unitary matrices as the vectors belong to them are orthonormal.

$$A = UDV^T.$$

This is a singular value decomposition. Now writing a matrix C in Eq. (3.16) in SVD form given by:

$$\sum_{i,j,k} \alpha_{i,j} = \sum_k u_{i,k} c_k v_{k,j}. \tag{3.17}$$

Substituting equation (3.17) in (3.14), we will get.

$$|\psi\rangle = \sum_{i,j,k} u_{i,k} c_k v_{k,j} (|i\rangle_A \otimes |j\rangle_B),$$

$$|\psi\rangle = \sum_k c_k (|k\rangle_A \otimes |k\rangle_B),$$

$$|\psi\rangle = \sum_k \sqrt{\lambda_k} (|k\rangle_A \otimes |k\rangle_B). \tag{3.18}$$

We have defined the orthonormal bases on the system A as $|k\rangle_A = \sum_i u_{i,k}|i\rangle_A$, on system B as $|k\rangle_B = \sum_j v_{k,j}|j\rangle_B$. These orthonormal bases are known as Schmidt bases. While $\lambda_k = c_k^2$ and this is known as Schmidt coefficient. The Schmidt coefficients λ_k are like eigenvalues of a matrix and unique for any state $|\psi\rangle$. We can extract the information related to the entanglement of state in quantum system from the factor of Schmidt coefficient.

Schmidt Coefficient and Its Importance in Checking Separability of State

The standard criteria to check separability of any state $|\psi\rangle$ is that if decomposed state contains one non-zero Schmidt coefficient, then we can say that the state must be separable. On the other hand, if there exists more than one non-zero Schmidt coefficients, then the state $|\psi\rangle$ is not separable [18]. As discussed above the Schmidt coefficients are very helpful in differentiating between entangled and separable states, therefore our main focus is how we can evaluate them. We can do this with the help of reduce density matrices, so then reduced density matrices are explicitly useful.

3.4.2 Purity

As with the help of Schmidt decomposition we can find out easily whether the state is separable or entangled. For checking the degree of entanglement of a state (how much state is entangled) we use the concept of purity. We can characterize the degree of entanglement by the degree of purity of either of subsystem. Purity of any normalized quantum state can be defined as the trace of the squared value of its density operator.

$$P = Tr(\rho^2), \tag{3.19}$$

where the range of a purity is

$$0 < P \leq 1.$$

For subsystem, let us suppose for a subsystem A, purity is defined as: $P = Tr(\rho_A^2) = \sum_k \lambda_k^2$. When purity is equal to one, that means our state of system is separable. And when purity is less than one, the state of a system is entangled.
Measurement of the entanglement can be define as the Schmidt number given by:

$$\kappa \equiv \frac{1}{P} = \sum_{k=0}^{\infty} \frac{1}{\lambda_n^2}. \tag{3.20}$$

In terms of Schmidt number κ, if $\kappa = 1$ this means that the state will be separable. For all other values of κ the state of a composite system will appear to be entangled.

3.5 Composite Boson Creation Operator

Now, for the composite two-particle system, using the vision of second quantization one can express the state vector in form of ladder operators as:

$$|\psi_c\rangle = \hat{c}^\dagger |0\rangle. \tag{3.21}$$

Hence, by comparing it with the equation (3.4) we can say that this creation operator which is creating a particle in a composite system can also be the combination of two other creation operators which can create sub-particle in the relevant subsystem therefore,

$$\hat{c}^\dagger = \sum_{ij}^{\infty} \alpha_{ij} \hat{a}_i^\dagger \hat{b}_j^\dagger, \tag{3.22}$$

where α_{ij} represents probability amplitude of having particle A in $|i\rangle$ basis and particle B in $|j\rangle$ basis. The creation operators for particle A and particle B are $\hat{a}_i^\dagger$ and $\hat{b}_j^\dagger$. In the perspective of entanglement theory, we use the process of decomposition to calculate the probability amplitude therefore we can rewrite the state expressed above as:

$$|\psi_c\rangle = \hat{c}^\dagger|0\rangle = \sum_{n=0}^{\infty} \sqrt{\lambda_n}\hat{a}_n^\dagger \hat{b}_n^\dagger|0\rangle, \tag{3.23}$$

where basis n is the superposition of i and j and $\sqrt{n}$ is the Schmidt coefficient which tells us about the probability of having both particles in the same basis n. The value of λ_n also provides the measure of entanglement as we have discussed in chapter 2. In terms of the Schmidt number κ provides us the following:

$$\kappa = \frac{1}{\sum\limits_{n=0}^{\infty} \lambda_n^2}. \tag{3.24}$$

Thus the operator for composite particle in terms of the Schmidt coefficient is written as

$$\hat{c}^\dagger = \sum_{n=0}^{\infty} \sqrt{\lambda_n}\hat{a}_n^\dagger \hat{b}_n^\dagger. \tag{3.25}$$

The operator $c^\dagger$ can be treated as the ladder operator for the composite particle and we can discuss its properties as well.

