Some Electrostatic Modes in Two Temperature Electron-Positron and Electron-Positron-Ion Plasma

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Dedicated to my beloved parents, loving brothers and to all my teachers

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Abstract

Linear electrostatic waves are investigated in a magnetized four-component, two temperature electron-positron plasma, with the hot species having the Boltzmann density distribution and the cold species dynamics is governed by the fluid equations. Only the electron plasma mode exist for single temperature electron-positron plasma, but both electron-acoustic and electron plasma modes for parallel propagation are attained with two temperatures. The dispersion characteristics of these modes are analyzed numerically at different density ratios, temperature ratios, and propagation angles. Similarly, in a magnetized three-component electron-positron-ion plasma, linear electrostatic waves are investigated in the low-frequency range. The dispersion relation for the electron-positron-ion plasma composed of cold ions and hot Boltzmann electrons and positrons is derived. It is shown that these two possible modes are always stable for a parallel and perpendicular propagation.

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Chapter 1

Introduction

The Greek word plasma was first used by Czech Jone Evangelista (1788-1868) for clear fluid in the blood [1]. In 1928, Tonks and Langmuir introduced this word to understand the properties of "ionized gas" in a discharge tube [2]. After this, research in this discipline gradually unfold into different directions, for example, radio broadcasting developments led to the discovery of an ionosphere (a partially ionized gas layer in earth's atmosphere) that absorbs and contorts radio waves from time to time [3]. Astrophysicists soon discovered that a large part of the world consists of plasma and thus needs a better grasp of plasma physics to understand astrophysical phenomenons. Around 1942, Hannes Alfven developed a concept of magnetohydrodynamics (MHD) in which plasma is considered as more of a coducting fluid [4]. This concept has been commonly used to investigate sunspots (darker spots on sun's photosphere), solar flares (an abrupt increase in brightness on the sun's surface), the solar wind (the supersonic flow of energetic charged particle, which comes from the sun), star formation, and several other fields of astrophysics.

The discovery of Van Allen's Earth's radiation belts, started out the systematic exploration of the Earth's magnetosphere[5]. The development of high powered lasers broadened the field of laser plasma physics in the 1960s. When a high strength laser interacts with a bulk target then depending upon absorption properties material is ablated from bulk to form plasma plume. The generation of the strong electric field by using a passage of two high power laser pulses in a plasma to speed up particles is an interesting addition to laser-plasma physics [6].

1.1 How can we define Plasma?

Plasma is the fourth form of matter, by heat treatment gas can convert into plasma. In plasma there is practice to call temperature in ev, so typically 1 eV is equal to 10000 degrees temperature. In a gas molecules have Maxwellian distribution, the average kinetic energy of the atom is 1ev, but there are substantial number of atoms with kinetic energies of the order 10 eV or more and then through collision they ionize each other. A simple scheme by which nature produces plasmas in our atmosphere is a process called photoionization. An even simpler scheme in the laboratory to produce a plasma is through discharge either by DC or RF discharges.

Plasmas are quasineutral conductive assemblies of charged particles, neutrals and fields that exhibit collective effects[7]. Quasineutrality means an equal number of $+$ive and $-$ive charge particles on a scale long compare to the debye length scale[5]. It means that in the absence of external disturbance, the plasma will be macroscopically neutral. The ionized gas under certain condition behave like plasma, These conditions are given as

- The Debye length λ_D, provides the measure of the distance over which the influence of the electric field of an individual charged particle is felt by the charged particles inside the plasma. Due to the collective behavior, the charged particles will arrange themselves in such a way as to effectively shield any electrostatic fields within a distance of the order of the debye length, The first criterion of the plasma is that the physical length "L" of the plasma should be greater than the Debye length $\lambda_D \ll L$.

- Number of particles N_D; which is much greater than unity in a sphere of Debye equivalent to $N_D \gg 1$.

- In a partially ionized plasmas, in order to make sure that the electrons are not affected by the neutral particles, the following condition is needed to be met,

$$\omega_{pe}\tau_{en} \gg 1,$$

where ω_{pe} is the plasma frequency, and τ_{en} is the mean time between the collision of electrons with the neutrals [7].

Plasma has a property to restore charge neutrality. If we have a uniform plasma which is made up of e^-s and ions, the mass of an e^- is very small compared to the mass of an ion (approximately 1836 times), so ions can be considered stationary. When electrons are displaced from their mean position by any means, the electric field will be developed between the stationary ions and the displaced electrons. Under the influence of this field, the electrons will move towards the stationary ions. Because of inertia electrons do not stop at their mean position and begin motions about the mean position. The repetative oscillation of e^- is known as the plasma frequency [4]. The plasma frequency is given by the

$$\omega_{pe} = \sqrt{\frac{ne^2}{\epsilon_o m}}$$

where n is the plasma number density, e is the charge on an electron, m is the mass and ϵ_o is free space permittivity. The plasma frequency directly depends on the number density of the plasma. Higher the density the greater will be the frequency of oscillation.
The conductivity of the plasma is given as

$$\sigma = \frac{ne^2}{m\nu}$$

In hot plasma density $n \sim 10^{20}m^{-3}$ and collisional frequency $\nu \sim 10^3 s^{-1}$ whereas in the case of metal density $n \sim 10^{28}m^{-3}$ and collisional frequency $\nu \sim 10^{12}s^{-1}$. As a result conductivity of plasma could be bigger than that of metal as there is 8 order of magnitude difference in density and 9 order of magnitude difference in collision frequency[5]. Electrons in plasma gain energy from electric fields and loss via collisions.

1.2 Various Plasma Environments

The universe's most abundant state of matter is plasma. These matters have different temperature T and density n_o. The properties and environment of plasma change with temperature T and density n_o [5].

1.2.1 Non-Relativistic and Relativistic Plasma Environment

When the thermal energy of particles is very small as in comparison to the rest mass energy the plasma environment is called non-relativistic plasma. If the thermal energy of particles is equal or nearly equal to the rest mass energy the relativistic effects are dominant, such a plasma environment is known as relativistic plasma [8].

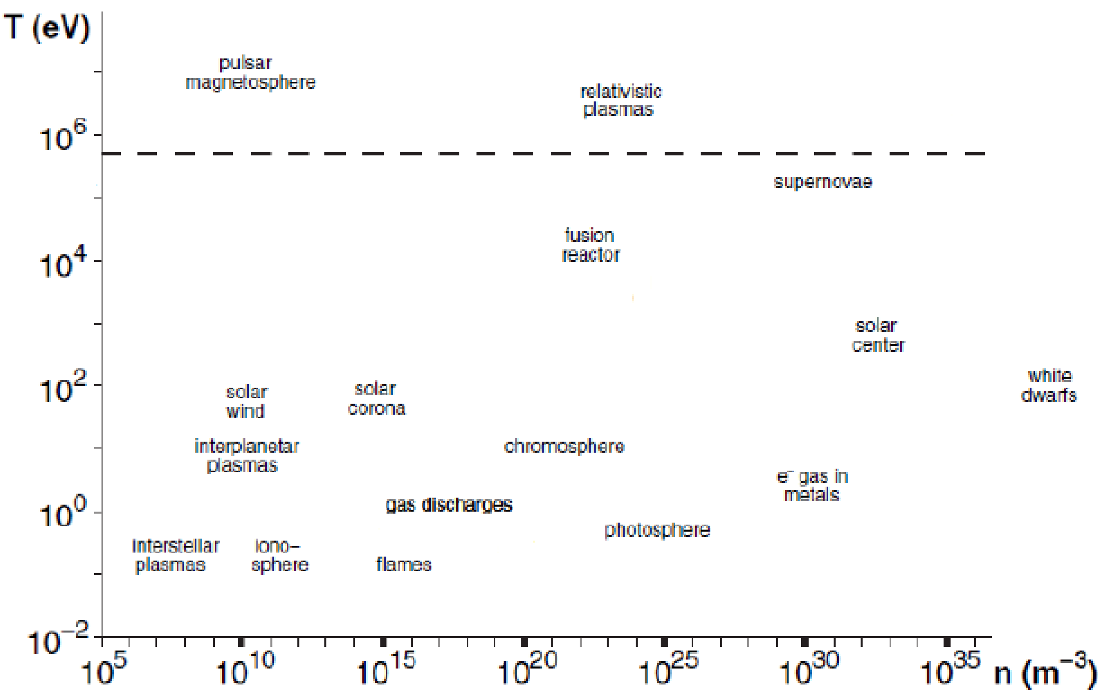

Figure 1.1: Plasma environment with density and temperature

1.2.2 Non-Relativistic Plasma Environments

Earth Ionosphere

Ionosphere is the part of the upper atmosphere of the earth sufficiently ionized by solar radiation to affect the propagation of electromagnetic waves through free electron concentrations. The number of charged particles in the lower layers are less but quite large in higher layers and have a maximum value at the altitude of 300 - 500km. This region of ionosphere is called F-layer. Values of plasma parameter for ionosphere, e.g. F-layer, the density of electron and ions is given as $n_e \approx n_i \approx 10^{12} m^{-3}$ and the temperature is rather high of the order of $(0.26 - 0.43)$ eV [9].

Solar Wind

The outer part of the sun's environment, the solar corona is made out of extremely hot plasma. The flow of energetic charged particles from solar corona into the interplanetary space is known as solar wind. These charged particles has high thermal energy so that sun's gravity cannot retain them in a confined static atmosphere. These particles flow like a wind in the entire solar system with very high velocity. The stream of these particles with very high speed is termed as solar wind. The plasma density for solar wind is $ne \approx ni \approx 10^{12} m^{-3}$ and the temperature is very high of the order of $(75 - 100)$ eV [9].

1.2.3 Relativistic Plasma Environments

Pulsar

Pulsars are very hot, dense and strongly magnetized rotating stars. It's surface temperature is around 51.72 eV and estimated range of magnetic fields on its surface is of the order of $(10^8 \ to \ 10^{15})$ gauss stronger than the earth's attractive field. Therefore the rotation of pulsar along with strong magnetic field generates an electric field. This induced electric field launches charged particles out of the pulsar surface. In all of these particles, electrons get relativistic velocity due to small inertia. Those electrons which move along the bent magnetic lines radiate gamma rays. When the energy of these rays is more than the twice of the rest energy of electrons, these rays are then converted into electron-positron pair. In the electric field this pair is also accelerated and gamma ray photons appear again. In this way, the surface of the pulsars is filled with relativistic electron-positron plasma [8].

1.3 Plasma Models

In plasma physics there are three basic models, the Magnetohydrodynamics (MHD), the Fluid Model and the Kinetic Model. The MHD theory based on the mass momentum and energy conservation laws along with the Maxwell's equations [10]. In the fluid model, the plasma species are considered as distinguishable fluids and all plasma variables are represented as a function of three spatial dimensions and time [4]. In Kinetic theory, there is a complete set of equations for the distribution function of the particles with combination of Maxwell equation. The kinetic model is a statistical explanation for plasma species as particles having different velocities, which figure the average motion of the enormous number of particles. For the statistical explanation of plasma, it is suitable to present a six-dimensional space, called phase space, which contains the position and velocity coordinates. For a large number of particles, it is significant to represent the particles average number density in a small volume element of phase space, which leads to the definition of distribution function. The evolution of distribution function in such a six-dimensional phase space can be described by Vlasov equation which gives us a complete structure for plasma waves and instabilities etc., in collisionless plasma [10].

1.4 Waves in Plasma

Waves that are created in plasmas comprised of fields and particles which oscillate in repeated periods. In the basic form, it is composed of thermal electrons and positive ions, however, it contains distinctive ion species which are: positrons, neutral, dust negative ions. Plasma supports a larger diversity of waves because of the complex arrangement of particles and fields. The investigation of these waves in plasma is very valuable for plasma diagnostics because the various wave modes depend on the plasma qualities. There are various kinds of waves in a plasma, relying upon the course of propagation in accordance with the electric and magnetic field. The waves which commonly exist in plasma are perpendicular, parallel, longitudinal, transverse, electrostatics and electromagnetics[4]. Here $\boldsymbol{E_o}, \boldsymbol{B_o}$ are encompassing electric and magnetic fields and $\boldsymbol{E_1}, \boldsymbol{B_1}$ are the disrupted electric and magnetic fields and $\boldsymbol{k}$ the propagation vector of the wave.

Following terminology is usually used in plasma dynamics.

$\boldsymbol{k} \parallel \boldsymbol{B_o} \rightarrow$ *Parallel propagating waves.*

$\boldsymbol{k} \perp \boldsymbol{B_o} \rightarrow$ *Perpendicular propagating waves.*

$\boldsymbol{k} \parallel \boldsymbol{E_o} \rightarrow$ *Longitudinal* waves.

$\boldsymbol{k} \perp \boldsymbol{E_o} \rightarrow$ *Transverse* waves.

