Model Order Reduction of non-linear power system using Projections Technique

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DEDICATION

This thesis is dedicated to *my beloved Parents & Siblings*.

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"Read. Read in the name of thy Lord who created; [He] created the human being from a blood clot. Read in the name of thy Lord who taught by the pen: [He] taught the human being what he did not know."

The importance of education lies in the first Revelation of the Quran and this proved to be my motivation. All praises to **ALLAH**, who bestowed me with the knowledge, patience, health, and ability to complete this thesis.

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LIST OF ABBREVIATIONS

MOR	Model Order Reduction
ROM	Reduced Order Model
BT	Balanced Truncation
IRKA	Iterative Rational Krylov Algorithm
SVD	Singular Value Decomposition
LTI	Linear Time Invariant
SISO	Single Input Single Output
MIMO	Multiple Input Multiple Output

Abstract

Stable and reliable operations of power systems are based on continuous or frequent monitoring and control of the systems by collecting and analyzing real-time data and optimizing its working. Multiple measurement devices are often deployed across various nodes of the power network to monitor the behavior. This framework of physically monitoring large-scale power network is significantly time consuming and inefficient in terms of human efforts and resources. An alternative method involves determining the mathematical model of the power system, simulating it, and analyzing the system's behavior to observe the desired outcomes. These mathematical models associated with large scale power systems involve differential as well as algebraic equations (DAE) and their simulation can be computationally cumbersome. Furthermore the dynamics of the system are often nonlinear which further adds to its complexity. A remedy to this problem is model order reduction, where the dynamics of original system are reduced such that its behaviour remain the same. In this thesis, we consider the problem of model order reduction for nonlinear power system models by constructing a reduced bilinear model from the original large-scale model with approximately the same behavior as the original model. Two specific approaches, bilinear balanced truncation (BBT) and bilinear iterative rational Krylov algorithm (BIRKA) has been utilized and compared. It is observed that the performance of the two approaches is almost comparable and they offer trade-off between accuracy and the size of the reduced order model. Two examples of bilinear power networks has been utilized from the literature for their comparison and analysis. Numerical results show that the reduced order models from the two approaches are highly accurate, stable, and significantly faster to simulate as compared to the original model. The BIRKA method is more useful than the BBT method in the sense that it canbe easily extended to very large-scale settings as it involves only matrix-vector multiplications. offer trade-off between accuracy and the size of the reduced order model. Two examples of bilinear power networks has been utilized from the literature for their comparison and analysis. Numerical results show that the reduced order models from the two approaches are highly accurate, stable, and significantly faster to simulate as compared to the original model. The BIRKA method is more useful than the BBT method in the sense that it can be easily extended to large-scale settings as it involves only matrix-vector multiplications.

Keywords: Model Order Reduction(MOR), Bilinear Power Systems, Projection-based Techniques.

Chapter 1

Introduction

This chapter presents a concise introduction to power systems, focusing on their key components crucial for distributing and transmitting power. It discusses the challenges of model order reduction and its practical applications. Furthermore, it outlines the objectives of the thesis and provides an overview of thesis contents

1.1 Electrical Power Systems

Electric power systems play a crucial role in modern society. They consist of a network of electrical components designed to generate, transmit, and distribute electrical energy for various uses. [1]. In general power systems are divided into the following sub-systems: Generating station, Transmission system, and Distribution systems as shown in Figure 1.1 [2]. The power plant, transformer, transmission line, substations, distribution line, and distribution transformer are six main components of power system. Electricity generation is multi-step process that transforms various raw materials like coal, natural gas, or nuclear fuel into usable electricity. These primary energy sources are often converted into a more usable form, mechanical energy, through gas turbines that burn fuel and spin a shaft. The generator, which uses electromagnetism to convert the mechanical energy into electrical energy, completes the electricity generation process. Turbines are the prime movers that convert steam energy into mechanical energy. One essential part of a turbine control system is a turbine governor. Its role is to control the turbine shaft's rotational speed so that it stays within a range that is both safe and effective. The transformers adjust the voltage levels so that the voltage is raised when power transmission is required and the voltage is dropped for power consumers in order to reduce the transmission losses. This means that power is efficiently transferred from one area to another by the use of transformers. With the exception of losses in the transformer, the power transfer from the secondary is roughly equivalent to main power. The portion of the power network that transfers high voltages from one region to another is known as transmission network. The transmission lines transmit the energy from generating stations to receiving stations after switching from one or more high voltage substations. Thus the transmission lines connect generating station to consumer (loaded) station and nearby substations. Similarly the network part that transfers relatively low voltages from the receiving stations to all the consumers in that region are called distribution network. They provide power to a few substations, which are often placed in handy locations close to the load centres. Power is distributed to home, commercial, and relatively small consumers through the substations.

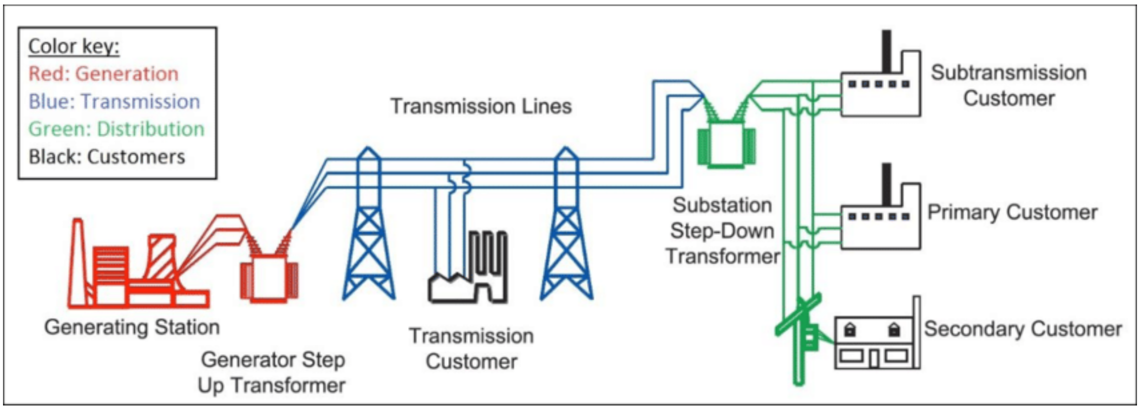


Figure 1.1: Electric Power System

1.2 Mathematical Modeling of Dynamical Systems

Mathematical modeling and simulations play major role for the analysis, design and optimization of power system [3]. These models capture the intricate relationships between different components of the physical system and act as virtual representations of it. One special form of modeling in time domain is state space modeling or matrix-vector representation of systems. State-space modelling provide a relationship between the input variables, output variables and internal state variables of the system. The model include a differential equation involving state variables and inputs and an algebraic equation of output involving states and inputs. Thus to compute the output for some specific input, we need the solution of differential equation to get the state variables and then use them to get the output.

For a given LTI system Σ, the state space representation of the system with p inputs, q outputs, and n state variables along with its representation is given as:

$$\begin{aligned} \dot{x}_L(t) &= \mathcal{A}x_L + \mathcal{B}u \\ y_L(t) &= \mathcal{C}x_L + \mathcal{D}u \end{aligned} \quad \rightarrow \quad \Sigma = \left(\begin{array}{c|c} \mathcal{A} & \mathcal{B} \\ \hline \mathcal{C} & \mathcal{D} \end{array}\right) \tag{1.2.1}$$

$\mathcal{A}$ is the state matrix with $\dim[\mathcal{A}] = n \times n, \mathcal{B}$ is the input matrix of $\dim[\mathcal{B}] = n \times p, \mathcal{C}$ is the output matrix of $\dim[\mathcal{C}] = q \times n$. The matrix with $\dim[D] = q \times p$. Also, the time derivative of $x(t)$ is denoted by $\dot{x}(t) := \frac{d}{dt}x(t)$.

Note that the first equation in (1.2.1) shows that the rate of change of the system state is dependent on the system state and the system input. Also, the second equation shows that the system output is dependent on the system state and the system input as well.

As most of the real-time systems are nonlinear and play a significant role in modeling several physical, biological, and engineering processes, nonlinear state space representations are often used for analysis[4]. Nonlinear systems may involve bilinear or quadratic or cubic or other polynomial nonlinear terms of inputs and states or in some cases their combination. In general, such nonlinear systems can be represented as:

$$\Sigma_{\text{nonlinear}} : \begin{cases} \dot{x}_N(t) = f(x_N, u) \\ y_N(t) = g(x_N, u) \end{cases} \tag{1.2}$$

To analyse such nonlinear dynamical systems, we need to solve the differential equation in (1.2) for $x(t)$ and utilize the output equation to observe the response of the system for specific input. For some specific nonlinear structures such as bilinear system or quadratic-bilinear systems, the nonlinear function $f(x, u)$ and $g(x, u)$ can be written in matrix-vector forms as well. Bilinear systems are similar to the linear system but they have additional terms involving product of state variables and control inputs. A large class of nonlinear systems can be represented in bilinear form and their are bilinearization processes that approximate the original nonlinear system to bilinear system. The dynamic behaviour of nonlinear power flow can also be represented/approximated by bilinear systems. The focus of this thesis is to analyse large-scale power systems that can be modelled by bilinear state space systems.

1.3 Model Order Reduction

Models for large scale systems involve a large set of differential equations and their solution is often computationally expensive. Model order reduction (MOR) is a methodology that aims to reduce the order or the number of differential equations so that less number of differential equations are required for computing a close approximation to the original solution[5]. This process facilitates efficient simulation and rapid analysis of complex systems. Thus the simulation of large-scale systems is more feasible especially in real-time applications with the use of MOR. A series of MOR methods were proposed to reduce the order of large scale systems to improve the efficiency of calculation in the field of science and engineering. MOR strategically simplifies these complex models into smaller, computationally efficient ”reduced-order models” (ROMs). These ROMs capture the essential dynamic behavior of the power system while being significantly faster to analyze. This translates to several benefits: faster simulations, streamlined control design, and the potential for real-time applications. The non-linear system are converted into bilinear system and then we use projection techniques to get the reduced version of original model. However, a successful application depends on selecting the appropriate projection subspace and evaluating the ROM's accuracy in relation to the original model. In general, projection techniques involve the following steps[6]:

- Compute basis matrices $V \in \mathbb{R}^{n\times r}$ and $W \in \mathbb{R}^{n\times r}$ for r dimensional subspaces $\mathcal{V}$ and $\mathcal{W}$, respectively - Approximate $x_L(t)$ by $Vx_{Lr}(t)$.

