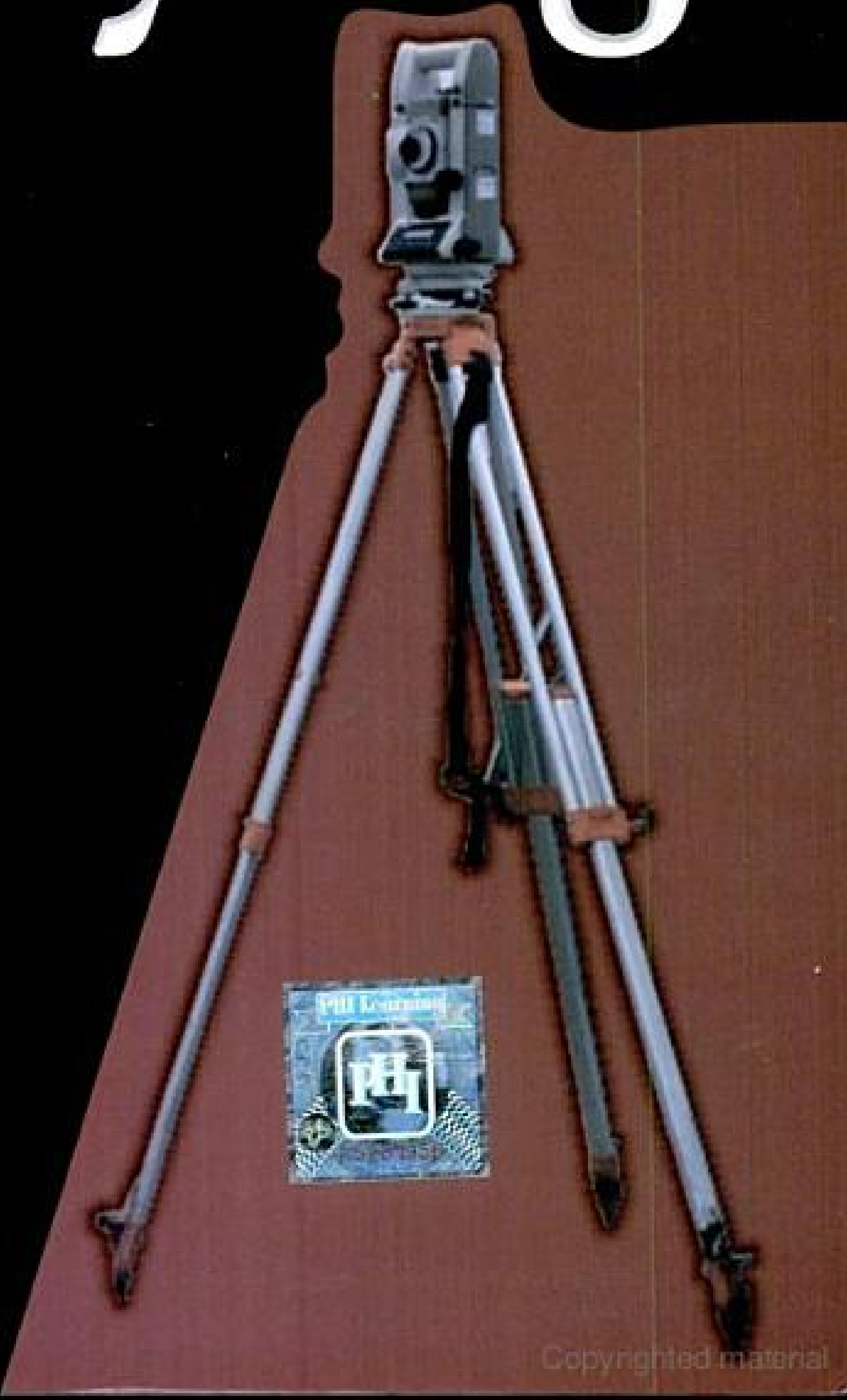
Eastern
Economy
Edition

Second Edition

Fundamentals of Surveying



S.K. Roy

PHI

CHAPTER

3

Measurement of Horizontal Distances

3.1 INTRODUCTION

One of the most important operations in surveying is measurement of horizontal distance between two points. If the points are at different elevations, the distance is the horizontal length between plumb lines at the points.

3.2 METHODS OF MEASURING HORIZONTAL DISTANCES

Depending on the accuracy desired and time available for measurement, there are many methods of measuring horizontal distances. They are: (i) Pacing, (ii) Odometer readings, (iii) Tacheometry, (iv) Electronic distance measurement, (v) Chaining, and (vi) Taping, While chaining and taping are most common in our country, electronic distance measurements (EDM) are gradually being increasingly used.

3.2.1 Pacing

Pacing is an approximate method of measuring distance. Initially the surveyor must walk a known distance a number of times in his own natural way so that his natural pace is known. To count the number of paces a pedometer or a passometer may be used.

3.2.2 Odometer

An odometer converts the number of revolutions of a wheel of a known circumference to a distance. This method can often be used to advantage, on preliminary surveys where precise distances are not necessary. Odometer distances should be converted to horizontal distance when the slope of the ground is steep.

3.2.3 Tacheometry

Here distance is measured not directly but indirectly with the help of an optical instrument called tacheometer. A theodolite with three cross hairs can also be used with the intercept on a levelling staff between the top and bottom cross hairs multiplied by a constant giving the horizontal distance. In subtense bar method, the angle subtended at the end of a line by a known horizontal base at the other end is measured and the horizontal length is geometrically obtained.

3.2.4 Electronic Distance Measurement (EDM)

This is a modem development in surveying where electromagnetic waves are utilized to measure distance. They are basically of two types: (i) Electro optical instruments which use light waves for measurement of distances such as geodimeter, mekometer and range master, (ii) Microwave equipment, which transmits microwaves with frequencies in the range of 3 to 35 GHz corresponding to wavelengths of about 1 dm to 8.6 mm.

3.2.5 Chains

Chains are used to measure distances when very great precision is not required. In our country it is frequently used though in other countries it is being gradually replaced by tapes. The chain is robust, easily read and easily repaired in the field if broken. It does not, however, give correct length owing to wear on the metal to metal surfaces, bending of the links, mud between the bearing surfaces, etc. Also the weight is a disadvantage when the chain has to be suspended.

In India link type surveying chains of 30 m lengths are frequently used in land measurement. Nomenclatures and dimensions of different parts of a chain are given in Fig. 3.1. Details of a 30 m chain are given in Fig. 3.2. For 5 and 10 m chains the shape of tallies and the corresponding distances are shown in Fig. 3.3. There is also 30 m chain with 100 links (instead of 150) so that each link is 0.3 m. There are tallies at every 3 m.

3.2.6 Tapes

Tapes are used for accurate work and may be of (i) cloth or linen, (ii) metal, (iii) steel, (iv) invar.

Tapes used for surveying are 30 m in length and graduated in meter, decimeter and centimeter. Cloth or metallic tapes are made of high grade linen with fine copper wires running length-wise to give additional strength and prevent excessive elongation. They come in enclosed reels and are not suitable for precise work. Steel tape is superior to metal tape, is usually 6 to 10 mm wide and is more accurately graduated. It cannot, however, withstand rough usage. If the tape gets wet, it should be wiped with a dry cloth and then with an oily rag. Invar tape is used for very precise work. It is made of 35% nickel and 65% steel. The coefficient of thermal expansion is very small, about 1/30 to 1/60 of that of an ordinary steel tape. The invar tape is soft and also very expensive.

3.3 CHAINING AND TAPING ACCESSORIES

The small instruments and accessories used with chain or tape are (i) Arrows, (ii) Pegs, (iii) Ranging rods, (iv) Offset rods, (v) Plumb bobs.

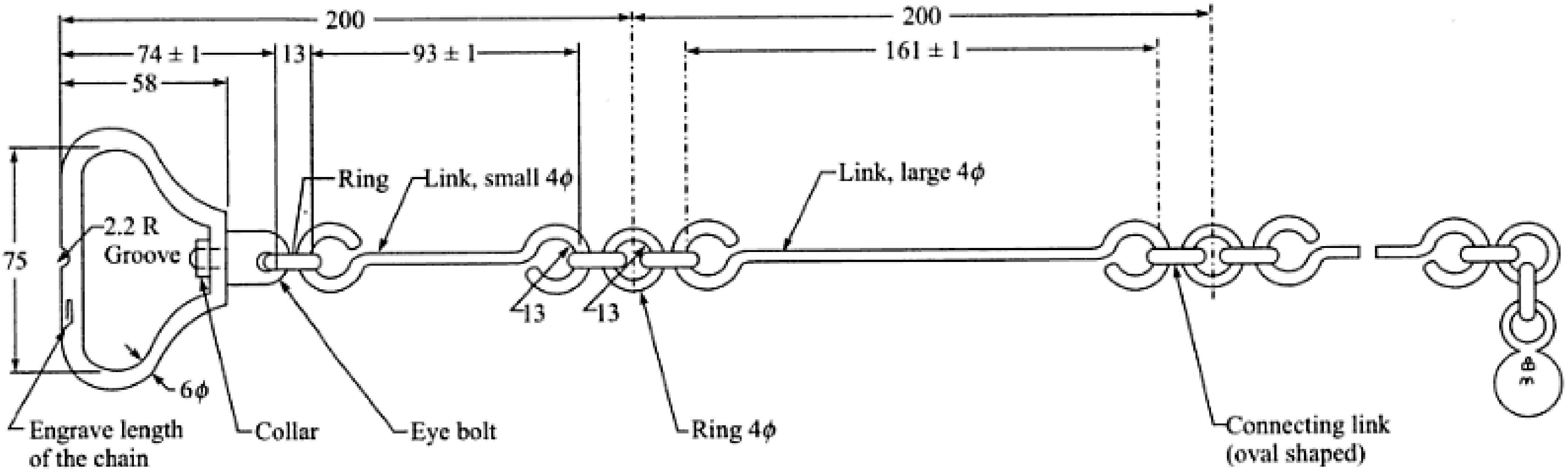


Fig. 3.1 Nomenclature and dimensions of different parts of chain (all dimension in mm).

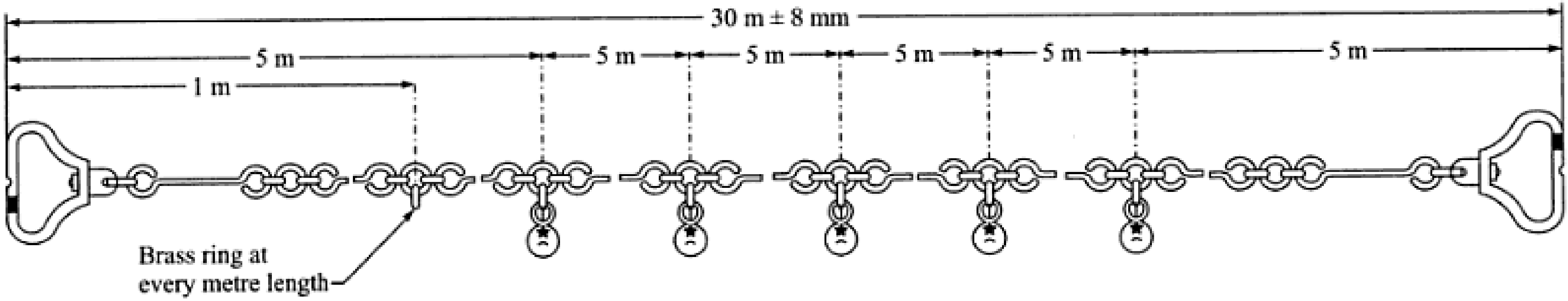


Fig. 3.2 30 Meter chain.

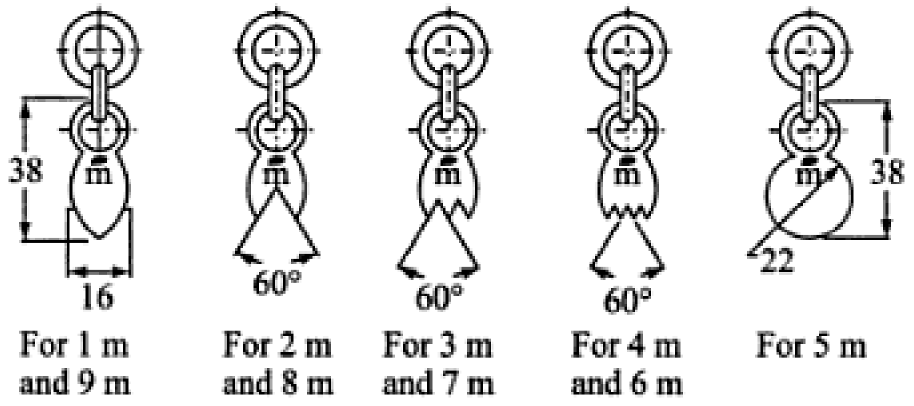


Fig. 3.3 Shapes of tallies for chains (5 m and 10 m).

Arrows or chain pins are used to mark the position of the ends of the chain on the ground. Details are shown in Fig. 3.4.

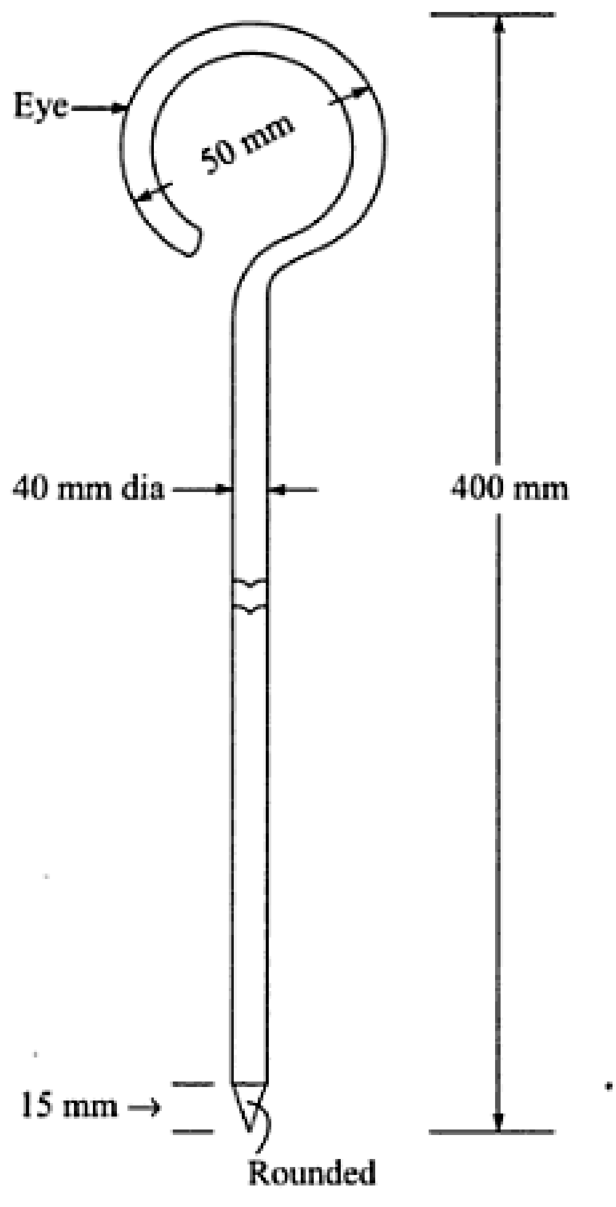


Fig. 3.4 Details of arrow or chain pin.

Wooden pegs are used to mark the positions of the survey stations or the end points of a survey line. The typical dimensions are 25 mm × 25 mm in cross section and 150 mm long with a nail at the top.

Ranging poles and rods are used to make measurements along a straight line. Details are shown in Fig. 3.5.

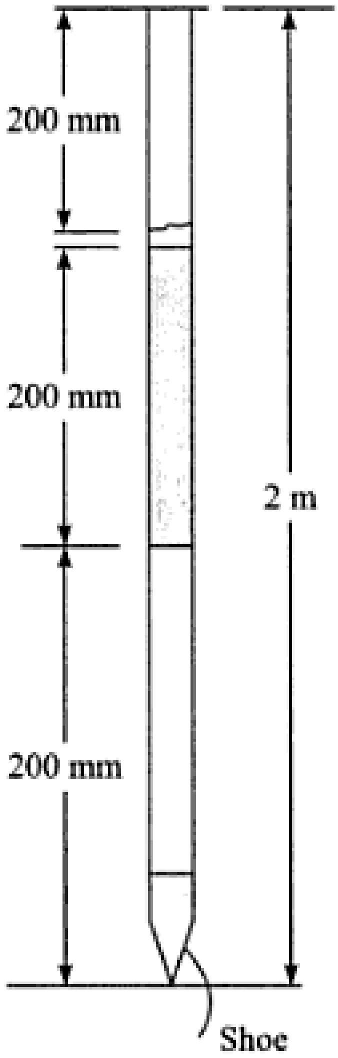


Fig. 3.5 Details of ranging rod.

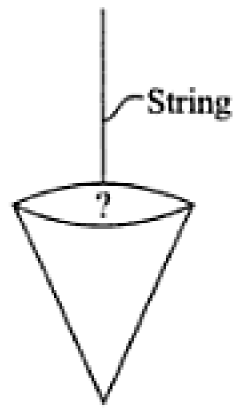


Fig. 3.6 Plumb bob.

Plumb bobs are used to project a point on the ground upto the tape or to project a point on the tape down to the ground. Details are shown in Fig. 3.6.

3.4 MEASUREMENT BY CHAIN

There are basically two types of measurements—(i) On level ground, (ii) On uneven ground.

In level ground the line to be measured is marked at both ends and at intermediate point where necessary so that a clear sight is obtained. Sometimes a theodolite is used for ranging. The follower holds the rear end of the chain at the station point and by movements of his arms directs the arrow or ranging rod held by the leader for the purpose into true alignment. The leader then pulls the chain taut and inserts an arrow in the ground to mark the end. After relevant work in this chain line is over, the leader again pulls on the chain leaving an arrow to mark the position of the end of the first length. The follower holds the rear end of the chain against this and directs the leader into alignment as before. After the chain has been pulled taut and the further end marked by the second arrow, the follower picks up the first and carries it with him. The number of arrows in the hand of the follower at any time will indicate the number of complete chain lengths measured. After 10 chains have been laid down the follower hands over the ten arrows to the leader and the same procedure is carried out for the next ten lengths.

In uneven or sloping ground the distance may be directly measured in small horizontal stretches or steps as shown in Fig. 3.7(a) or indirectly by measuring the sloping distance along the slope and then getting the horizontal distance analytically by measuring the slope by means of a clinometer or measure the difference in elevation between the points (Fig. 3.7b).

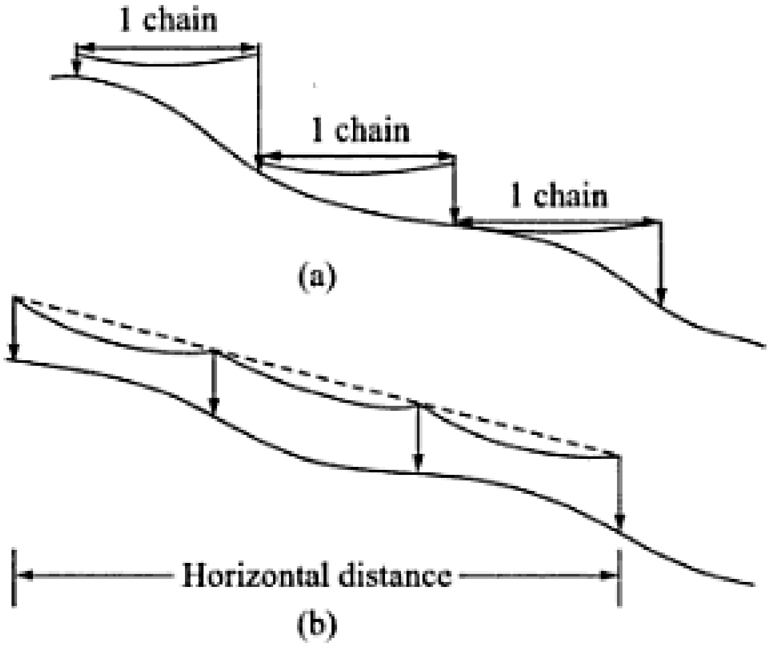


Fig. 3.7 Measurement on slope: (a) chain held horizontally, (b) chain held on slope.

For accurate measurements and in all important surveys, the lengths are now measured with a tape and not with a chain. For higher precision a taping tripod or taping buck must be used. The taping buck usually (i) is rigid in use, (ii) is easily aligned, (iii) is portable, (iv) permits easy transfer of chaining point to or from the ground, (v) can easily act as a back sight.

Since taping is usually done on the slope when tripods are used, the elevations of the tops of tripod must be ascertained simultaneously as the taping proceeds to determine the horizontal distance.

3.5 REDUCTIONS TO MEASUREMENT IN SLOPE

There are two ways in which this reduction can be made. When the slope angle α is known the horizontal distance is

$$H = S \cos \alpha \tag{3.1}$$

where S is the inclined length.

To determine the accuracy with which the vertical angle must be measured in order to meet a given relative accuracy in the resulting horizontal distance, we differentiate the above equation with respect to α and get

$$dH = -S \sin \alpha \, d\alpha \tag{3.2}$$

Relative accuracy is then

$$\frac{dH}{H} = -\frac{S \sin \alpha \, d\alpha}{S \cos \alpha} = -\tan \alpha \, d\alpha \tag{3.3}$$

When the slope is expressed in terms of difference in elevation as in Fig. 3.8, we have

$$H = \sqrt{S^2 - h^2} \tag{3.4}$$

The expression on binomial expansion becomes

$$H = S - \frac{h^2}{2S} - \frac{h^4}{8S^3} \tag{3.5}$$

If we neglect the 3rd term

$$C = S - H = \frac{h^2}{2S} \tag{3.6}$$

and

$$dC = \frac{hdh}{S} \tag{3.7}$$

If both sides are divided by S the relative accuracy becomes

$$\frac{dC}{S} = \frac{hdh}{S^2} \tag{3.8}$$

Slightly different expressions can be derived as follows:

$$\begin{aligned}\text{Correction} &= \text{hypotenusal allowance}\\ &= AC - AD = AD \sec \alpha - AD\\ &= AD\left(1 + \frac{\alpha^2}{2} + \frac{5\alpha^4}{24} + \dots\right) - AD\\ &\approx AD\,\frac{\alpha^2}{2}\end{aligned} \tag{3.9}$$

If the slope is expressed as 1 to n $\alpha = \frac{1}{n}$.

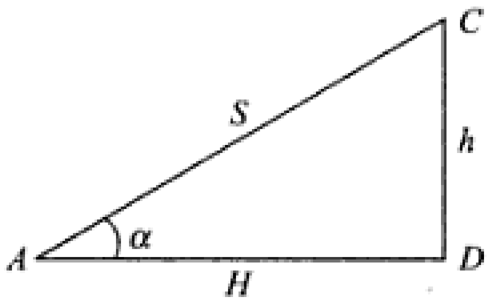


Fig. 3.8 Reduction in slope.

EXAMPLE 3.1 In chaining a lire what is the maximum slope (a) in degrees and (b) as 1 in n which can be ignored if the error from this source is not to exceed 1 in 1500?

Solution

(i) Let α be expressed in degrees.

$$\alpha^\circ = \frac{\alpha \cdot \pi}{180} \text{ radian}$$

Error is 1 in 1500 = $\dfrac{AD}{1500}$

$$\frac{AD}{1500} = \frac{AD}{2}\left(\frac{\alpha\pi}{180}\right)^2$$

$$\frac{\alpha\pi}{180} = \sqrt{\frac{2}{1500}}$$

$$\alpha = \frac{180}{\pi}\sqrt{\frac{2}{1500}}$$

$$\alpha = 2.092°$$

(ii)

$$\frac{AD\cdot\alpha^2}{2} = \frac{AD}{2n^2} = \frac{AD}{1500}$$

$$n = \sqrt{\frac{1500}{2}} = 27.386.$$

Therefore slope is 1 in 27.386.

EXAMPLE 3.2 With what accuracy must a difference in elevation between two ends of a 30 m tape be known if the difference in elevation is 2.28 m and the accuracy ratio is to be at least 1 in 25000.

Solution

we have

$$\frac{dC}{S} = \frac{hdh}{S^2} = \frac{1}{25000}$$

$$dh = \frac{S^2(1)\cdot(1)}{25000\times h}$$

$$= \frac{30^2(1)(1)}{(25000)(2.28)} = .0157894 \text{ m}$$

3.6 SYSTEMATIC ERRORS IN LINEAR MEASUREMENT BY CHAIN OR TAPE

The principal systematic errors in linear measurement made with a chain or tape are (i) Incorrect length, (ii) Tape or chain not horizontal, (iii) Fluctuations in temperature, (iv) Incorrect tension or pull, (v) Sag, (vi) Incorrect alignment, and (vii) Chain or tape not straight.

3.6.1 Incorrect Length

Incorrect length of a tape or chain is one of most important errors. It is systematic. A tape or chain is of nominal or designated length at the time of manufacture but with use it will seldom remain at its original length. It should be frequently compared with a standard length to find out the discrepancy. The correction to be applied is known as *absolute correction* C_a and is given by:

$$C_a = \text{True length} - \text{nominal length} \tag{3.10}$$

If the true length is shorter than the nominal length the correction should be subtracted while if the true length is greater than the nominal length, the correction is to be added.

3.6.2 Chain or Tape not Horizontal

When the chain or tape is inclined but assumed to be horizontal an error in measurement is introduced. The horizontal distance is always less than the inclined length, hence the correction is always subtractive. The correction is given by Eqs. (3.6) and (3.9).

3.6.3 Fluctuations in Temperature

A chain or tape is of standard length at a particular temperature. If the ambient temperature changes, the length of the tape also changes, the change being 1.15×10^{-5}/°C. The temperature correction C_t is, therefore

$$C_t = L\alpha\,(T - T_s) \tag{3.11}$$

where L is the length, α is the coefficient of thermal expansion, T is the temperature at which the measurement is made and T_s is the standardization temperature. Temperature effect is less pronounced on a cloudy day or early in the morning or late in the afternoon. Since the coefficient of invar is very small (3.6×10^{-7}/°C), invar tapes will give better result than steel and should be used in surveys of high order.

3.6.4 Incorrect Tension or Pull

A tape or chain is of standard length under a particular pull. In field operation the pull applied may be more or less which will introduce an error. Steel being elastic there will be extension or contraction given by the expression

$$\frac{(P - P_s)L}{A \cdot E} \tag{3.12}$$

which should be applied as pull correction C_P. Here P is the actual pull applied P_s is the standard pull, L is the length of the chain or tape, A is the cross sectional area and E is the modulus of elasticity of steel. E for steel is 2.1×10^7 N/cm^2. For important work a spring balance should be used to determine the exact pull. Otherwise, sometimes more pull or sometimes less pull will be applied though the tendency is to apply less pull than the standard.

3.6.5 Sag

A tape or chain supported at the two ends will always sag, i.e. the mid point will be at a lower level compared to the two ends. As a result the chord or horizontal length will be less than the curved length. Assuming the curve to take an approximate shape of a parabola, the difference between sagged length and chord length is given as

$$L_s - d = \frac{8v^2}{3d} \tag{3.13}$$

where

L_s = unsupported length of tape
d = chord length
v = sag in the middle.

Also by taking moment about one of the supports for half the tape length we get,

$$P_1 v = \frac{wd^2}{8} \tag{3.14}$$

where w is the weight of the tape/unit length and P_1 is the pull.

Combining the two equations, sag correction becomes

$$L_s - d = \frac{8v^2}{3d} = \frac{8}{3d}\left(\frac{wd^2}{8P_1}\right)^2 = \frac{w^2 d^3}{24P_1^2}$$

Substituting L. in place of d to simplify the result, we get

$$\text{Sag correction } C_s = \frac{w^2 L^3}{24P_1^2} \tag{3.15}$$

Sag correction is always negative as the correct length is always less than the measured length.

Normal tension when applied to a tape or chain will increase the length in such a way that sag correction will be compensated so that no sag correction will be necessary. This pull P_n is given by the expression:

$$P_n = \frac{0.204W\sqrt{AE}}{\sqrt{P_n - P_s}} \tag{3.16}$$

which is to be solved by trial and error.

Free tension is that tension which when applied will eliminate the need for corrections required due to tape standardization, temperature, sag and tension. This means

$$C_a + C_t - C_s + C_p = 0 \tag{3.17}$$

Like Eq. (3.16), Eq. (3.17) can also be solved by trial and error to find the free tension P_f.

Equation (3.16) can be derived as follows:

$$\text{Sag correction} = \frac{w^2 L^3}{24P_n^2} = \frac{W^2 L}{24P_n^2}$$

$$\text{Pull correction} = \frac{(P_n - P_s)L}{AE}, \text{ where } P_s \text{ is standard pull}$$

Equating

$$\frac{W^2 L}{24P_n^2} = \frac{(P_n - P_s)L}{AE}$$

$$P_n = \frac{\sqrt{AE}}{\sqrt{P_n - P_s}} \cdot W\left(\frac{1}{24}\right)^{1/2}$$

$$\frac{\sqrt{AE}\,W(0.204)}{\sqrt{P_n - P_s}} = \frac{0.204\,W\sqrt{AE}}{\sqrt{P_n - P_s}}$$

3.6.6 Incorrect Alignment

In taking a number of chain or tape measurements along a line the tape or chain may be off line and thus introduce systematic error. Equation (3.1) can be used to determine correct horizontal length with α being the horizontal off line angle instead of the slope angle. The correction is always subtractive.

3.6.7 Chain or Tape not Straight

When a chain or tape is not straight but gets bent due to bending of the links of the chain or bending of part of tape, the reading will always be more than the actual distance and the correction will be always subtractive. However, the magnitude is difficult to obtain unless compared with a standard chain or tape.

3.7 RANDOM ERRORS

The difference between systematic and random errors has already been explained. The systematic errors in chain or tape survey may become random when there is uncertainty about their magnitude and sign. Some of the random errors in chaining or taping are: (i) Incorrect determination of temperature, (ii) Incorrect application of pull, (iii) Deflection of plumb bob due to wind, (iv) Incorrect fixation of taping pin, (v) Incorrect reading.

