

Performance Improvement of Geysers

Final Year Project Report

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In Partial Fulfillment
of the Requirements for the Degree of
Bachelor of Mechanical Engineering

by

HAIDER ALI

ABDUL QADOOS

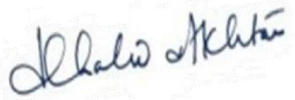


MUHAMMAD FAIZAN SAJID

EXAMINATION COMMITTEE

We hereby recommend that final year project report prepared under our supervision by:

HAIDER ALI	177611
ABDUL QADOOS	177778
MUHAMMAD FAIZAN SAJID	188784

Titled: “**PERFORMANCE IMPROVEMENT OF DOMESTIC GEYSER**” be accepted in partial fulfillment of the requirements for the award of **Bachelor of Mechanical Engineering** degree.

Supervisor: Dr. KHALID AKHTAR National University of Sciences and Technology.	 Dated: 26 th August, 2020
Committee Member: Dr. Emad-Ud din, (HOD) National University of Sciences and Technology.	 Dated: 9 th September, 2020
Committee Member: Engr. M. NAWEED HASSAN, National University of Sciences and Technology	 Dated: 19 th October, 2020



(Head of Department)

9th September, 2020

(Date)

COUNTERSIGNED

Dated: _____

(Dean / Principal)

ABSTRACT

Due to the Depletion of natural resources including natural gas there is a demand to develop new methods and techniques to efficiently use the existing resources. Gas geysers are being used for water heating at homes and on commercial scale. These geysers are cheap and being used most widely all over the country. These gas geysers are not efficient enough. Solar heating units are expensive as compared to traditional gas geysers, so people are forced to use these currently inefficient gas geysers. We are providing a midway solution.

We are going to devise ways to save gas consumption by improving the efficiency of geyser while analyzing existing ways being used to do so. One way is to minimize the fuel consumption to reach the desirable temperature that will be achieved by preheating of water.

ACKNOWLEDGMENTS

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ABBREVIATIONS

CAD	Computer Aided Design
Cp	Specific heat capacity
CPC	Concentrated Parabolic collector
EHPTs	Evacuated Heat Pipe Tubes
GI	Galvanized Iron
HVAC	Heating, ventilation, and air conditioning
MS	Mild Steel
MMBtu	Metric Million British Thermal Unit
NUST	National University of Sciences and Technology
Rs	Rupees

NOMENCLATURE

Burner design:

v_o	Velocity of gas out from orifice in ft/s
P	Difference between absolute and outside pressure
w	Initial density of gas on the orifice inlet (lb/ft ³)
v_e	Experimental velocity of gas at orifice outlet
G	Acceleration of gravity (32 ft / s ²)
h	Head in feet of column of a gas
H	Gas pressure on orifice outlet
d	Specific gravity (natural gas)
K	Discharge coefficient ($K = v_e / v_o$)
A	Area of gas orifice in in ²
Q	Rate of discharge in ft ³ / hr

A	Area of gas orifice in ft^2
R	Air to fuel ratio
R	momentum of gas to momentum of mixture ratio
X	Cross section area of pipe in in^2

CPC system

P	Reflectivity of CPC plate
v_e	Absorptivity of selective coating
v_d	Dust stratification factor
α_i	Absorptivity of selective coating
M	Mass flow rate

Insulation

K	Conductivity
r_1	Outer radius of geyser
r_2	Inner radius of cylinder
T_s	average surface temperature of geyser
D	Diameter of geyser tank
L	Length of geyser
V	Kinematic viscosity

CHAPTER 1: INTRODUCTION

Due to the Depletion of natural resources including natural gas there is a demand to develop new methods and techniques to efficiently use the existing resources. Gas geysers are being used for water heating at homes and on commercial scale. These geysers are cheap and being used most widely all over the country. These gas geysers are not efficient enough. Solar heating units are expensive as compared to traditional gas geysers, so people are forced to these currently inefficient gas geysers. We are providing a midway solution.

Problem Statement

Fuel consumption in the domestic sector for heating water has increased a lot in the last few years and continues to increase day by day which causes depletion of fossil fuels. This is a major problem our planet is facing today.

Aims and Objectives

The objective of the project is to save a major fossil fuel being depleted with a large pace that is Natural Gas; to do this we intend to improve the efficiency of domestic gas geyser that are being used at homes, hospitals and in any place that requires hot water.

To improve the efficiency of geyser, first understanding the geysers functioning along with all the subsystems and components being used in it need to be analyzed and studied extensively. The components work and how they contribute to the efficiency of geyser. All those possible ways where the components can be changed or redesigned to give the edge needed to improve the efficiency of the entire geyser are worth checking and will be analyzed.

Our motivation for this project is to explore ways for reducing extensive wastage of gas so that with less gas billing and expense we can produce more hot water. This not only produces a better economic aspect for the day to day consumer but also gives a lesser exhaust for the pollutant gases produced as a by-product for the combustion reaction that takes place in geyser. This helps to keep the environment better and healthy rather than ruin it further by exhausting more pollutant gases into the air. All this can simply be done by improving the efficiency of the domestic gas geyser that is used daily normally in winters and cold weathers.

Major Factors

Coming to the how the efficiency of geyser will be improved by this project. The major components being discussed in this project, the ones that are identified as the main way to increase the efficiency of the geyser are the Insulation, Burner and heat exchanger. The contribution of these major components as they provide the key role in heating the water in geyser and then keeping the heat stored in rather than letting it all escape. A brief introduction on how these different parts works is as follows:

Helical heat exchanger

Heat from the flue gases getting eliminated from the exhaust of the geyser, are wasted. These gases can be used to preheat cold water on the inlet of the geyser container. Hence, it reduces the time for heating water and save fuel consumption

CHAPTER 2: LITERATURE REVIEW

Typical water heating involves transferring heat from heat source whether it be energy from burning of fossil fuels or heat from natural source like sun to water for use domestically or industrially. In homes heated water is use when temperature is low and cold water can't be used for bathing, drinking, etc. Industrially hot water is used, or its steam is used for many purposes.

Types of water heaters

There are mainly two types of water heating system

1. Storage type water heating
2. Tank-less instantaneous water heater

Storage type water heater

Storage type water heaters are the traditional water heaters being used all over the world from old times. They have an oval shaped cylinder made of metal and can run on multiple energy sources. In case of gas geyser, it has burner inside its body on bottom which burns fuel as an energy source. Gas geysers are common in countries where gas is available in cheap price and is supplied to houses by pipes in low price. In most of countries including Pakistan vertical tanks are being used while in Spain and some other countries horizontal tank is used. Besides gas, there are other type of geysers which use electric resistance element. The heat from resisting electric current is used to heat water. Energy wasted when water is not use is called standby losses which will be studies later. Different capacity storage tanks are used according to need from 20 to 80 gallons. Fan assisted gas water heater is being used to reduce energy losses.

The image shows the normal working of storage type water heating system.

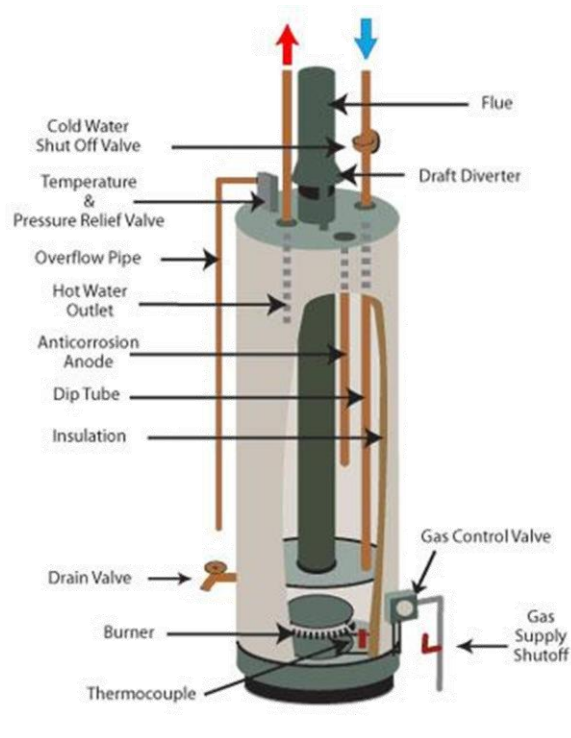


Figure 1: Storage Type Geyser

Also, there is another type of storage water heater system called point of use electric heater which is used to heat water at specific spots for instance kitchen or specific washroom, etc. They have low capacity as compared to normal storage tank storing water up to the range of 30L.

If we compare normal and point of use water geyser, for normal domestic use normal storage type heating system is good enough but for large buildings and plazas, point of use heating system is better to use to avoid long wait for hot water.

Tankless instantaneous water heater system

Tankless instantaneous water heating system is a new trend which has no tank and only water being stored is in coil of heat exchanger. Point of use tankless water heating system is more common than centrally installed water heating system.

Following [1] is the figure showing instantaneous water heating system

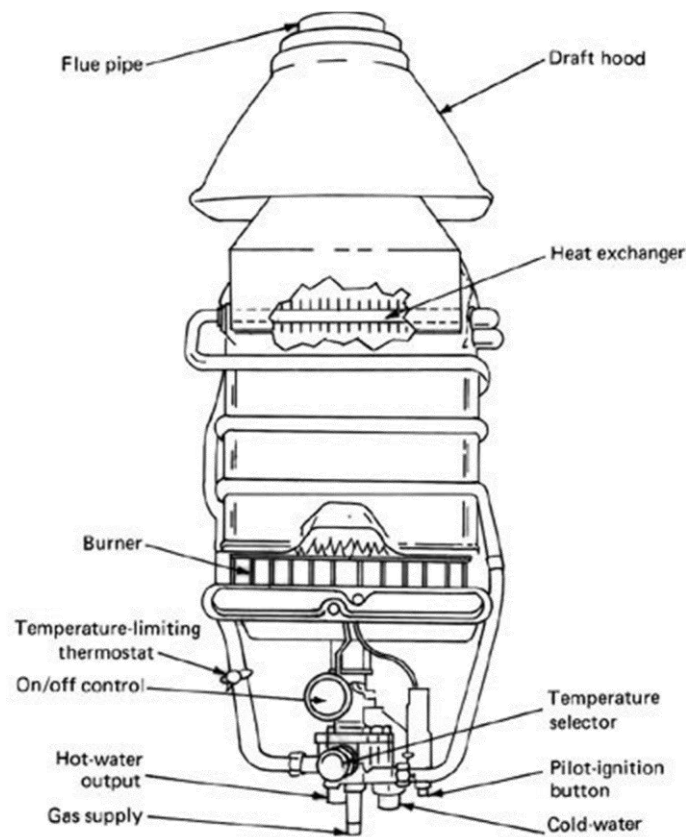


Figure 2: Tankless instantaneous geyser

It has flow sensors which turn on when water is flowing giving signal that hot water is now required as water is being used. There are many hybrid gas geysers available which combine tankless heaters with central heating systems or point of contact heating system.

Comparison of both types

If we do a comparison on storage water heater vs tankless water heater, we see both have their merits and demerits.

The merit of using tankless water heater is that it doesn't need delay to heat water. It heats and supplies water instantaneously without any wait. Unlike water storage tank, it doesn't lose energy with the passage of time. A lot of water is saved as we don't have to waste water to get hot water. When there is minimum use of water, it spends lesser energy.

