PERFOMANCE ENHANCEMENT OF SPACE TIME TURBO

TRELLIS CODES IN MIMO CHANNEL

MCS

By

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ABSTRACT

An effective and practical way to approaching the capacity of multiple input multiple output (MIMO) wireless channels is to employ space-time coding. Space time coding is a coding technique designed to use in a MIMO channel. There are various approaches in coding structures, including space-time turbo trellis codes, space-time block codes and layered space-time codes.

Space time turbo trellis codes were presented with recursive structure and were considered as full rate codes in MIMO channel because of the bandwidth efficient nature of Trellis codes, which were used as the component codes in design of space-time turbo trellis codes. The performance of the code was not much enhanced as compared to the component codes but the complexity was increased.

Our intended goal is to increase the performance and decrease the decoder complexity. The decoder performance depends on the minimum distance (D_{min}), we will try to increase the D_{min} through a better design of the component encoders. The feedback and feed-forward polynomials should be searched for better performance of the code, as they have great impact on increasing the Dmin and decreasing the required number of iterations at the decoder. We can increase the performance with better design of an interleaver, which can have the ability to maximize the spread and decorrelation. Permutations would be calculated algebraically by storing only few parameters on the fly.

Previous works done to imply turbo codes in MIMO with trellis as a constituent codes failed to give performance comparable to the constituent codes alone .Our intended goal is to achieve the enhanced performance by optimizing the interleaver design.

DEDICATION

All Praise And Thanks To Almighty Allah. To my parents who helped me carry on my studies despite of all hardships they faced.

ACKNOWLEDGEMENT

I bow my head in gratitude to Allah, the creator of the mind that seeks knowledge and sets this pen into motion.

I would like to thank my Supervisor Dr. Muhammad Arif Wahla, Military College of Signals, NUST who has consistently encouraged me and provided me with the necessary prospective to complete this task. I am also thankful to my guidance committee members DR. Faheem, Dr. Adnan Rashidi and Dr Adil Masood Siddique for their able guidance throughout the process. My thanks are also due to Dr. Ashraf Masood who created the competitive environment that is necessary for research.

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LIST OF ABBREVIATIONS

MIMO	Multiple Input Multiple Output antenna
STTC	Space Time Trellis codes
RSTTC	Recursive Space Trellis Codes
MAP	Maximum A-Posteriori Probability
QPSK	Quadrature Phase Shift Keying
MMSE	Minimum means square error
FER	Frame Error rate
SER	Symbol Error rate
HSD	High Spread Deterministic
Dmin	Minimum Hamming Distance
RSC	Recursive Systematic Convolutional
M-Psk	M-ary Phase Shift Keying
LSTC	Layered Space time codes
QAM	Quadrature amplitude modulation
SNR	Signal to noise ratio
SOVA	Soft –output Viterbi algorithm

Chapter 1

INTRODUCTION

1.1 Motivation

By the advent of MIMO in wireless communications the increasing demand of bandwidth seeks a hope to be fulfilled. BOD (Bandwidth on demand) and high streaming rates increase the demand of useable frequency spectrum in an optimized way so that the demands can be entertained.

Information Theory is used widely in different fields but when it comes to channel coding it gives us more degrees of freedom to enhance the capacity of the existing system and redundancy of the data. Different type of codes has been used in wireless systems from the beginning like convolution codes, block codes and turbo codes .They randomize the data in a way that cannot be perceived by the channel with minimum number of parity bits. According to Shannon, the ultimate code would be one where a message is sent infinite times, each time shuffled randomly. The receiver has infinite versions of the message albeit corrupted randomly. From these copies, the decoder would be able to decode with near error-free probability the message sent. This is the theory of an ultimate code, the one that can correct all errors for a virtually signal [1]. Researchers show that turbo codes approaches the Shannon capacity limit in systems with single antenna link [2]. Since their introduction in 1993, Turbo codes have gained a lot of attention from the research community, all around the globe. The iterative error correcting capability of these codes, enables the user to maintain the reliability of the information during transmitting it over fading channels. By using Trellis codes as constituent recursive codes of turbo another degree of freedom can be exploited in terms of bandwidth efficiency. Implementation of Space time turbo trellis codes can give us an increased capacity as well as the maximum usage of the available bandwidth

The basic motive behind our thesis is to increase the performance of the Space time turbo codes with a special emphasis on the optimized parameters like interleaver patterns which play a vital role in the performance of turbo codes.

1.2 Problem Statement

An effective and practical way to approaching the capacity of MIMO (multiple input multiple output) wireless channels is to employ space-time coding. Space time coding is a coding technique designed to use in a MIMO channel. Coding is performed in both spatial and temporal domains to introduce correlation between signals transmitted from various antennas at various time periods. The spatial and temporal correlation is used to exploit the fading and minimize transmission errors at the receiver .Space time coding can achieve transmit diversity and power gain over spatially un-coded systems without sacrificing the bandwidth. There are various approaches in coding structures, including space-time turbo trellis codes, space-time block codes and layered space-time codes.

Space time turbo trellis codes were presented with recursive structure and were considered as full rate codes in MIMO channel because of the bandwidth efficient nature of Trellis codes, which were used as the component codes in design of space-time turbo trellis codes. The performance of the code was not much enhanced as compared to the component codes but the complexity was increased.

Our intended goal is to increase the performance and decrease the decoder complexity. The decoder performance depends on the D_{min}, the focus is to increase the D_{min} through a better design of the component encoders. The feedback and feed-forward polynomials should be searched for better performance of the code, as they have great impact on increasing the Dmin and decreasing the required number of iterations at the decoder. So the selection of feed-back and feed forward polynomials would be done carefully. The increased performance depends on better design of an interleaver, high spread deterministic interleaver is considered which can provide best suitable permutations.

1.3 Thesis Objective

The field of space time coding is still a hot, active and open area of research because of increasing demand for high data rate at low power consumption and FER. Because of transmission of data at low power, the fading channel impairments in the data become very high and the transmitted symbols becomes highly correlated.

Therefore, the basic objective of the thesis is to devise a way to increase the system performance by incorporating different techniques to decrease the correlation of the transmitted symbols so that the channels effect can be catered in advance and as a result the decoding process should become easy and effective.

1.4 Thesis Organization

The thesis is organized as follows:

Chapter-1 gives an introduction to the work being carried out in this thesis. Basic information about the thesis the problem statement that is addressed and basic objectives of the thesis and goals to achieve them.

Literature review is presented in chapter 2&3 which clearly shows the capcity of MIMO and complexity of turbo codes. A brief but precise description of different types of Convolutional Encoders, systematic and non-systematic encoders, recursive, non-recursive encoders, there advantages/disadvantages and uses in different scenarios. Second portion of this chapter is dedicated to the Turbo decoders and different decoding algorithms. MAP decoding algorithm used in our work is explained with sufficient details with an introduction to SOVA algorithm along with their mathematical descriptions.

Proposed system model and results are discussed in chapter 4 which describes of Space time turbo trellis codes and there implementation with different number of antennas and also the impact of inter-leaver patterns used by the previous authors for the implementation of Space time turbo trellis codes in a MIMO environment. Especially the work of Branka Vcuetic and her research group has been focused for the better understanding of the system.

Chapter-6 gives overall findings of the research and shows direction for any future work that can be carried out in this field.

C h a p t e r 2

OVERVIEW OF MIMO SYSTEM

2.1 Introduction

The capacity of MIMO system and its different variants gained a lot of attention in the modern research which shall be discussed in this chapter.

2.2 Single Input Single Output (SISO)

Traditional radio transmitter uses one antenna at the transmitter and one antenna at the receiver. This system is termed Single Input Single Output (SISO) as shown in Figure 2.1.

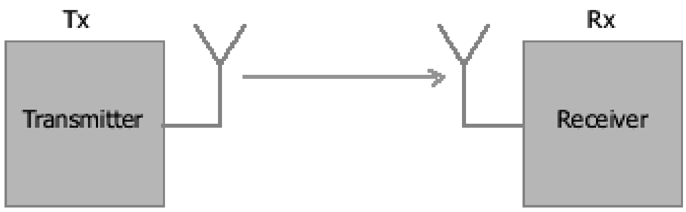

Figure 2.1: Single Input Single Output

SISO is comparatively simple and cost effective to implement and it has been used since the birth of radio technology. Its usage is in radio and TV broadcast and in personal wireless technologies e.g. Wi-Fi and Bluetooth.

2.3 Single Input Multiple Output (SIMO)

To enhance its performance, a multiple antenna technique has been developed. A system in which there is a single antenna at the transmitter and multiple antennas at the receiver is known as Single Input Multiple Output (SIMO) as shown in Figure 2.2. The receiver has options either to choose the best antenna to receive a stronger signal or to combine signals from all antennas in a particular way that maximizes SNR (Signal to Noise Ratio). The first technique is named as switched diversity or selection diversity. The latter one is named as maximal ratio combining (MRC).

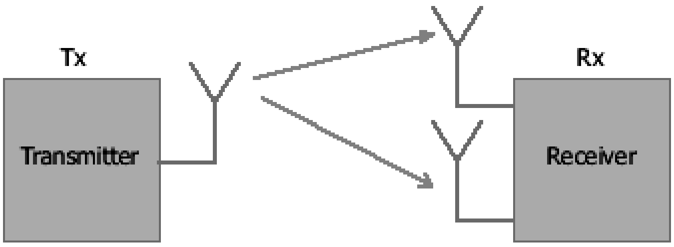

Figure 2.2:Single Input Multiple Output

2.4 Multiple Input Single Output

A system in which there are multiple antennas at the transmitter end and a single antenna at the receiver is known as Multiple Input Single Output (MISO) as shown in Figure 2.3. A technique named as Alamouti STC (Space Time Coding) is employed at the transmitter with two antennas. STC allows the transmitter to transmit signals (information) both in time and space, meaning the information is transmitted by two antennas at two different times simultaneously.

Multiple antennas of either SIMO or MISO are usually installed at a base station (BS). Benefit of this is that the cost of providing either a receive diversity (in SIMO) or transmit diversity (in MISO) can be shared by all subscriber stations (SSs) served by the base station which send control signals for all the serving base stations.

Including this it also controls the power and call establishment in case of voice communication.

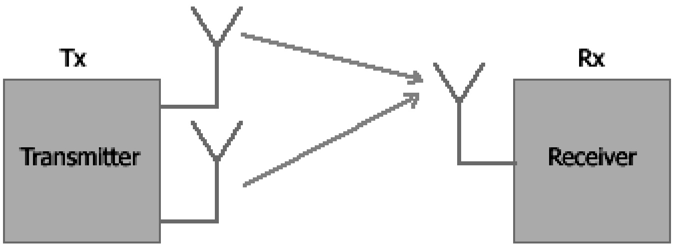

Figure 2.3:Multiple Input Single Output

2.5 Multiple Input Multiple Output (MIMO)

To multiply throughput of a radio link, multiple antennas are put at both the transmitter and the receiver end. This system is named as Multiple Input Multiple Output **(**MIMO). The credit of earliest ideas in the field of MIMO Figure 2.4 shows that A.R. Kaye and D.A. George (1970) and W. van Etten (1975, 1976). Jack Winters and Jack Salz at Bell Laboratories published several papers on beam forming related applications.

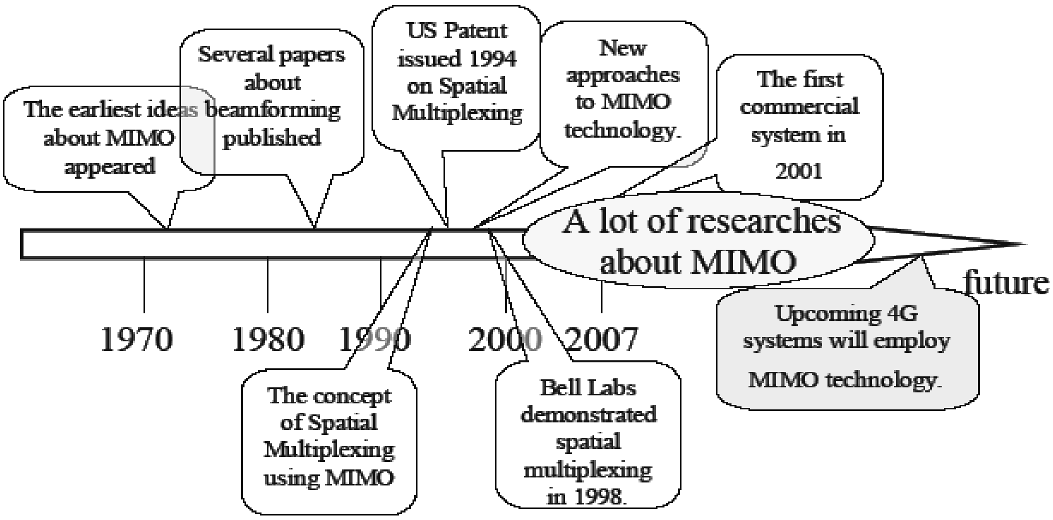

Figure 2.4: History of MIMO

Arogyaswami Paulraj and Thomas Kailath proposed the concept of Spatial Multiplexing (SM) by implementing the system MIMO in 1993.

A MIMO system with similar number of antennas at both the transmitter and the receiver in a point-to-point (PTP) link is capable to multiply the system throughput linearly with every other additional antenna. As shown in Figure 2.5, a 2x2 MIMO will double the throughput.