3.6 Commutation Relation for Bosonic Operator

Being ladder operator $\hat{c}$ and $\hat{c}^\dagger$, satisfies the Bosonic algebra [19]. If constituent particles A and B both are bosons then the commutation relation results as

$$[\hat{c}, \hat{c}^\dagger] = 1 + \sum_{n=0}^{\infty} \lambda_n(\hat{a}_n^\dagger \hat{a}_n + \hat{b}_n^\dagger \hat{b}_n). \tag{3.26}$$

If constituent particles are fermions then,

$$[\hat{c}, \hat{c}^\dagger] = 1 - \sum_{n=0}^{\infty}[\lambda_n(\hat{a}_n^\dagger \hat{a}_n + \hat{b}_n^\dagger \hat{b}_n)]. \tag{3.27}$$

Collectively, we can write the above relation as:

$$[\hat{c}, \hat{c}^\dagger] = 1 + s\Delta, \tag{3.28}$$

where
$s = +1$ wehn A and B both are bosonic particles.
$s = -1$ wehn A and B both are fermonic particles.
The Δ operator is defined as:

$$\Delta = \sum_{n=0}^{\infty} [\lambda_n (\hat{a}_n^\dagger \hat{a}_n + \hat{b}_n^\dagger \hat{b}_n)]. \tag{3.29}$$

Where the operator Δ apperas as it shows how much composite bosonic operator deviates from pure bosonic operator [20]. Δ should be minimum so that c and $c^\dagger$ will operate similar as pure bosonic operator.

3.7 N Particle State for Composite Particle

The N particle state for composite particle is

$$|N\rangle = \frac{1}{\sqrt{\chi_N}} \frac{(\hat{c}^\dagger)^N}{\sqrt{N!}} |0\rangle, \tag{3.30}$$

where χ_N is the normalization constant and it should be here because $\hat{c}^\dagger$ is not a perfect bosonic creation operator [21]. χ_N measures the bosonic quality of composite bosons overall. When $\chi_N = 1$ it means our composite system is like a pure bosonic system. when $\chi_N = 0$ means system is least bosonic and any intermediate values represents sub-bosonic quality. We can calculate this normalization constant by considering $\langle N|N\rangle = 1$. By taking projection of state $\langle N|$ with itself, we can write χ_N as:

$$\langle 0|\hat{c}^N (\hat{c}^\dagger)^N |0\rangle = N! \chi_N,$$

$$\chi_N = \frac{1}{N!} \langle 0|\hat{c}^N (\hat{c}^\dagger)^N |0\rangle. \tag{3.31}$$

In order to understand that how well c exhibits as bosonic operator, we check its action on the composite particle state $|N\rangle$. This is defined as

$$\hat{c}|N\rangle = \alpha_N \sqrt{N} |N-1\rangle + |\varepsilon_N\rangle, \tag{3.32}$$

where α_N is constant and ε_N is another term which appears to be orthogonal to $|N-1\rangle$. It is basically correction term which should appear here because the state of composite particle $|N\rangle$ is only subset itself of the whole Hilbert space for composite system. The value of α_N can be find out by using following equation:

$$\langle N-1|\hat{c}|N\rangle = \alpha_N \sqrt{N} \langle N-1|N-1\rangle + \langle N-1|\varepsilon_N\rangle. \tag{3.33}$$

$$\Longrightarrow \qquad \langle N-1|\hat{c}|N\rangle = \alpha_N\sqrt{N}. \tag{3.34}$$

Also;

$$\langle N-1|\hat{c}|N\rangle = \sqrt{N}\frac{\langle 0|\hat{c}^N(\hat{c}^\dagger)^N|0\rangle}{\sqrt{\chi_N}\sqrt{\chi_{N-1}}N!}. \tag{3.35}$$

Putting the value from Eq. (3.31) in Eq. (3.35),

$$\langle N-1|\hat{c}|N\rangle = \frac{\sqrt{\chi_N}\sqrt{N}}{\sqrt{\chi_{N-1}}}. \tag{3.36}$$

Comparing Eq. (3.34) and Eq. (3.36), we find the value of α_N,

$$\alpha_N = \sqrt{\frac{\chi_N}{\chi_{N-1}}}. \tag{3.37}$$

3.8 Conditions for Perfect Bosonic Operator

In Eq. (3.32), we can see, bosonic operator will be pure bosonic, if it satisfy the following two conditions:

$$\alpha_N \longrightarrow 1,$$
$$\langle\varepsilon_N|\varepsilon_N\rangle \longrightarrow 0,$$

where $\langle\varepsilon_N|\varepsilon_N\rangle$ can be derive using Eq. (3.32). We can write Eq. (3.32) as:

$$|\varepsilon_N\rangle = \hat{c}|N\rangle - \alpha\sqrt{N}|N-1\rangle.$$

Also

$$\langle\varepsilon_N| = \langle N|\hat{c}^\dagger - \alpha_N\sqrt{N}\langle N-1|.$$