- Ensure Petrov-Galerkin conditions

$$W^T\left(Vx_{Lr}(t) - AVx_{Lr}(t) - Bu(t)\right) = 0$$
$$\hat{y}L(t) = \mathcal{C}Vx_{Lr}(t) + \mathcal{D}u(t)$$

Thus the reduced system matrices are given by:

$$\mathcal{A}_r = W^T\mathcal{A}V, \mathcal{B}_r = W^T\mathcal{B}, \mathcal{C}_r = \mathcal{C}V, \mathcal{D}_r = \mathcal{D} \text{ and } W^TV = I$$

and the reduced system becomes

$$\dot{x}_{Lr}(t) = \mathcal{A}_r x_{Lr}(t) + \mathcal{B}_r u(t)$$
$$\hat{y}_L(t) = \mathcal{C}_{\mathrm{r}} x_{Lr}(t) + \mathcal{D}_{\mathrm{r}} u(t)$$

This shows that the reduced system depends on the choice of V and W, or equivalently the sub-spaces $\mathcal{V}$ and $\mathcal{W}$. That is, we need to identify a good choice of V and W for which the reduced system ensures that $\tilde{y}_L(t) \approx y_L(t)$.

1.4 Problem Statement

As discussed before, a large class of nonlinear systems can be represented in bilinear state space form. Also nonlinear systems can be transformed/approximated in bilinear form. Nonlinear power systems are also modelled as bilinear systems in the literature. The response of the bilinear system is more close to the original nonlinear model as compared to the linear model. The issue of computational complexity, however, restrict the application of bilinear systems to medium or small scale systems. Thus the problem is to explore the concept of model order reduction for bilinear systems so that the simulation or control of a large-scale bilinear system is possible in a computationally efficient way. Mathematically, our problem is to identify, for a given bilinear system

$$\begin{aligned} \dot{x}(t) &= A x(t) + N x(t) u(t) + B u(t) \\ y(t) &= C x(t) + D u(t) \end{aligned}$$

a reduced bilinear system

$$\begin{aligned} \dot{x}_r(t) &= A_r x_r(t) + N_r x_r(t) u(t) + B_r u(t) \\ \hat{y}(t) &= C_r x_r(t) + D_r u(t) \end{aligned}$$

such that the response of the two systems is approximately the same. That is, to compute A_r, B_r, N_r, C_r, D_r, for given A, B, N, C and D such that $\hat{y}(t) \approx y(t)$ for a fixed input $u(t)$.

1.5 Motivation

The desire for the details and accurate output results generate large scale complex models that require huge computational resources for their analysis and design. Model order reduction is important tool for the analysis, control and optimization of systems as it reduces the computational complexity and produces approximately the same response, much faster. Their use for structured nonlinear systems further extends their applications and importance. As a system engineer, this project gives a detailed insight of how large scale systems should be analyzed and designed as well as on the importance of computational time and fast response in large-scale systems.

1.6 Objectives

The research work's objectives are as follows:

- To implement and simulate the original bilinear power system's model and its response to general inputs, and validate the results from the literature.
- To develop two projection-based non-linear MOR algorithms for efficient and accurate model reduction of bilinear power systems.
- Evaluate ROMs performance by comparing their simulation outputs to the original model to assess accuracy.

1.7 Thesis Overview

The remaining portion of the thesis is organized as follows: Chapter 2 presents the literature review on model order reduction techniques for linear and bilinear systems. Chapter 3 illustrates the practical implementation of model order reduction techniques tailored for bilinear power system models. In chapter 4, results and discussions are presented. The conclusion part is discussed in chapter 5.

Chapter 2

Literature Review

This chapter focuses on different MOR techniques for linear system and their extension to bilinear systems. There are various techniques of MOR in the literature to reduce linear systems. Since the reduction techniques for bilinear systems are extension of linear versions, therefore linear techniques are reviewed first to provide a basis for understanding the reduction of bilinear systems. For linear system, MOR techniques can be classified into two categories: singular value decomposition (SVD) methods and interpolation based methods.

2.1 Model Reduction of LTI Systems

The classical model reduction theory was developed for LTI systems. Among the most popular methods, balanced truncation, and Krylov projection methods have been used to compute the reduced order model[7]. The choice of model order reduction technique depends on many factors such as the desired accuracy, computational resources, system size, and specific requirements of the problem. In the following, we briefly discuss the Balanced truncation method for LTI systems, followed by projection based model order reduction techniques.

2.1.1 Balanced Truncation

Balanced truncation technique is widely used to reduce the LTI systems. The effectiveness of balanced truncation for reducing linear time-varying systems is analyzed for both discrete and continuous time domains [8]. It is used to simplify analysis and controller design by using a lower-order approximation that preserves the dynamics of interest. This method depends on the ideas of observability and controllability grammians. The ability to use outside inputs to change the state of the system from any initial condition is known as controllability. Conversely, observability describes the capacity to reassemble the state of the system from its outputs. The balanced truncation method decomposes the controllability and observability gramians to compute the transformation matrix T. This approach considers only those states which have major contribution in determining the systems response.
With reference to the equation (1.1) and (1.2), Assuming the dynamical system is stable, the associated Lyapunov equations for the gramians are given as [9]:

$$\begin{aligned} AP^T + PA^T + \quad BB^T &= 0 \\ A^TQ + \quad QA + C^TC &= 0 \end{aligned} \tag{2.1.1}$$

where P denotes the system's controllability gramians and its size is $P \in \mathbb{R}^{n\times n}$. Q denotes observability gramians and its size is $Q \in \mathbb{R}^{n\times n}$. In balanced model,both grammians are equal and diagonal i.e $P = \quad Q = \sum = \quad diag(\sigma_i)$.

Cholesky factorization of gramians and SVD can be used to get the transformation matrix T which can transform any system into balanced system, that is $T = R^{-1}U\sum^{-1/2}$.

$$A_b = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},\ B_b = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix},\ C_b = \begin{pmatrix} C_1 & C_2 \end{pmatrix}$$

The truncated matrices define the ROM and can be expressed as

$$A_r \quad = A_{11},\ B_r \quad = B_1,\ C_r \quad = C_1,\ D_r = D$$

An important property of balanced truncation is that it gives an a prior error bound on the error system in H_∞ norm.

$$\sigma_{r+1} \leq \|G - G_r\|_\infty \leq 2 \sum_{i=r+1}^{n} \sigma_i$$

where G_r is the reduced model. The square-root (SR) method is based on Cholesky factorization of the Gramians P and Q, computes bases for the left and right eigenspaces of product PQ of reachability and controlability grammians [10]. These square roots, known as singular values, highlight the importance of certain states in the system's dynamics. The approach achieves balanced truncation by retaining states associated with larger singular values and eliminating those corresponding to smaller values The square root technique allows for the robust computation of the reduced model by computing the bases for the left and right eigenspaces of the product PQ. Square root method computes bases for the left and right eigenspaces of product PQ of reachability and controlability grammians which enables the robust computation of reduced model. The use of orthogonal transformations Basis V_L and V_R , tends to increase numerical stability in algorithms by computing cholesky factors directly without computing P and Q.
Choleskly factorization of grammians:

$$L_r L_r^T := P$$

$$L_o L_o^T := Q.$$

singular-value-decomposition of $L_o^T L_r$

$$U \Sigma_1 V^T = L_\theta^T L_r$$

$$\Sigma_1 = \operatorname{diag}(\sigma_1, \cdots, \sigma_m)$$

The columns of L_0U are the respective left eigenvectors associated with eigenvalues $\sigma_1^2, \cdots, \sigma_m^2$ of PQ.

$$V_{R.BIG} = L_r V \begin{bmatrix} I_k \\ 0 \end{bmatrix}, V_{L,BIG} = L_o U \begin{bmatrix} I_k \\ 0 \end{bmatrix}.$$

$$E_{BIG} = V_{L,BIG}^T V_{R,BIG} = \begin{bmatrix} I_k & 0 \end{bmatrix} U^T L_O^T L_r V \begin{bmatrix} I_k \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} I_k & 0 \end{bmatrix} \operatorname{diag}(\sigma_1, \cdots, \sigma_m) \begin{bmatrix} I_k \\ 0 \end{bmatrix} = \operatorname{diag}(\sigma_1, \cdots, \sigma_k) = \hat{\Sigma}_{\text{BAL}} \text{ so that}$$

$$S_{L,\text{BiG}} = L_0 U \begin{bmatrix} \Sigma_{\text{BAL}}^{-1/2} \\ 0 \end{bmatrix}, \quad S_{R,\text{BIG}} = L_r V \begin{bmatrix} \bar{\Sigma}_{\text{BAL}}^{-1/2} \\ 0 \end{bmatrix}.$$

State transformations:

$$T = S_{R,\text{BiG}}$$

$$T^{-1} = S_{L,\text{BiG}}$$

$$(A_r, B_r, C_r, D_r) \leftarrow \left(T^{-1}AT, T^{-1}B, CT, D\right)$$

A balanced and stable reduced system that satisfies the BT error bound is computed by the SR technique. In balanced truncation, calculation of balancing Transformation T is ill-conditioned when PQ's condition number is high and some modes are unobservable and uncontrollable.

In the following, a ten-state model is reduced to a four-state model using the SR (square root) method:

$$A = \begin{pmatrix} -6 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -10 & 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -13 & -3 & 9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -8 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -14 & -9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 \times 10^{-3} & 1 \times 10^{-2} \end{pmatrix}$$

$$C = \begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 5 \times 10^{5} \\ 0 & 0 & 0 & 0 & 0 & 0 & -6 & 1 & -2 & 5 \times 10^{5} \end{pmatrix}, \quad D = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

The Hankel singular values are given below:

σ_1	σ_2	σ_3
2.5001×10^2	2.5168×10^{-2}	5.5789×10^{-3}

σ_4	σ_5	σ_6
2.4142×10^{-3}	9.2182×10^{-4}	1.3086×10^{-5}

σ_7	σ_8	σ_9	σ_{10}
1.1386×10^{-7}	6.7051×10^{-9}	0	0

The balancing of tenth order system using BT is numerically infeasible because system is uncontrollable and unobservable due to small hankel singular values. Therefore, the square root method is employed to effectively reduce the system's complexity.

2.2 MOR Projection Techniques

Most practical methods of solving large-scale linear systems of equations currently involve a projection process. These projection-based model reduction techniques provide numerous benefits and are applicable to both linear and nonlinear systems. Interpolatory projection-based techniques are used for simulation but also in control because the reduced model is independent of the system's input where input variation is a common practice. Interpolation techniques construct reduced models whose frequency response is approximately equal to original model at some pre-defined frequencies[11].

