Energy Flow in Streaming Non-Maxwellain Plasmas With Parallel Electromagnetic Waves

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Saqib Ghani

Abstract

Energy Flow in Streaming Non-Maxwellian Plasmas with Parallel Electromagnetic Waves (PEMW) is studied by using the kinetic model. The wave-particle interaction is considered while studying energy flux. We study the effects of streaming velocity, temperature anisotropy $\left(\eta = \frac{T_\perp}{T_\parallel}\right)$, wave frequency, thermal velocity and the index kappa (κ) on the energy flux. It is observed, for smaller values of streaming velocity, wave frequency, thermal velocity and the index κ the PEMW transport its energy slowly over the long distance; while for smaller values of temperature anisotropy the wave rapidly transport its energy over the short distance. Thus the aforementioned parameters play a crucial role in the transmission of wave energy.

Contents

Chapter 1

INTRODUCTION

1.1 What is Plasma

A quasi-neutral gas of charged particles having collective behavior is called a plasma. Plasmas are generally composed of an equal number of positive and negative charged particles once a neutral gas has been ionized. When this happens, the oppositely charged fluids are very much bonded together and they cancel each other out in respect of electricity in terms of large scale lengths. This plasma termed quasineutral. Collective behaviour indicates that each individual particle (ions,electrons) affects the behaviour of the plasma. Sometimes we often stated that 95% (or 99% depending on who we wish to flatter) of the Universe is composed of plasma. This assertion quantifies the quantity of plasma with a high level of endearment to plasma physics and is non-falsifiable (or falsifiable). Nonetheless, we must recognize that most matter around us exists in form known as plasma. In the early universe, everything existed as plasma, and this state continues today, as plasma still fills nebulae, stars, and interstellar space. The solar system is abundant with plasma in the form of solar wind, and the Earth is entirely surrounded by plasma trapped within its magnetic field. Terrestrial plasmas are not difficult to find also. They are present in a variety of laboratory experiments, fluorescent lamps, lightning, and an expanding number of industrial activities [1].

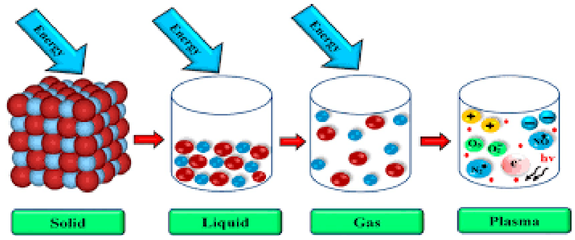

Figure 1.1. Four states of matter, plasma is created when energy is added to a solid, liquid or gas.[2]

1.2 Debye Shielding

A plasma behavior is fundamentally characterized by its ability to shield off electric potentials that are impressed upon it. Imagine we insert two charged balls connect to a battery to create an electric field inside a plasma. Particles of the opposite charge would be attracted by the balls, cloud of ion would revolve around the negative ball while the positive one would be surrounded with another cloud made of electrons. if there were no thermal motions in the cold plasma, there would have been equal numbers of charges on the ball itself and in the cloud and perfect shielding would exist, and there would be no electric field outside the plasma. Debye length is distance over which ions and electrons can be seperated in plasma. This is an essential physical feature that define the plasma. Debye length has direct relation with temperature square root and inverse relation with density square root, calculated as follows

$$\lambda_D = \sqrt{\frac{\epsilon_o KT}{ne^2}} \tag{1.1}$$

where λ_D is debye length.

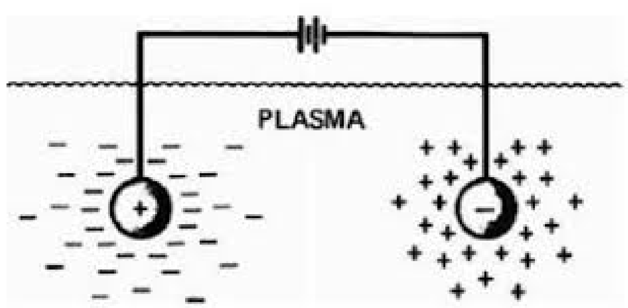

Figure 1.2. Debye Shielding

1.3 Conditions for Plasma

An ionized gas is classified as plasma if it meets the following three conditions [3].

(1) $\lambda_D << L$: The Debye length is very small compared to the overall length of the plasma, ensuring the plasma quasi-neutrality.

(2) $N_D >> 1$: This implies that the number of particles within the Debye sphere must be significantly greater than one. This condition ensures the fulfillment of the collective behavior of plasma.

(3) $\omega\tau >> 1$: In this context, ω represents the frequency of plasma oscillations, while τ denotes the average duration between collisions involving neutral atoms.

1.4 Plasma Models

Two main models are utilized for exploring plasma physics.

1.4.1 Fluid Model

Charged particles are found in plasma; electrons and ions, which undergo complex movements while in plasma. This movement generates electric and magnetic fields in the plasma [4]. It is hard to study bahavior of these complex movements, we use the fluid model in order to avoid studying all the complexities of the plasma particles by treating plasma as a fluid [3]. At the macroscopic level, fluid model is the simplest method used for the study of plasma. The fluid model is used to analyze plasma at the macroscopic level because it does not take into account the identity of individual particles but only focuses on how fluid elements move. Magneto-hydrodynamics (MHD) is an expansion of fluid models where plasma is regarded as a conductive fluid subject to the effects of magnetic and electric fields [5, 6].

Equations of Fluid Model

In fluid model, following equations are used

(1) Maxwell's equation

(2) Equation of Continuity

(3) Equation for Momentum Transport

1.4.2 Kinetic Model

In order to investigate microscopic-scale plasma physics, a kinetic model is employed. Landau damping, wave particle interactions, and temperature anisotropy are examples of the processes for which the use of fluid model is not appropriate. The kinetic model utilizes the velocity distribution function $f(v)$. A model that uses the velocity distribution function (VDF) to describe the behavior of a plasma, as well as each constituent species, is known as the kinetic model. Kinetic equations are solved to study spatial and temporal estimation. There is also the use of kinetic model for studying plasma in non thermal equilibrium cases [1, 5]. The best technique for theoretical studies of plasma is the kinetic model.

The particles distribution function in the kinetic model is determined by their position, time and velocity at any instant [3];

$$f(r, v, t) = f(x, y, z, v_x, v_y, v_z, t) \tag{1.2}$$

1.5 Waves in Plasma

Plasma has various type of waves. This multitude of waves shows changes in these two fields, electric field as well as magnetic field. However, they are grouped into two major groups among the many classes of plasma waves. These two include electrostatic waves and electromagnetic waves. Electric perturbations lead to the emission of electrostatic waves. If electric and magnetic perturbation exist simultaneously then plasma wave known as electromagnetic waves. There are also plasma waves which are categorized based on moving particles that oscillate such as electron waves are high frequency and ion waves are low frequency. This is due to electrons are more energetic than ions. However, electrons are extremely hot compared to ions. It happens that ion inertia supplies the uniform background which gives rise to high frequency of electron waves and low frequency of ion waves. Properties of plasma waves propagation are related to such other parameters as plasma waves frequency, frequency of collision, plasma density, as well as temperature. Plasma waves travel in the direction indicated by the wave number ($\mathbf{k}$) as it relates to the unperturbed magnetic field ($\mathbf{B_0}$) and perturbed electric fields ($\mathbf{E_1}$) [7].

1.5.1 Perpendicular Propagating Electromagnetic Waves

Waves where the wave number ($\mathbf{k}$) is perpendicular to the unperturbed magnetic field ($\mathbf{B_0}$) are referred to as perpendicular waves.

$$\mathbf{k} \perp \mathbf{B_0}$$

Lower hybrid mode, upper hybrid mode, ordinary mode (also called O-mode), Extraordinary mode (also called X-mode), Bernstein modes and magnetosonic mode all these are example of perpendicular waves.

Ordinary Wave (O-Mode)

In ordinary wave, propagation vector ($\mathbf{k}$) of EM waves is orthogonal to the unperturbed magnetic field ($\mathbf{B_0}$) and electric field ($\mathbf{E_1}$) is parallel to the $\mathbf{B_0}$. Dispersion relation of O-mode is

$$\omega^2 = \omega_p^2 + c^2 k^2 \tag{1.3}$$

The same outcome emerges if we set the electromagnetic problem to $\mathbf{B_0}$=0. A changing magnetic field doesn't affect ordinary waves.

Extraordinary Waves

In extraordinary waves, both perturbed electric field ($\mathbf{E_1}$) and propagation vector ($\mathbf{k}$) of EM waves is orthogonal to the unperturbed magnetic field ($\mathbf{B_0}$). This type waves are referred to as extraordinary wave. In this case the electric field ($\mathbf{E_1}$) polarization is elliptical in nature. It is comprised of a combination of longitudinal and transverse electromagnetic waves. The dispersion relation for these waves would be.

$$\frac{c^2 k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{(\omega^2 - \omega_p^2)}{(\omega^2 - \omega_h^2)} \tag{1.4}$$

The term ω_h refers to the upper hybrid frequency and it is determined as

$$\omega_h^2 = \omega_p^2 + \omega_c^2 \tag{1.5}$$

Where ω_c and ω_p denotes the cyclotron frequency and plasma frequency. In this specific instance,

is the factor that influences how electrons move. This mode resonates at the upper hybrid frequency ($\omega_h = \omega = \sqrt{\omega_p^2 + \omega_c^2}$) and preventing the wave from propagating any further.

1.5.2 Parallel Propagating Electromagnetic Waves

Waves with a wave number ($\mathbf{k}$) aligned with the direction of the unperturbed magnetic field ($\mathbf{B_0}$) are referred to as parallel waves.

$$\mathbf{k} \parallel \mathbf{B_0}$$

Examples of plasma waves that propagate parallel to the magnetic field include right-handed and left-handed circularly polarized waves (R-waves and L-waves), Langmuir waves (or electron plasma waves), ion acoustic waves, pure Alfvén waves, whistler waves, and helicon waves.

Electron Plasma Waves

These waves as a result of electrons being displaced during ions acting as a uniform background. The displacement of the electron establishes an electric field in a specific direction which in turn pulls the electrons back towards their true positions in a bid to restore plasma neutrality. Ions function as a uniform background for electrons because of their high mass in comparison with that of electrons. Due to electrons inertia, they will always rebound and as a result they vibrate around the average position or the equilibrium position at a frequency referred to as plasma frequency [8]. The speed of these oscillations is such that ions are unable to keep up with the oscillatory field, for this reason they are referred to as being stationary with a constant background. At high temperatures, thermal energies are so high that these kinds of oscillations are spreading throughout the whole system. These oscillations play crucial role in plasma diagnostics. In a solar corona, one observes this type of oscillations at larger scale [9]. The dispersion relation for these waves as

$$\omega^2 = \omega_{pe}^2 + \frac{3}{2} k^2 v_{th}^2 \tag{1.6}$$

where v_{th} is thermal velocity of electron $v_{th}^2 = \frac{2k_BT}{m}$. Speed of light is greater then group velocity in case of electron plasma waves.

Ion Acoustic waves

Without particles clashing with each other, ordinary sound wave can't be created but, on account of their charge, ions are capable of transmitting vibrations between themselves. Ions acoustic waves act as pressure waves. These are also slow moving waves for heavy ions in motion. In plasma with few collisions, such waves are produced. The main role in producing these waves is played by the ion dynamics. The dispersion relation as

$$\omega^2 = k^2(\frac{\gamma_i K_B T_i}{M} + \frac{K_B T_e}{M}) \tag{1.7}$$

Group velocity has the same value as the phase velocity for ion acoustic waves.

Right and Left-Handed Circularly Polarized Electromagnetic Waves

These waves propagate parallelly. In this case, wave number ($\mathbf{k}$) aligns with the unperturbed magnetic field ($\mathbf{B_0}$) and there are two transverse components to the perturbed electric field ($\mathbf{E_1}$). Dispersion relation for the R-wave as

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2 - \omega_c\omega} \tag{1.8}$$

In the same way for L-waves, dispersion relations become

$$\frac{c^2k^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2 + \omega_c\omega} \tag{1.9}$$

The only difference in their dispersion relations is whether there is a $+$ or $-$ sign with ω_c in the denominator of the right side. For the L-wave it is positive while for R-wave it is negative. The rotation of the **E** vector in the R-wave resembles that of a typical screw. Using Right-hand rule, you can determine the direction of the wave vector k and the direction of the rotation of the vector E. According to the right-hand rule, the direction of vector **E** is indicated by the curved fingers, while the direction of wave number (**k**) is indicated by the thumb. However, for L-wave, the rotation direction of vector E stands entirely opposite to that of R-wave. When it comes to R-waves, electrons resonate at $\omega = \omega_c$ and $\mathbf{k}$ goes to infinity. It is known as cyclotron resonance. Right-handed circularly polarized waves rotate in the same direction as electrons do. The accelerating electrons continuously lose energy as the wave propagates at resonance hence R-wave does not travel. This is not the case with L-waves since they rotate oppositely.

Whislter Waves

Low frequency R-waves are called whislter wave. Whistler waves are characterized by parallel propagation, where the wave number ($\mathbf{k}$) aligns parallel to the direction of the uniform magnetic field [9]. The signals are usually of a higher frequency while whistling sounds indicate that the waves are moving at relatively lower frequencies. The whistling sounds take place at audio frequencies. The ionosphere is subject to some sort of excitement and this results from lightning, which we can understand by means of electromagnetic radiations and normally, these waves are curbed near the field because the earth magnetic field deflects the waves. In southern hemisphere when lightning errupts, there are sounds on various frequencies. For example, high harmonic waves come faster than the waves with low harmonic frequencies [10]. The rotation direction of the R wave vector resembles that of a typical screw. Apply the right-hand rule in describing the direction of the wave number ($\mathbf{k}$) and vector $\mathbf{E}$. The direction of vector $\mathbf{E}$ is shown by curved fingers while the direction of propagation of vector ($\mathbf{k}$) is shown by a thumb. Whistler wavers have the same direction as electrons hence, whistler waves exhibit some resonance at ($\omega = \omega_{ce}$). The waves that are classified as right or left-handed, which are circularly polarized, are known to behave in unique ways during their interactions with moving particles along the magnetic field [11]. It is necessary to thoroughly explore these types of interactions for learning about the process of diffusion, the heating, damping and growth of particles.