Thus we get

$$\langle\varepsilon_N|\varepsilon_N\rangle = \langle N|\hat{c}^\dagger\hat{c}|N\rangle + {\alpha_N}^2 N - \alpha_N\sqrt{N}\langle N|\hat{c}^\dagger|N-1\rangle - \alpha_N\sqrt{N}\langle N-1|\hat{c}|N\rangle. \tag{3.38}$$

By solving Eq. (3.38), we get,

$$\langle\varepsilon_N|\varepsilon_N\rangle = N-(N-1)\left(1-\frac{\chi_{N+1}}{\chi_N}\right)+N\frac{\chi_N}{\chi_{N-1}}-N+(N)\left(1-\frac{\chi_N}{\chi_{N-1}}\right)-N\left(\frac{\chi_N}{\chi_{N-1}}\right).$$

$$\langle\varepsilon_N|\varepsilon_N\rangle = 1+(N-1)\left(\frac{\chi_{N+1}}{\chi_N}\right)-N\left(\frac{\chi_N}{\chi_{N-1}}\right). \tag{3.39}$$

Thus from Eq. (3.37) and Eq. (3.39), we can see that both conditions depend on ratio of normalization constant. Composite bosonic operator will be like pure bosonic when $\frac{\chi_{N\pm1}}{\chi_N} \rightarrow 1$.

3.9 Bounds on Normalization Factor

As normalization constant is given as:

$$\chi_N = \frac{1}{N!}\langle 0|\hat{c}^N(\hat{c}^\dagger)^N|0\rangle. \tag{3.40}$$

χ_N is derived [22] and is given by:

$$\chi_N^F = N! \sum_{p_1<p_2...<p_N} \lambda_{p_1}\lambda_{p_2}\lambda_{p_3}....\lambda_{p_N}, \tag{3.41}$$

and

$$\chi_N^B = N! \sum_{p_1\leq p_2...\leq p_N} \lambda_{p_1}\lambda_{p_2}\lambda_{p_3}....\lambda_{p_N}, \tag{3.42}$$

where χ_N^F refers to the normalization constant for composite particle is made up of pair of fermions, which is the elementary symmetric polynomial [23]. When our While χ_N^B is the normalization constant, when constituent particles are pair of bosons.
For the case of two particle wave function we can consider χ_N in terms of some specified Schmidt eigenvalues, which allows the very close and exact form to our system. Therefore Schmidt eigenvalue is

$$\lambda_n = (1-z)z^n, n = 0, 1, 2, 3, 4.... \tag{3.43}$$

Here z explains decrease of normalization constant and is defined in the range of $0 < z < 1$. To find normalization constant, we make some assumptions. let we take

$$p_1 = r_N,$$

$$p_2 = r_N + r_{N-1},$$

$$p_3 = r_N + r_{N-1} + r_{N-2},$$

$$\cdot$$

$$\cdot$$

$$\cdot$$

$$p_N = r_N + r_{N-1} + ... + r_1.$$

Substituting above values in Eq. (3.42), normalization constant for pair of bosons become

$$\chi^B{}_N = N!(1-z)^N \sum_{r_1=0}^{\infty}\sum_{r_2=0}^{\infty}\sum_{r_3=0}^{\infty} ...... \sum_{r_N=0}^{\infty} z^{r_1+2r_2+3r_3+.....Nr_N}. \tag{3.44}$$

$$\Rightarrow \quad \chi_N^B = N!(1-z)^N \sum_{r_1=0}^{\infty} z^{r_1} \sum_{r_2=0}^{\infty} (z^2)^{r_2} \sum_{r_3=0}^{\infty} (z^3)^{r_3} + ... \tag{3.45}$$

For Geometric series when it converges for $|r| < 1$

$$s = \sum_{k=0}^{\infty} r^k = 1/(1-r).$$

By solving series in Eq. (3.45), equation becomes:

$$\chi_N^B = \frac{N!(1-z)^N}{(1-z^1)(1-z^2)(1-z^3)......(1-z^N)}. \tag{3.46}$$

Similarly for normalization constant of composite boson made of pair of fermions Eq.(3.41) become

$$\chi_N{}^F = N! \sum_{p_N > p > ....p_2 > p_1} \lambda_{p_1} \lambda_{p_2} ....\lambda_{p_N}. \tag{3.47}$$

After few mathematical steps we reach at equation below

$$\chi^F{}_N = N!(1-z)^N \sum_{r_1=1}^{\infty} \sum_{r_2=1}^{\infty} \sum_{r_3=1}^{\infty} ...... \sum_{r_N=1}^{\infty} z^{r_1+2r_2+3r_3+.....Nr_N}. \tag{3.48}$$

$$\chi_N^F = \frac{N!(1-z)^N}{(1-z^1)(1-z^2)(1-z^3)......(1-z^N)} z^{N(N-1)/2} \tag{3.49}$$