Choices for subspaces V & W:

i- Proposition 1

If $V = [B, AB, \ldots, A^{k-1}B]$ and W be any left inverse of V, such that $W^T V = I$.

Then the reduced system matches the first K-Markov parameters of the original system as given bellow:

$$CA^{k-1}B = C_r A_r^{k-1} B_r$$

Proof. The following set of equations leads to the desired results:

$$\begin{aligned}
C_r A_r^j B_r &= CVW^T A^j VW^T B \\
&= CVW^T A^j P_{ob} B, \quad \text{where } P_{ob} = VW^T \\
&= CVW^T A^j B, \quad \text{as } B \in span\{V\},\ P_{ob}B = B \\
&= CP_{ob} A^j B \\
C_r A_r^j B_r &= CA^j B \quad \text{as } A^j B \in span\{V\},\ P_{ob} A^j B = A^j B,\ j = 1, \ldots, K
\end{aligned} \tag{2.2.1}$$

ii- Proposition 2

Interpolation points σ_i, $i = 1, \ldots, k$, if $V = [(\sigma_1 I_n - A)^{-1}B, \ldots, (\sigma_k I_n - A)^{-1}B]$ and W is any left inverse of V, such that $W^T V = I$. Then the transfer function of the reduced system interpolates the transfer function of original system at the points s_j.

$$H(\sigma_j) = C(\sigma_j I_n - A)^{-1}B = \hat{C}(\sigma_j I_k - \hat{A})^{-1}\hat{B} = \hat{H}(\sigma_j) \quad j = 1, \ldots, k$$

iii- Proposition 3

Let $\sigma_0 \in \mathbb{C}$, $V = [(\sigma_0 I_n - A)^{-1}B, (\sigma_0 I_n - A)^{-2}B, \ldots, (\sigma_0 I_n - A)^{-k}B]$ and W be any left inverse of V, such that $W^T V = I$. Then the transfer function of the reduced system interpolates the transfer function of original system at the points σ_0 as well as the first $K - 1$ derivative at the same point.

$$\left.\frac{d^{k-1}}{ds^{k-1}} H(\sigma)\right|_{\sigma=\sigma_0} = \left.\frac{d^{k-1}}{ds^{k-1}} \hat{H}(\sigma)\right|_{\sigma=\sigma_0}$$

iv-Proposition 4

$$V = \left[A, A^{k-1}B(\sigma_1 I_n - A)B, (\sigma_k I_n - A)B(\sigma_0 I_n - A)^{-1}B, \ldots, (\sigma_0 I_n - A)^{-k}B\right]$$

Σ matches K-Markov parameters.

$$CA^{k-1}B = C_r A_r^{k-1} B_r$$

Σ interpolates the transfer function of Σ at the points s_j.

$$H(\sigma_j) = C(\sigma_j I_n - A)^{-1}B = C_r(\sigma_j I_k - A_r)^{-1}B_r = H_r(\sigma_j) \quad j = 1, \dots, k$$

v-Proposition 5 Given $2k$ distinct points $\sigma_1, \sigma_2, \dots, \sigma_{2k}$, if

$$V = [(\sigma_1 I_n - A)^{-1}B, \dots, (\sigma_k I_n - A)^{-1}B]$$

$$W = [(\sigma_{k+1} I_n - A)^{-1}C^*, \dots, (\sigma_{2k} I_n - A)^{-1}C^*]$$

The transfer function of $\hat{\Sigma}$ interpolates the transfer function of Σ at the points σ_j, $j = 1, \dots, 2k$.

2.3 Iterative Rational Krylov Algorithm

The Iterative Rational Krylov Subspace (IRKA) approach is a model reduction methodology that lowers the dimensionality of a system by combining Krylov subspace methods and rational approximation. It is especially helpful in bringing down the complexity of large linear or nonlinear systems. The IRKA is highly beneficial for simulation and control tasks involving large-dimensional networks[12]. The algorithm generates a Krylov subspace, which is a lower-dimensional subspace within the state space of the high-dimensional system. IRKA revolves around an iterative process, by repeatedly applying the system matrix to a seed vector, the algorithm spans a Krylov subspace The controllability and observability properties of the system are then projected onto this subspace. IRKA obtains a lower-dimensional approximation of the original system by solving a tiny eigenvalue problem within this reduced space. Lastly, the technique refines the approximation by updating the parameter based on the eigenvalues of the original system. IRKA is a well-known method for generating locally optimal reduced-order H2 -approximations for linear time-invariant (LTI) dynamical systems. IRKA performs a Hermite type interpolation of the initial system transfer function at each iteration. The transfer function for the linear system given in (1.1), can be written as:

$$G(s) = C(sI - A)^{-1} \quad B \tag{2.2}$$

By using a stable rational transfer function G, MOR attempts to approximate the transfer function G_r.

$$G_r(s) = C_r(sI_r - A_r)^{-1} \quad B_r \tag{2.3}$$

IRKA Algorithm for Linear Systems [13]

Input: A, B, C and r

Output: A_r, B_r, C_r

1: Make initial r-fold shift selection: $\{\sigma_1, \ldots, \sigma_r\}$ that is closed under conjugation (i.e., $\{\sigma_1, \ldots, \sigma_r\} \equiv \{\bar{\sigma}_1, \ldots, \bar{\sigma}_r\}$ viewed as sets) and tangential directions $\hat{b}_1, \ldots, \hat{b}_r$ and $\hat{c}_1, \ldots, \hat{c}_r$, also closed under conjugation.

4: WHILE NOT CONVERGED:

1. Set iter = 1

2. Compute basis of span $\left((\sigma_1\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}, \ldots, (\sigma_r\mathbf{I} - \mathbf{A})^{-1} \quad \mathbf{B}\right) \to \mathbf{V}$

3. Compute basis of span $\left(\left(\sigma_1 \quad \mathbf{I} - \mathbf{A}^T\right)^{-1} \quad \mathbf{C}, \ldots, \quad \left(\sigma_r\mathbf{I} - \mathbf{A}^T\right)^{-1}\mathbf{C}\right) \to \mathrm{W}$

4. $\mathbf{A}_r = \quad \left(\mathbf{W}^T\mathbf{V}\right)^{-1} \quad \mathbf{W}^T\mathbf{AV}$

5. Compute eigenvalues$\lambda\left(\mathbf{A}_r\right) = \quad \left(\lambda_1\left(\mathbf{A}_r\right), \ldots, \lambda_r \quad \left(\mathbf{A}_r\right)\right)$

6. Compute (matching) distance ζ between the sets $\lambda\left(\mathbf{A}_r\right)$ and $-whiteT\sigma$

7. $\sigma_i \quad \longleftarrow -\lambda_i\left(\mathbf{A}_r\right), i = 1, \quad \ldots, r$

8. Repeat until ζ is sufficiently small

9. $\mathbf{B}_r = \quad \left(\mathbf{W}^T\mathbf{V}\right)^{-1}\mathbf{W}^T\mathbf{B}; \quad \mathbf{C}_r = \quad \mathbf{V}^T\mathbf{C}$ 10. The reduced order model is $(\mathbf{A}_r, \mathbf{B}_r, \quad \mathbf{C}_r)$

IRKA operates by iteratively refining its approximation of optimal interpolation points.

The iteration stops when the relative change in the set of shifts between two successive iterations drops below a specified tolerance threshold.

2.4 MOR for Bilinear Systems

This section presents a concise introduction to bilinear systems, which are characterized by nonlinear interactions between variables. These systems can be effectively analyzed using both frequency domain and time domain methods These systems exhibit linearity concerning the state and the control individually, but not jointly. The exploration focuses on MOR techniques applied specifically to bilinear dynamical systems using projection-based methods. With m inputs and p outputs, a MIMO bilinear dynamical system is represented in the time domain as follows [14]:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{m} N_i x(t) u_i(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{2.4}$$

where $x(t)$ is $n \times 1$ state vector, $u(t)$ is $m \times 1$ input vector, u_i is ith component of $u(t)$, $y(t)$ is $p \times 1$ output vector and $A, N_1, N_2, \ldots, N_m, B$, and C are actual matrices of appropriate size. Bilinear systems are employed in control theory to model systems exhibiting both linear and nonlinear behavior. In signal processing, bilinear systems are commonly used to simulate nonlinear distortion in devices such as amplifiers and filters. Bilinear systems are utilized in computer graphics for tasks such as interpolating between points and constructing smooth surfaces.

2.4.1 Balanced Truncation for Bilinear Systems

Although Balanced Truncation, a popular method of model reduction for linear systems, but it also finds the application to bilinear systems. Balanced truncation is a favored approach for simplifying the complexity of bilinear systems, as it effectively preserves essential system dynamics while significantly reducing computational demands[15]: Step 1. P and Q. The controllability and observability Grammian matrices are expressed as follows.

$$\mathbf{P} = \sum_{i=1}^{\infty} \int \cdots \int \boldsymbol{P}_i \boldsymbol{P}_i^* \ \mathrm{d}t_1 \ldots \mathrm{d}t_i$$

and

$$\mathbf{Q} = \sum_{i=1}^{\infty} \int \cdots \int \boldsymbol{Q}_i^* \boldsymbol{Q}_i \ \mathrm{d}t_1 \ldots \mathrm{d}t_i$$

The Grammians fulfill the following generalized algebraic Lyapunov equations:

$$\mathbf{AP} + \mathbf{PA}^\top + \sum_{i=1}^{m} \mathbf{N}_i \mathbf{P} \mathbf{N}_i^\top + \mathbf{BB}^\top = 0$$
$$\mathbf{A}^\top \mathbf{Q} + \mathbf{QA} + \sum_{i=1}^{m} \mathbf{N}_i^\top \mathbf{Q} \mathbf{N}_i + \mathbf{C}^\top \mathbf{C} = 0$$

Step 2.Using $\mathbf{L}_r$ and $\mathbf{L}_0$ to represent the lower triangular Cholesky factors of Grammians $\mathbf{P}$ and $\mathbf{Q}$, compute the Cholesky factors of the Grammians.

$$\mathbf{P} = \mathbf{L}_r \mathbf{L}_{\mathrm{r}}^{\mathrm{T}}, \quad \mathbf{Q} = \mathbf{L}_o \mathbf{L}_{\mathrm{o}}^{\mathrm{T}}$$