Its demerits are a few. Firstly, it consumes energy at a faster rate than storage tank heaters if it is run for longer durations. In Pakistan or other underdeveloped countries, it is a major concern for people. Secondly, when water is required at multiple points, it can supply water at fix spots. More initial investment is required in case of tankless water system, so it is out of purchasing power of most people of country.

Aspects to investigate for improving a domestic gas geyser

We had to study all the aspects of a domestic gas geyser.

- Combustion
- Solar heating
- Insulation/Geyser blankets

- Baffles
- Timing of use/ flow meter
- Heat Exchanger

Combustion

Combustion is the process in which fossil fuels reacts with oxygen to produce heat and water in an exothermic reaction. The measure of success of combustion process is dependent upon combustion efficiency. For hundred percent combustion efficiency, fuel should be completely burnt, and no carbon and hydrogen remain unburnt. It is dependent upon air to fuel ratio. We can calculate the theoretical air to fuel ratio but can't be achieved due to unreliable factors. When the combustion process is complete, the process becomes cheap and fuel is saved. More air also ensures that maximum of carbon monoxide reacts with oxygen.

In order to improve the combustion process, the burner design was to be considered. Dr. Gunther Bethold in Worgas Company gave the basics of stoichiometric reactions for the air to fuel reaction. Burner can be fan-powered which premix the air or atmospheric type which basically involves partial premixing of the air, John. H Eisman [2] wrote about the maximum limits which burner could be designed without blow off or flashback. Walter M. Berry [3] worked to provide a burner design, how the increasing or decreasing the quality of gas requires the injector tube and orifice to be set as optimal parameters. Most of the domestic and industrial water heating systems use burners with channel type orifice hence reducing the maximum velocity due to gas constriction. C.A Duff [4] presented the solution of waste of energy by suggesting point of contact geyser. Several papers regarding industrial burner design for educational purposes have been published by the American Gas Association. They don't illustrate much detail about the gas properties with numerical evidences that how these effects the flow change.

Solar heating/Types of Solar Heating

There are two types of solar thermal heating system

1. Passive heating system
2. Active type solar heating system

Passive solar heating system

It is also called direct gain type heating system is the one in which water is sent directly to collector to be heated. It has also two types.

Integrator collector storage (ICS) has water tank incorporated into collector. It is cheap to use but it has disadvantaged that significant amount of heat is lost in the night due to non-insulation of side of collector facing the sun.

Convection heat storage unit (CHU) is different from the ICS for the fact that it has separate water tank and collector. The heat transfer takes place by convection.

Active solar heating system

In indirect type solar heating system, the fluid mostly antifreeze is pumped into collector to heat and then transfer heat to water via a heat exchanger. The heat transfer fluid can be water but mostly it is antifreeze with corrosion inhibitor. Copper is very important element in the solar water heating system due to its high corrosion resistance and high conductivity.

Components of solar heating

1. Collector

Two types of collectors are

a) Flat plate collector

The glass of flat plate is the reason it is considered the most long-standing type of collector. The glass of flat plate collector is tempered, low iron which is very hard to break. Unglazed collector also being used which have no thermal insulation, hence they are less efficient than flat plate type. The fluid travels from lower pipe pumped and after being heated it absorbs heat in the absorber plate then travels from upper pipe back.

Following [5] is the image of a flat plate collector.

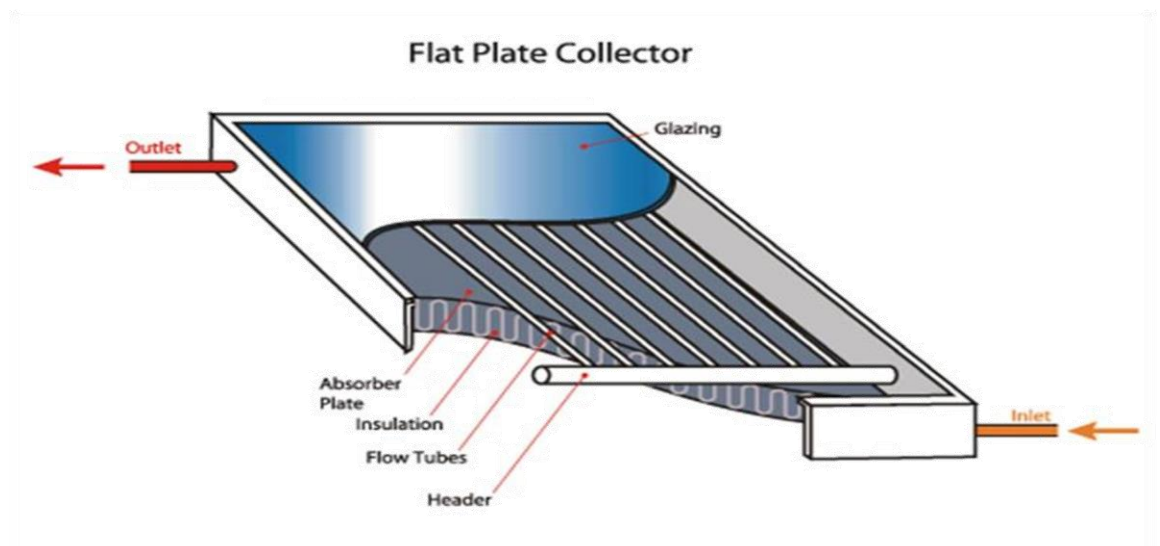


Figure 3: Flat plate collector

It is basically an improved version of flat plate collector. A lot of energy is lost from the flat plate collector is lost due to no insulation. As heat transfer due to convection can't happen due to vacuum, so once heat entered can't escape from tube. Heat transfer mechanism and procedure is basically same for flat plate collectors. There is another type of evacuated tube collector known as CPC (concentrated parabolic collector). CPC can not only accept and absorb radiations

from wide angles as compared to other collectors. Most of the flat plate collectors also use CPC and less attention has been focused on evacuated type solar collectors

Following diagram [6] shows the working of evacuated glass heater.

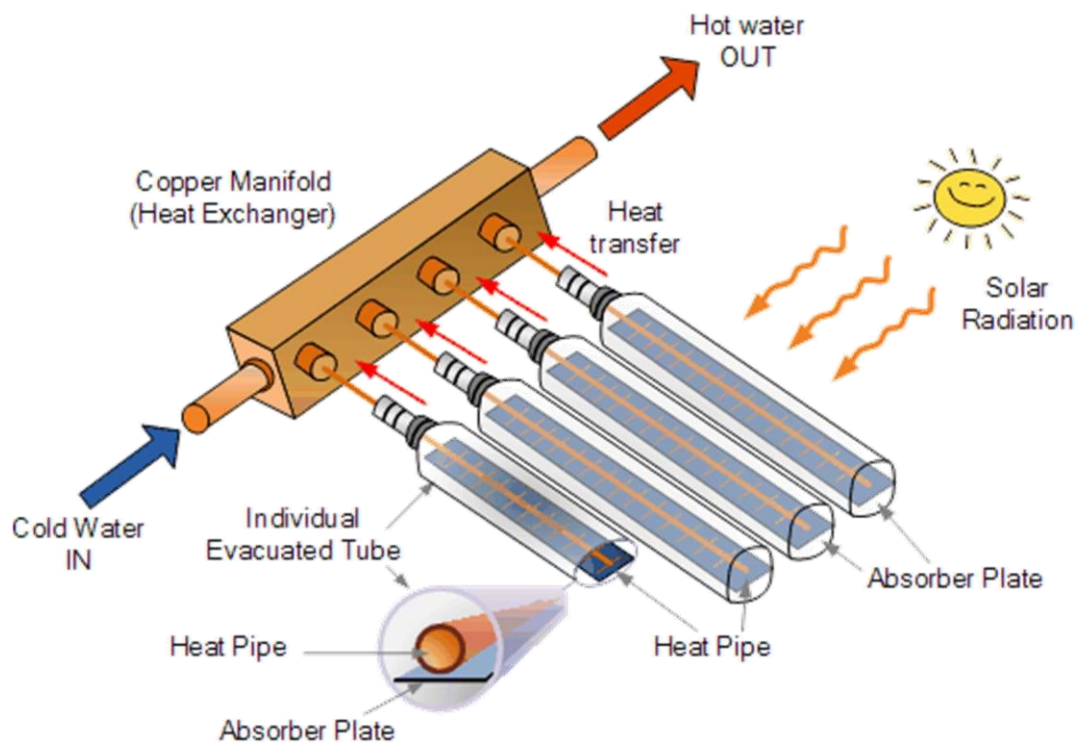


Figure 4: Evacuated tube type collector

b). Evacuated tube collector

Flat plate collectors are more efficient than evacuated type collectors under normal weather and conditions but under cloudy weather, its efficiency is greatly reduced due to energy loss.

2. Controller

The controller works on the principle of temperature difference between water in the collector and the water in the tank. When the temperature difference is significant, it starts the pump and it turns off the pump when the temperature difference is not no large. That makes sure pump doesn't turn on and off for excessive times.

3. Pump used to power an active system via photovoltaic panel.

One [7] analyzed and modeled that how the performance of a solar flat-plate air heater can be measured. Garg and Adhikari [8] did a performance evaluation of a single solar air heater with n number of sub collectors. Tchinda [9] showed a mathematical mode for CPC solar energy collectors, essentially how the air collectors thermal performance gets effected by quantification of heat transfer. Majid Azimi [10] gave the mathematical model as well as simulated the vacuum tube solar collector as a solution to Iran's energy problems. Bainbridge [11] gave a complete overview of solar collectors. As we know solar heating system are divided into active and passive type solar heating systems. Passive systems need no external power to support while active systems need external power or fan to operate. Passive solar heating system are further subdivided into two classes. Pin-Yang Wang [12] worked on a different type of gas tube that can be used within a compound parabolic concentrator (CPC). CPC can not only accept and absorb radiations from wide angles as compared to other collectors. Most of the flat plate collectors also use CPC and less attention has been focused on evacuated type solar collectors. CPC requires an expensive initial investment but is rather quite effective in its working and has a good payback time period, the processing of CPC is complex, so the additional charges are quite reasonable. Hottel and Whillier, [13] developed the flat plate collectors that are divided into three common types.

The types are

- (1) A flat plate absorber with darkened surface,
- (2) for reducing heat loss introducing covers that are transparent,
- (3) a fluid for transfer of heat (air, antifreeze or water; heat is removed from absorber)
- (4) Insulated back for reducing heat escape.

A polycarbonate cover or casing is used in the absorber to provide insulation to avoid heat loss. A thin absorber sheet is preferred that is often backed with a coil or grid of a fluid tubing, for the heating of water, the fluid is normally circulated in the tubes that gains heat through solar radiations, then from the fluid the heat is then transferred directly into water; a heat exchanger can also be used to avoid mixing of water and the heating fluid.

Another way is by using the multiple evacuated glass tubes that work as absorber plates, the heat pipe is fused with these glass tubes. Such a system is called Evacuated heat pipe tubes (EHPTs). The heat is transferred to fluid or a heat exchanger called ‘manifold’ insulation is majorly done by a plastic case and protection coating is done by sheet metal. The evacuated glass tubes types vary as well depending on glass-metal or glass-glass layer. In glass-metal the heat pipe is enclosed by a vacuum inside glass absorber; while in the glass-glass type there is glass on the both ends of the vacuum. The absorber and heat pipe themselves are kept at normal atmospheric pressures.

Heat exchangers

Heat exchangers are devices used to move heat energy starting with one fluid then onto the next. Different types of heat exchangers are experienced by us in our everyday lives incorporate condensers and evaporators utilized in cooling units and fridges. Boilers and condensers in thermal power plants are instances of huge heat exchangers. There are heat exchangers in our autos as radiators and oil coolers. Heat exchangers are additionally used in chemical and process industries. The figure shows a simple heat exchanger.