Now we can find normalization ratio, that is

$$\frac{\chi_{N+1}^B}{\chi_N^B} = \frac{(N+1)(1-z)}{1-z^{N+1}}, \tag{3.50}$$

$$\frac{\chi_{N+1}^F}{\chi_N^F} = \frac{z^N(N+1)(1-z)}{1-z^{N+1}}. \tag{3.51}$$

This normalization ratio shows deviation of composite particle from pure boson when further particles are added to the state. Result shows that the ratio of normalization constant for pair of boson is $\frac{\chi_{N+1}^B}{\chi_N^B} > 1$ and for pair of fermions is $\frac{\chi_{N+1}^F}{\chi_N^F} < 1$. Here the difference seen between the normalization ratios of fermions and bosons is because of their difference in nature [24]. Bosons are the particles that can lies in the same state but fermions behave opposite to them as fermions follow Pauli- exclusion principle.
As the value of integer z is between zero and one. We can see from our result that when z approaches to one, ratio of normalization constants approaches to one and

composite boson will behave like pure boson. For z less than one, composite will show deviation depending on the value of normalization ratio. In other words χ_{N+1}/χ_N is interconnected to the strength of correlation in composite boson. As particle behaves as a pure boson when z approaches to one, thus the quantum statistics associated to the constituent particles appears to be less important at that point.

3.9.1 Purity as a Bound for Bosonic Quality

Now we can relate quantum entanglement with the normalization constant by using the definition of quantum number κ. As for the Schmidt eigenvalues given by equation (3.43), Schmidt number κ defined in equation (3.24) becomes:

$$\kappa = \frac{1+z}{1-z},$$

κ increases monotonically in the range of $0 < z < 1$. Degree of entanglement can be related to bose enhancement factor $\frac{\chi^B_{N+1}}{\chi^B_N}$ and $\frac{\chi^F_{N+1}}{\chi^F_N}$ when we express them in terms of z because of the relation $\kappa = \frac{1+z}{1-z}$. We notice that by increasing Schmidt number κ, $\frac{\chi^B_{N+1}}{\chi_N}$ and $\frac{\chi^F_{N+1}}{\chi^F_N}$ approaches to one. Specifically, we can show for $\kappa \gg N$

$$\frac{\chi_{N+1}}{\chi_N} \approx 1 + \frac{sN}{\kappa}, \tag{3.52}$$

where $s = +1$ [21], for bosonic pair and $s = -1$ for fermionic pair.

In this section [25], we have discussed bipartite composite particle wave function, we have provided the basic information about the composite system which tells us that the composite character is directly related to the correlation between the constituent element. Therefore, we can apply a composite representation to those particles which are strongly entangled. Now consider composite particle composed of a fermionic pair [26], then we can explain the above assumptions as follows: Consider a composite particle, consists of a pair of fermions. To find the large entanglement between the two fermions, purity P will be small. Let N be the number of composite particles for the quantum state, therefore the composite particles behaves like an ordinary bosons if they satisfy the following conditions,

$$NP \ll 1.$$

Therefore, according to the above hypothesis, we can get the idea about the number of particles which we can add in any pure state from quantity 1/P, without looking to their composite behavior or before the interference of composite character with the independent ideal behavior of constituents. The ratio χ_{N+1}/χ_N is considered as the quantifier of the bosonic character where χ_N is basically a normalization factor which appears due to the presence of composite behavior which is different from the idea case. Ideally for pure bosons, $\chi_N = 1$ for all N.

Chapter 4

Collective Interference of Composite Bosons

4.1 Interferometry of Elementary Particles

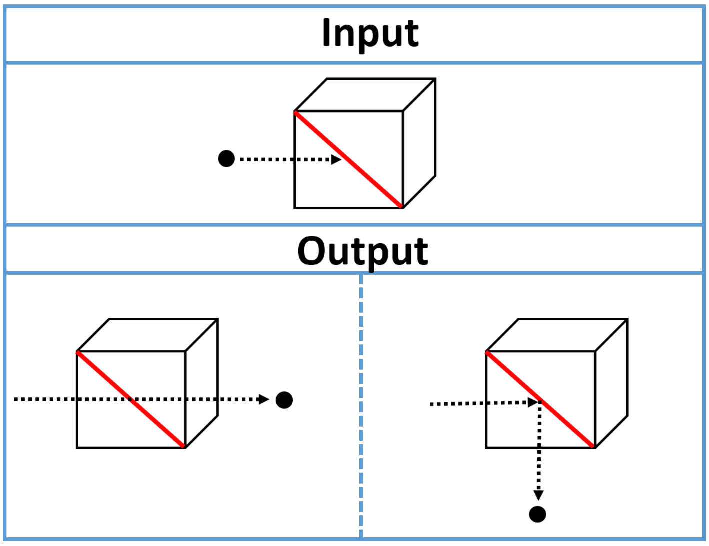

Figure 4.1: single elementary particle can go through of reflect by beam splitter