$\boldsymbol{B_1} = 0$ and $\boldsymbol{k} \parallel \boldsymbol{E_1} \rightarrow$ *Electrostatic waves.*

$\boldsymbol{B_1} \neq 0$ and $\boldsymbol{k} \perp \boldsymbol{E_1} \rightarrow$ *Electromagnetic waves.*

1.5 Types of plasma

There are different kinds of plasmas exists in a planetary and intergalactic environment such as electron positron plasma (EP), dusty plasma, multi ion plasma, electron ion plasma (EI), electron positron ion plasma (EPI) etc. A multi-component plasma made up of neutral, charged particle mixtures that meet the conditions of quasi-neutrality. Some of these plasmas can be produced in the laboratory. All of these have different properties because electrons, positrons, and ions have different temperature and densities. This dissertation will be mainly focused on EP and EPI plasmas.

1.6 Positron: An Abundant Antimatter Particle

Positrons are said to be present most commonly as an antiparticle of the visible universe, possessing a mass equal to the electron, but positive charged. In 1931 Dirac provided the theoretical explanation for positron and experimentally Anderson observed it on a cloud chamber photograph. The positrons detected by the Anderson were secondary particles produced by collisions with earth atmosphere molecules by high-energy particles of cosmic rays. Positrons have been identified in numerous astrophysical and earthbound situations.

1.6.1 Astrophysical sources of positrons

The data-based, theoretical studies show that various positron origins are milky way and in the other parts of the universe comprise of:

- **Galactic Cosmic Rays:**

The nuclear reaction of high-energy cosmic rays contribute to positron production. In the various astrophysical environment, relativistic cosmic protons and heavy nuclei, collide inelastically with the interstellar gas, thereby producing mesons. The positively charged mesons ultimately decay to produce secondary positrons [11].

- **Active Galactic Nuclei :**

Positrons also originate from super-massive black holes (SMBH), which reside in the core of several actively functioning galactic nuclei. Positrons are produced by (SMBH), following two possible mechanisms (a) the interaction between SMBH accelerated protons with the molecular clouds in the surrounding result in the formation of mesons which finally decay to produce positrons, (b)The interaction between the γ-ray from the hot inner disk and the X-rays from the outer disk ,result in the production of positrons from the photon. The relativistic wind initiated by the radiation pressure will then blown these positrons to the interstellar medium [11].

- **Solar and stellar flares:**

In the solar environment, positrons have been founded. Setallite borne indicators have discovered that when highly accelerated proton and ion of solar flare hit upon atomic nuclie of photosphere it results in production of radioactive nuclie and pions that decay into positrons [16].

1.6.2 Terrestrial sources of positrons

Positrons are also produced naturally in the earthbound environment and in the laboratories. Some of the earthbound sources include:

- **Van Allen belts:**

The earth is encircled by the two radiation belts which are called, inner and outer Van Allen belts. These are present in the magnetosphere of the earth. These regions include charged particles which are confined to the magnetic field of the earth. These energetic particles come from the primary and secondary cosmic rays and the solar wind. The interaction of the cosmic protons and high Z nuclei which are striking the molecules in the upper atmosphere of the earth produce positrons. The presence of positrons in the Van Allen belts was confirmed by satellite missions [12].

- **Positron traps:**

Numerous storage traps are now being created to further enhance and update storage capacity. In one strategy, positrons coming from Na^{22}, source is backed off by neon mediator, and magnetically led into a three phase Penning-Malmberg trap with a support gas (N_2) to trap and to cool the positrons. The preceeding technique has allowed the trap for roughly 8 minutes to reach $\sim 3 \times 10^8$ positrons with densities of $\sim 8 \times 10^7 cm^{-3}$at a temperature of $300K$. Near future multicell Penning-Malmberg trap is being built to hold a minimum of $\sim 10^{12}$ positrons in density of $\sim 8 \times 10^{10} cm^{-3}$ [13].

- **Radio isotopes:**

Various naturally found as well as artificially prepared radioisotopes, add value to the positron fraction of the earth by beta decay. The positrons from these sources are cooled down by utilizing the mediators (e.g., Nickle, Neon) to frame positron beam that can be used for a number of uses, for instance in electron-positron colliders, in storage rings.

1.6.3 Electron Positron Plasma

Pair plasma comprises of charged particles of the same mass with opposite charge. A positron is antiparticle of an electron having a positive charge. There are many sources of pair production in the laboratory as well as in astrophysical environments. Theoretical interest has been centered around the relativistic regime since much plasma in astrophysical bodies is relativistic in origin. In the laboratory, the pair production takes place through light-matter, light-light interactions. When an intense laser interacts with the solid target(light-matter) or when two high intensity laser beams (light-light) interact with each other then pair production occurs. In light-matter interaction pair production occurs through two processes, first is BH-process (Bethe-Heitler) and second is Trident process.The laser has accelerated the electrons into MeV energies during the BH method. These accelerated electrons then interact with nuclei of the target material to give bremsstrahlung radiation (Radiation resulting from deceleration of charged particles, the loss of kinetic energy of moving particles is converted into photons by complying with the energy conservation law), which then interact with cores to create electron-positron sets. Moreover, the accelerated electrons in the Trident system produce pairs immediately after interaction with target nuclei. In light-matter interaction when laser interacts with a solid target the density of positron as high as $10^{16}cm^{-3}$ and pair density approaching $10^{21}cm^{3}$ has been acheived[13].These processes occur at a fundamental frequency known as plasma frequency. When the frequency of the laser is greater then plasma frequency $\omega \gg \omega_p$ than laser interaction with plasma can occur otherwise laser cannot pass through the plasma. On the other hand, when counter propagating laser beam pass through dense plasma they accelerated the plasma electrons to ultra-relativistic speed, which thusly produce photons having energy practically identical to laser photons. The energy of these photons is $\sim 100MeV$, when they interact with laser photons then pairs are produced. Paul trap was one of the idea put forward for confining both signs of charge with long confinement times [14]. It is well known thatEP plasma is commonly seen in the early universe [15], and are and is often found in the Van Allen Belts, in active galactic nuclei, in the vicinity of pulsar magnetosphere [16].

1.6.4 Time for annihilation vs. time for plasma effects

The experimental study on EP and EPI plasmas requires the experimenters to be highly skilled. This is due to the annihilation of positrons with plasma electrons, resulting in reduced densities of electrons and positrons. There are two procedures that result in decreased e^- and e^+ densities (i) direct annihilation of positrons with plasma electrons and (ii) radiative capture of electron on to create a bound e^- and e^+ state known as positronium. Pair annihilate on a short time scale, Can an EP plasma be maintained long enough to study collective processes in EP plasma? The answer is a "yes" if the annihilation time scale is slower than the plasma effect time scale, that is reverse of the plasma frequency ($\omega_p \approx 10^9 GHz$)[7].

At low energy, positronium atom decay time for singlet state is $10^{-10} sec$, whereas triplet state decays in $10^{-7} sec$. At 300K the recombination rate is $10^{-11} cm^3 sec^{-1}$, thus for a plasma density of $10^{10} cm^{-3}$, the annihilation time scale for positronium atom formation will be 10 sec while the direct annihilation time will be 100 sec,i.e much longer than the plasma oscillation time $10^{-9} sec$.Direct anihilation is dominant than loss process for energies above 100 eV, i.e. for 10 keV positrons, the rate of direct annihilation is around $10^{-14} cm^3 sec^{-1}$i.e. lower than at low energies. So in either case, the EP plasma will live long enough for many collective oscillations before it is annihilated [17, 18].

1.6.5 Distinct Features of EP Plasma

EP plasma possesses some distinct features quite different from conventional electron ion plasma;

1. The ratio of electron mass (m_e) to ion mass (M) is heavily exploited in standard electron ion plasma, leading to differentiation between high (electron dominated) and low (ion dominated) frequency propagation. But in EP plasma, owing to the same mass and magnitude of charge, the dynamical behavior of electron and positron is identical.
2. The dynamic behavior of a EP pair could be different around massive objects, where gravity becomes significant, as the pair plasma is less bound to gravity than ions.
3. Unlike electron-ion plasma, e^- and e^+ gyrate in the presence of an ambient B_0 with the same frequencies but in opposite directions. For a condition $n_+ = n_-$, the EP plasma couple to the right and left circularly polarized waves is equal in contrast with EI plasma .
4. The velocity of Alfven wave in EP plasma is enhanced and Whistler mode is absent as compared to electron-ion plasma.

1.6.6 Electron-Positron-Ion Plasma

The EPI plasma having an equal amount of electrons and ions with some small concentration of positron. EPI plasma is present in the interstellar medium, the matter and radiation that exists in a galaxy's space between the star systems. This matter contains atomic, ionic and molecular gas and cosmic radiation. In Astrophysical objects, positrons are produced when a cosmic ray nuclei interact with atom, however in laboratory plasma a short relativistically strong laser pulse interact with matter as a result epi plasma is formed due to pair production.

$$\gamma + A \rightarrow A + e^+ + e^-$$

"Advanced Satellite for Cosmology and Astrophysics (ASCA)" has revealed the presence of a fraction of ion in astrophysical plasmas[19], therefore the equilibrium quasi-neutrality condition for epi plasma is,

$$n_{eo} \approx Zn_{io} + n_{po}$$

When ions are introduced in e-p plasma then there dynamic scales changes significantly, linear wave spectrum also increases and the wide range of frequencies are available in three component EPI plasma, as compared to two component (EP, EI) plasma. Another aspect of EPI plasma is that many nonlinear phenomena could happen and the possibility of wave-wave interaction increases. In EPI plasma the lifetime of the positron is greater than EP plasma. For many years, different authors investigated the waves propagation in EPI plasma to describe the dynamics properties of astrophysical and laboratory. The effects of Ion temperature on high amplitude ion acoustic waves were studied in EPI plasma by Nejoh [20]. The temperature of the electron and positron is greater than that of the ion, so that the electron and positron act either fluid or the distribution of Boltzmann while the ion is handled as fluid.

Chapter 2

Mathematical Model

A fluid model shows plasmas in terms of smooth quantities, which do not need velocity dependence information. In this theory, plasma is treated as a fluid containing charge particles. In that situation we can't talk about single particle but the charge and current density. In a usual fluid the constituent's particles go on moving always due to regular collisions among themselves. In plasma there are two interpenetrating fluids i.e. electrons and ions fluid. In a fluid model, each species is to be composed of particles with similar velocities. The fluid model is based on some fluid equations. The equation of continuity, which states that the fluid can't be produced nor be destroyed. For every fluid element velocity vector $v_\alpha(x,t)$ gives the velocity at the position x at interval of time t. Then continuity equation for any specie α can be

$$\frac{\partial n_\alpha (x,t)}{\partial t} + \nabla \cdot (n_\alpha \overrightarrow{v_\alpha}) = 0, \tag{2.1}$$

where n_α is number density of corresponding species and $\overrightarrow{\nabla}$ is the del operator in three dimensional space. The fluid theory lies in the fact that the dynamics of neutral fluid has been extensively studied and many aspects of their behaviors are well understood and describe most of basic plasma phenomena [4, 21].

2.1 The Fluid Equation Of Motion

2.1.1 Collisionless Plasma

Maxwell's equations explain the behavior of $\boldsymbol{E}$ and $\boldsymbol{B}$ in given state of plasma. Usually we consider the plasma containing two or more fluid charged particles in the fluid approximation for each species. There are two motion equations, one for positive ion and the other for negative ion fluid, when there is only one species in the pair-ion plasma. Electrons and ions interact with neutral fluid just because of collisions, but the opposite charged fluids interact with each other without collisions since they produce the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ [4]. The equation of motion for particle having single charge can be

$$m\frac{d\boldsymbol{v}\left(x,t\right)}{dt} = q\left(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}\right) \tag{2.2}$$

By assuming that the fluid does not have thermal movements and collisions. Then we can say that all the particles move together and their average velocity u is the single particle velocity v. The fluid equation for the particles is obtain by multiplying the Eq.(2.2) with number density

$$nm\frac{d\boldsymbol{v}\left(x,t\right)}{dt} = nq\left(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B}\right)$$

Consider $v(x,t)$ as any fluid property in 1D in x space. Changing speed v with time in the moving frame is a sum of two terms.

$$\frac{d\boldsymbol{v}\left(x,t\right)}{dt} = \partial_t\boldsymbol{v} + \partial_x\boldsymbol{v}\partial_t x = \partial_t\boldsymbol{v} + (\boldsymbol{v}\cdot\boldsymbol{\nabla})\boldsymbol{v}$$

Then equation of motion for α specie ($\alpha = e, p, i$) in the absence of collisions can be written as,

$$m_\alpha n_\alpha[\partial_t\boldsymbol{v_\alpha} + (\boldsymbol{v_\alpha}\cdot\boldsymbol{\nabla})\boldsymbol{v_\alpha}] = n_\alpha q_\alpha\left(\boldsymbol{E} + \boldsymbol{v_\alpha}\times\boldsymbol{B}\right) \tag{2.3}$$

2.1.2 Collisional Plasma

Suppose, if we consider a plasma with a tiny proportion of neutrals, the charge particles interact with neutrals through collisions and exchange their momentum. In any collision the loss of momentum is proportionate with the relative velocity $\boldsymbol{v} - \boldsymbol{v_o}$ where $\boldsymbol{v_o}$

is the neutral particle velocity [5]. The frequency of collision between charged particles is neu v, ∇p is pressure gradient term which arises due to nonuniformity in density and pressure, so momentum equation for collisonal plasma can be expressed as

$$m_\alpha n_\alpha \left[\partial_t \boldsymbol{v_\alpha} + (\boldsymbol{v_\alpha} \cdot \boldsymbol{\nabla})\boldsymbol{v_\alpha}\right] = n_\alpha q_\alpha \left(\boldsymbol{E} + \boldsymbol{v_\alpha} \times \boldsymbol{B}\right) - \boldsymbol{\nabla} p_\alpha - m_\alpha n_\alpha \nu(\boldsymbol{v} - \boldsymbol{v_o}) \quad (2.4)$$

2.2 Electrostatic Waves in Plasma

Electrostatic waves only have an electric field component without a magnetic field component. For waves like this, the propagating vector $\vec{k}$ is always in line with the electric field i.e. $\boldsymbol{k} \| \mathbf{E}$. Electrostatic waves are known to have only electrical field while the magnetic component of these waves is zero [5]. Some of the well known ES waves that accur in plasma are discussed below.