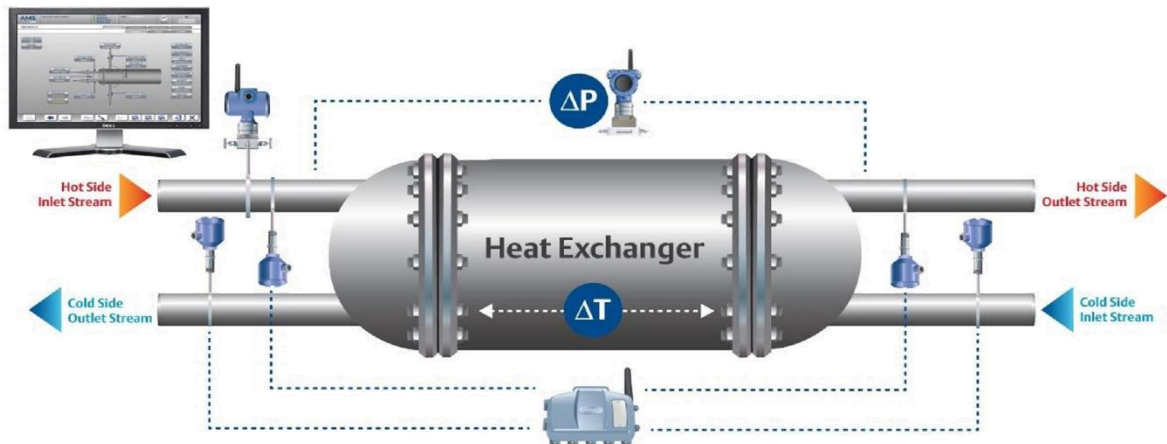


Figure 5: Simple Heat Exchanger

Straight Tube Heat Exchangers

There have been various exploration researches led on heat exchangers in the previous century to measure the boundaries influencing their heat move qualities. A heat exchanger utilizes two liquids with a temperature difference to move heat starting with one then onto the next, most regularly through a solid interface. Heat exchangers arrive in an varies of shapes and scale and are utilized in pretty much every industry including car, oil, semiconductor, HVAC, and elective vitality. One of the most well-known Heat exchangers being used is the concentric tube counter stream heat exchanger including a straight channel and a straight funnel shell with the internal and external liquids streaming the other way.

Helical Type Heat Exchangers

With an end goal to give a similar measure of heat move as a straight tube heat exchanger in a small space, engineers change the straight internal pipe with a helical coil. This takes into consideration more heat move from surface in a small length shell, yet increase the pressure drop over the heat exchanger. Helical coil heat exchangers

have a progressively complex stream design because of the geometrical arrangement of helical loops, which likewise give extra centrifugal force on the internal coil flow and expanding the pressure drop on the shell side.

In 1997, Yildiz et al. [10] conducted an experimental study on a helical tube in a shell heat exchanger containing inside springs. The springs were set inside the helical coil as an approach to increase the heat move inside the circle. The trial arrangement comprised of 5 mm helical tube with a diameter of 75 mm. The shell was all around protected, and air and water were utilized as the working liquids in the coil and shell sides, separately. It was not expressed whether the air was experiencing heating or cooling, we can accept it was experiencing warming. The data we are keen on for this current experiment is the consequences of the investigations without springs. Concerning the outcomes of the spring in the coil, an expansion in heat move viability of up to 30% is found in the heat exchanger while the pressure drop likewise ten times that of the empty tube flow.

In 1998, Guo et al. [11] experimentally studied the effects of pulsatile flow on heat transfer in helically coiled tubes. The investigation was led utilizing two-stage steam water as the working fluid. The test had a count of 102 thermocouples outwardly of the cylinder, differed pivotally just as peripherally. This gave a point by point description of the temperature field. Some portion of the test was led under consistent single-phase stream. The outcomes for consistent stream demonstrated that for $6000 < Re < 60,000$, the Seban MchLaughlin (1963) precisely described the framework. For bigger Reynolds numbers, the upgrading impact of the secondary stream because of the helical coil turned out to be less significant and indeed fell towards the anticipated qualities by the Dittus-Boelter relationship found in Equation.

Shell and tube heat exchanger can be used to preheat water in the geyser by using flue gases

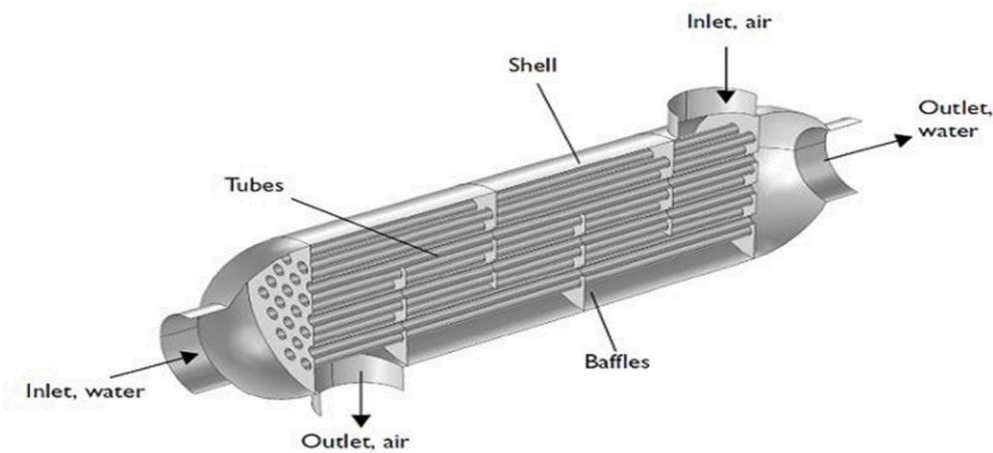


Figure 6: Shell and Tube Heat Exchanger

Insulation/ Geyser blanket

With the extension of knowledge, a large amount of materials has been identified that can be used as insulators so there comes a need for a standard comparison among them which are better and can be considered as best candidates for the insulation to be done. Geyser blanket is an external layer of insulation that can be used to reduce standby losses by wrapping around geyser. Although geyser blanket's typically is 50 to 75mm of thickness but it has also 100 to 150mm thickness for larger cylinders.

Dr. Mohammad S. Al-Homoud [14] has published a paper showing extensive comparison between the materials that how materials may provide a better advantage over others. Some even though seem quite feasible but still cannot be used in thermal insulation for the cylindrical wall of geyser such an example is Polyurethane/Polyisocyanurate-Foam whose R-value is very good but its disadvantage is that it shrinks down exposing the surface, in regard it is incapable to be used in some cases such as geyser insulation.

The comparison among the insulating material is done by the units of U-Values, R-Values and Thermal conductivity. A basic idea or a tip comparing these is that the

material with higher R-value is good and U-values need to be lower. R-value is measured by dividing the length of the material with its thermal conductivity. U-values are calculated by the addition of convection and radiation heat losses with the reciprocal of R-value. So, it can be concluded that U- values are the best since they give most accurate results by accounting the different ways heat loss can occur through the material. Omer Kaynakli [15] in his paper has provided the U-values of all the major materials being used along with how with thickness the heat transfer varies. This research gives us the major optimal thickness of all the insulation material that need to be used in walls of a building and how that gives reduction to the heat transfer. Azra Korjenic [16] has calculated U-values of materials composed of renewable resources. These materials are very vital since most of the insulators that are being used are mainly non-renewable and provide a great disadvantage. Bjørn Petter Jelle [17] has presented a huge research mainly upon all the new and major materials that provide ground breaking insulation among one of them is Aerogels produced by NASA used in their space shuttles .A.Harris [18] writes about how the standing losses and how they can be reduced in geysers by introduction of geysers blankets. Prop. Nabataea [19] shows in his paper the critical radius of all the given material can be calculated and it is quite important. As if the outer radius of a material falls short than the critical radius then material instead of working as an insulator, it works as a conductor. This aspect always needs to be the basis of analysis when installing a material that need to work as an insulator.

Baffles

Baffles [20] are extensively used in the industry among heat exchangers, mixing chambers, chemical plants, they provide an obstruction in the flow direction of fluid. They are in the shape of vanes and panels and are placed inside the vessel tanks. The implantation of baffles requires an expert analysis on the cost, size, and major effectiveness with or without the introduction inside the vessel. As geysers are heat exchangers, they are used to increase heat transfer. They provide support to the tubes by directing the flow. General classification of the baffles is as follows:

- Longitudinal Flow type baffle
- Orifice Baffle type
- Single segmental baffle type
- Double segmental baffle type

Table 1: Literature review summary

Study, Year Reference	Summary
Pin-Yang Wang (2013)	Using compound parabolic collector (CPC) to heat water beyond 100 C that other collectors can't do.
CA Duff & C Bradnum (2013).	In order to investigate the ways, the point of contact geyser works and fuel consumption estimate.
R.N.S.V. Ramakant *, Lakshmi Reddy (2015)	Study the design and heat transfer in baffles in shell and tube heat exchanger.
A.J. Piscor (2015)	Basics of air to fuel ratio control
Walter M. Berry et al. (1921).	Investigation of burner design and having a detailed view on observations compiled from thousands of experimental analyses including effects of many variables.

Harris et al. (2017) To study the shortcomings in use of geyser blankets wrapped around geysers. The study highlighted the problems including poorly secured blankets.

A. Kim (1985) To study the air to air heat exchanger and see if the flu gases can be used to heat the room.

Majid Azimi (2015) Study solar heating phenomenon using evacuated glass tubes collector for the purpose.

Energy observer (December 2008) Explained some method to reduce energy lose with the help of better insulation.

Ahmed Aboulmagd et Al. (2014) Study the performance of evacuated glass tube collector for heating for solar heating.

D. M. S. Al-Homoud (2004) Shows extensive comparison between the materials how materials may provide a better advantage over others. Some even tough seems quite feasible but still cannot be used in thermal insulation.

B. P. Jelle (2011) A huge research mainly upon all the new and major materials that provide groundbreaking insulations.

Prof.N.B. Totala (2013) Showed in his paper is the critical radius of all the given material can be calculated and it is quite important.

Peter Biermayer Make a study on 15% flue gas losses in 60% efficient geysers.

and Jim Lutz

Devendra Borse, Study different aspects of
 •Secondary flow
Jayesh V. Bute •Efficient Mixing
 •Increase Heat transfer coefficient
 •Large heat transfer area in a small space

A. Harris, M. study different short comings:
 •poorly secured blankets;
Kilfoil and E-A. •inadequate covering of the piping
 •inaccurate determination of the water temperatures
Uken

Conclusion after literature review

1. The process of combustion is needed to be overviewed as no improvement in gas burner design of gas geysers is observed from past several years. Changing the burner design brings about difficulties in maintenance.
2. The concept of baffles has been explored considerable times in the past and is being used in geysers in market, so no need to explore it.
3. Flow meters can be used to make efficient use of geyser but require expensive sensors, which increase considerable price.
4. Solar heating is the way of future so probably a hybrid of solar and gas that doesn't increase the price significantly.
5. Best insulations are already being used.
6. Heat Exchangers can be installed inside the geyser to preheat the water as heat from the flue gases is being wasted.