Table 3.1 summarizes different characteristic of the types of errors discussed. Errors can be instrumental (*I*), natural (*N*) or personal (*P*):

Table 3.1 Classification of Errors

Type of errors	*Classification*	*Systematic (S) or random (R)*
Tape length	I	S
Temperature	*N*	S or R
Pull	*P*	S or R
Sag	*N, P*	S
Alignment	*P*	S
Tape not level	*P*	S
Plumbing	*P*	R
Marking	*P*	R
Interpolation	*P*	R

EXAMPLE 3.3 A 30 m tape weighs 12 g/m and has a cross sectional area of 0.020 cm^2. It measures correctly when supported throughout under a tension of 85 newton and at a temperature of 13.5°C. When used in the field, the tape is only supported at its ends, under a tension of 85 newton. The temperature is 20°C. What is the distance of zero and 30 mark under these conditions?

Solution There is variation from standardized condition as regards (i) Temperature, (ii) Support at the ends instead of throughout.

(i) Correction for temperature

$$= L \cdot \alpha (T - T_s)$$
$$= 30 \times 1.15 \times 10^{-5}(20 - 13.5)$$
$$= \mathbf{.00224\ m.}$$

This correction is additive as the length of the tape is more than the standard because of rise in temperature.

(ii) Sag correction $= \dfrac{w^2 L^3}{24P^2}$

Weight of tape = 12 g/m = .012 kg/m = 0.12 N/m.

$$C_s = \frac{(0.12)^2 (30^3)}{(24)(85^2)}$$
$$= \mathbf{.00224\ m.}$$

The sag correction is negative as the correct length is always less than the measured length.

$$\text{Total correction} = 0.00224 - .00224 = 0$$

Distance between 0 and 30 mark = 30 m.

EXAMPLE 3.4 A 30 m steel tape measured 30.0150 m when standardized fully supported under a 70 newton pull at a temperature of 20°C. The tape weighed 0.90 kg (9N) and had a cross-sectional area of 0.028 cm^2. What is the true length of the recorded distance AB for the following condition? (Assume all full tape lengths except in the last one.)

Recorded distance AB	Average temperature	Means of support	Tension	Elevation difference per 100 m.
114.095 m	12°	Suspended	100 N	2.5 m

Solution

(i) Correction for absolute length $= +\dfrac{(30.0150 - 30.00)}{30.00} \times 114.095$

$$= +\,0.0570\ \text{m}$$

(ii) Temperature correction $= L\alpha (T - T_s)$

$$= -114.095 \times 1.15 \times 10^{-5} \times (12° - 20°)$$
$$= -0.01049674\ \text{m}$$

(iii) Pull correction $= \dfrac{(P - P_s)L}{AE}$

$$= \frac{(100 - 70)(114.095)}{(0.028)(2.1 \times 10^7)} = +\,.0058211\ \text{m}$$

(iv) Sag correction $= \dfrac{W^2L}{24P^2}$ (negative)

$$= \left(\frac{9^2 \times 30}{24 \times 100^2}\right)(3) + \frac{\left(\frac{9}{30} \times 24.095\right)^2 (24.09)}{24 \times 100^2}$$

$= .030375 + .005245 = .0356208$ (negative)

(v) Correction for slope $= -\dfrac{d^2}{2L}$

$$= -\frac{2.5 \times 2.5}{2 \times 100} = -\ .03125 \text{ m/100 m}$$

For 114.095 m, slope correction

$$= -\frac{.03125}{100} \times 114.095 = -\ .0356546 \text{ m}$$

Hence total correction

$= +\ .0570 - .01049 + .0058 - .0356 - .03565$
$= -\ .01894.$

Corrected length = 114.076 m.

EXAMPLE 3.5 A steel tape of length 30 m standardized on the flat under a pull of 49 N has a width of 12.70 mm and a thickness of 0.25 mm. It is to be used on the site to measure lengths of 30 m to an accuracy of ± 1/10000. Assuming that the ends of the tape are held at the same level and that the standardizing temperature for the tape obtains on the site, determine the increases in tension to be applied to realize that accuracy. Take the density of steel as 7750 kg/m^3, Young's Modulus as 20700 MN/m^2 and the acceleration due to gravity as 9.806 m/s^2. [Salford]

Solution

Permissible error on 30 m $= \pm \dfrac{30 \times 1000}{10{,}000} = \pm\ 3$ mm

Weight of unit length of tape $= .0127 \times .00025 \times 1 \times 7750$
$= .024606$ kg/m $= .24129$ N/m

Let P be the pull applied.

$$\text{Pull correction} = \frac{(P - 49)(30)}{(.0127 \times .00025)(20700)(10^6)}$$

$$\text{Sag correction} = \frac{(.24129)^2 \times 30^3}{24P^2}$$

When the error is $\pm \dfrac{1}{10{,}000}$,

$$\pm 3\text{ mm} = \frac{(P-49)(30)(10^3)}{(.0127\times.00025)(20700)(10^6)} - \frac{(.24129)^2\times 30^3}{24P^2}\times 10^3$$

$$= (P-49)(.45646) - \frac{65498.472}{P^2}$$

Solving by trial and error, when

$P = 50$ RHS = .45646 – 26.199 = – 25.7425
$P = 100$ RHS = 23.2794 – 6.5498 = 16.7297

– 3 will lie between 50 and 100 App. value = 76.77 N
+ 3 will lie between 50 and 100 App. value = 83.8369 N.

EXAMPLE 3.6 A tape which was standardized on the flat under a tension P_s was used in catenary to measure the length of a base line. Show that the nominal corrections for pull and sag must be modified by factors of $\pm\ \delta P/(P - P_s)$ and $\mp\ 2\delta P/P$ respectively if an error of $\pm\ \delta P$ occurred in the applied field tension P.

The length of the line was deduced as 659.870 m, the apparent field tension being 178 N. Determine (i) The nominal corrections for pull and sag which would have been evaluated for each 30 m tape length, and (ii) The corrected length of the line if the actual Field tension was 185 N.

The tape which had a mass of 0.026 kg/m and a cross-sectional area of 3.20 mm^2 was standardized on the flat under a pull of 89 N. Take Young's modulus as 155000 MN/m^2 and the acceleration due to gravity as 9.806 m/s^2.

Solution

Theoretical Part

(i) Pull correction $C_P = \dfrac{(P-P_s)L}{AE}$

$$\delta C_P = \frac{\delta P\cdot L}{AE}$$

$$\pm\frac{\delta C_P}{C_P} = \pm\frac{\delta PL}{AE}\,\frac{AE}{L}\,\frac{1}{(P-P_s)}$$

$$= \pm\frac{\delta P}{(P-P_s)}$$

(ii) Sag correction $C_s = \dfrac{w^2L^3}{24P^2}$

$$\delta C_s = -\frac{w^2L^3}{24}\cdot\frac{\delta P}{P^3}\cdot 2$$

$$\pm\frac{\delta C_s}{C_s} = \mp\frac{w^2L^3}{24}\cdot\frac{\delta P}{P^3}\cdot\frac{24P^2}{w^2L^3}\cdot 2 = \mp\frac{2\delta P}{P}.$$

Mathematical Part

$$\text{Pull correction for 178 N} = \frac{(178-89)(30)\times 1000}{3.2\times 155000}$$

$$= +\,5.383 \text{ mm}$$

$$\text{Sag correction for 178 N} = -\frac{w^2L^3}{24P^2}$$

$$= -\frac{(.026\times 9.806)^2\times 30^3}{24\times 178^2}\times 1000 = -\,2.3 \text{ mm}$$

$$\text{Modification factor for pull} = \pm\frac{\delta P}{P-P_s}$$

$$= +\frac{(185-178)}{(178-89)} = +\,.07865$$

$$\text{Modification factor for sag} = -\frac{2\delta P}{P} = -\frac{2\times 7}{178} = -\,.07865$$

$$\text{Change in correction for 30 m tape} = (5.38)(.07865) - (2.3)(.07865)$$

$$= .423137 - .180895 = .242242$$

$$\text{No. of 30 m tapes} = \frac{659.87}{30} \approx 22$$

$$\text{Correction} = .242242\times 22 = 5.329324 \text{ mm}$$

$$\text{Correct length} = 659.870 + .005 = 659.875 \text{ m}$$

EXAMPLE 3.7 A steel tape, 30 m long was standardized on the flat, under a pull of 89 N. If the tape had a cross-sectional area of 3 mm^2 and a mass of 0.024 kg/m, determine the field tension to be applied in order that the correction in tension was equal in magnitude to the correction for sag. What error was induced in the sag correction by an error of + 6 N in that tension? Young's modulus = 155000 MN/m^2.

Solution $$\text{Pull correction} = \frac{(P-P_s)L}{AE} = \frac{(P-89)(30)(1000)}{(3)(155000)}$$

$$\text{Sag correction} = \frac{w^2L^3}{24P^2} = \frac{(.024\times 9.806)^2(30^3)}{24P^2}(1000) \text{ mm}$$

By trial $P = 140$ N $\quad \left.\begin{array}{l}\text{Pull correction} = 3.29\\ \text{Sag correction} = 3.16\end{array}\right\}$

$P = 138$ N $\quad \left.\begin{array}{l}\text{Pull correction} = 3.16\\ \text{Sag correction} = 3.27\end{array}\right\}$

$$P = 139 \text{ N} \qquad \left.\begin{array}{l}\text{Pull correction} = 3.22\\ \text{Sag correction} = 3.22\end{array}\right\}$$

$$\text{Change in sag correction} = -3.22 \times \frac{2\delta P}{P}$$

$$= -\frac{3.22 \times 2 \times 6}{139} = -0.27798 \text{ mm.}$$

EXAMPLE 3.8 A copper transmission line 12.7 mm diameter is stretched between two points 300 m apart at the same level, with a tension of 5 kN when the temperature is 35°C. It is necessary to define its limiting positions when the temperature varies. Making use of the corrections for sag, temperature, and elasticity normally applied to base line measurements in catenary, find the tension at a temperature of –15°C and the sag in the two cases. Young's modulus for copper is 68950 MN/m^2, its density 8890 kg/m^3 and its coefficient of linear expansion 15×10^{-6}/°C.

[London University]

Solution Weight of transmission line/m

$$= \frac{\pi}{4}(12.7^2)\left(\frac{1}{10^6}\right)(1)(8890)(9.806) = 11.043125 \text{ N/m}$$

Initial length of line

$$= 300 + \frac{(11.043125)^2 (300)^3}{(24)(5^2)(10^6)} = 305.48777 \text{ m}$$

With this length of line a better approximation for sag

$$= \frac{(11.043125)^2 (305.48777)^3}{(24)(5)^2(10^6)} = 5.79403 \text{ m}$$

Hence correct length of line = 305.79403 m

$$\text{Amount of sag} = \frac{wL^2}{8T} = \frac{(11.043125)(305.794)^2}{(8)(5)(1000)} = 25.81 \text{ m}$$

When the temperature falls to –15°C, let T_1 be the tension.

Total present length of transmission line

$$300 + \frac{w^2L^3}{24T_1^2} = 300 + \frac{(11.043125)^2 (305.794)^3}{(24)(10^6)(T_1)^2} = 300 + \frac{145.29}{T_1^2}$$

Contraction of wire $L\alpha t$

$$= (305.794)(15)(10^{-6})(35 - (-15)) = 0.2293455 \text{ m}$$

$$\text{Extension due to increase in tension} = \frac{(T_1 - 5)(305.794)(10)^3}{(\pi/4)(12.57)^2(68950)}$$

Equating,

$$300 + \frac{145.29}{T_1^2} = 305.79403 - 0.2293455 + 0.03573(T_1 - 5)$$

By trial and error $T_1 = 5.11$ kN

$$\text{New sag} = \frac{(11.043125)\,(305.564)^2}{(8)\,(5.11)\,(1000)} = 25.222 \text{ m}$$

EXAMPLE 3.9 A tape of nominal length 30 m is standardized in catenary at 40 N tension and found to be 29.8850 m. If the mass of the tape is 0.015 kg/m, calculate the horizontal length of a span recorded as 16 m.

Solution Standardized chord length = 29.8850 m

$$\text{Sag correction } C_s = \frac{(.015 \times 9.806/40)^2 \times 30^3}{(24)}$$

$$= +\,.0152 \text{ m}$$

$$\text{Standardized arc length} = 29.9002$$

$$\text{Standardization error per 30 m} = -\,.0998 \text{ m}$$

$$\text{Recorded arc length} = 16.000 \text{ m}$$

$$\text{Standardization error} = \frac{(16.000)\,(-\,.0998)}{30} = -\,.0532 \text{ m}$$

$$\text{Standardized arc length} = 15.9468 \text{ m}$$

$$\text{Sag correction} = C_s \times \left(\frac{16.00}{30}\right)^3 = -\,.0023 \text{ m}$$

$$\text{Standardized chord length} = 15.9468 - .0023 = 15.9445 \text{ m}$$

(i) *Tape used vertically for measurement*

When a measurement is taken keeping the tape vertical say, in mining operations, the tape will extend under its own weight which can be obtained as follows (Fig. 3.9).

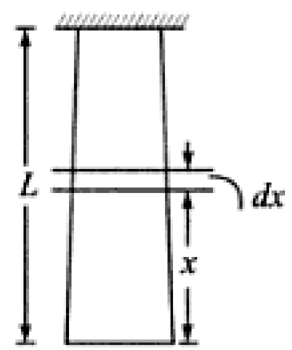


Fig. 3.9 Tape measured vertically.

Let m = mass of tape/unit length
g = acceleration due to gravity
A = cross sectional area of the tape
E = Young's modulus

Force acting on element $dx = mg \cdot x$

$$\text{Extension } \delta e = \frac{mg \cdot x \cdot \delta x}{AE}$$

$$\text{Integrating } e = \frac{mg \cdot x^2}{2AE} + C$$

When x varies from 0 to L.

$$e = \frac{mg \cdot L^2}{2AE}$$

(ii) *Sag correction with the supports not at the same level*
When the supports are not at the same level, sag correction

$$C_s' = C_s \cos^2\theta\left(1 + \frac{wL}{P}\cdot\sin\theta\right) \tag{3.18}$$

when tension P is applied at the higher end and is equal to

$$C_s \cos^2\theta\left(1 - \frac{wL}{P}\cdot\sin\theta\right) \tag{3.19}$$

when tension P is applied at the lower end. Here θ is the angle of inclination with the horizontal and when θ is small C_s' becomes $C_s \cos^2\theta$. The above formulae (3.18) and (3.19) include the effect of slope and as such separate slope correction is not necessary.

EXAMPLE 3.10 Calculate the elongation of a 30 m tape suspended under its own weight at (a) 30 m from top; (b) 10 m from top. Given that $E = 20.7(10^{10})\text{N/m}^2$, mass of the tape is 0.0744 kg/m and the cross sectional area is $9.6(10^{-6})\text{m}^2$.

Solution

(i) $$e = \frac{mg\cdot L^2}{2AE} = \frac{(.0744)(9.806)(30)^2}{(2)(9.6)(10^{-6})(20.7)(10^{10})} = 0.0001652 \text{ m.}$$

(ii) At 10 m from top $x = 20$ m.

$$e = \frac{(.0744)(9.806)(20)^2}{(2)(9.6)(10^{-6})(20.7)(10^{10})} = .00007342 \text{ m.}$$

EXAMPLE 3.11 A nominal distance of "30 m" was set out with a 30 m tape from a mark on the top of one peg to a mark on the top of another, the tape being in catenary under a pull of 90 N. The top of one peg was 0.370 m below the other. Calculate the horizontal distance between the marks on the two pegs. Assume density of steel $7.75(10^3)$ kg/m^3, section of tape 3.13 mm by 1.20 mm, Young's modulus $2(10)^5$ N/mm^2.
Solution Weight of tape/unit length = $(3.13)(1.2)(10)^{-6}(7.75)(10^3) = .029109$ kg/m.

$$\text{Correction for catenary} = \frac{w^2L^3}{24P^2}$$

$$= \frac{(2.9109\times 9.806)^2(30)^3(10)^{-4}}{(24)(90)^2} = .0113 \text{ m}$$

$$\theta = \tan^{-1}\frac{.37}{30} = 0.7066°$$

$$\sin\theta = 0.01233,\ \cos\theta = 0.9999$$

$$\cos^2\theta = 0.9998$$

When tension is applied at the top end

$$C_s' = -(.0113)(.9998)\left(1 + \frac{(.029109)(9.806)(.01233)\,30}{90}\right)$$

$$= -(.0113)(.9998)(1 + 0.001173) = -.0113110 \text{ m}$$
$$\text{Horizontal distance} = 30 - .0113110 = 29.9886890 \text{ m.}$$

When tension is applied at the lower end

$$C_s' = -.0112844 \text{ m.}$$
$$\text{Horizontal distance} = 30 - .0112844 = 29.9887156 \text{ m.}$$

3.8 CHAIN AND TAPE SURVEY OF A FIELD

A field may be completely surveyed by a chain and tape or by tape only. In fact this was the only method available before instruments for measuring angles were developed. Now EDM equipments have brought this method to use again.

The method consists of dividing a field into a number of triangles and measuring the sides of each triangle. The field may be covered by a chain of triangles as in Fig. 3.10 or by a number of triangles with a central station as in Fig. 3.11. The triangles should be such that lengths of the sides do not differ widely when they become well conditioned triangles. If they differ widely the triangle is "ill conditioned". This type of survey is suitable for surveys of small extent on open ground with simple details. However, the following basic principles should be followed:

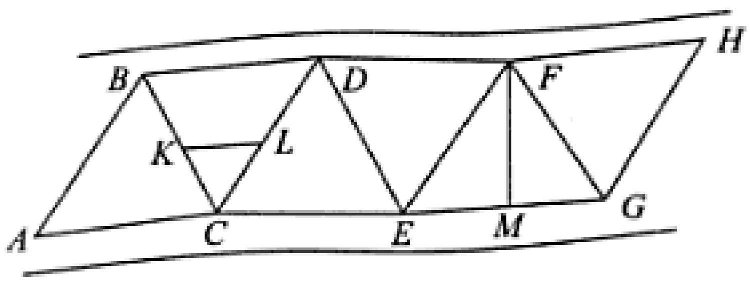


Fig. 3.10 Chain of triangles.

1. Always work from the whole to the part. The area should always be covered with as big triangles as possible. The tie line can then be plotted to fix details.
2. Always make provisions for adequate checks. Hence we have check lines. In Fig. 3.10. *A*, *B*, *C*, *D*, ... are station points, *AB*, *BC*, *AC*, *BD*, ... are chain lines, *FM* is a check line and *KL* is a tie line.

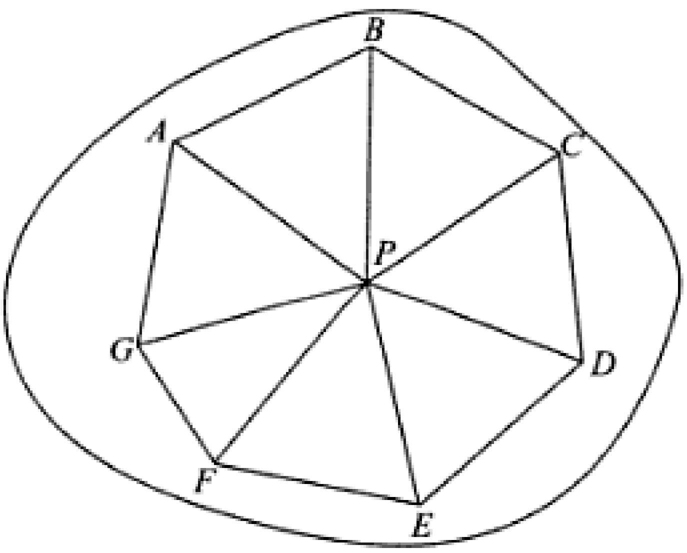


Fig. 3.11 Polygon with central station.

The interior details are usually plotted with respect to the chain line by taking measurements perpendicular to them when we have perpendicular offsets and sometimes taken at an angle to the chain line when we have oblique offsets.

In Fig. 3.12 AB is the chain line, PQ and RS are perpendicular offsets. In Fig. 3.12 P can be plotted if AQ and PQ are known where PQ is the perpendicular offset and AQ is the chain length. Similarly in Fig. 3.13 P can be plotted if AP, PQ and AQ are known where AP, PQ are oblique offsets and AQ is the chain length. To avoid error offsets should be as small as possible.

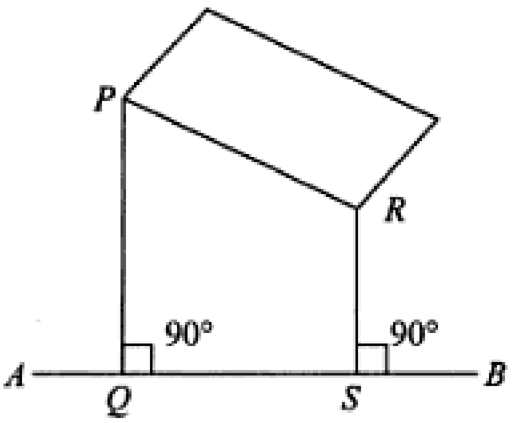


Fig. 3.12 Perpendicular offsets.

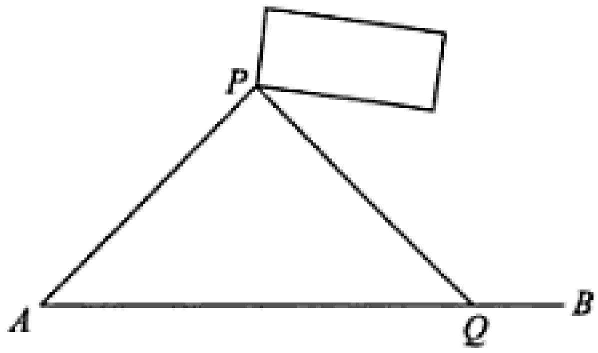


Fig. 3.13 Oblique offsets.

3.9 ERROR IN OFFSET

The offset may not be set exactly at right angle to the chain line but deviate from right angle through a small angle α. The horizontal displacement PP_2 in plotting is then $l \sin \alpha / S$ cm, where l = length of offset in metres and S = scale (1 cm = S metre) as shown in Fig. 3.14.

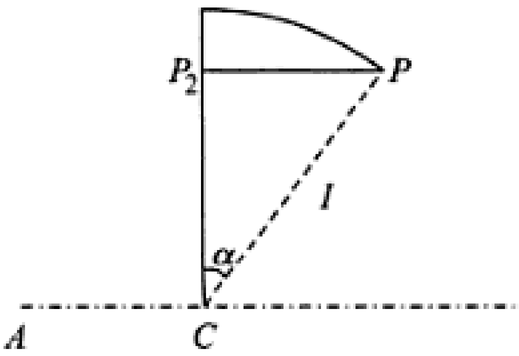


Fig. 3.14 Error in offset angle.

When there is error both in length and direction the total error is PP_2 as shown in Fig. 3.15. Taking PP_1P_2 as a right angled triangle

$$PP_2 = \sqrt{PP_1^2 + P_1P_2^2}$$

$$PP_1 = \delta l \quad \text{and} \quad P_1P_2 = l \sin \alpha$$

giving

$$PP_2 = \sqrt{\delta l^2 + (l \sin \alpha)^2}$$

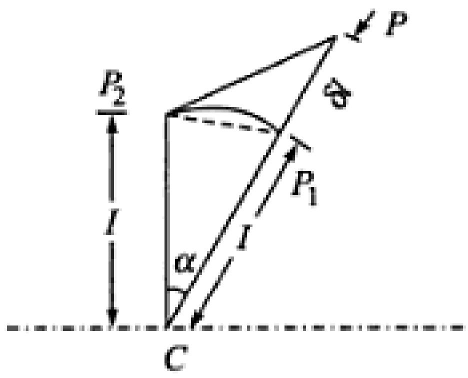


Fig. 3.15 Error in length and angle of offset.

EXAMPLE 3.12 Find the maximum permissible error in laying off the direction of offset so that the maximum displacement may not exceed 0.25 mm on the paper, given that length of the offset is 10 metres, the scale is 20 m to 1 cm and the maximum error in the length of the offset is 0.3 m.

Solution $$PP_2 = \sqrt{\delta l^2 + (l \sin \alpha)^2}$$

Here $$PP_2 = 0.25 \text{ mm}$$

$$\delta l = \frac{0.3}{20} \times 10 \text{ mm} = 0.15 \text{ mm}$$

$$l = \frac{10}{20} \times 10 \text{ mm} = 5 \text{ mm}$$

$$0.25 = \sqrt{(0.15)^2 + (5 \sin \alpha)^2}$$

or $$(5 \sin \alpha)^2 = 0.25^2 - 0.15^2$$

or $$\sin \alpha = \frac{\sqrt{0.25^2 - 0.15^2}}{5}$$

or $$\alpha = 2.292° = 2°17'31''$$

3.10 INSTRUMENTS FOR SETTING OUT RIGHT ANGLES

The instruments used to set offsets at right angles to the chain line are (i) Cross staff, (ii) Optical square, and (iii) Prism square.

3.10.1 Cross Staff

The cross staff is the simplest instrument for setting out right angles. Two types of cross staff are shown in Fig. 3.16(a) and (b). Figure 3.16(a) shows the open cross staff with two pairs of vertical slits giving two lines of sights at right angles to each other. Figure 3.16(b) shows a French cross staff. It is essentially an octagonal brass box with slits cut in each face so that opposite pairs form

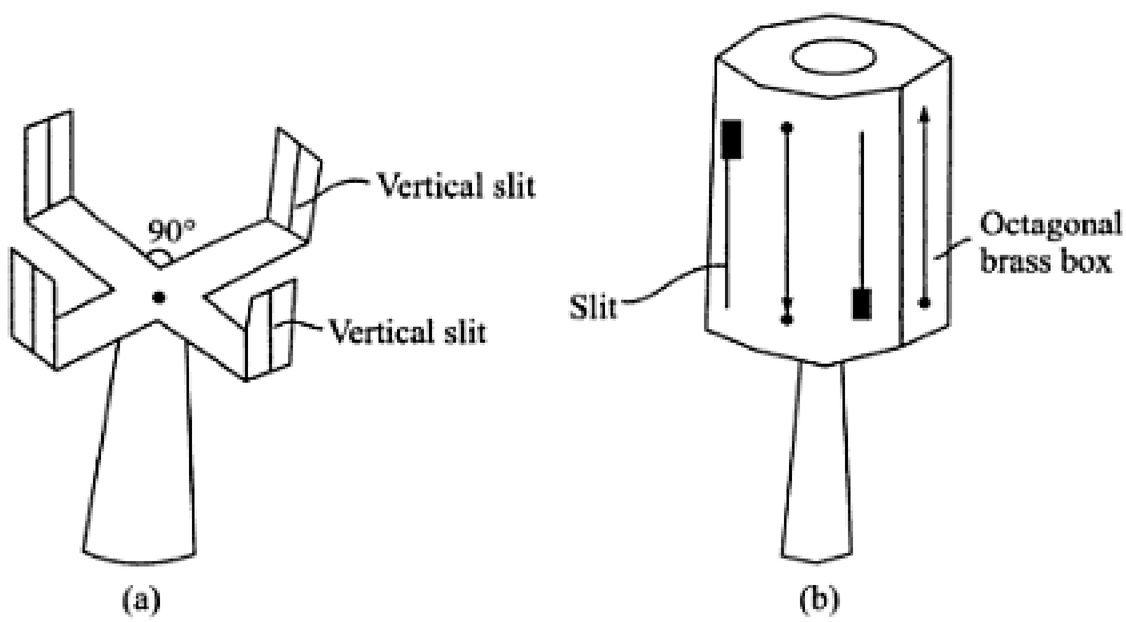


Fig. 3.16 (a) Schematic diagram of open cross staff. (b) French cross staff.

sight lines. The instrument is mounted over a short ranging rod and two sights, are observed through slits at right angles to each other. The other two pairs enable angles of 45° and 135° to be set out.

3.10.2 Optical Square

This is a handy instrument with three openings and is based on optical principle as shown in Fig. 3.17(a). If two mirrors A and B are fixed at an angle of 45°, rays from a point P will get reflected at first mirror A and then again get reflected at B to meet the eye E. The lines PA and BE are at right angles which can be seen from Fig. 3.17(b).

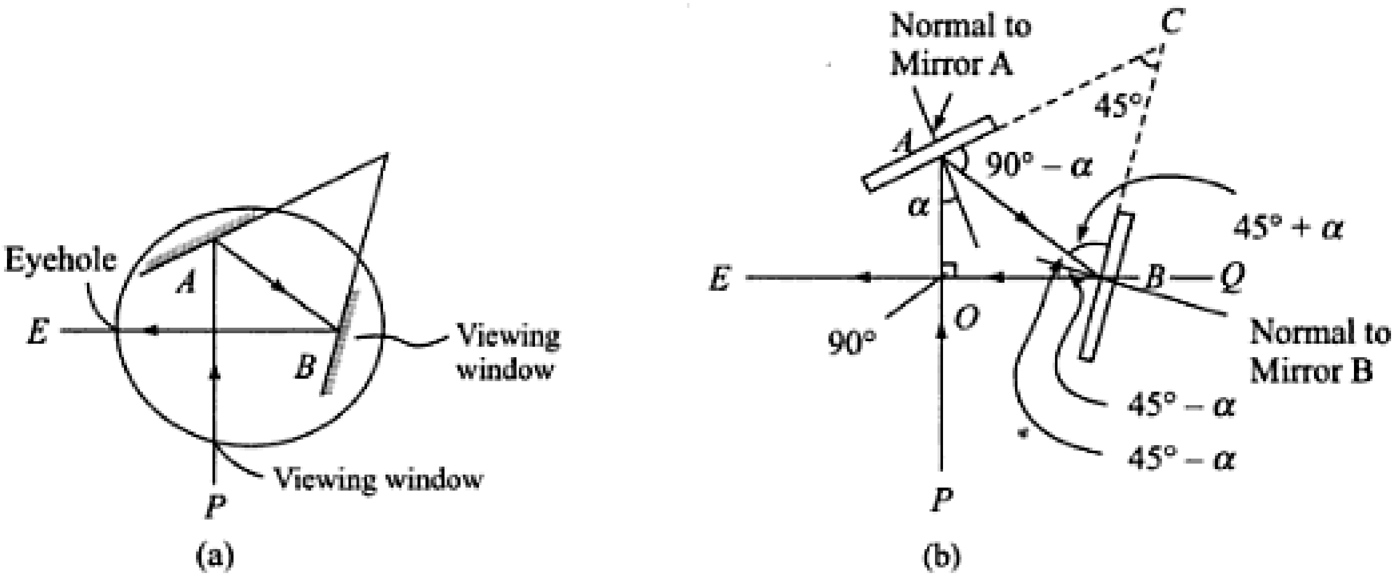


Fig. 3.17 (a) Optical square. (b) Path of rays in optical square

If the incident ray PA makes an angle α with the normal, the reflected ray AB will also make the same angle. Hence

$$\angle CAB = 90° - \alpha$$

with $$\angle C = 45°$$

$$\angle ABC = 45° + \alpha$$

and therefore $$\angle ABE = 90° - 2\alpha$$

Hence $$\angle AOB = 90°.$$

In the optical square the mirror B is half silvered. To see whether the two lines are at right angles, observer at E sees a pole at Q through the unsilvered portion and the image of the pole at P through silvered portion of the mirror B. When the two poles appear coincident the two lines PO and EQ are at right angle and O is the foot of the perpendicular from P on EQ at O.