1.6 Damping of Plasma Waves

One of plasma physics most beautiful results is the fact that plasma waves suffer damping without any loss of energy arising from collisions among particles. The most common examples of Landau damping and cyclotron damping illustrate this particular phenomenon. Landau was the first person to provide a justification for collision-less damping in the case of electron plasma waves in early 1945. This was experimentally verified in 1960. The concept of wave particle interactions helps offer insight into this damping process at its physical level. It's possible for a strong wave-particle interaction to occur when the particle speeds are such that they closely match the phase velocity of the wave. When the particle's velocity surpasses that of the wave, it will slow down but it will accelerate when the speed of the wave is higher. In the Maxwellian velocity distribution case, the wave loses energy in wave-particle interaction with electrons by transferring its energy to them when many electrons have velocities lower than the wave phase velocity. This decreases the amplitude of the wave, thus causing the wave to damp.

To solve the problem of damping of waves, one will need either complex values of $\mathbf{k}$ or of ω. There are two kinds of damping, which are spatial and temporal damping. Spatial damping is when the damping is known as if $\mathbf{k}$ is complex while ω is real; besides, temporal damping happens if only $\mathbf{k}$ is real with an imaginary component of ω. In the spatial damping process, there is a decrease in the amplitude that happens with increasing distance from where the wave was generated along its path of travel. In steady-state heating we observe spatial damping experimentally. This damping happens when a medium is excited by a continuous driver of fixed frequency. Howveer, temporal damping occurs whenever the medium itself gets spontaneously perturbed and waves amplitude decrease over a propagating wave as time goes by. Temporal damping happens even on the sun atmosphere characterized by wave excitation through spontaneous perturbation [12].

1.7 Plasma Instabilities

In plasma there are many instabilities that exist and it can be defined by "anything that disturbs an almost balanced state of a system; this helps in lowering the free energy of the system thereby allowing it to attain true thermodynamic status". We shall categorize these instabilities into two groups namely the macro instabilities related to configuration or low wave number, and the micro instabilities associated with kinetics or high wave number. The simplest model for these macro instabilities are in terms of fluid models. Examples of macro-instability behavior include Rayleigh-Taylor instability, Kelvin-Helmholtz instability, firehose instability, mirror instability, helical instability, kink instability, and pinch instability. Micro instabilities are created at short wavelengths. Analyzing micro instability problems typically involves utilizing an analytical model such as the Vlasov equation. The Weibel instability, Harris instability, and Whistler instability represent some of the most common examples of micro-instabilities. Dismissal of these instability criteria is because they all depend on the shape of free energy source and velocity distribution function (VDF) in plasma. Various outcomes may result from different sources of free energy. Understanding the nature of the free energy source in plasma is critical for studying the type of instability. Factors include temperature anisotropy as well as streaming of particles with respect to (w.r.t) each other in a magnetic configuration within plasmas which result to free energy source. Similarly, it has been proved that kinetic instability reduces this free energy and scatters particles within the plasma while at the same time preventing any increase caused by temperature anisotropy [13, 14, 15, 16, 17]. In collision-less magnetized plasma, the particle distribution function exhibits anisotropy. Depending on the magnitude and sign of the temperature anisotropy, different types of instabilities appear. For example, when the temperature perpendicular is less than the temperature parallel $\left(T_\perp < T_\parallel\right)$, it's possible to have two instabilities ordinary or fire-hose instability. When the temperature perpendicular to the specified direction exceeds the temperature parallel to it $\left(T_\perp > T_\parallel\right)$, then it gives birth to whistler instability, characterized by a right-handed circularly waves beneath gyro-frequency. While transitioning toward a field free region, this becomes purely Weibel instability.

1.8 Energy Flow

In electromagnetic waves, the Poynting vector provides the energy flux density. What happens to the energy as the wave travels through a particular region is determined by the divergence of Poynting vector [18]. To understand energy transfer rates within a plasma, one needs to understand energy flow. These energy transfers arise from collisions between charged ions and electrons that are enclosed in these plasmas as well interactions with external agents such as electric currents and magnetic fields. Understanding energy flow enables scientists manipulate fabrication processes like plasma etching used to produce microchips or power exchange processes in fusion reactors. When scientists measure the energy flux density at the surface, they can tell how plasma parameters affect processes such as thin film deposition. Moreover, knowledge of the effects of plasma density and external fields on the energy flow makes it possible to devise diagnostic instruments for monitoring and optimizing plasma behavior.

Chapter 2

KINETIC MODEL OF PLASMA

The fluid model is employed to examine plasma, treating it as a fluid entity [3], which is the simplest model applied in studying dynamics of plasma on macroscopic level. In fluid model, individual particles do not have any identity; instead what is considered here is the movement of fluid elements as used to study plasma in this model at macroscopic level. In contrast, kinetic model is used when studying the plasma at the microscopic level. Landau damping and temperature anisotropy are examples of phenomena for which fluid models can no longer be applied. Kinetic model utilizes velocity distribution function. A model that studies plasma phenomena and behavior of each species using velocity distribution function is called kinetic model. Kinetic theory is used to study behavior of each species. Kinetic equations have been derived for solving temporal and spatial estimations. Kinetic model is an alternate framework to understand the properties of plasma in non-equilibrium scenarios [3, 5]. Kinetic model offers most accurate description of plasma under theoretical scenario. In plasma and neutral gases, the particles have random speeds. It is hard to define an exact speed for a particle. The distribution of velocities in the system is best explained by statistical method when it comes to particles moving in plasma. The velocity distribution functions (VDFs) is a demonstration of a statistical approach which is mathematical [19]. A function resembling a distribution function, encompassing seven variables $f(x, y, z; v_x, v_y, v_z; t)$, represents the density of particles within the phase space per unit volume for a single particle [3]. There is large number of particles in plasma. Plasma comprises particles traveling at different velocities. Deriving equations of motion for individual particles is very hard. And that is why the statistical method is applied.

Therefore, we employ the radial velocity distribution function, providing the density of particle's divide by the unit volume in ordinary space or alternatively the density of particles divide by the m^3 near the particular point x and moment t having the components of velocity v_1, v_2 , v_3 to $v_1 + dv_1$, $v_2 + dv_2$, $v_3 + dv_3$. In a mathematical form, it reads;

$$f(x, v, t) = \frac{dN}{d^3x d^3v} \tag{2.1}$$

"d^3x" is the elemental volume of a space, while "d^3v" is the elemental velocity volume. Every particle can be situated at any point in the range from x to $x + dx$ for position, from v to $v + dv$ for velocity.

The expression of the number of particles per unit volume is as.

$$n\left(x, t\right) = \frac{\partial N}{\partial V} = \frac{\partial N}{\partial x^3} \tag{2.2}$$

The density of particles divide by the unit volume is denoted by $n\left(x, t\right)$, ∂N represents the total density of particles, while ∂V denotes the volume. From the equations provided above, we can express it as follows

$$n\left(x, t\right) = \frac{\partial N}{\partial x^3} = \frac{1}{d^3x} \int f\left(x, v, t\right) d^3x d^3v$$

$$n\left(x, t\right) = \int f\left(x, v, t\right) d^3v \tag{2.3}$$

The integration above can be used for any velocities [3, 5].

The Boltzmann Vlasov equation describes the temporal evolution of the distribution function within phase space. The Vlasov equation represents a specific instance of the Boltzmann equation applicable to collision-less scenarios. The expression for the relativistic Vlasov equation is as follows,

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{x}} + \frac{q}{m}(\mathbf{E} \; + \mathbf{v} \times \mathbf{B}).\frac{\partial f}{\partial \mathbf{v}} = 0 \tag{2.4}$$

The electrochemical oscillations produced in plasma were first studied using the Vlasov theory. Kinetic theory can be used to describe instabilities and plasma waves, including the wave-particle interaction such as Landau damping. It was demonstrated on the basis of kinetic theory that the fluid-like behavior of the Langmuir oscillations is determined by the electron density at which these oscillations occur. We can determine such, that the density and velocity of an electron are functions with respect to space and time within the fluid theory. Therefore, between temperature and density we can postulate a relationship in order for us to achieve this; however, this is not necessary according to kinetic theory since the distribution function could be calculated without making any assumptions. This way it becomes easy for us to compute the velocity, temperature and density as they are nothing more than velocity moments of distribution function.

The Maxwells curl equations and the Vlasov equation describe the behavior of electromagnetic waves. These curl equations are follows as

$$\boldsymbol{\nabla} \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t}; \quad \boldsymbol{\nabla} \times \mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} - \frac{\partial \mathbf{E}}{\partial t}\right) \tag{2.5}$$

2.1 Maxwellian Velocity Distribution Function

In plasmas with high temperature and low density, the Maxwellian distribution is employed. It can also be used in systems that are at thermal equilibrium. A gas containing multiple random velocities is known as a thermal equilibrium gas hence the reason why Maxwellian distribution function best fits this type of system. Plasma has several uses in geophysics, space physics, and astrophysics. In these systems, the plasma is said to be in thermal equilibrium. Particles that have higher energie's within the system will get to ''maxwellize" their energies through collision within a particular time frame denoted as relaxation time [3, 5]. An optimized form for this law can be expressed mathematically through the use of equations as stated below.

$$f_{0\beta} = \sqrt{\frac{m_\beta}{2\pi k_B T_\beta}} \exp\left(-\frac{1}{2}\frac{m_\beta v_\beta^2}{k_B T_\beta}\right)$$

or

$$f_{0\beta} = \frac{1}{\pi^{\frac{3}{2}} v_{th\beta}^2} \exp\left(-\frac{v_\beta^2}{v_{th\beta}^2}\right)$$

where β is subscript for species (electrons and ions), and v_{th} denote thermal velocity.

2.2 Bi-Maxwellian Velocity Distribution Function

In case of temperature differences in plasma, the Maxwellian distribution function gives way to the Bi-Maxwellian type. This function has two velocity components, $v_{th\|\beta}$ and $v_{th\perp\beta}$ component arising from the presence of two temperatures in different directions.

The velocity component direction is in diection of constant magnetic field. Space plasma contains an abundance of bi-Maxwellian distribution. Cosmic plasma parameters are similar to those of solar wind plasma due to near packed condensed objects in accretion disks, interstellar medium within interclusters of about heavy particles [20, 21]. It has been observed that ion distribution in solar wind closely resembles an electron distribution pattern namely bi-Maxwellian distribution which has two distinct temperature for two distinct directions. This can be mathematically denoted as:

$$f_{0\beta} = \frac{1}{\pi v_{th\perp\beta}^2 \sqrt{\pi v_{th\|\beta}}} \exp\left(-\frac{v_{\perp\beta}^2}{v_{th\perp\beta}^2} - \frac{v_{\|\beta}^2}{v_{th\|\beta}^2} \right)$$

or

$$f_{0\beta} = \frac{1}{\pi^{\frac{3}{2}} v_{th\perp\beta}^2 v_{th\|\beta}} \exp\left(-\frac{v_{\perp\beta}^2}{v_{th\perp\beta}^2} - \frac{v_{\|\beta}^2}{v_{th\|\beta}^2} \right)$$

where

$$v_{th\perp\beta}^2 = \frac{2T_{\perp\beta}}{m_\beta}$$

$$v_{th\|\beta}^2 = \frac{2T_{\|\beta}}{m_\beta}$$

$v_{th\perp\beta}$ and $v_{th\|\beta}$ are the perpendicular and parallel thermal velocity components, while m_β denotes the mass of the particle.

In the context of momentum, the bi-Maxwellian distribution function is expressed as

$$f_{0\beta} = \frac{1}{2\pi m_\beta T_{\perp\beta}\left(2\pi m_\beta T_{\parallel\beta}\right)^{\frac{1}{2}}} \exp\left(-\frac{p_{\perp\beta}^2}{2m_\beta T_{\perp\beta}} - \frac{p_{\parallel\beta}^2}{2m_\beta T_{\parallel\beta}}\right)$$

2.3 Non-Maxwellian Velocity Distribution Function

When particles have very high energy, we can describe the particle distribution better using the velocity power law than with an exponential decay like in Maxwellian distribution. In systems where particles possess high energy levels, the distribution ceases to be Maxwellian. Non-Maxwellian distributions come in handy for such systems. The plasma particles do not possess thermal equilibrium under these distributions. The distribution functions which violate Maxwellian theory are caused by the interaction of waves with particles. The suprathermal radiation field contains more than its fair share of thermal particles. Others are found in galactic comic rays, solar flares, and the solar wind too in outer space. Whenever shock waves free of any collisions pass through an area for a moment there is no balance involving heat anymore due to these waves penetrating into a collisionless state. Some forms of such distributions are: bi-Kappa velocity distribution function, kappa Maxwellian velocity distribution function, loss cone kappa velocity distribution function, and product bi-Kappa velocity distribution function.

In this thesis, we utilized a bi-Kappa velocity distribution function, thus we provide a concise overview of this function below.

2.3.1 Streaming Bi-Kappa Velocity Distribution Function

The streaming bi-Kappa velocity distribution function also known as the bi-Lorentzian velocity distribution function, can be mathematically expressed as:

$$f_{0\beta} = \frac{n_{0\beta}\Gamma\left(\kappa+1\right)}{\pi^{\frac{3}{2}}\theta_{\perp\beta}^2\theta_{\parallel\beta}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\left[1-\frac{v_{\perp\beta}^2}{k\theta_{\perp\beta}^2}+\frac{\left(v_{\parallel\beta}-v_0\right)^2}{k\theta_{\parallel\beta}^2}\right]^{-\kappa-1}$$

where

$$\theta_{(\perp,\parallel)\beta}^2 = \left(\frac{2\kappa-3}{\kappa}\right)v_{th_{(\perp,\parallel)\beta}}^2 \; ; \; v_{th_{(\perp,\parallel)\beta}} = \sqrt{\frac{T_{(\perp,\parallel)\beta}}{m_\beta}}$$

Spectral index 'κ' serves as the descriptor of particle kappa distribution. It shows the gradient of those high-speed tails that are seen in the velocities distribution of plasma particles beyond the thermal speed. For values approaching critical limit ($\kappa \rightarrow \infty$), plasma particles' distribution tends Maxwellian alike. Provided kappa number is $\kappa > \frac{3}{2}$, this prevents distribution from breaking and allowing to define an equivalent temperature or otherwise the aforementioned parameters would be meaningless. It was said before that one uses Maxwellian velocity distribution function for such a system which is in thermal equilibrium. The specifics of this dependency are responsible for physical properties such as fluctuations in temperature or magnetic field strengths existing within plasma created under laboratory conditions or around stars, respectively. The former case includes power law models whereas latter type involves exponential forms also known as bi-Maxwellians they describe low-energy/thermalized particles within those energetic/intermediate ranges above (high energy). These kind of distributions occur due to particle interactions . A bi-Maxwellian distribution function is able to well describing these non-Maxwellian high energy or such thermaly suprathermal tails. This is an interactional result of particles [22, 23]. Such distributions are frequently observed in plasmas characterized by infrequent collisions among charged particles and low plasma density. For example, kappa distribution exists when $6 > \kappa > 2$ found using satellite data in the solar wind [24, 25]. Foreshocks of the Earth are best fitted by distributions with $6 > \kappa > 3$ [26, 27, 28]. Kappa index spectral of $10 > \kappa > 5$ are the optimal match for the solar corona [29].