Step 3. Determine how to calculate the product of the Cholesky factors' singular value decomposition:

$$\mathbf{L}_{\mathrm{o}}^{\mathrm{T}} \mathbf{L}_{\mathrm{r}} = \mathbf{U} \Sigma \mathbf{V}^{\mathrm{T}}$$

Step 4. Create the balanced transition, matrix T

$$\mathbf{T} = \mathbf{L}_r \mathbf{V} \boldsymbol{\Sigma}^{-1/2}$$

Step 5. The balanced system of (2.4) has a state-space representation that is

$$\dot{\boldsymbol{x}}_{\mathrm{bb}}(t) = \mathbf{A}_{\mathrm{bb}}\boldsymbol{x}_{\mathrm{bb}}(t) + \sum_{\mathrm{i}=1}^{m} \mathbf{N}_{\mathrm{bb}i}\boldsymbol{x}_{\mathrm{bb}}(t)u_i(t) + \mathbf{B}_{\mathrm{bb}}\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = \mathbf{C}_{\mathrm{bb}}\boldsymbol{x}_{\mathrm{bb}}(t)$$

where

$$\mathbf{A}_{\mathrm{bb}} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}, \quad \mathbf{N}_{\mathrm{bb}i} = \mathbf{T}^{-1}\mathbf{N}_{\mathrm{i}}\mathbf{T}$$

$$\mathbf{B}_{\mathrm{bb}} = \mathbf{T}^{-1}\mathbf{B}, \quad \mathbf{C}_{\mathrm{bb}} = \mathbf{C}\mathbf{T}$$

Let's split the balanced system given in equation (2.5) as follows to get a reduced order model:

$$\begin{bmatrix} \dot{\boldsymbol{x}}_{\mathrm{bb1}} \\ \dot{\boldsymbol{x}}_{\mathrm{bb2}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{\mathrm{bb1}} \\ \boldsymbol{x}_{\mathrm{bb2}} \end{bmatrix}$$

$$+ \sum_{i=1}^{\mathrm{m}} \begin{bmatrix} \mathbf{N}_{11i} & \mathbf{N}_{12i} \\ \mathbf{N}_{21i} & \mathbf{N}_{22i} \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{\mathrm{bb}} \\ \boldsymbol{x}_{\mathrm{bb2}} \end{bmatrix} u_i + \begin{bmatrix} \mathbf{B}_1 \\ \mathbf{B}_2 \end{bmatrix} \boldsymbol{u}$$

$$\boldsymbol{y} = \begin{bmatrix} \mathbf{C}_1 & \mathbf{C}_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}_{\mathrm{bb1}} \\ \boldsymbol{x}_{\mathrm{bb2}} \end{bmatrix}$$

$$\mathbf{\Sigma} = \begin{bmatrix} \mathbf{\Sigma}_1 & 0 \\ 0 & \mathbf{\Sigma}_2 \end{bmatrix}$$

where $\mathbf{\Sigma}_1 = \operatorname{diag}[\sigma_1, \ldots, \sigma_r]$ and $\mathbf{\Sigma}_2 = \operatorname{diag}[\sigma_{r+1}, \ldots, \sigma_n]$. If $\sigma_r/\sigma_{r+1} \gg 1$, then the subsystem represented by the most controllable and observable states can be effectively decoupled from the rest of the system.

$$\dot{\boldsymbol{x}}_{\mathrm{bbr}}(t) = \mathbf{A}_{11}\boldsymbol{x}_{\mathrm{bbr}}(t) + \sum_{i=1}^{m} \mathbf{N}_{11i}\boldsymbol{x}_{\mathrm{bbrr}}u_i(t) + \mathbf{B}_1\boldsymbol{u}(t)$$

$$\hat{\boldsymbol{y}}(t) = \mathbf{C}_1\boldsymbol{x}_{\mathrm{bbr}r}(t)$$

These equations outline how the original system matrices are transformed into reduced matrices using the transformation matrix. The $\mathcal{H}_2$ norm for bilinear systems quantifies the system's performance by evaluating the energy gain from disturbances, expressed through the reachability and observability Grammians P and Q.

$$\|\Sigma_B\|_{H_2} = \sqrt{CPC^T} = \sqrt{B^TQB}$$

where C is output matrix, B is input matrix.

2.4.2 Bilinear Iterative Rational Krylov Algorithm (BIRKA)

This section explores the BIRKA and its application in MOR for bilinear dynamical systems. BIRKA is designed as a locally optimal algorithm aimed at approximating the behavior of the original system within a limited region of the state space. BIRKA iteratively constructs a rational Krylov subspace tailored for bilinear systems. A Krylov subspace is a subspace within the state space that comprises vectors generated from the initial state vector through successive applications of the bilinear operator. It is highly advantageous for reducing the complexity of bilinear dynamical systems while preserving accuracy. The BIRKA algorithm is based on the interpolatory projection approach, is a very well-known, standard, and mathematically acceptable algorithm for bilinear MOR.

$$\dot{x}_r(t) = A_r x_r(t) + N_r x_r(t) u + B_r u, \quad \dot{y}_r(t) = C_r x_r(t), \quad x_r(0) = 0 \tag{2.8}$$

where

$$\dot{y}_r(t) \approx y(t)$$

If BIRKA converges, it will produce a locally H_2 optimal reduced system that satisfies the necessary conditions for H_2 optimality. The following section explains the algorithm BIRKA for constructing the required H_2-optimal reduced bilinear system [16].

Algorithm BIRKA

1: Given input bilinear dynamical system $A, N1, \ldots, Nm, B, C$.

2: Select initial guess for the reduced system as $\bar{A}, \bar{N}_1 1, \ldots, \bar{N}_m, \bar{B}, \bar{C}$.

Also select stopping tolerance btol.

3: while relative change in eigenvalues of $\bar{A} \geq$ btol.

a. $RAR^{-1} = \breve{A}, \breve{B} = \breve{B}^T R^{-T}, \breve{C} = \breve{C}R, \breve{N}_k = R^T \breve{N}_k R^{-T}$ for $k = 1, \ldots, m$.

b. $\text{vec}(V) = \left(-\Lambda \otimes I_n - I_r \otimes A - \sum_{k-1}^{m} \overset{\bullet}{N}_k^T \otimes N_k\right)^{-1} \left(B^T \otimes B\right) \text{vec}\,(I_m)$.

c. $\text{vec}(W) = \left(-\Lambda \otimes I_n - I_r \otimes A^T - \sum_{k-1}^{m} N_k \otimes N_k^T\right)^{-1} \left(C^T \otimes C^T\right) \text{vec}\,(I_p)$.

d. $V_r = \text{orth}(V), W_r = \text{orth}(W)$.

e.

$$\bar{A} = \left(W_r^T V_r\right)^{-1} W_r^T A V_r, \quad \bar{N}_k = \left(W_r^T V_r\right)^{-1} W_r^T N_k V_r$$

$$\bar{B} = \left(W_r^T V_r\right)^{-1} W_r^T B, \quad \bar{C} = C V_r$$

f.

$$A_r = \bar{A}, \quad N_{k_r} = \bar{N}_k, \quad B_r = \bar{B}, \quad C_r = \bar{C}$$

2.4.3 Research Gap

Existing literature on MOR for k-power bilinear systems highlights a significant gap in systematic approaches that can effectively balance computational efficiency with accurate representation of their complex nonlinear interactions. This study proposes the implementation of two algorithms using projection-based techniques for reducing the model complexity of k-power bilinear systems. The comparative analysis focuses on evaluating the computational time required by each algorithm to generate reduced-order models across varying dimensions. This analysis aims to determine which algorithm offers optimal computational efficiency while maintaining satisfactory model accuracy. Moreover, the study investigates the trade-offs between computational time and model fidelity, providing valuable insights for enhancing computational performance in practical applications.

Chapter 3

Design and Methodology

This chapter applies MOR techniques to bilinear power system models, demonstrating their effectiveness through two benchmark examples from literature. It highlights MOR's ability to simplify complex models while maintaining essential dynamics, aiming to improve computational efficiency in representing bilinear power system behaviors.

3.1 Bilinear Representation of Power Systems

This section begins with the comprehensive overview of how power systems can be effectively modeled using bilinear state space models, drawing upon examples from existing literature to illustrate the application and utility of this approach in understanding and analyzing the dynamic behavior of power systems.

3.1.1 Two-area Interconnected Power System

A two-area interconnected, power system is vital for ensuring reliable and efficient electricity delivery by enabling resource sharing between regions. It consists of two distinct regions, each with its own independent generating tion and load characteristics. These areas are then strategically linked via dedicated transmission lines known as tie-lines as shown in schematic depiction of power system in Figure 3.1 [17]. In addition to enabling the efficient use of generation resources and offering redundancy which is essential for averting outages this interconnected structure improves system stability.