3.10.3 Prism Square

The same principle as described in optical square is followed in the working of prism square as shown in Fig. 3.18. The advantage of prism square is that the angle 45° is always fixed and needs no adjustment unlike in the optical square.

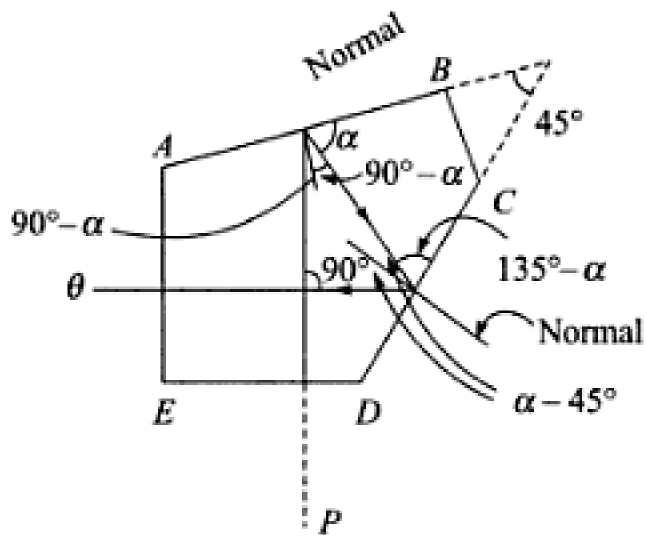


Fig. 3.18 Path of rays in prism square.

3.11 MISCELLANEOUS PROBLEMS IN CHAINING

In practical surveying many types of obstacles are encountered which can be classified as (i) Obstacles to ranging but not chaining, (ii) Obstacles to chaining but not ranging, and (iii) Obstacles to both chaining and ranging.
Case (i) can be further subdivided into two groups:

(a) When both ends of the line may be visible from intermediate points on the line.
(b) When both ends are not visible from intermediate points.

In case (a) recourse is to be taken to *reciprocal ranging*. As shown in Fig. 3.19 two intermediate points M_1 and N_1 are selected such that from M_1, N_1 and B are visible. Similarly from N_1 both M_1 and A are visible. First a range man at M_1 will ask the range man at N_1 to move to N_2 such that M_1N_2B are in one line. Similarly range man at N_2 will ask range man M_1 to move to M_2 such that AM_2N_2 are in one line. The process will be repeated till A, M, N, B are in one line as shown in Fig. 3.19.

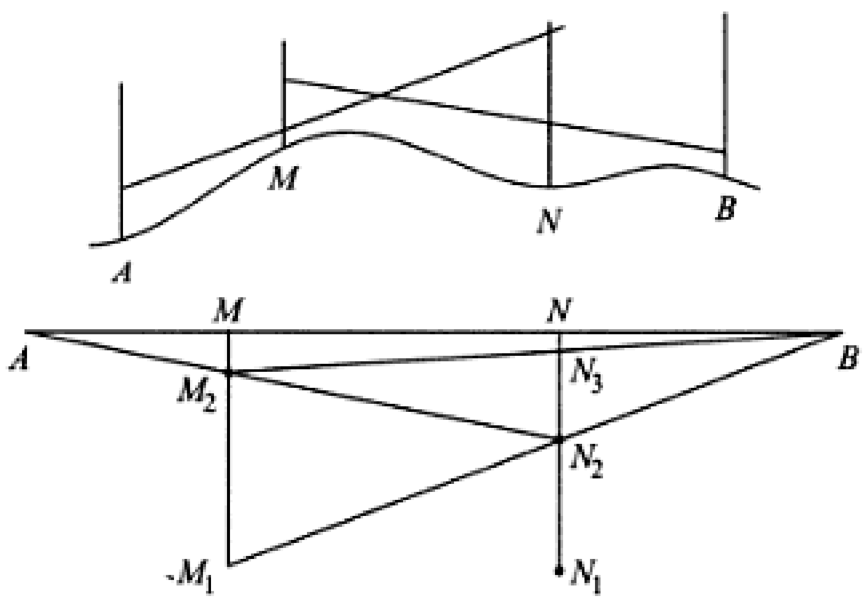


Fig. 3.19 Reciprocal ranging (two ends visible).

In case (b) as shown in Fig. 3.20, a random line AB_1 should be chosen such that B_1 is visible from B and BB_1 is perpendicular to the random line. Then computing C_1C and D_1D from consideration of proportionate triangle C and D can be plotted. Finally CD is joined and prolonged.

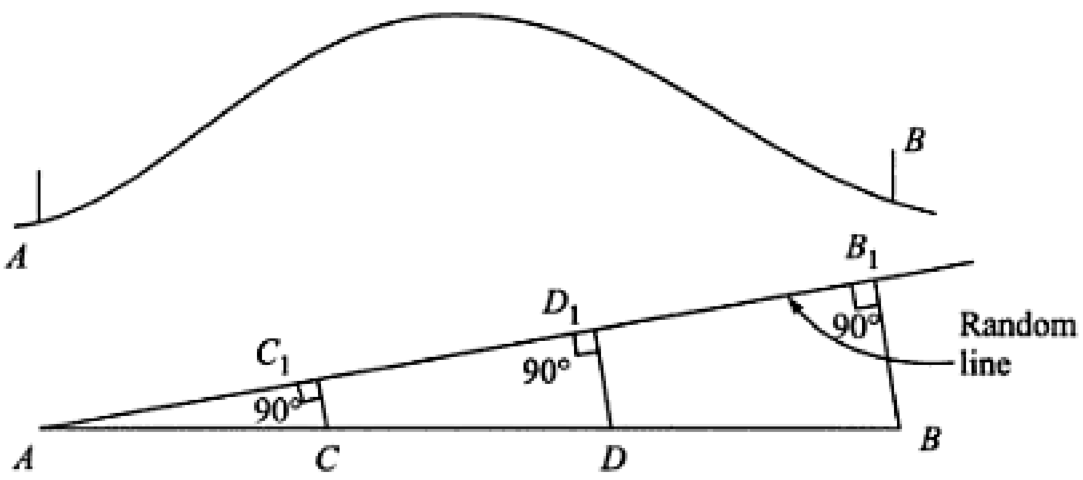


Fig. 3.20 Reciprocal ranging (two ends not visible).

Obstacles to chaining but not ranging are encountered while crossing rivers. Obstacles to both chaining and ranging occur while chaining across a building. These are exemplified by solving a few typical problems.

EXAMPLE 3.13 A survey line ABC crossing a river at right angles cuts its banks at B and C (Fig. 3.21). To determine the width BC of the river, the following operation was carried out.

A line BE 60 m long was set out roughly parallel to the river. Line CE was extended to D and mid-point F of DB was established. Then EF was extended to G such that $FG = EF$. And DG was extended to cut the survey line ABC at H. GH and HB were measured and found to be 40 m and 80 m respectively. Find the width of the river.

[AMIE, Summer 1981]

Solution As $BF = FD$ and $EF = FG$, BE and GD are parallel and equal.

Hence $GD = 60$ m

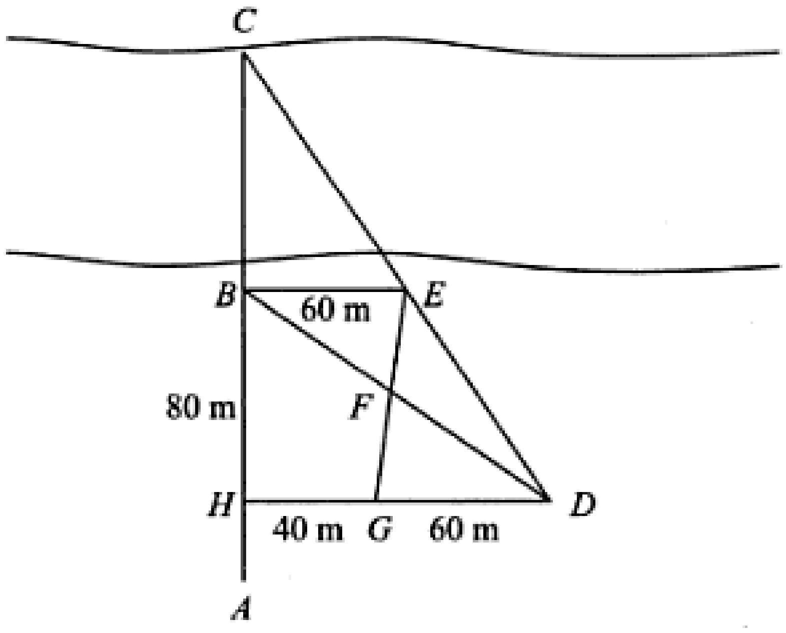


Fig. 3.21 Example 3.13.

From ratio and proportion

$$\frac{BE}{HD} = \frac{CB}{CH} = \frac{60}{100}$$

or

$$\frac{CB}{CH - CB} = \frac{60}{100 - 60} = \frac{60}{40}$$

or

$$CB = \frac{80 \times 60}{40} = 120 \text{ m.}$$

EXAMPLE 3.14 A river is flowing from west to east. For determining the width of the river two points *A* and *B* are selected on southern bank such that distance *AB* = 75 m (Fig. 3.22). Point *A* is westwards. The bearings of a tree *C* on the northern bank are observed to be 38° and 338° respectively from *A* and *B*. Calculate the width of the river.

[AMIE, Summer 1982]

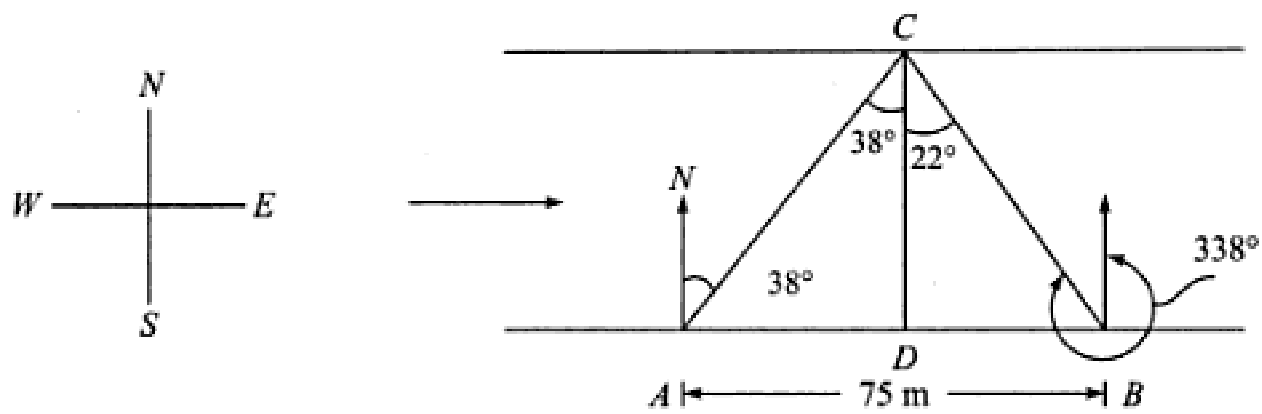


Fig. 3.22 Example 3.14.

Solution Let *CD* be the width of the river.

$$\frac{AD}{CD} = \tan 38°$$

$$\frac{DB}{CD} = \tan 22°$$

Hence

$$AD = CD \tan 38°$$
$$DB = CD \tan 22°$$

Adding

$$AD + DB = 75 = CD(\tan 38° + \tan 22°)$$

or

$$CD = \frac{75}{\tan 38° + \tan 22°} = 63.27 \text{ m}$$

EXAMPLE 3.15 *AB* is a chain line crossing a lake. *A* and *B* are on the opposite sides of the lake. A line *AC*, 800 m long is ranged to the right of *AB* clear of the lake. Similarly another line *AD*, 1000 m long is ranged to the left of *AB* such that the points *C*, *B*, and *D* are collinear (Fig. 3.23). The lengths *BC* and *BD* are 400 m and 600 m respectively. If the chainage at *A* is 1262.44 m, calculate the chainage of *B*. [AMIE, Winter 1985]

Solution

$$CD \cos 30^\circ = 121.92 \text{ m}$$

$$CD = \frac{121.92}{\cos 30^\circ} = 140.78 \text{ m}$$

$$CE = \frac{121.92}{\cos 40^\circ} = 159.16 \text{ m}$$

$$BD = 121.92 \tan 30^\circ = 70.39 \text{ m}$$

$$\text{Chainage of } D = \text{Chainage of } B + BD$$
$$= 95.10 + 70.39 = 165.49 \text{ m}$$

3.12 FIELD WORK FOR CHAIN SURVEYING

In chain surveying only linear measurements can be taken with the help of chain or tape. No angular measurement is possible. Hence the principle of chain survey or *chain triangulation* as it is sometimes called, is to provide a skeleton or framework of a number of connected triangles as triangle is the only simple figure that can be plotted from the lengths of sides measured in the field. The intersection points of the sides are called stations and these are established by placing ranging rods at station points after reconnaissance survey of the site. The following points should be considered while selecting survey stations or survey lines.

1. Survey stations should be mutually visible.
2. Number of survey lines should be as small as possible.
3. There should be atleast one long back bone line in the survey upon which the surveyor forms the triangles.
4. The lines should preferably run through level ground.
5. The triangles formed should be well conditioned.
6. There should be sufficient checklines.
7. The offsets should be as short as possible. Hence the survey lines should pass close to the objects.

3.12.1 Booking the Survey

The data obtained in the field are recorded systematically in an oblong book of size about 200 mm by 120 mm which is known as field book. It opens lengthwise and usually has two lines spaced about 15 to 20 mm apart ruled down the middle of each page. This is double line field book and the distance along the chain line is entered within the double lines. The important steps before starting a survey are:

1. Make a rough sketch in the field book showing the locations of chosen stations and chain lines.
2. The bearing from true or magnetic north of atleast one of the lines should be shown.
3. The stations should be located from three or more points and enough information should be plotted so that they can be relocated if necessary.

The following are the guide lines in recording a field book.

1. Begin each line at the bottom of a page.
2. Sufficient space should be kept in the field book between different chainages. Plotting in the field book need not be to scale.
3. Small details should be plotted in an exaggerated scale.
4. Clear sketches of all details should be shown in the field book. Nothing should be left to memory.
5. Bookings should be done systematically starting with the side having more details.

Figures 3.25, 3.26 and 3.27 show the rough sketch of a plot, the station points and the double line entry in the field book.

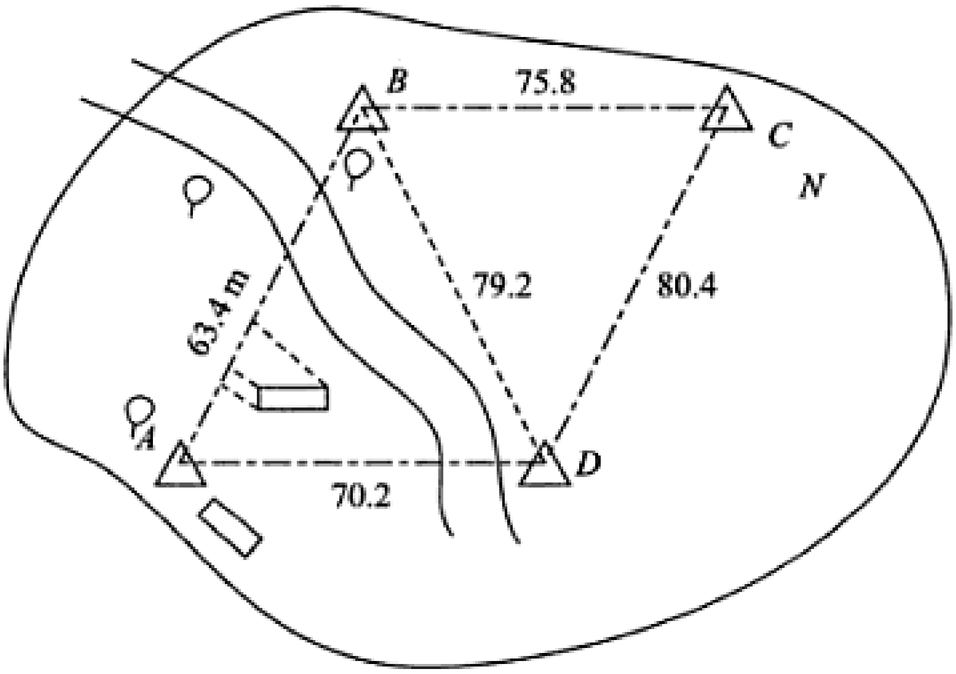


Fig. 3.25 Rough sketch of the plot, stations and chain lines.

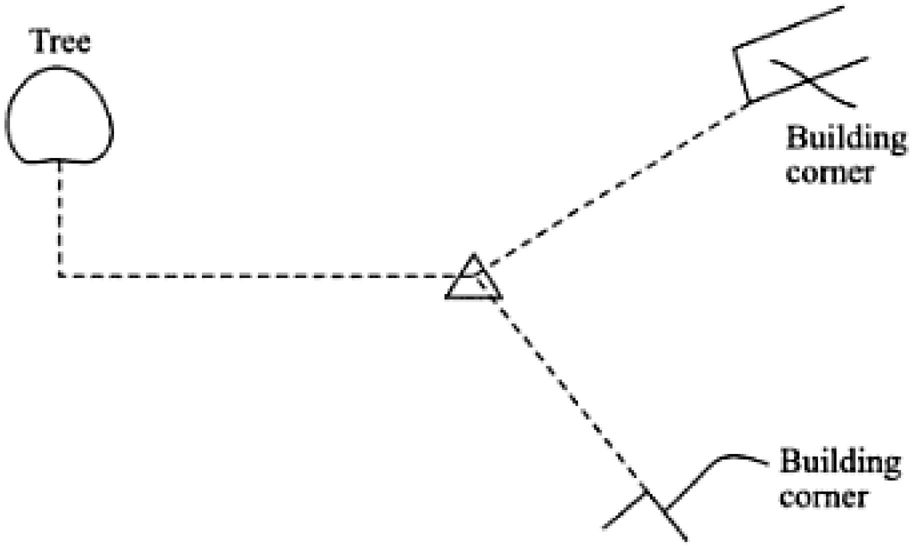


Fig. 3.26 Fixing of a station.

3.12.2 Conventional Symbols

Different features in survey are represented by different symbols and colours. Figure 3.28 shows some conventional symbols commonly used.

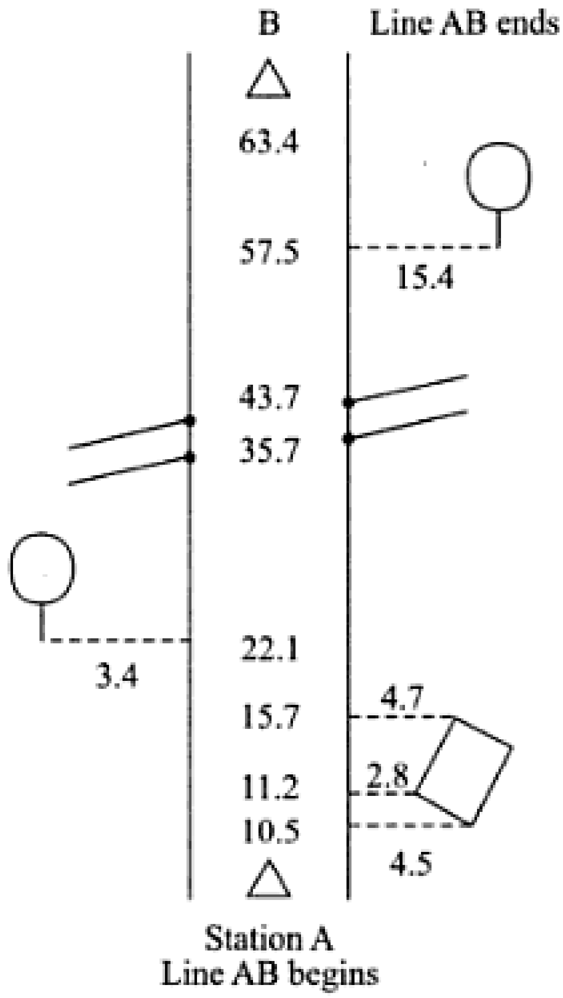


Fig. 3.27 A typical page of double entry field book.

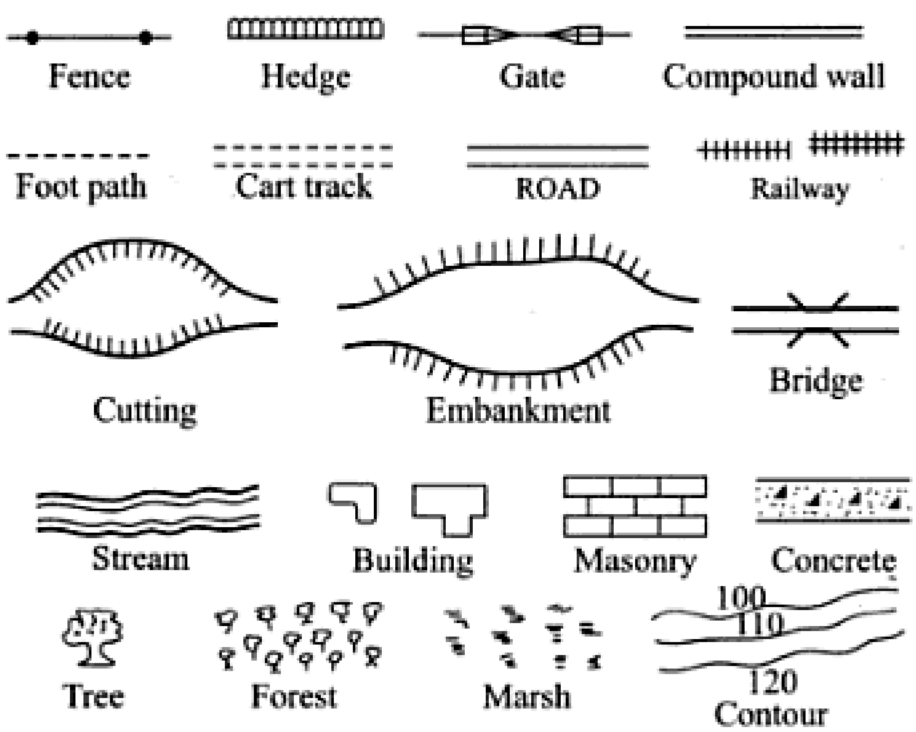


Fig. 3.28 Conventional signs or symbols.

3.12.3 Degree of Accuracy of Chaining

The degree of accuracy which can be attained depends on (i) fineness of graduation of the chain, (ii) nature of the ground, (iii) time and money available, (iv) field conditions, (v) technical competence of the staff, etc. The following accuracy of chaining can usually be obtained under most suitable conditions.

1. For measurements with chain on rough or hilly ground — 1 in 250.
2. For measurement with tested chain, plumb bob — 1 in 1000.
3. For measurement with steel tape — 1 in 2000 to 1 in 20000.
4. For measurement with invar tape — 1 in 20000 to 1 in 1000000.

The accuracy varies from 1 in 100 to 1 in 200 when measurement is done through pacing or pedometer.

PROBLEMS

3.1 Give different methods of measuring horizontal distances. Give an advantage and a disadvantage of each.

3.2 Explain the principle of chain surveying.

3.3 What is a well conditioned triangle? Why it is necessary to use well conditioned triangle?

3.4 State the principles involved in choosing stations for a chain and tape survey.

3.5 What is an offset? What are different types of offsets? Why offsets should be as small as possible?

3.6 Explain the various points which you will keep in mind while recording entries in a field book. Give a neat sketch of a page of the field book.

3.7 A 30 m chain was found to be 15 cm long after chaining 1524 m. The same chain was found to be 30.5 cm too long after chaining a total distance of 3048 m. Find the correct length of the total distance chained assuming the chain was correct at the commencement of chaining.

[AMIE, May 1966]

3.8 *B* and *C* are two points on the opposite banks of a river along a chain line *ABC* which crosses the river at right angles to the bank. From a point *P* which is 45.720 m from *B* along the bank, the bearing of *A* is 215°30′ and the bearing of *C* is 305°30′. If the length *AB* is 60.960 m find the width of the river. [AMIE, May 1966]

3.9 A tape 100 m long was of standard length under a pull of 4 kg at 12°C. It was then used in catenary in three equal spans of 100/3 m each to measure a level line which was found to measure 3400 m. Calculate the true length of the line from the following data:

$$\text{Pull on tape} = 10 \text{ kg}$$
$$\text{Cross section of tape} = 5 \text{ mm} \times \frac{1}{2} \text{ mm}$$
$$\text{Weight of tape per cubic cm of steel} = 7.7 \text{ gm}$$
$$\text{Mean field temperature} = 20°\text{C}$$
$$\text{Coefficient of expansion} = 0.0000113$$
$$E = 21 \times 10^5 \text{ kg/cm}^2$$

[AMIE, Advanced Surveying, Winter 1978]

3.10 (a) Explain the principle, construction and use of an optical square.
(b) When is it necessary to adopt method of reciprocal ranging? Describe the procedure in detail.

(c) Explain briefly the method of chaining on sloping ground.

[AMIE, Surveying, Winter 1982]

3.11 (a) A survey line *AB* is running along different slopes as detailed below: There is a downward slope of 1 in 10 from station *A* to chainage 238 m. The ground has an angle of elevation of 8°15′ from chainage 238 m to chainage 465 m. There is a rise of 25 m from chainage 465 m to station *B* having chainage of 665 m. All the measurements of chainages have actually been taken along the ground. It was also found that the 20 m chain used for chaining was 5 cm too long throughout the work. Calculate the correct horizontal distance from station *A* to station *B*.

(b) State clearly the degree of accuracy required to be achieved in measuring horizontal distances under different conditions.

(c) Draw explanatory sketches to show (i) Well conditioned triangle, (ii) Tie line, (iii) Check line. [AMIE, Surveying, Summer 1983]

3.12 (a) What factors should be considered in selecting stations of a chain survey?

(b) What are offsets? Discuss the relative merits of different types of offsets? Why is it desirable that offsets should be as short as possible?

(c) *A* and *B* are two points 150 m apart on the near bank of a river which flows from east to west. The bearings of a tree on the far bank as observed from *A* and *B* are N 50°E and N 43°W respectively. Determine the width of the river.

[AMIE, Surveying, Winter 1984]

3.13 (a) Explain with sketches how to chain past (i) a river, (ii) a building.

(b) Two ranging rods, one of 2.50 m and the other of 1.00 m length were used in order to find the height of an inaccessible tower. In the first setting, the rods were so placed that their tops were in line with the top of the tower. The distance between the rods was 15 m. In the second setting the rods were ranged on the same line as before. This time the distance between the rods was 30 m. If the distance between the two longer rods was 90 m, find the height of the tower.

[AMIE, Surveying, Winter 1984]

3.14 (a) Describe the method of ranging a line across a ridge when the terminal stations are not intervisible

(b) A big pond obstructs the chain line *ab*. A line *al* was measured on the left of line *ab* for circumventing the obstacle. The length *al* was 901 m. Similarly another line *am* was measured on the right of *ab* whose length was 1100 m. Points *m*, *b* and *l* are on the same straight line. Lengths of lines *bl* and *bm* are 502 m and 548 m respectively. Find the distance *ab*.

[AMIE, Winter 1977]

3.15 (a) Draw the conventional signs, as used on topography map for the following objects and indicate their colour:

(i) Pucca building, (ii) Temple, (iii) Surveyed tree, (iv) Embankment, (v) Road culvert on a drain, (vi) Railway bridge over river.

(b) Describe any three of the following operations in a chain surveying:

(i) Measurement of lengths on sloping ground,

(ii) Criteria for selection of chain survey stations,

(iii) Crossing a wide river as an obstacle to chaining,

(iv) Booking chainage and offset measurement entries in a field book.

3.16 (a) Define surveying. What are the principles of surveying? Explain them briefly.

(b) Discuss briefly the different types and sources of error in surveying.

(c) A line was measured with a steel tape which was exactly 30 m at a temperature of 20°C and pull of 10 kg. The measured length was 1650 m. The temperature during measurement was 30°C and the pull applied was 15 kg. Find the true length of the line if the cross sectional area of the tape was 0.025 cm^2. The coefficient of expansion of the material of the tape per °C is 3.5×10^{-6} and modulus of elasticity of the material of the tape is 2.1×10^6 kg/cm^2.

[AMIE, Summer 2006]

3.17 A 30 m steel tape weighing 6 newton is used in sag in a sloping ground having 1 : 100 downgrade. What is the plan length between two supports if field pull is 100 N.

[AMIE, Summer 2007]

3.18 A steel tape was exactly 30 m long at 18°C when supported throughout its length under a pull of 8 kg. A line was measured with a tape under a pull of 12 kg and found to be 1602 m. Mean temperature during measurement was 26°C. Assuming the tape was supported at every 30 m, calculate the true length of the line, given crosssectional area of the tape = 0.04 cm^2. Weight of 1 cubic cm is 0.0077 kg $\alpha = 0.0000\ 12°/°C$ $E = 2.1 \times 10^6$ kg/cm^2.

CHAPTER 4

Electronic Distance Measurements

4.1 INTRODUCTION

Electronic distance measuring instruments, as the name implies utilizes electromagnetic energy for measuring distances between two points. To facilitate understanding, basic electronic concepts are first discussed.

4.2 BASIC CONCEPTS

Electromagnetic waves can be represented in the form of periodic sinusoidal waves as shown in Fig. 4.1.