Consider a single elementary particle as an input of the beam-splitter (BS) whose hamiltonian is given by:

$$H_{BS} = a_L^\dagger a_R + a_R^\dagger a_L,$$

where L and R represents left and right mode of beam splitter. Input particle incident on the right mode transforms as:

$$a_R^\dagger |0\rangle \rightarrow \frac{1}{\sqrt{2}}(a_R^\dagger - i a_L^\dagger) |0\rangle . \tag{4.1}$$

Similarly, input particle incident on the left mode transforms as:

$$a_L^\dagger |0\rangle \rightarrow \frac{1}{\sqrt{2}}(a_L^\dagger - i a_R^\dagger) |0\rangle . \tag{4.2}$$

So we can say that a single particle can go through or reflect by beam-splitter(BS).

4.2 Interferometry of Two Non-interacting Patricles

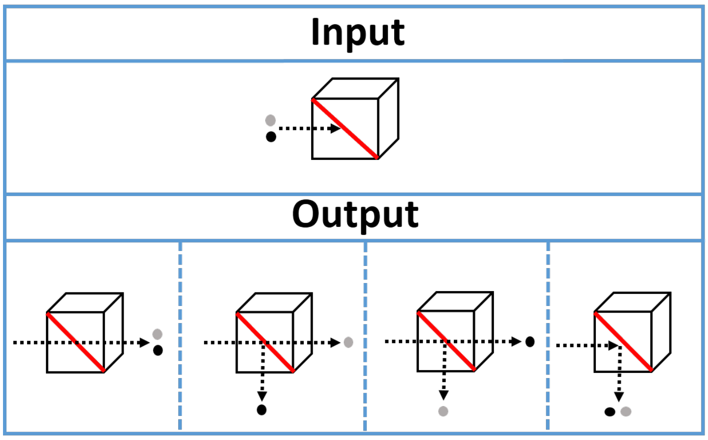

Figure 4.2: Two non-interacting patricles evolve independently, lead to four possible outcomes

Consider two fermions described by two degrees of freedom $a_{i,X}^\dagger$ $a_{i,Y}^\dagger$, where i=1,2,....d and X , Y = R , L. The independent hamiltonain of each particle is given as:

$$H_A = \sum_{i=1}^{d} (a_{i,L}^\dagger a_{i,R}^\dagger + a_{i,R}^\dagger a_{i,L}^\dagger),$$

$$H_B = \sum_{i=1}^{d} (b_{i,L}^\dagger b_{i,R}^\dagger + b_{i,R}^\dagger b_{i,L}^\dagger).$$

If the non-interacting pair of particles is indcident on the left mode of BS then it transforms as:

$$\begin{aligned} c_L^\dagger |0\rangle &= \frac{1}{\sqrt{2}} \sum_{i=1}^{d} a_{i,L}^\dagger b_{i,L}^\dagger |0\rangle \,, \\ &= \frac{1}{\sqrt{2d}} \sum_{i=1}^{d} (a_{i,L}^\dagger b_{i,L}^\dagger - i a_{i,R}^\dagger b_{i,L}^\dagger - i a_{i,L}^\dagger b_{i,R}^\dagger - a_{i,R}^\dagger b_{i,R}^\dagger) |0\rangle \,, \end{aligned} \tag{4.3}$$

where we see that the two particles evolve independently, which leads to four possible outcomes and decay of composite boson.

4.3 Interferometry of Two Interacting Patricles

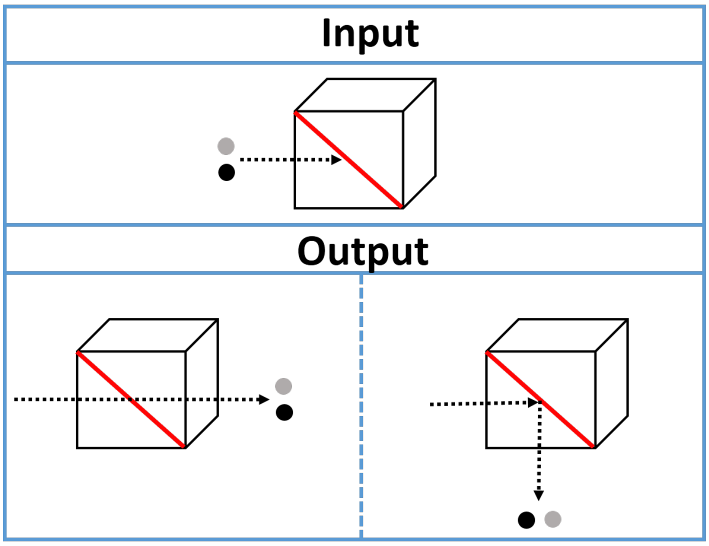

Figure 4.3: Two interacting particles either both go through or both reflects from beam splitter