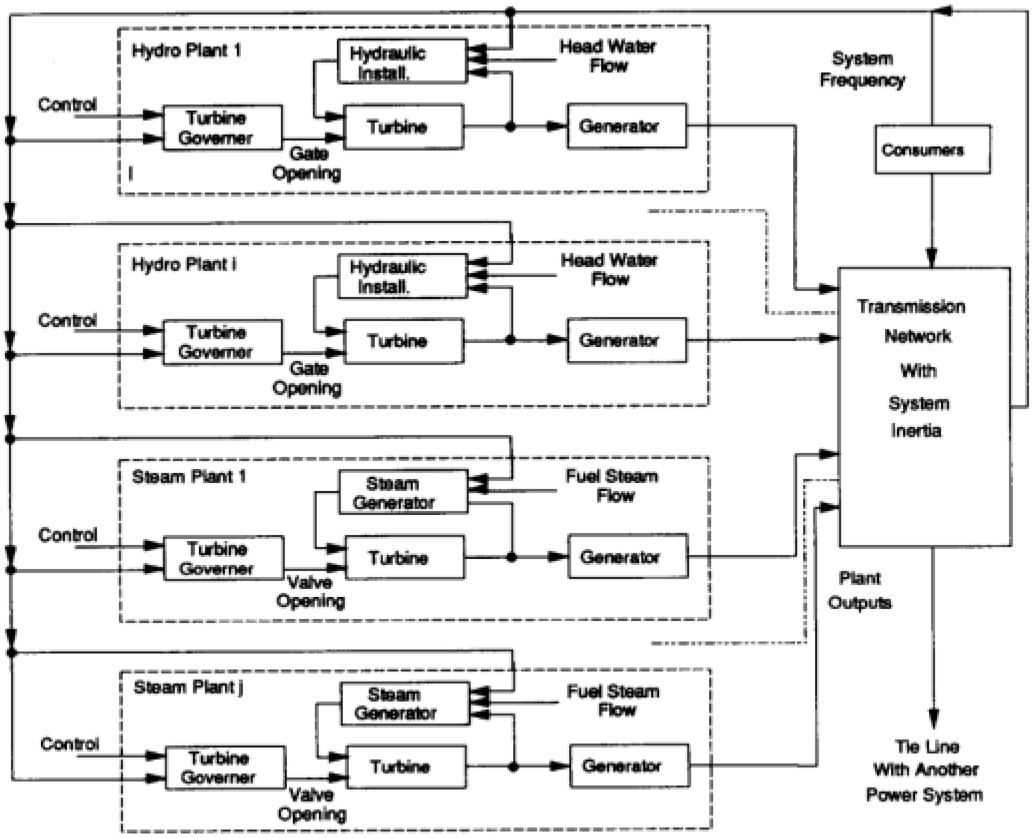


Figure 3.1: Two area interconnected Power System

The exchange of power between the two areas is controlled by a power management system. Each area has one hydro plant and one steam plant. All four plants have four control inputs that is the change in Δf_{mT1}, Δf_{mH1}, Δf_{mH2} and Δf_{mT2}. Where Δf_{mH1} and Δf_{mH2} are hydro unit control input variations. Δf_{mT1} and Δf_{mT2} are steam unit control input variations. The three outputs are frequency variations and tie-line exchange variations that are represented by Δf_1, Δf_2 and ΔP_{12} respectively. The two-area interconnection comprises regions linked by tie lines, enabling electricity transfer between them.

When a power plant exceeds its capacity during peak demand, other connected plants can share the extra load. Power systems interconnect to improve efficiency and ensure reliable supply. Interconnected networks have unique challenges such as load sharing, minimizing frequency errors, and maintaining reliable power supply compared to stand-alone systems. Connecting stand-alone power systems involves linking areas via tie-lines, with each area controlling its own load variations. This system, based on the physical interactions between various components, will be used to construct mathematical models of individual elements and the power system as a whole. The elements to be considered are: The core components listed are critical for modeling and managing a robust power system. A model for two area interconnected power networks with one steam and one hydro unit is as follows: **System 1 Steam Plant**

Pulverized coal is introduced into the boiler and combusted in the furnace. The turbine's rotor is connected to a generator, which rotates at the same speed as the turbine. The turbine converts mechanical energy into electrical energy through the generator. After leaving the turbine, the steam flows into the condenser, where it is condensed back into water. The condensed water, along with the feed water, is routed to the economizer. In the economizer, the feed water is preheated before entering the boiler, enhancing the overall efficiency of the boiler.

A model for steam plant is represented as follows:

$$\Delta\dot{\omega}_{T1} = \frac{-r_{T1}}{T_{ST1}}\Delta\omega_{T1} - \frac{1}{T_{ST1}}\Delta f_1 + \frac{1}{T_{ST1}}\Delta f_{mT1}$$

$$\Delta\dot{P}_{t1}^{(1)} = \frac{K_{t1}}{T_{t1}}\Delta\omega_{T1} - \frac{1}{T_{t1}}\Delta P_{t1}^{(1)}$$

$$\Delta\dot{P}_{t2}^{(1)} = \frac{1}{T_{n1}}\Delta P_{t1}^{(1)} - \frac{1}{T_{n1}}\Delta P_{t2}^{(1)}$$

$$\Delta\dot{P}_{t3}^{(1)} = \frac{1}{T_{n1}}\Delta P_{t2}^{(1)} - \frac{1}{T_{n1}}\Delta P_{t3}^{(1)}$$

$\Delta\omega_{T1}$ is the variation of steam turbine valve opening in the system, ΔP_{t1} is the variation of steam turbine at high pressure output, ΔP_{t2} is the variation of steam turbine at intermediate pressure output, and ΔP_{t3} is the variation of steam turbine at low pressure output.

There are also some coefficients being used; for details, check [18]. Similarly, we can write a representation for the second steam plant.

For System 1 Hydro Plant

A hydroelectric power plant, or hydro plant, generates electricity by harnessing the energy of moving or falling water. A consistent water source, such as a river or a reservoir, is necessary for a hydroelectric power plant. A generator is connected to the rotating turbine. As the turbine spins, it drives the rotor of the generator, which is surrounded by a stationary stator. The relative motion between the rotor and stator creates an electromagnetic field, which generates electricity.

A model for hydro plant is represented as follows:

$$\begin{aligned}
\Delta \dot{a}_{\mathrm{H1}} =& \frac{-\varepsilon_{\mathrm{T1}} C_{\mathrm{V1}} T_{\mathrm{al}}}{T_1 T_{\mathrm{SH1}}} \Delta P_{\mathrm{t1}}^{(1)} - \frac{\varepsilon_{\mathrm{T1}} \left(1 - C_{\mathrm{V1}}\right) C_{\mathrm{S1}} T_{\mathrm{a1}}}{T_1 T_{\mathrm{SH1}}} \Delta P_{\mathrm{t2}}^{(1)} \\
&- \frac{\varepsilon_{\mathrm{T1}} \left(1 - C_{\mathrm{V1}}\right) \left(1 - C_{\mathrm{S1}}\right) T_{\mathrm{al}}}{T_1 T_{\mathrm{SH1}}} \Delta P_{\mathrm{t3}}^{(1)} + \frac{1}{T_{\mathrm{SH1}}} \left(\frac{\varepsilon_{\mathrm{H1}} K_{\mathrm{w1}} T_{\mathrm{al}}}{T_1} - r_{\mathrm{H1}} - r_1' \right) \Delta a_{\mathrm{H1}} \\
&+ \frac{1}{T_{\mathrm{SH1}}} \Delta V_1 - \frac{\varepsilon_{\mathrm{H1}} K_{\mathrm{q1}} T_{\mathrm{a1}}}{T_1 T_{\mathrm{SH1}}} \Delta q_1 + \frac{e_1 T_{\mathrm{a1}} - T_1}{T_1 T_{\mathrm{SH1}}} \Delta f_1 \\
&+ \frac{T_{\mathrm{a1}}}{T_1 T_{\mathrm{SH1}}} \Delta P_{12} + \frac{1}{T_{\mathrm{SH1}}} \Delta f_{\mathrm{mH1}} + \frac{T_{\mathrm{a1}}}{T_1 T_{\mathrm{SH1}}} \Delta P_{\mathrm{L}\alpha} \\
\Delta \dot{V}_1 =& \frac{r_1'}{T_{\mathrm{e1}}} \Delta a_{\mathrm{H1}} - \frac{1}{T_{\mathrm{el}}} \Delta V_1 \\
\Delta \dot{q}_1 =& \frac{1}{T_{\mathrm{w1}}} \Delta a_1 - \frac{1}{T_{\mathrm{q1}}} \Delta q_1 + \frac{1}{T_{\mathrm{f1}}} \Delta f_1
\end{aligned}$$

The state-space variables of the two coupled mixed power systems are as follows.

$$\begin{aligned}
&x_1 = \Delta a_{\mathrm{T1}}, \quad x_2 = \Delta P_{\mathrm{t1}}, \quad x_3 = \Delta P_{\mathrm{t2}}, \quad x_4 = \Delta P_{\mathrm{t3}} \\
&x_5 = \Delta a_{\mathrm{H1}}, \quad x_6 = \Delta V_1, \quad x_7 = \Delta q_1, \quad x_8 = \Delta f_1 \\
&x_9 = \Delta P_{12}, \quad x_{10} = \Delta f_2, \quad x_{11} = \Delta q_2, \quad x_{12} = \Delta V_2 \\
&x_{13} = \Delta a_{\mathrm{H2}}, \quad x_{14} = \Delta P_{\mathrm{t3}}^{(2)}, \quad x_{15} = \Delta P_{\mathrm{t2}}^{(2)}, \quad x_{16} = \Delta P_{\mathrm{t1}}^{(2)} \\
&x_{17} = \Delta a_{\mathrm{T2}}
\end{aligned}$$

The relationship between state space variables is given by literature. The dynamics of x_1 and x_2 can be written as

$$\dot{x}_1 = A_1x_1 + A_2x_4 + B_1u_1, \quad \dot{x}_2 = A_3x_1 + A_4x_2$$

Since the state equations x_5, x_8, x_{10} and x_{13} have non-linearities, this system can be expressed in the form given by equation (2.1).

The two terms z_1 and z_2 represent the parameters that capture the bilinear interactions between the input vector u and state vector x.

$$z_1 = k_1x_5u_1 + k_2x_8u_2, \quad z_2 = k_3x_{10}u_3 + k_4x_{13}u_4.$$

3.2 Model Order Reduction of k-Power Systems

This research focuses on the application of two reduction techniques, namely Balanced Truncation and BIRKA reduction, to k-power systems. These methods are utilized to simplify the dynamic models of k-power systems, aiming to enhance computational efficiency and facilitate analysis and control tasks[19]. An example of k-power systems from the literature is discussed to illustrate this approach.

3.2.1 k-Power Bilinear System

The k-power system is identified as a specific type of bilinear system made up of several lower-order subsystems. This property highlights its ability to be decomposed or represented as interconnected subsystems, each potentially exhibiting simpler dynamics. This structural feature makes the k-power system distinct in the realm of dynamic systems analysis. A subclass of bilinear systems is characterized by an input-output map that exhibits homogeneity with respect to the input u(t), where the degree of homogeneity is k. This condition is expressed as

$$y(\alpha u(t)) = \quad \alpha^k y(u(t))$$

A k-power bilinear system adheres to this equation, indicating that its input output behavior satisfies the homogeneity property. Such systems are also known as zero state responses of internally bilinear systems and describe configurations where linear subsystems are interconnected multiplicatively. The minimal bilinear realization of k-power system is given by[20]:

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \vdots \\ \dot{x}_k(t) \end{bmatrix} = \begin{bmatrix} A_1 & 0 & \cdots & 0 \\ 0 & A_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & A_k \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_k(t) \end{bmatrix} + \sum_{i=1}^{m} \begin{bmatrix} 0 & 0 & \cdots & 0 \\ N_{1i} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & N_{(k-1)i} & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_k(t) \end{bmatrix} u_i(t)$$
$$+ \begin{bmatrix} B_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} u(t)$$

where $x_j \in \mathbb{R}^{n_j}$, $A_j \in \mathbb{R}^{n_j \times n_j}$, $N_{ji} \in \mathbb{R}^{n_{j+1} \times n_j} (j = 1, 2, \ldots, k-1), x_k \in \mathbb{R}^{n_k}, A_k \in \mathbb{R}^{n_k \times n_k}$.