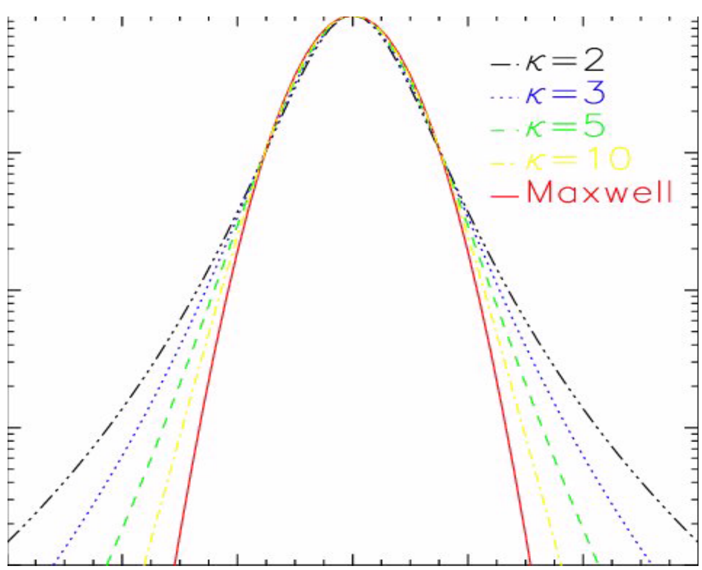

Figure 2.1: Kappa velocity distribution function for various values of the kappa parameters.

2.4 Derivation of Generalized Dielectric Tensor

By using relativistic vlasov equation

$$\frac{\partial f}{\partial t} + \mathbf{v}.\frac{\partial f}{\partial \mathbf{x}} + q.(\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c}).\frac{\partial f}{\partial \mathbf{p}} = 0 \tag{2.6}$$

The relationship between relativistic momentum $\mathbf{p}$ and velocity $\mathbf{v}$ is expressed as follows

$$\begin{aligned} \mathbf{p} &= \gamma m \mathbf{v} \qquad \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} = \sqrt{1 + \frac{\mathbf{p}^2}{m^2 c^2}} \\ \mathbf{v} &= \frac{c\mathbf{p}}{\sqrt{m^2 c^2 + \mathbf{p}^2}} \end{aligned} \tag{2.7}$$

The relativistic vlasov equation accompanied by the Maxwell equations, explain complete dynamics of the plasma system.

Faraday's law

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \tag{2.8}$$

Ampere's law

$$\nabla \times \mathbf{B} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{J} \tag{2.8}$$

By linearization we get,

$$\begin{aligned} (\mathbf{E}, \mathbf{B},\ f) &= \int_0^\infty dt\ e^{-ut} \int_{-\infty}^{\infty} \frac{1}{(2\pi)^3} e^{-i\ \mathbf{k}\ .\ \mathbf{x}}.dx\ \ (\mathbf{E}_1, \mathbf{B}_1, f_1)\,, \\ \mathcal{L}\left(\frac{\partial\,(\mathbf{E}_1, \mathbf{B}_1, f_1)}{\partial t}\right) &= u\mathcal{L}\,(\mathbf{E}_1, \mathbf{B}_1,\ f_1) \end{aligned}$$

and

$$\mathcal{F}\left(\frac{\partial\left(\mathbf{E}_1,\mathbf{B}_1,\ f_1\right)}{\partial t}\right)=i\ \mathbf{k}\ \mathcal{F}\left(\mathbf{E}_1,\mathbf{B}_1,\ f_1\right)\ ;\ u=-i\omega$$

the Vlasov Eq (2.6) and the equation of wave from Maxwell's curl Eqs (2.7, 2.8) become

$$(u+i\ \mathbf{k}.\mathbf{v})\ \ f_1+\frac{q}{c}(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial f_1}{\partial\mathbf{p}}+q(\mathbf{E}_1-\frac{i}{u}\mathbf{v}\times(\mathbf{k}\times\mathbf{E}_1)\cdot\frac{\partial f_0}{\partial\mathbf{p}}=0 \tag{2.9}$$

and

$$\left[\left(u^2+c^2.k^2\right).\delta mn\ -\ c^2k_mk_n+4\pi u\sigma_{mn}\right]E_n=0$$

or

$$\left[\left(\delta_{mn}+\frac{4\pi}{u}\sigma_{mn}\ \right)+\frac{c^2k^2}{u^2}-\ \frac{ck_m}{u}\frac{ck_n}{u}+\right]E_n=0$$

or

$$\left[\gamma_{mn}-N^2\left(\delta_{mn}-\ \frac{N_mN_n}{N^2}\right)\right]E_n=0 \tag{2.10}$$

Where $\gamma_{mn}\equiv\delta_{mn}+\frac{4\pi}{u}\sigma_{mn}$ represents the dielectric tensor, $N_{m,\ n}=\dfrac{ck_{m,\ n}}{\omega}$ is the total refractive index, and $mth\ /nth$ subscript indicates the respective refractive index.

Given that the wave number **k** is in the x-z plane, by changing the differential equation (2.9) using cylindrical coordinates

$$\frac{\partial f_1}{\partial\varphi}-\frac{\left(u+ik_\parallel v_\parallel+ik_\perp v_\perp\cos\varphi\right)}{\Omega}f_1=\frac{\Psi(\varphi)}{\Omega} \tag{2.11}$$

$$\frac{q}{c}(\mathbf{v}\times\mathbf{B}_0).\frac{\partial f_1}{\partial\mathbf{p}}=-\Omega\frac{\partial f_1}{\partial\varphi}$$

Also

$$\Psi(\varphi)=q\left(\mathbf{E}_1-\frac{i}{u}\mathbf{v}\times(\mathbf{k}\times\mathbf{E}_1)\right)\cdot\frac{\partial f_0}{\partial\mathbf{p}}$$

And

$$\Omega=\frac{q\mathbf{B}_0}{\gamma m\ c}=\frac{\Omega_0}{\gamma}$$

Vlasov equation, in equilibrium as

$$\frac{q}{c}(\mathbf{v}\times\mathbf{B}_0)\cdot\frac{\partial f_0}{\partial \mathbf{p}}=0$$

gives

$$\frac{\partial f_0}{\partial \varphi}=0 \ \text{ i.e., } f_0 \text{ depends on } \mathbf{p}_\perp \ \text{ and } \mathbf{p}_\parallel \text{ but independent of } \varphi$$

Alternatively, it is possible to express $\Psi(\varphi)$ is

$$\begin{aligned}
\Psi(\varphi') &= q\left\{\mathbf{E}_1-\frac{i}{u}\,\mathbf{v}\times(\mathbf{k}\times\mathbf{E}_1)\right\}\cdot\frac{\partial f_0}{\partial \mathbf{p}'} \qquad \text{, where } \mathbf{p}'(p_\perp,\varphi',p_\parallel)\\
&= q\left[\mathbf{E}_1\cdot\frac{\partial f_0}{\partial \mathbf{p}'}-\frac{i}{u}\left\{\left(\mathbf{k}\cdot\frac{\partial f_0}{\partial \mathbf{p}'}\right)(\mathbf{v}\cdot\mathbf{E}_1)-\left(\mathbf{E}_1\cdot\frac{\partial f_0}{\partial \mathbf{p}'}\right)(\mathbf{v}\cdot\mathbf{k})\right\}\right]\\
&= q\left[\mathbf{E}_1\cdot\frac{\partial f_0}{\partial \mathbf{p}'}\left(1+\frac{i}{u}(\mathbf{k}\cdot\mathbf{v})\right)-\frac{i}{u}\left(\mathbf{k}\ \cdot\frac{\partial f_0}{\partial \mathbf{p}'}\right)(\mathbf{v}\cdot\mathbf{E}_1)\right]\\
&= q\left[\left\{1+\frac{i}{u}\left(k_\perp v_\perp\cos\varphi'+k_\parallel v_\parallel\right)\right\}\left\{\left(E_{x1}\cos\varphi'+E_{y1}\sin\varphi'\right)\frac{\partial f_0}{\partial p_\perp}+E_{z1}\frac{\partial f_0}{\partial p_\parallel}\right\}\right.\\
&\quad\left.-\frac{i}{u}\left(k_\perp\cos\varphi'\frac{\partial f_0}{\partial p_\perp}+k_\parallel\frac{\partial f_0}{\partial p_\parallel}\right)\left\{\left(E_{x1}\cos\varphi'+E_{y1}\sin\varphi'\right)v_\perp+E_{z1}v_\parallel\right\}\right]\\
&= q\left[\frac{\partial f_0}{\partial p_\perp}+\frac{i}{u}k_\parallel\left\{v_\parallel\frac{\partial f_0}{\partial p_\perp}-v_\perp\frac{\partial f_0}{\partial p_\parallel}\right\}\right]E_{x1}\cos\varphi'\\
&\quad+q\left[\frac{\partial f_0}{\partial p_\perp}+\frac{i}{u}k_\parallel\left\{v_\parallel\frac{\partial f_0}{\partial p_\perp}-v_\perp\frac{\partial f_0}{\partial p_\parallel}\right\}\right]E_{y1}\sin\varphi'\\
&\quad+q\left[\frac{\partial f_0}{\partial p_\parallel}-\frac{i}{u}k_\perp\cos\varphi'\left\{v_\parallel\frac{\partial f_0}{\partial p_\perp}-v_\perp\frac{\partial f_0}{\partial p_\parallel}\right\}\right]E_{z1}
\end{aligned}$$

or

$$\Psi(\varphi')=q(\zeta_1.\sin\varphi',\zeta_1.\cos\varphi',\zeta_2+\cos\varphi_3'\zeta).\mathbf{E}_1$$

Where

$$\begin{aligned}
\zeta_1 &= \left[\frac{\partial f_0}{\partial p_\perp}+\frac{ik_\parallel}{u}\left(v_\parallel\frac{\partial f_0}{\partial p_\perp}-v_\perp\frac{\partial f_0}{\partial p_\parallel}\right)\right]\\
\zeta_2 &= \frac{\partial f_0}{\partial p_\parallel} \quad \text{and} \quad \zeta_3=-\frac{ik_\perp}{u}\left(v_\parallel\frac{\partial f_0}{\partial p_\perp}-v_\perp\frac{\partial f_0}{\partial p_\parallel}\right)
\end{aligned}$$

for finding the solution of in-homogeneous differential Eq. (2.6), First, we solve the homogeneous part of this equation i.e.,

$$\frac{\partial f_1}{\partial \varphi} - \left(\frac{u + ik_{\|}.v_{\|} + ik_{\perp} v_{\perp} \cos \varphi}{\Omega} \right) f_1 = 0$$

we get the final form is

$$\begin{aligned} H(\varphi, \varphi') &= \exp \left[\frac{1}{\Omega} \int_{\varphi'}^{\varphi} \left(u + ik_{\|} v_{\|} + ik_{\perp} v_{\perp} \cos \varphi \right) d\varphi'' \right] \\ &= \exp \left[\frac{1}{\Omega} \left\{ (u + ik_{\|} v_{\|})(\varphi - \varphi^{'}) + ik_{\perp} v_{\perp} (\sin \varphi - \sin \varphi^{'}) \right\} \right] \end{aligned}$$

We get the solution of Eq. (2.11) by using integration by factor technique as

$$f_1 = \frac{1}{\Omega} \int_{\pm\infty}^{\varphi} H(\varphi, \varphi') \Phi(\varphi') d\varphi' \tag{2.12}$$

For the function $H(\varphi, \varphi')$ to converge, the choice of sign at the lower bound is essential; it is therefore desirable to have a $-\infty$ limit when Ω is positive and a $+\infty$ limit when Ω is negative.