Consider a coboson, whose two constituent particles are interacting. The interaction hamiltonian is given by:

$$H_{int} = -\gamma \sum_{X=R,L} \sum_{i=1}^{d} a_{i,X}^\dagger a_{i,X} b_{i,X}^\dagger b_{i,X}. \tag{4.4}$$

If the interacting pair of particles is indcident on the left mode of BS, then it transforms as:

$$\hat{c}_L^\dagger |0\rangle = \frac{1}{\sqrt{2}} \sum_{i=1}^{d} \hat{a}_{i,L}^\dagger \hat{b}_{i,L}^\dagger |0\rangle ,$$
$$= \frac{1}{\sqrt{2}} (\hat{c}_L^\dagger - \hat{c}_R^\dagger) |0\rangle . \tag{4.5}$$

The two interacting particles stay together, they collectively go through or reflect from BS which allows to treat them as a single particle.

4.3.1 Two Photon Case

Let us consider a case, in which two photons are incident on the same port of beam-splitter (BS), e.g. $|2,0\rangle$. This state undergoes the transformation as:

$$|2,0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0,2\rangle - i\,|2,0\rangle),$$

here the two photons behave as a single boson system (a bi-boson) which also behave as a pure boson, albeit one with twice the energy and half the wavelength of a single boson.

4.4 Interferometry of Two Cobosons

Consider two cobosons, one as an input on left and other on the right mode of the BS. The initial state is given by:

$$|\psi\rangle_i = c_L^\dagger c_R^\dagger |0\rangle ,$$
$$= \frac{1}{\sqrt{d}} \sum_{i,j=1}^{d} a_{i,L}^\dagger b_{i,L}^\dagger a_{j,R}^\dagger b_{j,R}^\dagger |0\rangle .$$

The two cobosons undergo a transformation and the final state is given as:

$$|\psi\rangle_f = \frac{1}{d\sqrt{\chi_2}} \sum_{i,j=1}^{d} (a_{i,L}^\dagger b_{i,L}^\dagger a_{j,L}^\dagger b_{j,L}^\dagger + a_{i,R}^\dagger b_{i,R}^\dagger a_{j,R}^\dagger b_{j,R}^\dagger),$$
$$= \frac{(c_L^\dagger)^2 + (c_R^\dagger)^2}{2\sqrt{\chi_2}} |0\rangle , \tag{4.6}$$

where

$$\chi_2 = 1 - P.$$

This indicates that the two cobosons bunch together on the same port, just like elementary bosons do.

4.4.1 Two Pairs of Photon

Consider a state of the form $|2,2\rangle$. The Hamiltonian,

$$H = (a^\dagger)^2 b^2 + (b^\dagger)^2 a^2, \tag{4.7}$$

will transform this state to $|4,0\rangle$ and $|0,4\rangle$. Now the states $|4,0\rangle$, $|2,2\rangle$, $|0,4\rangle$ will span a 3-dimensional subspace, lets denote this subspace as H_3. Upon diagonalizing the Hamiltonian we find the eigenvectors of form,

$$\begin{aligned} |\lambda_0\rangle &= \frac{1}{\sqrt{2}}(|4,0\rangle - |0,4\rangle), \\ |\lambda_\pm\rangle &= \frac{1}{2}(|4,0\rangle \pm \sqrt{2}\,|2,2\rangle + |0,4\rangle), \end{aligned}$$

where $H\,|\lambda_\pm\rangle = \pm 3\sqrt{4}\,|\lambda_\pm\rangle$ and $H\,|\lambda_0\rangle = 0$. The three states $|4,0\rangle$, $|2,2\rangle$, $|0,4\rangle$, can be expressed in terms of new basis, as follows,

$$\begin{aligned} |4,0\rangle &= \frac{1}{2}(|\lambda_+\rangle + |\lambda_-\rangle - \sqrt{2}\,|\lambda_0\rangle\,, \\ |2,2\rangle &= \frac{1}{\sqrt{2}}(|\lambda_+\rangle + |\lambda_-\rangle), \\ |4,0\rangle &= \frac{1}{2}(|\lambda_+\rangle + |\lambda_-\rangle + \sqrt{2}\,|\lambda_0\rangle\,. \end{aligned}$$

The evolution operator must have the form,

$$U(t) = \exp(-iHt) = e^{-i\lambda t}\,|\lambda_+\rangle\,\langle\lambda_+| + e^{i\lambda t}\,|\lambda_-\rangle\,\langle\lambda_-| + |\lambda_0\rangle\,\langle\lambda_0|\,. \tag{4.8}$$

Consider here, one photon pair as an input on 1st port of beam-splitter and other photon pair on 2nd port of beam-splitter [28], i.e. the state is $|2,2\rangle$. So we can say that, we have one composite boson as an input on each port of beam-splitter. Thus the state $|2,2\rangle$ evolves under the Hamiltonian as,

$$U(t)\,|2,2\rangle = (e^{-i4\sqrt{3}t}\,|\lambda_+\rangle + e^{i4\sqrt{3}t}\,|\lambda_-\rangle)/\sqrt{2}. \tag{4.9}$$