CHAPTER 3: METHODOLOGY

Heat exchanger

The burner is used to heat water in the geyser. The burner runs on oil or gas. The flue gases on burning of gases get wasted in normal geyser. We will use the heat recovery method to improve the efficiency of geyser. Very hot gases leave the geyser at the exhaust, so a portion of energy is lost without utilizing its potential. In order to minimize this loss, we will install a shell and tube heat exchanger with tube being helical. The water will use the thermal energy of the exhaust gases and pre heat the water before entering the main heat exchanger (Tank) for heating. A helical copper coil carries water is located inside the shell carrying hot exhaust gases.

CAD Model

Following are the labeled Diagrams of the CAD model:

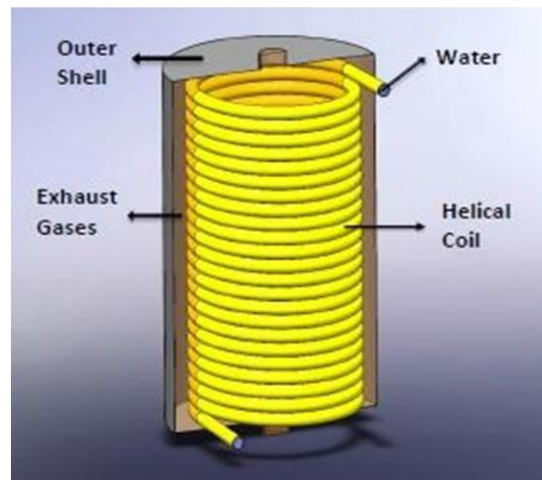


Figure 7: Modified Heat Exchanger Kit

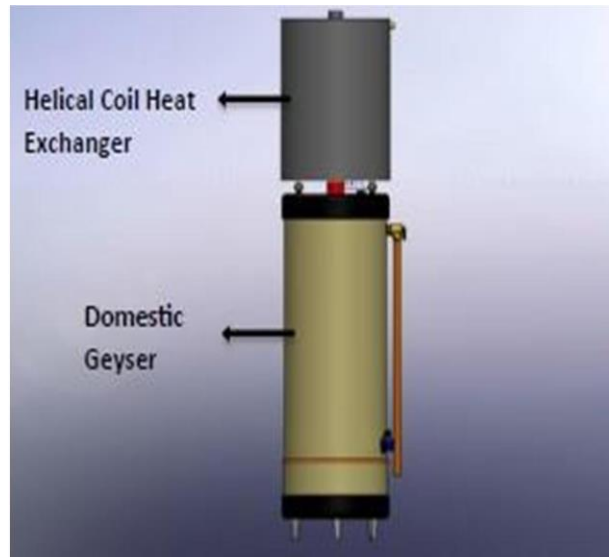


Figure 8: Combined Kit & Heat Exchanger



Figure 9: Manufactured Helical Tube

Theoretical/software calculations

Inner flow

We can find the Nusselt number of the flow in our heat exchanger by [21]

$$\text{Nu}_{\text{water}} = 0.023 \text{ Re}^{0.85} \text{ Pr}^{0.4} \delta^{0.1}$$

After this heat transfer coefficient is calculated by:

$$h_{\text{water}} = \frac{k_{\text{water}} \times \text{Nu}_{\text{water}}}{d_{\text{inner tube}}}$$

After this we can calculate the thermal resistance of water flow inside the tube by:

$$R_{\text{water}} = \frac{1}{(2\pi r_{\text{inner}} L) h_{\text{water}}}$$

Tube

We will calculate the thermal resistance of tube by:

$$R_{\text{tube}} = \frac{\ln\left(\frac{r_{\text{outer}}}{r_{\text{inner}}}\right)}{2\pi L_{\text{tube}} k_{\text{tube}}}$$

Exhaust Flow

We can the Nusselt number for outer flow in our heat exchanger by [21]

$$\text{Nu}_{\text{exhaust}} = 0.023 \times \text{Re}^{0.8} \times \text{Pr}^{0.4}$$

After this heat transfer coefficient is calculated by:

$$h_{\text{exhaust}} = \frac{K_{\text{exhaust}} \times \text{Nu}_{\text{exhaust}}}{d_{\text{inner shell}}}$$

After this we will calculate the thermal resistance of water flow inside the tube by:

$$R_{\text{exhaust}} = \frac{1}{(2\pi r_{\text{inner shell}} L) h_{\text{exhaust}}}$$

Total Thermal Resistance

$$R_{\text{total}} = R_{\text{water}} + R_{\text{tube}} + R_{\text{exhaustSS}}$$

Exit Temperature

Exit Temperature of water is calculated by:

$$T_{\text{exit}} = T_{\text{surface}} - (T_{\text{surface}} - T_{\text{inlet}}) e^{-hA/\dot{m}C_p}$$

Logarithmic Mean Temperature Difference (LMTD)

The LMTD is calculated by:

$$\Delta T_{\text{ln}} = \frac{T_i - T_e}{\ln\left[\frac{T_s - T_e}{T_s - T_i}\right]} = \frac{\Delta T_e - \Delta T_i}{\ln[\Delta T_e / \Delta T_i]}$$

Heat Transfer Rate

$$\text{Heat Transfer Rate} = \Delta T_{\text{ln}} \times R_{\text{total}}$$

Assumptions

- The Flow is supposed to be counter flow
- Constant surface heat flux model is assumed
- Flow is supposed to be turbulent

CHAPTER 4: RESULTS AND DISCUSSIONS

Analytical Results and Discussion

These graphs show the relationship between the different design parameters of helical coil heat exchanger.