Alternatively, it is possible to express the above Eq.(2.12) as

$$\begin{aligned} f_1 &= \frac{q}{\Omega} \int_{\pm\infty}^{\varphi} \exp \left[\left(\frac{u + ik_{\|} v_{\|}}{\Omega} \right) (\varphi - \varphi^{'}) + i \frac{k_{\perp} v_{\perp}}{\Omega} (\sin \varphi - \sin \varphi^{'}) \right] \times \\ & \times (\zeta_1 \cos \varphi', \zeta_1 \sin \varphi', \zeta_2 + \cos \varphi_3' \zeta) d\varphi^{'} . E_1 \end{aligned} \tag{2.13}$$

Now, let's express the current density function using the perturbed distribution function.

$$\mathbf{J} = \overleftrightarrow{\boldsymbol{\sigma}} . \mathbf{E}_1 = \sum q n_0 \int \mathbf{v} . f_1 d^3 \mathbf{p}$$

using above eq in equation. (2.10) as

$$
\begin{aligned}
4\pi u \overleftrightarrow{\boldsymbol{\sigma}}.\mathbf{E}_1 &= 4\pi u \sum q^2 n_0 \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} dp_\perp p_\perp \int_0^{2\pi} d\varphi \int_{\infty}^{\varphi} d\varphi' \\
&\times \frac{1}{\Omega} \exp\left[\frac{1}{\Omega}\left\{\left(u + ik_\parallel v_\parallel\right)\left(\varphi - \varphi'\right) + ik_\perp v_\perp \left(\sin\,\varphi - \sin\,\varphi'\right)\right\}\right] \times \\
&\left(v_\perp \cos\,\varphi,\ v_\perp \sin\,\varphi,\ v_\parallel\right) \times \left(\cos\,\varphi_1' \zeta_1,\ \sin\,\varphi'\zeta_1,\ \zeta_2 + \cos\,\varphi'\zeta_3\right).E_1 \quad (2.14)
\end{aligned}
$$

We introduce new variable for $\theta = \varphi - \varphi'$ as an alternative of φ' and then by performing φ-integration first we have.

$$
\int_0^{2\pi} d\varphi \exp[i.(\sin\varphi - \sin(\varphi - \theta))z]
$$

$$
\times \begin{pmatrix} v_\perp \cos\varphi \cos(\varphi-\theta)\zeta_1 & v_\perp \cos\varphi \sin(\varphi-\theta)\zeta_1 & v_\perp \cos\varphi\,(\zeta_2 + \cos(\varphi-\theta)\zeta_3) \\ v_\perp \sin\varphi \cos(\varphi-\theta)\zeta_1 & v_\perp \sin\varphi \sin(\varphi-\theta)\zeta_1 & v_\perp \sin\varphi\,(\zeta_2 + \cos(\varphi-\theta)\zeta_3) \\ v_\parallel \cos(\varphi-\theta)\zeta_1 & v_\parallel \sin(\varphi-\theta)\zeta_1 & v_\parallel(\zeta_2 + \cos(\varphi-\theta)\zeta_3) \end{pmatrix}
$$

Hence

$$
= 2\pi \sum_{j=-\infty}^{\infty} \exp[ij\theta] \times \tag{2.15}
$$

$$
\times \begin{pmatrix} \frac{j^2}{z^2} v_\perp [J_j(z)]^2 \zeta_1 & \frac{ij}{z} v_\perp J_j(z) J'_j(z)\zeta_1 & v_\perp \frac{j}{z}[J_j(z)]^2 \left(\zeta_2 + \frac{j}{z}\zeta_3\right) \\ -\frac{ij}{z} v_\perp J_j(z) J'_j(z)\zeta_1 & v_\perp [J'_j(z)]^2 \zeta_1 & -iv_\perp J_j(z) J'_j(z) \left(\zeta_2 + \frac{j}{z}\zeta_3\right) \\ \frac{j}{z} v_\parallel [J_j(z)]^2 \zeta_1 & iv_\parallel J_j(z) J'_j(z)\zeta_1 & v_\parallel [J_j(z)]^2 \left(\zeta_2 + \frac{j}{z}\zeta_3\right) \end{pmatrix}
$$

where we utilized the Bessel's function identities.

$$
\exp[i\sin\varphi.z] = \sum_{j=-\infty}^{\infty} \exp[ij\varphi] J_j(z) \ \text{ and } \ \exp[-\,i\sin(\varphi-\theta).z] = \sum_{i=-\infty}^{\infty} \exp[-ii(\varphi-\theta)] J_i(z)
$$

with $z = \frac{k_\perp v_\perp}{\Omega}$.

We now carry out the θ integration

$$\int_0^{-\infty} d\theta \exp\left[\frac{1}{\Omega}(u + ik_\parallel v_\parallel + ij\Omega)\theta\right] = \frac{-\Omega}{u + ik_\parallel v_\parallel + ij\Omega} \tag{2.16}$$

The equation (2.10) can be rewritten using the result of θ and φ integrations from Eq.(2.14).

$$\left[\gamma_{mn} - N^2\left(\delta_{mn} - \frac{NmNn}{N^2}\right)\right] E_n = 0 \tag{2.17}$$

Where γ_{mn} is dielectric tensor and write as

$$\begin{aligned}\gamma_{mn} &= \delta_{mn} - \frac{2\pi}{u}\sum m\,\omega_p^2 \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} p_\perp dp_\perp \sum_{j=-\infty}^{\infty} \frac{1}{u + ik_\parallel v_\parallel + ij\Omega} \times \\ &\begin{pmatrix} \frac{j^2}{z^2} v_\perp \left[J_j(z)\right]^2 \zeta_1 & \frac{ij}{z} v_\perp J_j(z) J'_j(z)\zeta_1 & v_\perp \frac{j}{z}[J_j(z)]^2\left(\zeta_2 + \frac{j}{z}\zeta_3\right) \\ -\frac{ij}{z} v_\perp J_j(z) J'_j(z)\zeta_1 & v_\perp [J'_j(z)]^2\zeta_1 & -iv_\perp J_j(z) J'_j(z)\left(\zeta_2 + \frac{j}{z}\zeta_3\right) \\ \frac{j}{z}[J_j(z)]^2 v_\parallel \zeta_1 & iJ_j(z)J'_j(z) v_\parallel \zeta_1 & v_\parallel [J_j(z)]^2\left(\zeta_2 + \frac{j}{z}\zeta_3\right)\end{pmatrix}\end{aligned} \tag{2.18}$$

Further we can note the next relationship in order to simplify the tensor γ_{mn}

$$\frac{v_\perp\left(\zeta_2 + \frac{j}{z}\zeta_3\right)}{u + ik_\parallel.v_\parallel + ij\Omega} = \frac{v_\parallel \zeta_1}{u + ik_\parallel.v_\parallel + ij\Omega} - \frac{1}{u}\left(v_\parallel \frac{\partial f_0}{\partial p_\perp} - v_\perp \frac{\partial f_0}{\partial p_\parallel}\right)$$

And using the Bessel's function properties

$$\sum_{n=-\infty}^{\infty} n[J_n(z)]^2 = 0, \quad \sum_{n=-\infty}^{\infty} J_n(z)J'_n(z) = 0, \quad \sum_{n=-\infty}^{\infty} [J_n(z)]^2 = 1$$

After performing the steps above, we have Eq. (2.17) as:

$$\gamma_{mn} = \delta_{mn} - \frac{2\pi}{u}\sum m\,\omega_p^2 \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} p_\perp dp_\perp \zeta_1 \sum_{j=-\infty}^{\infty} \frac{M_{mn}}{u + ik_\parallel v_\parallel + ij\Omega} + L_{mn} \tag{2.19}$$

where

$$L_{mn} = \begin{pmatrix} \frac{j^2}{z^2} v_\perp \left[J_j(z)\right]^2 & \frac{i.j}{z} v_\perp J_j(z) J'_j(z) & \frac{j}{z} v_\parallel [_j(z)]^2 \\ -\frac{ij}{z} v_\perp J_j(z) J'_j(z) & v_\perp [J'_j(z)]^2 & -iv_\parallel J_j(z) J'_j(z) \\ \frac{j}{z} v_\parallel [J_j(z)]^2 & iv_\parallel J_j(z) J'_j(z) & \frac{v_\parallel^2}{v_\perp} [J_j(z)]^2 \end{pmatrix}$$

L_{mn} has only non-zero component

$$L_{zz} = \frac{2\pi}{u^2} \sum m\,\omega_p^2 \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} p_\perp dp_\perp \left[\frac{v_\parallel}{v_\perp} \left(v_\parallel \frac{\partial f_0}{\partial p_\perp} - v_\perp \frac{\partial f_0}{\partial p_\parallel} \right) \right]$$

We also noted here

$$\zeta_1 = \left\{ \frac{\partial f_0}{\partial p_\perp} + \frac{i\,k_\parallel}{u} \left(v_\parallel \frac{\partial f_0}{\partial p_\perp} - v_\perp \frac{\partial f_0}{\partial p_\parallel} \right) \right\}$$

Equation (2.21) conveys the generalized dielectric tensor in a plasma that is uniform, on the condition that the corresponding equilibrium distribution function f_0 can be arbitrary. A striking characteristic of the former lies in its Onsager symmetry; the symmetry properties are always fulfilled by the dielectric tensor of hot plasma.

$$\gamma_{xy} = -\gamma_{xy}, \quad \gamma_{xz} = \gamma_{zx} \quad \text{and } \gamma_{yz} = -\gamma_{zy}$$

Assuming the equilibrium distribution function f_0 is held constant.

We will derive the generalized dielectric tensor for a bi-Maxwellian plasma that is not fast-moving. The distribution function for a bi-Maxwellian plasma that is not fast-moving is

$$f_0 = \frac{1}{2\pi m T_\perp \left(2\pi m T_\parallel\right)^{\frac{1}{2}}} \exp\left[-\frac{p_\perp^2}{2mT_\perp} - \frac{p_\parallel^2}{2mT_\parallel} \right] \tag{2.20}$$

The generalized dielectric tensor in equation (2.17) changes to the above Eq.(2.19)

$$\gamma_{mn} = \delta_{mn} - 4\pi \sum \frac{\omega_p^2}{\omega} \int_{-\infty}^{\infty} dp_\parallel \int_0^{\infty} p_\perp dp_\perp \sum_{j=-\infty}^{\infty} M_{mn} \frac{\left\{ 1 - \frac{k_\parallel v_\parallel}{\omega} \left(1 - \frac{T_\perp}{T_\parallel} \right) \right\} f_0}{\omega - k_\parallel v_\parallel - j\Omega} + L_{mn} \tag{2.21}$$

where

$$L_{mn} = \begin{pmatrix} \frac{j^2\Omega^2}{k_\perp^2 v_{th\perp}^2}[J_j(z)]^2 & \frac{ij\Omega}{k_\perp v_{th\perp}^2} v_\perp J_j(z) J'_j(z) & \frac{j\Omega}{k_\perp v_{th\perp}^2} v_\parallel [J_j(z)]^2 \\ -\frac{ij\Omega}{k_\perp v_{th\perp}^2} v_\perp J_j(z) J'_j(z) & \frac{v_\perp^2}{v_{th\perp}^2}[J'_j(z)]^2 & -i\frac{v_\perp v_\parallel}{v_{th\perp}^2} J_j(z) J'_j(z) \\ \frac{j\Omega}{k_\perp v_{th\perp}^2} v_\parallel [J_j(z)]^2 & i\frac{v_\perp v_\parallel}{v_{th\perp}^2} J_j(z) J'_j(z) & \frac{v_\parallel^2}{v_{th\perp}^2}[J_j(z)]^2 \end{pmatrix}$$

and

$$L_{zz} = \frac{1}{\omega^2} 4\pi \sum \frac{\omega_p^2}{v_{th\perp}^2} \left(1 - \frac{T_\perp}{T_\parallel}\right) \int_{-\infty}^{\infty} dp_\parallel v_\parallel^2 \int_0^{\infty} dp_\perp p_\perp f_0$$

At the equation above (2.22), we used $u = -i\omega$. The fact is, for a non relativistic bi-maxwellian distribution, parallel and perpendicular integrations are decoupled. So, in the sequel, we will handle them separately.

2.4.1 $p_\parallel$ Integration

For parallel transport $k_\perp = 0$, by taking limit of $k_\perp \to 0$ in the previous equation. And in this limit $\frac{1}{z}[J_j(z)]^2 \to 0$ and $J_j(z) J'_j(z) \to 0$ due to $J_j(z) \to \frac{1}{j!}(\frac{z}{2})^j$ additionally $J_j(z) = (-1)^j$ $J_j(z)$. Thus the γ_{mn} matrix would change such that xz and yz components will be zero making it 2×2 matrix then gives R-L waves while zz component results into electrostatic modes i.e. Langmuir and ion-Acoustic modes. In order to perform $p_\parallel$- integration, we will need these integrals

$$(A_1, A_2, A_3, A_4) \equiv \frac{1}{\sqrt{2\pi m T_\parallel}} \int_{-\infty}^{\infty} \frac{dp_\parallel \, \exp\left[-\frac{p_\parallel^2}{2mT_\parallel}\right]}{\omega - k_\parallel v_\parallel - j\Omega} \left(1,\, v_\parallel,\, v_\parallel^2,\, v_\parallel^3\right)$$

Presenting the familiar plasma dispersion function

$$Z(x) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} dU \frac{e^{-U^2}}{(U - x)}$$

we get

$$(A_1, A_2, A_3, A_4) \;=\; \left(\frac{-Z\left(\xi_j\right)}{k_\| v_{th\|}}, \quad \frac{Z'\left(\xi_j\right)}{2k_\|}, \quad \frac{v_{th\|}\, \xi_j Z'\left(\xi_j\right)}{2k_\|}, -\frac{v_{th\|}}{2k_\|} \left[1 - \xi_j Z'\left(\xi_j\right)\right] \right)$$

where we substitute

$$U^2 = \frac{p_\|^2}{2mT_\|} \quad \text{and defined} \quad v_{th\|} = \sqrt{\frac{2T_\|}{m}}, \quad \xi_j = \frac{\omega \; - \; j\Omega}{k_\| v_{th\|}}$$

We also find the A_5 integral as

$$A_5 \equiv \int_{-\infty}^{\infty} dp_\| \frac{v_\|^2 \exp[-\frac{p_\|^2}{2mT_\|}]}{\sqrt{2\pi m T_\|}} = \frac{v_{th\|}^2}{2}$$

2.4.2 $p_\perp$ Integration

Similarly, if perturbed wave propagates perpendicular to magnetic field $k_\| = 0$, then integration of $p_\|$ leads to zero for γ_{xz} and γ_{yz} for the same components. In this case, the 2×2 matrix gives the x-mode and the O-mode results from zz component. In order to perform $p_\perp$- integration, we will need these integrals

$$\begin{aligned}
(A_6, A_7, A_8) \;&\equiv\; \frac{1}{2\pi}\frac{1}{mT_\perp}\int_0^\infty dp_\perp p_\perp \exp[-\frac{p_\perp^2}{2mT_\perp}] \left([J_j(z)]^2, \quad p_\perp J_j(z) J_j^{'}(z), \; p_\perp^2 [J_j^{'}(z)]^2\right) \\
&=\; \frac{1}{2\pi}\exp[-\lambda] \Bigg(I(\lambda), m v_{th\perp}\sqrt{\frac{\lambda}{2}}\left(I_j^{'}(\lambda) - I_j(\lambda)\right), \\
&\qquad \frac{m^2 v_{th\perp}^2}{2\lambda}\left\{ j^2 I_j(\lambda) - 2\lambda^2 \left(I_j^{'}(\lambda) - I_j(\lambda)\right)\right\}\Bigg)
\end{aligned}$$

where

$$\lambda = \frac{k_\perp^2 v_{th\perp}^2}{2\Omega_0^2}$$

and

$$A_9 \equiv \frac{1}{2\pi m T_\perp}\int_0^\infty p_\perp dp_\perp \;\exp\left[-\frac{p_\perp^2}{2mT_\perp}\right] = \frac{1}{2\pi}$$