For simplicity, we assume a zero initial condition. With the above canonical realisation, K-power systems can be reformulated as coupled systems

$$\begin{cases} \dot{x}_1(t) = A_1 x_1(t) + B_1 u(t) \\ \dot{x}_2(t) = A_2 x_2(t) + \sum_{i=1}^{m} N_{1i} x_1(t) u_i(t) \\ \cdots \\ \dot{x}_k(t) = A_k x_k(t) + \sum_{i=1}^{m} N_{(k-1)i} x_{k-1}(t) u_i(t) \\ y(t) = C_k x_k(t) \end{cases}$$

This approach ensures that the reduction process preserves coupled structure of the system, thereby accurately capturing the interdependencies and interactions between subsystem.

3.2.2 k-Power Model Reduction Algorithm

In this section, the application of Balanced Truncation and BIRKA algorithms to k-power systems, as discussed in Chapter 2, is explored. Projection-based techniques utilize projection matrices to construct smaller models while preserving the structural characteristics of the original systems.

Balanced Truncation for k-Power Bilinear System

This section will discuss an algorithm for

$$
\begin{gathered}
A_1 P_{11} + P_{11} A_1^T + B_1 B_1^T = 0 \\
A_j P_{jj} + P_j A_j^T + \sum_{i=1}^{m} N_{(j-1)i} P_{(j-1)(j-1)} N_{(j-1)i}^T = 0 \\
\vdots \\
A_k^T Q_{kk} + Q_{kk} A_k + C_k^T C_k = 0 \\
A_j^T Q_j + Q_k A_j + \sum_{i=1}^{m} N_{ji}^T Q_{(j+1)(j+1)} N_{ji} = 0
\end{gathered}
$$

while $j = k-1, k-2, \ldots, 2, 1$

ii. Compute Cholesky factors of P_{ij} and Q_{ij} : Let L_{rj} and L_{oj} denote the lower triangular Cholesky factors of P_{ij} and Q_{ij}, i.e.,

$$P_{ij} = L_{rj} L_{rj}^T, \quad Q_{jj} = L_{oj} L_{oj^*}^T$$

iii. Compute the singular value decomposition

$$L_{oj}^T L_{rj} = U_j \Sigma_j V_j^T$$

iv. Form the balancing transformation for the subsystems

$$T_j = L_{rj} V_j \Sigma_j^{-1/2}$$

v. Form the balancing transformation

$$T = \operatorname{diag}\left[T_1, T_2 \ldots, T_k\right]$$

vi. Form the balanced state-space matrices

$$A_{bj} = T_j^{-1} A_j T_j, \quad B_{bl} = T_1^{-1} B_1, \quad C_k = C_k T_k, \quad j = 1, 2, \ldots, k$$
$$N_{bjl} = T_{j+1}^{-1} N_{ji} T_j \quad j = 1, 2, \ldots, k-1 \quad i = 1, 2, \ldots, m$$

To obtain a reduced order model, balanced matrices can be partitioned as

$$A_{bj} = \begin{bmatrix} A_{bj11} & A_{bj12} \\ A_{bj21} & A_{bj22} \end{bmatrix}, \quad N_{bji} = \begin{bmatrix} N_{bjil1} & N_{bji12} \\ N_{bji21} & N_{bji22} \end{bmatrix}$$
$$B_{b1} = \begin{bmatrix} B_{b11}^T & B_{b12}^T \end{bmatrix}^T, \quad C_{bk} = \begin{bmatrix} , C_{bk11} & C_{bk12} \end{bmatrix}$$

Also, let Σ_j be partitioned :

$$\mathbf{\Sigma}_j = \begin{bmatrix} \mathbf{\Sigma}_{j1} & 0 \\ 0 & \mathbf{\Sigma}_{j2} \end{bmatrix}$$

where $\Sigma_{j1} = \operatorname{diag}\left[\sigma_{jl}, \ldots, \sigma_{j]}\right]$ and $\Sigma_{j2} = \operatorname{diag}\left[\sigma_{j(r+1)}, \ldots, \sigma_{jv}\right]$. If $\sigma_{jr}/\sigma_{j(r+1)} > 1$ for $j = 1, 2, \ldots, k$, then the reduced order model is given by

$$\dot{x}_{br}(t) = A_{br} x_{br}(t) + \sum_{i=1}^{m} N_{bri} x_{br} \mu_i(t) + B_{br} \mu(t)$$
$$\hat{y}(t) = C_{br} x_{br}(t)$$

where

$A_{br} = \text{diag}\left[A_{bII}, A_{b211}, \ldots, A_{klI}\right]$

$$\mathrm{B}_{br} = \begin{bmatrix} B_1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, N_{bri} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ N_{1i11} & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & N_{(k-1)i11} & 0 \end{bmatrix}$$
$$C_{br} = \begin{bmatrix} 0 & 0 & \ldots & 0 & C_{bkl} \end{bmatrix}$$

Bilinear Iterative Rational Krylov Algorithm (BIRKA)

Given an input bilinear dynamical k-power system $A, N_1, \ldots, N_m, B, C$ where k is the power of the system[21].

$$N_m = \begin{bmatrix} 0 & 0 \\ N_{(j-1)m} & 0 \end{bmatrix}$$

for 2-power bilinear system

$$N_1 = \begin{bmatrix} 0 & 0 \\ N_{11} & 0 \end{bmatrix}, \quad N_2 = \begin{bmatrix} 0 & 0 \\ N_{12} & 0 \end{bmatrix}$$

2: Select initial guess for reduced system as $\bar{A}, \bar{N}_1, \ldots, \bar{N}_m, \bar{B}, \dot{C}$. Also select the stopping tolerance btol.

3. While (relative change in eigenvalues of $\bar{A} \geq$ btol):

a. $R\Lambda R^{-1} = \check{A}, \quad \breve{B} = \check{B}^T R^{-T}, \check{C} = \check{C}R, \check{N}_k = R^T \check{N}_k R^{-T}$ for $k = 1, \ldots, m$.

b. $\text{vec}(V) = \left(-\Lambda \otimes I_n - I_r \otimes A - \sum_{k=1}^{m} \dot{N}_k^T \otimes N_k\right)^{-1} \left(\check{B}^T \otimes B\right) \text{vec}\left(I_m\right)$.

c. $\text{vec}(W) = \left(-\Lambda \otimes I_n - I_r \otimes A^T - \sum_{k=1}^{m} N_k \otimes N_k^T\right)^{-1} \left(\check{C}^T \otimes C^T\right) \text{vec} \left(I_p\right)$.

d. $V_r = \text{orth}(V), \quad W_r = \text{orth}(W)$.

e.

$$\breve{A} = \left(W_r^T V_r\right)^{-1} \quad W_r^T A V_r$$
$$\breve{N}_k = \left(W_r^T V_r\right)^{-1} W_r^T N_k V_r$$
$$\breve{B} = \left(W_r^T V_r\right)^{-1} W_r^T B$$
$$\breve{C} = C V_r$$

4. $A_r = \quad \breve{A}, N_{k_r} = \breve{N}_k, B_r = \quad \breve{B}, C_r = \quad \breve{C}$.

5.

$$N_{r1} = N_{kr}(:, 1 : r)$$
$$N_{r2} = N_{kr}(:, r + 1 : 2r)$$

By iteratively improving rational Krylov subspaces, the BIRKA accurately preserves system dynamics and maximizes computational efficiency when reducing bilinear systems.

Chapter 4

Results and Discussion

This chapter details the implementation of MOR for k-power systems using two benchmark examples from literature. It compares the output responses of the original and reduced systems, presents absolute error plots, and calculates computational times. The analyses demonstrate MOR's effectiveness in power system simulations.

4.1 Example 1: SISO k-Power Bilinear System

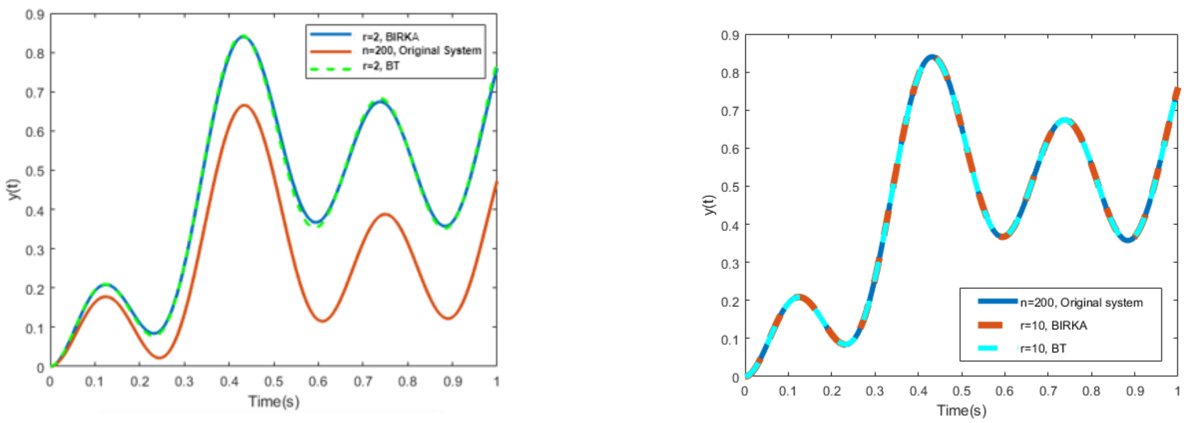
Consider a single-input-single-output (SISO) 2-power bilinear system consisting of two subsystems as described in [22].

$$A_1 = \begin{bmatrix} -10 & 2 & & & \\ 7 & -10 & 2 & & \\ & \ddots & \ddots & \ddots & \\ & & & 7 & -10 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -5 & 2 & & & \\ 2 & -5 & 2 & & \\ & \ddots & \ddots & \ddots & \\ & & & 2 & -5 \end{bmatrix}, N_{11} = \begin{bmatrix} 2 & 1 & & & \\ -1 & 2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & & -1 & 2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}^{\mathrm{T}}$$

The original system has been reduced to 10 th-order and 2 nd-order model. The 10 th-order reduced model, acquired through the BT and BIRKA algorithms, demonstrate remarkable efficacy in encapsulating the original system dynamics. In contrast, the 2nd-order reduced model, derived using both the BT and BIRKA methodologies, exhibits pronounced deviation from the original system's response. This divergence is evident across various aspects of system behavior, encompassing both transient dynamics and steady-state characteristics. Despite the computational efficiency offered by low-order models, the inherent simplifications imposed by the 2nd-order reduction fail to encapsulate the full complexity of the system dynamics.