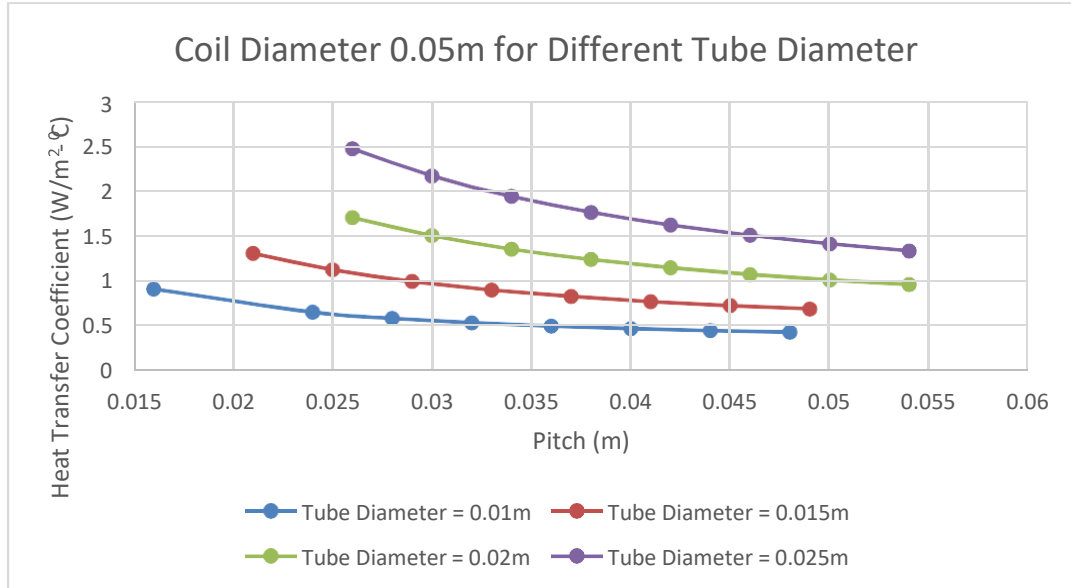


Figure 10: Heat Transfer Coefficient vs Pitch for coil of 0.05m

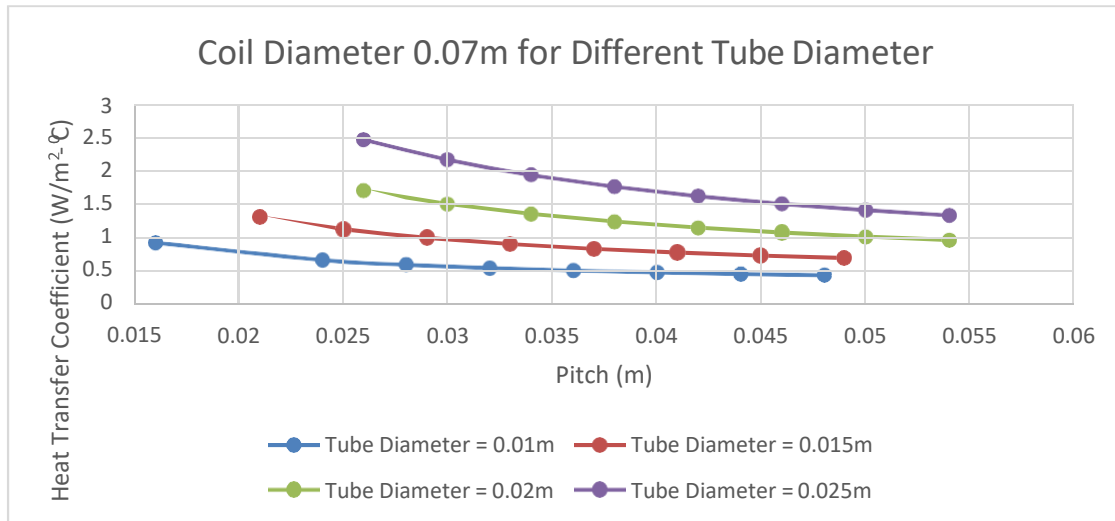


Figure 11: Heat Transfer Coefficient vs Pitch for coil of 0.07m

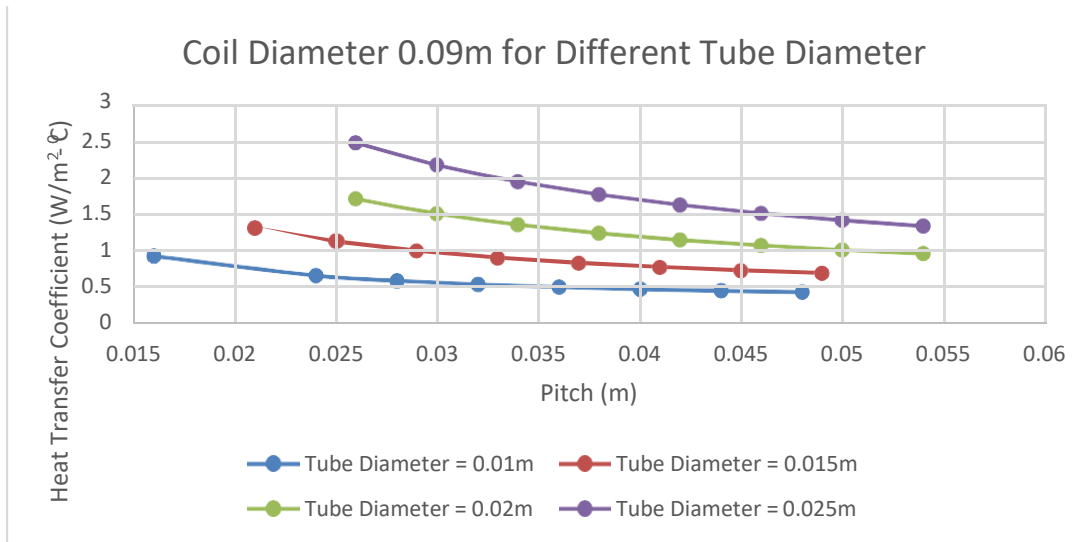


Figure 12: Heat Transfer Coefficient vs Pitch for coil of 0.09m

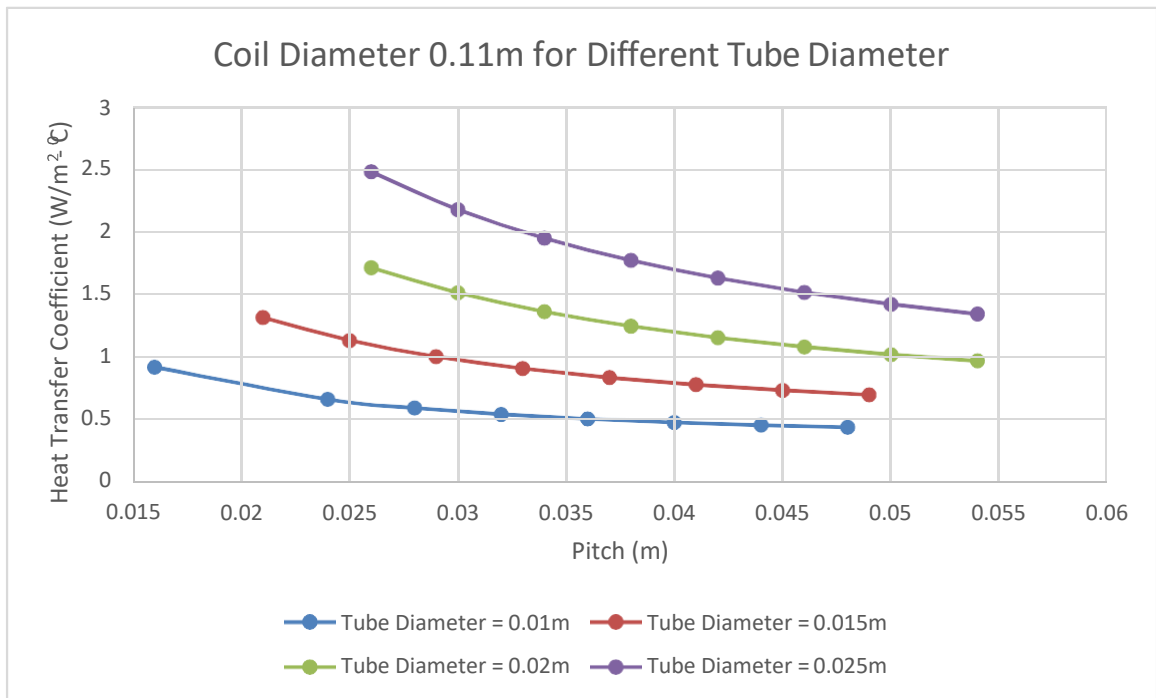


Figure 13: Heat Transfer Coefficient vs Pitch for coil of 0.11m

The above four graphs (Figure:9-12) are between the pitch and heat transfer coefficient with tubes of different diameters. It has been observed that increase in pitch decreases heat transfer coefficient, keeping coil diameter and tube diameter constant because the amount of water decreases, and it has less time for preheat. The other reason is that as the pipe becomes straighter, it minimizes the effect of secondary flow. It's clear that the heat transfer coefficient is directly proportional to tube diameter for same pitch and coil diameter. The reason is that it increases the volume of water acquired by the coil. By increase the diameter of tube, the pitch also decreases so the heat transfer coefficient increases. The coil diameter has marginal effect on the heat transfer coefficient.