The above integrations as well as Onsager symmetry relationships were used to arrive at this result along with the condition that wave number vector $\overrightarrow{k}$ lies on x-z plane. A magnetized non relativistic bi-Maxwellian plasma the generalized dispersion relation reads

$$\begin{vmatrix} \gamma_{xx} - N_{\parallel}^2 & \gamma_{yx} & \gamma_{xz} + N_{\perp}N_{\parallel} \\ -\gamma_{xy} & \gamma_{yy} - N^2 & \gamma_{yz} \\ \gamma_{xz} + N_{\perp}N_{\parallel} & -\gamma_{yz} & \gamma_{zz} - N_{\perp}^2 \end{vmatrix} = 0 \tag{2.22}$$

where

$$\gamma_{xx} = 1 + \sum \frac{\omega_p^2}{\omega^2} \sum_{j=-\infty}^{\infty} \frac{j^2}{\lambda} \Gamma_j(\lambda) \left\{ \xi_0 \, Z(\xi_j) - \frac{Z'(\xi_j)}{2} \left(\frac{T_{\perp}}{T_{\parallel}} - 1 \right) \right\} \tag{2.23}$$

$$\gamma_{yy} = 1 + \sum \frac{\omega_p^2}{\omega^2} \sum_{j=-\infty}^{\infty} \left(\frac{j^2 \Gamma_j(\lambda)}{\lambda} - 2\lambda \, \Gamma_j'(\lambda) \right) \left\{ \xi_0 \, Z(\xi_j) - \frac{Z'(\xi_j)}{2} \left(\frac{T_{\perp}}{T_{\parallel}} - 1 \right) \right\} \tag{2.24}$$

$$\gamma_{zz} = 1 - \sum \frac{\omega_p^2}{\omega^2} \xi_0 \sum_{j=-\infty}^{\infty} \Gamma_j(\lambda) \xi_j Z'(\xi_j) \left\{ 1 + \frac{j\Omega}{\omega} \left(\frac{T_{\parallel}}{T_{\perp}} - 1 \right) \right\} \tag{2.25}$$

$$\gamma_{xy} = i \sum \frac{\omega_p^2}{\omega^2} \sum_{j=-\infty}^{\infty} j \Gamma_j'(\lambda) \left\{ \xi_0 \, Z(\xi_j) - \frac{Z'(\xi_j)}{2} \left(\frac{T_{\perp}}{T_{\parallel}} - 1 \right) \right\} \tag{2.26}$$

$$\gamma_{xz} = \sum \frac{\omega_p^2}{\omega^2} \frac{v_{th\parallel}}{v_{th_{\perp}}} \sum_{j=-\infty}^{\infty} \frac{j \Gamma_j(\lambda)}{\sqrt{2\lambda}} \left\{ \xi_0 + \xi_j \left(\frac{T_{\perp}}{T_{\parallel}} - 1 \right) \right\} Z'(\xi_j) \tag{2.27}$$

$$\gamma_{yz} = i \sum \frac{v_{th\parallel}}{v_{th_{\perp}}} \frac{\omega_p^2}{\omega^2} \sum_{j=-\infty}^{\infty} \sqrt{\frac{\lambda}{2}} \, \Gamma_j'(\lambda) \left\{ \xi_0 + \xi_j \left(\frac{T_{\perp}}{T_{\parallel}} - 1 \right) \right\} Z'(\xi_j) \tag{2.28}$$

The equations mentioned above represent the components of the dielectric tensor, with each component corresponding to different types of waves.

Derive the general dispersion relation for right handed circularly polarized EM waves for two scenarios: $\xi_j \ll 1$ and $\xi_j \gg 1$. For the case $\xi_j \ll 1$ the dispersion relation will take the form.

$$Z(\xi_j) = i\sqrt{\pi}e^{-\xi_j^2} - \frac{1}{\xi_j} - \frac{1}{2\xi_j^3} - \frac{3}{4\xi_j^5} + \cdots = -\sum_{l=0}^{\infty} \frac{(2l+1)!!}{(2l+1)\ 2^l} \left(\frac{k_\| v_{th_\|}}{\omega - j\Omega} \right)^{2l+1} \text{ for } \xi_j \gg 1$$

and

$$Z(\xi_j) = i\sqrt{\pi}e^{-\xi_j^2} - 2\xi_j \left(1 - \frac{2\xi_j^2}{3} + \frac{4\xi_j^4}{15} + ... \right) = i\sqrt{\pi} + \sum_{l=0}^{\infty} \frac{(-2)^{l+1}}{(2l+1)!!} \left(\frac{\omega - j\Omega}{k_\| v_{th\|}} \right)^{2l+1} \text{ for } \xi_j \ll 1$$

Chapter 3

MATHEMATICAL WORK

3.1 Dispersion Relation of Parallel Electromagnetic Waves (PEMW) by Using Kinetic Model

The disperion relation for PEMW in magnetized plasmas is,

$$\omega^2 = c^2k^2 - \pi\omega\omega_{pe}^2 \int_{-\infty}^{\infty} dv_{\|} \int_0^{\infty} v_{\perp}^2 dv_{\perp} \times \left[\frac{\left(\frac{\partial f_0}{\partial v_{\perp}} - \frac{k}{\omega} \left(v_{\|} \frac{\partial f_0}{\partial v_{\perp}} - v_{\perp} \frac{\partial f_0}{\partial v_{\|}} \right) \right)}{(\omega - k_{\|} v_{\|} - \omega_{ce})} \right] \tag{3.1}$$

Where $k_{\|}$ represents the wave-number, ω represents wave frequency, ω_{ce} is the electron gyro-frequency, f_0 is the arbitrary function and $v_{\perp}$or $v_{\|}$ is the perpendicular and parallel velocity of the electron to the unperturbed magnetic field.

We are using kinetic approach in this research article. We suppose that a streaming velocity is present and moving parallel to the EM wave. Here we used the temperature anisotropic streaming bi-Kappa velocity distribution function expressed as,

$$f_0 = \frac{n_0 \Gamma(\kappa + 1)}{\pi^{\frac{3}{2}} \theta_{\perp}^2 \theta_{\|} \Gamma\left(\kappa - \frac{1}{2}\right) \kappa^{\frac{3}{2}}} \left[1 - \frac{v_{\perp}^2}{\kappa \theta_{\perp}^2} + \frac{(v_{\|} - v_0)^2}{\kappa \theta_{\|}^2} \right]^{-\kappa - 1} \tag{3.2}$$

where v_0 denotes the streaming velocity whose direction is parallel to the electromagnetic waves. Spectral index κ is limited to $\kappa > \frac{3}{2}$ and Γ is gamma function.

$$\theta_{(\perp,\|)}^2 = \left(\frac{2\kappa - 3}{\kappa} \right) v_{th_{(\perp,\|)}}^2 ; \; v_{th_{(\perp,\|)}} = \sqrt{\frac{T_{(\perp,\|)}}{m}} \tag{3.3}$$

Let

$$A = \frac{n_0 \Gamma\left(\kappa+1\right)}{\pi^{\frac{3}{2}} \theta_{\perp}^{2} \theta_{\parallel} \Gamma\left(\kappa - \frac{1}{2}\right) \kappa^{\frac{3}{2}}} \tag{3.4}$$

Distribution function becomes

$$f_0 = A \left[1 - \frac{v_{\perp}^2}{\kappa\theta_{\perp}^2} + \frac{\left(v_{\parallel} - v_0\right)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa-1}$$

Differentiate f_0 with respect to $v_{\perp}$

$$\frac{\partial f_0}{\partial v_{\perp}} = -2A\left(\kappa+1\right)\frac{v_{\perp}}{\kappa\theta_{\perp}^2}\left[1 - \frac{v_{\perp}^2}{\kappa\theta_{\perp}^2} + \frac{\left(v_{\parallel} - v_0\right)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa-2} \tag{3.5}$$

Differentiate f_0 with respect to $v_{\parallel}$

$$\frac{\partial f_0}{\partial v_{\parallel}} = -2A\left(\kappa+1\right)\frac{\left(v_{\parallel} - v_0\right)}{\kappa\theta_{\parallel}^2}\left[1 - \frac{v_{\perp}^2}{\kappa\theta_{\perp}^2} + \frac{\left(v_{\parallel} - v_0\right)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa-2} \tag{3.6}$$

put equation (3.5) and (3.6) in equation (3.1)

$$\begin{aligned}
\omega^2 \;=\;& c^2k^2 - \pi\omega\omega_{pe}^2 \int_{-\infty}^{\infty} dv_{\parallel} \int_{0}^{\infty} v_{\perp}^2 dv_{\perp} \times \\
& \frac{\left[\begin{array}{c} -2A\left(\kappa+1\right)\frac{v_{\perp}}{\kappa\theta_{\perp}^2}\left[1 - \frac{v_{\perp}^2}{\kappa\theta_{\perp}^2} + \frac{\left(v_{\parallel}-v_0\right)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa-2} - \frac{k}{\omega}\times \\ \left\{-2v_{\parallel}A\left(\kappa+1\right)\frac{v_{\perp}}{\kappa\theta_{\perp}^2}\left[1 - \frac{v_{\perp}^2}{\kappa\theta_{\perp}^2} + \frac{\left(v_{\parallel}-v_0\right)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa-2} - \left(-2v_{\perp}A\left(\kappa+1\right)\frac{\left(v_{\parallel}-v_0\right)}{\kappa\theta_{\parallel}^2}\left[1 - \frac{v_{\perp}^2}{\kappa\theta_{\perp}^2} + \frac{\left(v_{\parallel}-v_0\right)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa-2}\right.\right. \end{array}\right.}{\left(\omega - kv_{\parallel} - \omega_{ce}\right)}
\end{aligned}$$

By taking common

$$\begin{aligned}
\omega^2 \;=\;& c^2k^2 + 2A\pi\omega\omega_{pe}^2\left(\kappa+1\right) \int_{-\infty}^{\infty} dv_{\parallel} \int_{0}^{\infty} v_{\perp}^2 dv_{\perp} \times \\
& \frac{\left[\left\{\frac{v_{\perp}}{\kappa\theta_{\perp}^2} - \frac{k}{\omega}\left(v_{\parallel}\left(\frac{v_{\perp}}{\kappa\theta_{\perp}^2}\right) - v_{\perp}\left(\frac{\left(v_{\parallel}-v_0\right)}{\kappa\theta_{\parallel}^2}\right)\right)\right\}\left[1 - \frac{v_{\perp}^2}{\kappa\theta_{\perp}^2} + \frac{\left(v_{\parallel}-v_0\right)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa-2}\right]}{\left(\omega - kv_{\parallel} - \omega_{ce}\right)}
\end{aligned}$$

After simplification

$$\omega^2 = c^2k^2 + \pi\omega\omega_{pe}^2 2A\frac{(\kappa+1)}{\kappa\theta_\perp^2}\int_{-\infty}^{\infty} dv_\parallel \int_0^\infty v_\perp^3 dv_\perp \times \frac{\left[\left\{1-\frac{k}{\omega}\left(v_\parallel - \left(\frac{(v_\parallel - v_0)\kappa\theta_\perp^2}{\kappa\theta_\parallel^2}\right)\right)\right\}\left[1-\frac{v_\perp^2}{\kappa\theta_\perp^2}+\frac{(v_\parallel-v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa-2}\right]}{(\omega - kv_\parallel - \omega_{ce})}$$

$$\omega^2 = c^2k^2 + \pi\omega\omega_{pe}^2 \frac{2A(\kappa+1)}{\kappa\theta_\perp^2}\int_{-\infty}^{\infty} dv_\parallel \int_0^\infty v_\perp^3 dv_\perp \times \left[\frac{\left\{1-\frac{k}{\omega}\left(v_\parallel - (v_\parallel - v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}\left[1-\frac{v_\perp^2}{\kappa\theta_\perp^2}+\frac{(v_\parallel-v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa-2}}{(\omega - kv_\parallel - \omega_{ce})}\right]$$

So

$$\omega^2 = c^2k^2 + \pi\omega\omega_{pe}^2 \frac{2A(\kappa+1)}{\kappa\theta_\perp^2}\int_{-\infty}^{\infty} dv_\parallel \times \left[\frac{\left\{1-\frac{k}{\omega}\left(v_\parallel - (v_\parallel - v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{(\omega - kv_\parallel - \omega_{ce})}\int_0^\infty v_\perp^3\left[1-\frac{v_\perp^2}{\kappa\theta_\perp^2}+\frac{(v_\parallel-v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa-2} dv_\perp\right]$$

finally we obtained

$$\omega^2 = c^2k^2 + \pi\omega\omega_{pe}^2 \frac{2A(\kappa+1)}{\kappa\theta_\perp^2}\int_{-\infty}^{\infty} dv_\parallel \times \left[\frac{\left\{1-\frac{k}{\omega}\left(v_\parallel - (v_\parallel - v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{(\omega - kv_\parallel - \omega_{ce})}\right] I_1 \tag{3.7}$$

where

$$
\begin{aligned}
I_1 &= \int_0^\infty \left[1 - \frac{v_\perp^2}{\kappa\theta_\perp^2} + \frac{(v_\parallel - v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa-2} v_\perp^3 dv_\perp \\
&= \frac{1}{2\kappa\left(\kappa+1\right)}\kappa^2\theta_\perp^4 \left[1 + \frac{(v_\parallel - v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}
\end{aligned}
$$

After solving perpendicular integration part we get

$$
I_1 = \frac{1}{2\left(\kappa+1\right)}\kappa\theta_\perp^4 \left[1 + \frac{(v_\parallel - v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}
$$

Put the above equation in equation (3.7)

$$
\begin{aligned}
\omega^2 &= c^2k^2 + \pi\omega\omega_{pe}^2 \frac{2A\left(\kappa+1\right)}{\kappa\theta_\perp^2} \int_{-\infty}^{\infty} dv_\parallel \times \\
&\left[\frac{\left\{1 - \frac{k}{\omega}\left(v_\parallel - (v_\parallel - v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{(\omega - kv_\parallel - \omega_{ce})}\right] \frac{1}{2\left(\kappa+1\right)}\kappa\theta_\perp^4 \left[1 + \frac{(v_\parallel - v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}
\end{aligned}
$$

$$
\begin{aligned}
\omega^2 &= c^2k^2 + A\pi\omega\omega_{pe}^2\theta_\perp^2 \int_{-\infty}^{\infty} dv_\parallel \times \\
&\left[\frac{\left\{1 - \frac{k}{\omega}\left(v_\parallel - (v_\parallel - v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{(\omega - kv_\parallel - \omega_{ce})}\right] \left[1 + \frac{(v_\parallel - v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}
\end{aligned}
$$