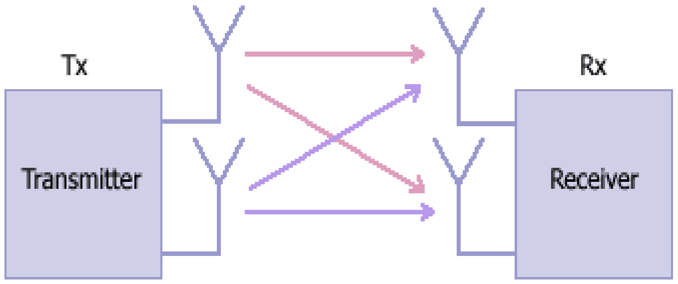

Figure 2.5: Multiple Input Multiple Output(MIMO),2x2

MIMO often employs spatial multiplexing to help signal to be transmitted across different spatial domains. However, Mobile WiMAX supports multiple MIMO modes i.e. using either SM or STC or both to maximize spectral efficiency without compromising the coverage area. The dynamic switching between these modes based on channel conditions is called Adaptive MIMO Switching (AMS). If AAS (Adaptive Antenna System) is combined with, MIMO can further increase WiMAX performance.

2.5.1 Multipath in MIMO

Multipath in wireless channel is known as a wireless propagation phenomenon that results in a signal reaching the receiving antenna by two or more paths. Due to the presence of different scatters and reflectors the duplicated copies of the symbol received at receiver with some delay as shown in Figure 2.6. When receiver receives theses delayed versions it becomes very hard for the receiver to decide and reject the symbols, different techniques are used to cater the effect of multipath. Multicarrier schemes give very good results in multipath channel due to narrow band.

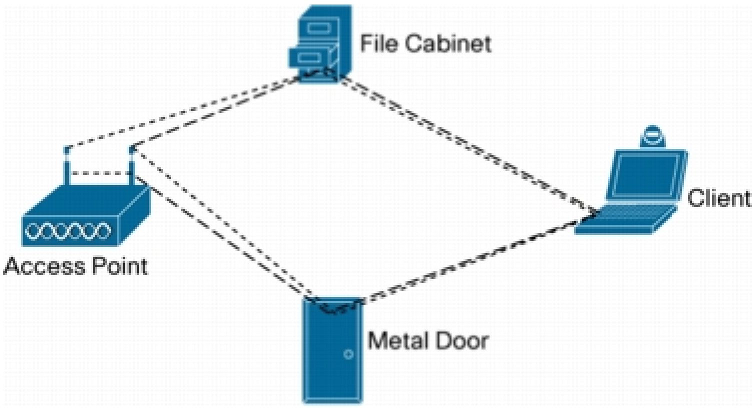

Figure 2.6: Multipath Propagation

Multipath fading happens when a transmitted signal divides and takes more than one path to a receiver and some of the signals are received out of phase, resulting in a weak or faded signal.

A basic feature of MIMO systems is the ability to turn multipath propagation, traditionally a downfall of wireless transmission, into a benefit for the user. Each multipath route is evaluated as a separate channel, creating many “virtual wires” over which to transmit signals. Traditional radios are confused by this multipath, Mean while MIMO takes advantage of these “echoes” to increase the throughput.

2.5.2 Capacity of MIMO

Channel capacity is the maximum data rate that a channel can support with an arbitrarily low probability of error. It is also defined as “the maximum mutual information between vectors x and y, where the maximization is taken over all possible probability distributions of the random vector x”[3]. Using the system model where the channel matrix H is random, the randomness of the channel also makes the information rate associated with the MIMO channel random. Two variants of capacity: Ergodic Capacity and Outage Capacity are thus discussed below

2.5.2.1 Ergodic Capacity

The ergodic capacity of a MIMO channel is the "ensemble average of the information rate over the distribution of the elements of the channel matrix H" [4]. When the channel is blind to the transmitter but known to the reciver, the ergodic capacity is given by [5,6]

$$C = E_H[\log_2 \det(I_r + \frac{\rho}{N_t} HH^H)] \quad bps/Hz \qquad (2.1)$$

Where H^H is the transpose-conjugate of H and ρ is the SNR at any receive antenna; it is assumed that the transmitted signal vector x is composed of N_t statistically independent equal power components each with a Gaussian distribution.

To gain some insight on the significance of the above result it can be rewritten as [5]

$$C = \sum_{i=1}^{m} E[\log_2(1 + \frac{\rho}{N_t}\lambda_i)] \quad bps/Hz \qquad (2.2)$$

Where $m = min\ (N_t, N_r)$ and $\lambda_1 \ldots\ \lambda_m$ are the eigen values of the matrix HH^H. Here, the capacity of the MIMO channel is expressed as the sum of the capacities of m parallel SISO channels ($C_{SISO} = \log_2(1 + \rho\,|h|^2) \quad bps/Hz$), each having power gain λ_i and SNR equal to ρ.

In this scenario when the channel is time-varying and the channel **H** is not known at the transmitter but known at the receiver (CSIT is not know and CSIR is known). Figure 2.7 shows the significant gains in capacity when using multiple antennae as opposed to single antenna systems. So, when the channel is time varying and known at the receiver, which helps to mitigate the effect of channel from the information with the help CSIR. The capacity gain of know CSI in MIMO channel is considered to be better than the system which does not cater for the CSI.

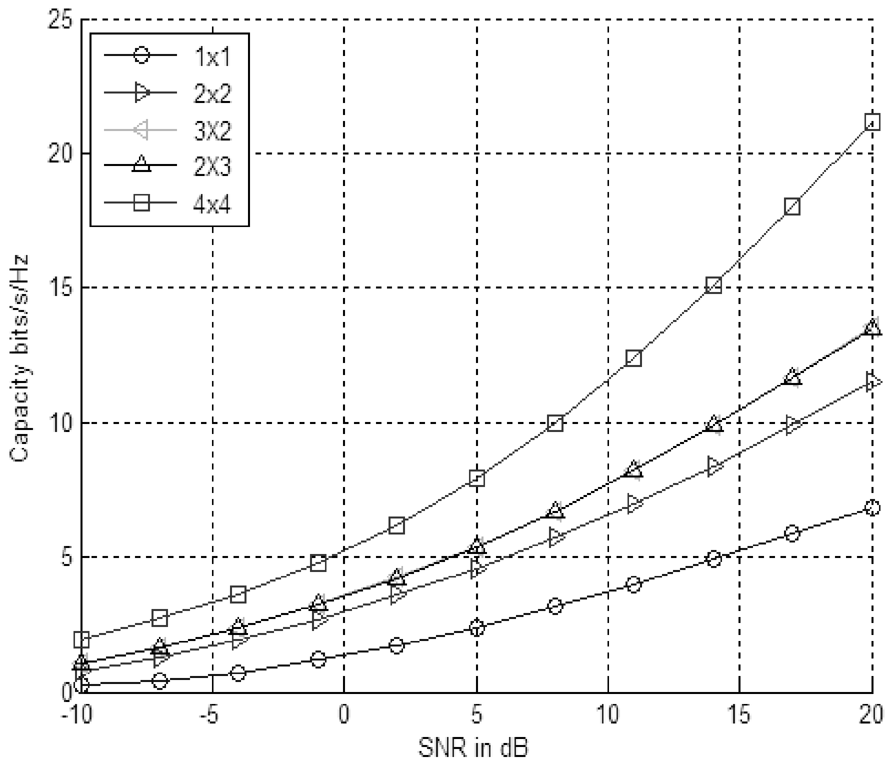

Figure 2.7: Ergodic Capacity of MIMO Systems

It can be shown that when both N_t and N_r increase, the capacity grows linearly with min (N_t, N_r). On the other hand, if N_t is fixed and N_r is allowed to increase, the capacity increases logarithmically with N_r; whereas if N_r is fixed and N_t is increased, the capacity saturates at some fixed value.

So far it was assumed that the channel state information is known only at the receiver. Another possible scenario is that the channel state information is known at both the transmitter and the receiver; this assumes the presence of an ideal feedback link from the receiver to the transmitter and a very slowly fading channel for this assumption to be feasible in practice. With this knowledge of the channel, the total transmitted power can now be allocated in the most efficient way over the different transmitting antenna to achieve the highest possible bit rate; this is done using the water-filling algorithm [7].

The ergodic capacity is then given by

$$C = E[\sum_{i=1}^{t} \log_2(\nu\lambda_i))^+] \quad bps/Hz \tag{2.3}$$

Where λ_i is the ith eigen value of HH^H and the parameter ν is chosen such that it satisfies the instantaneous power constraint $P_t = \sum_{i=1}^{t}(\nu - \frac{1}{\lambda_i})^+$. The notation a^+ denotes max (0, a). It is expected that the ergodic capacity when the channel is known to the transmitter to be higher than when the channel is unknown.

2.5.2.2 Outage Capacity

Since in a Rayleigh fading environment the channel matrix H changes randomly, the capacity is also random. One way to express the capacity of such a channel is the ergodic expression of (2.2). However, if a bad realization of H occurs, then no matter how small the rate that is attempted to be communicated at, there is a non-zero probability that this realized H is incapable of supporting this rate no matter how long the code length is taken. For such a scenario, the concept of outage probability q is introduced, which is the fraction of time the capacity falls below a given threshold C_{outage} and is given by

$$q = P_r\left\{C \le C_{outage}\right\} \tag{2.4}$$

where C is the instantaneous capacity given by $C = \log_2(I_r + \frac{\rho}{t}HH^H)$.

A capacity of 20 bps/Hz with 1% outage probability means that a data rate of 20bps/Hz is supportable for 99% of the time. As in the case of ergodic capacity, the outage capacity increases with SNR and is higher for larger antennae configurations.

2.5.3 Function of MIMO

This section will briefly discuss the spatial multiplexing, ergodiac capacity and the effect of a coded MIMO system on the capacity.

2.5.3.1 Spatial Multiplexing

SM requires MIMO antenna configuration. In spatial multiplexing, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel. If these signals arrive at the receiver antenna array with sufficiently different spatial signatures, the receiver can separate these streams, creating parallel channels free.

Spatial multiplexing is a very powerful technique for increasing channel capacity at higher SNR. The maximum number of spatial streams is limited by the lesser in the number of antennas at the transmitter or receiver. Spatial multiplexing can be used with or without transmit channel knowledge.

2.5.3.1.1 Bell Labs Layered Space–Time (BLAST)

BLAST architecture was one of the first spatial multiplexing systems. Researchers at Bell-Labs developed the first MIMO architecture for high-speed wireless communications, i.e. the BLAST system. In a BLAST system, the input data stream is demultiplexed into N_t sub streams independent bit-to-symbol mapping of each substream is performed at each of the N_t transmit antennae. The generated continuous-time waveforms are then simultaneously launched into the wireless channel overlapping in time and frequency. The signals are received by the N_r receive antennae as shown in Figure. 2.8 Signal processing at the receiver attempts to unmix the received signals and recover the transmitted data. Measurement campaigns of the BLAST system showed the great increase in spectral efficiency at reasonable SNR and BER vs. a SISO system [8].

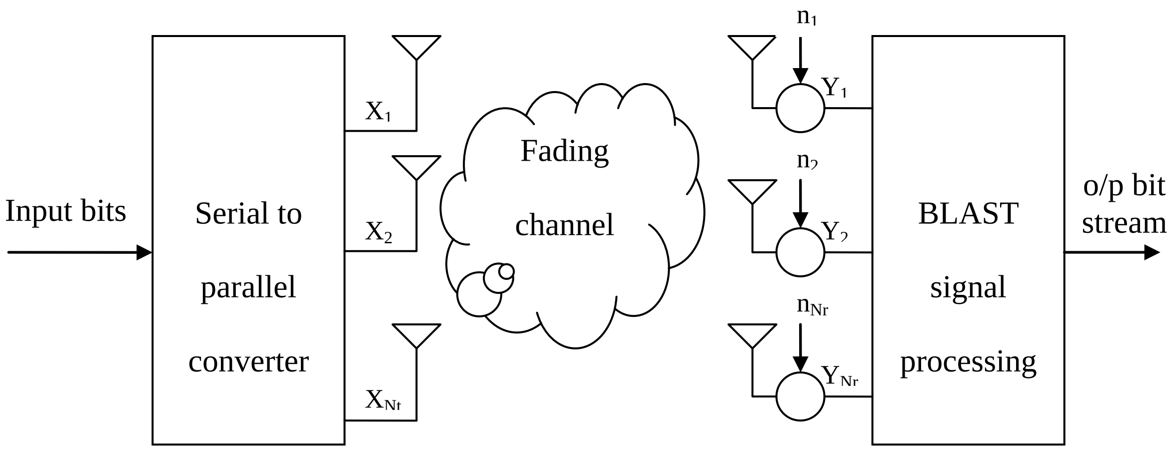

Figure 2.8: Schematic Representation of the BLAST System

2.5.3.2 Diversity through Coding

Diversity coding techniques are used when there is no channel knowledge at the transmitter. In diversity methods a single stream (unlike multiple streams in spatial multiplexing) is transmitted, and the signal is coded using techniques known as space-time coding. The signal is emitted from the transmit antennas using certain principles of full or near orthogonal coding.

Diversity exploits the independent fading in the multiple antenna links to enhance signal diversity. Because there is no channel knowledge, there is no beamforming or array gain from diversity coding. The diversity achieved through this technique should helps to gain the maximum advantage of the resources in hand.