2.2.1 Electron Plasma Waves

In electron plasma oscillation, the $e^{-}s$ are oscillating about their mean position with frequency ω_{pe} with no thermal motion. This implies that the propagating vector k is null. When $k \sim 0$ results in group velocity being zero, which indicates that they are not transmitting information in plasma from one position to another. But it can propagate the plasma oscillations when the thermal motion of the electrons is included. The exchange or transfer of such information is known as the Langmuir waves via electron thermal motion. Analytically such waves can be obtained with the inclusion of pressure term i.e. $-\nabla P_e$ (due to thermal motion of electrons) in the equation of momentum. For Langmuir waves the dispersal relation is expressed as

$$\omega^2 = \omega_{pe}^2 + \frac{3}{2} k^2 v_{th}^2, \quad (2.5)$$

where $\omega_{pe} = \sqrt{\frac{n_0 e^2}{\varepsilon_0 m}}$ is the plasma frequency and $v_{th} = \sqrt{\frac{T_e}{m}}$ is thermal speed of electrons [4].

2.2.2 Electrostatic Electron Waves Perpendicular to $\boldsymbol{B_0}$

Electrons respond to high frequency oscillations. Ions are massive so that they are generally considered stationary and generate a uniform positive charge background. We consider electrostatic case so $\boldsymbol{B_1} = 0$, and k parallel to $\boldsymbol{E_1}$ so they are longitudinal plane wave propagating perpendicular to $\boldsymbol{B_0}$. Under these assumptions $\boldsymbol{E_0} = 0 = v_0$, $k_B T_e = 0$ (we neglect thermal motion as their is no perturbation in density or uniform plasma) we get the dispersion relation,

$$\omega^2 = \omega_{pe}^2 + \omega_{ce}^2 = \omega_{UH}^2 \tag{2.6}$$

where ω_{pe} is the oscilation frequency of electron, ω_{ce} is the cyclotron frequency of electron and define as

$$\omega_{pe} = \frac{n_o e^2}{m\epsilon_o}$$

$$\omega_{ce} = \frac{eB_0}{m}$$

where ω_{UH} is the upper hybrid frequency. It is ' hybrid ' as it is a combination of plasma frequency and cyclotron frequency. ES electron waves which are moving across the $\boldsymbol{B_0}$ have upper hybrid frequency and the waves which are moving along $\boldsymbol{B_0}$ have only plasma frequency. The magnetic field exerts force on electrostatic electron which are propagating perpendicular to $\boldsymbol{B_0}$ and changes their direction into an elliptical path, instead of oscillating along a straight line. When the electrons are relocated from their mean position, the electric field will be developed in such way that it opposes the motion of electrons, but in the initial stage magnetic forces are strong compared to the electric force and motion of particles is governed due to magnetic force. When the speed of particles increases the Lorentz force also increases. As the motion of particles is against the electric field so they lose energy. There are two forces acting on charged particles that propagate across the $\boldsymbol{B_0}$. One is known as the electrostatic force and the other is the Lorentz force. This additional Lorentz force gives an increase in the frequency.

2.2.3 Acoustic Waves

The mechanical longitudinal vibrations can pass through all kinds of matter, and can be interpreted as sound. The matter supporting the sound is called the medium. Sound waves exist as a variation of pressure in a medium. Because sound is the vibration of matter, it does not travel through a vacuum.

• Sound Waves

A sound wave causes alternate regions of compression and rarefactions at each point in the fluid. We can produce sound in air and can communicate with each other by producing a region of compression and rarefaction.
To describe a sound wave we use the ideal fluid equation in the simplest form.

Density balance equation is

$$\partial_t \rho_m + \boldsymbol{\nabla} \cdot (\rho_m \vec{v}) = 0$$

Momentum balance equation has no lorentz force term, as they all are neutrals

$$\rho_m \partial_t \vec{v_1} + (\vec{v} \cdot \nabla)\vec{v} = \boldsymbol{\nabla} P$$

Now linearizing these equation for small perturbation about equilibrium.Since the oscillations are small, so the nonlinear term $(\vec{v} \cdot \nabla)\vec{v}$ is neglected. ρ_{m_o} is the density of fluid which is constant.
After linearization

$$\partial_t \rho_{m_1} + \rho_{m_o} \boldsymbol{\nabla} \cdot \vec{v_1} = 0 \tag{2.7}$$

$\boldsymbol{\nabla} \cdot \vec{v_1}$ is fluid velocity compression to carry sound wave.

$$\rho_{m_o} \partial_t \vec{v_1} = -\gamma \frac{P_o}{\rho_{m_o}} \boldsymbol{\nabla} \rho_{m_1} \tag{2.8}$$

All perturbed quantity are varying as $\exp(\iota(kx - \omega t))$,
$\partial_t \to -\iota\omega$, $\partial_x \to \iota \boldsymbol{k}$

Applying Fourier-Laplace transformation on the Eqs (2.7) *and* (2.8).

$$\rho_{m_1} = \frac{\iota \boldsymbol{k} \cdot \vec{v_1}}{\omega} \rho_{m_o} \tag{2.9}$$

$$\vec{v_1} = -\gamma \frac{P_o}{\omega \rho_{m_o}^2} \boldsymbol{k} \rho_{m_1} \tag{2.10}$$

Eqs (2.10) says that gradient in density causes the fluid velocity to change in a compressible way. By inserting the value of fluid velocity in Eqs(2.9)

We get,

$$\omega^2 = k^2 v_s^2$$

Where $v_s = \sqrt{\frac{\gamma P_o}{\rho_{m_o}}} = \sqrt{\frac{\gamma K_B T}{M}}$. These are waves in which information propagates at same speed as the phase velocity.

• Ion Acoustic Waves (IAWs)

The frequency of ES electron wave is very high in comparison to both plasma and cyclotron frequencies, but because of the large masses the response of ions in a field is very low. Therefore electrostatic ion waves are low frequency waves. Here we let, $\boldsymbol{E_0} = \boldsymbol{v_0}$, $K_B T_i \neq 0$. The dispersion of electrostatic ion acoustic waves is given as,

$$\omega = k \sqrt{\frac{\gamma_i K_B T_i + K_B T_e}{M + m}}$$

where $v_s = \sqrt{\frac{\gamma_i K_B T_i + K_B T_e}{M+m}}$, is ions acoustic speed.

Electron Acoustic Waves (EAWs)

Electron acoustic wave is another notable mode. In EAWs the electrons are thermally divided into two distinct groups, respectively hot and cold electrons. Inertia is supplied by cold electrons while the hot electron pressure produces the restoring force and the ions are supposed to be fixed in the background.[22]. Here these two independent components have different concentrations and temperatures. The colder electron population is indicated by subscript c, the hotter one by subscript h. From the study of quasineutality, the undisturbed densities satisfy the relation

$$n_{oi} \approx n_{oc} + n_{oh}, \tag{2.11}$$

while the phase velocity ranging is

$$v_{ti} \sim v_{tc} \ll \frac{\omega}{k} \ll v_{th} \tag{2.12}$$

The thermal speed is specified for any α species i.e. $v_{t\alpha} = \sqrt{\frac{K_B T_\alpha}{m_\alpha}}$. The dispersion relation for EAWs can be formulated along with the above assumptions [4, 21],

$$\omega_{ea}^2 = \frac{\omega_{pc}^2}{1 + 1/k^2\lambda_{Dh}^2}\left(1 + 3k^2\lambda_{Dc}^2 + \frac{3n_{0h}}{n_{0c}}\frac{T_c}{T_h}\right). \tag{2.13}$$

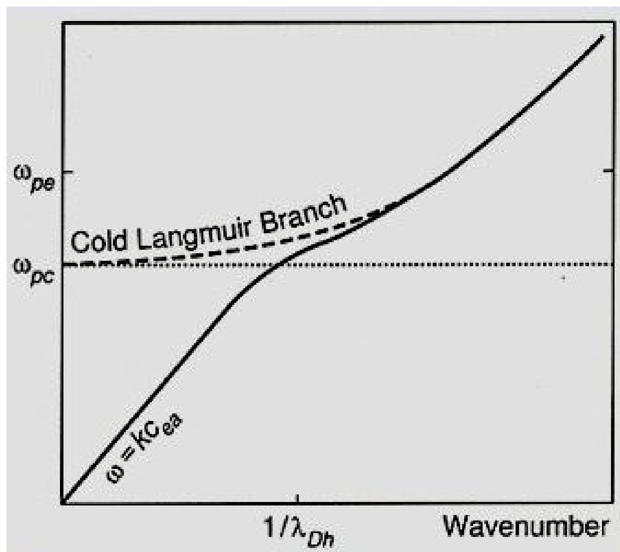

Figure 2.1: Electron acoustic waves

In Eq. (2.13) ion contribution has been ignored and the wave mode becomes entirely electronic. From long wavelength approximation, $k^2\lambda_{Dh}^2 \ll 1$, the dispersion relation becomes an acoustic wave. Defining electron acoustic speed as,

$$c_{ea} = \left(\frac{n_{0c}}{n_{0h}}\right)^{\frac{1}{2}} v_{th}, \tag{2.14}$$

EAWs satisfy the relation for a long wavelength

$$\omega^2 = k^2 c_{ea}^2 \tag{2.15}$$

From short wavelength approximation, Eq. (2.13) becomes a modified Langmuir wave,

$$\omega_{ea}^2 = \omega_{pc}^2 \left(1 + 3k^2\lambda_{Dc}^2 + \frac{3n_{0h}}{n_{0c}}\frac{T_c}{T_h}\right)$$

The above two ideas of long wavelength and short wavelength are given in Fig. 1.2, which gives an idea about the different regions of waves. In this figure EAWs propagate like sound waves by using long wavelength approximation, while from short wavelength we get the cold electron Langmuir wave.

It was evaluated that Landau damping in this system would reject their creation[23]. Lev Davidovich for the first time gave the clue about Landau damping which explained the collisionless wave particle interaction [24]. By the population of trapped electrons, the un-damped EAWs can occur which was discussed in Ref.[25]. The EAWs heavily damped by electron Landau damping. There are two types of regimes one is weakly damped and another one is a strongly damped regime, the electron acoustic mode is weakly damped when $T_c = T_i$. Classically, for weakly damped one must have $\frac{T_h}{T_c} > 10$, but at high concentration of cold electrons, no weakly damped regime exists. For a strongly damped region, we use the idea of short wavelength approximation [21].

Chapter 3

Electrostatic Modes in EP and EPI Plasma

This chapter uncovers linear electrostatic modes in EP and EPI plasma. For this purpose, magnetized plasma consists of cold and hot species. Fluid equations are used to govern the dynamics of cooler species and hot species are intended to have Boltzmann distribution. For different electrostatic modes, a linear dispersion relation is obtained.

3.0.1 Linear Elecrostatic Wave in Two Temperture EP Plasma:

To study linear ES modes in EP plasma, magnetized plasma consisting of four components with two temperature electron positron is studied. These different populations must coexist in time scales that are shorter than species thermalization period. So we have, cool electrons and cool positrons at the same temperature T_c with equal number densities of n_{oc}, and hot electron positron at the same temperature T_h with equal number densities of n_{oh}. Temperatures are expressed in eV. In the xy-plane, wave propagation is carried out at an angle θ to the external magnetic field B_0, that is supposed to be in direction of x- axis. It is presumed that the hot isothermal species are not magnetized and have the Boltzmann distribution. We will derive a boltzmann distribution for hot species having thermal velocity greater then phase velocity of the mode.