Put eq.(3.3) in above equation we get

$$
\begin{aligned}
\omega^2 &= c^2k^2 + \frac{\Gamma\left(\kappa+1\right)\pi\omega\omega_{pe}^2\theta_\perp^2}{\pi^{\frac{3}{2}}\theta_\perp^2\theta_\parallel\Gamma\left(\kappa - \frac{1}{2}\right)\kappa^{\frac{3}{2}}} \int_{-\infty}^{\infty} dv_\parallel \times \\
&\left[\frac{\left\{1 - \frac{k}{\omega}\left(v_\parallel - (v_\parallel - v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{(\omega - kv_\parallel - \omega_{ce})}\right] \left[1 + \frac{(v_\parallel - v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}
\end{aligned}
$$

$$\omega^2 = c^2k^2 + \frac{n_\circ \Gamma(\kappa+1)\,\omega\omega_{pe}^2}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}} \int_{-\infty}^{\infty} dv_\parallel \times$$

$$\left[\frac{\left\{1-\frac{k}{\omega}\left(v_\parallel-(v_\parallel-v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{(\omega-kv_\parallel-\omega_{ce})}\right]\left[1+\frac{(v_\parallel-v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}$$

By doing some mathematical arrangements

$$\omega^2 = c^2k^2 + \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega\omega_{pe}^2 \int_{-\infty}^{\infty} dv_\parallel \times$$

$$\left[\frac{\left\{1-\frac{k}{\omega}\left(v_\parallel-(v_\parallel-v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{k\theta_\parallel\left(\frac{\omega-kv_\parallel-\omega_{ce}}{k\theta_\parallel}\right)}\right]\left[1+\frac{(v_\parallel-v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}$$

$$\omega^2 = c^2k^2 + \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega\omega_{pe}^2 \int_{-\infty}^{\infty} dv_\parallel \times$$

$$\left[\frac{\left\{1-\frac{k}{\omega}\left(v_\parallel-(v_\parallel-v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{k\theta_\parallel\left(\frac{\omega-\omega_{ce}}{k\theta_\parallel}-\frac{v_\parallel}{\theta_\parallel}\right)}\right]\left[1+\frac{(v_\parallel-v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}$$

Hence

$$\omega^2 = c^2k^2 + \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega\omega_{pe}^2 \int_{-\infty}^{\infty} dv_\parallel \times$$

$$\left[\frac{\left\{1-\frac{k}{\omega}\left(v_\parallel-(v_\parallel-v_0)\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}{k\theta_\parallel\left(\frac{\omega-\omega_{ce}}{k\theta_\parallel}-\frac{v_\parallel}{\theta_\parallel}+\frac{v_0}{\theta_\parallel}-\frac{v_0}{\theta_\parallel}\right)}\right]\left[1+\frac{(v_\parallel-v_0)^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}$$

Finally we get

$$\omega^2 = c^2k^2 + \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_{\parallel}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega\omega_{pe}^2\int_{-\infty}^{\infty} dv_{\parallel} \times$$
$$\left[\frac{\left\{1-\frac{k}{\omega}\left(v_{\parallel}-(v_{\parallel}-v_0)\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)\right\}}{k\theta_{\parallel}\left(\frac{\omega-\omega_{ce}}{k\theta_{\parallel}}-\frac{v_0}{\theta_{\parallel}}-\frac{v_{\parallel}}{\theta_{\parallel}}+\frac{v_0}{\theta_{\parallel}}\right)}\right]\left[1+\frac{(v_{\parallel}-v_0)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa}$$

$$\omega^2 = c^2k^2 + \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_{\parallel}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega\omega_{pe}^2\int_{-\infty}^{\infty} dv_{\parallel} \times$$
$$\left[\frac{\left\{1-\frac{k}{\omega}\left(v_{\parallel}-(v_{\parallel}-v_0)\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)\right\}}{k\theta_{\parallel}\left(\frac{\omega-\omega_{ce}-kv_0}{k\theta_{\parallel}}-\left(\frac{v_{\parallel}-v_0}{\theta_{\parallel}}\right)\right)}\right]\left[1+\frac{(v_{\parallel}-v_0)^2}{\kappa\theta_{\parallel}^2}\right]^{-\kappa}$$

Here some variables are defined as

$$B = \frac{v_{\parallel}-v_0}{\theta_{\parallel}}$$

$$v_{\parallel} = B\theta_{\parallel} + v_0$$

$$dv_{\parallel} = \theta_{\parallel} dB$$

$$\xi = \frac{\omega-\omega_{ce}-kv_0}{k\theta_{\parallel}}$$

Put these variable's in eq.(3.8)

$$\omega^2 = c^2k^2 + \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_{\parallel}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega\omega_{pe}^2\int_{-\infty}^{\infty}\theta_{\parallel} dB \times \left[\frac{\left(1-\frac{k}{\omega}\left(B\theta_{\parallel}+v_0-\left(B\theta_{\parallel}\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)\right)\right)}{k\theta_{\parallel}(\xi-B)}\right]\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}$$

$$\omega^2 = c^2k^2 + \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega\omega_{pe}^2 \int_{-\infty}^{\infty} dB \times \left[\frac{\left(1-\frac{k}{\omega}\left(v_0+\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)B\theta_\parallel\right)\right)}{k(\xi-B)}\right]\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}$$

By taking common $\frac{k}{\omega}$

$$\omega^2 = c^2k^2 - \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2 \int_{-\infty}^{\infty} dB \times \left[\frac{\left(\frac{\omega}{k}-\left(v_0+\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)B\theta_\parallel\right)\right)}{(B-\xi)}\right]\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}$$

$$\omega^2 = c^2k^2 - \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2 \int_{-\infty}^{\infty} dB \times \left[\frac{\left(\frac{\omega}{k}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}-\left(v_0+\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)B\theta_\parallel\right)\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}\right)}{(B-\xi)}\right]$$

$$\omega^2 = c^2k^2 - \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2 \int_{-\infty}^{\infty} dB \times \left[\frac{\left(\frac{\omega}{k}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}-v_0\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}-\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)B\theta_\parallel\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}\right)}{(B-\xi)}\right]$$

After simplification

$$\omega^2 = c^2k^2 - \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\theta_\parallel \Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2 \int_{-\infty}^{\infty} dB \times \left[\frac{\left(\frac{\omega-kv_0}{k}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}-\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)B\theta_\parallel\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}\right)}{(B-\xi)}\right]$$

$$\begin{aligned}\omega^2 \quad = \quad & c^2k^2 - \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2 \frac{\omega-kv_0}{\theta_\parallel k}\int_{-\infty}^{\infty}\frac{1}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB + \\ & \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\int_{-\infty}^{\infty}\frac{B}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB\end{aligned}$$

$$\omega^2 = c^2k^2 - \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\frac{\omega-kv_0}{\theta_\parallel k}\int_{-\infty}^{\infty}\frac{1}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB + I_2 \tag{3.9}$$

where

$$I_2 = \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\int_{-\infty}^{\infty}\frac{B}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB$$

$$I_2 = \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\int_{-\infty}^{\infty}\frac{B-\xi+\xi}{(B-\xi)}[1+\frac{B^2}{\kappa}]^{-\kappa}dB$$

$$I_2 = \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\int_{-\infty}^{\infty}\left(\frac{B-\xi}{B-\xi}+\frac{\xi}{(B-\xi)}\right)\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB$$

$$I_2 = \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\int_{-\infty}^{\infty}\left(1+\frac{\xi}{(B-\xi)}\right)\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB$$

After separating

$$I_2 = \frac{\Gamma(\kappa+1)\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\int_{-\infty}^{\infty}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB+\frac{\Gamma(\kappa+1)\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\int_{-\infty}^{\infty}\frac{\xi}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB$$

By solving integration we get,

$$I_2 = \left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\omega_{pe}^2 + \frac{\Gamma(\kappa+1)\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\int_{-\infty}^{\infty}\frac{\xi}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB$$

Put value of I_2 in eq.(3.9)

$$\begin{aligned}\omega^2 \quad = \quad & c^2k^2 - \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\frac{\omega-kv_0}{\theta_\parallel k}\int_{-\infty}^{\infty}\frac{1}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa}dB + \\ & \left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\omega_{pe}^2 + \frac{\Gamma(\kappa+1)\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\int_{-\infty}^{\infty}\frac{\xi}{(B-\xi)}\left[1+\frac{B^2}{\kappa\theta_\parallel^2}\right]^{-\kappa}dB\end{aligned}$$

$$\omega^2 = c^2k^2 + \left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)\omega_{pe}^2 - \frac{\Gamma(\kappa+1)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\frac{\omega - kv_0}{\theta_\parallel k}\int_{-\infty}^{\infty}\frac{1}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa} dB \tag{3.10}$$

$$\frac{\Gamma(\kappa+1)\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)}{\pi^{\frac{1}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)\kappa^{\frac{3}{2}}}\omega_{pe}^2\int_{-\infty}^{\infty}\frac{\xi}{(B-\xi)}\left[1+\frac{B^2}{\kappa}\right]^{-\kappa} dB$$

$Z(\xi)$ is modified plasma dispersion function defined as [30, 31, 32]

$$Z(\xi) = \frac{\Gamma(\kappa+1)}{\sqrt{\pi}\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\int_{-\infty}^{\infty}\frac{\left(1+\frac{B^2}{\kappa}\right)^{-\kappa} dB}{B-\xi}$$

$$\omega^2 = c^2k^2 + \left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\omega_{pe}^2 - \omega_{pe}^2\left(\frac{\omega - kv_0}{\theta_\parallel k}\right)Z(\xi) + \omega_{pe}^2\xi\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)Z(\xi)$$

$$\omega^2 = c^2k^2 - \omega_{pe}^2\left(\frac{\omega - kv_0}{\theta_\parallel k}\right)Z(\xi) + \omega_{pe}^2\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)[1+\xi Z(\xi)]$$

$$\frac{c^2k^2}{\omega^2} = 1 + \frac{\omega_{pe}^2}{\omega^2}\left[\left(\frac{\omega - kv_0}{k\theta_\parallel}\right)Z(\xi) - \left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)[1+\xi Z(\xi)]\right] \tag{3.11}$$

Equation (3.11) presents the dispersion relation for an R-wave which comes with temperature anisotropy and that streaming in the direction parallel to the axis.

In literature, many authors have studied the dispersion relation of EM waves that propagate in a parallel way to explore damping/instability of the waves. This research paper focus on how the energy flow of EM waves propagating in parallel across a distance in streaming bi-Kappa distributed plasmas. To the best of our knowledge, this work has not been studied by any authors before. In this work, analytical approach is employed to solve equation (3.11) for the complex wave number to obtain the imaginary wave-number (k_i) from which it is used in the Poynting flux theorem.

3.2 Resonant Case

The plasma dispersion function expansions for resonant case ($\xi << 1$) is given by

$$Z(\xi) = \frac{\Gamma(\kappa+1)}{\sqrt{\pi}\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)} \int_{-\infty}^{\infty} \frac{\left(1+\frac{B^2}{\kappa}\right)^{-\kappa} dB}{B-\xi}$$

$$Z(\xi) = \frac{\Gamma(\kappa+1)}{\sqrt{\pi}\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)} \int_{-\infty}^{\infty} \frac{1}{B} \times \frac{1}{\left(1-\frac{\xi}{B}\right)} \left(1+\frac{B^2}{\kappa}\right)^{-\kappa} dB$$

$$Z(\xi) = \frac{\Gamma(\kappa+1)}{\sqrt{\pi}\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)} \int_{-\infty}^{\infty} \frac{1}{B} \times \left(1+\frac{B^2}{\kappa}\right)^{-\kappa} \left(1-\frac{\xi}{B}\right)^{-1} dB$$

By using binomial theorem

$$Z(\xi) = \frac{\Gamma(\kappa+1)}{\sqrt{\pi}\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)} \int_{-\infty}^{\infty} \frac{1}{B} \times \left(1+\frac{B^2}{\kappa}\right)^{-\kappa} \left(1+\frac{\xi}{B}+\left(\frac{\xi}{B}\right)^2+...\right) dB$$

$$Z(\xi) = i\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{\xi^2}{B}\right]^{-\kappa} - \sqrt{\pi}\frac{\Gamma\left(\kappa+\frac{1}{2}\right)}{\kappa\Gamma\left(\kappa-\frac{1}{2}\right)\Gamma\left(\frac{3}{2}\right)}\xi + \sqrt{\pi}\frac{\Gamma\left(\kappa+\frac{1}{2}+1\right)}{\kappa^2\Gamma\left(\kappa-\frac{1}{2}\right)\Gamma\left(1+\frac{3}{2}\right)}\xi^3 + ...$$

By using small argument expansion of plasma dispersion relation in equation (3.11)we get,

$$\frac{c^2k^2}{\omega^2} = 1 + \frac{\omega_{pe}^2}{\omega^2}\left[\begin{array}{c} \left(\frac{\omega-kv_0}{k\theta_\parallel}\right) i\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{\xi^2}{B}\right]^{-\kappa} - \\ \left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\left\{1+\xi i\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{\xi^2}{B}\right]^{-\kappa}\right\} \end{array}\right]$$

$$\frac{c^2k^2}{\omega^2} = 1 + \frac{\omega_{pe}^2}{\omega^2}\left[\left\{\frac{\omega-kv_0}{k\theta_\parallel} - \xi\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\} i\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{\xi^2}{B}\right]^{-\kappa} - \left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right]$$

Now we put $\xi = \frac{\omega-\omega_{ce}-kv_\circ}{k\theta_\parallel}$ in the above equation

$$\frac{c^2k^2}{\omega^2} = 1 + \frac{\omega_{pe}^2}{\omega^2}\left[\left\{\frac{\omega-kv_0}{k\theta_\parallel} - \frac{\omega-\omega_{ce}-kv_0}{k\theta_\parallel}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\} i\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{\xi^2}{B}\right]^{-\kappa} - \left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right]$$

$$\frac{c^2k^2}{\omega^2}=1+\frac{\omega_{pe}^2}{\omega^2}\left[\left\{\frac{\omega-kv_0}{k\theta_{\parallel}}-\frac{\omega-\omega_{ce}-kv_0}{k\theta_{\parallel}}+\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\frac{\omega-\omega_{ce}-kv_0}{k\theta_{\parallel}}\right\}i\sqrt{\pi}\frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{\xi^2}{B}\right]^{-\kappa}-\left(1-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)\right]$$

$$\frac{c^2k^2}{\omega^2}=1+\frac{\omega_{pe}^2}{\omega^2}\left[\left\{\frac{\omega-kv_0}{k\theta_{\parallel}}-\frac{\omega-kv_0}{k\theta_{\parallel}}+\frac{\omega_{ce}}{k\theta_{\parallel}}+\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\frac{\omega-\omega_{ce}-kv_0}{k\theta_{\parallel}}\right\}i\sqrt{\pi}\frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[1+\frac{\xi^2}{B}\right]^{-\kappa}-\left(1-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)\right]$$

After simplification we get

$$\frac{c^2k^2}{\omega^2}=1+\frac{\omega_{pe}^2}{\omega^2}\left[\left\{\frac{\omega_{ce}}{k\theta_{\parallel}}+\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\frac{\omega-\omega_{ce}-kv_0}{k\theta_{\parallel}}\right\}i\sqrt{\pi}\frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}-\left(1-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)\right]$$

$$k^2=\frac{\omega^2}{c^2}+\frac{\omega_{pe}^2}{c^2}\left[i\sqrt{\pi}\frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left\{\frac{\omega_{ce}}{k\theta_{\parallel}}\left(1-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)+\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\frac{\omega}{k\theta_{\parallel}}-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\frac{v_0}{\theta_{\parallel}}\right\}-\left(1-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)\right] \quad (3.12)$$

We aim to explore strong wave-particle interactions by focusing on the small argument ($\xi << 1$) of the plasma dispersion function, while for large arguments ($\xi >> 1$) the wave-particle interactions is weak and the imaginary part of the wave number becomes insignificant.