2.5.3.3 Space-Time Coding

Space-time coding is a set of practical signal design techniques aimed at approaching the information theoretic capacity limit of MIMO channels. The fundamentals of space-time block coding have been established by Tarokh et al. in 1998 [9]. Space time codes may be split into two main types first of them is Space–time block codes (STBCs) and the second is Space–time trellis codes (STTCs).

2.5.3.3.1 Space-Time Block Codes

As the name suggests, the space-time block encoder operates on a block of input symbols producing a code matrix. This encoding operation is explained by discussing the pioneering Alamouti scheme [10], which is one of the simplest and most elegant space-time codes as the Figure 2.9 shows.

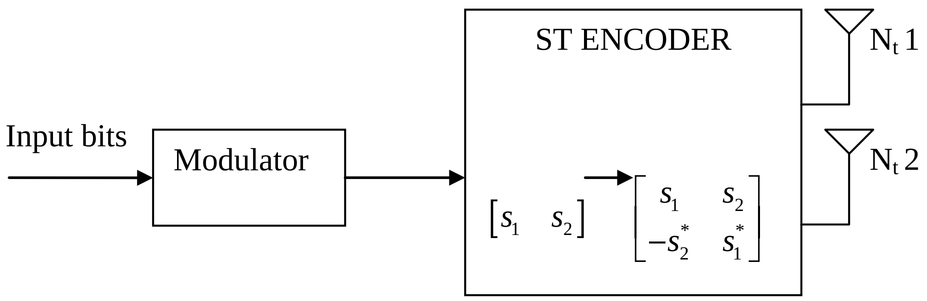

Figure 2.9: Schematic Representation of the Alamouti Space-Time Encoder

The information bits are first modulated according to some M-ary modulation scheme. The encoder takes a block of two modulated symbols, s_1 and s_2, and produces the corresponding code matrix S. The rows of S represent transmission over space and its columns represent transmission over time; thus, in the first signaling interval, s_1 is transmitted from transmit antenna 1(N_t 1) and s_2 is transmitted from transmit antenna 2(N_t 2); in the next signaling interval, $-s_2^*$ is transmitted from N_t 1 and s_1^* from N_t 2. A close examination shows that matrix S is orthogonal, this result in a diversity gain of order 2 Nr for a 2 × Nr MIMO system with Alamouti encoding. Moreover, the orthogonality of the code matrix greatly simplifies the decoder design on the receiving end of the channel. The simplicity and efficiency of the Alamouti scheme inspired its generalization to more than two transmit antennae; the new codes, referred to in the literature as space-time block codes, maintain the orthogonality property of the code matrix thus achieving full diversity with a very simple decoding scheme [11].

2.5.3.3.2 Space-Time Trellis Codes

Space-time block codes do not generally provide coding gain, unless concatenated with an outer code. Space-time trellis codes, on the other hand, provide coding gain that depends on the complexity of the code, in addition to providing diversity gain. STTCs were first introduced by Tarokh, Seshadri and Calderbank in 1998 [12], they are an extension of trellis coded modulation. A STTC encoder maps binary data into modulation symbols according to a trellis diagram, then spreads the coded symbols over space and time as in STBCs. Their disadvantage is that they are extremely hard to design and generally require high complexity encoders and decoders.

STTCs decoding is usually implemented using a Viterbi algorithm whose complexity grows exponentially with the number of states in the trellis [12].

2.6 Conclusion

This chapter briefly discusses the ways by which the capacity of MIMO can be achieved by the help of coding and different techniques. Thus impact of each technique can help us in achieving maximum benefits from the employed resources in a spatially distributed system with best available coding scheme and different modulation schemes. The capacity introduced by MIMO system is much more then the previously introduced systems that enhances the usage of MIMO system widely.

Chapter 3

TURBO CODES

3.1 Introduction

The advent of Forward Error Correction (FEC) coding belongs to Shannon's pioneering work [13] in 1948, suggesting that reliable communication is achievable with the help of FEC, by adding redundancy to information in the transmitted messages.

The known channel capacity equation devised by Claud.A.Shanon for a band-limited channel with additive white Gaussian Noise is given by

$$C = B\log_2(1+SNR) \tag{3.1}$$

where C is in bits/sec, B is bandwidth of channel. This formula is not the only thing introduced by Shannon for the improvement of channel capacity, he also introduced different channel coding schemes for better exploitation of channel capacity.

Furthermore, the amount of redundancy introduced increases as the associated information delay increases, he failed to specify the maximum delay that may have to be tolerated, and in order to achieve the Shannonian limits [14].

The first Forward Error Correcting (FEC) codes were Hamming codes developed in 1950[15] capable of correcting single bit errors. Convolution FEC codes were introduced in 1955 [16] by Elias, Wozencraft and Reiffen [17,18], while Fano [19] and Massey [20], introduced there own decoding algorithm. A major advancement in the history of convolution error correction coding was the introduction of a maximum likelihood (ML) sequence estimation algorithm by Viterbi [21] in 1967. Another variant of the Viterbi Algorithm (VA) was given by Forney's work [22]. One of the initial practical applications of convolution codes was proposed by Heller and Jacobs

[23] during the 1970s. The above mentioned work was focused on achieving the channel capacity limit set by Shannon in 1948.

The turbo codes principle when introduced in 1993 by C. Berrou, Glavieux and Thitimajashima [24] in Geneva congress were the first ever error correcting code working within the range of 0.5 dB of the Shannon capacity limit. Turbo codes have gained a lot of acceptance and these codes have been used with various communication techniques for error correction purposes, from the day of there introduction.

3.2 Composition of Turbo Encoder

Usually there are two types of Encoders used in a communication system

Source Encoder converts the input information sequence into a sequence of binary digits with minimum redundancy i.e Huffman Coding.

Channel Encoder adds redundancy to the source encoded information sequence bits to minimize the impact of channel. Convolution Encoder, Turbo Encoder are the most common examples.

This chapter focuses on the Turbo Channel Encoder. Since their introduction in 1993, several schemes of Turbo Codes have been proposed but the basic structure of all was the same i.e. "Combination of two or more parallely concatenated Recursive Systematic Convolution (RSC) encoders, all separated by an interleaver". A Turbo Code Encoder with rate 1/3 is shown in Figure 3.1.

The generator matrix for the description of rate ½ component RSC encoder is given as

$$G(D) = \begin{bmatrix} 1 & \dfrac{g_1(D)}{g_0(D)} \end{bmatrix} \tag{3.2}$$

here $g_0(D)$ and $g_1(D)$ are feedback and feedforward polynomial of degree v. In the encoder the same information sequence is encoded twice with same generator polynomials but in

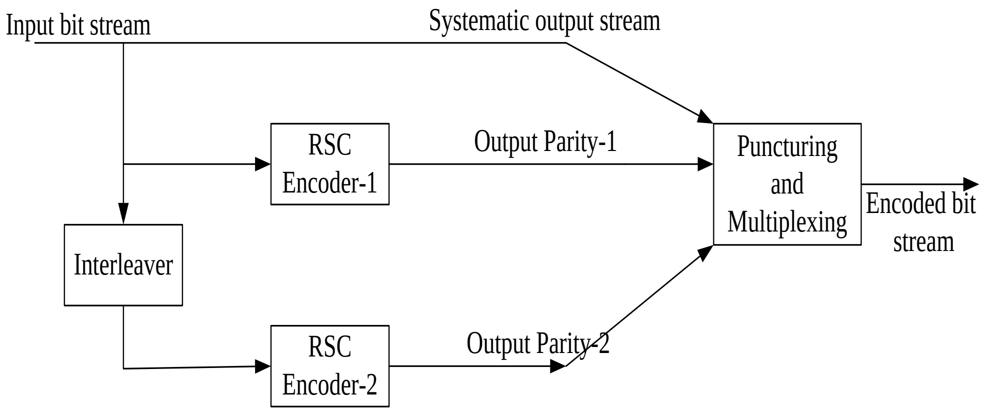

Figure 3.1:Turbo Encoder

an interleaved fashion, the reason of which is given in the section 3.2.1. The first component RSC encoder of rate 1/2 works directly on the input bit stream, it has two outputs, systematic and parity bit 1. In the second encoder the interleaved version of the same information sequence is fed. But only the parity sequence is transmitted for RSC convolutional encoder-2. The information sequence and the parity check sequences are multiplexed and punctured to generate the turbo coded sequence of rate 1/3.

The functionality of the interleaver is to scramble the bits into a predetermined random fashion that is to be recovered at the receiver side. The choice of interleaver plays an important role in the overall system performance and will be discussed in detail in section in next chapter. This becomes the base our work the role of interleaver and its pattern helps us to randomize the information in totally different way which helps in increasing the minimum distance of the symbols which in return gives the decoder a good convergence.

3.2.1 Why Parallel Concatenation

The parallel concatenation of the RSC Encoders is important for two reasons. The first reason is the tendency of producing high weight output codes because of efficient decoding of bit 1. So if the input to the second encoder is interleaved, the output produced by the second encoder is less correlated from the output of the first encoder. So if the output produced by one of the encoder is low weight then there are very less chances that the output produced by the second encoder will also be of less weight. So the chances of production of a low weight output code word are very less.

Second reason is the employment of divide-and-conquer strategy for decoding. If the input to the second decoder is scrambled, its output will be uncorrelated from the output of the first encoder. Thus the corresponding two decoders will gain more from information exchange.

3.2.2 System Design

The component encoder used in our simulation for Turbo Encoder is RSC Encoder. The structure of the RSC encoder is derived from a non-recursive Convolutional encoder by feeding back one of its parity bit output to the input[21]. Figure 3.2 shows the structure of a non-recursive convolution encoder of rate ½.

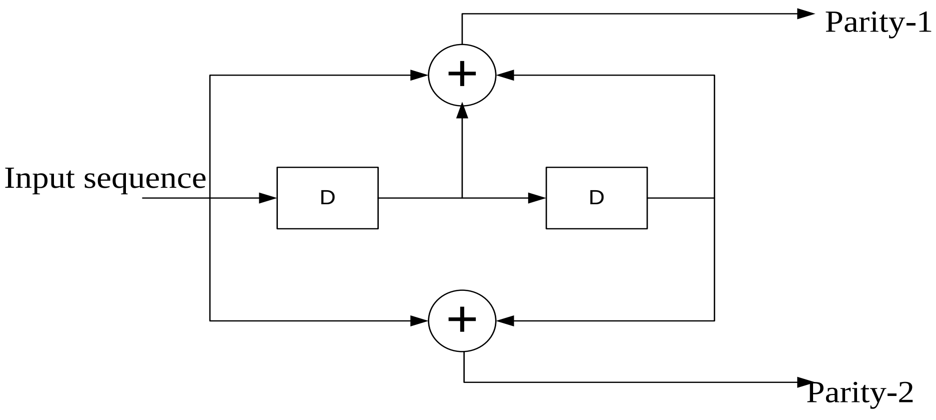

Figure 3.2: A Non-Recursive Convolutional Encoder with Rate ½

This convolutional encoder is represented by the generator sequence g1=[1 1 1] and g2=[1 0 1]. The corresponding Generator matrix is represented as G=[g1 g2]. A generator matrix is a basis for a linear code which generates all the possible code words for a given [n,k] code.

For the corresponding RSC convolutional encoder, the generator matrix is represented as G=[1, g1/g2] where the first output represented by g1 is fed back to the input. In the above representation, 1 denotes the systematic output, g2 represents the feed-forward output g1 is the input to the RSC encoder. It is called systematic code because the input sequence is exactly been replicated in the output as well. Figure 3.3 depicts the resulting RSC encoder.

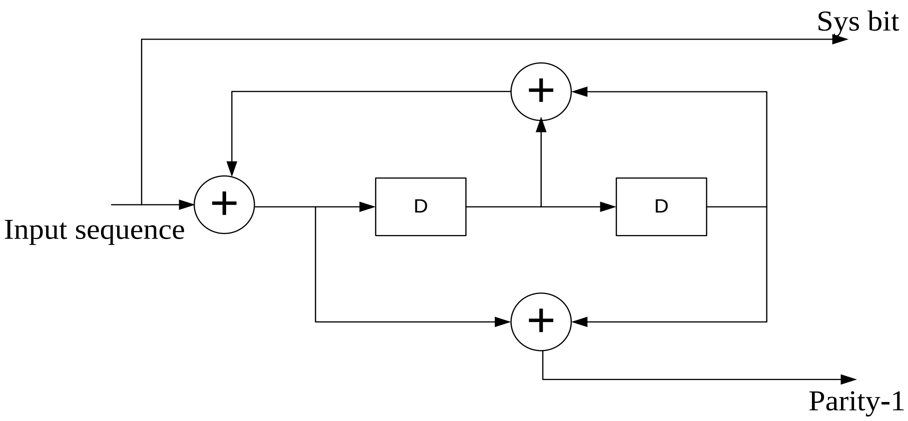

Figure 3.3: An RSC Convolutional Encoder with Rate ½

In an RSC encoder it transmit all the systematic bits from the first RSC encoder and then one parity bit from each of the two component RSC encoder of rate ½ so the overall code rate comes out to be 1/3. The generator sequences used were [1 1 1 1] and [1 1 0 1] and have been borrowed from the work of Benedetto[22]. Output sequence from the encoder is

$$S_1 P_1^1 P_1^2 \; S_2 P_2^1 P_2^2 \; S_3 P_3^1 P_3^2 \; S_4 P_4^1 P_4^2 \; S_5 P_5^1 P_5^2 \; ---$$

Where the subscript represents bit number and the superscript represents the component RSC Convolutional Encoder number.