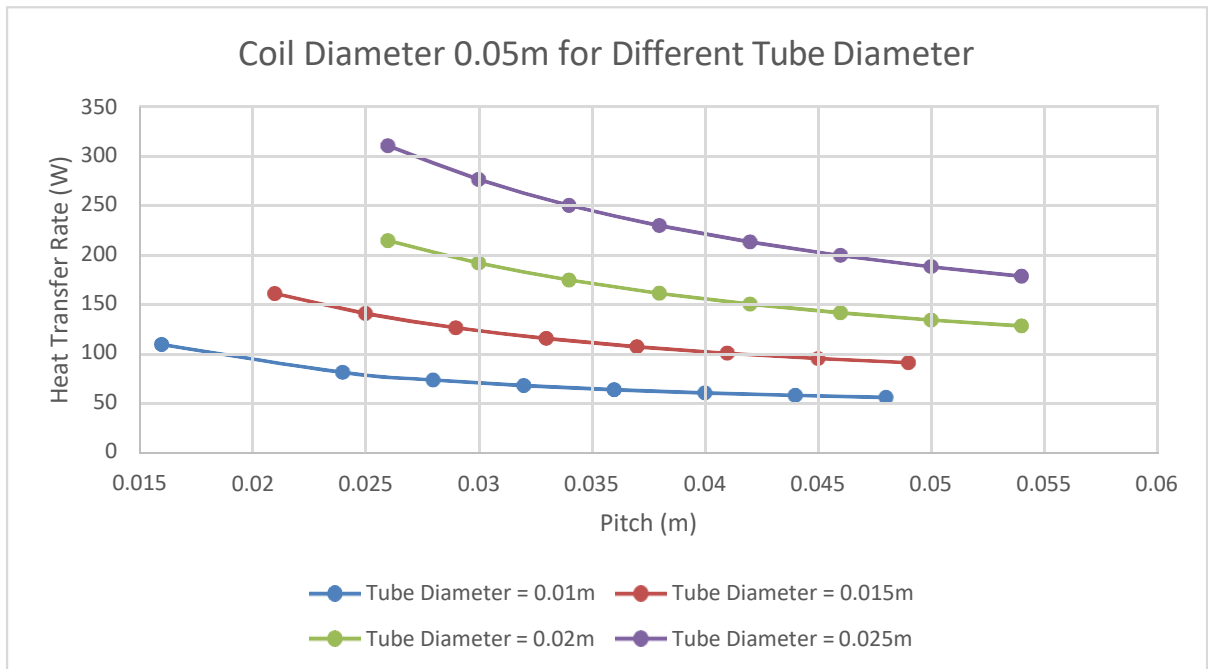


Figure 14: Heat Transfer Rate vs Pitch for coil of 0.05m

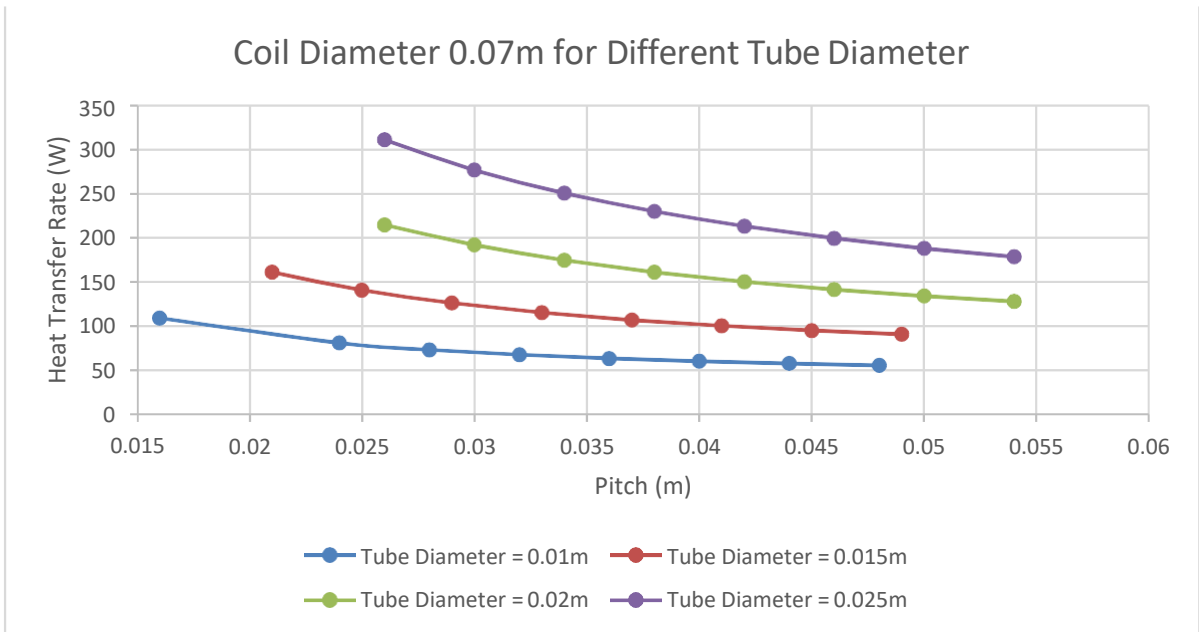


Figure 15: Heat Transfer Rate vs Pitch for coil of 0.07m

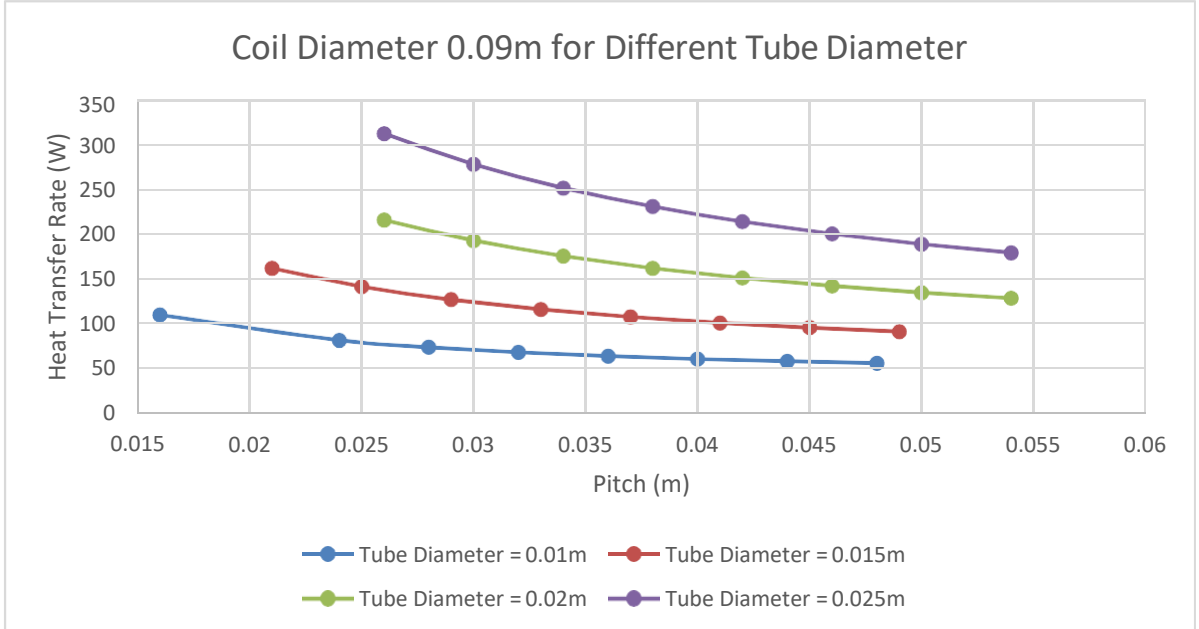


Figure 16: Heat Transfer Rate vs Pitch for coil of 0.09m

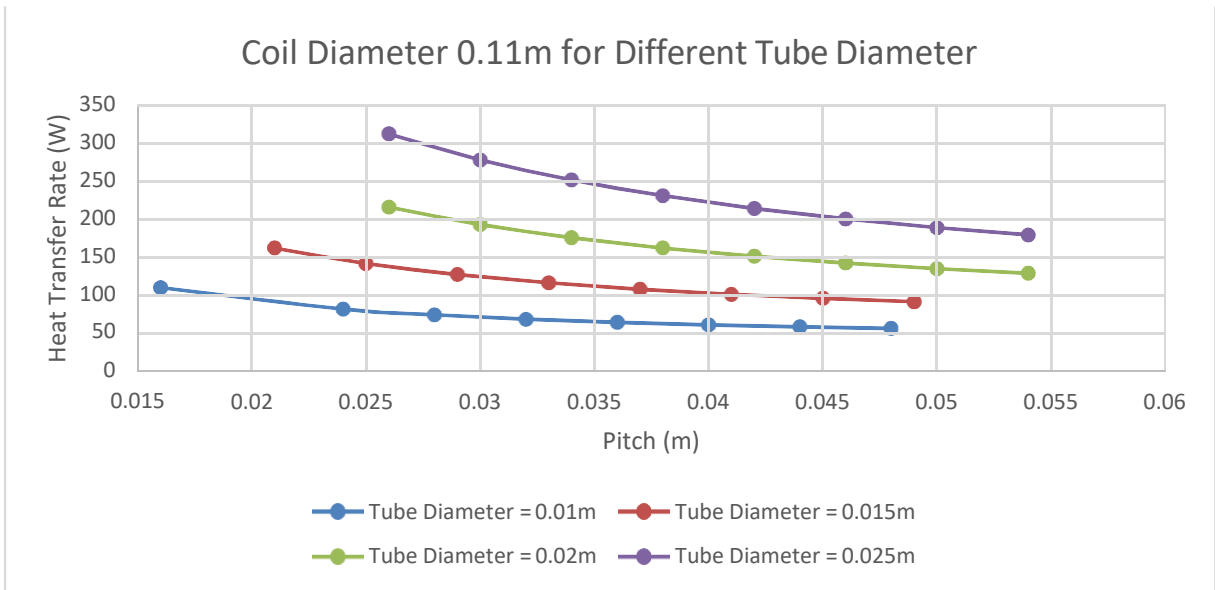


Figure 17: Heat Transfer Rate vs Pitch for coil of 0.11m

Next four graphs (Figure:13-16) are between pitch and heat transfer rate for different tube diameters. Heat transfer rate is inversely proportional to pitch and directly proportional to tube diameter. Heat transfer coefficient and heat transfer rate have same behavior.

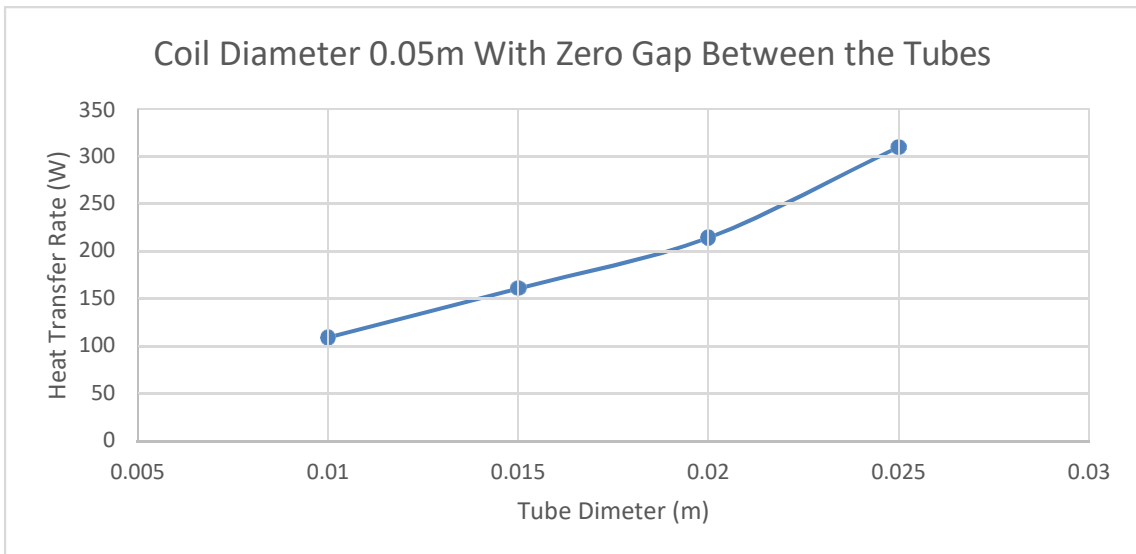


Figure 18: Heat Transfer Rate vs Tube Diameter for coil of 0.05m

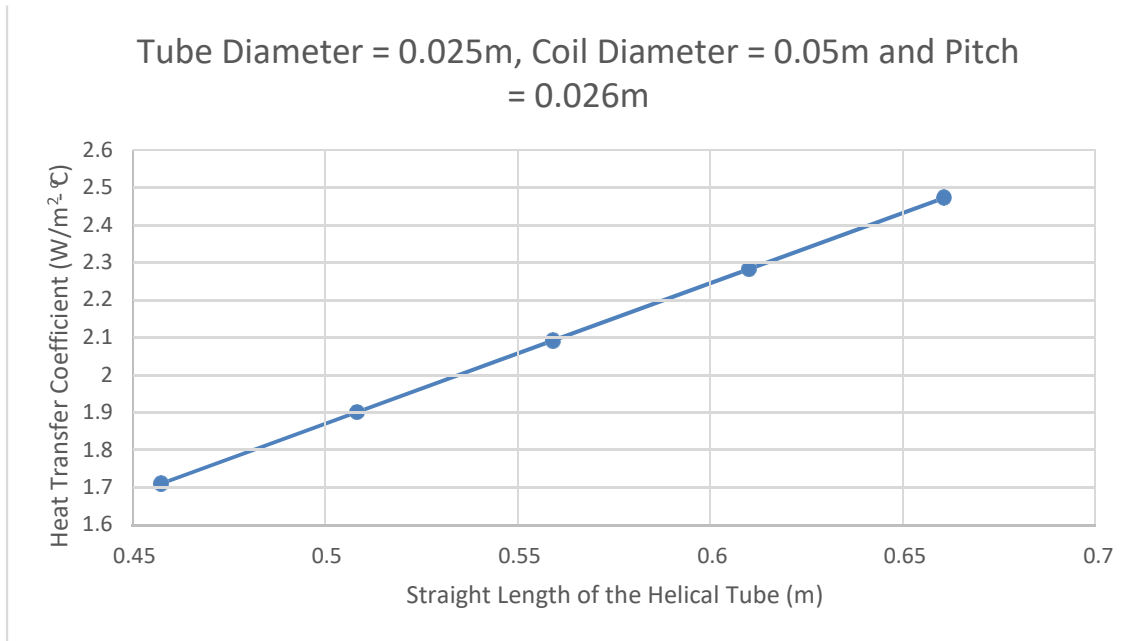


Figure 19: Heat Transfer Coefficient vs Straight Length of the Helical Tube

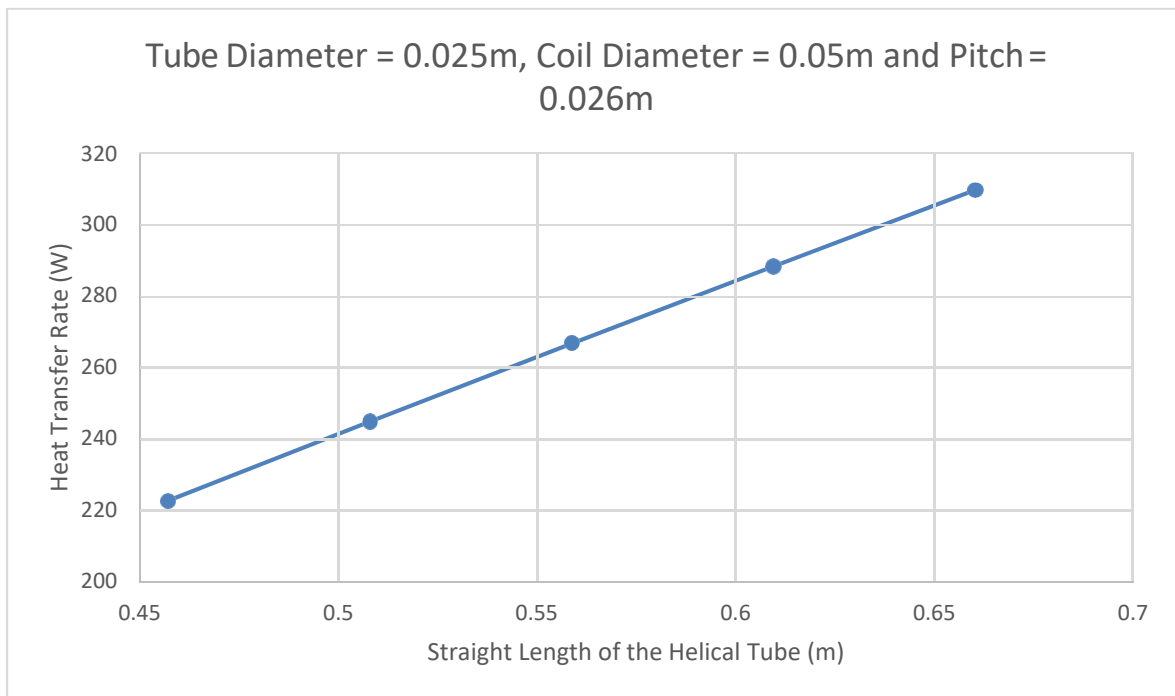


Figure 20: Heat Transfer Rate vs Straight Length of the Helical Tube

It is noted that the heat transfer coefficient and heat transfer rate is directly proportional to the straight length of the helical tube because with the increase in straight length of the helical tube the volume and the mass of water contained by the tube is more than the tube with less length, due to which the heat transfer rate and heat transfer coefficient increases.

Numerical Simulation Results:

Transient heat analysis in Ansys simulation at the outlet of the heat exchanger showed the temperature contours as well as outlet temperature at the heat exchanger. Results were obtained for the water flowing in the outer pipe at a mass flowrate of 0.54 kg/s.