Assume that k is complex and ω is real.

$$k=\left(k_r+ik_i\right)$$

$$\left(k\right)^2=\left(\left(k_r\right)^2-\left(k_i\right)^2+2i\left(k_r\right)\left(k_i\right)\right)$$

Put value of k in equation (3.12)

$$\left(k_r\right)^2-\left(k_i\right)^2+2i\left(k_r\right)\left(k_i\right)=\frac{\omega^2}{c^2}+\frac{\omega_{pe}^2}{c^2}\left[i\sqrt{\pi}\frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left\{\begin{array}{c}\frac{\omega_{ce}}{(k_r+ik_i)\theta_{\parallel}}\left(1-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)+\\ \frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\frac{\omega}{(k_r+ik_i)\theta_{\parallel}}-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\frac{v_0}{\theta_{\parallel}}\end{array}\right\}-\left(1-\frac{\theta_{\perp}^2}{\theta_{\parallel}^2}\right)\right]$$

We take assumption $k_r^2 >> k_i^2$

$$k_r^2 + 2ik_rk_i = \frac{\omega^2}{c^2} + \frac{\omega_{pe}^2}{c^2}\left[i\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left\{\begin{array}{c}\frac{\omega_{ce}}{\theta_\parallel}\frac{k_r-ik_i}{k_r^2-k_i^2}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)+\\ \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\theta_\parallel}\frac{k_r-ik_i}{k_r^2-k_i^2}-\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{v_0}{\theta_\parallel}\end{array}\right\}-\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right]$$

$$k_r^2 + 2ik_rk_i = \frac{\omega^2}{c^2} + \frac{\omega_{pe}^2}{c^2}\left[i\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left\{\begin{array}{c}\frac{\omega_{ce}}{k_r\theta_\parallel}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)+\\ \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{k_r\theta_\parallel}-\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{v_0}{\theta_\parallel}\end{array}\right\}-\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right] \tag{3.13}$$

The real and imaginary part of eq.(3.13) is

$$k_r^2 = \frac{\omega^2}{c^2} - \frac{\omega_{pe}^2}{c^2}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)$$

$$k_r = \sqrt{\frac{\omega^2}{c^2} - \frac{\omega_{pe}^2}{c^2}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)} \tag{3.14}$$

$$2k_rk_i = \frac{\omega_{pe}^2}{c^2}\left[\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left\{\frac{\omega_{ce}}{k_r\theta_\parallel}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)+\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{k_r\theta_\parallel}-\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{v_0}{\theta_\parallel}\right\}\right]$$

Divid both sides by $2k_r$

$$k_i = \frac{\omega_{pe}^2}{2c^2}\left[\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left\{\frac{\omega_{ce}}{k_r^2\theta_\parallel}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)+\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{k_r^2\theta_\parallel}-\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{v_0}{k_r\theta_\parallel}\right\}\right]$$

$$k_i = \frac{\omega_{pe}^2}{2c^2k_r^2}\left[\sqrt{\pi}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left\{\frac{\omega_{ce}}{\theta_\parallel}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)+\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\theta_\parallel}-\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{k_rv_0}{\theta_\parallel}\right\}\right]$$

Put value of k_r in the above equation

$$k_i = \frac{\sqrt{\pi}\omega_{pe}^2}{2c^2\left\{\frac{\omega^2}{c^2}-\frac{\omega_{pe}^2}{c^2}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[\frac{\omega_{ce}}{\theta_\parallel}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)+\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\theta_\parallel}-\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{v_0\sqrt{\frac{\omega^2}{c^2}-\frac{\omega_{pe}^2}{c^2}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)}}{\theta_\parallel}\right]$$

$$k_i = \frac{\sqrt{\pi}\omega_{pe}^2}{2\left\{\omega^2 - \omega_{pe}^2\left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right\}} \frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa - \frac{1}{2}\right)} \left[\begin{array}{c} \frac{\omega_{ce}}{\theta_\parallel}\left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right) + \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\theta_\parallel} - \\ \frac{\theta_\perp^2}{\theta_\parallel^2} \frac{v_0\sqrt{\frac{\omega^2}{c^2} - \frac{\omega_{pe}^2}{c^2}\left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)}}{\theta_\parallel} \end{array} \right] \tag{3.15}$$

Eq.(3.15) show the imaginary part of wave vector number (k) where in low magnetized plasma $\omega > \omega_{ce}$ and $\frac{\theta_\perp^2}{\theta_\parallel^2} > 1$.

3.3 Derivation of Energy flow

In order to ascertain how the EM waves transfer their energy to the particles, we employ the Poynting flux theorem under uniform state conditions as [33, 34]

$$\boldsymbol{\nabla}.\mathbf{S} = -P$$

$$P = \frac{1}{2}\,\mathrm{Re}(\mathbf{J}^*.\mathbf{E})$$

$$\mathbf{S} = \frac{\mathrm{Re}}{\mu_o}(\mathbf{E}^* \times \mathbf{B})$$

where P represents the power dissipation, S represents the poynting vector, μ_o is the magnetic permeability, **J** ia current density, and **E** is the perturbed electric.

The geometry of our problem is such that the unperturbed magnetic field and wave propagation both are along the z-axis, while the magnetic field and electric field perturbation is in the x-y plane.

When we simplified the expressions for S and P, we revisited the energy flux theorem,

$$\frac{dS}{dz} = -k_i S_z \tag{3.16}$$

Taking integration on both sides

$$\int_0^Z \frac{dS}{S_z} = -k_i \int_0^Z dz$$

$$\ln \left[S_z \right]_0^Z = -k_i \left[Z \right]_0^Z$$

$$\ln \left(S\left(z\right) - S\left(0\right) \right) = -k_i \left(Z - 0 \right)$$

$$\ln \left(\frac{S\left(z\right)}{S\left(0\right)} \right) = -k_i Z$$

Taking expontial on both sides

$$\frac{S\left(z\right)}{S\left(0\right)} = \exp\left(-k_i Z\right) \tag{3.17}$$

Here, $S(0)$ represents the energy at the origin of wave propagation, and equation (3.15) defines k_i. It is essential to note the way in which the energy flux theorem shows the spatial damping factor k_i.

Put eq.(3.15) in eq(3.17)

$$\frac{S\left(z\right)}{S\left(0\right)} = \exp\left(-\frac{\sqrt{\pi}\omega_{pe}^2}{2\left(\omega^2 - \omega_{pe}^2\left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right)} \frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa - \frac{1}{2}\right)} \left\{ \begin{array}{c} \frac{\omega_{ce}}{\theta_\parallel}\left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right) + \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\theta_\parallel} - \\ \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{v_0\sqrt{\frac{\omega^2}{c^2} - \frac{\omega_{pe}^2}{c^2}\left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)}}{\theta_\parallel} \end{array} \right\} Z \right)$$

$$\frac{S\left(z\right)}{S\left(0\right)} = \exp\left(-\frac{\sqrt{\pi}}{2\left(\frac{\omega^2}{\omega_{pe}^2} - \left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right)} \frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa - \frac{1}{2}\right)} \left\{ \begin{array}{c} \frac{\omega_{ce}}{\theta_\parallel}\left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right) + \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\theta_\parallel} - \\ \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega_{pe}}{c}\frac{v_0\sqrt{\frac{\omega^2}{\omega_{pe}^2} - \left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)}}{\theta_\parallel} \end{array} \right\} Z \right)$$

$$\frac{S\left(z\right)}{S\left(0\right)} = \exp\left(-\frac{\sqrt{\pi}}{2\left(\frac{\omega^2}{\omega_{pe}^2} - \left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right)} \frac{\Gamma\left(\kappa+1\right)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa - \frac{1}{2}\right)} \frac{\omega_{pe}}{\theta_\parallel} \left\{ \begin{array}{c} \frac{\omega_{ce}}{\omega_{pe}}\left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right) + \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\omega_{pe}} - \\ \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{v_0}{c}\sqrt{\frac{\omega^2}{\omega_{pe}^2} - \left(1 - \frac{\theta_\perp^2}{\theta_\parallel^2}\right)} \end{array} \right\} Z \right)$$

$$\frac{S(z)}{S(0)} = \exp\left(-\frac{\sqrt{\pi}}{2\left(\frac{\omega^2}{\omega_{pe}^2} - \left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)\right)}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left\{\begin{array}{c}\frac{\omega_{ce}}{\omega_{pe}}\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)+\frac{\theta_\perp^2}{\theta_\parallel^2}\frac{\omega}{\omega_{pe}}- \\ \frac{\theta_\perp^2}{\theta_\parallel^2}\frac{v_0}{c}\sqrt{\frac{\omega^2}{\omega_{pe}^2}-\left(1-\frac{\theta_\perp^2}{\theta_\parallel^2}\right)}\end{array}\right\}\frac{\omega_{pe}}{\theta_\parallel}Z\right)$$

we know that

$$\theta^2_{(\perp,\parallel)} = \left(\frac{2\kappa-3}{\kappa}\right)v^2_{th_{(\perp,\parallel)}} \;;\; v_{th_{(\perp,\parallel)}} = \sqrt{\frac{T_{(\perp,\parallel)}}{m}}$$

putting values in above equation

$$\frac{S(z)}{S(0)} = \exp\left(-\frac{\sqrt{\pi}}{2\left(\frac{\omega^2}{\omega_{pe}^2} - \left(1-\frac{T_\perp}{T_\parallel}\right)\right)}\frac{\Gamma(\kappa+1)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[\begin{array}{c}\frac{\omega_{ce}}{\omega_{pe}}\left(1-\frac{T_\perp}{T_\parallel}\right)+\frac{T_\perp}{T_\parallel}\frac{\omega}{\omega_{pe}}- \\ \frac{T_\perp}{T_\parallel}\frac{v_0}{c}\sqrt{\frac{\omega^2}{\omega_{pe}^2}-\left(1-\frac{T_\perp}{T_\parallel}\right)}\end{array}\right]\frac{\omega_{pe}}{\sqrt{\left(\frac{2\kappa-3}{\kappa}\right)}v_{th\parallel}}Z\right)$$

$$\frac{S(z)}{S(0)} = \exp\left(-\frac{\sqrt{\pi}}{2\left(\frac{\omega^2}{\omega_{pe}^2} - \left(1-\frac{T_\perp}{T_\parallel}\right)\right)\sqrt{2\kappa-3}}\frac{\kappa^{\frac{1}{2}}\kappa\Gamma(\kappa)}{\kappa^{\frac{3}{2}}\Gamma\left(\kappa-\frac{1}{2}\right)}\left[\begin{array}{c}\frac{\omega_{ce}}{\omega_{pe}}\left(1-\frac{T_\perp}{T_\parallel}\right)+\frac{T_\perp}{T_\parallel}\frac{\omega}{\omega_{pe}}- \\ \frac{T_\perp}{T_\parallel}\frac{v_0}{c}\sqrt{\frac{\omega^2}{\omega_{pe}^2}-\left(1-\frac{T_\perp}{T_\parallel}\right)}\end{array}\right]\frac{\omega_{pe}}{c\frac{v_{th\parallel}}{c}}Z\right)$$

$$\frac{S(z)}{S(0)} = \exp\left(-\frac{\sqrt{\pi}}{2\left(\frac{\omega^2}{\omega_{pe}^2} - \left(1-\frac{T_\perp}{T_\parallel}\right)\right)\frac{v_{th\parallel}}{c}\sqrt{2\kappa-3}}\frac{\Gamma(\kappa)}{\Gamma\left(\kappa-\frac{1}{2}\right)}\left[\left(\begin{array}{c}\frac{\omega_{ce}}{\omega_{pe}}\left(1-\frac{T_\perp}{T_\parallel}\right)+\frac{T_\perp}{T_\parallel}\frac{\omega}{\omega_{pe}}- \\ \frac{T_\perp}{T_\parallel}\frac{v_0}{c}\sqrt{\frac{\omega^2}{\omega_{pe}^2}-\left(1-\frac{T_\perp}{T_\parallel}\right)}\end{array}\right)\right]\frac{\omega_{pe}}{c}Z\right)$$

By defining some variables

$$v = \frac{v_{th\parallel}}{c},\; w^2 = \frac{\omega^2}{\omega_{pe}^2},\; s = \frac{\omega_{ce}}{\omega_{pe}},\; \eta = \frac{T_\perp}{T_\parallel}$$

$$\frac{S(z)}{S(0)} = \exp\left(-\frac{\sqrt{\pi}}{2\left(w^2-(1-\eta)\right)v\sqrt{2\kappa-3}}\frac{\Gamma(\kappa)}{\Gamma\left(\kappa-\frac{1}{2}\right)}\left[\left(s(1-\eta)+\eta w-\eta V\sqrt{w^2-(1-\eta)}\right)\right]\frac{\omega_{pe}}{c}Z\right) \tag{3.18}$$

Chapter 4

RESULTS AND DISCUSSION

To study the effect of temperature anisotropy, streaming velocity and the kappa index (κ) on the energy flow in streaming bi-Kappa distributed plasma with PEMW for the resonant case ($\xi << 1$), we plot equation (3.18). The finding are applicable for the solar wind region.