3.3 Turbo Decoding

A basic structure of a Turbo decoder consists of two component decoders that exchange soft information amongst each other to improve the final estimate about the decoded bits. Each of this component decoder is based on one of the two basic Turbo decoding algorithms namely the Maximum A Posteriori (MAP) Algorithm and Soft Output Viterbi Algorithm (SOVA)

We have used MAP decoding algorithm in our work and this will be explained with due details in the following lines, SOVA will be briefly explained to complete the discussion.

3.3.1 Maximum A Posteriori (MAP) Algorithm

The iterative turbo decoding consists of two component decoders serially concatenated via an interleaver, identical to the one in the encoder as shown in Figure 3.4.

The first MAP decoder takes as input the received systematic information sequence C^s and the received parity sequence generated by the first encoder $C^{1,p}$. The decoder then produces a soft output in the form of Log-Likelihood ratio (LLR) which is interleaved and used to produce an improved estimate of the a-priori probabilities of the information sequence for the second decoder. The LLR produced by Deocder-1 is given by

$$LLR(d) = \ln\left(\frac{P(d=1)}{P(d=0)}\right) \tag{3.3}$$

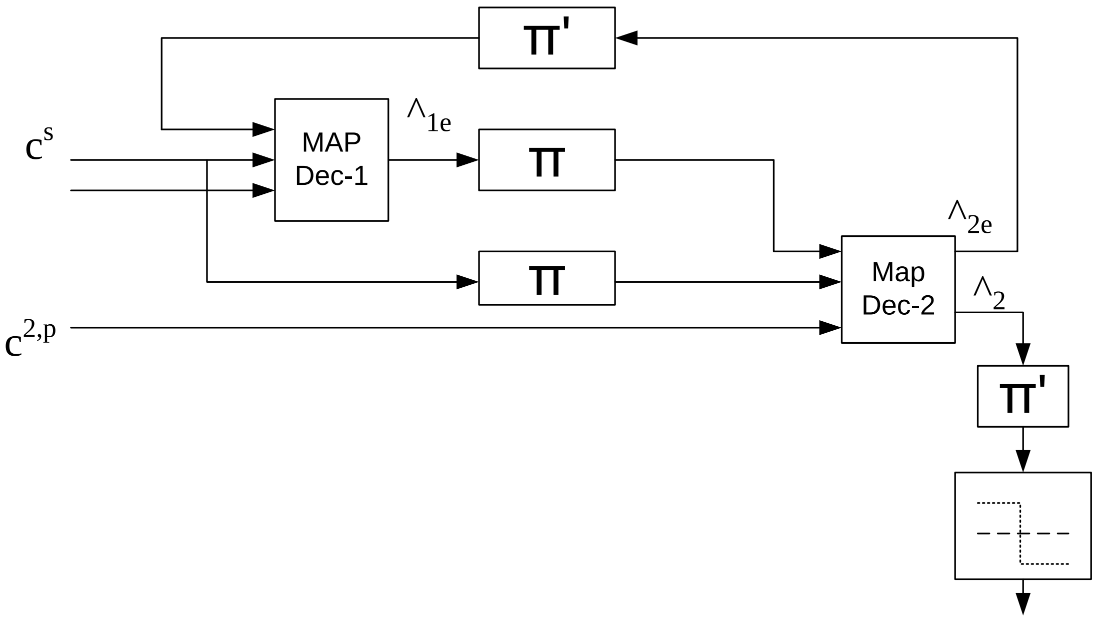

Figure 3.4: A Turbo Decoder Based on Two Component MAP Decoders

The third input to the first decoder is the a-priori probability of the second decoder which was taken as 0.5 in the first iteration.

For the MAP decoder-2, the two inputs are the interleaved received sequence $\overset{\upsilon}{C}_s$ and the received parity sequence produced by the second encoder $C^{2,p}$. The second MAP decoder produces its own soft output which is used to improve a priori probability of the information sequence at the input of the first MAP decoder. Thus the final estimate of the decoder keeps on improving relative to a single operation serially concatenated encoder. The extrinsic information produced by the two decoders diverges from the mean 0.5 as the iterations carry on and after n number of iterations a hard decision is taken at the output of MAP decoder # 2. A +ive value of the output of the MAP decoder depicts a received 1 and a –ive value represents a 0 bit .

Now the LLR produced by the first MAP decoder for rate 1/n component code is given mathematically as

$$\Lambda_1(C_t) = \log \frac{\sum_{l',l=0}^{M_s-1} \alpha_{t-1}(l')\, p_t^1(1) \exp\left(-\frac{\sum_{j=0}^{n-1}(r_{t,j} - x_{t,j}^1(l))^2}{2\sigma^2}\right)\beta_t(l)}{\sum_{l',l=0}^{M_s-1} \alpha_{t-1}(l')\, p_t^1(0) \exp\left(-\frac{\sum_{j=0}^{n-1}(r_{t,j} - x_{t,j}^0(l))^2}{2\sigma^2}\right)\beta_t(l)} \tag{3.4}$$

where $p_t^1(o)$ and $p_t^1(1)$ are the a priori probability at the input of MAP dec-1 taken as 0.5 each in the first iteration. $p_t^2(o)$ and $p_t^2(1)$ are the a priori probability at the input of the MAP dec-2.

The terms, α,β,γ are written as

$$\alpha_t(l) = P_r\{S_t = l, \mathbf{r}_1^t\}$$

$$= \sum_{l'=0}^{M_s-1} P_r\{S_{t-1} = l', S_t = l, \mathbf{r}_1^t\}$$

The α_t can be calculated for total number of states by putting each state in the above equation systematically, then as result α_t can be written in terms of backward recursive variable γ_t^i

$$= \sum_{l'=0}^{M_s-1} \alpha_{t-1}(l').\sum_{i\in(0,1)} \gamma_t^i(l',l) \tag{3.5}$$

for t=1,2,3,4,……….τ

for t=0 I have the boundary conditions $\alpha_o(0)=1$ and $\alpha_o(l)=0$ for $l\neq 0$

Similarly, $\beta_t(l)$ can be written as $\beta_t(l) = P_r\{\mathbf{r}_{t-1}^\tau \mid S_t = l\}$

$$= \sum_{l'=0}^{M_s-1} P_r\{S_{t+1} = l', \mathbf{r}_{t+1}^\tau \mid S_t = l\}$$

The β_t can be calculated for total number of states, by putting each state and then it can be written in terms of backward recursive variable γ_t^i

$$= \sum_{l'=0}^{M_s-1} \beta_{t+1}(l') \sum_{i\in(0,1)} \gamma_{t+1}^{i}(l,l') \tag{3.6}$$

for t=τ-1, …………..2,1,0.

The boundary conditions are $\beta_\tau(0)=1$ and $\beta_\tau(l)=0$ for $l \neq 0$

And γ can be written as

$$\gamma_t^i(l',l) = P_r\{C_t = i, S_t = l\}, \mathbf{r}_t \mid S_{t-1} = l'$$

$$= \frac{P_r\{\mathbf{r}_t, c_t = i, S_t = l, S_{t-1} = l'\}}{P_r\{S_{t-1} = l'\}}$$

The branch transition probability is then calculated as

$$= \left\{ \begin{matrix} p_t(i)\exp(-\dfrac{\sum_{j=0}^{n-1}(r_{1,j}^i - x_{t,j}^i(l))^2}{2\sigma^2})\, for(l,l') \in B_t^i \\ 0 otherwise \end{matrix} \right\} \tag{3.7}$$

Now by rewriting equation (3.5)

$$\Lambda_1(c_t) = \log\frac{p_t^1(1)}{p_t^1(0)} + \log\frac{\sum_{l',l=0}^{M_s-1}\alpha_{t-1}(l')\exp(-\dfrac{(r_{t,0}-x_{t,0}^1)^2\sum_{j=0}^{n-1}(r_{t,j}-x_{t,j}^1(l))^2}{2\sigma^2})\beta_t(l)}{\sum_{l',l=0}^{M_s-1}\alpha_{t-1}(l')\exp(-\dfrac{(r_{t,0}-x_{t,0}^0)^2\sum_{j=0}^{n-1}(r_{t,j}-x_{t,j}^0(l))^2}{2\sigma^2})\beta_t(l)} \tag{3.8}$$

Since the code is systematic so $x_{t,0}^1 = 1$ and $x_{t,0}^0 = -1$. Thus $\Lambda_1(C_t)$ could be further decomposed into

$$\Lambda_1(c_t) = \log\frac{p_t^1(1)}{p_t^1(0)} + \frac{2}{\sigma^2}r_{t,0} + \Lambda_{1e}(c_t) \tag{3.9}$$

$$\Lambda_{1e}(c_t) = \log \frac{\sum_{l',l=0}^{M_s-1} \alpha_{t-1}(l') \exp(-\frac{\sum_{j=0}^{n-1}(r_{t,j} - x_{t,j}^1(l))^2}{2\sigma^2}) \beta_t(l)}{\sum_{l',l=0}^{M_s-1} \alpha_{t-1}(l') \exp(-\frac{\sum_{j=0}^{n-1}(r_{t,j} - x_{t,j}^1(l))^2}{2\sigma^2}) \beta_t(l)} \quad (3.10)$$

where

Here $\Lambda_{1e}(C_t)$ is called extrinsic information and this extrinsic information when interleaved and fed to the 2nd MAP decoder becomes it's a priori information. Similarly the 2nd MAP decoder calculates its extrinsic information, the de-interleaved version of which acts as a priori information for MAP dec-1 improving its estimate. In this way the two decoders gain from each-other's information. In the final iteration, depending upon the sign of the LLR generated by MAP decoder-2, the bit is decoded as 1 or 0. A +ive sign of the LLR represents the decoded bit to be a 1 and a –ive sign represents a decoded bit 0.

3.3.1.1 Computational Complexity

The computational complexity of Turbo decoder is given by the Table 3.1:

Table 3.1: Table for Total Number of Operations in a Turbo MAP Decoder

Operation	Maximum *a posteriori* Algorithm
Maximization	2M-1
Addition	4M
Multiplication	10M
Table Look Up	0
Total Operation	14M
Total # of Operations	30M-1

M = Total states of decoder. For constraint length=3 the total number of states , M=8

The scope of our research is limited to MAP decoding algorithm, SOVA has been briefly explained here for completion of the discussion regarding turbo decoding algorithms.

3.3.2 Soft Output Viterbi Algorithm (SOVA)

The Soft Output Viterbi Algorithm (SOVA) is an extension of the predecessor Viterbi Algorithm (VA). But its increased computational complexity and degraded performance makes MAP algorithm a first choice for the researchers. The Viterbi algorithm finds the most likely path through trellis diagram given the received signal bits. The conditional probability is given by the Baye's Rule as:

$$P(x/y) = \frac{P(y/x)P(x)}{P(y)} \tag{3.11}$$

in the above equation, $P(x)$ and $P(y)$ represents the a-priori probability of transmitted and received symbol respectively. Assuming that the a-priori probabilities of the bits are statistically independent, and $P(x)$ is replaced by $P(u)$ which is the a priori probability of sign vector *u* then equation (3.11) can be written as

$$P(x/y) = \frac{1}{P(y)} \prod_{i=1}^{N} p(y_i / x_i) \prod_{k=1}^{K} p(u_k) \tag{3.12}$$

For the antipodal binary signaling, $x_i \in \{\pm 1\}$, I term the a priori LLR vectors as:

$$L_i^c = \log \frac{p(y_i / x_i = +1)}{p(y_i / x_i = -1)} \quad ; \quad L_k^c = \log \frac{p(u_k = +1)}{p(u_k = -1)} \tag{3.13}$$

The channel LLR can also be written as

$$L^c = \frac{2}{\sigma^2} A_y \tag{3.14}$$

Thus the conditional probability of any sequence that has been transmitted is given by

$$P(x/y) = C \exp(\frac{1}{\sigma^2} u(x)) \tag{3.15}$$

Here C is a constant and *u(x)* is a criterion defined by

$$u(x) = \frac{\sigma^2}{2} (x.L^c + u.L^a) \tag{3.16}$$

if I have only two possible antipodal results of an event, x and $\hat{x}$. Then

$$P(\hat{x}/y) = 1 - P(x/y) \qquad (3.17)$$

and thus the LLR is given by

$$L(\hat{x}) = \log \frac{P(\hat{x}/y)}{P(x/y)} = \frac{1}{\sigma^2}(u(\hat{x}) - u(x)) \qquad (3.18)$$

This is the value of LLR that is exchanged as soft information between the SOVA Decoders to improve the final estimate.

3.4 Conclusion

Turbo codes from the day one got ample attention from researchers and scientist due there complex nature and promising capacity. Used in different modern day applications for the facilitation of user demands. Turbo codes are usually considerd very complex to deal with not because of the encoder but due the extreme complexity of the iterative decoding procedure. This chapter discusses the different types of turbo encoders usually used and the iterative process's also with the help of both MAP and SOVA algorithm which briefly describes the complexity involved.