For all flows, Ansys Fluent solves conservation equations for mass and momentum. For flows involving heat transfer or compressibility, an additional equation for energy conservation is solved. The detailed Equation are listed in [Appendix-II: Equations](#)

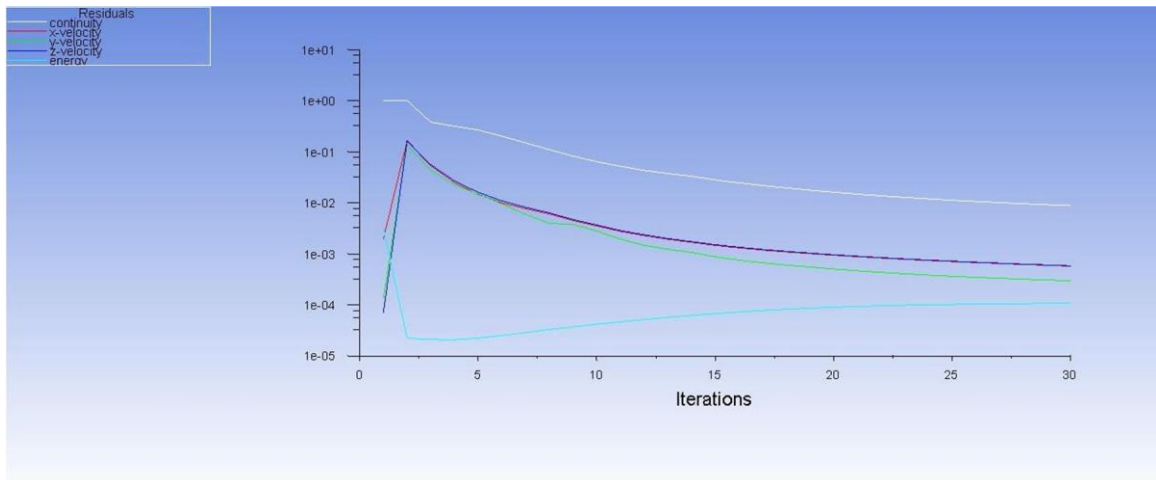


Figure 21: Flow parameters across 30 Iterations

The figures show following flow parameters across 30 Iterations:

- Continuity
- X-Velocity
- Y-Velocity
- Z-Velocity
- Energy

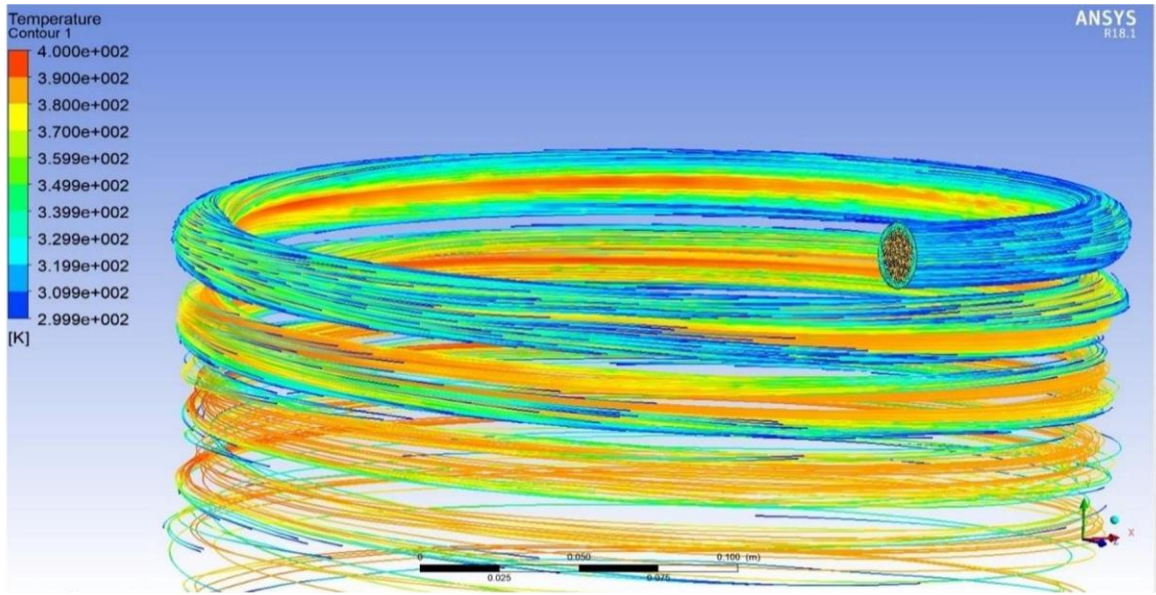


Figure 22: Streamline Flow Profile of water for 1000 particles

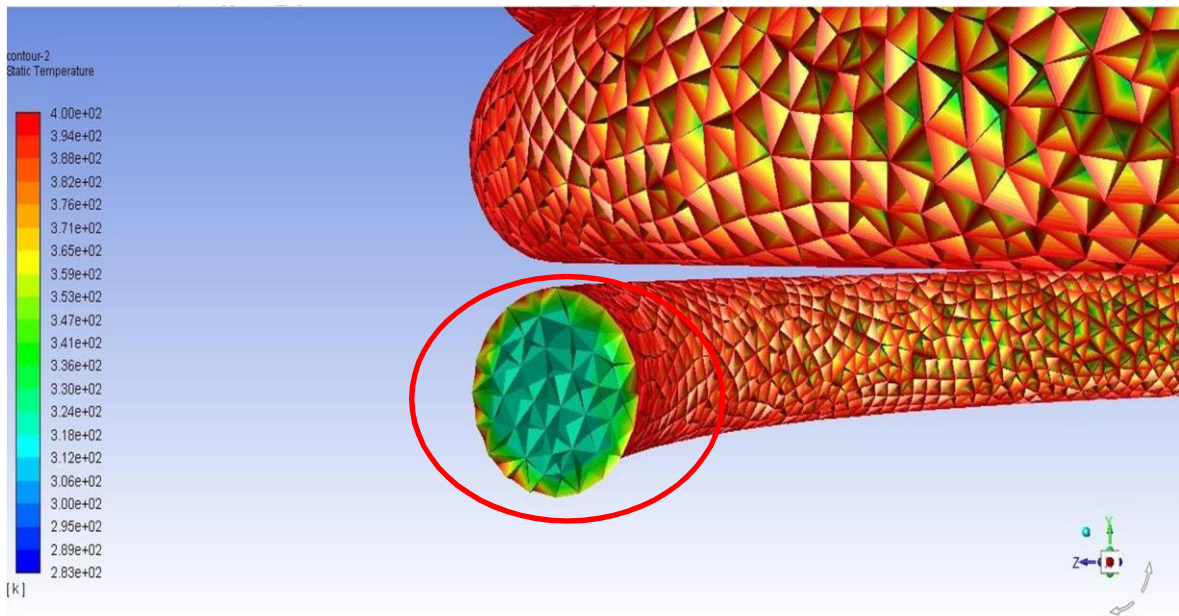


Figure 23: Outlet water Temperature Contour

The Temperature obtained from the Ansys calculation is about 312 k, which means that the rise of the water temperature is about 12 degrees while there is a difference of temperature that was calculated theoretically and that is due to fact that the whole helical coil was considered to be cylinder as the rings of copper tubes are tightly packed and the temperature used for the calculation was used average temperature.

Table 2: Comparison between Analytical and Ansys Simulation

Parameters	Analytical Method	Ansys Simulation
Coil Temperature (K)	400K	400K
Initial water temperature (K)	283	300
Water Mass Flowrate (kg/s)	.54	0.54
Outlet Water Temperature(K)	293.5	312.7
Rise in water Temperature(⁰ C)	10.5	12.7

There is a difference of temperature that was calculated theoretically and that is due to fact that the whole helical coil was cylinder as the rings of copper tubes are tightly packed and the temperature used for the calculation was used average temperature.

Final Design

By using the above results, selected model has following features:

Mass flow rate of water in the helical tube = 0.54kg/s

Straight length of the tube in the helical shape = 0.6604m (26inches)

Height of the shell = 0.7112m (28inches)

Diameter of the shell = 0.254m (10inches)

Inlet Temperature of the water = 10 °C

Material of Tube = Copper

Material of Inner Shell = Mild Steel

Insulation of Shell = Glass Wool

Material of Outer Shell = Galvanized Iron

Tube Thickness = 0.0033m

Tube inner Diameter = 0.018923m (0.745inches)

Coil Diameter = 0.2032m (8inches)

Pitch = 0.024923m

Note: We use this tube diameter because it is the maximum size of soft copper available in the market which can be bent to form a helical coil. Hard copper cannot bend. Shell is insulated by glass wool with sheet of mild steel on the inner side of the tube and galvanized iron sheet on the outside to protect the shell from the rusting.

Results

Increase in heat transfer rate = 205.5245W

Overall heat Transfer coefficient = 1.621750558

Effectiveness of the heat exchanger = 87.75%

Cost analysis

The purpose of cost analysis to optimize the value and check the geyser/heat exchanger (our project) marketplace. The scope includes cost optimization and a better product in a reasonable cost. The major outcomes of this analysis are forecasting budget of our project

and how quality varies with budget, and to reach at a optimize point. One other outcome is that we may find good approach to carry out our project.

Assumptions

Gas consumed for one cycle of geyser = 0.67 m^3

Geyser is turned on for minimum 2 times/day.

Gas consumed for whole day = 1.34 m^3

Gas consumed for one month = 40.3 m^3

Geyser works for 4 months' winter season in Pakistan.

$161.2 \text{ m}^3 = 1612$ rupees (Gas price w.r.t volume) Gas

consumed for simple geyser whole year = 201.66 m^3

Cost of gas for simple geyser whole year = 2016.6 Rs.

Estimated Cost Analysis of Modified Geyser

Cost of gas consumed by modified geyser:

1 MMBTU = 300 RS

1 MMBTU = 28.263682 m^3

$1 \text{ m}^3 = 10.61$ RS

$161.2 \text{ m}^3 = 1612$ rupees (Gas price w.r.t volume)

Payback Period

Difference between the gas bill for a year = 404.6 rupees

Price of simple geyser = 15000 rupees

Price of modified geyser = 18000 rupees

Payback period for difference in price = 7.4 years

Here is the graph showing gas consumption of a family in Pakistan within a year

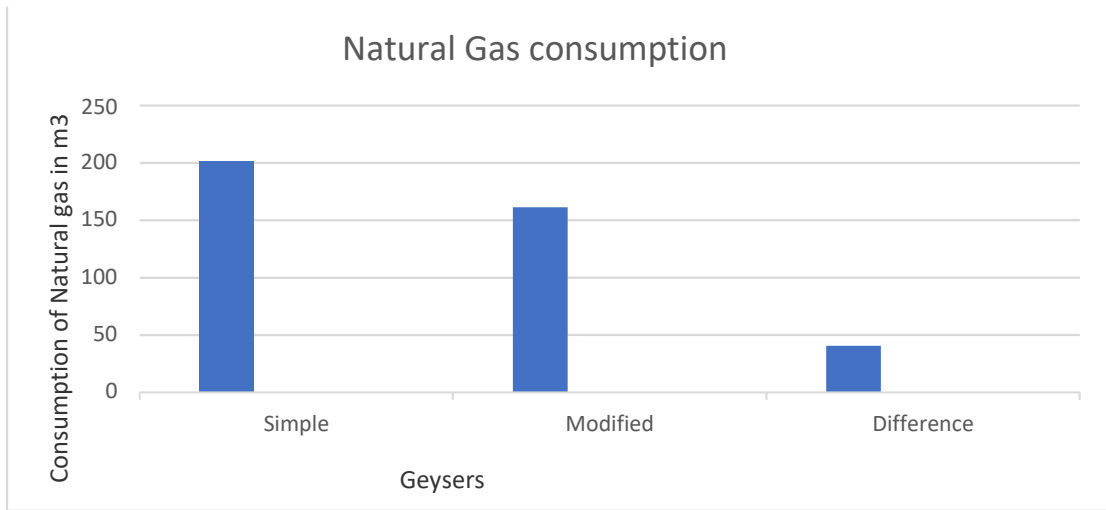


Figure 24: Gas consumption of a family of 4 across the year

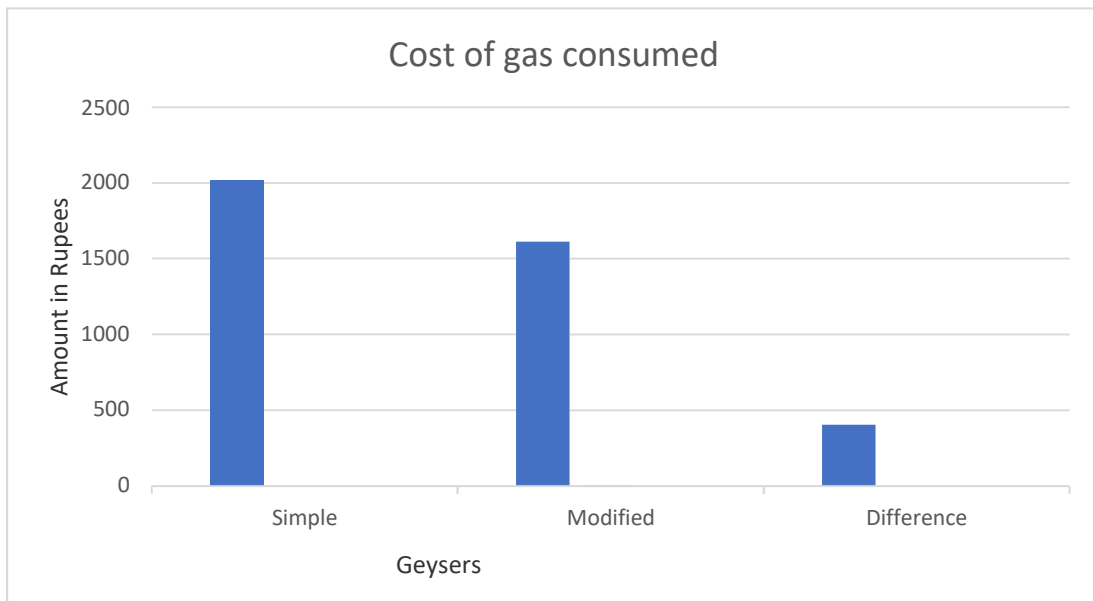


Figure 25: Comparison of savings done by simple and modified geyser

Theoretical Efficiency

$$\epsilon = (mC_p \Delta T)/Vh$$

m is the mass of water

C is the specific heat capacity V

is volume flowrate of the gas.

h is the enthalpy of combustion of fuel.