Equation of motion:

$$mn[\partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v}] = -en\vec{E} - \nabla P$$

$$\partial_t \vec{v}_{1e} = -\frac{e}{m}\vec{E}_1 - \frac{\gamma T_h}{(n_o + n_1)m}\nabla n_{1eh}$$

All perturbed quantity are varying as $\exp(\iota(kx - \omega t))$

$$-i\omega\vec{v}_{1e} = -\frac{e}{m}i\vec{k}\phi_1 - \frac{v_{th}^2}{n_o}i\vec{k}n_{1eh}$$

$$\vec{v}_{1e} = -\frac{e}{m\omega}\vec{k}\phi_1 + \frac{v_{th}^2}{n_o\omega}\vec{k}n_{1eh} \tag{1}$$

Continuity equations:

$$\partial_t n_{eh} + \nabla \cdot (n_{eh}\vec{v}_e) = 0$$

$$\partial_t n_{1eh} + n_o(\nabla \cdot \vec{v}_{1e}) = 0$$

$$n_{1eh} = n_o\frac{\vec{k} \cdot \vec{v}_{1e}}{\omega}$$

Using vale of $\vec{v_{1e}}$ from Eq.1

$$n_{1eh} = n_o\left[\frac{-e}{m\omega^2}k^2\phi_1 + \frac{v_{th}^2}{n_o\omega^2}k^2 n_{1eh}\right]$$

$$n_{1eh} = \frac{-n_o e k^2\phi_1}{m(\omega^2 - k^2 v_{th}^2)}$$

For $\omega^2 \ll k^2 v_{th}^2$, we get

$$n_{1eh} = \frac{n_o e\phi_1}{T_h}$$

The cosequence is that the electron can quickly follow the variation in the potential of wave. Boltzmann's assumption of the distribution for hot $e^-(e^+)$ is justified, as long the temperature is enough higher than colder species so that their thermal velocities parallel to the magnetic field surpass phase velocity of the mode so they can set up the Boltzmann distribution.

$$n_{1eh} = n_{oh} \exp\left(\frac{e\phi}{T_h}\right) \tag{3.1}$$

$$n_{1ph} = n_{oh} \exp\left(\frac{-e\phi}{T_h}\right) \tag{3.2}$$

where n_{1eh}, n_{1ph} is the number density of the hot electrons (positrons) and ϕ is the electrostatic potential. However, if the perturbed wavelength is shorter than their gyro radii, it is justified to treat hot species as unmagnetized, so that both hot electrons and positrons take mainly straight line orbits across the direction of the magnetic field. The magnetic field effect on hot species is not sensed in such condition. Fluid equations govern the dynamics of cold isothermal species

For electron:

Continuity equations:

$$\partial_t n_{ec} + \nabla \cdot (n_{ec}\vec{v}_e) = 0$$

As $\nabla n_o = 0, \partial_t n_o = 0, v_o = 0$

$$\partial_t n_{1ec} + n_{oec}(\nabla \cdot \vec{v}_{1e}) = 0$$

$$-\iota\omega n_{1ec} = -n_{oec} i(k_\parallel v_{1ex} + k_\perp v_{1ey})$$

$$n_{1ec} = \frac{n_{oec}(k_\parallel v_{1ex} + k_\perp v_{1ey})}{\omega} \tag{3.3}$$

Equation of motion:

$$mn[\partial_t\vec{v}+(\vec{v}\cdot\nabla)\vec{v}] = -en\left[\vec{E}+\vec{v}\times\vec{B}\right]-\nabla P$$

$$\partial_t\vec{v}_{1e}+(\vec{v}_{1e}\cdot\nabla)\vec{v}_{1e} = -\frac{e}{m}\left[\vec{E}_1+\vec{v}_{1e}\times\vec{B}_o\right]-\frac{\gamma T_c}{(n_o+n_1)m}\nabla n_{1ec}$$

$$\partial_t\vec{v}_{1e} = -\frac{e}{m}\left[-\nabla\phi_1+\vec{v}_{1e}\times\vec{B}_o\right]-\frac{\gamma T_c}{n_o m}\nabla n_{1ec} \tag{3.4}$$

$x-$component of Eq. (3.4) takes the following form

$$\partial_t v_{1ex} = \frac{e}{m}\partial_x\phi_1-\frac{\gamma T_c}{n_o m}\partial_x n_{1ec}$$

$$v_{1ex} = -\frac{ek_{\parallel}\phi_1}{m\omega}+\frac{v_{tc}^2 k_{\parallel} n_{1ec}}{n_o m\omega} \tag{i}$$

$y-$ component of Eq. (3.4) takes the following form

$$\partial_t v_{1ey} = \frac{e}{m}\partial_y\phi_1-\frac{eB_o v_z}{m}-\frac{\gamma T_c}{n_o m}\partial_y n_{1ec}$$

$$-i\omega v_{1e} = \frac{e}{m}ik_{\perp}\phi_1-\Omega v_z-\frac{v_{tc}^2}{n_o}ik_{\perp}n_{1ec} \tag{ii}$$

$z-$component of Eq. (3.4) takes the following form

$$\partial_t v_{1ez} = \frac{eB_o}{m}v_{1ey}$$

$$v_{1ez} = -\frac{\Omega v_{1ey}}{i\omega} \tag{iii}$$

To find v_{iey} we use Eqs.(ii) and (iii)

From Eq. (iii)

$$v_{1ez} = \frac{\Omega}{-\iota\omega} v_{1ey} \tag{iv}$$

Using relation (iv) in equation (ii), we get:

$$i\omega v_{1ey} = \frac{e}{m} i k_{\perp} \phi_1 + \frac{\Omega^2 v_{1ey}}{i\omega} - \frac{v_{tc}^2}{n_o m} i k_{\perp} n_{1ec}$$

$$(\omega^2 - \Omega^2) v_{1ey} = \frac{-e}{m} \omega k_{\perp} \phi_1 + \frac{v_{tc}^2}{n_o m} \omega k_{\perp} n_{1ec}$$

$$v_{1ey} = \frac{\omega \left(\frac{-e k_{\perp} \phi_1}{m} + \frac{v_{tc}^2 k_{\perp} n_{1ec}}{n_o m} \right)}{(\omega^2 - \Omega^2)} \tag{v}$$

Using v_{1ex} and v_{1ey} in Eq. (3.3)

$$n_{1ec} = n_o \left(k_{\parallel} \left(-\frac{e k_{\parallel} \phi_1}{m\omega^2} + \frac{v_{tc}^2 k_{\parallel} n_{1ec}}{n_o m \omega^2} \right) + k_{\perp} \frac{\left(\frac{-e k_{\perp} \phi_1}{m} + \frac{v_{tc}^2 k_{\perp} n_{1ec}}{n_o m} \right)}{m(\omega^2 - \Omega^2)} \right)$$

$$n_{1ec} = -\frac{n_o e k_{\parallel}^2 \phi_1}{m\omega^2} + \frac{v_{tc}^2 k_{\parallel}^2 n_{1ec}}{m\omega^2} - \frac{n_o e k_{\perp}^2 \phi_1}{m(\omega^2 - \Omega^2)} + \frac{v_{tc}^2 k_{\perp} n_{1ec}}{(\omega^2 - \Omega^2)}$$

$$n_{1ec} = -\frac{n_o e k_{\parallel}^2 \phi_1}{m\omega^2} - \frac{n_o e k_{\perp}^2 \phi_1}{m(\omega^2 - \Omega^2)} + \frac{v_{tc}^2 k_{\parallel}^2 n_{1ec}}{m\omega^2} + \frac{v_{tc}^2 k_{\perp} n_{1ec}}{(\omega^2 - \Omega^2)}$$

$$n_{1ec} = \frac{n_o e \phi_1 (-k_{\parallel}^2 (\omega^2 - \Omega^2) - k_{\perp}^2 \omega^2)}{m\omega^2 (\omega^2 - \Omega^2)} + \frac{n_{1ec} v_{tc}^2 (k_{\parallel}^2 (\omega^2 - \Omega^2) + k_{\perp}^2 \omega^2)}{\omega^2 (\omega^2 - \Omega^2)}$$

$$n_{1ec} = \frac{-n_o e \phi_1 ((\omega^2 k^2 - \Omega^2 k_{\parallel}^2)}{m\omega^2 (\omega^2 - \Omega^2)} + \frac{n_{1ec} v_{tc}^2 (\omega^2 k^2 - \Omega^2 k_{\parallel}^2)}{\omega^2 (\omega^2 - \Omega^2)}$$

$$n_{1ec} = \frac{-ek^2 n_o \phi_1(\omega^2 - \Omega^2 \cos^2\theta)}{m\omega^2(\omega^2 - \Omega^2)} + \frac{k^2 v_{tc}^2 n_{1ec}(\omega^2 - \Omega^2\cos^2\theta)}{\omega^2(\omega^2 - \Omega^2)}$$

$$n_{1ec}\left[\frac{\omega^2(\omega^2 - \Omega^2) - k^2 v_{tc}^2(\omega^2 - \Omega^2\cos^2\theta)}{\omega^2(\omega^2 - \Omega^2)}\right] = \frac{-ek^2 n_o \phi_1(\omega^2 - \Omega^2\cos^2\theta)}{m\omega^2(\omega^2 - \Omega^2)}$$

$$n_{1ec} = \frac{-ek^2 n_o \phi_1(\omega^2 - \Omega^2\cos^2\theta)}{m[\omega^2(\omega^2 - \Omega^2) - k^2 v_{tc}^2(\omega^2 - \Omega^2\cos^2\theta)]}$$

$$n_{1ec} = \frac{-ek^2 n_o \phi_1(\omega^2 - \Omega^2\cos^2\theta)}{m[\omega^4 - \omega^2(k^2 v_{tc}^2 + \Omega^2) + k^2 v_{tc}^2 \cos^2\theta)]} \tag{3.5}$$

Now we will do a calculation for positron density perturbations

For positron:

Continuity equations:

$$\partial_t n_{pc} + \nabla \cdot (n_{pc}\vec{v_p}) = 0$$

As $\nabla n_o = 0, \partial_t n_o = 0, v_o = 0$

All perturbed quantity are varying as $\exp(\iota(kx - \omega t))$,after linearization

$$\partial_t n_{1pc} + n_{opc}(\nabla \cdot \vec{v}_{1p}) = 0$$

$$-\iota\omega n_{1pc} = -n_{opc} i(k_\parallel v_{1px} + k_\perp v_{1py})$$

$$n_{1pc} = \frac{n_{opc}(k_\parallel v_{1px} + k_\perp v_{1py})}{\omega} \tag{3.6}$$

Equation of motion:

$$mn[\partial_t \vec{v} + (\vec{v}\cdot\nabla)\vec{v}] = en\left[\vec{E} + \vec{v}\times\vec{B}\right] - \nabla P$$

$$\partial_t \vec{v}_{1p} + (\vec{v}_{1p}\cdot\nabla)\vec{v}_{1p} = \frac{e}{m}\left[\vec{E}_1 + \vec{v}_{1p}\times\vec{B}_o\right] - \frac{\gamma T_c}{(n_o+n_1)m}\nabla n_{1pc}$$

$$\partial_t \vec{v}_{1p} = \frac{e}{m}\left[\vec{E}_1 + \vec{v}_{1p}\times\vec{B}_o\right] - \frac{\gamma T_c}{n_o m}\nabla n_{1pc} \tag{3.7}$$

$x-$component of Eq. (3.8) takes the following form

$$\partial_t v_{1px} = \frac{-e}{m}\partial_x \phi_1 - \frac{\gamma T_c}{n_o m}\partial_x n_{1ec}$$

$$v_{1px} = \frac{ek_\parallel \phi_1}{m\omega} + \frac{v_{tc}^2 k_\parallel n_{1pc}}{n_o m\omega} \tag{v}$$

$y-$ component of Eq. (3.8) takes the following form

$$\partial_t v_{1py} = -\frac{e}{m}\partial_y \phi_1 - \frac{eB_o v_z}{m} - \frac{\gamma T_c}{n_o m}\partial_y n_{1pc}$$

$$-i\omega v_{1py} = \frac{-e}{m} ik_\perp \phi_1 + \Omega v_z - \frac{v_{tc}^2}{n_o m} ik_\perp n_{1pc} \tag{vi}$$

$z-$component of Eq. (3.8) takes the following form

$$\partial_t v_{1pz} = \frac{-eB_o}{m} v_{1py}$$

$$v_{1pz} = \frac{\Omega v_{1py}}{i\omega} \tag{vii}$$

To find v_{1py}, we use Eqs.(vi) and (vii)