The probabilities of finding the photons in a particular state at time t is given by,

$$\begin{aligned} P_{22}(t) &= |\,\langle 2,2|\,U(t)\,|2,2\rangle\,|^2 = \cos^2(4\sqrt{3}t), \\ P_{40}(t) &= |\,\langle 4,0|\,U(t)\,|2,2\rangle\,|^2 = \frac{1}{2}\sin^2(4\sqrt{3}t), \\ P_{04}(t) &= |\,\langle 0,4|\,U(t)\,|2,2\rangle\,|^2 = \frac{1}{2}\sin^2(4\sqrt{3}t). \end{aligned}$$

Here if select the time to be $\pi/(8\sqrt{3}$, then $P_{22} = 0$, while $P_{40} = P_{04} = 1/2$. Thus probability of finding photons in both output ports will be zero. So the output state, at $\pi/(8\sqrt{3}$, is given as,

$$U(\frac{\pi}{8\sqrt{3}})\,|2,2\rangle = \frac{-i}{\sqrt{2}}(|4,0\rangle - |0,4\rangle). \tag{4.10}$$

In the above equation, the term corresponding to two photons on each output port $|2,2\rangle$ is missing. This result is analogous to the HOM effect.

4.5 Non-local Bunching

In order to prove he above conjecture, one can consider a scenario where particles a and b are separated, but still entangled [29]. Consider a bell type setup, to show

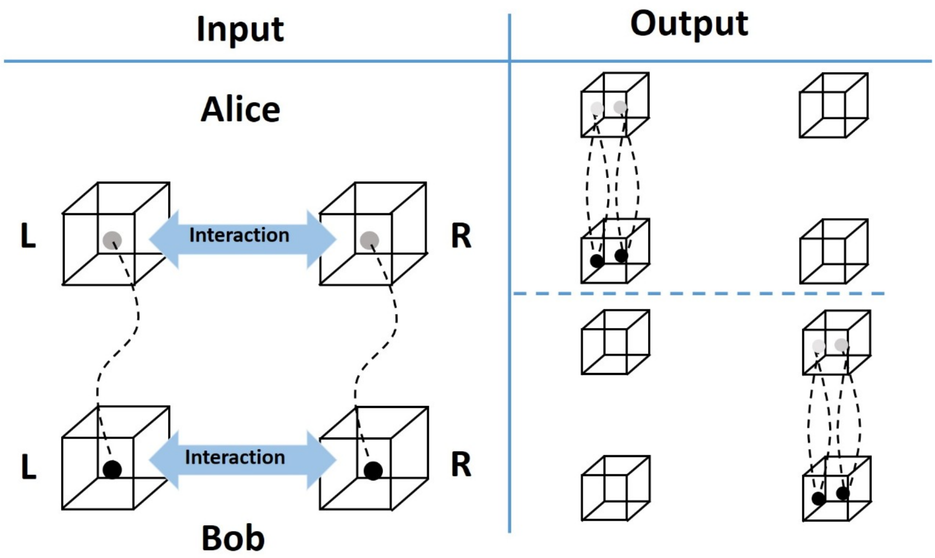

Figure 4.4: Schematic representation of the non-local bunching of two cobosons

the transformation is realisble. We have two spatially separated experimenters, Alice and Bob, who share two cobosons. Each cobosons is split into two basic constituents, fermions of a goes to Alice whereas fermions of b goes to Bob. The entanglement between a and b is still present. Undergoes the transformation as:

$$|\psi\rangle_i = c_L^\dagger c_R^\dagger\,|0\rangle = \frac{1}{d}\sum_{i,j=1}^{d} a_{i,L}^\dagger b_{i,L}^\dagger a_{j,R}^\dagger b_{j,R}^\dagger\,|0\rangle\,, \tag{4.11}$$

The intraction Hamiltonian of particles is given by:

$$H_A = \sum_{i,j=1}^{d} (a^\dagger_{i,L} a^\dagger_{j,L} a_{j,R} a_{i,L} + a^\dagger_{i,R} a^\dagger_{j,R} a_{i,R} a_{j,L} + a^\dagger_{i,L} a^\dagger_{j,R} a_{j,L} a_{i,L} + a^\dagger_{j,L} a^\dagger_{i,R} a_{j,R} a_{i,R}),$$

$$H_B = \sum_{i,j=1}^{d} (b^\dagger_{i,L} b^\dagger_{j,L} b_{j,R} b_{i,L} + b^\dagger_{i,R} b^\dagger_{j,R} b_{i,R} b_{j,L} + b^\dagger_{i,L} b^\dagger_{j,R} b_{j,L} b_{i,L} + b^\dagger_{j,L} b^\dagger_{i,R} b_{j,R} b_{i,R}).$$