ΔT is the rise in temperature.

Efficiency of simple geyser = 19.3%

Efficiency of improved geyser = 21.5%

CHAPTER 5: CONCLUSION AND RECOMMENDATION

Helical Coil heat exchanger is used to recover waste heat from exhaust gases and used to preheat water before entering in the main tank. The effect of different parameters on heat transfer of helical coil heat exchanger is investigated. The theoretical results are shown in the form of graphs and results from Ansys are shown in the form of contours. From the results, it is concluded that the heat transfer coefficient is inversely proportional to the pitch and directly proportional to the tube diameter keeping other factors as constant. The heat transfer coefficient and heat transfer rate have same behavior. The coil diameter has marginal effect on the heat transfer. The heat exchanger coupled with the domestic geyser is thermal efficient and cost effective, yet some adjustments are can be done for making it commercially useable. Detailed cost analysis shows the cost effectiveness of the modified geyser followed by a reasonable payback period. There are some differences between theoretical and software results due to difference in solving model and working conditions.

It is recommended that:

- Further work can be done in optimizing the size of heat exchanger. This can be done by installing the baffles in the heat exchanger and study its effect. The other way can be using fins on coil tubes to increase heat transfer area.
- Further studies can be carried on optimizing or modifying the design of burner like optimizing nozzle design of burner to achieve swirl flow which may lead to efficient combustion and saving the fuel consumption in return.

REFERENCES

- [1] "Wikipedia," [Online]. Available:
<https://encyclopedia2.thefreedictionary.com/instantaneous-type+water+heater>.
- [2] E. R. W. F. A. S. John H. Eiseman, "A method for determining the most favorable design of gas burner," in *Bureau of Standards Journal of Research*, pp. 673-679.
- [3] V. B. F. M. B. S. Walter M. Berry, "Design of atmospheric gas burners," in *Technologic papers bureau of standards*, 1921.
- [4] C. B. CA Duff, "Design of domestic water heating system to save water and electricity," 2013.
- [5] "Go green heat solution," [Online]. Available:
<http://www.gogreenheatsolutions.co.za/sites/default/files/flat20plate20collector1.png>.
- [6] "Alternate energy tutorials," [Online]. Available: <http://www.alternative-energy-tutorials.com/images/stories/heating/alt36.gif>.
- [7] O. KS, "Thermal performance of solar air heaters: mathematical model and," *Energy*, 1995.
- [8] A. R. Garg HP, "Performance evaluation of a single solar air heater with n-sub collectors.," *Energy*, pp. 403-14, 1999.
- [9] T. R., "Thermal behavior of solar air heater with compound parabolic concentrator," *Energy conversion and management*, pp. 529-40, 2008.
- [10] S. S. M. A. M. Majid Azimi, "Simulation and Optimization of Vacuum Tube Solar Collector Water Heating System in Iran," *Journal of Science and Engineering*, vol. 07, no. 01, 2015.

- [11] D. A. Bainbridge, *The Integral Passive Solar Water Heater Book*, 1981.
- [12] H.-Y. G. Z.-H. L. G.-S. W. ., F. Z. H.-S. X. Pin-Yang Wang, "High temperature collecting performance of a new all-glass evacuated tube tubular solar air heater with U-shaped tube heat exchanger," *Energy Conversion and Management*, 2013.
- [13] "Wikipedia," 2018. [Online]. Available:
https://en.wikipedia.org/wiki/Solar_thermal_collector.
- [14] D. M. S. Al-Homoud, "Performance characteristics and practical applications of common building thermal insulation materials," *Building and Environment*, 2004.
- [15] O. Kaynakli, "A review of the economical and optimum thermal insulation thickness for building applications," *Renewable and Sustainable Energy Reviews*, 2011.
- [16] V. P. J. Z. J. H. Azra Korjenic, "Development and performance evaluation of natural thermal-insulation materials composed of renewable resources," *Energy and Buildings*, 2011.
- [17] B. P. Jelle, "Traditional, state-of-the-art and future thermal building insulation materials and solutions – Properties, requirements and possibilities," *Energy and Buildings*, 2011.
- [18] M. a. E.-A. A.Harris, "Domestic energy savings with geyser blankets," 2017.
- [19] A. A. R. K. R. S. R. Prof.N.B.Totala, "Analysis for critical radius of insulation for a cylinder," *Journal of Engineering*, vol. 3, no. 09, 2013.
- [20] "Wikipedia," [Online]. Available:
[https://en.wikipedia.org/wiki/Baffle_\(heat_transfer\)](https://en.wikipedia.org/wiki/Baffle_(heat_transfer)).
- [21] M. G. E. R. K. Mohamad Ramadan, "Parametric analysis of air–water heat recovery concept," *Elsevier*, 2015.

- [22] "ENEAGRID Data Space," [Online]. Available:
<https://www.afs.enea.it/project/neptunius/docs/fluent/html/th/node11.htm>.
- [23] P. R. R.N.S.V.Ramakanth, "Design of helical baffle in shell and tube heat exchanger with using copper oxide Nano particle.," *International Journal of Engineering Sciences and Research Technology*, 2015.
- [24] X. X. a. J. Z. RSM Thomas, "Energy efficient geyser," 2017.
- [25] "Wikipedia," [Online]. Available:
https://en.wikipedia.org/wiki/Storage_water_heater.
- [26] "Engineering toolbox," [Online]. Available:
https://www.engineeringtoolbox.com/conductive-heat-loss-cylinder-pipe-d_1487.html.

APPENDIX I: TABLES

Table 3: Details of different dimensionless numbers

Sr No.	Dimensional less numbers	Details
1	Reynolds Number	$Re = \rho V d / \mu$
2	Nusselt Number	$Nu = h d / k$
3	Helix Number	$De = Re (d / D)^{1/2}$
4	Dean Number	$He = De / [1 + (b/2\pi a)^2]^{1/2}$

Table 4: Properties of Hot and cold fluid of helical coil heat exchanger

Sr No.	Properties	Cold water	Hot water
1	Specific heat (Cp) (J/Kg °C)	4178	4179.2
2	Viscosity (μ) (N-s/m ²)	0.000753	0.000620
3	Density (ρ) (kg/m ³)	997	990
4	Thermal conductivity (k) W/m ² K	0.6216	0.6352
5	Prandtl No. (Pr)	6.048	4.08

Table 5: Usable sizes of different helical tubes available in market.

Size	Actual outer diameter, inches	Type		
		K	L	M
		Actual inner diameter, inches		
3/8	1/2	0.402	0.430	0.450
1/2	5/8	0.528	0.545	0.569
5/8	3/4	0.652	0.668	0.690
3/4	7/8	0.745	0.785	0.811
1	1 1/8	0.995	1.025	1.055
1¼	1 3/8	1.245	1.265	1.291
1½	1 5/8	1.481	1.505	1.527
2	2 1/8	1.959	1.985	2.009
available in these forms	drawn temper	20-foot lengths	20-foot lengths	20-foot lengths
	annealed temper	60-foot, 100-foot, and 200-foot coils	60-foot, 100-foot, and 200-foot coils	not made

Table 6: Fouling resistances

Sr No.	Thermal resistances due to Fouling	Values (m ² ·K/W)
1	Inner surface of helical tube	0.040856
2	Outer surface of helical tube	0.1021

APPENDIX II: EQUATIONS

The Mass Conservation Equation [22]

The equation for conservation of mass, or continuity equation, can be written as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = S_m \quad (1.2-1)$$

For 2D axisymmetric geometries, the continuity equation is given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v_x) + \frac{\partial}{\partial r}(\rho v_r) + \frac{\rho v_r}{r} = S_m \quad (1.2-2)$$

Momentum Conservation Equations

Conservation of momentum in an inertial (non-accelerating) reference frame is described by

$$\frac{\partial}{\partial t}(\rho \vec{v}) + \nabla \cdot (\rho \vec{v} \vec{v}) = -\nabla p + \nabla \cdot (\bar{\bar{\tau}}) + \rho \vec{g} + \vec{F} \quad (1.2-3)$$

The stress tensor $\bar{\bar{\tau}}$ is given by

$$\bar{\bar{\tau}} = \mu \left[(\nabla \vec{v} + \nabla \vec{v}^T) - \frac{2}{3} \nabla \cdot \vec{v} I \right] \quad (1.2-4)$$

For 2D axisymmetric geometries, the axial and radial momentum conservation equations are given by

$$\begin{aligned} \frac{\partial}{\partial t}(\rho v_x) + \frac{1}{r} \frac{\partial}{\partial x}(r \rho v_x v_x) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho v_r v_x) = -\frac{\partial p}{\partial x} + \frac{1}{r} \frac{\partial}{\partial x} \left[r \mu \left(2 \frac{\partial v_x}{\partial x} - \frac{2}{3} (\nabla \cdot \vec{v}) \right) \right] \\ + \frac{1}{r} \frac{\partial}{\partial r} \left[r \mu \left(\frac{\partial v_x}{\partial r} + \frac{\partial v_r}{\partial x} \right) \right] + F_x \end{aligned} \quad (1.2-5)$$

And

$$\begin{aligned} \frac{\partial}{\partial t}(\rho v_r) + \frac{1}{r} \frac{\partial}{\partial x}(r \rho v_x v_r) + \frac{1}{r} \frac{\partial}{\partial r}(r \rho v_r v_r) = -\frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial x} \left[r \mu \left(\frac{\partial v_r}{\partial x} + \frac{\partial v_x}{\partial r} \right) \right] \\ + \frac{1}{r} \frac{\partial}{\partial r} \left[r \mu \left(2 \frac{\partial v_r}{\partial r} - \frac{2}{3} (\nabla \cdot \vec{v}) \right) \right] - 2 \mu \frac{v_r}{r^2} + \frac{2}{3} \frac{\mu}{r} (\nabla \cdot \vec{v}) + \rho \frac{v_z^2}{r} + F_r \end{aligned} \quad (1.2-6)$$

where

$$\nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_r}{\partial r} + \frac{v_r}{r} \quad (1.2-7)$$