PROPOSED MODEL AND SIMULATION RESULTS

4.1 Introduction

Turbo codes when decode by iterative decoder they can show outstanding BER performance [23]. The turbo encoder constitute of two recursive convolutional encoders which by the use of a very authoritative decoding algorithm like MAP can achieve the desired performance. By combining the bandwidth efficient natured trellis codes with the turbo codes as a constituent encoder were presented with parity check puncturing [24] which involves parallel concatenation of two recursive constituent Ungerboeck type trellis codes [25]. To assure the bandwidth efficiency of k bits/sec/HZ for a signal set 2k+1 points, the puncturing is performed on the output of the symbols alternatively. This is comparable to alternately puncturing parity symbols from the constituent codes. The scheme uses symbol inter-leaving in the turbo encoder/decoder. Parallely concatenated two recursive convolutional codes with puncturing of systematic bits is proposed [25]. The puncturing scheme operates in such a way that the information bits appear in the output only on one occasion. This scheme uses bit inter-leaving/de-interleaving of incoming sequences in the encoder/decoder. In this chapter construction of space-time coding system with HSD interleaver which unites the coding gain of turbo coding with the diversity advantage of space-time coding as well as the bandwidth efficiency of coded modulation is considered. Bandwidth efficiency can be achieved in space-time turbo trellis code (ST turbo TC) alternate parity symbol puncturing and symbol interleaving [26][27] or by information puncturing and bit interleaving [28]. By enhancing the symbol de-correlation with the help of HSD interleaver the decoder performance should be increased.

Whereas the decoding is done by the help of MAP algorithm the component decoders are separated with the same interleaver and deinterleaver which helps the decoder to converge below threshold. The inteleaver pattern plays an important role for the ease of decoder, because more distinguished the symbols are there's less probability of error event.

4.2 Proposed System Model

Space time turbo trellis codes are presented by different authors in the recent years but the work done by Branka Vcuteic is acknowledged as mile stone in the completion of the recursive space time turbo trellis coded system. The structure of Space time turbo trellis codes presented by branka vcuteic promises the bandwidth efficiency but failed to convey the expected performance by trellis as a constituent code, although the system shows the promised behavior of decoder convergence at lower SNR .The transmitter side contains the two paralleley connected recursive systematic coders separated by an inter-leaver.

The input symbols are from a set of binary inputs which then are convolved with the generator polynomials which are recursive in nature. These generators polynomials were found by exhaustive search (Branka), and are considered to be the best generators yet discovered. As these generators are known best for the performance of an iterative decoder as they can guarantees the maximum uncorrelated symbols to be delivered out of the encoder. But when these symbols are transmitted in the channel which is normally slow fading channel plays an important role in the degradation of the performance and due to symbols deterioration it becomes difficult to decode these symbols and hence a large amount of information is lost.

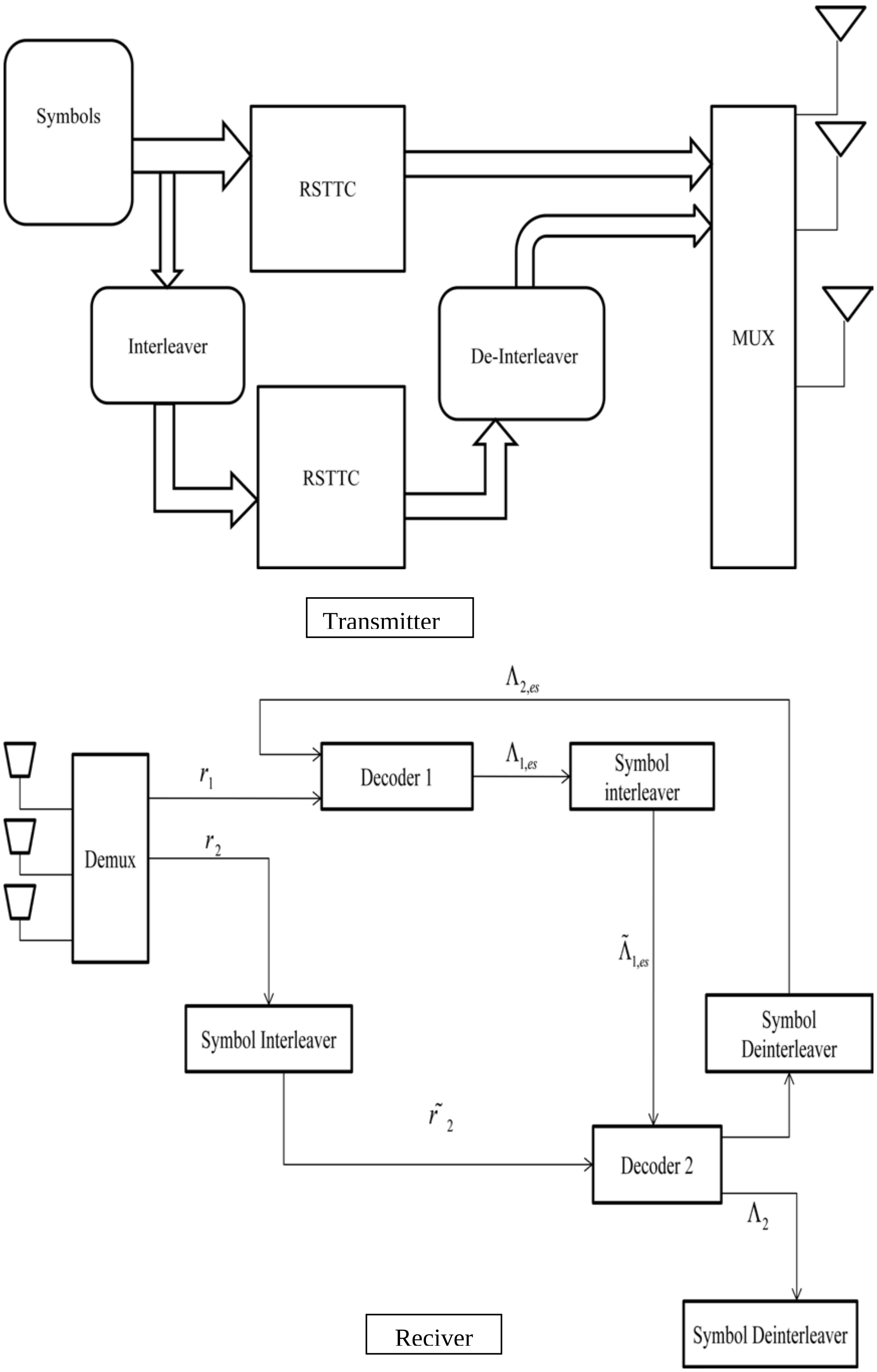

Figure 4.1: ST Turbo TC with HSD Interleaver

These genrators are actually found by making the STTC genrators recursive in nature ,to make these generators recursive in nature the feedforward genrator polynomials are divided by suitable feedback polynomials. The criteria for feedback coefficients are that they should have to be equivalent to the degree of the memory order. Hence the generator matrix G(D) can be converted in to a recursive matrix by dividing it with a feedback polynomial of degree less than or equal to the memory order of the branch

$$G_i(D)=\left[\frac{G_i^1(D)}{G_i^2(D)}\right] \tag{4.9}$$

Where memory order *v is*

$$v=\sum_{k=1}^{m} v_k$$

This generator matrix can be converted into an equivalent recursive matrix by dividing it by a binary polynomial q(D) of a degree equal or less than *v*. However, if *q(D)* a primitive polynomial is chosen, the resulting recursive code should have a high minimum distance. The generator polynomial for antenna *i* can be represented as

$$G_i(D)=\left[\frac{\dfrac{G_i^1(D)}{q(D)}}{\dfrac{G_i^2(D)}{q(D)}}\right] \tag{4.10}$$

Whereas q(d) is

$$q(D)=q_{0+}\ q_1 D+q_2 D^2+\ldots.q_{v1} D^{v1}$$

and q_j, j = 0, 1,2,... , vi, are binary coefficients from (0, 1).

The codes generated by this construction method, with the feedforward coefficients as in the corresponding feedforward STTC, have the same diversity and coding gain as these feedforward STTC. The feedback polynomials are considered to be key player in the decoder convergence. These generators are considered to perform best at low SNR and lead decoder towards convergence in a very systematic way. These generator polynomials discussed are given in the Table 4.1 for QPSK and Table 4.2 for 8- PSK modulation presented by branka vucuteic in her work in [16].

Table 4.1: Generator Polynomials for QPSK Modulation

n^t	v	Feedforward Coefficients		Feedback Coefficients		d_E^2	R
		g^1	g^2	q^1	q^2		
2	2	[(0,2)(1,2)]	[(2,3)(2,0)]	3	3	10	2
	3	[(2,2)(2,1)]	[(2,0)(1,2)(0,2)]	3	7	12	2
	4	[(1,2)(1,3)(3,2)]	[(2,0)(2,2)(2,0)]	7	5	16	2
3	2	[(0,2,1)(1,2,2)]	[(2,3,2)(2,0,1)]	3	3	16	2
	3	[(2,2,2)(2,1,3)]	[(2,0,1)(1,2,0)(02,2,)]	3	7	20	2
	4	[(1,2,3)(1,3,2)(3,2,3)]	[(2,0,2)(2,2,0)(2,0,2)]	7	7	24	2
4	2	[(0,2,1,0)(1,2,2,2)]	[(2,3,2,3)(2,0,1,1)]	3	3	20	2
	3	[(2,2,2,2)(2,1,3,2)]	[(2,0,1,1)(1,2,0,1)(0,2,2,3)]	3	7	26	2
	4	[(1,2,3,3)(1,3,2,2)(3,2,3,3)]	[(2,0,2,2)(2,2,0,0)(2,0,2,2)]	7	7	32	>2

Table 4.2: Generator Polynomials for 8-PSK Modulation

n^t	v	Feedforward Coefficients			Feedback Coefficients			d_E^2	r
		g^1	g^2	g^3	q^1	q^2	q^3		
2	3	[(2,1)(3,4)]	[(4,6)(2,0)]	[(0,4)(0,4)]	3	3	3	7.2	2
	4	[(2,4)(3,7)]	[(4,0)(6,6)]	[(7,2)(0,7)(4,4)]	3	3	5	8	2
	5	[(0,4)(4,4)]	[(0,2)(2,3)(2,2)]	[(3,0)(2,2)(3,7)]	3	5	5	8.6	2
3	3	[(2,1,2)(3,4,6)]	[(4,6,1)(2,0,4)]	[(0,4,4)(4,0,0)]	3	3	3	12	2
	4	[(2,4,6)(3,7,2)]	[(4,0,4)(6,6,4)]	[(7,2,7)(0,7,6)(4,4,1)]	3	3	7	12.7	2
4	3	[(2,1,2,5)(3,4,6,1)]	[(4,6,1,4)(2,0,4,6)]	[(0,4,4,5)(4,0,0,4)]	3	3	3	16.6	2
	4	[(2,4,6,6)(3,7,2,6)]	[(4,0,4,4)(6,6,4,5)]	[(7,2,7,4)(0,7,6,6)(4,4,1,2)]	3	3	7	18	2

Each encoder operates on a message block of L groups of *m* information bits, where *L* is the interleaver size. The block size considered in this model is of 128 symbols .The message sequence c is given by $\mathbf{c} = (\mathbf{c}_0, \mathbf{c}_1, \mathbf{c}_2, \ldots, \mathbf{c}_t, \ldots)$ where c_t is a group of *m* information bits at time *t*, given by $\mathbf{c}_t = (c_t^1, c_t^2, \ldots, c_t^m)$,which are then fed to both the encoders separately ,the first encoder gets the info bits without interleaving whereas the second encoder gets the input from an interleaver. The encoder output for transmit antenna *i at* time *t,* denoted by *x,* can be computed as

$$x_t^i = \sum_{n=0}^{m} \sum_{j=0}^{v_k} g_{j,i}^k c_{t-j}^k$$

These outputs are elements of an *M*-PSK signal set. Modulated signals from the space-time symbol transmitted at time *t*

$$x_t = (x_t^1, x_t^2, \ldots\ldots, x_t^{nT})^T$$

The space-time trellis coded *M*-PSK can achieve a bandwidth efficiency of *m* bits/s/Hz. Where $m = \log_2 M$, which becomes $m=2$ for QPSK modulation.

The QPSK modulated symbols are fed into the first encoder without interleaving where these info bits are coded with the help of generator polynomials. Second decoder gets its input from an interleaver which interleaves the data in deterministic way to increase the D_{min} . Then these symbols are de-interleaved and punctured to transmit on different antennas with the same rate.

When these symbols are transmitted into the channel through different antennas the distance between the antennas play an important role for saving the transmitted symbols from the channel's impact. This adds another degree of freedom to this system model which is already taking advantage of channel encoding and interleaving. Slow fading and fast fading comes into play when dealing with a MIMO system, because in MIMO the user is considered to be stationary and nomadic. For designing these codes rank and determinant criteria has been used.

4.2.1 Anatomy of the Interleaver

Random interleavers are used previously for ST Turbo TC's which shows very insignificant gains of coding and diversity as the system is capable of delivering [28]. Which brought us to a point to choose an another kind of interleaver, as it plays an important role in to increase the de-correlation of the symbols .Whereas the random type of interleavers doesn't show good results for short block length, as in proposed simulated model the block length is 128 symbols which is quiet short. As the interleavers with larger block length are not considered to be used for practical applications as they becomes a reason of latency which is not tolerable in some practical applications such as voice.

Increasing the interleaver size may not be a desirable solution as it increases the latency of the system to the unacceptable limits for some real time applications. The error floor can be reduced by increasing the block size L or by increasing the d_{min}. So the second option that is, increasing the d_{min} can prove to be a good solution for proposed model which clearly claims to outperform the previously proposed models. But by only increasing the d_{min} with the help of interleaver can give rise to another situation that it starts affecting the decoder convergence although the generator polynomials have an inherited property of convergence. By using an interleaver that only increase the d_{min} can surely stop us to exploit the encoder property of convergence. So the selection of interleaver for the purposed system is very critical.