The hamiltonian H_A generates the following transformations as:

$$\begin{aligned} a^\dagger_{i,L} a^\dagger_{j,R} &\rightarrow -i a^\dagger_{i,L} a^\dagger_{j,L} \quad for\ all\ \ i > j, \\ a^\dagger_{j,L} a^\dagger_{i,R} &\rightarrow -i a^\dagger_{i,R} a^\dagger_{j,R} \quad for\ all\ \ i > j, \\ a^\dagger_{i,L} a^\dagger_{i,R} &\rightarrow a^\dagger_{i,L} a^\dagger_{i,R}, \end{aligned}$$

Similar transformations are generated by H_B, and the state is transformed as:

$$= \frac{1}{d}(\sum_{i>j=1}^{d} a^\dagger_{i,L} b^\dagger_{i,L} a^\dagger_{j,L} b^\dagger_{j,L} + a^\dagger_{i,R} b^\dagger_{i,R} a^\dagger_{j,R} b^\dagger_{j,R} + \sum_{k=1}^{d} a^\dagger_{k,L} b^\dagger_{k,L} a^\dagger_{k,R} b^\dagger_{k,R}) \left|0\right\rangle , \tag{4.12}$$

$$= (-\frac{(c^\dagger_L)^2 + (c^\dagger_R)^2}{2} + \frac{1}{d} \sum_{k=1}^{d} a^\dagger_{k,L} b^\dagger_{k,L} a^\dagger_{k,R} b^\dagger_{k,R} \left|0\right\rangle , \tag{4.13}$$

$$= -\sqrt{1-P} \left|\psi\right\rangle_f - \sqrt{P} \left|\gamma\right\rangle , \tag{4.14}$$

where

$$\left|\psi\right\rangle_f = \frac{(c^\dagger_L)^2 + (c^\dagger_R)^2}{2\sqrt{\chi_2}} \left|0\right\rangle , \tag{4.15}$$

$$\left|\gamma\right\rangle = \frac{1}{d} \sum_{k=1}^{d} a^\dagger_{k,L} b^\dagger_{k,L} a^\dagger_{k,R} b^\dagger_{k,R} \left|0\right\rangle . \tag{4.16}$$

We found the required state with probability $1 - P$. The probability of required state depends on the degree of entanglement inside the coboson. In case of large entanglement (d » 1), the probability of success approaches to one, since $P \rightarrow 0$. This confirms our claim that bosonic quality is related to degree of entanglement.

Chapter 5

Summary and Conclusion

Summary

Elementary particles are of two types either fermions or bosons. First of all we discuss the interferometry of the elementary fermions and bosons. Bosons show bunching while fermions show anti-bunching phenomena while passing through Beam-splitter. Then we move towards the formalism of composite particles specially bi-partite composite system with Qunatum Information theory tools. We defined a bosonic creation operator $\hat{c}^\dagger = \sum_{n=0}^{\infty} \sqrt{\lambda_n}\hat{a}_n^\dagger \hat{b}_n^\dagger$, this creation operator which will create a pair of A and B particles in the state. The N-composite bosons state is given as $|N\rangle = \frac{1}{\sqrt{\chi_N}}\frac{(\hat{c}^\dagger)^N}{\sqrt{N!}}|0\rangle$, where χ_N is a normalization constant which is necessary to put here since $\hat{c}^\dagger$ is not a perfect bosonic creation operator. The ratio of normalization constants $\frac{\chi_{N\pm1}}{\chi_N}$ is the indicator whether the composite particle will behave like pure boson or not. To find the large entanglement between the two constituent particles purity P will be small. Let N be the number of composite particles for the quantum state. Therefore the composite particles behave like an ordinary bosons if they satisfy the condition $NP \ll 1$. After studying the formulasim of composite particles, then we discuss the interferometry of composite particles using Beam-splitter. In the interferometry part of this thesis first of all we discuss the interferomety of the elementary particles both for fermions as well as bosons. Then we move towards the formalism of composite particles specially bi-partite composite system with Qunatum Information theory. At the end we discuss the problem of collective interference of composite bosons and proves that interaction in important for the stability of composite system.

Conclusion

We deal with the problems of collective interference of composite bosons. First we study the case of interferometry of composite bosons whose constituent particles are interacting while undergoing Beam-splitter transformation, the two interacting particles stay together, they collectively go through or reflect from BS which allows to treat them as a single particle. Then we move towards the interferometry of composite bosons whose constituent particles are non-interacting, where the final state after transformation shows that the two particles evolve independently, which leads to the decay of composite boson. From the results of these two cases one can say that interaction is important for the stability of composite system. Also the interferometry of two cobosons in state $\hat{c}_L^\dagger \hat{c}_R^\dagger |0\rangle$ shows the bunching phenomena as in the standard case two elementary bosons bunch without an interaction but in case of cobosons an interaction is necessary to provide stability of the system for any evolution involving a BS.

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