Table: Parameters for solar wind at 1AU [34].

$n_e(cm^{-3})$	$T_e(k)$	$B_\circ(G)$	$\omega_{pe}(s^{-1})$	$\omega_{ce}(s^{-1})$	$v_\circ(cms^{-1})$
10	10^5	5×10^{-5}	2×10^{-5}	10^2	3×10^7

Where n_e is number density of electron, T_e is temperature, $B_\circ$ is ambient magnetic field, ω_{pe} is plasma frequency, ω_{ce} cyclotron frequency, and $v_\circ$ is streaming velocity of electron in streaming plasma.

4.1 Graphical Illustration

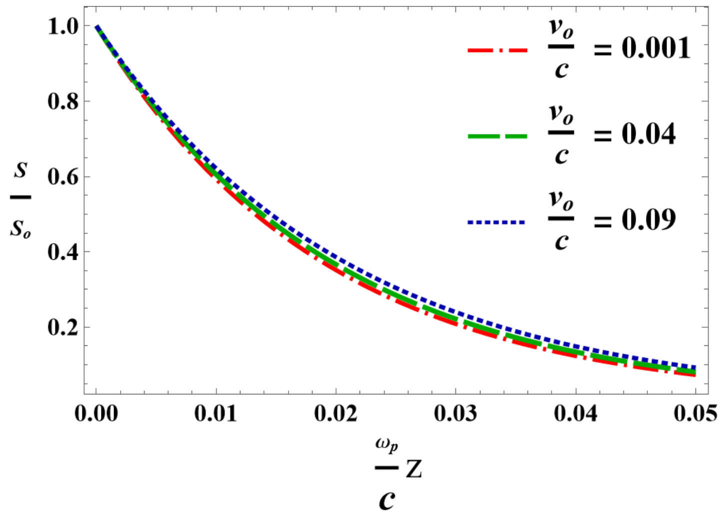

Figure 4.1: Poynting flux ($\frac{S}{S_{\circ}}$) versus distance ($\frac{\omega_p}{c}z$) for fixed value of temperature anisotropy ($\eta = 1$), kappa index ($\kappa = \infty$), and thermal velocity ($\frac{v_{t\parallel}}{c} = 0.004$)

Figure 1 show that the effectiveness of wave-particle interactions in isotropic Maxwellian plasma significantly depends on the electron streaming velocity. When the streaming velocity is low, the interactions are robust, leading to rapid wave damping over short distances. On the other hand, when the streaming velocity is high, the interactions are weaker, causing waves to carry their energy more slowly over longer distances.

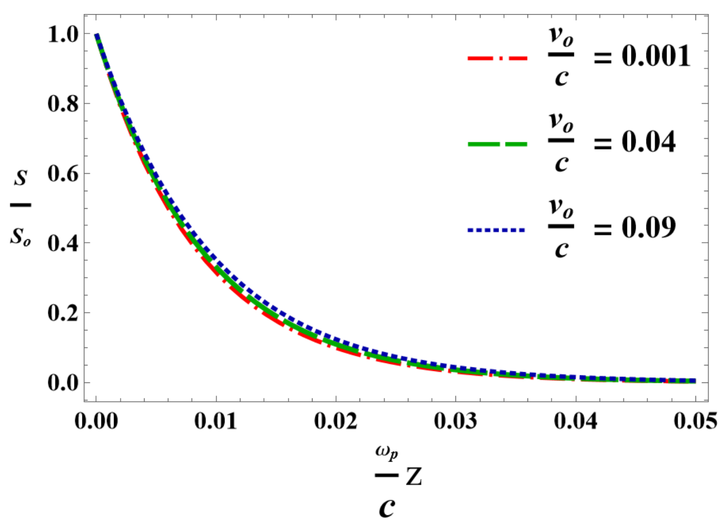

Figure 4.2: Poynting flux ($\frac{S}{S_{\circ}}$) versus distance ($\frac{\omega_p}{c}z$) for fixed value of temperature anisotropy ($\eta = 2$), kappa index ($\kappa = 3$), and thermal velocity ($\frac{v_{t\parallel}}{c} = 0.004$)

Figure 2. show that when we fixed temperature anisotropy, thermal velocity, and index kappa the wave rapidly transport their energy across shorter distances.

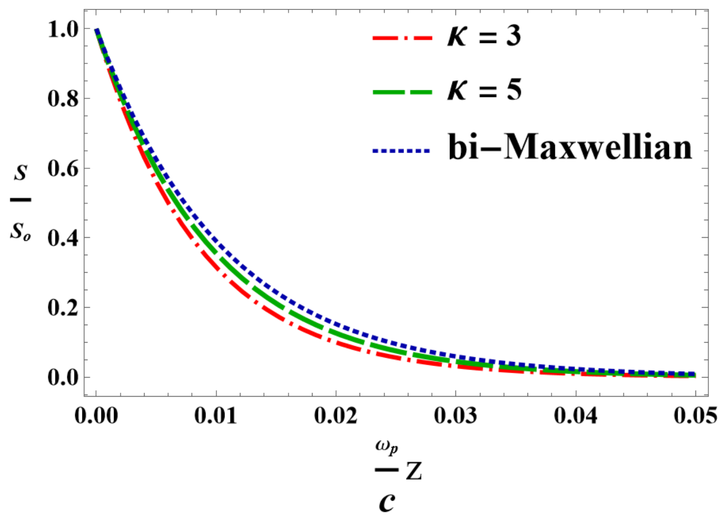

Figure 4.3: Poynting flux ($\frac{S}{S_\circ}$) versus distance ($\frac{\omega_p}{c}z$) for fixed value of temperature anisotropy ($\eta = 2$), streaming velocity ($\frac{v_\circ}{c} = 0.001$), and thermal velocity ($\frac{v_{t\|}}{c} = 0.004$)

From figure 3, we can see that for small value of kappa index (κ), the wave deliver their energy rapidly. The reason may be the efficient conversion of thermal energy to particle motion in streaming plasma.

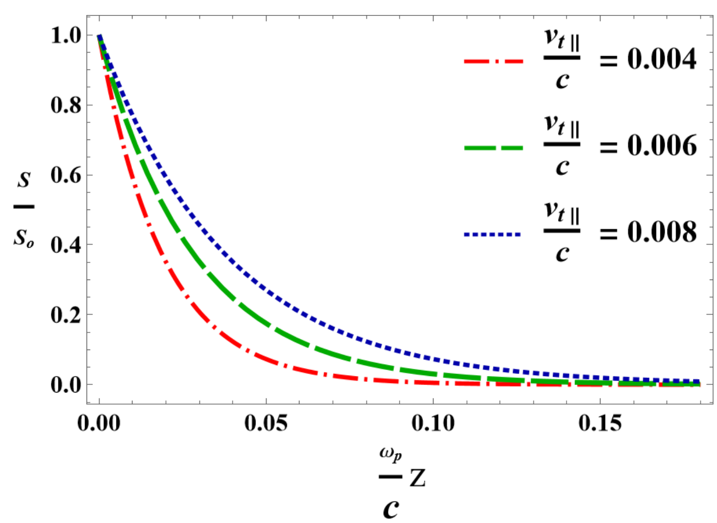

Figure 4.4: Poynting flux ($\frac{S}{S_\circ}$) versus distance ($\frac{\omega_p}{c}z$) for fixed value of kappa index ($\kappa = \infty$), temperature anisotropy ($\eta = 1$), and streaming velocity ($\frac{v_o}{c} = 0.001$)

Figure 4 show the isotropic Maxwellian plasmas, we can see that if the thermal velocity is low the energy of the waves will be carried more quickly but for short distances; Whereas if the thermal velocity is high these waves will take more time to carry their energy to longer distances.

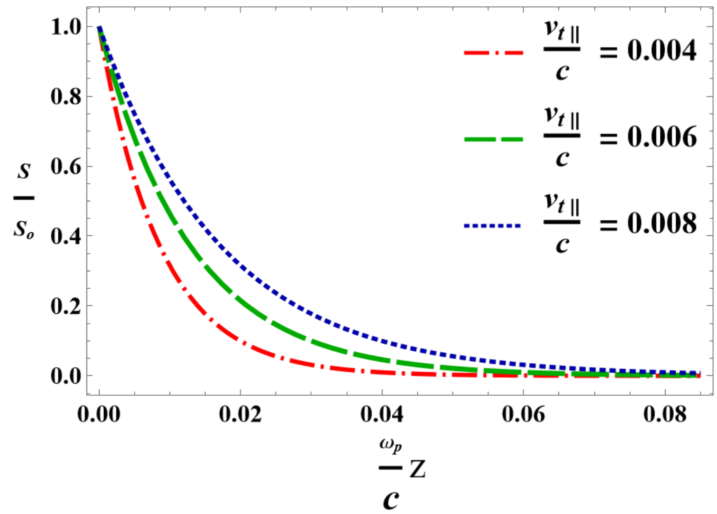

Figure 4.5: Poynting flux ($\frac{S}{S_\circ}$) versus distance ($\frac{\omega_p}{c}z$) for fixed value of temperature anisotropy ($\eta = 2$), index kappa ($\kappa = 3$), and streaming velocity ($\frac{v_o}{c} = 0.001$)

Figure 5 show that when we fixed temperature anisotropy, streaming velocity, and index kappa the waves transport their energy across shorter distances.

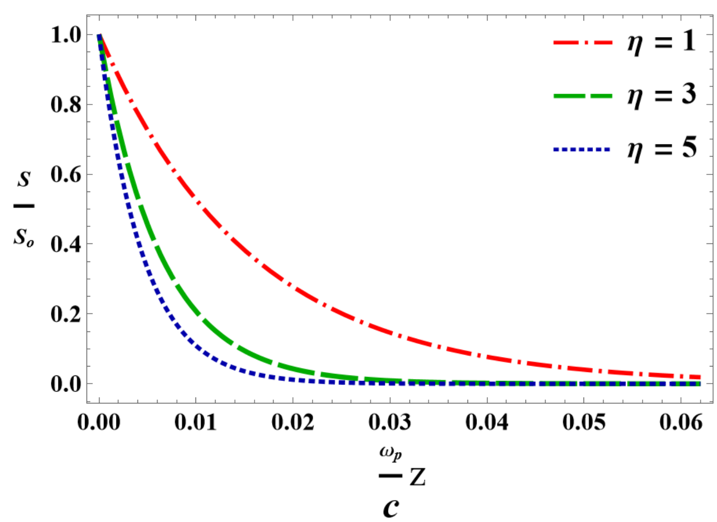

Figure 4.6: Poynting flux ($\frac{S}{S_\circ}$) versus distance ($\frac{\omega_p}{c}z$) for fixed value of thermal velocity ($\frac{v_{t\parallel}}{c} = 0.004$), index kappa ($\kappa = 3$), and streaming velocity ($\frac{v_o}{c} = 0.001$)

Figure 6 show that when temperature anisotropy of electron increases, waves rapidly transport their energy over the smaller distance. In temperature anisotropic plasma, there are more resonant particles, enabling the wave to quickly transfer its energy over smaller distances.

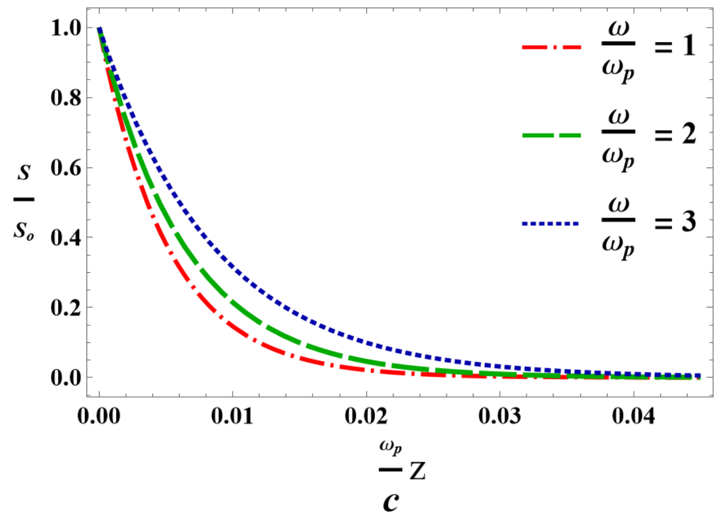

Figure 4.7: Poynting flux ($\frac{S}{S_\circ}$) versus distance ($\frac{\omega_p}{c}z$) for fixed value of thermal velocity ($\frac{v_{t\parallel}}{c} = 0.004$), index kappa ($\kappa = 3$), and streaming velocity ($\frac{v_o}{c} = 0.001$)

In fig.7. we observe that when the values of wave frequency of electrons is small then the wave deliver their energy slowly over longer distances. It may be due to electron with lower energy move at a slow pace and have a lesser probability of colliding with other particles.

4.2 Conclusion

Kinetic approach has used to examine the energy flow in streaming non-Maxwellian plasmas with parallel electromagnetic waves. We observed that with the change in streaming velocity, temperature anisotropy, wave frequency, thermal speed and the kappa index (κ) the energy flux changes significantly over distances. From figures 1, 2 and 3 we examine that wave transport its energy across long distances for greater values of streaming velocity, index kappa, and thermal speed. It can be seen in figures 4, and 6 that the wave deliver its energy across long distances for greater values of thermal speeds and wave frequencies. Figure 5 show that wave transport its energy across shorter distance for larger value of temperature anisotropy. It is also examined that the wave delivered their energy quickly in streaming non-Maxwellian plasma, it may be due to streaming cause instabilities in the plasma, where the wave energy can be transmitted to streaming particles making them even more energetic, this may result in additional wave growth and turbulence.

This research may be relevant in various applications, including space physics , astrophysics, laboratory plasma experiments. In astronomy it could be used to explain phenomena such as plasma waves in the interstellar medium or the behavior of plasma around stars or black holes. It should be useful in space physics to understand how solar wind and planetary magnetospheres interact. In laboratory plasma researchers may examine the behavior of non-Maxwellian plasmas in order to enhance plasma confinement.

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