Most of the turbo code applications use S-random type of interleaver [30]. An S-random interleaver is a "semi-random" interleaver which is works by following method. Comparison of randomly selected integers is done with S previously selected random integers. The random integer is rejected on the basis of, if the diffrence between the current selection and previous selection is less than S. This process is repeated until L distinct integers have been selected. Different simulations shows that if $s \leq \sqrt{L/2}$, then this process converges [31] in an acceptable time. This interleaver design guarantees that short cycle events are avoided. A short cycle event occurs when two neighboring bits remains neighbor both before and after interleaving. But the problem with the S-random interleaver is that there is a possibility of error event when the c_t is equal to $\Pi(c_t)$, means the symbol remains at the same place before and after the interleaving.

Recently some deterministic interleavers have been proposed [32–34]. These interleavers are designed to achieve maximum spread to correct single error events and certain amount of controlled disorder or vectorial fluctuation is provided to correct multiple error events. The HSD interleaver is algebraically designed by selecting the best permutation generator in a way that the points in smaller subsets of the interleaved output are uniformly spread over the entire range of the data frame. This technique improves the d_{min} as compared to the one achievable by the interleavers in [33,34]. Finally the length specific circular shift is introduced in design becomes a reason to decrease the correlation between the parity bits corresponding to the original and interleaved data frames. The idea of combining the two design criteria to construct a deterministic interleaver is influenced by two step S-random design [6] but the generation of the interleaver is based on arithmetic equation instead of S-random algorithm. The interleaver is designed by using very simple rules and provides enhanced performance and has an advantage of simplest hardware implementation as compared with long pseudo random and deterministic interleavers reported previously in literature. The High Spread deterministic interleaver discussed above is used to get the desired results in proposed model. Following discussion will show the effect of HSD interleaver.

4.2.2 Interleaving Function

The design of interleaver laid its foundation on two major criteria's: first the distance spectrum properties reflecting the weight allocation of the code, and second the correlation between the soft output of the decoder corresponding to its parity bits and the information data sequence. The second criterion is referred as iterative decoding suitability (IDS) criterion [21]. This is a measure of efficiency of iterative decoding based on the fact that if two data sequences are less correlated, then the performance of the iterative decoding algorithm is enhanced. A soft output decoding algorithm such as maximum a posteriori probability (MAP) is used to decode turbo codes in an iterative fashion. The MAP algorithm is assumed to be equivalent to maximum likelihood (ML) decoder. However, the MAP decoder infrequently makes non ML decisions when used in iterative fashion. The performance of iterative decoding improves if the information that is sent to each decoder from the other decoders is less correlated with the input information data sequence. Hokfelt proposed the IDS criterion and designed an interleaver based on this criterion [35].

Turbo codes utilize an iterative decoding process based on the MAP or other algorithms that can provide a soft output. At each decoding step, some information related to the parity bits of one decoder is fed into the other decoder together with the systematic data sequence and the parity bits corresponding to that decoder. Figure 4.1 shows this iterative decoding scheme. The inputs to each decoder are the input data sequence, the parity bits, and the logarithm of the likelihood ratio (LLR) associated with the parity bits from the other decoder, which is used as *a priori* information. All these inputs are utilized by the decoder to create three outputs corresponding to the weighted version of these inputs. In Figure 4.1, r represents the weighted version of the input data sequence, c. Also r_2 in the same figure demonstrates the fact that the input data sequence is fed into the second decoder after interleaving. The input to each decoder from the other decoder is used as *a priori* information in the next decoding step and corresponds to the weighted version of the parity bits. This information will be more effective in the performance of iterative decoding if it is less correlated with the input data sequence (or interleaved input data sequence). Therefore, it is reasonable to use this as a criterion for designing the interleaver. For large block size interleavers, most random interleavers provide a low correlation between Λ and input data sequence *r*. The interleaver designed on these criteria's will show us a significant improvement of results as compared to the previously used random interleavers.

From all of the above discussion the HSD interleaver caters the both criteria's of distance spectrum and iterative decoder suitability in a simple fashion and the way it conducts its operations. First of all the upper bound of distance spectrum and the way it's been approached by the help of HSD interleaver. The distance spectrum is catered with the help of multiplication factor denoted by MF and the IDS criteria is catered with the help of circular shift which is introduced also to save the system from short length error events which usually becomes a trouble for decoder.

4.2.3 Multiplication Factor

The multiplication factor MF is introduced to increase the distance spectrum in the incoming stream of data by using a permutation generator of its own kind. The upper bound of distance spectrum is defined by considering a simple mathematical analogy. Let L denotes the length of information block of the code and $\mathbf{\Pi}$ denotes the permutation function that associates an index $\mathbf{\Pi}\ (j)$ in the interleaved order with an index j in the natural order. The spread between two symbols j_1 and j_2 is defined as

$$S(j_1, j_2) = |j_1 - j_2| + |\Pi(j_1) - \Pi(j_2)| \qquad (4.11)$$

Let minimum spread S_{min} be the minimum value of $S(j_1, j_2)$ for all possible pairs of j_1 and j_2

$$S_{\min} = \underbrace{\min}_{j_1, j_2}\{S(j_1, j_2)\} \qquad (4.12)$$

The upper bound on S_{min} can be easily related to the sphere packing bound (SB) for D-dimensional turbo codes as below Boutillon.

$$S_{\min} \leq \left| L^{(D-1)} D \right|^{1/D} \qquad (4.13)$$

The SB gives an upper bound of the minimum spread, but there is no indication on how tightly it approaches S_{min} in D-dimensional case. The sphere packing density can be defined as S_{min} /SB. The minimum spread is equal to upper bound of SB if, and only if, the sphere packing density is equal to one. This condition can be achieved in two dimensions. For two dimensional case, the SB is equal to $\sqrt{2L}$. This SB can be achieved with deterministic interleavers. For example, if $L=2n^2$, where n is any positive integer, the minimum spread is $S_{min}=2n$.

The construction of HSD interleaver is done in a very simple and mathematical way by considering.

$$i = \prod(j) = jk \qquad \text{mod} \quad L \qquad (4.14)$$

where i represents the interleaved indices at the output of the interleaver and j is the original sequence of the information frame. The upper bound defined above can be achieved by setting

$$k = 2n - 1 \qquad (4.15)$$

the parameter k will be often referred as the permutation generator. $2n$ — 1 is not the only choice for the permutation generator. The value of k that maximizes $S_{\min}$, can be calculated as

$$k = M(2n) \pm 1 \qquad k < L \qquad (4.16)$$

where M is a multiplication factor of $2n$. The value of M is small compared with $S_{\min}$ and prime relative to L. In order to meet the restriction on k in eq(4.16),that k should have to be less then L, now the need is to restrict *M* as that it can be never across the limits .

So that the permutation generated will remain under the same subset of block size L otherwise it will be difficult to spread the symbols on the whole sample space. But while choosing the subset it should have also been taken care of that the subset should not to be small then a minimum number of allowed symbols which is always greater than one.

The upper limit on *M* can be derived as follows

$$k < L$$
$$M(2n) \pm 1 < L$$

This refers to

$$\begin{aligned} &M(2n) \le L - 1 \\ &M \le \frac{L}{2n} - 1 \\ &M \le \frac{2n^2}{2n} - 1 \\ &M \le n - 1 \end{aligned} \qquad (4.17)$$

Different values of *M* can generate different interleavers with no effect on S_{min}. The question is which value of *k* should be chosen by selecting an appropriate value of *M* in (4.16) to construct an interleaver with good permutations. Let $\square_L$ be a set of information bits and $\square_L^P = \prod(\square_L)$ be a set of permutations of $\square_L$. The set $\psi = \square_n^P, 1 \le n \le l$, is only a small subset of $\square_L^P$ for *l*<<*L*. In this case, which points (indices) of $\square_L^P$ should be in our opinion is that the points in Ψ should be uniformly spread over the entire range of $\square_L$. This is our main criterion for discriminating among the permutation generators presented by different values of *k* and *M* for constructing what is called ‘‘good permutations’’

The interleaver designed in this manner provides regular permutations and meets the upper bound on S_{min}, which can efficiently handle the single error events. Another class of codeword's is made up of multiple error events. It is believed that certain amount of non uniformity or controlled disorder has to be introduced to the regular permutation function in equation (4.14) to correct multiple error events. To overcome this problem in the case of HS interleaver, two solutions have been proposed [33,34] by introducing diagonal dither or a vectorial fluctuation around the locations given by regular permutation. When the dither is applied to the HS interleaver, it is designated as high spread random (HSR) interleaver. However, the USD interleaver provides uniform spread to the regular permutations, which has two effects on the iterative decoding of the turbo codes. Firstly, it provides the enlarged $d_{\min}$ and reduces its multiplicity. Secondly, it reduces the correlation between the parity bits corresponding to the original and interleaved data frames by mapping the indices of the input sequence to the interleaved indices for apart from their original locations. The reduced correlation enables the decoder to negotiate multiple error events caused by compound RTZ sequences. Therefore, the controlled disorder or dither recommended to introduce non uniformity to regular permutations in [33,34] is not required in the case of the USD interleaver. The interleaver designed with M = 1 is designated as HS interleaver and the interleaver designed with the appropriate value of $M > 1$ as the USD interleaver.

4.2.4 Displacement

The S-random interleaver which is believed to be the best in literature, guarantees that if $|j_1 - j_2| < S$ then $|\prod(j_1) - \prod(j_2)| > S$. However, some information bits are mapped to itself in the interleaved output vector. That is, mapping of index $j \rightarrow \prod(j)$, where $j = \prod(j)$ is a valid assignment for S-random interleaver. The interleavers designed for regular permutations avoid such assignments but the distance $\left| j_1 - \prod(j_1) \right|_{\min}$ is always one. Such assignment can degrade the performance of iterative decoding in turbo codes. Let us define the minimum displacement distance D_{min} as

$$D_{\min} = \left| j_1 - \prod(j_1) \right|_{\min} \qquad (4.18)$$

The larger the (D_{min}), the smaller the correlation between the extrinsic information from one decoder to the information data sequence at the input of the other decoder.

In the next step, by applying a circular displacement in (4) the D_{min} is maximized as

$$i = \prod(j) = jk + d \quad \text{mod} \quad L; \qquad 0<d<S_{min}/2 \qquad (4.19)$$

The upper bound for on D_{min} can be calculated by placing d=0,which gives the lowest values of displacement and shift which can be possible. So the maximum value of d which helps in maximizing the D_{min} can be calculated as

$$d = \frac{S_{\min}}{2} - 1 \qquad (4.20)$$

And the achievable D_{min}= S_{min}/2.

4.3 Decoding Mechanism

The decoder block diagram for the encoder is shown in Figure 4.2 At time *t*, the signal received by antenna *j*, where j = 1,2,... , n_R, can be represented as

$$r_t^j = \sum_{i=1}^{n^T} h_{i,j}^t x_{p,t}^i + n_t^j \qquad (4.21)$$

where $x^i_{p.t}$ is the output of the component encoder p at time t, where p = 1 for odd time instants t and $p = 2$ for even time instants t.

The received string at each antenna j, j = 1,2,... , n_r, is demultiplexed into two vectors, denoted by r^1_j and r^2_j, contributed by the upper and lower encoder, respectively. These vectors are applied to the first and second decoder, respectively the punctured symbols in these decoder input vectors are represented by erasures. They are given by

$$r_1^j = (r^j_1, 0, r^j_3, 0, r^j_5, \ldots\ldots)$$

$$r_2^j = (0, r^j_2, 0, r^j_4 \ldots\ldots\ldots)$$

The vector r^j is fed into the first decoder directly, while the vector r_t^j is fed to the second decoder via the symbol interleaver, identical to the one in the encoder.

The decoding process is very similar to the binary turbo code except that the symbol probability is used as the extrinsic information rather than the bit probability. The **MAP** decoding algorithm for nonbinary trellises is called symbol-by-symbol **MAP** algorithm. As proposed model is based on the symbol rate the symbol probabilities are used as the extrinsic information for the decoding process, which gives us the brief look of the decoding process discussed in the following discussion. These probabilities are calculated here and used for the iterative decoding process and the information passed from HSD interleaver with increased D_{min}.

The **MAP** decoder computes the LLR log-likelihood ratio of each group of information bits $c_t = i$. The soft output $\Lambda(c_t = i)$ is given by [27]

$$\Lambda(c_t = i) = \log \frac{P_r\{c_t = i/r\}}{P_r\{c_t = o/r\}} \qquad (4.22)$$

$$= \log \frac{\sum_{(l',l)\in\beta_t^i} \alpha_{t-1}(l')\gamma_t^i(l',l)\beta_t(l)}{\sum_{(l',l)\in\beta_t^o} \alpha_{t-1}(l')\gamma_t^o(l',l)\beta_t(l)}$$

where *i* denotes an information group from the set, {0, 1, 2,... , 2^m-1}, r is the received sequence, B_t^i is the set of transitions defined by $S_{t-1} = I' \rightarrow S_t = I$, that are caused by the input symbol *i*, where S_t is a trellis state at time t, and the probabilities $a_t(l)$, $\beta_t(l)$ and $\gamma_t(l',l)$ can be computed recursively [24].