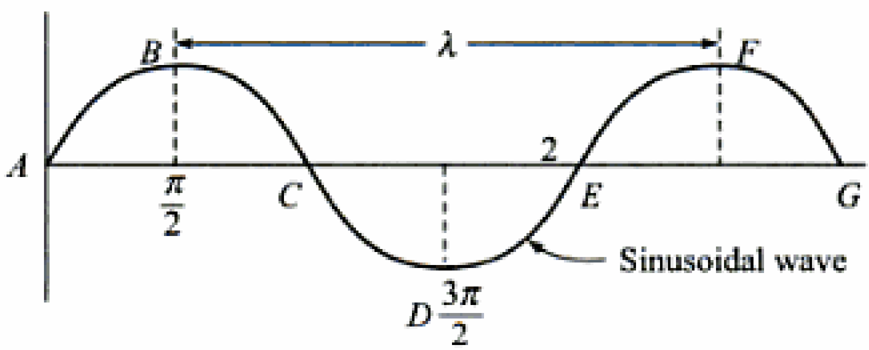


Fig. 4.1 Wavelength.

The time taken for an alternating current to go through one complete cycle of values is called *period* of the wave. One cycle of the wave motion is completed when one period has been completed and the number of cycles per unit of time is called *frequency*. The unit of frequency is hertz (Hz) which is one cycle per second. The linear length λ of a wave is the wavelength which can be determined as a function of the frequency f and the velocity of electromagnetic radiation C as $\lambda = C/f$.

Oscillators convert *DC* drawn from an energy source (battery) into an *AC* of continuous sinewaves. A continuous wave does not include any information or intelligence but it can be effectively used as a carrying agent.

The fundamental information transmitted by the carrierwave in electronic surveying is a sinusoidal wave form to be used for measurements. The process of superimposing the desired sinewave or other periodic signal on to the carrierwave is called *modulation.*

Three main types of modulation are frequently used in electronic distance measurements. They are: (i) Amplitude modulation, (ii) Frequency modulation, and (iii) Phase modulation.

In amplitude modulation, the frequency and the phase of the carrierwave do not change but the strength and amplitude V_c of the carrierwave, $V = V_c \sin \omega_c \cdot t$ alternates sinusoidally with an amount $V_c' = \alpha \cdot V_c$. The coefficient α indicates the depth or the *degree* of modulation; it is defined as V_c'/V_c where V_c' is the amplitude of modulation wave. During modulation, the amplitude of the carrierwave thus alternates between the limits $V_c + V_c'$ and $V_c - V_c'$ as shown in Fig. 4.2.

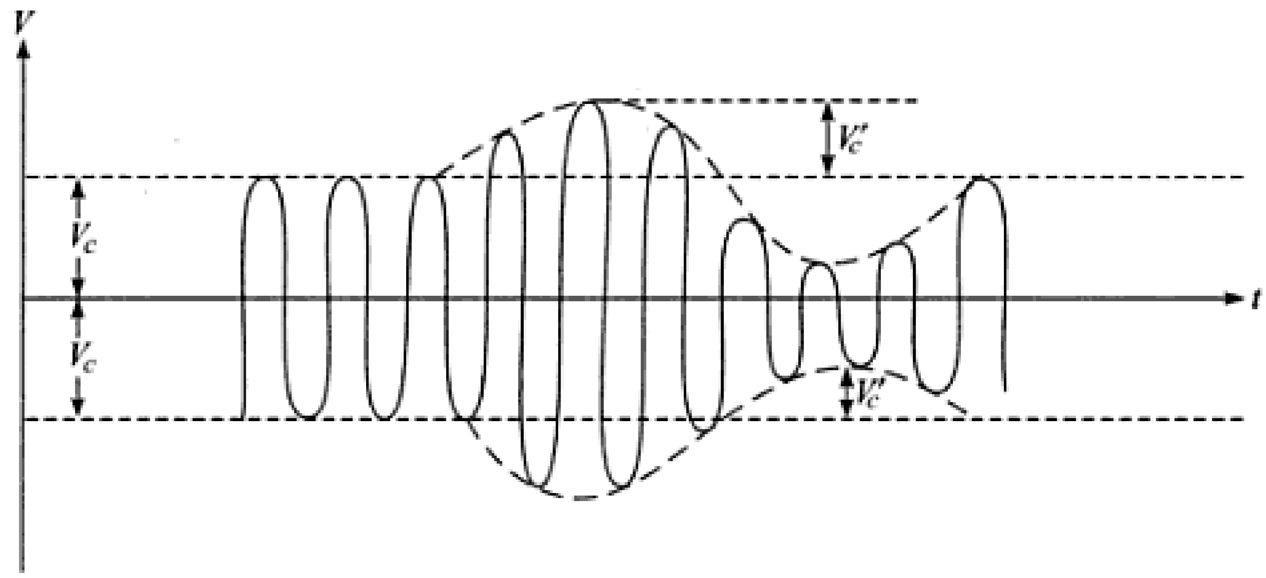
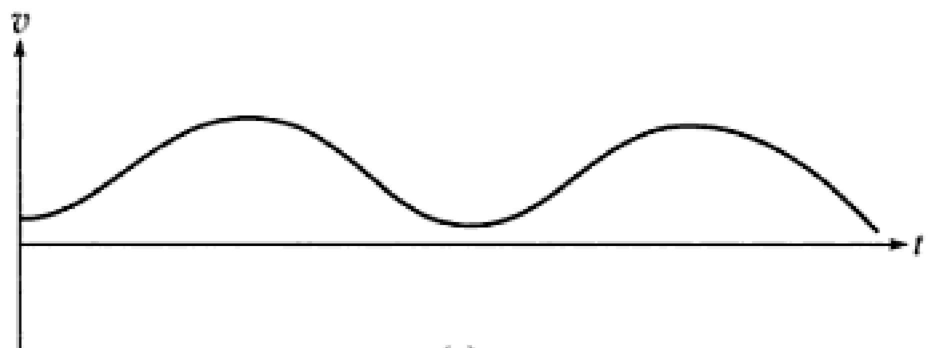


Fig. 4.2 Amplitude modulation.

In *frequency modulation* (Fig. 4.3) the amplitude of the carrierwave is kept constant but the frequency varies according to the amplitude and polarity of the modulation signal. The carrier frequency is increased during one half cycle of the modulation signal and decreased during the other half cycle; thus the frequency is least positive and highest when the modulation is most positive.

In *phase modulation* (Fig. 4.3) as in frequency modulation the amplitude of the carrierwave remains constant but the phase of the carrierwave is varied according to the phase of the modulation wave.



(a)

velocity c is given the approximate value of $c = 3(10^5)$, km/s. Table 4.2 gives a commonly used classification of different frequency and wavelength.

Table 4.2 Commonly Used Frequencies and Wavelengths (Ref. 2)

Classification	*Symbol*	*Frequency*	*Wavelength*
Very low frequency	VLF	10–30 kHz	30,000–10,000
Low frequency	LF	30–300 kHz	10,000–1,000
Medium frequency	MF	300–3000 kHz	1,000–100
High frequency	HF	3–30 MHz	100–10
Very high frequency	VHF	30–300 MHz	10–1.0
Ultra high frequency	UHF	300–3000 MHz	1.0–0.1
Super high frequency	SHF	3–30 GHz	0.1–0.01
Extremely high frequency	EHF	30–GHz	0.01–

4.4 BASIC PRINCIPLE OF ELECTRONIC DISTANCE MEASUREMENT

In measuring the distance between two points electronically, an alternating signal travels from one point to the other, it is reflected or returned in some manner and then is compared with the phase of the original signal to determine the travel time for the round trip. If the distance is to be measured by direct time comparison with an accuracy corresponding to the nearest cm, a time of approximately $67(10^{-12})$ sec must be distinguishable. This interval of time is difficult to measure directly but it can be resolved by a phase measurement of a signal with a period of $67(10^{-9})$ sec corresponding to 15 mc per sec. This signal frequency is too low for direct transmission, so a much higher carrier frequency is employed and the signal appears as a modulation frequency. Depending on the type of carrierwave employed, EDM instruments can be classified as:

1. Microwave instruments.
2. Visible light instruments.
3. Infrared instruments.

Light frequencies permit the use of optical corner reflectors at the section stations but requires an optically clear path between the two stations. Microwave systems can operate through fog and clouds although an optically clear path is required if the vertical angle between two stations must be determined to convert into a horizontal distance. The presence of fog or clouds may cause a loss in accuracy and prevent a reliable estimate.

4.5 COMPUTING THE DISTANCE FROM THE PHASE DIFFERENCES

One complete cycle at the modulation frequency corresponds to the time required for the signal to travel one half wavelength in both direction. The distance being measured corresponds to many half wavelengths at the modulation frequency plus some fraction of a half wavelength. All the instruments use data from several frequencies to overcome this ambiguity.

Four modulation frequencies are used in determining the total length being measured. Several measurements are made at one of these frequencies in order to reduce inherent electronic errors in the system. The four frequencies are f_A, f_B, f_c and f_D. The corresponding half lengths are $\lambda_A/2$, $\lambda_B/2$, $\lambda_c/2$ and $\lambda_D/2$. The wavelength λ is related to c, the velocity of propagation and the frequency f by

$$\lambda = \frac{c}{f} \tag{4.1}$$

Representative nominal values for these quantities are given in Table 4.3.

Table 4.3 Nominal Frequency and Half Wavelengths

Frequency in cycle/sec	*Half wavelength in* m
$f_A = 10.00(10^6)$	$\frac{\lambda_A}{2} \cong \frac{3(10)^8}{2(10)^7} = 15$
$f_B = 9.99(10^6)$	$\frac{\lambda_B}{2} = \frac{f_A}{f_B} \cdot \frac{\lambda_A}{2} \cong \frac{15}{0.99a}$
$f_c = 9.90(10^6)$	$\frac{\lambda_C}{2} \cong \frac{15}{0.99}$
$f_D = 9.0(10^6)$	$\frac{\lambda_D}{2} \cong \frac{15}{0.9}$

With the exception of λ_A, the modulation wavelengths are not useful directly but the wavelengths associated with the frequency differences allow the ambiguity introduced by the long path to be resolved. These relations are given in Table 4.4.

Table 4.4 Frequency Differences and Equivalent Half Wavelengths

Frequency differences in cycles/sec	*Equivalent half wavelength in* m
$f_A - f_B = 10^4$	$\frac{\lambda_{A-B}}{2} \cong \frac{3\times10^8}{2\times10^4} \cong 15{,}000$
$f_A - f_c = 10^5$	$\frac{\lambda_{A-C}}{2} \cong 1{,}500$
$f_A - f_D = 10^6$	$\frac{\lambda_{A-D}}{2} \cong 150$
$f_A - f_o = 10^7$	$\lambda_A \cong 15.$

Phase differences needed in the determination of distance d can be derived as follows:

$$A - B = \frac{2d}{\lambda_A} - \frac{2d}{\lambda_B} = \frac{2d}{\lambda_A}\left(1 - \frac{\lambda_A}{\lambda_B}\right) = \frac{2d}{\lambda_A}\left(1 - \frac{f_B}{f_A}\right) \tag{4.2}$$

Equation (4.2) can be rearranged to obtain an expression for the distance d.

$$A - B = 2d\left(\frac{f_A}{c} - \frac{f_B}{c}\right) = 2d \cdot \frac{f_A - f_B}{c} = \frac{d}{\lambda_{A-B}/2}$$

or $$d = (A - B)\frac{\lambda_{A-B}}{2}$$

in which half wavelengths are obtained from Table 4.4 and the phase differences from the instrument readings. A hypothetical example of how distances are derived and total distance obtained is shown in Table 4.5. It can be seen from the table that only the first figure to the right of the decimal point is used in calculating the coarse contributions to d since other significant figures are included in later terms of the summation. All significant figures beyond the decimal point in the $A - O$ term are used. The final length is obtained by summing all the part lengths.

Table 4.5 Summation of Distances Contribution from Phase Difference

Hypothetical phase difference in cycles	*Equivalent* $\lambda/2$ *in* m	*Distance contribution in* m	
$A - B = 0.92$	15,000	$\frac{0.9(\lambda_{A-B})}{2}$ =	13,500.00
$A - C = 9.21$	1,500	$0.2\frac{(\lambda_{A-C})}{2}$ =	300.00
$A - D = 92.11$	150	$\frac{0.1(\lambda_{A-D})}{2}$ =	15.00
$A - O = 921.124$	15	$0.124\frac{(\lambda_{A-O})}{2}$ =	1.86
			13,816.86 m

Modern EDMs use the decade modulation technique. When the modulation frequency is 15 MHz, the half wavelength is 10 m. The phase meter reading then gives distance between 0 and 9.999 m. When the modulation frequency is brought down to 1.5 MHz, the half wavelength is 100 m and the phase meter gives tens of meters. When it is still brought down to 0.15 MHz, the half wavelength is 1000 m and we get hundreds of meters. Finally a 15 kHz frequency will give the number of thousand meters. The distance 13,816.86 m will then be obtained as summation of 6.86, 10.00, 800 and 13,000.

4.6 BRIEF DESCRIPTION OF DIFFERENT TYPES OF INSTRUMENTS

Geodimeters. All geodimeters employ visible light as the carrier. The measuring set consists of an active transmitter and receiver at one end of the line to be measured and a passive retro directive prism reflector at the other end. Continuous light emission in the transmitter is intensity (amplitude) modulated, using a precision radio frequency generator and an electro-optical shutter to form sinusoidal light intensity waves. The distance is obtained by comparing the phases of outgoing modulation waves with those received by the receiving component after reflection from the distant reflector. All reflectors in the modern instruments are based on the retro directivity principle. Each unit in the reflector is a retro directive prism made of three mutually perpendicular reflecting surfaces.

The prism is often called a *corner reflector* because, for stability, it is made by cutting a corner from a solid glass cube. Light entering the prism reflects from each of the three surfaces and afterwards this triple reflection returns to the instrument parallel to the incident beam. As regards illumination power, the alignment of the reflector with respect to the geodimeter instrument is not critical and alignment errors of the order of 10° may be tolerated.

Tellurimeter. The tellurimeter uses microwaves at about 3, 10 or 35 GHz as the carrier. The measuring set consists of two active units with a transmitter or receiver; one is called the master and the other the remote unit. The carrier frequencies of the two units differ slightly making it possible to utilize intermediate frequency (IF) amplification. Because the carriers are microwaves, the beam widths are narrow—between 2 and 20°. Measuring can be carried on either at night or day time, through haze or light rain, although heavy rainfall may reduce the working range. The bare outlines of the measuring principle consist of a frequency modulated carrierwave from the master station being sent to the remote station where it is received and retransmitted to the master station. There the phase difference between the transmitted and received modulation or pattern waves is compared. Knowing the phase difference or by decade modulation technique distance can be determined.

Hewlett-Packard 3800. This is a modern EDM instrument. Block diagram of the instrument is shown in Fig. 4.4. The transmitter uses a GaAs diode which emits amplitude modulated (AM) infrared light. Frequency of modulation is precisely controlled by a crystal oscillator. The intensity

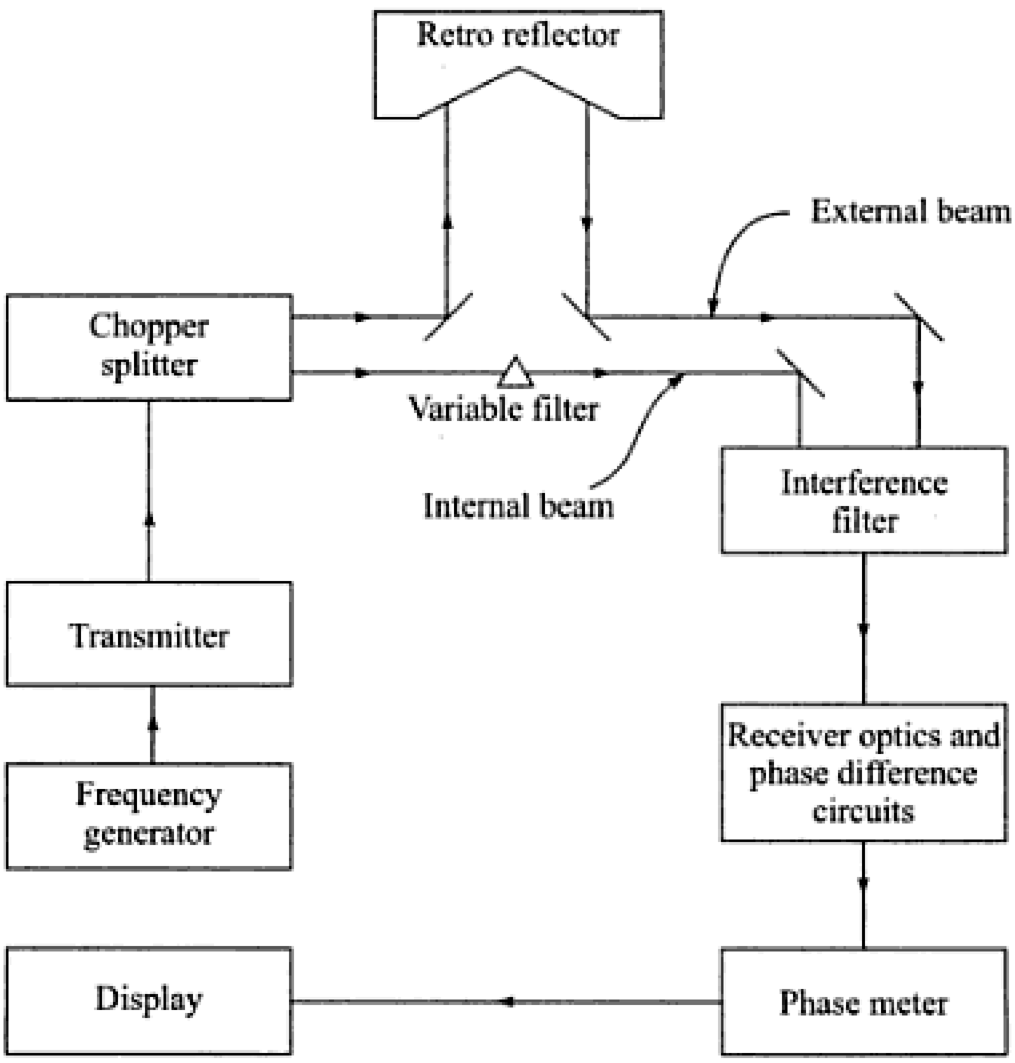


Fig. 4.4 General flow diagram of Hewlett-Packard 3800.

variation or amplitude variation is properly represented by sine waves. Environmental correction factor can be directly dialed into the transmitter to slightly vary the frequency so that a constant wavelength is maintained despite atmospheric variations. Hence no adjustment of distance is necessary at a later stage. Humidity has little effect on the propagation of infrared light and hence is not measured.

With the refractive index n = 1.0002783 and c_0 = 299792.5 km/sec the basic modulation frequency representing 10 m distance is f_1 = 14.985 MHz. The other modulation frequencies are:

$$f_2 = 1.4985 \text{ MHZ}$$
$$f_3 = 149.85 \text{ kHz}$$
$$f_4 = 14.985 \text{ kHz}$$

The time sharing between the transmitted signals and the reference signals is made in the chopper beam splitter which divides the modulated light coming from the Ga–As transmitted diode into two separate beams alternately. The light signal to be transmitted is then focussed into a beam. This beam is sent to the distant reflector. Both external and internal light signals then pass through an interference filter located just in front of the receiver diode. This filter helps to reject signals of other wavelengths (e.g. visible sunlight) without eliminating the modulation signal from the carrier. None of the beam splitting, chopping and filtering processes will cause phase shifts between the transmitted and reference light signals. The internal and external beams are then converted to electrical energy. A phase meter converts the phase difference into direct current having a magnitude proportional to the differential phase. This current is connected to a null meter which automatically adjusts itself to null the current. The fractional wavelength is converted to distance during the nulling process and displayed on instrument dials.

Modern version of HP 3800 is fully automatic and a built-in computer averages the distance measurements without being affected by interruptions of the beam. The digital light emitting diode display gives the operator the distance and classifies its measuring quality in three ways. A steady numerical display indicates a good, solid measurement within the instruments specified accuracy; a flashing display means that conditions are such that the measurements are marginal, and a flashing "O" display warns the operator that under present conditions a valid measurement cannot be made.

4.7 TOTAL STATION INSTRUMENTS

Modern surveying system typically consists of an electronic total station, electronic field book and softwares used in the office for processing data.

The total station's function is to measure horizontal and vertical angles and slope distances in a single integrated unit. It is usually connected to an electronic field book. The field system (total station and field book) is usually controlled via the field book. The principal reason for this is that the field book keyboard does not transfer the force used to press the keys to the total station.

In operation the total station is set up over the required point and its height over the survey station measured. Then the operator points at a prism/target and initiates a reading. Usually this is done by pressing a key on the field book. In some systems it may be a key on the electronic total station.

While the basic data sent by the electronic total station consist of slope distance, horizontal angle and vertical angle, other data may be included in the data stream. This may include units settings, parts per million (ppm) value (for the electronic distance meter EDM), prism constant being used, etc. Additionally, calculated values such as coordinates, azimuths, and horizontal distances may be transmitted. Electronic total stations can also have a variety of functions to improve efficiency and accuracy. Some of these may be corrections for collimations, curvature and refraction and horizontal and vertical angles to compensate for the tilt of the vertical axis.

The electronic field book's basic function is to store the raw data gathered in the field, including horizontal and vertical angles, slope distances, heights of instruments and targets, temperature and pressure, point numbers and descriptive codes.

One of the most crucial aspects of the electronic data collection concept is data flow. Traditional surveying techniques force one to view surveying as a data gathering activity. This view does not recognize the fact that once a design is completed based on the survey data there usually is a need to transfer this design on to the topography. In modern surveying, therefore, setting out is given equal importance as data gathering.

4.8 EFFECT OF ATMOSPHERIC CONDITIONS ON WAVE VELOCITY

The velocity of electromagnetic radiation is constant in vacuum (at the velocity of light) but when affected by the atmosphere it is retarded in direct proportion to the density of air. Because of refraction the direction and speed change. The refractive index usually symbolized as n is related to the dielectric constant μ of the air in the following way:

$$n = \sqrt{\mu}$$

The instantaneous velocity c of the radiation at any point within the atmosphere is a function of the speed of light c_0 and the refractive index n and is given as

$$c = c_0/n$$

where c_0 is a constant and is taken as 299,792.5 km/s.

Refractive index is a function of temperature, pressure and humidity. Humidity is given as the partial pressure of the water vapour in air. In field observations it is almost always obtained by the simultaneous observations of wet and dry bulb readings of a psychrometer.

In electro-optical instruments the light forms groups of waves of slightly different lengths even if it is monochromatically filtered. Since the velocity of such a group wave differs a little from the velocity of equivalent or effective wave, a special group index of refraction must be determined. International Association of Geodesy General Assembly (1963) has recommended that the Barrell and Sears (1939) formula may be used to calculate n_g or group index of refraction when λ is the equivalent or effective wavelength of radiation in micrometers.

$$(n_g - 1)10^7 = 2876.04 + \frac{3(16.288)}{\lambda^2} + \frac{5(0.136)}{\lambda^4}$$

For the kinds of light used in EDM's the values of λ are:

(a) Mercury vapour = 0.5500
(b) Incandescent = 0.5650
(c) Red laser = 0.6328
(d) Infrared = 0.900–0.930

To compute the ambient refractive index for lightwaves, the Barrell and Sears (1939) formula is usually applied as follows:

$$N_a = \left[\frac{n_g - 1}{1+\alpha t}\frac{P}{760} - \frac{5.5(10)^{-8}}{1+\alpha t}e\right]10^6$$

where $N_a = (n_a - 1)10^6$
P = Total pressure
e = Partial pressure of water vapour in millimeters of mercury
α = Heat expansion coefficient of air 0.00367
t = Temperature in degree Centigrade.

The effect of water vapour is small on propagation of lightwaves but is of great significance when microwaves are used. For microwaves Essen-Froome formula can be used which is as follows:

$$N_m = \frac{103.49}{T}(P-e) + \frac{86.26}{T}\left(1+\frac{5748}{T}\right)e$$

Where T is in Kelvin units, p and e are in mm of Hg.

To investigate the sensitivity of the atmospheric parameters T, P and e or their partial effect on N, partial differentiation of the formulae is to be done. If error is to be restricted to one part per million (ppm) the allowable standard errors in observing T, P and e are:

For lightwave

$$m_T = \pm\ 1.0°\text{C}$$
$$m_p = \pm\ 3.6 \text{ mb (millibar)}$$
$$m_e = \pm\ 25.6 \text{ mb (millibar)}$$

For microwave

$$m_T = \pm\ 0.8°\text{C}$$
$$m_P = \pm\ 3.7 \text{ mb}$$
$$m_e = \pm\ 0.23 \text{ mb}$$

From above, it is obvious that the allowable uncertainty in observing temperature and pressure is of the same magnitude in each case. Humidity is the most critical parameter to be observed in connection with measurements made by using radiowaves. The required accuracy by which humidity must be known is about 100 times greater in radiowave propagation than it is when light emission is utilized. In high precision geodetic or geophysical applications, therefore, light is the carrying agent of preference.

4.9 INSTRUMENTAL ERRORS IN EDM

Apart from error due to atmospheric refraction, error in EDM may occur for not having the effective centre of the reflector plumbed over the far end of the line as shown in Fig. 4.5.

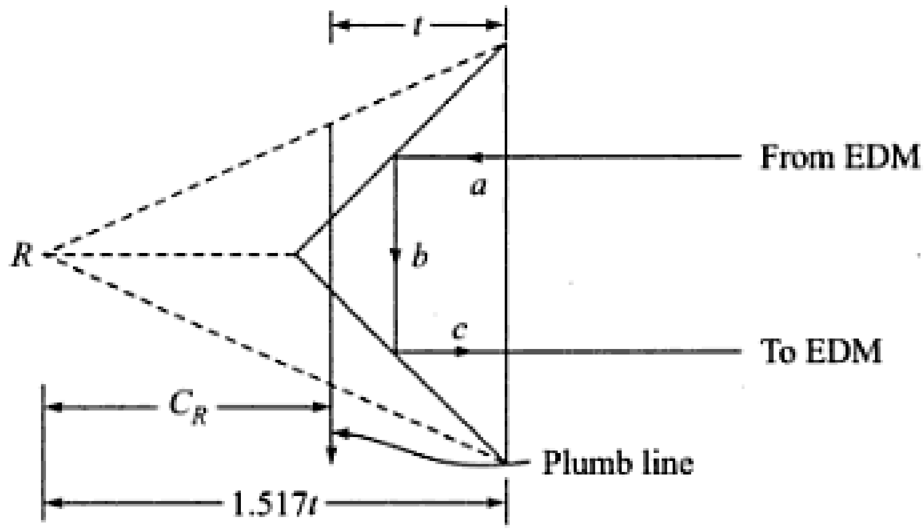


Fig. 4.5 Reflector correction.

The distance through which light travels in the glass tube during retro reflection is $a + b + c$ which in turn is equal to $2t$. The distance t is measured from the face of the reflectors to the corner of the glass tube. The equivalent air distance through which the light travels is 1.517 ($2t$) on account of the refractive index of glass.

The effective corner of the cube is at R and represents the end of the line and hence C_R is the correction to be applied to the measured line. As different combination of reflectors are used at different times of measurement the "reflector constant" is not the same for all setting of the instrument. This has to be determined for each reflector-instrument combination known as C_1. This can be obtained by measuring a distance electronically and also very accurately by means of an invar tape.

Another way of determining C_1 is to take measurements AB, BC and AC all in a straight line. Then

$$(\text{Measured } AB + C_1) + (\text{Measured } BC + C_1) = \text{Measured } AC + C_1$$

or

$$C_1 = \text{Measured } AC - (\text{Measured } AB + \text{Measured } BC)$$

Microwave instruments may suffer from a phenomenon known as *Ground Swing*. This is due to multiple reflection of microwaves from ground or water surface. Errors from this source can be reduced by elevating the master and remote units as high above ground as possible and averaging a number of measurements taken from both ends.

4.10 REDUCTION OF SLOPE MEASUREMENTS IN EDM

EDM measures slope distance between stations. Many instruments automatically reduce the horizontal distance. In some cases, it is to be done manually. Reduction of slope distance to horizontal can be based on difference in elevation or in vertical angle. The method is explained with the help of examples.

EXAMPLE 4.1 (i) If electromagnetic energy travels 299792.5 km/sec under given conditions what unit of distance corresponds to each millimicro second of time?

(ii) The speed of electromagnetic energy through the atmosphere at a standard barometric pressure of 760 mm of mercury is accepted as 299792.5 km/sec for measurements with an EDM instrument. What time lag in the equipment will produce an error of 15 m in the distance to a target 80 km away?

(iii) (a) If an EDM has a purported accuracy capability of ± (5 mm + 5 ppm) what error can be expected in a measured distance of 800 m? (b) If a certain EDM instrument has an accuracy capability of ± (7 mm + 7 ppm) what is the precision of measurements in terms of *l/x* for line length of 3000 m?

Solution

(i) 1 millimicro second = 10^{-9} sec

Distance travelled/sec = 299,792.5 km

Distance travelled in 1 millimicro second

$$= 299{,}792.5(10^{-9})\text{ km}$$
$$= 2.997925(10^{-4})\text{ km}$$
$$= 2.997925(10^{-1})\text{ m}$$
$$= 29.97\text{ cm}$$

(ii) 15 m = .015 km

Velocity of light = 299,792.5 km/sec

$$\text{Hence, time lag} = \frac{2 \times .015}{299{,}792.5}\text{sec} = 10.00690 \times 10^{-8}\text{ sec}$$

(iii) (a) Accuracy = ± (5 mm + 5 ppm)

$$= \pm\left(5 + \frac{5(800)\,(10^3)}{10^6}\right) = \pm\ 9\text{ mm}$$

(b) Accuracy = ± (7 mm + 7 ppm)

$$= \pm\left(7\text{ mm} + \frac{7(3000)\,(10^3)}{10^6}\right)$$
$$= \pm\ 28\text{ mm}$$

$$\text{In terms of } 1/x = \frac{28}{(3000)\,(10^3)} = \frac{1}{107{,}142}$$

$$\approx \frac{1}{107{,}000}$$

EXAMPLE 4.2 What is the refractive index of red laser light at a temperature of 20°C and barometric pressure of 710 torr? Neglect the effect of vapour pressure. What is the velocity through this air? What is the modulated wavelength if the modulating frequency is 24 MHz?