As the order of a system is reduced, it is commonly observed that the associated absolute error increases, as shown in figure 4.6 and 4.7. The absolute error for 10 th order models at the high frequencies, error is very low approximately 10^{-9} and, at low frequencies converges below 10^{-6}. For 2 nd order model, absolute error for 10 th order models at the high frequencies, error is very low approximately 10^{-5} and, at low frequencies converges below 10^{-1}.



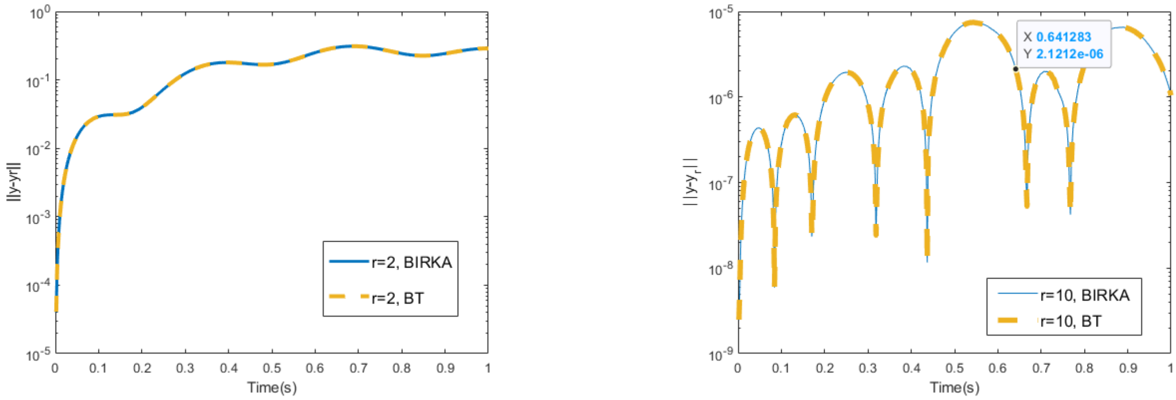
(a) The output y(t) behaviour of system to step input u

(b) The output y(t) behaviour of system to step input u

Figure 4.1: The simulation results of output responses original k-power bilinear system, 2nd order and 4th order reduced model.

Computational Time

In comparative analysis of two model reduction algorithms, a notable difference is observed



(a) Absolute Error of 2nd order ROM

(b) Absolute Error of 10th order ROM

Figure 4.2: Absolute error plots of 2nd order and 4th order reduced model.

in their computational times. For the 10th order ROM, algorithm BT demonstrated superior time efficiency, completing the model reduction task in 0.0393 seconds, whereas algorithm BIRKA required 0.1716 seconds. The BT algorithm takes approximately 85.33% less time than the original system and Birka algorithm takes approximately 35.94% less time than the original system. These findings underscore both algorithms' enhanced computational efficiency, with the BT algorithm providing the most substantial time savings.

Sr.no	**System Type**	**Order**	**Computational time (s) using BT**	**Computational time (s) using BIRKA**
1	Original	200	0.2679	0.2679
2	Reduced	50	0.1027	0.1970
3	Reduced	10	0.0393	0.1716
4	Reduced	6	0.0363	0.0389
5	Reduced	4	0.0335	0.0359
6	Reduced	2	0.0317	0.2036

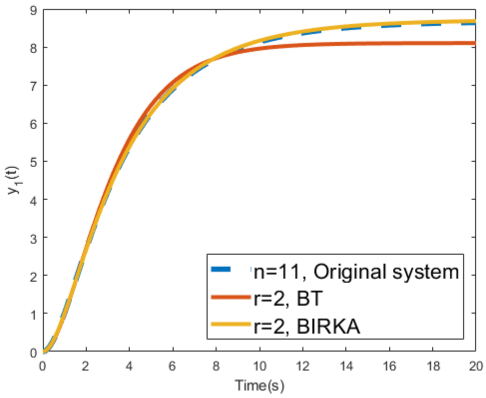
Table 4.1: Computational Time Comparison: Original system vs. ROMs

4.2 Example 2: MIMO k-Power Bilinear System

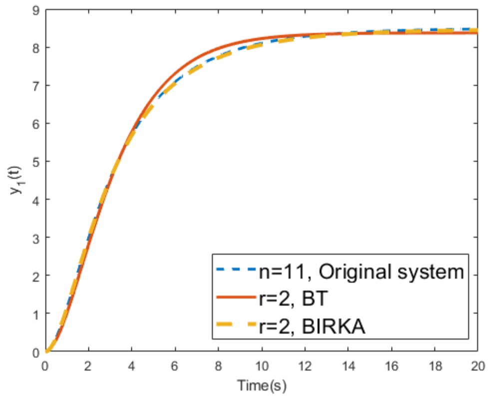
In this section, the aforementioned algorithm in chapter 3 will be applied to an eleventh-order 2-power system [23] with two inputs and two outputs. This example aims to illustrate the results and interpretation of the derived model reduction algorithm. The system can be represented by two subsystems in series and the bilinear representation with coefficient matrices A, N_1, N_2, B, and C can be utilized.

$$A = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \quad N_1 = \begin{bmatrix} 0 & 0 \\ N_{11} & 0 \end{bmatrix}$$
$$N_2 = \begin{bmatrix} 0 & 0 \\ N_{12} & 0 \end{bmatrix} \quad B = \begin{bmatrix} B_1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & C_2 \end{bmatrix}$$

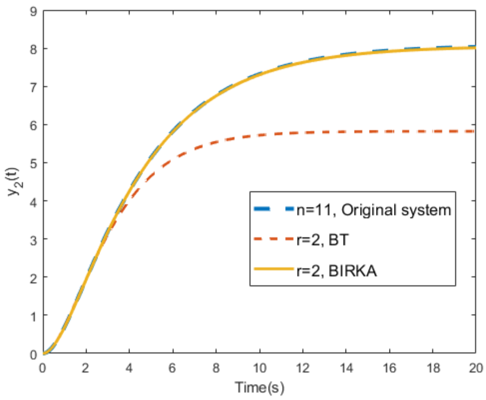
Assuming zero initial conditions, the original system and the reduced-order models were simulated with a step input signal for 20 seconds. Utilizing the BT and BIRKA MOR algorithms, the original model was reduced to second-order and fourth-order models, each consisting of two subsystems with orders 1 and 2, respectively. Response y1 and y2 are shown in figure 4.3 for the step input u1 and u2 respectively, for the original 11th order 2-power system, and 2nd order reduced model. The 2nd order ROM obtained via the BIRKA method closely aligns with the original system, nearly overlapping it, indicating an excellent approximation that captures both transient and steady-state behavior effectively. For the second-order model obtained via the BT method , the response closely aligns with the original system during the transient state but diverges in the steady state. The BIRKA method demonstrates good approximation compared to the BT algorithm.



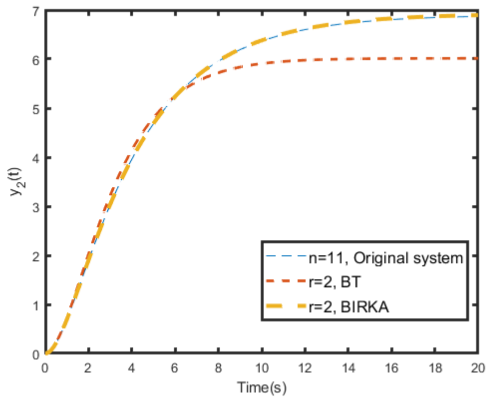
(a) The output $y_1(t)$ behaviour of the system to step input u_1.



(b) The output $y_1(t)$ behaviour of the system to step input u_2.



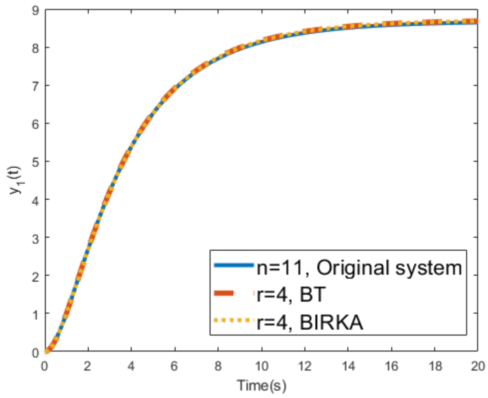
(c) The output $y_2(t)$ behaviour of the system to step input u_1.



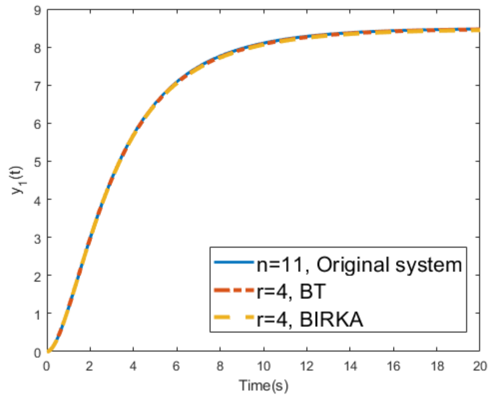
(d) The output $y_2(t)$ behaviour of the system to step input u_2.

Figure 4.3: The simulation results of output responses of 2-power bilinear system and 2nd order reduced model

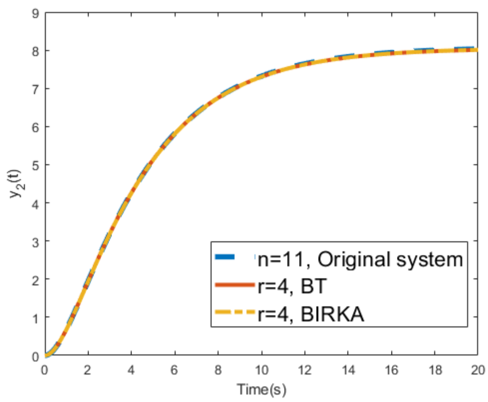
The fourth-order ROMs obtained via the BIRKA and BT methods in figure 4.4 closely align with the original system, nearly overlapping it. The reduced models effectively capture both the transient and steady-state behaviors, demonstrating that the essential dynamics of the original system are preserved.