From Eq.(vii)

$$v_{1pz} = \frac{\Omega}{\iota\omega} v_{1py} \tag{viii}$$

Using relation (viii) in Eq. (vi), we get:

$$-i\omega v_{1py} = \frac{-e}{m} ik_\perp \phi_1 + \frac{\Omega^2 v_{1py}}{i\omega} - \frac{v_{tc}^2}{n_o m} ik_\perp n_{1pc}$$

$$(\omega^2 - \Omega^2) v_{pey} = \frac{e}{m} \omega k_\perp \phi_1 - \frac{v_{tc}^2}{n_o m} \omega k_\perp n_{1pc}$$

$$v_{1ey} = \frac{\omega \left(\frac{ek_\perp \phi_1}{m} + \frac{v_{tc}^2 k_\perp n_{1ec}}{n_o m} \right)}{(\omega^2 - \Omega^2)} \tag{xi}$$

Using v_{1px} and v_{1py} in Eq. (3.6)

$$n_{1pc} = n_o \left(k_\parallel \left(\frac{ek_\parallel \phi_1}{m\omega^2} + \frac{v_{tc}^2 k_\parallel n_{1pc}}{n_o m \omega^2} \right) + k_\perp \frac{\left(\frac{ek_\perp \phi_1}{m} + \frac{v_{tc}^2 k_\perp n_{1pc}}{n_o m} \right)}{m(\omega^2 - \Omega^2)} \right)$$

$$n_{1pc} = \frac{n_o e k_\parallel^2 \phi_1}{m\omega^2} + \frac{v_{tc}^2 k_\parallel^2 n_{1pc}}{m\omega^2} + \frac{n_o e k_\perp^2 \phi_1}{m(\omega^2 - \Omega^2)} + \frac{v_{tc}^2 k_\perp n_{1pc}}{(\omega^2 - \Omega^2)}$$

$$n_{1pc} = \frac{n_o e k_\parallel^2 \phi_1}{m\omega^2} + \frac{n_o e k_\perp^2 \phi_1}{m(\omega^2 - \Omega^2)} + \frac{v_{tc}^2 k_\parallel^2 n_{1ec}}{m\omega^2} + \frac{v_{tc}^2 k_\perp n_{1pc}}{(\omega^2 - \Omega^2)}$$

$$n_{1pc} = \frac{n_o e \phi_1 (k_\parallel^2 (\omega^2 - \Omega^2) + k_\perp^2 \omega^2)}{m\omega^2 (\omega^2 - \Omega^2)} + \frac{n_{1pc} v_{tc}^2 (k_\parallel^2 (\omega^2 - \Omega^2) + k_\perp^2 \omega^2)}{\omega^2 (\omega^2 - \Omega^2)}$$

$$n_{1pc} = \frac{n_o e \phi_1 ((\omega^2 k^2 - \Omega^2 k_\parallel^2)}{m\omega^2 (\omega^2 - \Omega^2)} + \frac{n_{1pc} v_{tc}^2 (\omega^2 k^2 - \Omega^2 k_\parallel^2)}{\omega^2 (\omega^2 - \Omega^2)}$$

$$n_{1pc} = \frac{ek^2 n_o \phi_1(\omega^2 - \Omega^2 \cos^2\theta)}{m\omega^2(\omega^2 - \Omega^2)} + \frac{k^2 v_{tc}^2 n_{1pc}(\omega^2 - \Omega^2 \cos^2\theta)}{\omega^2(\omega^2 - \Omega^2)}$$

$$n_{1pc}\left[\frac{\omega^2(\omega^2 - \Omega^2) - k^2 v_{tc}^2(\omega^2 - \Omega^2 \cos^2\theta)}{\omega^2(\omega^2 - \Omega^2)}\right] = \frac{ek^2 n_o \phi_1(\omega^2 - \Omega^2 \cos^2\theta)}{m\omega^2(\omega^2 - \Omega^2)}$$

$$n_{1pc} = \frac{ek^2 n_o \phi_1(\omega^2 - \Omega^2 \cos^2\theta)}{m[\omega^2(\omega^2 - \Omega^2) - k^2 v_{tc}^2(\omega^2 - \Omega^2 \cos^2\theta)]}$$

$$n_{1pc} = \frac{ek^2 n_{oc} \phi_1(\omega^2 - \Omega^2 \cos^2\theta)}{m[\omega^4 - \omega^2(k^2 v_{tc}^2 + \Omega^2) + k^2 v_{tc}^2 \cos^2\theta)]} \qquad (3.8)$$

Poisson equation:

$$\epsilon_o \nabla \cdot E = e(n_{pc} - n_{ec} + n_{ph} - n_{eh})$$

The uniform number density of cold species are equal and the same applies to hot species.

$$\epsilon_o \nabla \cdot E_1 = e(n_{1pc} - n_{1ec} + n_{1ph} - n_{1eh})$$

$$k^2 \epsilon_o \phi_1 = e(n_{1pc} - n_{1ec} + n_{1ph} - n_{1eh}) \qquad (3.9)$$

From Eqs (3.1) and (3.2) the perturbed densities for hot species are provided as

$$n_{1eh} = n_{oh}\frac{e\phi_1}{T_h}$$

$$n_{1ph} = -n_{oh}\frac{e\phi_1}{T_h}$$

Using values of n_{1ec},n_{1pc},n_{1eh} and n_{1ph} in Eq. (3.11)

$$k^2\epsilon_o\phi_1 = \frac{e^2k^2n_{oc}\phi_1(\omega^2-\Omega^2\cos^2\theta)}{m[\omega^4-\omega^2(k^2v_{tc}^2+\Omega^2)+k^2v_{tc}^2\cos^2\theta)]}+\frac{e^2k^2n_{oc}\phi_1(\omega^2-\Omega^2\cos^2\theta)}{m[\omega^4-\omega^2(k^2v_{tc}^2-\Omega^2)+k^2v_{tc}^2\cos^2\theta)]}-n_{oh}\frac{e^2\phi}{T_h}$$

$$k^2\epsilon_o = \frac{2e^2k^2n_{oc}(\omega^2-\Omega^2\cos^2\theta)}{m[\omega^4-\omega^2(k^2v_{tc}^2+\Omega^2)+k^2v_{tc}^2\cos^2\theta)]}-\frac{2n_{oh}e^2}{T_h}$$

$$\frac{k^2T_h}{m}+\frac{2n_{oh}e^2m}{m\epsilon_o} = \frac{2e^2k^2n_{oc}T_h(\omega^2-\Omega^2\cos^2\theta)}{m\epsilon_o[\omega^4-\omega^2(k^2v_{tc}^2+\Omega^2)+k^2v_{tc}^2\cos^2\theta)]}$$

$$m(k^2v_{tc}^2+2\omega_{ph}^2) = \frac{2e^2k^2n_{oc}T_h(\omega^2-\Omega^2\cos^2\theta)}{m\epsilon_o[\omega^4-\omega^2(k^2v_{tc}^2+\Omega^2)+k^2v_{tc}^2\cos^2\theta)]}$$

$$(k^2v_{th}^2+2\omega_{ph}^2)[\omega^4-\omega^2(k^2v_{tc}^2+\Omega^2)+k^2v_{tc}^2\cos^2\theta)] = \frac{2e^2k^2n_{oc}T_h(\omega^2-\Omega^2\cos^2\theta)}{m^2\epsilon_o}$$

$$(k^2v_{th}^2+2\omega_{ph}^2)(\omega^2(\omega^2-\Omega^2))-k^2v_{tc}^2(\omega^2-\Omega^2\cos^2\theta)(k^2v_{th}^2+2\omega_{ph}^2) = 2k^2\omega_{pc}^2v_{th}^2(\omega^2-\Omega^2\cos^2\theta)$$

$$\omega^2(\omega^2-\Omega^2)-k^2v_{tc}^2(\omega^2-\Omega^2\cos^2\theta)-\frac{2k^2\omega_{pc}^2v_{th}^2(\omega^2-\Omega^2\cos^2\theta)}{(k^2v_{th}^2+2\omega_{ph}^2)} = 0$$

$$\omega^2(\omega^2-\Omega^2)-\left(k^2v_{tc}^2+\frac{2k^2\omega_{pc}^2v_{th}^2}{(k^2v_{th}^2+2\omega_{ph}^2)}\right)(\omega^2-\Omega^2\cos^2\theta) = 0 \qquad (3.10)$$

where $\omega_{ph}^2 = \frac{n_{oh}e^2}{m\epsilon_o}$ is plasma frequency of hot species ,$\omega_{pc}^2 = \frac{n_{oc}e^2}{m\epsilon_o}$ is plasma frequency of cold species and $v_{tc} = \sqrt{\frac{T_h}{m}}$ is thermal speed of cold species.

Now applying low frequency condition as $\omega \ll \Omega\cos\theta$ and $T_c \ll T_h$

$$-\omega^2\Omega^2 - \left(k^2v_{tc}^2 - \frac{2k^2\omega_{ph}^2v_{th}^2}{(k^2v_{th}^2 + 2\omega_{ph}^2)}\right)(-\Omega^2\cos^2\theta) = 0$$

$$\omega^2 = \frac{2k^2\omega_{ph}^2v_{th}^2\cos^2\theta}{\omega_{ph}^2(\frac{k^2v_{th}^2}{\omega_{ph}^2} + 2)}$$

where $\lambda_{dh} = \frac{v_{th}^2}{\omega_{ph}^2}$ is debye length of hot species.

$$\omega^2 = \frac{2n_{oc}k^2v_{th}^2\cos^2\theta}{n_{oh}(k^2\lambda_{dh} + 2)}$$

Where $v_{ea} = \sqrt{\frac{n_{oc}}{n_{oh}}}v_{th}$ is electron acoustic speed.

$$\omega^2 = \frac{k^2v_{ea}^2\cos^2\theta}{(\frac{1}{2}k^2\lambda_{dh} + 1)}$$

In the absences of hot species,insert $\omega_{ph} = 0$ in Eq. (12)

$$\omega^4 - \omega^2\Omega^2 - \left(k^2v_{tc}^2 + \frac{2k^2\omega_{pc}^2v_{th}^2}{k^2v_{th}^2}\right)(\omega^2 - \Omega^2\cos^2\theta) = 0$$

$$\omega^4 - \omega^2\Omega^2 - (k^2v_{tc}^2 + 2\omega_{pc}^2)(\omega^2 - \Omega^2\cos^2\theta) = 0$$

$$\omega^4 - \omega^2\Omega^2 - k^2v_{tc}^2\omega^2 + k^2v_{tc}^2\Omega^2\cos^2\theta - 2\omega_{pc}^2\omega^2 + \omega_{pc}^2\Omega^2\cos^2\theta = 0$$

$$\omega^4 - \omega^2(\Omega^2 + 2\omega_{pc}^2 + k^2 v_{tc}^2) + (k^2 v_{tc}^2 + 2\omega_{pc}^2)\Omega^2 \cos^2\theta = 0$$

$$\omega^4 - \omega^2(\omega_{UH}^2 + k^2 v_{tc}^2) + (k^2 v_{tc}^2 + 2\omega_{pc}^2)\Omega^2 \cos^2\theta = 0$$

Where $\omega_{UH}^2 = \Omega^2 + 2\omega_{pc}^2$ is the upper hybrid frequency linked with the colder species, the two extreme limits of Eq.(12) will be considered in order to obtain physical understanding of the dispersion relation.

Perpendicular propagation $(\theta = 90^o)$

$$\omega^2(\omega^2 - \Omega^2) - \left(k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{(k^2 v_{th}^2 + 2\omega_{ph}^2)}\right)\omega^2 = 0$$

$$\omega^2\left(\omega^2 - \Omega^2 - k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{(k^2 v_{th}^2 + 2\omega_{ph}^2)}\right) = 0$$

There are two normal modes to above equation ,The first is $\omega = 0$, non-propagating mode and second is the electron cyclotron mode in e-p plasma with the input of both adiabatic colder species thermal motion and acoustic motion corresponding to two distinct temperature species.

$$\omega^2 = \Omega^2 + k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{(k^2 v_{th}^2 + 2\omega_{ph}^2)}$$

Parallel propagation $(\theta = 90^o)$

$$\omega^2(\omega^2 - \Omega^2) - \left(k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{(k^2 v_{th}^2 + 2\omega_{ph}^2)}\right)(\omega^2 - \Omega^2) = 0$$

$$(\omega^2 - \Omega^2)\left[\omega^2 - \left(k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{(k^2 v_{th}^2 + 2\omega_{ph}^2)}\right)\right] = 0$$

Similarly above dispersion relation has two modes,

$$\omega^2 = \Omega^2$$

and

$$\omega^2 = k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{k^2 v_{th}^2 + 2\omega_{ph}^2}$$

This mode is the electron cyclotron mode. When there is no hot species, $\omega_{ph} = 0$, this mode turned into electron plasma mode resulting from the motion of colder species, which indicates that electron acoustic mode appears due to the presences of hot species.

$$\omega^2 = k^2 v_{tc}^2 + 2\omega_{pc}^2$$

3.0.2 Electrostatic modes in three-component EPI plasma:

For the study of linear electrostatic modes, we considered collision-free, three-component plasma, consist of cold ions and hot electron positron. In the xy-plane, wave propagation is carried out at an angle θ to the ambient magnetic field B_0, that is supposed to be in the x direction. The set of fluid equations for colder species (ion) and Boltzmann densities distributed of hot species are given as follows,

$$n_{1eh} = n_{0h} \exp\left(\frac{e\phi}{T_h}\right) \tag{3.11}$$

$$n_{1ph} = n_{0h} \exp\left(\frac{-e\phi}{T_h}\right) \tag{3.12}$$

where n_{1eh}, n_{1ph} are the number density of the hot electrons (positrons) and ϕ represents the electrostatic potential.