Solution For red laser light $\lambda = 0.6328$. Hence refractive index of standard air for the laser carrier is given by

$$(n_g - 1)10^7 = 2876.04 + \frac{(3)(16.288)}{\lambda^2} + \frac{5(0.136)}{\lambda^4}$$

$$= 2876.04 + \frac{3(16.288)}{(0.6328)^2} + \frac{5(0.136)}{(0.6328)^4}$$

$$= 3002.3078$$

$$n_g = 1.0003002.$$

Refractive index in air under given atmospheric condition neglecting effect of vapour pressure.

$$N_a = \left[\frac{n_g - 1}{1 + \alpha t} \cdot \frac{P}{760}\right] 10^6$$

$$= \left(\frac{.0003002}{1 + (0.00367)(20)}\right)\left(\frac{710}{760}\right)(10^6)$$

$$= (.000261272)(10^6)$$

$$(n_a - 1)(10^6) = (.000261272)(10^6)$$

$$n_a = 1.000261272.$$

The velocity of light through this atmosphere

$$V_a = \frac{c_0}{n_a}$$

where c_0 is the velocity of light in vacuum taken as 299792.5 km/s

$$= \frac{299792.5}{1.000261272} = 299714.21 \text{ km/s}$$

Modulated wavelength

$$= \frac{299714.21}{(24)(10^6)} = (12488.092)(10^{-6}) \text{ km}$$

$$= 12.488092 \text{ m}$$

EXAMPLE 4.3 Microwaves are modulated at a frequency of 70 MHz. They are propagated through an atmosphere at a temperature of 12°C, atmospheric pressure of 712 torr, and a vapour pressure of 7.6 torr. What is the modulated wavelength of these waves?

Solution For microwaves,

$$N_m = \frac{103.49}{T}(P - e) + \frac{86.26}{T}\left(1 + \frac{5748}{T}\right)e$$

$$= \frac{103.49}{285}(712 - 7.6) + \frac{86.26}{285}\left(1 + \frac{5748}{285}\right)7.6$$

$$= 255.78371 + 48.693013$$
$$= 304.47672$$
$$n_m = 1 + .0003044$$
$$= 1.0003044$$

$$V_m = \frac{299792.5}{1.0003044} = 299702.47 \text{ km/sec}$$

Modulated wavelength

$$\lambda = \frac{299{,}702.47}{(70)(10^6)} = 4.2814638 \text{ m}$$

EXAMPLE 4.4 In a straight line *ABC*, *AB* measures 354.384 m, *BC* measures 282.092 m and *AC* measures 636.318 m using a particular EDM reflector combination. A line measures 533.452 m with this instrument-reflector combination. What is the correct length of the line?

Solution C_I = Measured *AC* – (Measured *AB* + Measured *BC*)

$$= 636.318 - (354.384 + 282.092)$$
$$= -0.158 \text{ m.}$$

Correct length of the line = 533.452 – 0.158

= 533.294 m.

EXAMPLE 4.5 The height of an EDM set up at *M* is 1.495 m. The height of the reflector set up at *P* is 1.295 m. The height of the theodolite at *M* used to measure the vertical angle is 1.615 m. The height of the target at *P* on which the vertical sight is taken is 1.385 m. The slope distance after meterological corrections is 1650.452 m. The measured vertical angle is +3°02′32″. What is the horizontal distance between *M* and *P*?

Solution In Fig. 4.6

E = Position of EDM
T = Position of theodolite
R = Position of reflector
S = Position of target

From the data given

$$ME = 1.495 \text{ m} \quad MT = 1.615 \text{ m}$$

Therefore, $TE = MT - ME = 1.615 - 1.495 = 0.12$ m

$$PR = 1.295 \quad PS = 1.385$$

Therefore, $SR = 1.385 - 1.295 = 0.09$ m

From *E*, *ER′* is drawn parallel to *TS* giving $RR' = 0.12 - 0.09 = .03$ m

$$\Delta\alpha \text{ in sec} = \frac{RR' \cos\alpha}{ER'} = \frac{(0.03)(\cos 3°02'32'')}{1650.452} (206265) \text{ sec}$$
$$= 3.74396''$$
$$\alpha + \Delta\alpha = 3°02'35.74''$$

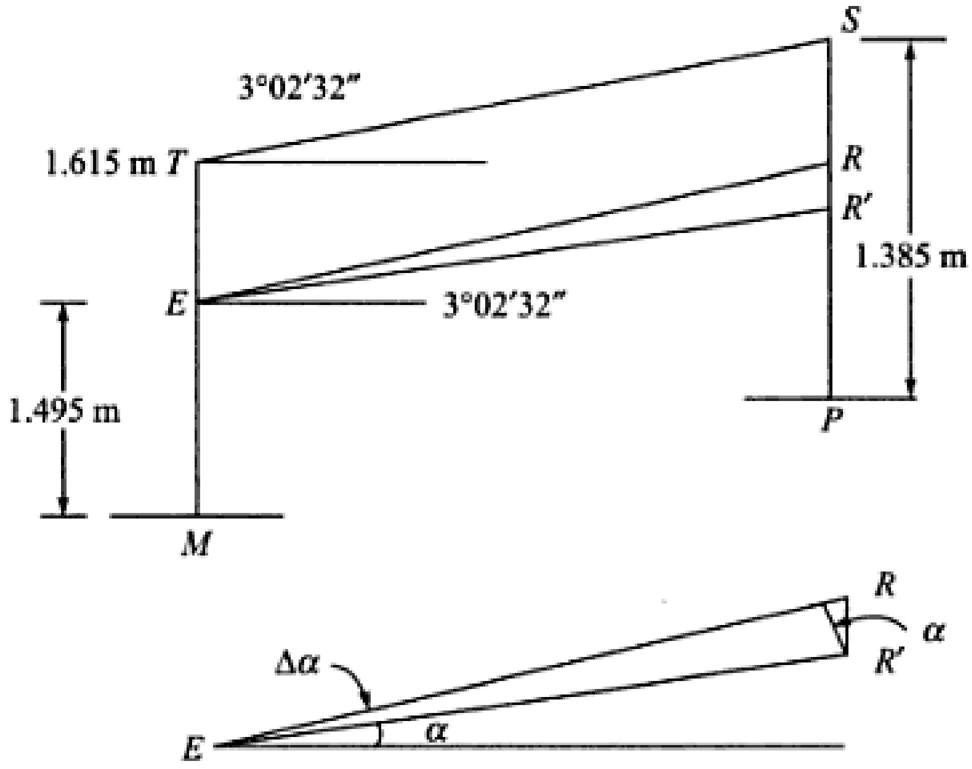


Fig. 4.6 Example 4.5.

Horizontal distance = $ER \cos(\alpha + \Delta\alpha)$

$= 1650.452 \cos 3°02'35.74''$

$= 1648.1244$ m

EXAMPLE 4.6 In Fig. 4.7 a vertical angle of – 8°06′20″ was recorded. The EDM instrument was standard mounted and offset a distance of 0.20 m above the theodolite axis. If the theodolite and reflector heights are equal, what is the corrected horizontal distance for a recorded slope distance of 75.65 m?

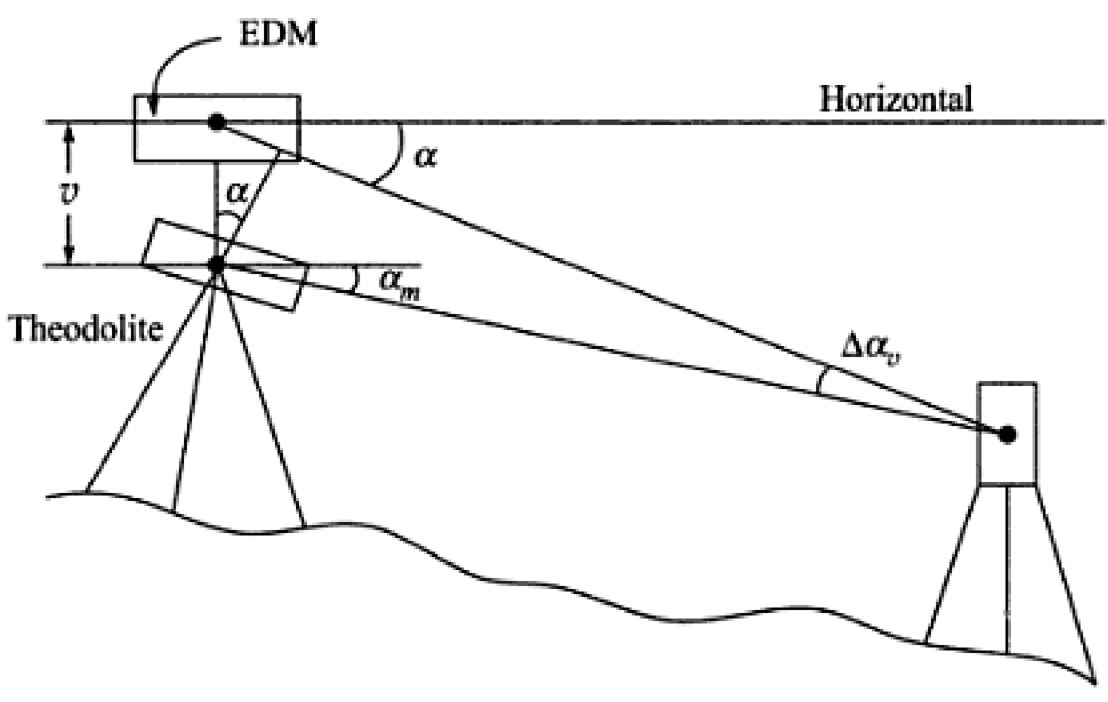


Fig. 4.7 Example 4.6.

Solution Measured angle α_m (by Theodolite) = – 8°06′20″

$$\Delta\alpha_v'' = \frac{0.20 \cos 8°06'20''}{75.65}(206265)'$$
$$= 539.87''$$
$$\alpha = \alpha_m + \Delta\alpha_v = 8°06'20'' + 08'59.87''$$
$$= 8°15'19.87''$$
$$\text{Horizontal length} = 75.65 \cos 8°15'19.87'' = 74.866 \text{ m}$$

EXAMPLE 4.7 A slope distance of 940.07 m (corrected for meteorological conditions) was measured from *A* to *B* whose elevations were 643.41 m and 568.39 m above datum respectively (Fig. 4.8). Find the horizontal length *AB* if heights of the EDMI and reflector were 1.205 m and 1.804 m above their respective stations.

Solution Here

$$CD = L \quad h_e = 1.205 \text{ m}$$
$$h_r = 1.804 \text{ m}$$
$$d = 643.410 + h_e - (\text{Elev. of } B + h_r)$$
$$= 643.410 + 1.205 - (568.39 + 1.804)$$
$$= 74.421 \text{ m}$$
$$H = \sqrt{L^2 - d^2} = \sqrt{940.07^2 - 74.421}$$
$$= 937.1196 \text{ m}$$

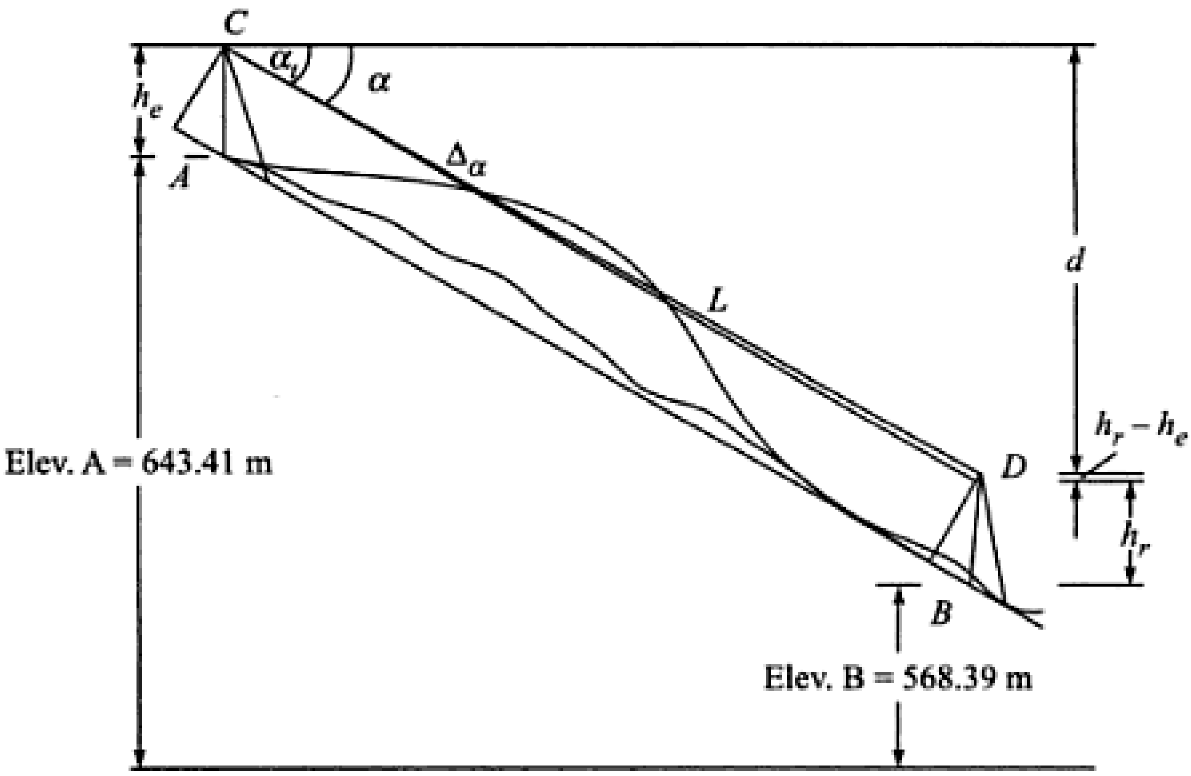


Fig. 4.8 Example 4.7.

EXAMPLE 4.8 The formula given in a manufacturer's instruction manual for computing the atmospheric correction (C_m) to measure electro-optical distance measurement is

$$C_m = \frac{1.00028195}{\left[\dfrac{0.000294335}{1+0.00366086t} \times \dfrac{P}{1013} + 1\right]}$$

where t = ambient atmospheric temperature (°C)

P = ambient atmospheric pressure (mb)

Corrected slope distance = measured slope distance × C_m

The modulated wavelength of the instrument (λ_s) is 20.00000 m corresponding to a frequency of 14.985400 MHz at specified meteorological reference data of 12°C *(t)* and 1013 mb *(P)* and carrier wavelength (λ) of 0.860 μm.

A survey line forming part of a precise test network was measured with the instrument and a mean value of 2999.097 m recorded. The mean ambient temperature t and pressure P were 13.4°C and 978.00 mb respectively.

Compute the atmospheric correction using the formula given in the instruction manual and from first principles, and compare the results. Assume the velocity of electromagnetic radiation in free space to be 299792.5 km/s.

It was later discovered that the field barometer was in error by + 24 mb. Compute the correction in the distance due to this error. What conclusions can be drawn from these calculations?
Aide memoire:

$$n_a = 1 + \frac{n_g - 1}{\alpha T} \times \frac{P}{1013.25}$$

$$n_g = 1 + \left[28760.4 + \frac{3 \times 162.88}{\lambda^2} + 5 \times \frac{1.36}{\lambda^4}\right] \times 10^{-8}$$

$$\lambda_s = \frac{C_0}{f n_s}$$

where

n_a = group refractive index of atmosphere,
n_g = group refractive index of white light (1.000294),
n_s = group refractive index for standard conditions,
$\alpha = 3.661 \times 10^{-3}\ \text{K}^{-1}$,
T = ambient temperature (K),
P = ambient pressure (mb),
λ_s = modulated wavelength (20.000000 m),
λ = carrier wavelength (0.860 μm)
C_0 = velocity of electromagnetic radiation in free space (299792.5 km/sec)
f = modulation frequency (14.98540 MHz) [Eng. Council]

Solution From manufacturer's formula:

$$C_m = \frac{1.00028195}{\left[\dfrac{0.000294335}{1+0.00366086t} \times \dfrac{P}{1013} + 1\right]}$$

$$= \frac{1.00028195}{\left[\frac{0.000294335}{1+0.00366086\times 13.4}\times\frac{978.00}{1013}+1\right]}$$
$$= 1.000011.$$

From 1st principles:

$$n_g = 1+\left[28760.4+\frac{3\times 162.88}{\lambda^2}+\frac{5\times 1.36}{\lambda^4}\right]\times 10^{-8}$$

when $\lambda = 0.860$

$$n_g = 1+\left[28760.4+\frac{3\times 162.88}{.860^2}+\frac{5\times 1.36}{0.860^4}\right]\times 10^{-8}$$
$$= 1.00029433513.$$

With temperature of 12°C and $P = 1013$ mb.

$$n_a = 1+\frac{0.0002943}{3.661\times 10^{-3}\times 285}\times\frac{1013}{1013.25}$$
$$= 1.000281993$$
$$\lambda_s = \frac{299792500}{1.000281993\times 14985400} = 20.00$$

At temperature of 13.4°C and 978.00 mb of pressure,

$$n_a = 1+\frac{0.0002943}{3.661\times 10^{-3}\times 286.4}\times\frac{978}{1013.25}$$
$$= 1.000270919$$
$$\text{ratio} = \frac{1.000281993}{1.000270919} = 1.000011$$
$$n_a = 1+\frac{n_g-1}{\alpha T}\times\frac{P}{1013.25}$$
$$\delta n_a = \frac{n_g-1}{\alpha T}\times\frac{\delta P}{1013.25}$$
$$= \frac{.00029433513}{3.661\times 10^{-3}\times 286.4}\times\frac{\delta P}{1013.25}$$
$$= 2.77046\times 10^{-7}\,\delta P$$
$$\delta n_a\times 10^6 = .277\,\delta p$$

with $\delta P = +24$

$$\delta n_a\times 10^6 = .277\times 24 = 6.648$$

With length 2999.097 m

$$\text{Correction in distance} = 2999.097\times 6.648\times 10^6$$
$$= .02 \text{ m}$$

Error due to incorrect reading of pressure is small.

PROBLEMS

4.1 Explain the principles of electronic distance measurement.

4.2 How does electro-optical instrument differ from EDM instrument?

4.3 If an EDM instrument has a purported accuracy capability of ± (5 mm + 5 ppm) what error can be expected in a measured distance of (a) 600 m (b) 3 km?

4.4 If a certain EDM instrument has an accuracy capability of ± (7 mm + 7 ppm) what is the precision of measurements, in terms of $1/x$, for line lengths of (a) 30 m, (b) 150 m, (c) 2000 m?

4.5 To calibrate an EDM instrument, distances *AC*, *AB* and *BC* along a straight line were measured as 2436.24 m, 1205.45 m and 1230.65 m respectively. What is the instrument constant for this instrument? Compute the length of each segment corrected for the instrument constant.

4.6 Discuss the errors in electronic distance measurements.

4.7 Which causes a greater error in a line measured with an EDMI?
(a) A 2°C variation of temperature from the standard.
(b) A neglected atmospheric pressure difference from standard of 2 mb of mercury.

4.8 Calculate the horizontal length between *A* and *B* if in Example 4.7, h_e, h_r, elev_A, elev_B and the measured slope length *L* are 1.7 m, 1.45 m, 275.25 m, 329.12 m and 428.09 m respectively.

4.9 Calculate the horizontal length in Example 4.6 if the vertical angle is + 10°45′30″. EDM instrument is standard mounted and offset a distance of 0.25 m vertically above the theodolite axis and the recorded slope distance is 59.83 m.

4.10 What is the velocity of mercury vapour light at a temperature of 10°C and barometric pressure of 710 torr?

4.11 Microwaves are propagated through an atmosphere of 75°F, atmospheric pressure of 715 torr and a vapour pressure of 12.5 torr. If the modulating frequency is 30 MHz, what is the modulated wavelength?

4.12 Determine the velocity of red laser light through an atmosphere at 30°C and elevation 1700 m.

REFERENCES

1. Harrison, A.E., "Electronic Surveying: Electronic Distance Measurements", *Journal of the Surveying and Mapping Division,* Proceedings of the American Society of Civil Engineers, Vol. 89, No. 503, October 1963, pp. 97–116.
2. Laurila Simo, H., *Electronic Surveying and Navigation,* New York: John Wiley & Sons, 1976.

CHAPTER 5

Levelling I

5.1 INTRODUCTION

Levelling involves measurements in vertical direction. With the help of levelling difference in elevation between two points or level of one point with respect to another point of known elevation can be determined. Levelling helps in (i) knowing the topography of an area, (ii) in the design of highways, railways, canals, sewers, etc., (iii) locating the gradient lines for drainage characteristics of an area, (iv) laying out construction projects, and (v) calculating volume of earth work, reservoir, etc.

5.2 BASIC DEFINITIONS

Figure 5.1 illustrates some of the basic terms defined below as used in levelling.

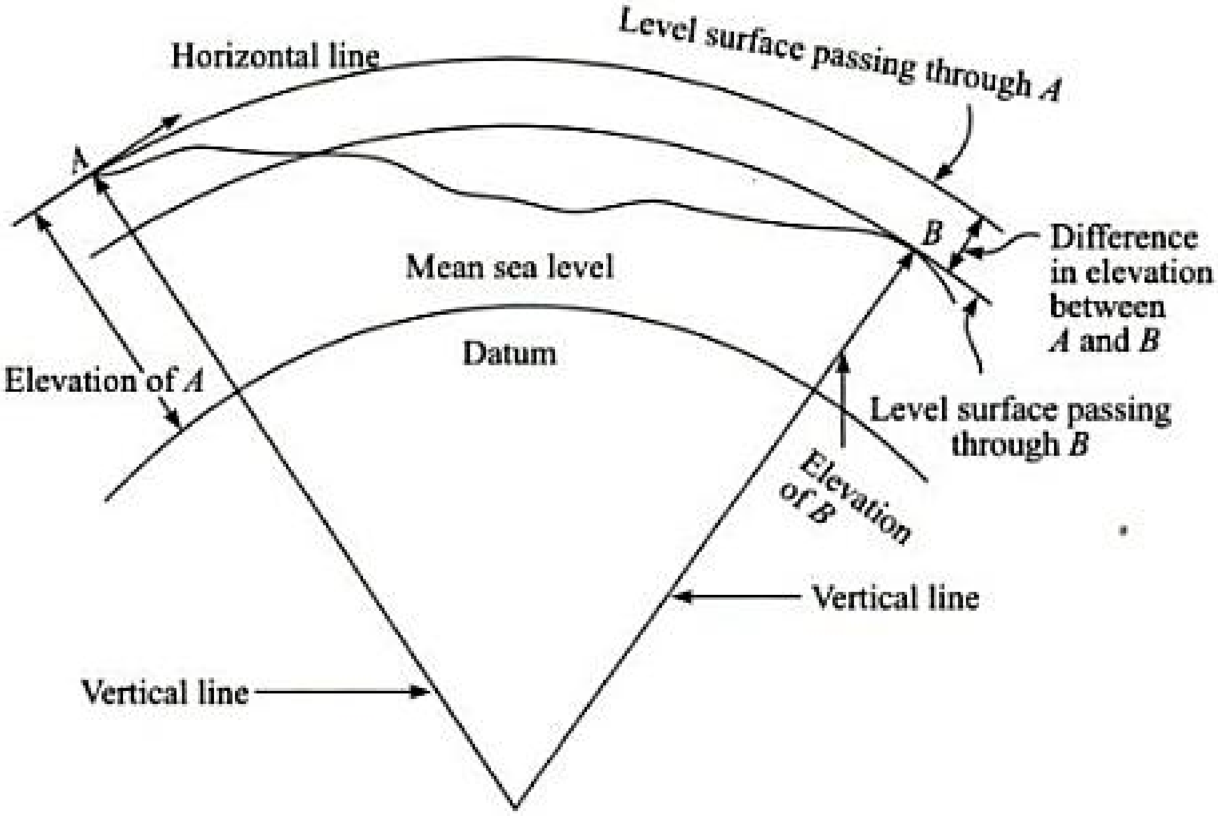


Fig. 5.1 Basic terms in levelling.

Vertical Line. It follows the direction of gravity at any point on the earth's surface and is indicated by a plumb at that point.

Horizontal Line. A line at any point which is perpendicular to the vertical line at that point.

Level Surface. It is a continuous surface that is perpendicular to the plumb line. A large body of still water unaffected by tidal waves is the best example of level surface. For small areas level surface is taken to be a plane surface.

Mean Sea Level. The average height of the sea's surface for all stages of the tide over a very long period (usually 19 years).

Datum. Any level surface to which elevations are referred (for example, mean sea level).

Bench Mark (B.M.). It is a point of known elevation above or below a datum. It is usually a permanent object, e.g. top of a metal disc set in concrete, top of a culvert, etc.

5.3 CURVATURE AND REFRACTION

From Fig. 5.1 it is apparent that difference in level between A and B is measured by passing level lines through the points A and B. However, levelling instruments provide horizontal line of sight and as a result curvature error occurs. In addition due to refraction in the earth's atmosphere the ray gets bent towards the earth introducing refraction error. Figure 5.2 illustrates these errors.

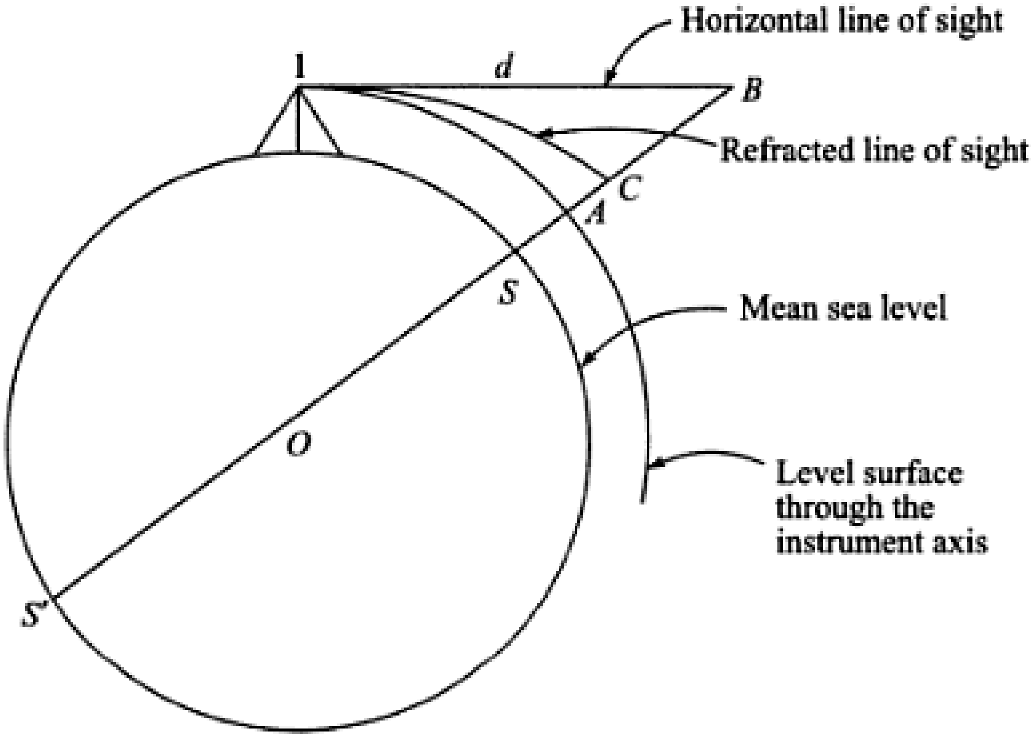


Fig. 5.2 Curvature and refraction correction: I = instrument station; S = staff station; AB = curvature error; BC = error due to refraction; AC = combined error due to curvature and refraction; SB = staff normal to earth's surface; $IB = d$, distance of the staff from the instrument.

Neglecting small instrument height SA, OA can be taken as the radius of the earth.
From geometry of circle

$$AB(2R + AB) = d^2$$

As AB is very small compared to diameter of the earth

$$AB \cdot 2R = d^2$$

or

$$AB = \frac{d^2}{2R} \tag{5.1}$$

The diameter of the earth is taken as

12734 km

Hence curvature correction

$$AB = \frac{d^2}{12734} \text{ km}$$

$$= 0.078\, d^2 \text{ m} \tag{5.2}$$

when d is expressed in km.

The radius of the ray IC bent due to refraction is taken as seven times the radius of the earth. Consequently the refraction correction is taken as 1/7th of the curvature correction. From Fig. 5.1, it can be seen that refraction correction reduces the curvature correction and hence the combined correction is 6/7th of $.078d^2$ m, i.e. $0.067d^2$ m when d is expressed in km.

The correction is subtractive from the staff reading.

EXAMPLE 5.1 Determine the distance for which the combined correction is 5 mm.

Solution Correction in $m = 0.067d^2$, where d is in km

$$d^2 = \frac{.005}{.067}$$

or

$$d = \sqrt{\frac{.005}{.067}} = 0.273 \text{ km}$$

$$= 273 \text{ m}$$

EXAMPLE 5.2 What will be the effect of curvature and refraction at a distance of (i) 100 m (ii) 1 km (iii) 50 km (iv) 100 km?

Solution

(i) $E_a = 0.067(.01)^2 = 6.7(10^{-6})$ m
(ii) $E_b = .067(1)^2 = .067$ m
(iii) $E_c = .067(50)^2 = 167.5$ m
(iv) $E_d = .067(100)^2 = 670$ m

From the above result, it is seen that curvature and refraction correction may be neglected for small lengths of sights but should invariably be taken for long sights.

EXAMPLE 5.3 A sailor standing on the deck of a ship just sees the top of a light house. The top of the light house is 30 m above sea level and the height of the sailor's eye is 5 m above sea level. Find the distance of the sailor from the light house.