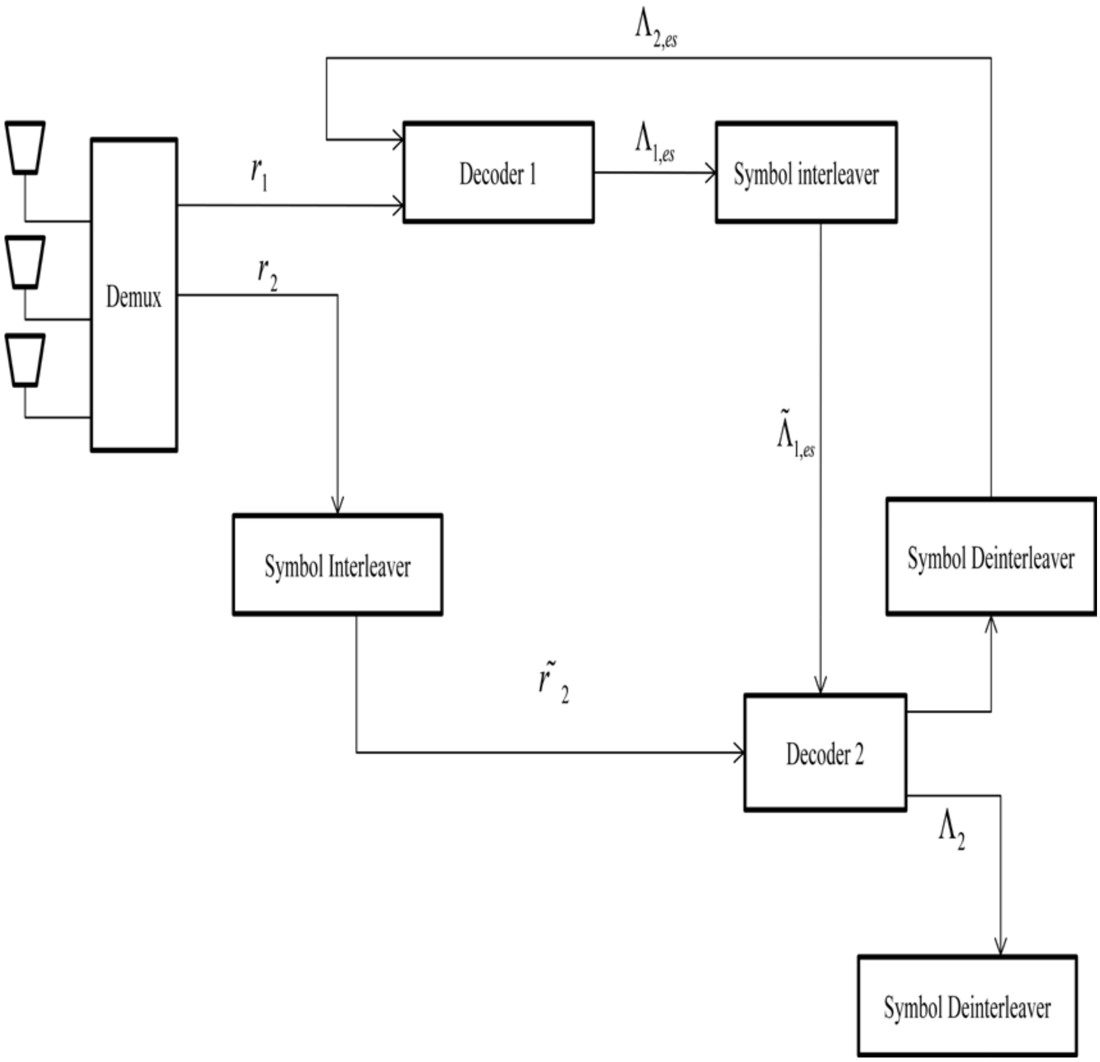

Figure 4.2: Decoder with Symbol Interleaving

The symbol *i* with the largest log-likelihood ratio in Eq. (4.22), *i* € {0, 1, 2,... , 2^m - 1}, is chosen as the hard decision output.

The decoder operates on a trellis with M_s states. The forward recursive variables can be computed as follows

$$\alpha_t(l) = \sum_{l'=0}^{Ms-1} \alpha_{T-1}(l') \sum_{i=0}^{2^m-1} \gamma_t^i(l', l) \quad l=0,1,\ldots\ldots\ldots\ldots Ms-1 \tag{4.23}$$

with the initial condition

$$\alpha_0(0) = 1$$

$$\alpha_0(l)=0 \qquad l\neq 0$$

and the backward recursive variables can be computed as

$$\beta_t(l) = \sum_{l'=0}^{Ms-1} \beta_{t+1}(l') \sum_{i=0}^{2^m-1} \gamma_{t+1}^i(l', l) \quad l=0,1,\ldots\ldots\ldots\ldots M_s-1 \tag{4.24}$$

with the initial condition

$$\beta\tau(0)=1$$

$$\beta\tau(l)=0 \qquad l\neq 0$$

The branch transition probability at time t, denoted by $\gamma_t{}^i(l', l)$, is calculated as

$$\gamma_t^i(l', l) = \left\{ \frac{p_t(i)}{p_t(0)} \exp\left[\begin{matrix} -\dfrac{\sum_{j=1}^{n_r} \left| r_t^j - \sum_{n=1}^{n_T} h_{j,n} x_t^n \right|^2}{2\sigma^2} \\ 0 \end{matrix} \right] \right\} \qquad \text{for}\,(l', l) \in B_i^t$$

where r^t_j is the received signal by antenna j at time t, $hj,_n$ is the channel attenuation between transmit antenna n and receive antenna j, $x_n{}^t$ is the modulated symbol at time t, transmitted from antenna n and associated with the transition $S_{t-1}= I'$ to $S_t = I$, and $p_t(i)$ is the a priori probability of $c_t = i$.

The iterative process of the symbol-by-symbol MAP algorithm for space-time turbo trellis codes is similar to that of binary turbo decoders. However, for binary turbo decoders, a soft output can be split into three terms. They are the a priori information generated by the other decoder, the systematic information generated by the code information symbol and the extrinsic information generated by the code parity symbols. The extrinsic information is independent of the a priori and systematic information. The extrinsic information is exchanged between the two component decoders. For space-time turbo trellis codes, regardless whether component codes are systematic or nonsystematic, it is not possible to separate the influence of the information and the parity-check components within one received symbol, as the symbols transmitted from various antennas interfere with each other. The systematic information and the extrinsic information are not independent. Thus both systematic and extrinsic information will be exchanged between the two component decoders. The joint extrinsic and systematic information of the first MAP decoder, denoted by $\Lambda_{1,es}(c_t = i)$, can be obtained as

$$\Lambda_{1,es}(c_t = i) = \Lambda_1(c_t = i) - \log \frac{p_t(i)}{p_t(0)} \tag{4.25}$$

The joint extrinsic and systematic information $\Lambda_{1,es}(c_t = i)$, is used as the estimate of the a priori probability ratio at the next decoding stage. After interleaving, it is denoted by $\Lambda_{1,es}(c_t = i)$,. The joint extrinsic and systematic information of the second decoder is given by

$$\Lambda_{2,es}(c_t = i) = \Lambda_2(c_t = i) - \tilde{\Lambda}_{1es}(c_t = i) \tag{4.26}$$

In the next iteration the a priori probability ratio in Eq. (4.15) is replaced by the de-interleaved joint extrinsic and systematic information from the second decoding stage, denoted by $\Lambda_{2,es}(c_t = i)$.

Note that each decoder alternately receives the noisy output of its own encoder and that of the other encoder. That is, the parity symbols in every second received signal belong to the other encoder and need to be treated as punctured.

Let's consider the first decoder, for every odd received signal, the decoding operation proceeds as for the systematic binary turbo codes when the decoder receives the symbol generated by its own encoder, except that the extrinsic information is replaced by the joint extrinsic and systematic information. However, for every even received signal, the decoder receives the punctured symbol in which the parity components are generated by the other encoder. The decoder in this case ignores this symbol by setting the branch transition metric to zero. The only input at this step in the trellis is the a priori component obtained from the other decoder. This component contains the systematic information.

For bit interleaving, decoding can be carried out by converting the joint systematic and extrinsic information computed for a symbol to a bit level, since the exchange of the information between the decoders is on a bit level not on symbols level. After interleaving/deinterleaving operations, the a priori probabilities need to be converted to a symbol level since they will be used in the branch transition probability calculations.

Let us consider a symbol of a group of *m* information bits given by

$$\mathbf{C} = (c_0, c_1 ...., c_{m-1}) \qquad (4.27)$$

where $c_j = 0,1$, $j = 0,1, 2,... , m - 1$. If the extrinsic information of the symbol c is denoted by $\Lambda_e(c)$, the extrinsic information of extrinsic information the jth bit can be represented by [36].

$$\Lambda_e(c_j) = \log \frac{\sum_{c:c_j=1} e^{\Lambda_e(c)}}{\sum_{c:c_j=0} e^{\Lambda_e(c)}} \tag{4.28}$$

After the interleaving/deinterleaving operations, the a priori probability of any symbol can be given by [26]

$$P(c = (c_0, c_1 \ldots\ldots\ldots, c_{m-1})) = \prod_{j=0}^{m-1} \frac{e^{c_j . \tilde{\Lambda}_e c(j)}}{1 + e^{\tilde{\Lambda}_e (c_j)}}$$

4.4 Decoder Convergence

Consider a turbo code with two component codes. The decoder is based on two component modules, as shown in Figure 4.3.

The iterative decoder can be viewed as a nonlinear dynamic feedback system [31][32]. Each component decoder can be described by a nonlinear characteristic representing the output versus the input SNR associated with the extrinsic information, denoted by λ. These characteristics are referred to as extrinsic information transfer (EXIT) charts [31]. The pdf of *k* can be approximated by a Gaussian distribution. These characteristics are denoted by G_1 and G_2, for the first and second decoder, respectively. For a given channel Eb/N_0, the output SNR of each encoder is a nonlinear function of its input. Thus I have

$$\mathrm{SNR1}_{\mathrm{out}} = \mathrm{G}_1(\mathrm{SNR1}_{\mathrm{in}}, E_b/N_0)$$

and

$$\mathrm{SNR2}_{\mathrm{0Ut}} = \mathrm{G}_2(\mathrm{SNR2}_{\mathrm{in}}, E_b/N_0)$$

Also I have

$$\mathrm{SNR2}_{\mathrm{in}} = \mathrm{SNR1}_{\mathrm{out}}$$

And

$$\mathrm{SNR2}_{\mathrm{out}} = \mathrm{G}_2(\mathrm{G}_1(\mathrm{SNR1}_{\mathrm{in}}, E_b/N_0), E_b/N_0)$$

Note that a nonzero E_b/N_0 from the channel enables the first encoder to produce a nonzero output SNR, denoted by $SNRl_{0UT}$ though it starts with a zero input SNR, denoted by SNR 1 in. The second decoder gives a zero output SNR for a zero input SNR, since its information bit is punctured. Figure 4.3 shows the decoder convergence as a function of iterations. The SNR values of the extrinsic information follow a staircase path between the curves corresponding to G_1 and G_2^{-1}. The steps are large when the curves are far apart and small when they are close. That is, the

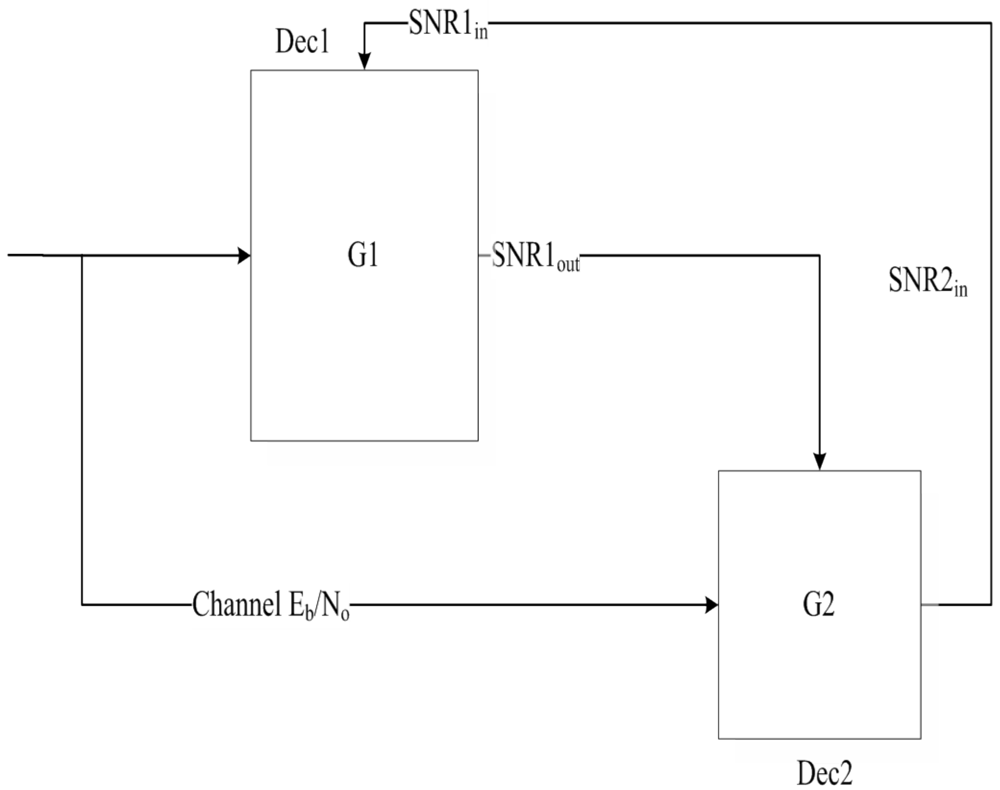

Figure 4.3: Block Diagram of an Iterative Decoder

convergence and consequently the bit error rate improvement, is large when the curves are far apart. The convergence rate is most critical in the narrow passage called the **decoding tunnel.** If the decoder passes the tunnel the convergence becomes fast.