For ion:

Continuity equations:

$$\partial_t n_i + \nabla \cdot (n_i \vec{v}_i) = 0$$

As $\nabla n_o = 0, \partial_t n_o = 0, v_o = 0$

All perturbed quantity are varying as $\exp\left(\iota(kx - \omega t)\right)$,after linearization

$$\partial_t n_{1i} + n_{oi}(\nabla \cdot \vec{v}_{1i}) = 0$$

$$-\iota\omega n_{1i} = -n_{opc} i(k_{\parallel} v_{1ix} + k_{\perp} v_{1iy})$$

$$n_{1ic} = \frac{n_{oi}(k_{\parallel} v_{1ix} + k_{\perp} v_{1iy})}{\omega} \tag{3.13}$$

Equation of motion:

$$mn[\partial_t \vec{v} + (\vec{v} \cdot \nabla)\vec{v}] = en\left[\vec{E} + \vec{v} \times \vec{B}\right] - \nabla P$$

$$\partial_t \vec{v}_{1i} + (\vec{v}_{1i} \cdot \nabla)\vec{v}_{1i} = \frac{e}{m}\left[\vec{E}_1 + \vec{v}_{1i} \times \vec{B}_o\right] - \frac{\gamma T_i}{(n_{oi} + n_{1i})m_i} \nabla n_{1i}$$

$$\partial_t \vec{v}_{1i} = \frac{e}{m}\left[\vec{E}_1 + \vec{v}_{1i} \times \vec{B}_o\right] - \frac{\gamma T_i}{n_{oi} m_i} \nabla n_{1i} \tag{3.14}$$

$x-$component of Eq. (3.14) takes the following form

$$\partial_t v_{1ix} = \frac{-e}{m_i}\partial_x \phi_1 - \frac{3T_i}{n_o m_i}\partial_x n_{1i}$$

$$v_{1ix} = \frac{ek_{\parallel}\phi_1}{m_i\omega} + \frac{3v_{ti}^2 k_{\parallel} n_{1i}}{n_o\omega} \tag{a}$$

$y-$ component of Eq. (3.16) takes the following form

$$\partial_t v_{1iy} = -\frac{e}{m_i}\partial_y \phi_1 - \frac{eB_o v_z}{m} - \frac{3T_i}{n_o m_i}\partial_x n_{1i}$$

$$-i\omega v_{1iy} = \frac{-e}{m_i} ik_{\perp}\phi_1 + \Omega v_z - \frac{3v_{ti}^2}{n_o} ik_{\perp} n_{1i} \tag{b}$$

$z-$component of Eq. (3.16) takes the following form

$$\partial_t v_{1iz} = \frac{-eB_o}{m_i} v_{1iy}$$

$$v_{1iz} = \frac{\Omega_i v_{1iy}}{i\omega} \tag{c}$$

To find v_{1iy}, we use Eqs.(b) and (c)

From Eq.(c)

$$v_{1iz} = \frac{\Omega_i}{\iota\omega} v_{1iy} \tag{d}$$

Using relation (d) in Eq. (b), we get:

$$-i\omega v_{1iy} = \frac{-e}{m_i} ik_\perp \phi_1 + \frac{\Omega_i^2 v_{1iy}}{i\omega} - \frac{3v_{ti}^2}{n_{oi}} ik_\perp n_{1i}$$

$$(\omega^2 - \Omega_i^2) v_{1iy} = \frac{e}{m_i} \omega k_\perp \phi_1 - \frac{3v_{ti}^2}{n_{oi}} \omega k_\perp n_{1i}$$

$$v_{1iy} = \frac{\omega \left(\frac{ek_\perp \phi_1}{m_i} + \frac{3v_{ti}^2 k_\perp n_{1i}}{n_{oi}} \right)}{(\omega^2 - \Omega_i^2)} \tag{e}$$

Using v_{1px} and v_{1py} in Eq. (3.13)

$$n_{1i} = n_{oi} \left(k_\parallel \left(\frac{ek_\parallel \phi_1}{m_i \omega^2} + \frac{3v_{ti}^2 k_\parallel n_{1i}}{n_{oi} \omega^2} \right) + k_\perp \frac{\left(\frac{ek_\perp \phi_1}{m_i} + \frac{3v_{ti}^2 k_\perp n_{1i}}{n_{oi}} \right)}{(\omega^2 - \Omega_i^2)} \right)$$

$$n_{1i} = \frac{n_{oi} e k_\parallel^2 \phi_1}{m_i \omega^2} + \frac{3v_{ti}^2 k_\parallel^2 n_{1i}}{\omega^2} + \frac{n_{oi} e k_\perp^2 \phi_1}{m_i(\omega^2 - \Omega_i^2)} + \frac{3v_{ti}^2 k_\perp n_{1i}}{(\omega^2 - \Omega_i^2)}$$

$$n_{1i} = \frac{n_{oi} e k_\parallel^2 \phi_1}{m_i \omega^2} + \frac{n_{oi} e k_\perp^2 \phi_1}{m_i(\omega^2 - \Omega_i^2)} + \frac{3v_{ti}^2 k_\parallel^2 n_{1i}}{\omega^2} + \frac{3v_{ti}^2 k_\perp n_{1i}}{(\omega^2 - \Omega_i^2)}$$

$$n_{1i} = \frac{n_{oi} e \phi_1 (k_\parallel^2 (\omega^2 - \Omega_i^2) + k_\perp^2 \omega^2)}{m_i \omega^2 (\omega^2 - \Omega_i^2)} + \frac{n_{1i} 3v_{ti}^2 (k_\parallel^2 (\omega^2 - \Omega_i^2) + k_\perp^2 \omega^2)}{\omega^2 (\omega^2 - \Omega_i^2)}$$

$$n_{1i} = \frac{n_{oi} e \phi_1 ((\omega^2 k^2 - \Omega_i^2 k_\parallel^2)}{m_i \omega^2 (\omega^2 - \Omega_i^2)} + \frac{n_{1i} 3v_{ti}^2 (\omega^2 k^2 - \Omega^2 k_\parallel^2)}{\omega^2 (\omega^2 - \Omega_i^2)}$$

$$n_{1i} = \frac{e n_{oi} \phi_1 k^2 (\omega^2 - \Omega_i^2 \cos^2 \theta)}{m_i \omega^2 (\omega^2 - \Omega_i^2)} + \frac{3v_{ti}^2 k^2 n_{1i} (\omega^2 - \Omega_i^2 \cos^2 \theta)}{\omega^2 (\omega^2 - \Omega_i^2)}$$

$$n_{1i} \left[\frac{\omega^2 (\omega^2 - \Omega_i^2) - 3v_{ti}^2 k^2 (\omega^2 - \Omega_i^2 \cos^2 \theta)}{\omega^2 (\omega^2 - \Omega_i^2)} \right] = \frac{e n_{oi} \phi_1 k^2 (\omega^2 - \Omega_i^2 \cos^2 \theta)}{m_i \omega^2 (\omega^2 - \Omega_i^2)}$$

$$n_{1i} = \frac{en_{oi}\phi_1 k^2(\omega^2 - \Omega_i^2\cos^2\theta)}{m_i[\omega^2(\omega^2 - \Omega^2) - 3v_{ti}^2 k^2(\omega^2 - \Omega_i^2\cos^2\theta)]}$$

$$n_{1i} = \frac{en_{oi}\phi_1 k^2(\omega^2 - \Omega_i^2\cos^2\theta)}{m_i[\omega^4 - \omega^2(3v_{ti}^2 k^2 + \Omega^2) + 3v_{ti}^2 k^2\cos^2\theta)]} \tag{3.15}$$

Poisson's equation:

$$\epsilon\nabla\cdot E = e(n_{ph} - n_{eh} + n_i)$$

Uniform number density of electron is equal to combine number density of ion and positron.

$$\epsilon_o\nabla\cdot E_1 = e(n_{1ph} - n_{1eh} + n_{1i})$$

$$k^2\epsilon_o\phi_1 = e(n_{1ph} - n_{1eh} + n_{1i}) \tag{3.16}$$

From Eqs. (3.13) and (3.14), the perturbed densities for hot species are expressed as

$$n_{1eh} = n_{oh}\frac{e\phi_1}{T_h}$$
$$n_{1ph} = -n_{oh}\frac{e\phi_1}{T_h}$$

Using values of n_{1ph},n_{1eh} and n_{1i} in Eq. (3.19)

$$k^2\epsilon_o\phi_1 = \frac{e^2k^2n_{oi}\phi_1(\omega^2 - \Omega_i^2\cos^2\theta)}{m_i[\omega^4 - \omega^2(3k^2v_{ti}^2 + \Omega^2) + 3k^2v_{ti}^2\cos^2\theta)]} - n_{oh}\frac{e^2\phi_1}{T_h} - n_{oh}\frac{e^2\phi_1}{T_h}$$

$$k^2\epsilon_o = \frac{e^2k^2n_{oi}(\omega^2 - \Omega_i^2\cos^2\theta)}{m_i[\omega^4 - \omega^2(3k^2v_{ti}^2 + \Omega^2) + 3k^2v_{ti}^2\cos^2\theta)]} - \frac{2e^2n_{oh}}{T_h}$$

$$\frac{k^2T_h\epsilon_o}{e^2n_{oh}} + 2 = \frac{k^2n_{oi}T_h(\omega^2 - \Omega_i^2\cos^2\theta)}{m_in_{oh}[\omega^4 - \omega^2(3k^2v_{ti}^2 + \Omega^2) + 3k^2v_{ti}^2\cos^2\theta)]}$$

$$(\lambda_{Dh}^2k^2 + 2)(\omega^2(\omega^2 - \Omega_i^2) - 3k^2v_{ti}^2(\omega^2 - \Omega_i^2\cos^2\theta) = \frac{k^2n_{oi}T_h(\omega^2 - \Omega_i^2\cos^2\theta)}{m_in_{oh}}$$

$$(\lambda_{Dh}^2k^2 + 2)(\omega^2(\omega^2 - \Omega_i^2) - 3k^2v_{ti}^2(\omega^2 - \Omega_i^2\cos^2\theta) = k^2v_{ia}^2(\omega^2 - \Omega_i^2\cos^2\theta)$$

where $v_{ia} = \sqrt{\frac{n_{oi}T_h}{n_{oh}m_i}}$ is ion acoustic speed

$$\omega^2(\omega^2 - \Omega_i^2) - \left(3k^2v_{ti}^2 + \frac{k^2v_{ia}^2}{2(1 + \frac{1}{2}\lambda_{Dh}^2k^2)}\right)(\omega^2 - \Omega_i^2\cos^2\theta) = 0 \tag{3.17}$$

Applying limit $\omega \ll \Omega_i\cos\theta$, Eq. (3.20) yields

$$\omega^2 = \left(3k^2v_{ti}^2 + \frac{k^2v_{ia}^2}{2(1 + \frac{1}{2}\lambda_{Dh}^2k^2)}\right)\cos^2\theta$$

Applying the short wavelength limit $1 \ll \lambda_{Dh}^2k^2$, Eq.(3.20) yields

$$\omega^2(\omega^2 - \Omega_i^2) - \left(3k^2v_{ti}^2 + \frac{k^2v_{ia}^2}{\lambda_{Dh}^2k^2}\right)(\omega^2 - \Omega_i^2\cos^2\theta) = 0$$

$$(\omega^4 - \omega^2\Omega_i^2) - 3k^2v_{ti}^2 + \frac{k^2v_{ia}^2}{\lambda_{Dh}^2k^2}](\omega^2 - \Omega_i^2\cos^2\theta) = 0$$

where $\omega_{pi}^2 = \frac{v_{ia}^2}{\lambda_{Dh}^2}$

$$\omega^4 - \omega^2\Omega_i^2 - \omega^2\omega_{pi}^2 - 3k^2v_{ti}^2\omega^2 + 3k^2v_{ti}^2\Omega_i^2\cos^2\theta + \omega_{pi}^2\Omega_i^2\cos^2\theta = 0$$

$$\omega^4 - \omega^2\omega_{UH}^2 - 3k^2v_{ti}^2\omega^2 + 3k^2v_{ti}^2\Omega_i^2\cos^2\theta + \omega_{pi}^2\Omega_i^2\cos^2\theta = 0$$

where $\omega_{UH}^2 = \Omega_i^2 + \omega_{pi}^2$

$$\omega^4 - \omega^2(\omega_{UH}^2 + 3k^2v_{ti}^2) + (3k^2v_{ti}^2 + \omega_{pi}^2)\Omega_i^2\cos^2\theta = 0$$