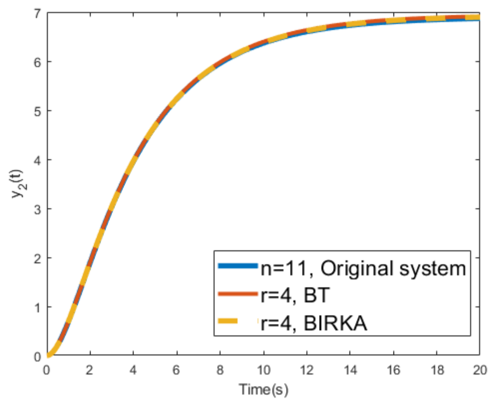
(a) The output $y_1(t)$ behaviour of the system to step input u_1.



(b) The output $y_1(t)$ behaviour of the system to step input u_2.



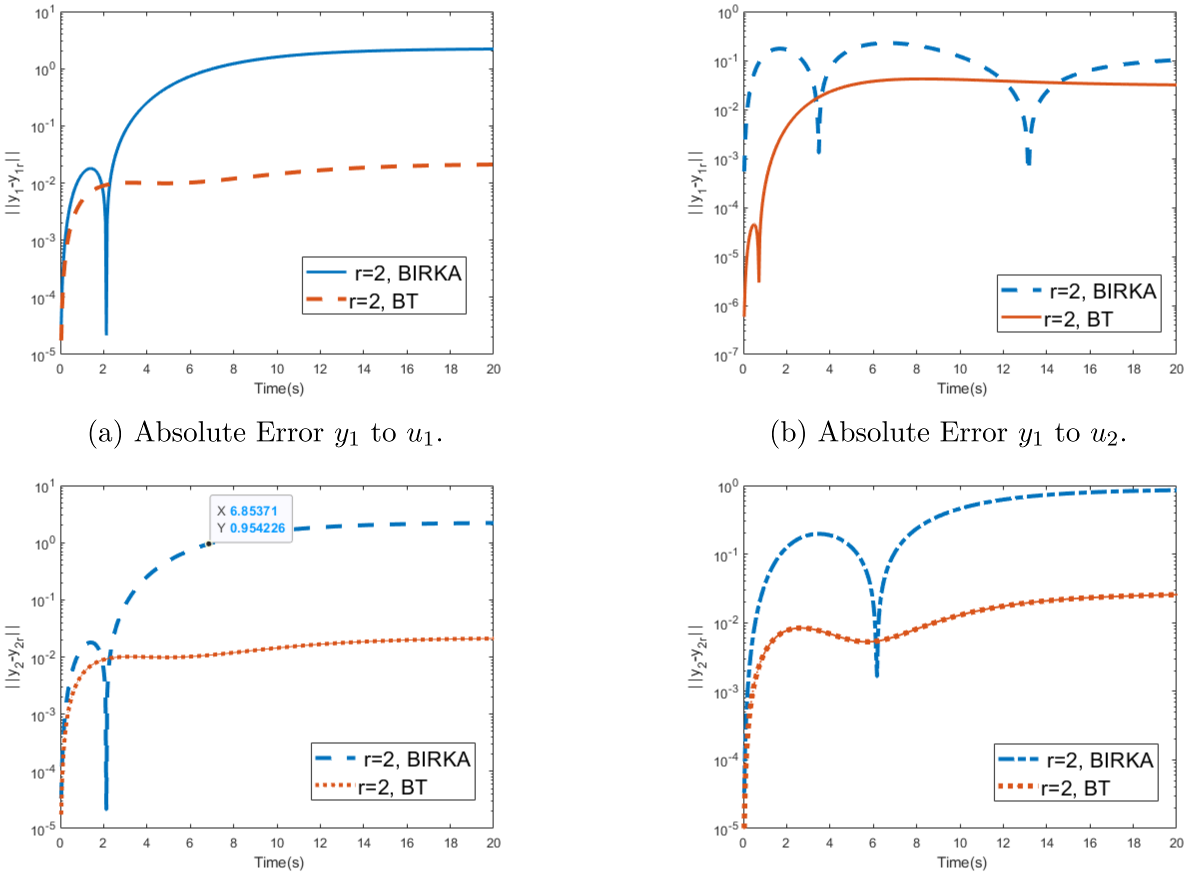
(c) The output $y_2(t)$ behaviour of the system to step input u_1.



(d) The output $y_2(t)$ behaviour of the system to step input u_2.

Figure 4.4: The simulation results of output responses of 2-power bilinear system and 2nd order reduced model

Figure 4.5 shows the absolute errors plotted against time. The absolute error using BT method stays close to 10^0 for 2 nd order model throughout the simulation time interval, $T = \begin{bmatrix} 0 & 20 \end{bmatrix}$ seconds. Since at the high frequencies, error is very low approximately 10^{-4} and, at low frequencies converges below 10^0. The absolute error for BIRKA algorithm converges to 10^{-2}. The absolute error graph reveals that the BT algorithm, despite its efficiency, has higher error values, indicating a less accurate representation of the original system. The BIRKA algorithm shows lower absolute errors, suggesting a more accurate reduced model.



(a) Absolute Error y_1 to u_1.

(b) Absolute Error y_1 to u_2.

(c) Absolute Error y_2 to u_1

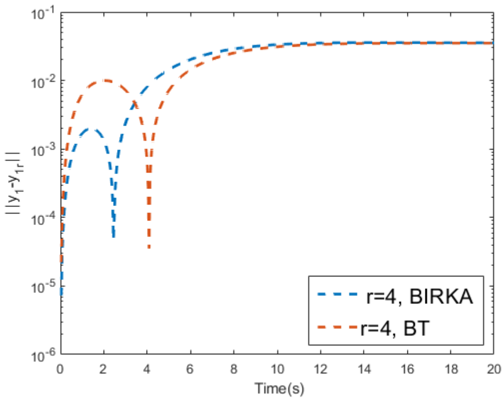
(d) Absolute Error y_2 to u_2

Figure 4.5: The simulation results absolute errors of 2nd order model

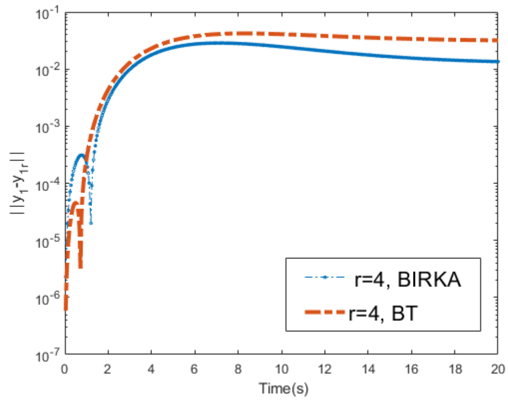
When the original system is reduced to a 4th order model, both the BT and BIRKA algorithms achieve identical absolute errors, indicating that the reduced models have the same level of accuracy. This demonstrates that both algorithms are equally effective in preserving the essential dynamics of the original system. However, the BT algorithm completes the reduction process significantly faster than the BIRKA algorithm.

Computational Time

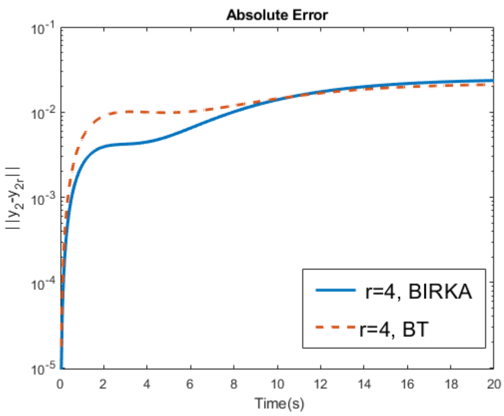
Tic toc function was used to measure the computational time. To calculate the computational time, original and reduced order models were simulated using ode15s solver in MATLAB, and the time taken in simulation was measured. The Balanced Truncation (BT) algorithm



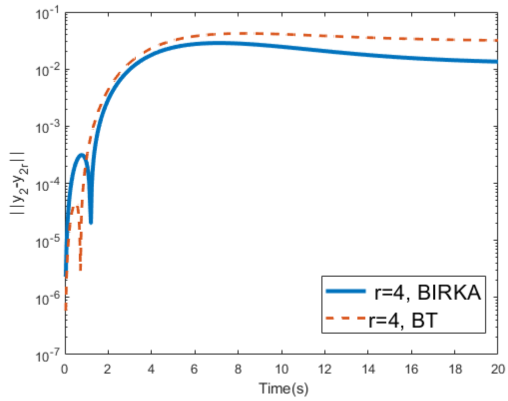
(a) Absolute Error y_1 to u_1.



(b) Absolute Error y_1 to u_2.



(c) Absolute Error y_2 to u_1



(d) Absolute Error y_2 to u_2

Figure 4.6: The simulation results absolute errors of 4th order model

takes approximately 63.23% less time than the BIRKA algorithm for reducing the system to a 4th order model. Specifically, BT takes 0.0158 seconds compared to BIRKA's 0.03334 seconds. The BT algorithm takes approximately 55.79% less time than the BIRKA algorithm for generating the second-order reduced model. The BT algorithm performs better in terms of computational efficiency, as it completes the reduction process significantly faster than the BIRKA algorithm.

Sr.no	**System Type**	**Order**	**Computational time (s) using BT**	**Computational time (s) using BIRKA**
1	Original	11	0.555350	0.555350
2	Reduced	10	0.0177	0.041734
3	Reduced	8	0.0168	0.037642
4	Reduced	6	0.0163	0.035158
5	Reduced	4	0.0158	0.033340
6	Reduced	2	0.0139	0.031430

Table 4.2: Computational Time Comparison: Original system vs. ROMs

Chapter 5

Conclusion and Future Work

In this research, two structure-preserving dimension reduction algorithms based on projection techniques for k-power bilinear systems are introduced. These algorithms ensure an approximate error bound and maintain BIBO stability under specific conditions.
As the system order decreases, the BT algorithm demonstrates faster computational times but may deviate from the original system's response. This efficiency makes BT suitable for applications requiring rapid iterative analyses or real-time simulations.
While BT is efficient, BIRKA shows competitive accuracy, providing overlapping results. The choice between BT and BIRKA depends on the application's specific needs. For scenarios prioritizing computational speed with acceptable accuracy for lower-order models, BT is robust. Conversely, for applications where precision is critical and longer simulation times are acceptable, BIRKA is effective.
Future research will involve modeling and simulating an actual physical power system, applying these model order reduction techniques to achieve similar benefits. This approach aims to reduce complexity and simulation time, while improving computational efficiency, manageability, scalability, and applicability in real-time.

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