[AMIE, Summer 1979]

Solution $$h = 0.067\,d^2\ \text{m}$$

where h is in m, d is in km

$$h_1 = 30 = .067\,D_1^2$$

or $$D_1 = 21.16\ \text{km}$$

Similarly $$D_2 = \sqrt{\frac{5}{.067}} = 8.64\ \text{km}$$

Hence total distance $D = 21.16 + 8.64 = 29.80$ km

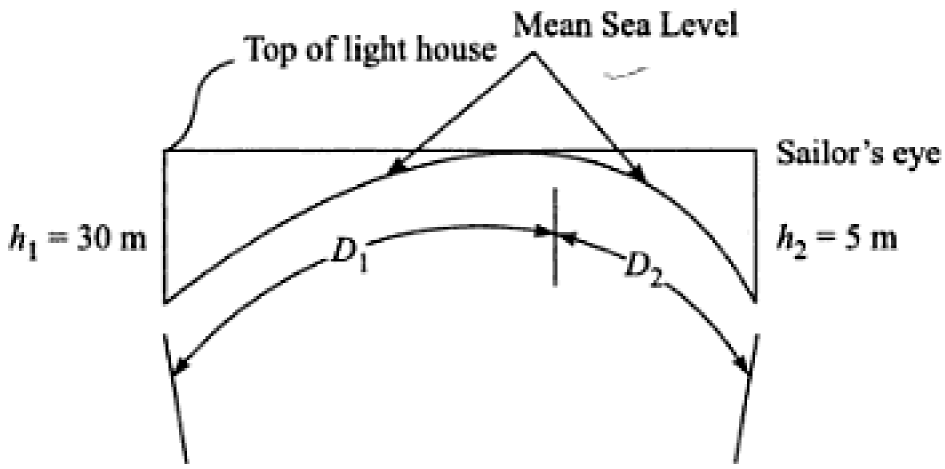


Fig. 5.3 Example 5.3.

5.4 LEVELLING INSTRUMENTS

In levelling, distant objects are to be viewed and measurements taken. A measuring telescope and *not* a viewing telescope forms the main part of a levelling instrument. Telescopes are broadly of two types. Figure 5.4(a) shows a Kepler's or astronomical telescope. Rays from the object *AB* after refraction from the objective *O*, are brought to focus before they enter the eyepiece *E* and in consequence a real inverted image is formed in front of the eyepiece. If the lens is so placed that *ba* is situated within the focal length, the rays after refraction at *E* appear to the eye to proceed from *b′a,′* a virtual image conjugate to *ba*. The object *AB* thus appears magnified, inverted and placed at *b′a′*,. In Galileo's telescope (Fig. 5.4(b)) the rays refracted by the objective *O* are intercepted by a concave eyepiece *E* before a real image is formed. On entering the eye, they therefore appear to diverge from the vertical image *ab* which is magnified and erect.

For viewing purpose Galileo's telescope is more suitable than Kepler's telescope as an erect image is obtained in the former. However, for measuring purposes Kepler's telescope is more suitable as a real inverted image is formed in front of the eyepiece. In surveying telescope there is a diaphragm carrying crosshairs placed in front of the eyepiece. The line joining the intersection of the crosshairs with the centre of the objective provides a definite line of sight known as *line of collimation*. In surveying telescope, the real image is formed in the plane of the crosshairs. The eyepiece magnifies both the image and the crosshairs simultaneously and distortion or other defects produced in the passage of the rays through the eyepiece affects both to the same degree.

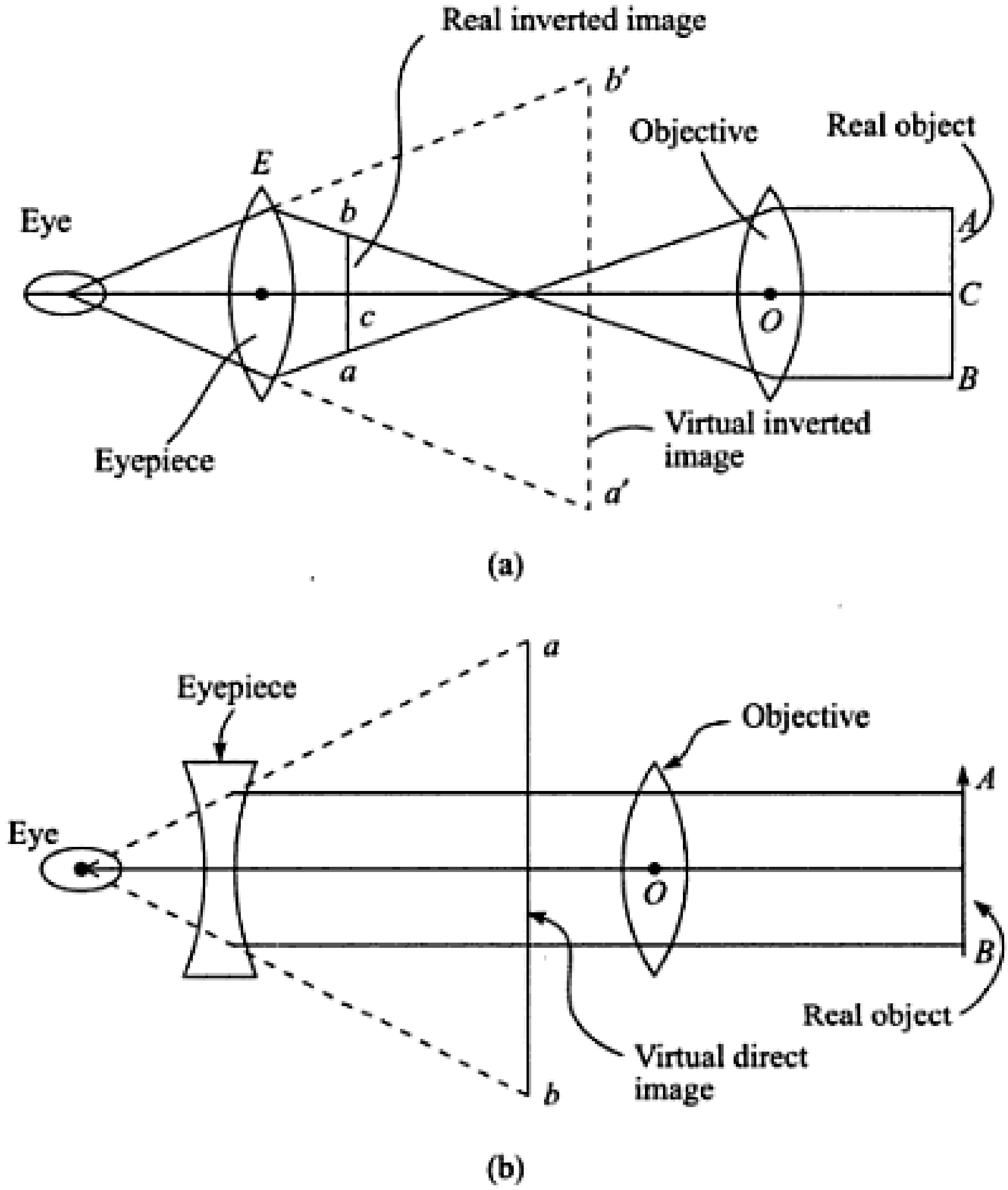


Fig. 5.4 (a) Kepler's telescope. (b) Galileo's telescope.

5.5 CLASSIFICATION OF SURVEYING TELESCOPE

The surveying telescope is broadly of two types—(i) External focussing, (ii) Internal focussing.

By focussing is meant bringing the image of the object in the plane of the crosshairs. If it is done by changing the position of the objective relative to the crosshair, it is *external focussing*. If it is done by moving an additional concave lens between the object and crosshairs it is *internal focussing*. Figure 5.5(a) shows a section through an external focussing telescope while Fig. 5.5(b) shows a section through an internal focussing telescope. In external focussing telescope the objective which is mounted on the inner tube can be moved with respect to the diaphragm which is fixed inside the outer tube. The movement is done by a rack and pinion arrangement operated by focussing screw. As can be seen from Fig. 5.5(b) in the internal focussing telescope, both the objective and the diaphragm carrying crosshairs are mounted inside the outer tube and the distance between them is fixed. An additional double concave lens is mounted on a short tube which can move to and fro between the diaphragm and the objective by means of focussing screw.

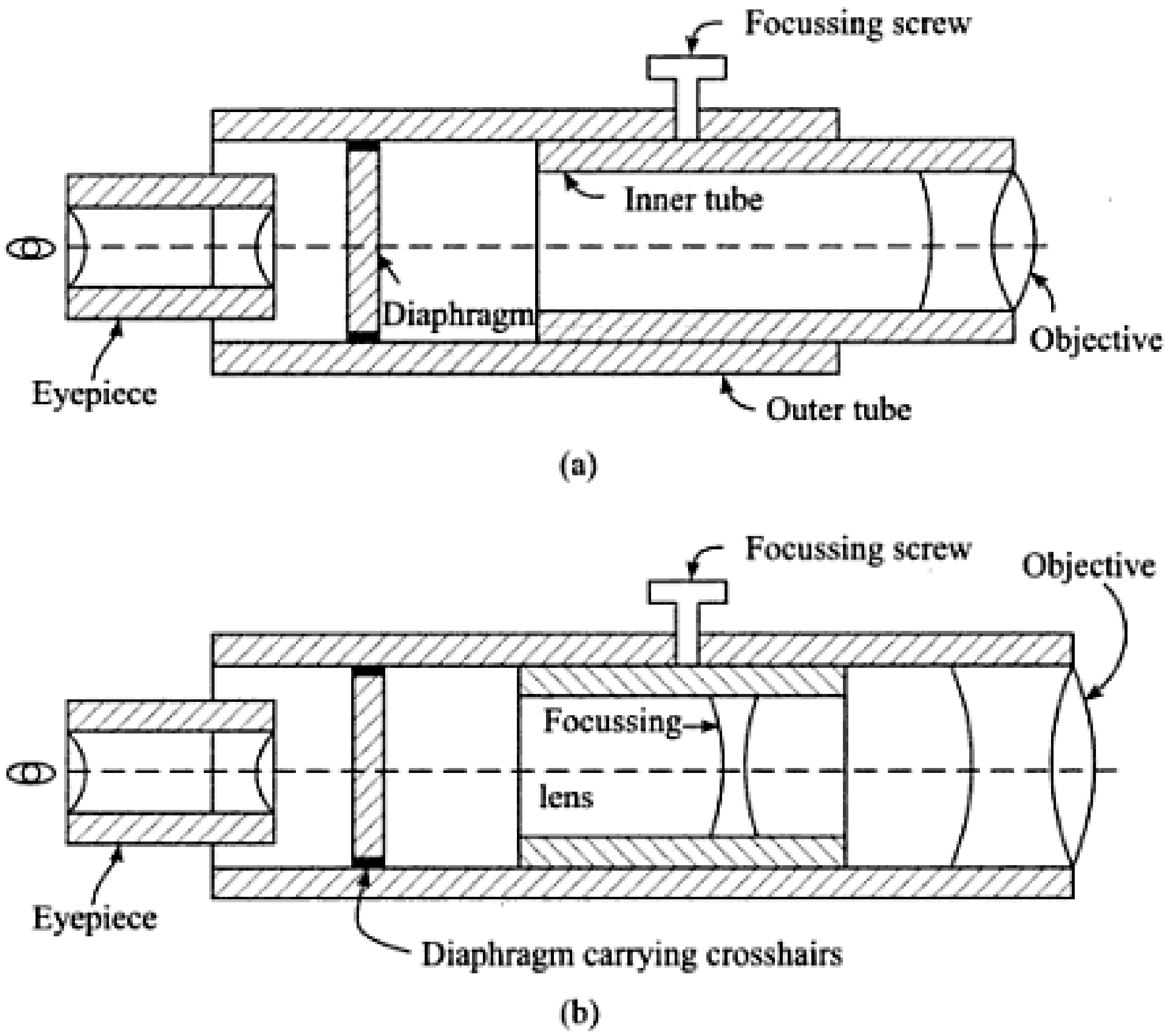


Fig. 5.5 (a) External focussing telescope. (b) Internal focussing telescope.

5.6 LENS FORMULA

The lens formula $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ can be applied to find v when u and f are known. The conventions to be followed in this book are:

Distances are measured inwards towards the lens, and

u = object distance and is positive in a direction opposite to the direction of rays coming from the object

v = image distance and is positive in the direction of rays

f = focal length, positive for convex lens and negative for concave lens

Figure 5.6(a) gives the ray diagram for external focussing telescope. Figure 5.6(b) gives that for an internal focussing telescope. The advantages of internal focussing are:

(1) Less overall length of the tube of the telescope.
(2) Less imbalance.
(3) Less wear of rack and pinion.
(4) Better optical properties.
(5) Line of sight is not affected much during focussing operation.
(6) When used as a tacheometer, additive constant is very small.

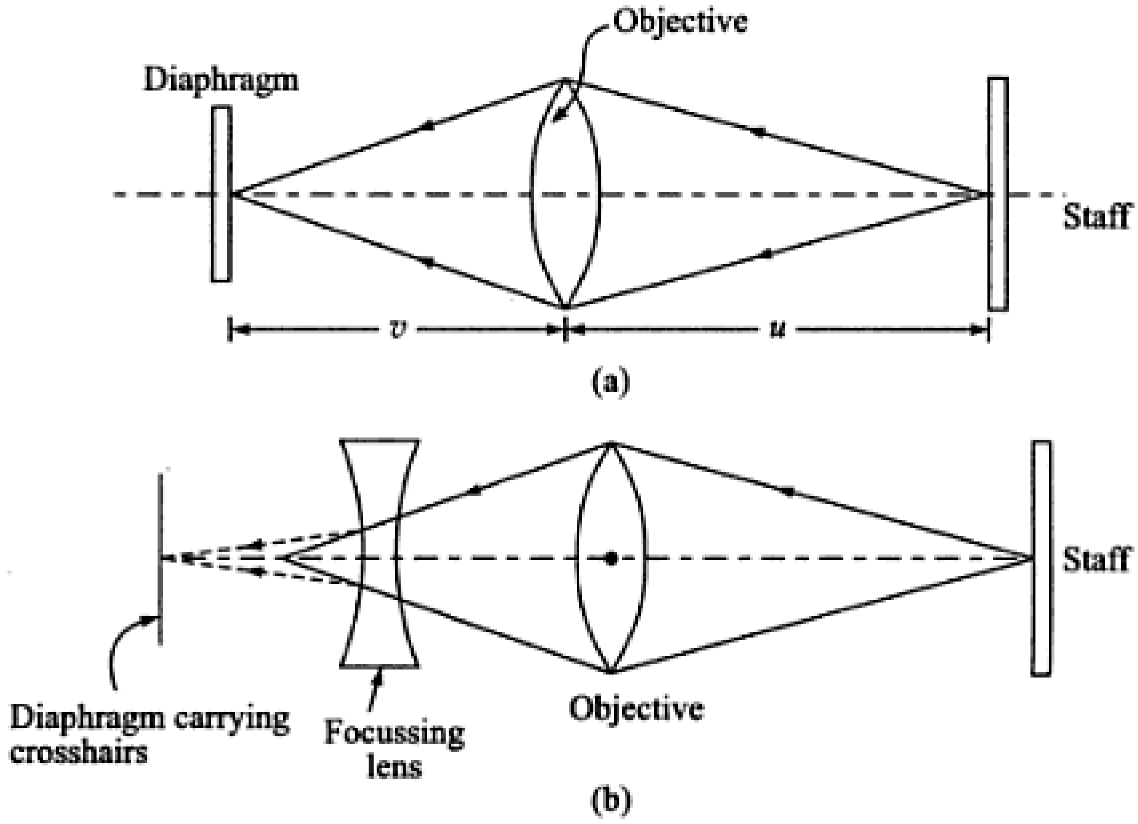


Fig. 5.6 (a) Ray diagram for external focussing telescope. (b) Rays in an internal focussing telescope.

Its disadvantages are:

(i) Less brightness of the image, (ii) Interior of the telescope cannot be cleared and repaired in the field.

EXAMPLE 5.4 An internal focussing lens has an object glass of 200 mm focal length. The distance between the object glass and the diaphragm is 250 mm. When the telescope is at infinity focus the internal focussing lens is exactly midway between the objective and diaphragm. Determine the focal length of the focussing lens.

At infinity focus the optical centre of the focussing lens lies on the line joining the optical centre of the objective and the crosshairs but deviates laterally 0.025 mm from it when the telescope is focussed at 7.5 m. Calculate the angular error in seconds due to this cause.

[L.U.]

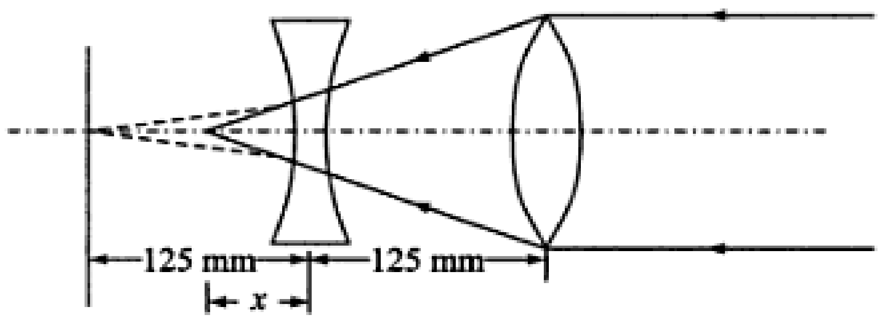


Fig. 5.7(a) Example 5.4.

Solution Using the lens formula

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Without the focussing lens

$$u = \infty \text{ and is positive}$$

$$f = 200$$

$$\frac{1}{v} + \frac{1}{\infty} = \frac{1}{200}$$

giving $v = 200$ mm

$$x = 75 \text{ mm}$$

For the focussing lens, object distance is negative as they are measured towards the lens in the direction of light but image distance is positive,

$$-\frac{1}{75} + \left(\frac{1}{+125}\right) = \frac{1}{f}$$

or $\frac{1}{f} = -\frac{1}{75} + \frac{1}{125} = -\frac{2}{375}$

or $f = -187.5$ mm (concave)

when the telescope is focussed at 7.5 m without the focussing lens

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{+7500} = \frac{1}{200}$$

or $\frac{1}{v} = \frac{1}{200} - \frac{1}{7500}$

or $v = 205.479$ mm

with the shifting of the focussing lens. Let

$$u = x$$

then $v = x + 250 - 205.479$

$$= x + 44.521$$

Using the lens formula

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{x + 44.521} - \frac{1}{x} = -\frac{1}{187.5}$$

which gives $x = 71.77$ mm

The distance between the lens and objective then becomes 205.479 – 71.77 = 133.709 mm.

When the focus is at 7.5 m with lateral displacement of concave lens .025 mm the position of the lenses are as shown in Fig. 5.7(b). xx_1 is the displacement at the level of 1st image because of lateral displacement of 0.025 mm of the concave lens.

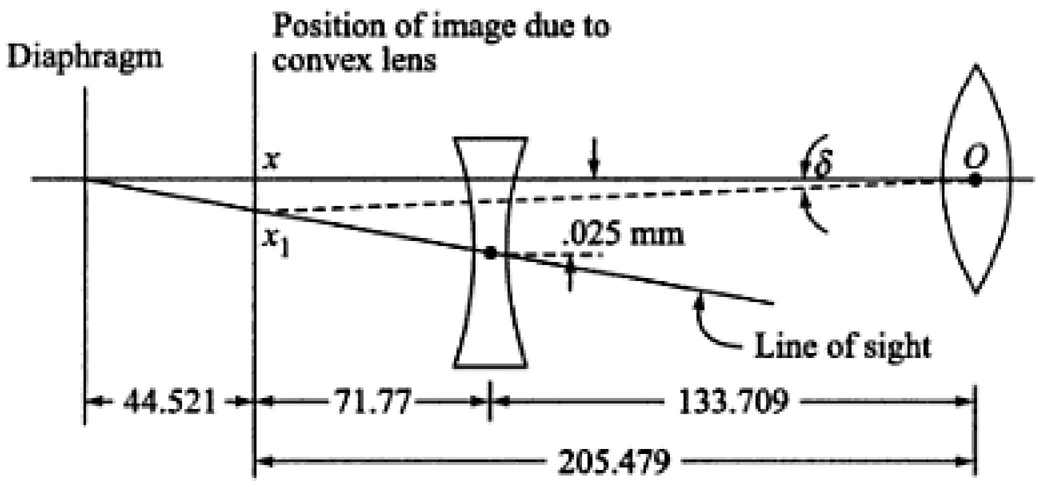


Fig. 5.7(b) Example 5.4.

From Fig. 5.7(b)

$$\frac{xx_1}{44.521} = \frac{.025}{44.521 + 71.77}$$

or

$$xx_1 = .00957 \text{ mm}$$

$$\text{Angular error } \delta = \frac{xx_1}{ox} = \frac{.00957}{205.479}(206265'')$$

$$= 9.62''$$

5.7 ENGINEER'S LEVELS

There are three general types of engineer's levels. These are: (i) Dumpy level, (ii) Tilting level, and (iii) Automatic or self levelling level.

Though the design of the three types differs, the operating principle is the same.

The major parts of a dumpy level are:

1. A telescope
2. A bubble tube
3. A vertical spindle
4. A levelling head
5. A tripod

The schematic diagram of a dumpy level is shown in Fig. 5.8.

The telescope tube and the vertical spindle are cast together as one piece. The spindle revolves in the socket of the levelling head. The levelling head consists of two parallel plates held apart by three levelling screws. The upper parallel plate is called the *tribrach.* The lower plate, known as *trivet stage,* is screwed on top of a tripod when the instrument is to be used. The telescope can be rotated in the horizontal plane about its vertical axis.

Telescope of a dumpy level is normally internal focussing. It is a metal tube containing four main parts:

(a) Objective lens
(b) Negative lens
(c) Diaphragm or reticule, and
(d) Eyepiece

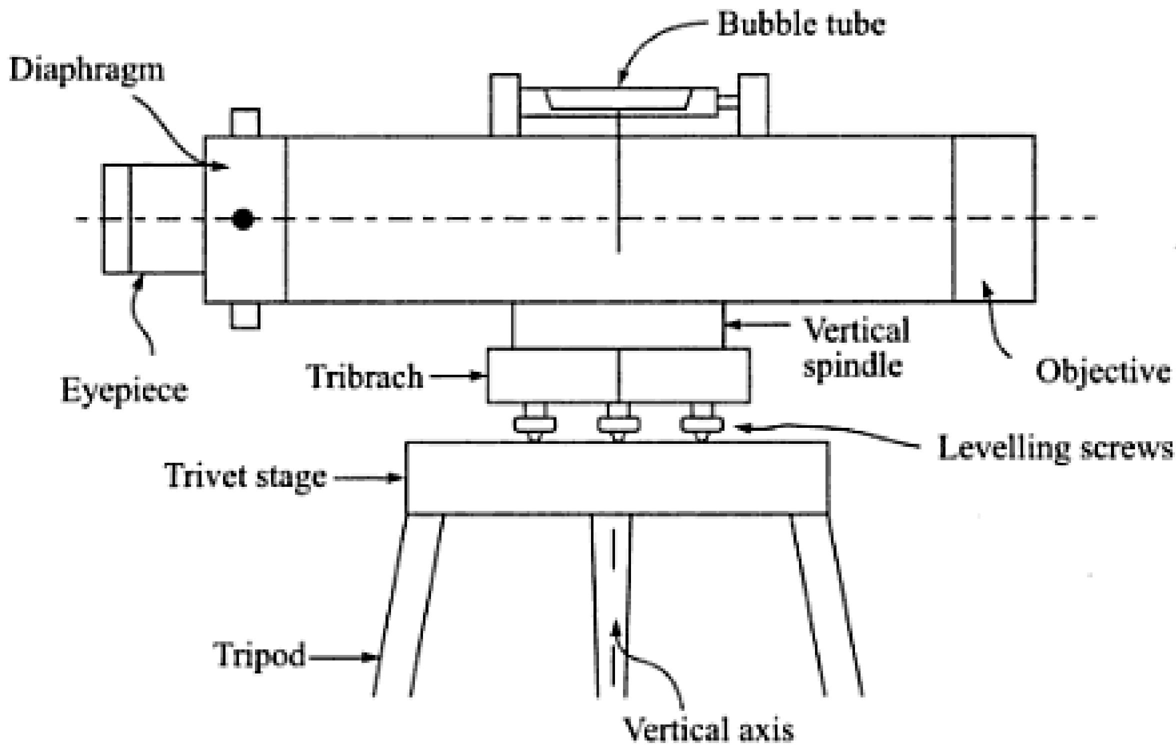


Fig. 5.8 Dumpy level.

Objective Lens. A single lens has many defects like (i) chromatic aberration, (ii) spherical aberration, (iii) coma, (iv) astigmatism, (v) curvature of field, and (vi) distortion. To avoid the first two defects as much as possible the objective lens is composed of both crown and flint glass as shown in Fig. 5.9.

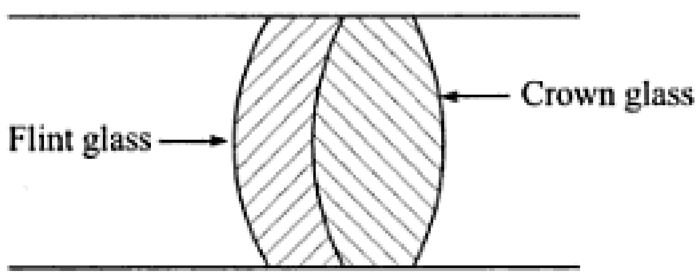


Fig. 5.9 Objective lens.

As a result, chromatic aberration and spherical aberration are nearly eliminated. To minimize loss due to reflection, the lenses are given a thin uniform coating which has an index of refraction smaller than that for glass.

Negative Lens. The negative lens should be mounted on a sliding tube co-axially inside the tube carrying the objective lens. The optical axis of both the lenses should be the same and movement of the negative lens during focussing operations should not introduce deviation of either of the lens axes.

Diaphragm. The diaphragm carries the reticle containing a horizontal and vertical hair. The diaphragm is fitted inside the main tube by means of four capstan headed screws with the help of which the position of the crosshairs inside the tube can be adjusted slightly, both horizontally and vertically and a slight rotational movement is also possible. Previously the crosshairs were made of spider web or filaments of platinum or glass stretched across an annular ring. In many modern instruments, a thin glass plate with lines ruled or etched and filaments of dark metal deposited in them, serves as reticle. Sometimes, two additional horizontal lines are added, one above and another below the usual horizontal hair. The additional hairs are known as stadia hairs and are used in computing distances by stadia tacheometry. Figure 5.10 shows the diaphragm and reticle. Figure 5.11 shows different arrangements of crosshairs.

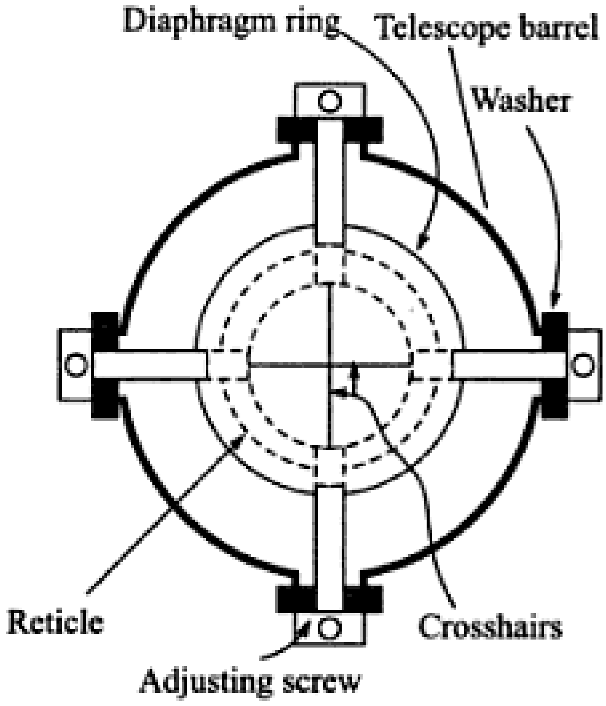


Fig. 5.10 Diaphragm carrying crosshairs.

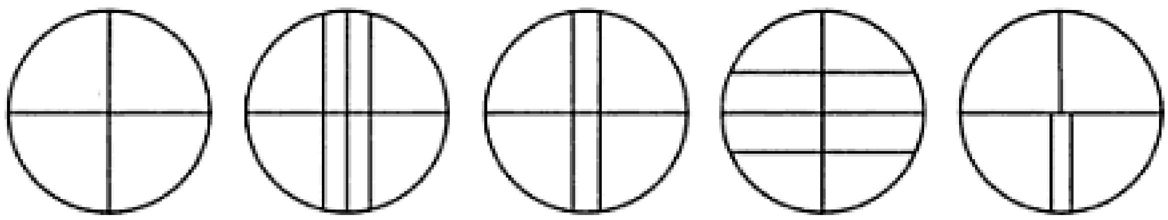


Fig. 5.11 Different types of crosshairs.

Eyepiece. The eyepiece lenses magnify the image together with the crosshairs in order to give the surveyor ability to sight and read accurately the levelling rod graduations. The image formed by the objective and the focussing lens is inverted. Some eyepiece erect the image to give a normal view when it is known as *erecting* eyepiece. Normally, however, the image is seen magnified and inverted through the eyepiece.

Ideally, the eyepieces should reduce chromatic and spherical aberration. Lenses of the same material are achromatic if their distance apart is equal to the average of their focal lengths, i.e.

$$d = \frac{1}{2}(f_1 + f_2)$$

If their distance apart is equal to the differences between their focal lengths, spherical aberration is reduced, i.e. $d = f_1 - f_2$.

For surveying purposes the diaphragm must be between the eyepiece and the objective. The most suitable is Ramsden's eyepiece (Fig. 5.12).