We now focus on extreme limits to analyze Eq. (3.20)

Parallel propagation $\theta = 0^o$

$$\omega^2(\omega^2 - \Omega_i^2) - \left(3k^2v_{ti}^2 + \frac{k^2v_{ia}^2}{2(1+\frac{1}{2}\lambda_{Dh}^2k^2)}\right)(\omega^2 - \Omega_i^2) = 0$$

$$(\omega^2 - \Omega_i^2)\left(\omega^2 - 3k^2v_{ti}^2 + \frac{k^2v_{ia}^2}{2(1+\frac{1}{2}\lambda_{Dh}^2k^2)}\right) = 0$$

There are two modes of the above dispersion relation. One is not propagating, constant frequency, ion cyclotron mode,

$$\omega^2 = \Omega_i^2$$

The other one is ion acoustic mode,

$$\omega^2 = 3k^2v_{ti}^2 + \frac{k^2v_{ia}^2}{2(1+\frac{1}{2}\lambda_{Dh}^2k^2)}$$

By taking a short wavelength limit of $1 \ll \lambda_{Dh}^2k^2$, above Eq. becomes an ion plasma mode resulting from cold ion motion.

$$\omega^2 = 3k^2v_{ti}^2 + \omega_{pi}^2$$

Perpedicular propagation $\theta = 90^o$

$$\omega^2(\omega^2 - \Omega_i^2) - \left(3k^2 v_{ti}^2 + \frac{k^2 v_{ia}^2}{2(1+\frac{1}{2}\lambda_{Dh}^2 k^2)}\right)\omega^2 = 0$$

$$\omega^2 \left(\omega^2 - \Omega_i^2 - 3k^2 v_{ti}^2 - \frac{k^2 v_{ia}^2}{2(1+\frac{1}{2}\lambda_{Dh}^2 k^2)}\right) = 0$$

Above dispersion relation has two modes, one is not propagating mode

$$\omega^2 = 0$$

The one is ion cyclotron mode, which results from both cold and hot species.

$$\omega^2 = \Omega_i^2 + 3k^2 v_{ti}^2 + \frac{k^2 v_{ia}^2}{2(1+\frac{1}{2}\lambda_{Dh}^2 k^2)}$$

For the limit $1 \ll \lambda_{Dh}^2 k^2$ above mode truns into upper hybrid mode.

$$\omega^2 = \omega_{UH}^2 + 3k^2 v_{ti}^2$$

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Chapter 4

Results and Discussion

In this chapter, we work on the effect of temperature and density for two specific wave propagations. The normalized form (3.12) is used for this purpose. The following Normalizations are used here: fluid speeds are normalized by the thermal veocity,the particle density by total equilibruim plasma density, $n_o = n_{oc} + n_{oh}$, temperature by T_h, the spatial length by $\lambda_D = (\frac{\epsilon_o T_h}{n_{oh} e^2})^{\frac{1}{2}}$, and the time by $\omega_p^{-1} = (\frac{n_{oh} e^2}{\epsilon_o m})^{-\frac{1}{2}}$. To obtain normalized dispersion relation, divide Eq (3.12) by ω_p^4.

$$\frac{\omega^2(\omega^2 - \Omega^2)}{\omega_p^4}) - \frac{1}{\omega_p^4}\left(k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{(k^2 v_{th}^2 + 2\omega_{ph}^2)}\right)(\omega^2 - \Omega^2\cos^2\theta) = 0$$

$$\frac{\omega^4}{\omega_p^4} - \frac{\omega^2}{\omega_p^4}\left(\Omega^2 + k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{(k^2 v_{th}^2 + 2\omega_{ph}^2)}\right) + \frac{\Omega^2\cos^2\theta}{\omega_p^4}\left(k^2 v_{tc}^2 + \frac{2k^2\omega_{pc}^2 v_{th}^2}{(k^2 v_{th}^2 + 2\omega_{ph}^2)}\right) = 0$$

$$\omega'^4 - \omega'^2\left(\frac{1}{R^2} + k'^2\frac{T_c}{T_h} + \frac{k'^2 n'_{oc}}{n'_{oh}(1+\frac{1}{2}k'^2)}\right) + \frac{\cos^2\theta}{R^2}\left(k'^2\frac{T_c}{T_h} + \frac{k'^2 n'_{oc}}{n'_{oh}(1+\frac{1}{2}k'^2)}\right) = 0 \quad (4.1)$$

where $\omega' = \frac{\omega}{\omega_p}$, $k' = k\lambda_D$, $n'_{oh} = \frac{n_{oh}}{no}$, $n'_{oc} = \frac{n_{oc}}{n_o}$ and $R = \frac{\omega_p}{\Omega}$. The effect of temperature ratios, propagation angle, and density ratios of hot and cold species wave mode is studied.

4.0.1 Effect of Variation in $\frac{n_{oc}}{n_o}$ for perpendicular propagation

We investigate the perpedicular propagation of waves by using (4.1). Figure (4.1) displays the variation of normalized value of real frequency wave number $\lambda_D k$ for distinct values of normalized cold particle density ratios ($\frac{n_{oc}}{n_o}$) for other specified parameters, $R = 0.333$, $\lambda_D k = 0.01$, and $\theta = 90^o$. The range for $\lambda_D k$ is taken $0.3 - 0.8$ so that all our conditions, namely $\lambda_{Dh} k \ll 1$, $\lambda_{Dc} k \ll 1$,and $1 \ll \frac{kv_{th}}{\omega}$ are fulfilled. The frequency rises as the density of cold species rises. This is the cyclotron mode modified by the existence of finite cold species (second term in (2.16)) and acoustic effect resulting from the existence of hot and cold species. The mode reaches the upper hybrid frequency at high numbers of (n_{oc}/n_o), i.e. for smaller amount of hot species.

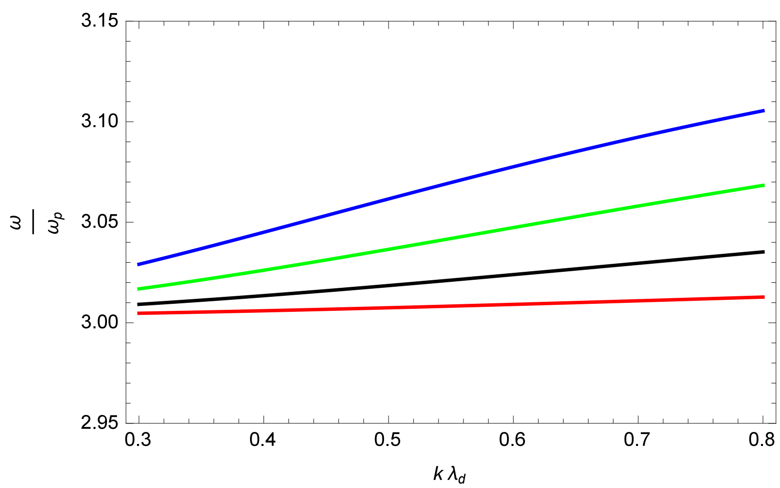

Figure 4.1: Normalized real frequency vs normalized wavenumber.Varying parameter is label as $\frac{n_{oc}}{n_o} = 0.1$ (red), $\frac{n_{oc}}{n_o} = 0.3$ $(black)$ $\frac{n_{oc}}{n_o} = 0.5$ $(green)$ and $\frac{n_{oc}}{n_o} = 0.6$ $(blue)$.

4.0.2 Effect of Varying temerature ratio $\frac{T_c}{T_h}$ for perpendicular propagation

Figure (4.1) displays the variation of normalized value of real frequency wave number $k\lambda_D$ for distinct values of normalized cold to hot species temperature ratios $\frac{T_c}{T_h}$ and other set parameters are, R = 0.333 , $\frac{n_{oc}}{n_o} = 0.1$, and $\theta = 90^o$. There is the electron cyclotron mode, as the curve is note near to $\frac{\omega}{\omega_p} = \frac{1}{R} = 3$ and frequency of mode rises with the rise in $\frac{T_c}{T_h}$. It is worth stressing that curve behavior in Fig.(4.2) is a feature of a four-component electron-positron plasma with a two-temperature, and it has not been mentioned before in the literature.

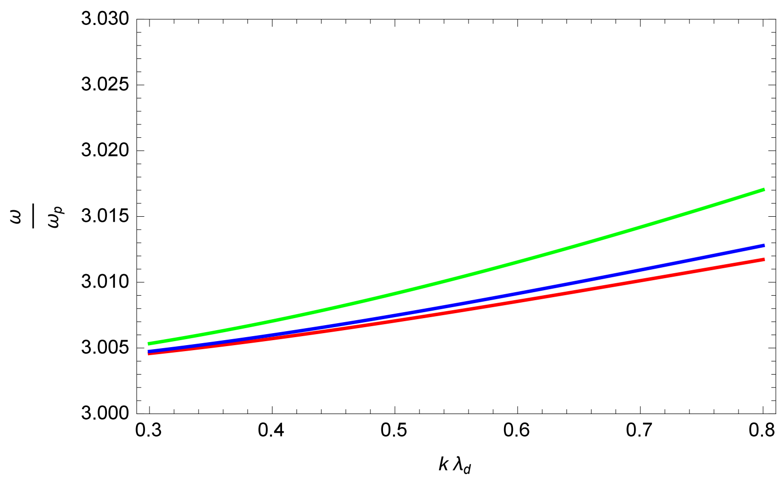

Figure 4.2: Normalized real frequency vs. normalized wavenumber.Varying parameter is label as $\frac{T_c}{T_h} = 0.0$ (red), $\frac{T_c}{T_h} = 0.01$ $(blue)$ and $\frac{T_c}{T_h} = 0.05$ $(green)$.

4.0.3 Effect of Variation in $\frac{n_{oc}}{n_o}$ for parallel propagation

Figure (4.3) shows the effect of varying cold species concentration for parallel propagation. For this purpose, graph is plotted between normalized frequency and wavenumber for the specified parameters of Fig. (4.1). It is observed that the mode frequency rises as $\frac{n_{oc}}{n_o}$ rises.The mode is recognized as an acoustic electron mode from the dispersion curves (cf. (2.19)). This is a characteristic of four-component, two-temperature EP plasma and is due to the contribution of the second species. The electron-acoustic mode can not occur in a single-temperature electron-positron plasma.

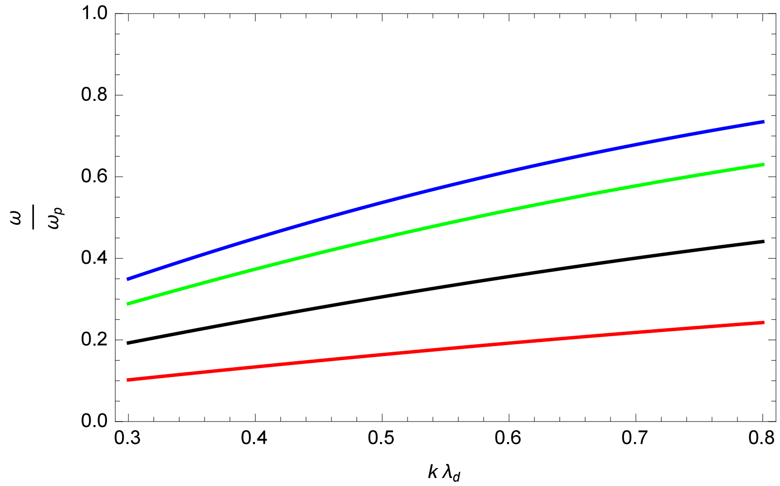

Figure 4.3: Normalized real frequency vs. normalized wavenumber.Varying parameter is label as $\frac{n_{oc}}{n_o} = 0.1$ (red), $\frac{n_{oc}}{n_o} = 0.3$ ($black$) $\frac{n_{oc}}{n_o} = 0.5$ ($green$) and $\frac{n_{oc}}{n_o} = 0.6$ ($blue$).

4.0.4 Effect of Varying Angle

Figure (4.4) shows a curve between normalized real frequency and wavenumber with a range of propagation angles for electron-acoustic branch. It is observed that the curve slope is much lower than Zank and Greaves ' (1995) single-temperature electron-positron model. The electron-acoustic mode frequency (Fig. 4) is getting smaller with the angle of propagation and eventually dies out at $\theta = 90$.