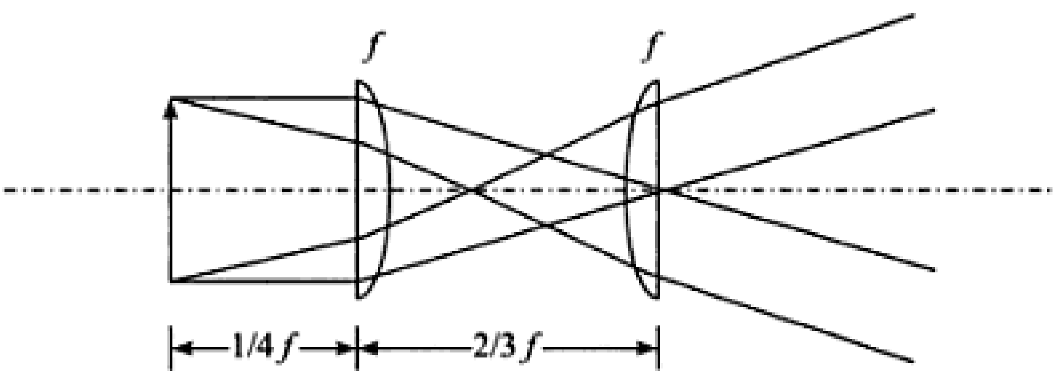


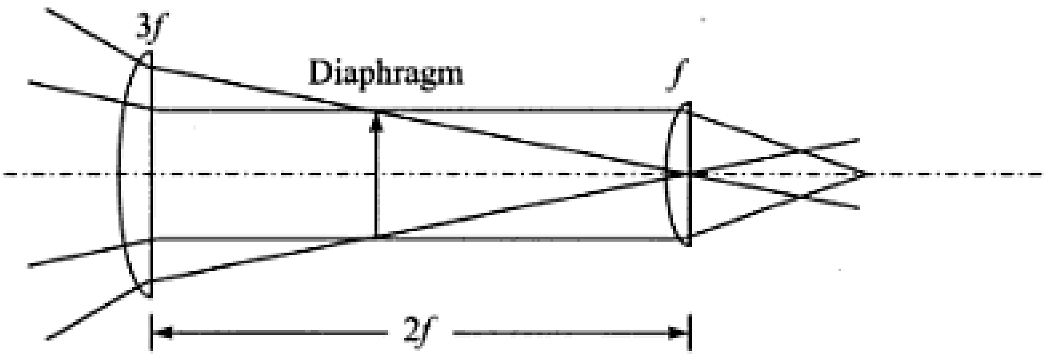
Fig. 5.12 Ramsden's eyepiece.

It can be seen that this eyepiece satisfies the condition for elimination of neither chromatic aberration nor spherical aberration. Here,

$$d = \frac{2}{3}f \quad \text{instead of } \frac{1}{2}(f + f) = f$$

$$= \frac{2}{3}f \quad \text{instead of } f - f = 0$$

Huygen's eyepiece (Fig. 5.13) satisfies the conditions but the focal plane lies between the lenses. This eyepiece, however, is not generally used with the telescopes for measuring instruments because it does not correct the image of the diaphragm which is put between the two lenses and is thus only viewed through one of them with the consequence that its image is distorted. This introduces error in measurement. However it is used in Galileo telescope.



Chromatic condition $\frac{1}{2}(3f + f) = 2f = d$
Spherical condition $3f - f = 2f = d$

Fig. 5.13 Huygen's eyepiece.

The third type of eyepiece is erecting eyepiece (Fig. 5.14). As seen in the figure, it consists of four plano-convex lenses. It gives erect image of the object. Its disadvantages are: (i) Loss of brilliancy of the image due to two additional lenses, (ii) Uncertain definition, and (iii) Larger length of the eyepiece.

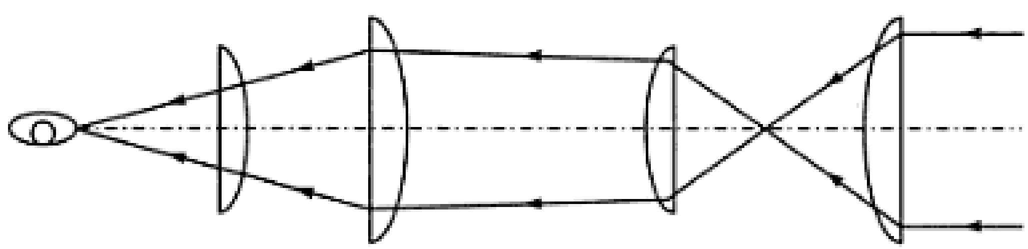


Fig. 5.14 Erecting eyepiece.

Level tube works on the principle that the surface of a still liquid, being at every point normal to the direction of gravity is a level surface. It is of glass tube sealed at both ends that contains a sensitive liquid and small air bubble. The liquid must be non-freezing, quick acting and relatively stable in length for normal temperature variation. Earlier, alcohol, chloroform or sulphuric acid or petroleum ether was used as a liquid. Nowadays, purified synthetic alcohol is used. The upper surface of the tube—and sometimes also the lower surface—is ground to form a longitudinal circular curve. The sectional elevation and the plan of a level tube are shown in Fig. 5.15. The capstan headed screws at the ends help in adjustment of the level tube.

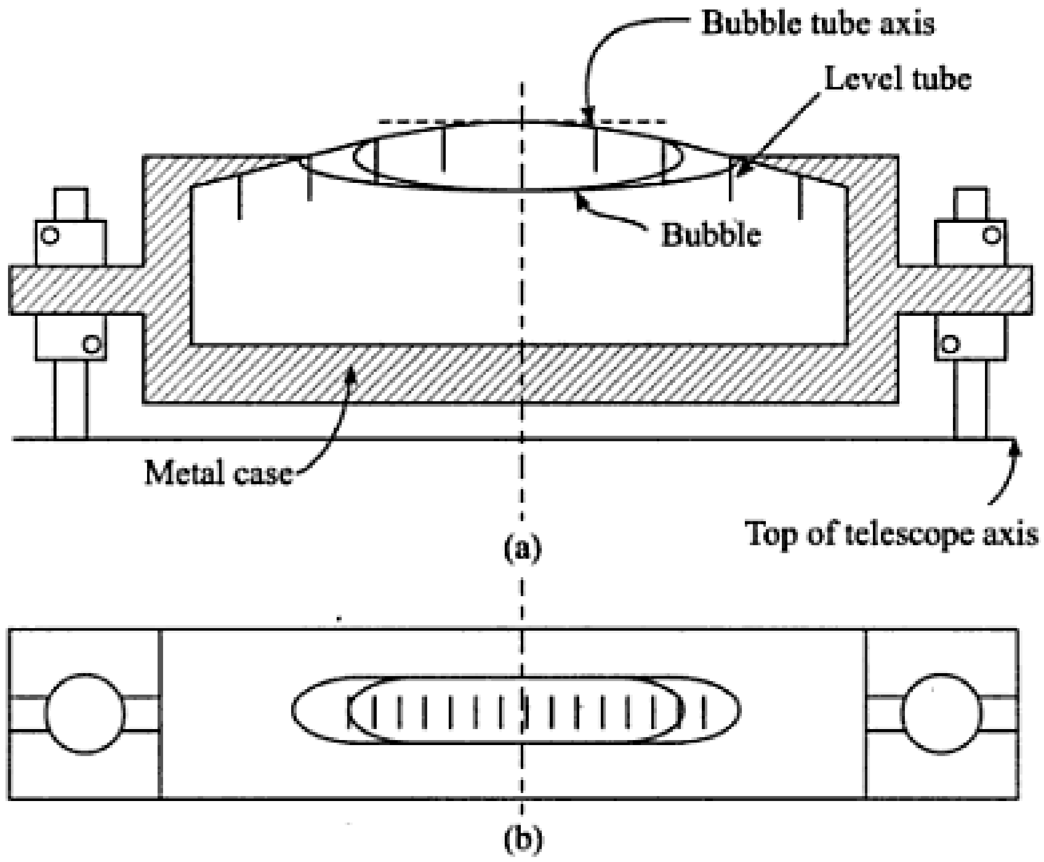


Fig. 5.15 A bubble tube: (a) Cross section. (b) Plan.

The sensitivity of the level tube depends on the radius of curvature (R) and is usually expressed as an angle (θ) per unit division (d) of the bubble scale. This angle may vary from 1″ to 2″ in the case of a precise level, upto 10″ to 30″ on an engineer's level. The radius to which the tube of an engineer's level is ground is usually between 25 to 50 m. This can be determined in the field by observing the staff readings at a known distance from the level by changing the bubble position by means of a foot screw or tilting screw as shown in Fig. 5.16. From Fig. 5.16

$$\tan n\theta = \frac{S}{l}$$

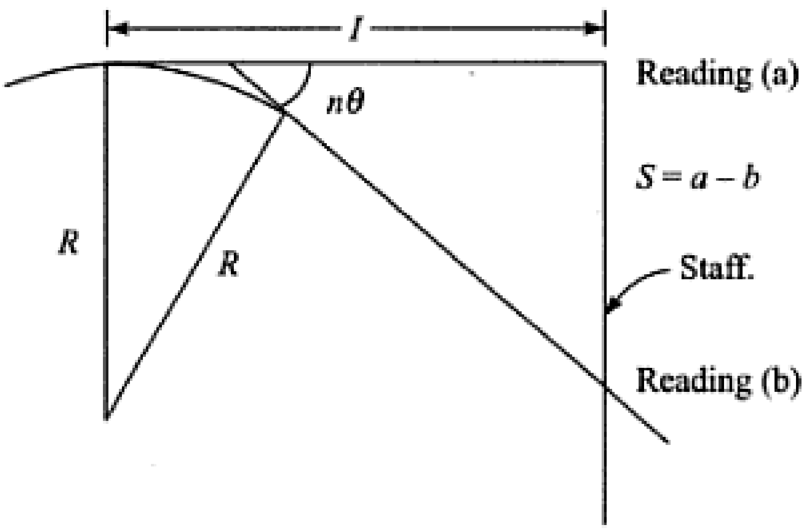


Fig. 5.16 Sensitivity of bubble tube.

Since θ is very small, $\tan n\theta \approx n\theta$

or $$n\theta_{rad} = S/l$$

$$\theta_{rad} = \frac{S}{nl}$$

$$\theta_{sec} = \frac{206265S}{nl}$$

where S = difference in staff readings a and b

n = number of divisions the bubble is displaced between readings

l = distance of staff from instrument

If d = length of one division of the bubble tube then

$$d = R\theta_{rad}$$

or

$$R = d/\theta$$

$$= \frac{ndl}{S}$$

A tube is said to be more sensitive if the bubble moves by more divisions for a given change in the angle. The sensitiveness of a bubble tube can be increased by:

(a) Increasing internal radius of the tube.
(b) Increasing diameter of the tube.
(c) Increasing length of the bubble.
(d) Decreasing roughness of the wall.
(e) Decreasing viscosity of the liquid.

The sensitivity of the bubble tube should tally with the accuracy achievable with other parts of the equipment. If the bubble is graduated from the centre then an accurate reading is possible by taking readings at the objective and eyepiece ends (Fig. 5.17).

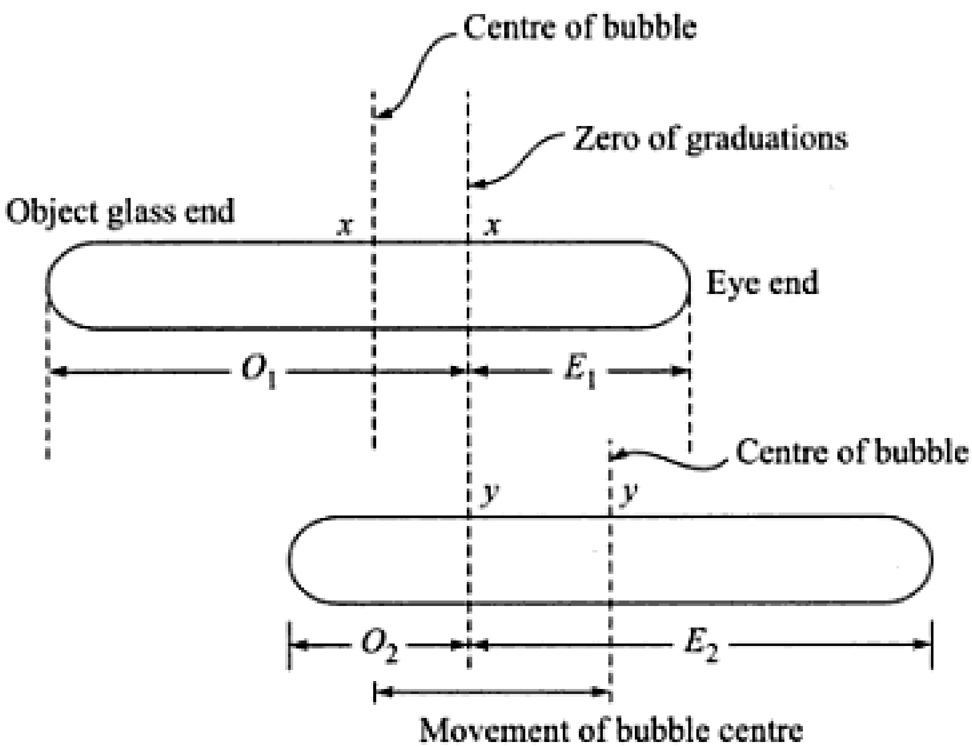


Fig. 5.17 Displacement of bubble.

From Fig. 5.17

$$L = \text{Length of bubble}$$
$$= O_1 + E_1 = O_2 + E_2$$
$$XX = \frac{O_1 + E_1}{2} - E_1$$
$$YY = \frac{O_2 + E_2}{2} - O_2$$

$$\text{Total movement } n = \frac{O_1 - E_1}{2} + \frac{E_2 - O_2}{2}$$

Figures 5.18 and 5.19 show the details of an adjustable leg of a tripod stand and a fixed leg tripod as per I.S. An adjustable tripod is advantageous for set ups in rough terrain but the type with fixed leg may be slightly more rigid. A sturdy tripod in good condition is necessary to obtain the best results for a fine instrument.

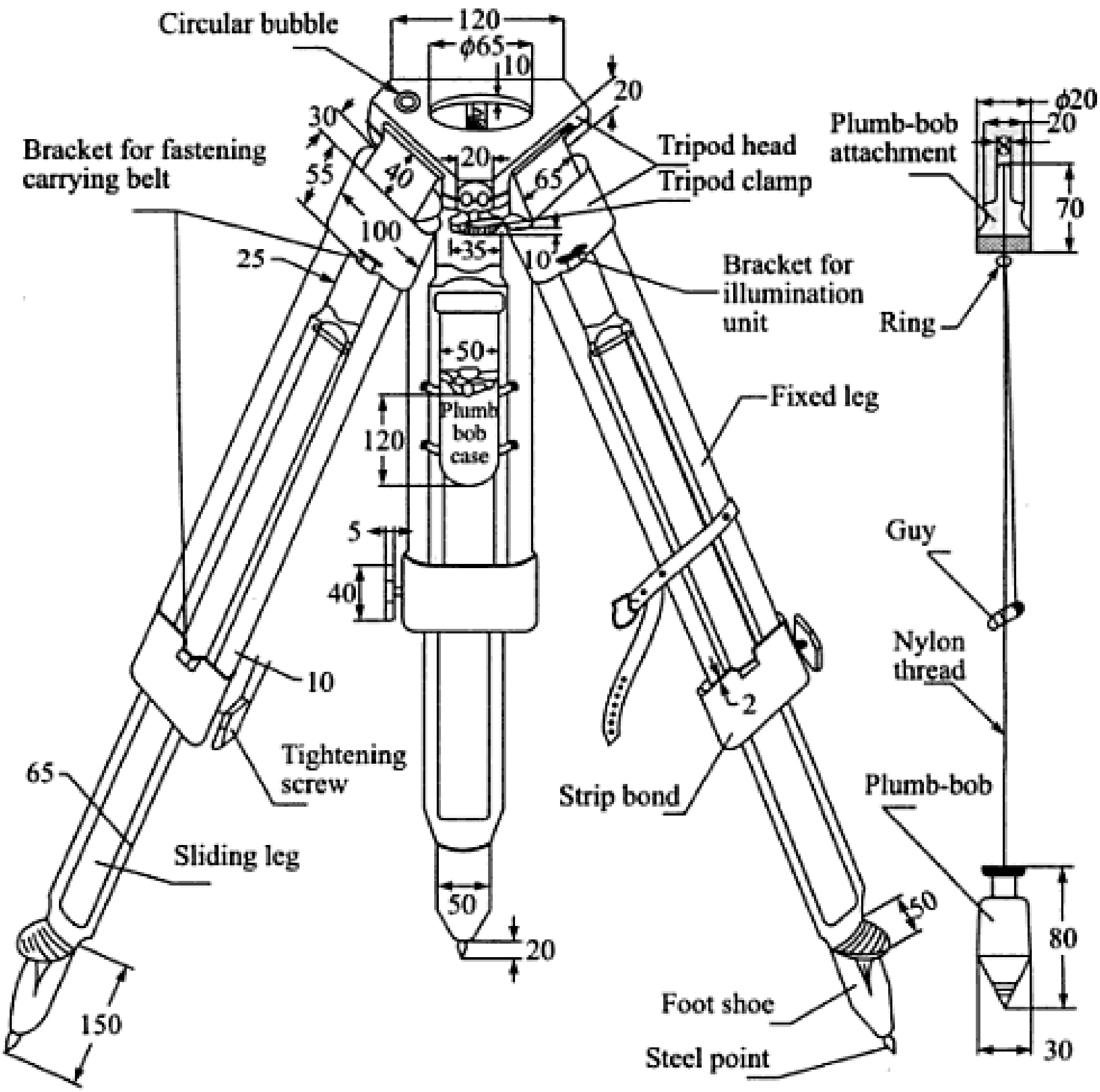
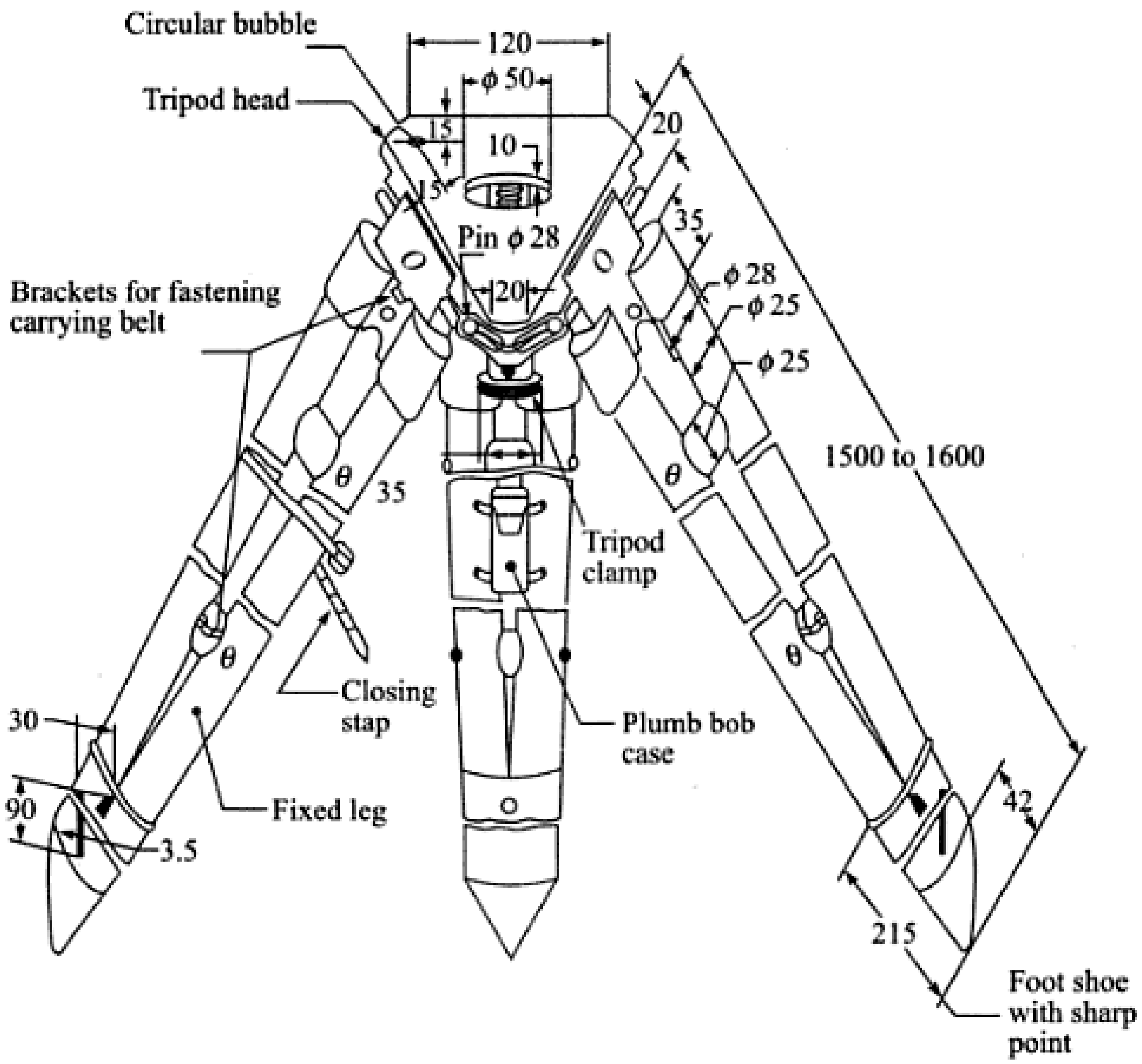


Fig. 5.18 Dimensions and nomenclature of tripod for surveying instruments (adjustable leg).



All dimensions in millimeters

Fig. 5.19 Dimensions and nomenclature for fixed leg tripod for surveying instruments.

EXAMPLE 5.5 A three screw dumpy level, setup with the telescope parallel to two foot screws is sighted on a staff 100 m away. The line of sight is depressed by manipulating the foot screws until the bubble on the telescope reads 4.1 at the object glass end and 14.4 at the eye piece end, these readings representing divisions from a zero at the centre of the bubble tube. The reading on the staff was 0.930 m. By similarly elevating the sight the bubble readings were—*O* 12.6, *E* 5.7 and staff reading 1.025 m.

Determine the sensitivity of the bubble and the radius of curvature of the bubble tube if the length of one division is 2.50 mm. [L.U.]

Solution

$$n = \frac{O_1 - E_1}{2} + \frac{E_2 - O_2}{2}$$

$$= \frac{4.1 - 14.4}{2} + \frac{5.7 - 12.6}{2}$$

$$= -\frac{10.3 + 6.9}{2} = -\frac{17.2}{2} = -\ 8.6 \text{ divisions}$$

(Negative because the line of sight is depressed and the bubble moves to the eyepiece end initially.)

$$\text{Sensitivity of bubble } \theta_{sec} = \frac{206265S}{nl}$$

$$= \frac{206265(1.025 - 0.930)}{8.6(100)}$$

$$= 22.78''$$

$$R = \frac{n \cdot d \cdot l}{S} = \frac{(8.6)(2.5)(100)}{.095}$$

$$= 22631.5 \text{ mm}$$

$$= 22.631 \text{ m}$$

5.8 TILTING LEVEL

In dumpy level, if the level is in adjustment and if the line of sight is made horizontal by bringing the bubble to the centre of its run, the vertical axis automatically becomes truly vertical. In tilting level, the line of sight can be made horizontal by a tilting screw even though the vertical axis is not exactly vertical. It was initially developed for precise levelling work but nowadays is used for general purpose. A bull's eye (circular) spirit level is available for quick approximate levelling or a ball and socket arrangement (with no levelling screws) permits the head to be tilted and locked nearly level. The exact level is obtained by tilting or rotating the telescope slightly in a vertical plane about a fulcrum at the vertical axis of the instrument without changing its height. A micrometer screw under the eyepiece controls this movement. When the level is not horizontal, the observer sees the main level tube as two half images of opposite ends of the bubble. These half images are brought into superposition and made visible by a prismatic arrangement directly over the bubble. The observer then tilts the telescope until the two half images are made to coincide in which position the bubble is centred. Figure 5.20 shows a split bubble before and after coincidence.

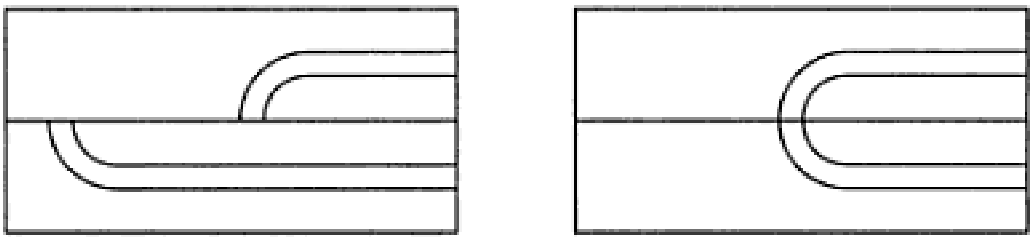


Fig. 5.20 Coincidence bubble.

Advantages of tilting level are accuracy and quickness. The level can be made horizontal just before the observation. Figure 5.21 shows schematically a tilting level.

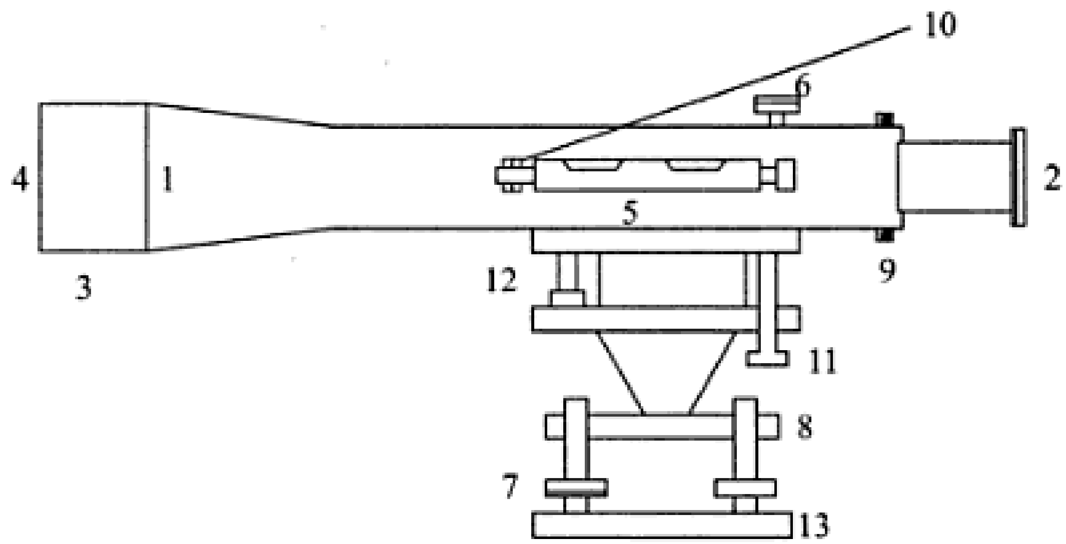


Fig. 5.21 Tilting level: 1. Telescope, 2. Eyepiece, 3. Ray shade, 4. Objective end, 5. Level tube, 6. Focussing screw, 7. Foot screw, 8. Tribrach, 9. Diaphragm adjusting screws, 10. Bubble tube fixing screws, 11. Tilting screws, 12. Spring loaded plunger, 13. Trivet stage.

5.9 AUTOMATIC OR SELF-LEVELLING LEVEL

This is the most popular variety of levels. The ease and rapidity with which the instrument makes error free readings has made it popular.

All levelling operations depend on the establishment of a line of collimation perpendicular to the direction of gravity. In making a conventional level ready for operation the skill and time of the operator is needed for accurate centring of its sensitive telescope bubble. But in making an auto level ready for operation only approximate levelling of its circular bubble is needed and then its in-built compensator takes over and makes the level ready for operation automatically in no time.

As apparent from Fig. 5.22, the compensator (which freely hangs in correct vertical position) takes the horizontal ray from the staff to the centre of diaphragm for correct readings inspite of any possible residual tilt. Technical data of an Indian automatic level is as follows:

Telescope

Image – Erect
Magnification – 24 times
Multiplication constant – 100
Shortest reading distance – 1.5 m
Longest reading distance – 200 m
Clear objective aperture – 35 mm

Circular level

Sensitivity – 10 mm/2 mm

Automatic levelling

Setting accuracy = ± 0.8 sec
Compensator range = ± 15 min

Horizontal circle

Diameter – 110 mm
Graduation interval – 1°

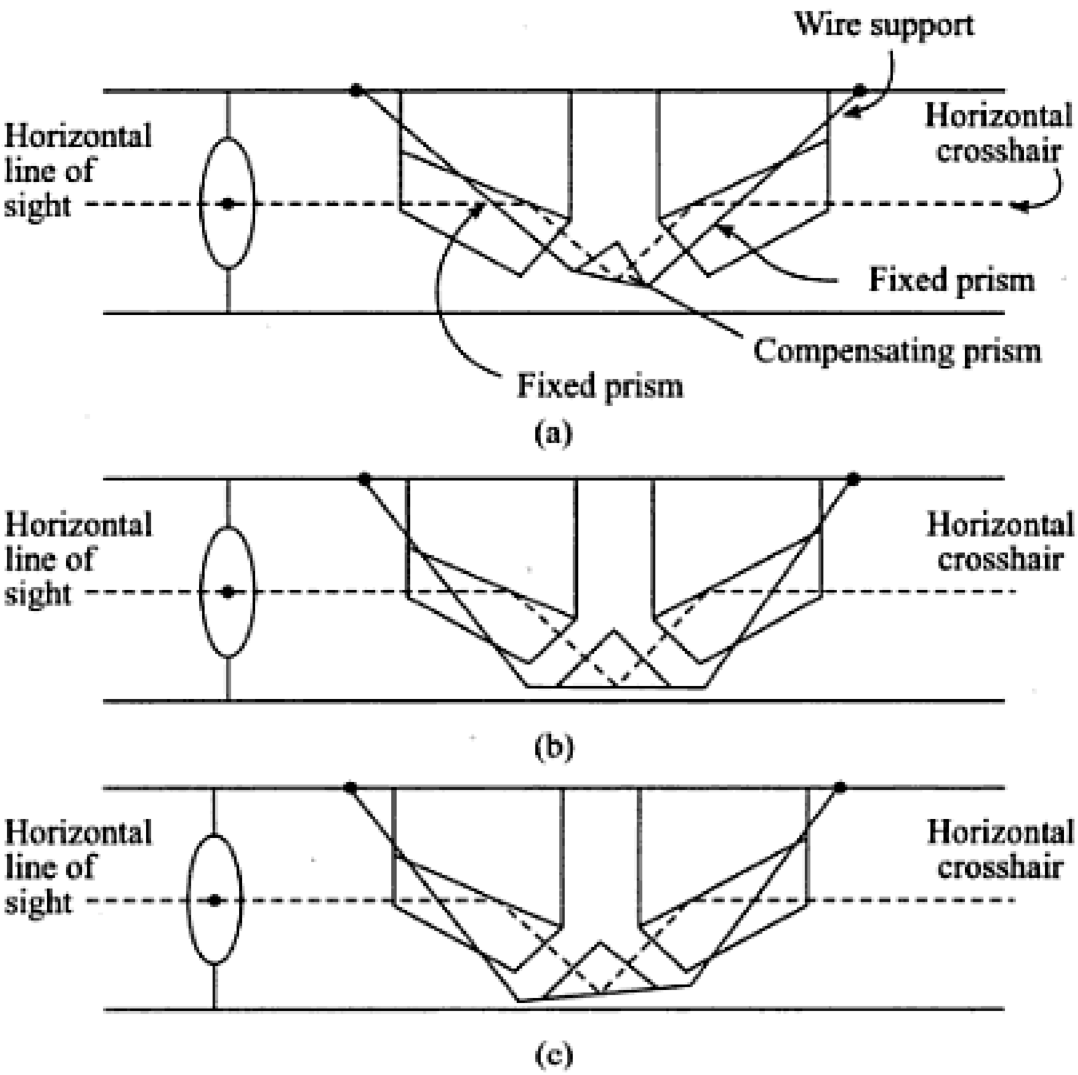


Fig. 5.22 Compensator of a self-levelling level: (a) When telescope tilts up, compensator swings backward. (b) Telescope horizontal. (c) When telescope tilts down compensator swings forward.