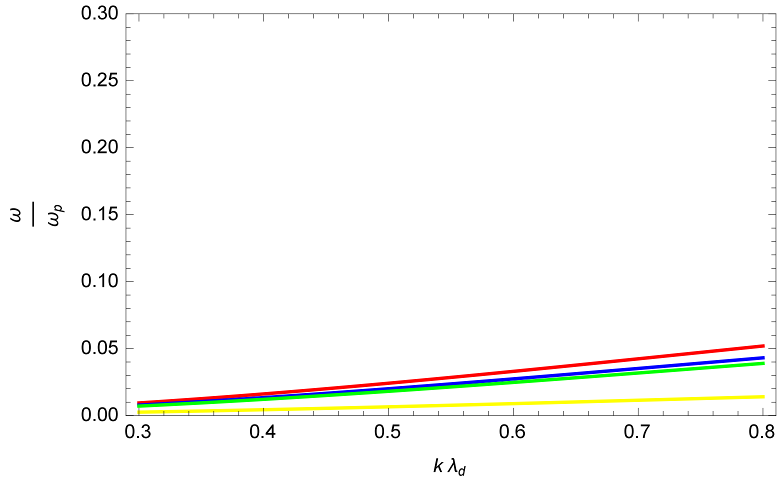

Figure 4.4: Normalized real frequency vs. normalized wavenumber.Varying parameter is label as $\theta = 0^o$,(red) $\theta = 9^o$ (blue) , $\theta = 22.5^o$ (green) and, $\theta = 45^o$(yellow). The specified parameters are $R = 0.333$, $\frac{n_{oc}}{n_o} = 0.1$ and $\frac{T_c}{T_h} = 0.01$.

4.1 Numerical Results of EPI Plasma

In this part, we will work on the density and temperature effectss of two specific wave propagations, i.e. parallel propagation and perpendicular propagation. .For this purpose normalized formed of Eq (3.20) is used. Normalization used here is the same as used in equation 4.1. To obtain normalized dispersion relation, divide Eqs (3.20) by ω_p^4.

$$\frac{\omega^2(\omega^2 - \Omega_i^2)}{\omega_p^4} - \frac{1}{\omega_p^4}\left(3k^2 v_{ti}^2 + \frac{k^2 v_{ia}^2}{2(\frac{1}{2}k^2\lambda_{Dh}^2 + 1)}\right)(\omega^2 - \Omega_i^2\cos^2\theta) = 0$$

$$\frac{\omega^4}{\omega_p^4} - \frac{\omega^2}{\omega_p^4}\left(\Omega_i^2 + 3k^2 v_{ti}^2 + \frac{k^2 v_{ia}^2}{2(\frac{1}{2}k^2\lambda_{Dh}^2 + 1)}\right) + \frac{\Omega_i^2\cos^2\theta}{\omega_p^4}\left(3k^2 v_{ti}^2 + \frac{k^2 v_{ia}^2}{2(\frac{1}{2}k^2\lambda_{Dh}^2 + 1)}\right) = 0$$

$$\omega'^4 - \omega'^2\left(\frac{\frac{m}{m_i}}{R^2} + 3k'^2\frac{mT_c}{m_iT_h} + \frac{k'^2 n'_{oc}\frac{m}{m_i}}{2n'_{oh} + k'^2}\right) + \frac{\cos^2\theta}{R^2}\left(3k'^2\frac{mT_c}{m_iT_h} + \frac{k'^2 n'_{oc}\frac{m}{m_i}}{2n'_{oh} + k'^2}\right) = 0 \quad (4.2)$$

This equation is analogous to Lazarus et al. but involves the $\frac{m}{m_i}$. mass ratio.

4.1.1 Effect of Variation in $\frac{n_{oc}}{n_o}$ for perpendicular propagation

We investigate the perpendicular propagation of waves for variation in cold species concentration. The curves of the different density ratios for the following set of parameters $R = 0.333$, $\frac{T_c}{T_h} = 0.01$ and $\theta = 90^o$ are presented in Fig.(4.6). With an increase of $\frac{n_{oc}}{n_o}$, the frequency rises. The ion cyclotron mode comes from the cold and hot species. The mode reaches the upper hybrid frequency for small values of the hot species.

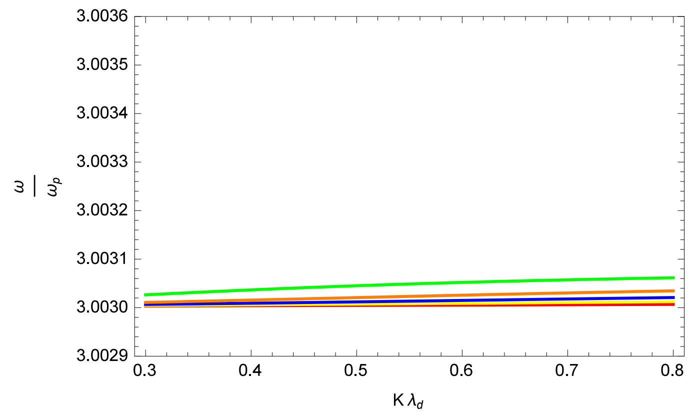

Figure 4.5: Normalized real frequency vs. normalized wavenumber.Varying parameter is label as $\frac{n_{oc}}{n_o} = 0.1$ (red), $\frac{n_{oc}}{n_o} = 0.3$ $(yellow)$ $\frac{n_{oc}}{n_o} = 0.5$ $(blue)$, $\frac{n_{oc}}{n_o} = 0.7$ $(orange)$ and $\frac{n_{oc}}{n_o} = 0.9$ $(green)$.

4.1.2 Effect of Varying temerature ratio $\frac{T_c}{T_h}$ for perpendicular propagation

Figure (4.4) displays the variation of normalized real frequency with normalized wavenumber $\lambda_D k$ at varying temerature ratio $\frac{T_c}{T_h}$ for other set parameters, $R = 0.333$ $, = 0.1$, and $\theta = 90^o$. This is the ion-cyclotron mode and the frequency rises as $\frac{T_c}{T_h}$ increases.

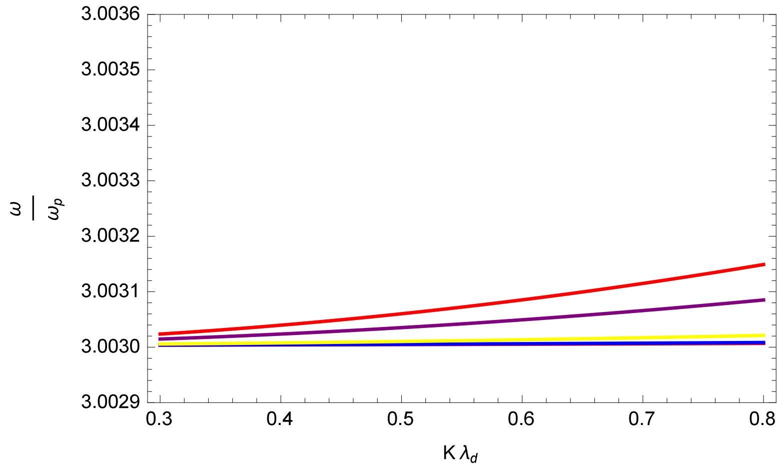

Figure 4.6: Normalized real frequency vs. normalized wavenumber.Varying parameter is label as $\frac{T_c}{T_h} = 0.02$ (*blue*), $\frac{T_c}{T_h} = 0.1$ (*yellow*), $\frac{T_c}{T_h} = 0.5$ (*pueple*) and $\frac{T_c}{T_h} = 0.9$ (*red*).

4.1.3 Effect of Variation in $\frac{n_{oc}}{n_o}$ for parallel propagation

Figure (4.8) displays the curve of normalized frequency versus normalized wavenumber for parallel propagation at different values of cold species density. The specified parameters are similar to those of Fig (4.8). The Equation (4.2) contains the $\frac{m}{m_i}$ electron to ion mass, which is not present in e-p plasma. The $\frac{m}{m_i}$ ratio significantly decreases that frequency, which is much lower than the EP plasma results.

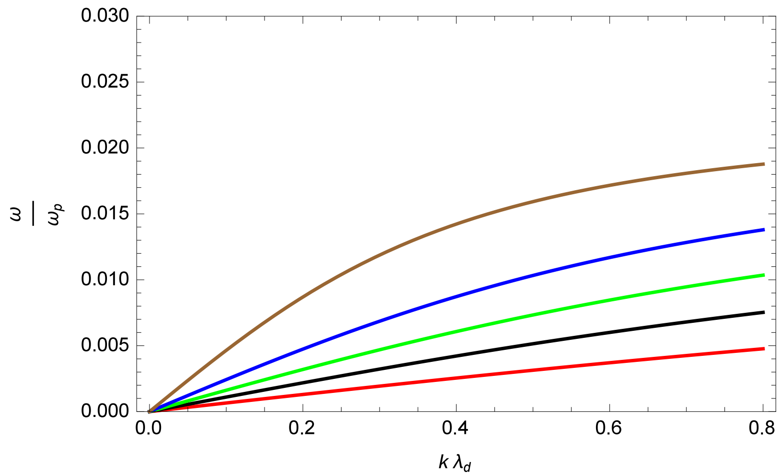

Figure 4.7: Normalized real frequency vs. normalized wavenumber.Varying parameter is label as $\frac{n_{oc}}{n_o} = 0.11$ (red), $\frac{n_{oc}}{n_o} = 0.43$ $(black)$ $\frac{n_{oc}}{n_o} = 0.1$ $(green)$, $\frac{n_{oc}}{n_o} = 0.23$ $(blue)$ and $\frac{n_{oc}}{n_o} = 9$ $(brown)$.

4.1.4 Effect of Varying Angle

The ion-acoustic and cyclotron branches for different propagation angles are shown in Figs. (4.8).By increasing the propagation angle, the ion-acoustic mode decreases and disappears at $\theta = 90^o$.

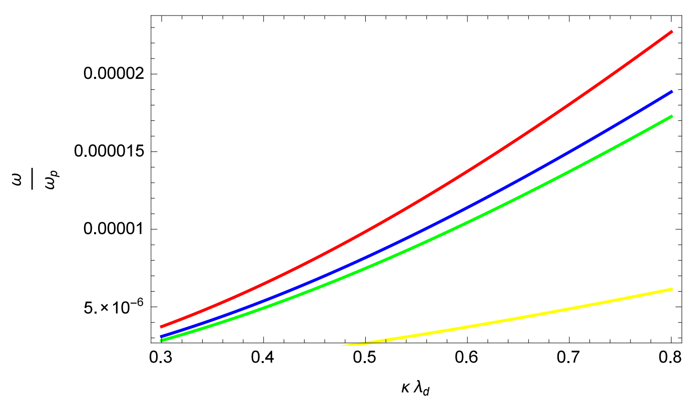

Figure 4.8: Normalized real frequency vs. normalized wave number.Varying parameter is label as $\theta = 0^o$ (red) , $\theta = 9^o (blue)$, $\theta = 22.5^o$ $(blue)$ and $\theta = 45^o (yellow)$. The specified parameters are $R = 0.333$, $\frac{n_{oc}}{n_o} = 0.11$ and $\frac{T_c}{T_h} = 0.01$.

4.2 Conclusion

In a two temperature EP plasma, the corresponding dispersion relation yield the electron acoustic, upper hybrid, electron plasma and electron cyclotron modes, which were investigated as a function of several plasma parameters. For perpendicular wave propagation, an electron cyclotron mode exists with contributions from both cold and hot species and hence influencing the dispersive properties of the wave. In the absence of hot species, this mode goes over to the upper hybrid mode where the cold species contribute to the wave dynamics, as expected and reported earlier by Zank and Greaves (1995). On the other hand, for parallel propagation, the solutions display a dominant electron acoustic mode, which goes over to an electron plasma mode when hot species are absent. Further, this mode is decoupled from the electron cyclotron mode.

Similarly, linear electrostatic wave oblique propagation in magnetized three-component EPI plasma was explored. We investigated the effects of the density ratio and temperature ratio on the structure of electrostatic waves. Parallel wave propagation identified the ion-acoustic wave mode and the interpretation showed that the frequency of the ion-acoustic mode increased as a cold and hot density ratio increased. On the other side, it shows the presence of a cyclotron mode for perpendicular propagation due to both the cold and hot species. The cold species density plays an essential part in modifying the wave characteristics. It is observed that the introduction of the ion species into the EP plasma results in a significant decrease in the frequency range. Our results may apply to pulsar magnetosphere cusp areas comprising EPI plasmas to understand pulsar magnetosphere radio emissions. However, these findings may also be relevant to future experiments on EPI plasmas and may justify some of the outstanding characteristics of the laboratory-generated electrostatic waves.

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