5.10 SOME IMPORTANT OPTICAL TERMS

Resolving Power. It is the ability of lens for distinguishing details. Its value is usually stated as the maximum number of lines per millimeter that can be seen as separate lines in the image. The resolving power depends on the diameter of the objective lens actually used (effective aperture). It is given by the empirical formula

$$R = \frac{140 \text{ sec}}{D}$$

where D is the diameter of the lens aperture in mm. If D is 30 mm R comes to 4.67. To distinguish details human eye requires a minimum resolving power of 60. This can be obtained with an instrument with $R = 4.67$ if it is magnified 13 times.

Magnification. It is the ratio between the angle subtended at the eye by the virtual image and that subtended by the object. Magnifying power of telescope is measured as the ratio of the focal length of the objective to that of the eye piece. Large magnification causes (i) Reduction of brilliancy of image, (ii) Waste of time in focussing, and (iii) Reduction of the field of view. Usually, therefore, magnification is restricted to 2 to 3 times $60/R$.

Definition. The quality of definition in a telescope is its capacity to produce a sharp image. It depends on eliminating optical defects like chromatic aberration and spherical aberration from the eyepiece and objective.

5.11 SOME IMPORTANT OPTICAL DEFECTS

Chromatic Aberration. When white light is refracted through a glass prism it is split into its component colours, the red end of the spectrum being refracted less than the violet end. This phenomenon, known as *dispersion,* makes accurate focussing difficult, the image being surrounded by a rainbow like boundary. This is chromatic aberration is shown in Fig. 5.23(a). To remedy this defect two lenses, one concave of flint glass and the other a convex lens of crown glass are cemented together with balsam as already explained in Section 5.10.

Spherical Aberration. It arises due to the spherical surface of the lens and prevents accurate focussing due to the rays incident on the lens being refracted more than the rays incident at the centre (Fig. 5.23(b)). This can be remedied by using only the central portion of the lens which also cuts down the amount of light entering the eye. Usually, therefore a combination of lenses is used so that aberration of one eliminates that of the other. For example, in objective a convex and a concave lens are cemented together. In Ramsden's eyepiece two piano convex lenses are kept at a fixed distance apart.

Fig. 5.23 (a) Chromatic aberration. (b) Spherical aberration.

5.12 THE LEVELLING STAFF

A level staff is a graduated rod of rectangular section. It is usually made of teakwood. It may also be of fibre glass or metal. Two main classes of rod are:

1. Self-reading which can be read by the instrument operator which sighting through the telescope and noting the apparent intersection of the cross wires on the rod. This is the most common type.
2. Target rods having a movable target that is set by a rod person at the position indicated by signals from the instrument—man.

A levelling staff can be of

(a) Solid, i.e. of one piece—Fig. 5.24(a).
(b) Folding when it can be folded to smaller length—Fig. 5.24(b).
(c) Telescopic when the staff can be shortened by putting one piece inside another—Fig. 5.24(c).

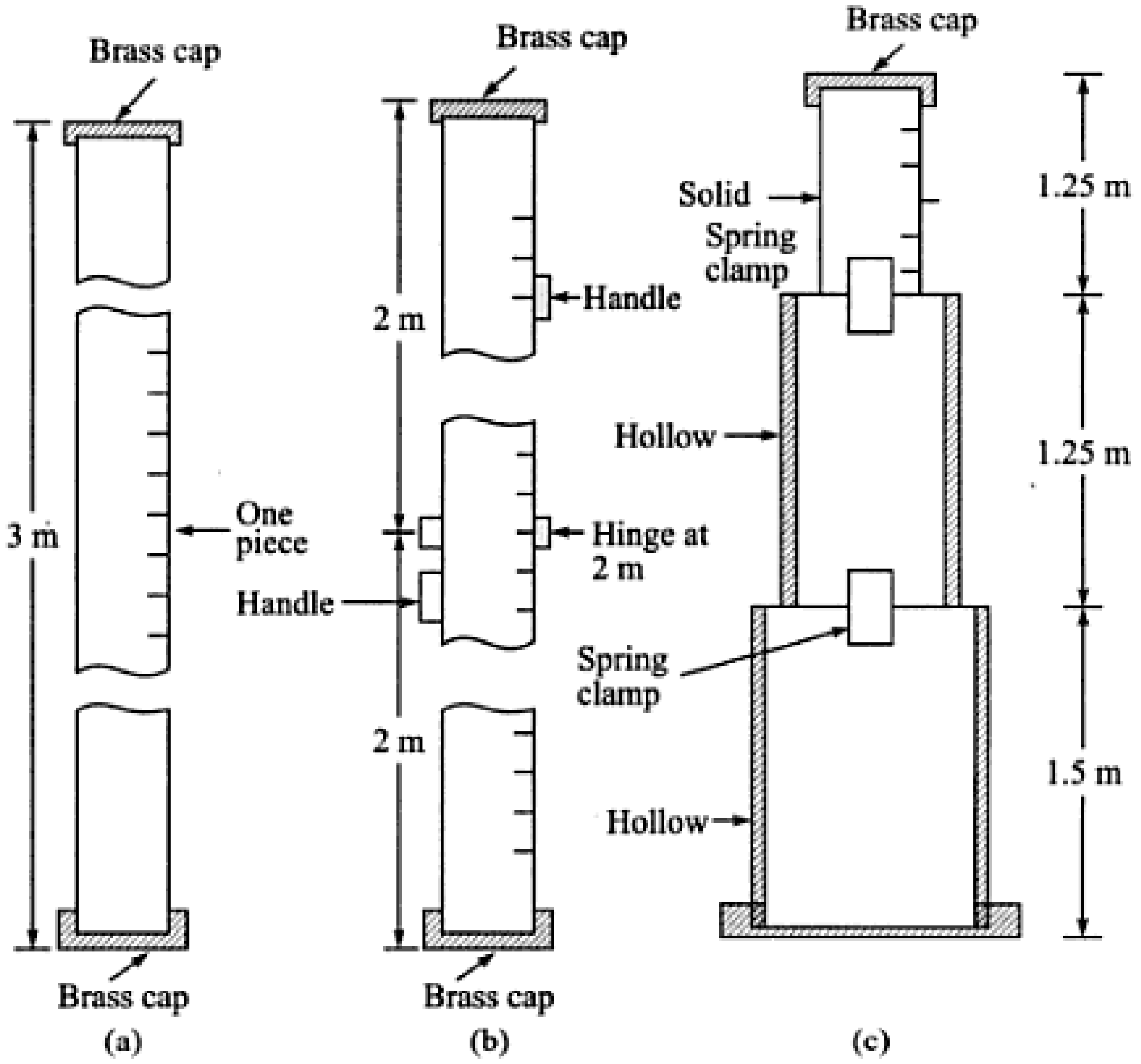


Fig. 5.24 Different types of levelling staff.

Solid staff being of one piece gives more accurate reading. Folding staff is light and convenient to handle. As per IS-1779–1961, the width and thickness of staff are 75 mm and 18 mm respectively. The staff can be folded to 2 m length. To ensure verticality the staff has a circular bubble of 25-minute sensitivity. Each meter is subdivided into 200 subdivisions, the thickness of the graduation being 5 mm. Details of a levelling staff (Folding type) are shown in Fig. 5.25(a) and (b). In telescopic staff shown in Fig. 5.24(c), the topmost part is solid and the other two parts are hollow.

The two top pieces when pulled up are kept in position by brass flat spring clamps at the back of each piece fixed at its lower end. While using the telescope staff care should be taken to ensure that the three parts are fully extended. The telescopic staff is not as accurate as a folding staff because of possible slippage between the parts.

Target staff has sliding target equipped with vernier. It is used for long distance sighting when it becomes difficult to take staff reading directly. The target is a small metal piece of circular or oval shape about 125 mm diameter. It is painted red and white in alternate quadrants. For taking reading the level man directs the staffman to raise or lower the target till it is bisected by the line of sight. The staff holder then clamps the target and take the reading. Apparent advantage of target staff is accurate reading but it takes more time. On the other hand self-reading staff is

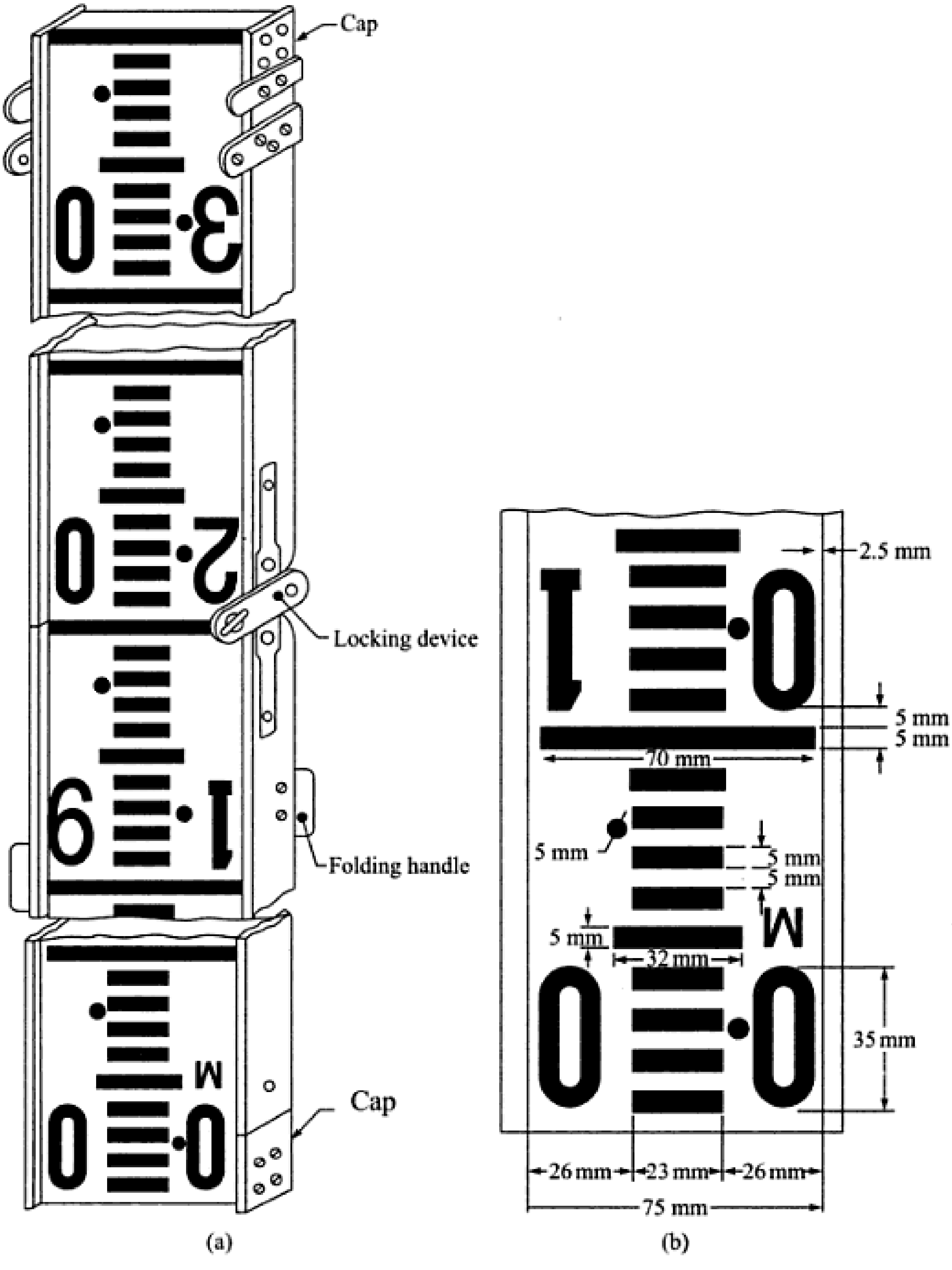


Fig. 5.25 (a) Levelling staff (folding type). (b) Typical details of graduations.

quicker. Moreover, for self-reading staff only one trained staff, that is, instrument man is required but for target staff reading both instrument man and target man should be adequately trained. Fig. 5.26 gives details of a target staff.

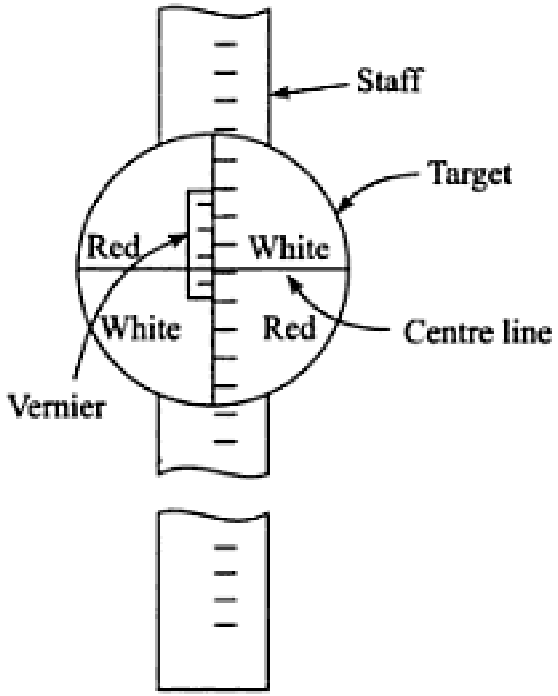


Fig. 5.26 Target staff.

5.13 PARALLEL PLATE MICROMETER

As shown in Fig. 5.27(a) parallel glass plate is usually fitted in front of the objective of a precise or geodetic level. It enables the interval between the horizontal crosshair and the nearest staff division to be read directly to 0.1 mm. The parallel glass plate can be tilted forward or backward by means of a micrometer head at the eye end of the telescope. Due to refraction a ray of light parallel to the telescope axis is displaced upwards and downwards by an amount proportional to

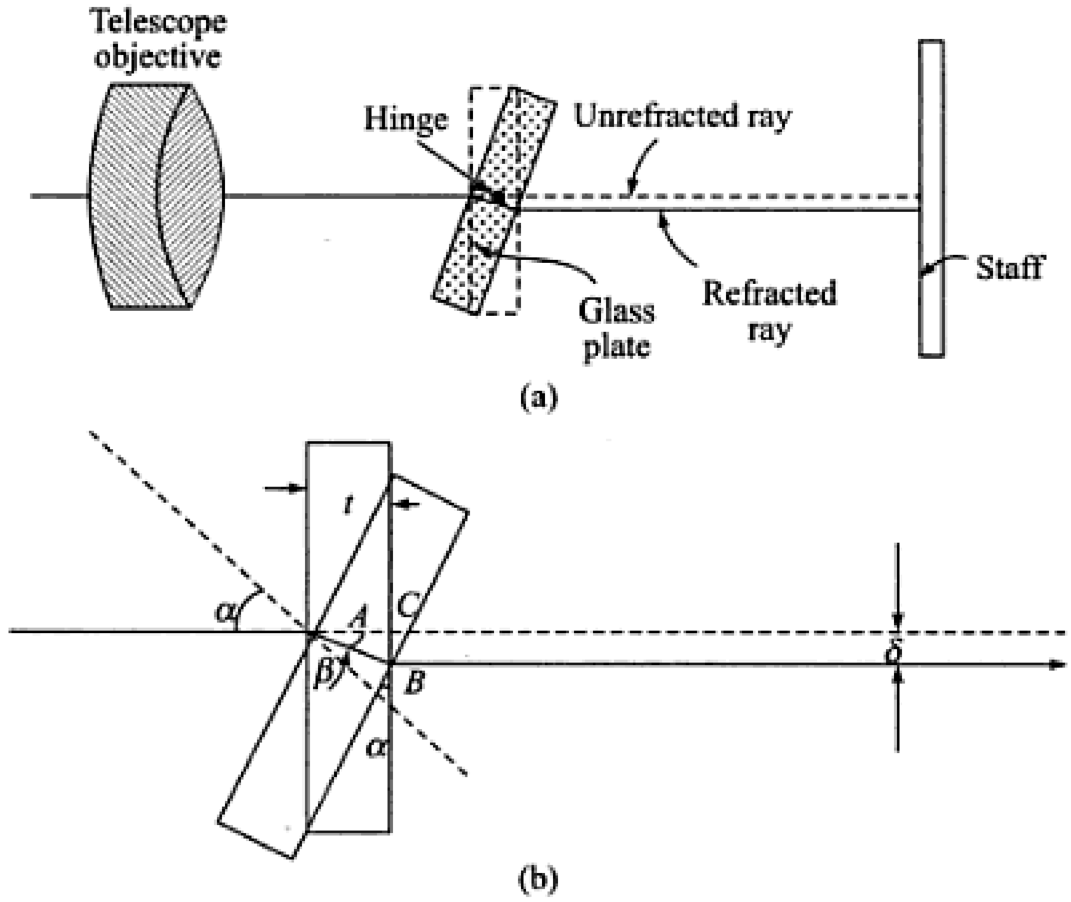


Fig. 5.27 (a) Parallel plate micrometer. (b) Derivation of equation.

the amount of tilt. When the plate is vertical no displacement occurs. The plate is tilted till a full reading of the staff coincides with the crosshair. The displacement d gives the fractional reading which is obtained directly from the micrometer drum. The theory can be derived as follows:

Let μ be the refractive index of the glass used in the plate (Fig. 5.27b). From ΔABC,

$AB \cos \beta = AC = t$, thickness of the glass plate

$$AB = \frac{t}{\cos \beta}$$

$$BC = \delta = AB \sin(\alpha - \beta)$$

$$= \frac{t}{\cos \beta} \cdot \sin(\alpha - \beta)$$

$$= \frac{t}{\cos \beta} \cdot (\sin \alpha \cos \beta - \cos \alpha \sin \beta)$$

$$= t\left(\sin \alpha - \cos \alpha \frac{\sin \beta}{\cos \beta}\right)$$

$$= t \sin \alpha \left(1 - \frac{\cos \alpha}{\sin \alpha} \cdot \frac{\sin \beta}{\cos \beta}\right)$$

From laws of refraction

$$\frac{\sin \alpha}{\sin \beta} = \mu \quad \text{or} \quad \sin \beta = \frac{\sin \alpha}{\mu} \quad \text{or} \quad \cos \beta = \sqrt{1 - \left(\frac{\sin \alpha}{\mu}\right)^2}$$

which gives

$$\delta = t \sin \alpha \left(1 - \frac{\cos \alpha}{\sin \alpha} \cdot \frac{\sin \alpha / \mu}{\sqrt{1 - (\sin \alpha / \mu)^2}}\right)$$

$$= t \sin \alpha \left(1 - \frac{\sqrt{1 - \sin^2 \alpha}}{\sqrt{\mu^2 - \sin^2 \alpha}}\right)$$

If α is small $\sin \alpha \to \alpha$, and $\sin^2 \alpha \to 0$, and

$$\delta = t\alpha\left(1 - \frac{1}{\mu}\right)$$

$$= t\alpha \cdot \frac{\mu - 1}{\mu}$$

$$= k\alpha \quad \text{where} \quad k = t \cdot \frac{\mu - 1}{\mu}$$

which shows that the displacement is directly proportional to the angle of rotation α of the plate provided. The angle α is small.

EXAMPLE 5.6 If the index of refraction from air to glass is 1.6 and the parallel plate prism is 16 mm thick, calculate the angular rotation of the prism to give a vertical displacement of the image of 1 mm.

Solution

$$\delta = t \cdot \alpha \cdot \frac{\mu - 1}{\mu}$$

$$1 = 16 \cdot \alpha \cdot \frac{1.6 - 1}{1.6}$$

$$\alpha = \frac{1.6}{16(0.6)} = 0.1667 \text{ rad}$$

$$= 9°33'$$

5.14 TEMPORARY ADJUSTMENTS OF A DUMPY LEVEL

Temporary adjustments are done at every setting of the instrument in the field. They are:

1. *Setting up*: Initially the tripod is set up at a convenient height and the instrument is approximately levelled. Some instruments are provided with a small circular bubble on the tribrach to check approximate levelling. At this stage the levelling screw should be at the middle of its run.
2. *Levelling up*: The instrument is then accurately levelled with the help of levelling screws or foot screws. For instruments with three foot screws the following steps are to be followed:

   (a) Turn the telescope so that the level tube is parallel to the line joining any two levelling screws as shown in Fig. 5.28(a).
   (b) Bring the bubble to the centre of its run by turning the two levelling screws either both inwards or outwards.
   (c) Turn the telescope through 90° so that the level tube is over the third screw or on the line perpendicular to the line joining screws 1 and 2. Bring the bubble to the centre of its run by the third foot screw only rotating either clockwise or anticlockwise Fig. 5.28(b).
   (d) Repeat the process till the bubble is accurately centred in both these conditions.
   (e) Now turn the telescope through 180° so that it is again parallel to levelling screws 1 and 2 Fig. 5.28(a). If the bubble still remains central, the adjustment is allright. If not, the level should be checked for permanent adjustments.

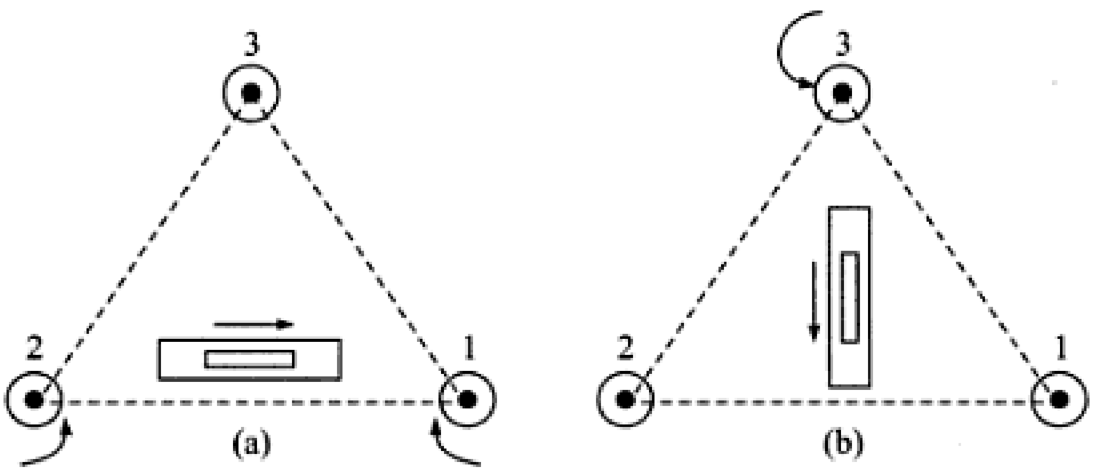


Fig. 5.28 Turning foot screws to level bubble tube.

3. *Focussing*: This is done in two steps. First step is focussing the eyepiece. This is done by turning the eyepiece either in or out until the crosshairs are sharp and distinct. This will vary from person to person as it depends on the vision of the observer. The next step is focussing the objective. This is done by means of the focussing screw where by the image of the staff is brought to the plane of the crosshairs. This is checked by moving the eye up or down when reading the crosshair does not change with the movement of the eye as the image and the crosshair both move together.

5.15 TERMS USED IN LEVELLING

The following terms are frequently used in levelling

1. *Station*: This is a point where a levelling staff is held for taking observations with a level.
2. *Height of the instrument* (H.I.): This has two meanings. It may mean height of the instrument above the ground at the station where the instrument is placed. However, usually it means elevation of the line of sight or line of collimation with respect to the datum. Line of collimation is an imaginary line joining the optical centre of the objective with the intersection of crosshairs and its continuation.
3. *Backsight* (B.S.): It is the first reading taken at a station of known elevation after setting up of the instrument. This reading gives the height of the instrument (elevation of the line of collimation) as.

   Elevation of line of collimation = Known elevation + Backsight

4. *Intermediate sight* (I.S.): As the name suggests these are readings taken between the 1st and last reading before shifting the instrument to a new station.
5. *Foresight* (F.S.): This is the last reading taken before shifting an instrument to a new station.
6. *Turning point or change point*: For levelling over a long distance, the instrument has to be shifted a number of times. Turning point or change point connects one set of instrument readings with the next set of readings with the changed position of the instrument. A staff is held on the turning point and a foresight is taken before shifting the instrument. From the next position of the instrument another reading is taken at the turning point keeping the staff undisturbed which is known as back sight.
7. *Reduced Level* (R.L.): Reduced level of a point is its height relative to the datum. The level is calculated or reduced with respect to the datum.

5.16 DIFFERENT METHODS OF LEVELLING

In levelling it is desired to find out the difference in level between two points. Then if the elevation of one point is known, the elevation of other point can be easily found out. In Fig. 5.29, the instrument is placed at *C* roughly midway between two points *A* and *B*. The staff readings are shown in the figure.

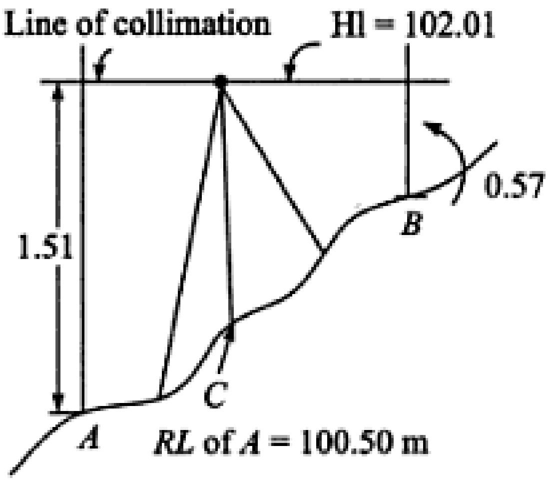


Fig. 5.29 Direct levelling

From the figure the reduced level of B can be derived as $100.50 + 1.51 - 0.57 = 101.44$ m. From the readings it can also be observed that if the second reading is smaller than the 1st reading, it means that the second point is at a higher level than the first. This is also known as *direct levelling*.

In trigonometrical levelling the difference in elevations is determined indirectly from the horizontal distance and the vertical angle. Since trigonometric relations are utilized in finding the difference in elevation it is known as trigonometrical levelling. It is used mainly to determine elevations of inaccessible points such as mountain peaks, top of towers, etc. as shown in Fig. 5.30.

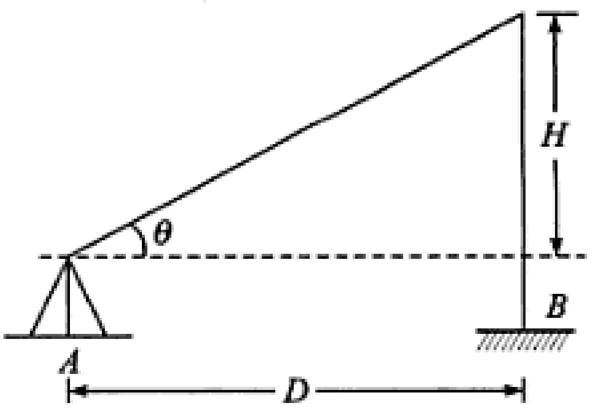


Fig. 5.30 Trigonometrical levelling.

In barometric levelling, the principle that pressure decreases with rise in elevation is used. Hence it is possible to determine the difference in elevation between two points by measuring the pressure difference between the points by either mercury barometer or aneroid barometer. As the aneroid barometer is strong and sturdy it is preferred to the mercury barometer which is fragile and cumbersome. However, aneroid barometer is less accurate compared to the mercury barometer.

PROBLEMS

5.1 Explain how surveyors and engineers can often ignore the error caused by curvature and refraction in levelling work.

5.2 What errors may be introduced in using telescope's focussing screws?

5.3 Define optical axis of a lens.

5.4 List in tabular form, for comparison, the advantages and disadvantages of a tilting level versus an automatic level.

5.5 Describe the method of operation of a parallel plate micrometer in precise levelling. If the index of refraction from air to glass is 1.6 and the parallel plate prism is 15 mm thick, calculate the angular rotation of the prism to give a vertical displacement of the image of 0.0001 m.

5.6 Describe with the aid of a sketch the function of an internal focussing lens in a surveyor's telescope and state the advantages and disadvantages of internal focussing as compared with external focussing.

In a telescope, the object glass of focal length 178 mm is located 230 mm from the diaphragm. The focussing lens is midway between these when a staff 20 m away is focussed. Determine the focal length of the focussing lens.

(L.U., BSc.)

5.7 Define the following terms.

(a) level surface, (b) horizontal plane, (c) vertical plane, (d) vertical line, (e) elevation of a point, (f) line of collimation, (g) benchmark, (h) change point, (i) datum, (j) back sight, (k) fore sight.

5.8 Give the definition of sensitiveness of bubble tube in levelling and its effect in accuracy of levelling. [AMIE, November 1964]

5.9 Sketch a modern tilting level, name its parts and describe step by step how it is used. What is the main advantage of this type of level over the Dumpy level?

5.10 Work out the true difference in level between points A and B if curvature and refraction effects are taken into account in the following case:

Level set up over point A
Staff held over point B
R.L. of point A = 100.000 m
Height of instrument at point A = 1.000 m
Reading at staff on point B = 2.000 m
Distance AB = 300 m
Assume diameter of earth = 12,742 km

[AMIE, May 1969]

5.11 (a) Explain the effects of curvature and refraction in levelling.

(b) An observer standing on the deck of a ship just sees a lighthouse. The top of the lighthouse is 64 m above sea level and the height of the observer's eye is 9 m above sea level. Find the distance of the observer from the lighthouse.

5.12 If the bubble tube of a level has a sensitiveness of 35″ per 2 mm division, find the error in staff reading on a vertical staff at a distance of 100 m caused by the bubble reading 1½ divisions out of centre.