If Eb/N_0 is reduced from the value in, at some point curves G_1 and G_2^{-1} will touch each other. That value represents the iterative **decoding threshold.** The decoder converges for $Ef,/N_0$ only above this threshold value.

4.5 Results

In this section, results of the Space time turbo coded system with different interleavers will be shown. This includes the results of Matlab simulated environment with different number of transmit and receive antennas with QPSK modulation. Different states for trellis has also been consider showing the comparison results with the work proposed in [27]. The complete work proposed in [27] is simulated locally to confirm the understanding established by me and to keep the similarity of work for different interleavers.

4.5.1 System Parameters

As discussed the design and size of interleaver plays an important role in the performance of turbo codes and some results are also shown for the deterministic interleaver which outperforms the random interleaver for short block length .The authors of [28] have shown their results with the block size of 130 symbols which is considered very small block length by using random two-step interleaver which works separately on even and odd position of the frame. In [28] another type of interleaver has been proposed which also works in two-steps but in a deterministic fashion, so for the performance comparison of both the interleavers and the block size 128 symbols per frame to calculate the frame error rate to show the fair comparison. The generator polynomials proposed in [27] are used to simulate the environment and are given in the Table 4.2 and Table 4.3.

4.5.2 ST Turbo TC with Random Interleaver

Under this section the performance curves previously presented by branka vucuteic with random interleaver for two, three and four transmit antennas are presented. Figure 4.4 shows the Frame error rate on a 2 transmit and 2 receive antennas with four state space time turbo trellis coded system.

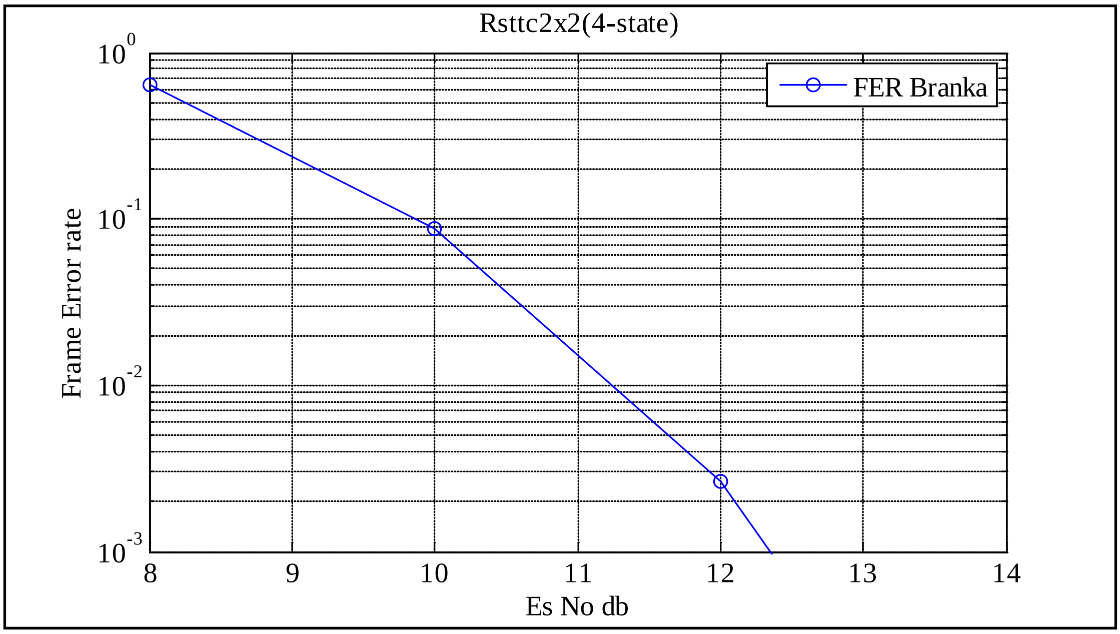

Figure 4.4: Space Time Turbo Trellis Code 2x2 with 4 State Trellis

The above result shows that the SNR threshold value for QPSK is 12.4dB for 10e-3 FER. This curve shows the behavior of space time turbo trellis codes with S-random interleaver on a slow fading channel by using QPSK modulation scheme for a four state trellis. The same parameters when applied for different states of trellis they produce the different results slightly degraded.

Figure 4.5 illustrates the performance curve of 2 transmit and 2 receive antenna system in which recursive STTC's structure is implemented with 8 state trellis with a random interleaver.

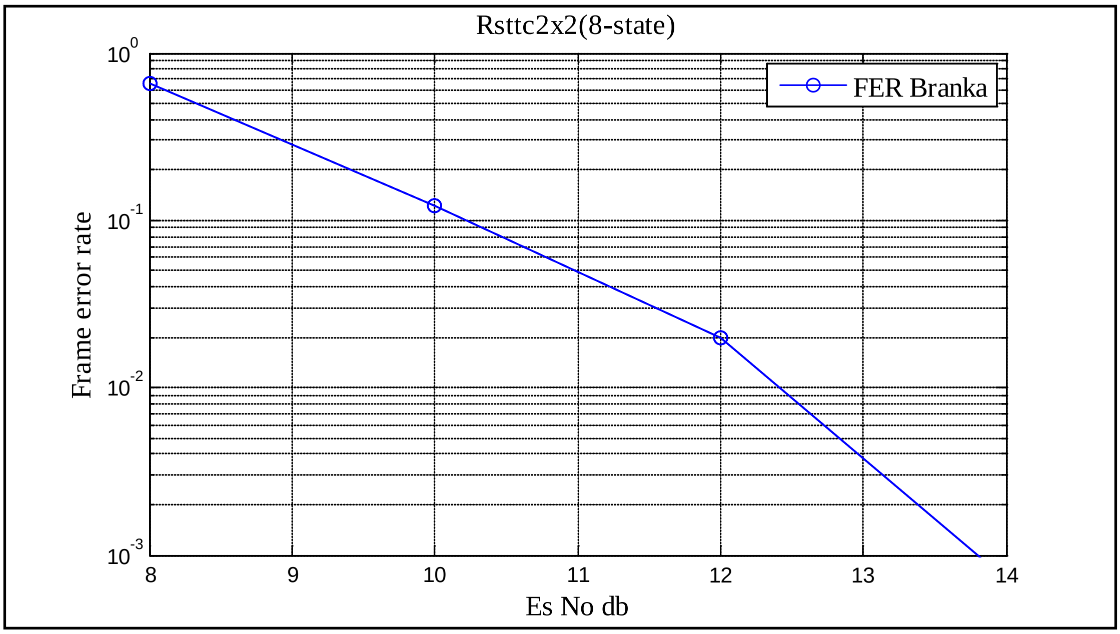

Figure 4.5: Space Time Turbo Trellis codes 2x2 with 8 State Trellis

The cutoff near 14db shown in Figure 4.6 shows the performance curve of a 2x2 MIMO system with 16 state trellis structure, this poor performance in both the curves as compare to 4 state trellis is due to the increased decoder complexity

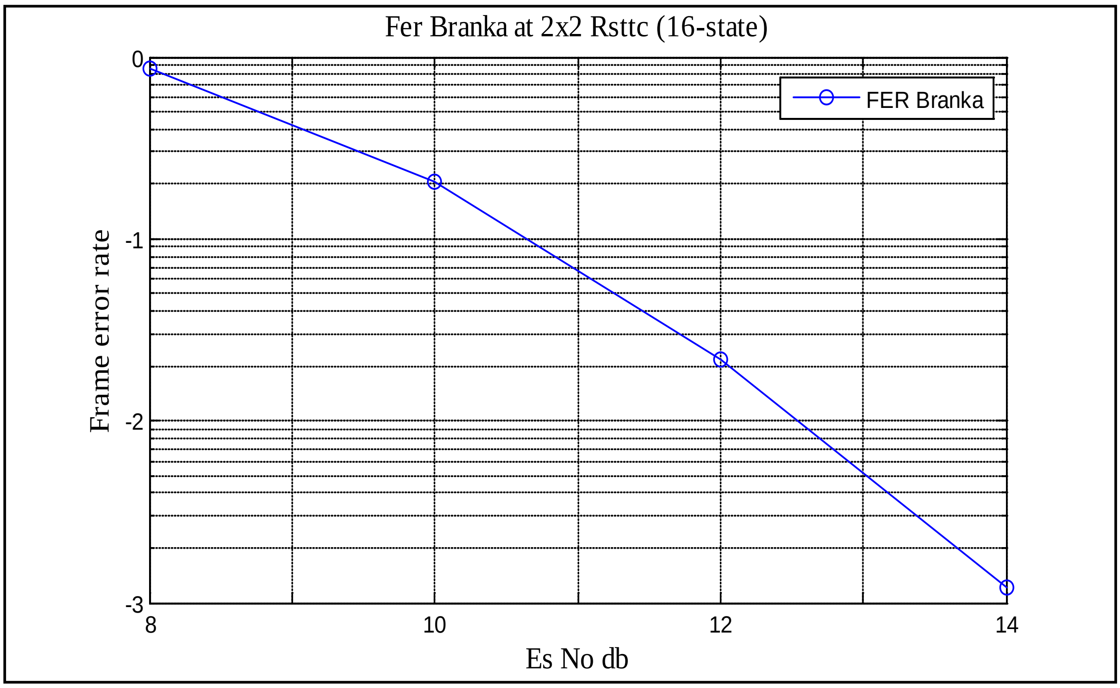

Figure 4.6: Space Time Turbo Trellis Codes 2x2 with 16 State Trellis

These all graphs show the behavior of random interlaver in space time turbo trellis codes. The use of random interleaver in this system can separate the odd and even symbols which does not help the decoder very much and the added states of trellis again becomes a reason for degraded performance .

4.5.3 ST Turbo TC with HSD Interlaever

This section shows the results obtained by simulating the environment with a newly proposed interleaver which is deterministic in nature. These results are obtained by the induction of HSD interlaever in the same environment in which previous results are obtained. In Figure 4.7 the performance curve of a space time turbo trellis codes in a 2x2 MIMO channel with 4 state trellis.

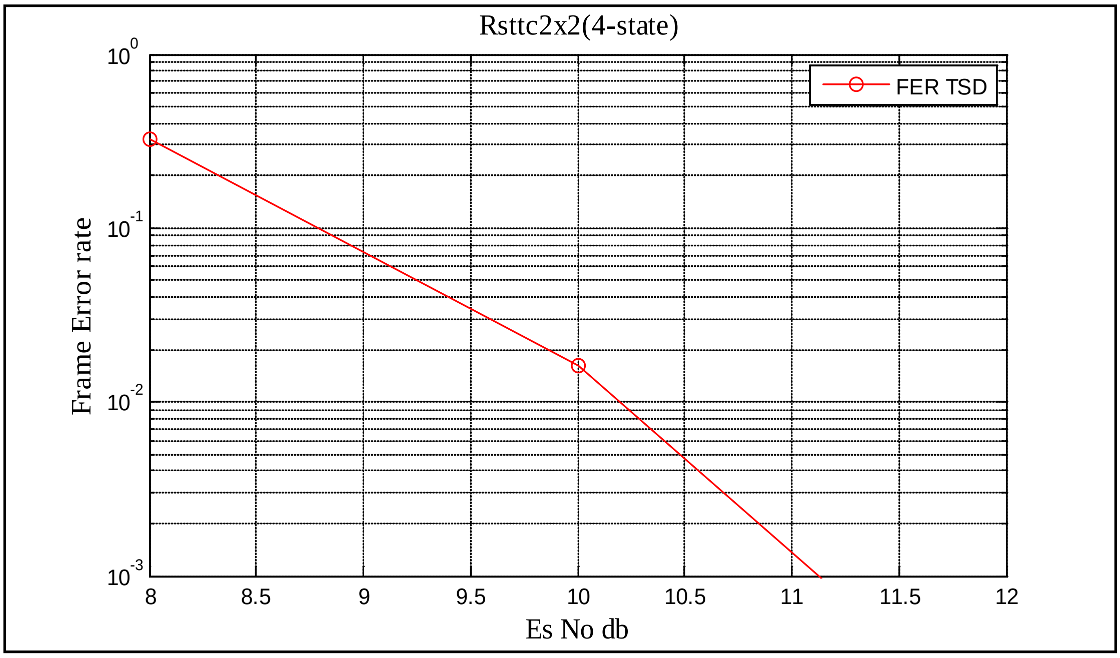

Figure 4.7: Space Time Turbo Trellis Codes 2x2 with 4-States by using HSD

The above graph shows the behavior of STTTC with a different interleaver type which is supposed to give increased hamming distance in the encoded systems .The FER performance of STTTC with HSD interleaver shows a cutoff at 11.2db.The other results in this section shows a bit degraded performance as compared to the 4 state trellis. But these results are considered very promising as compared to the already proposed model with random interleaver. The comparison will be discussed in the next section.

Figure 4.8 illustrates the result for 2x2 STTTC with 8 state trellis by using QPSK modulation. The results shows cutoff at almost around 13db SNR which almost comparable to 16 state performance of STTTC with HSD interleaver.

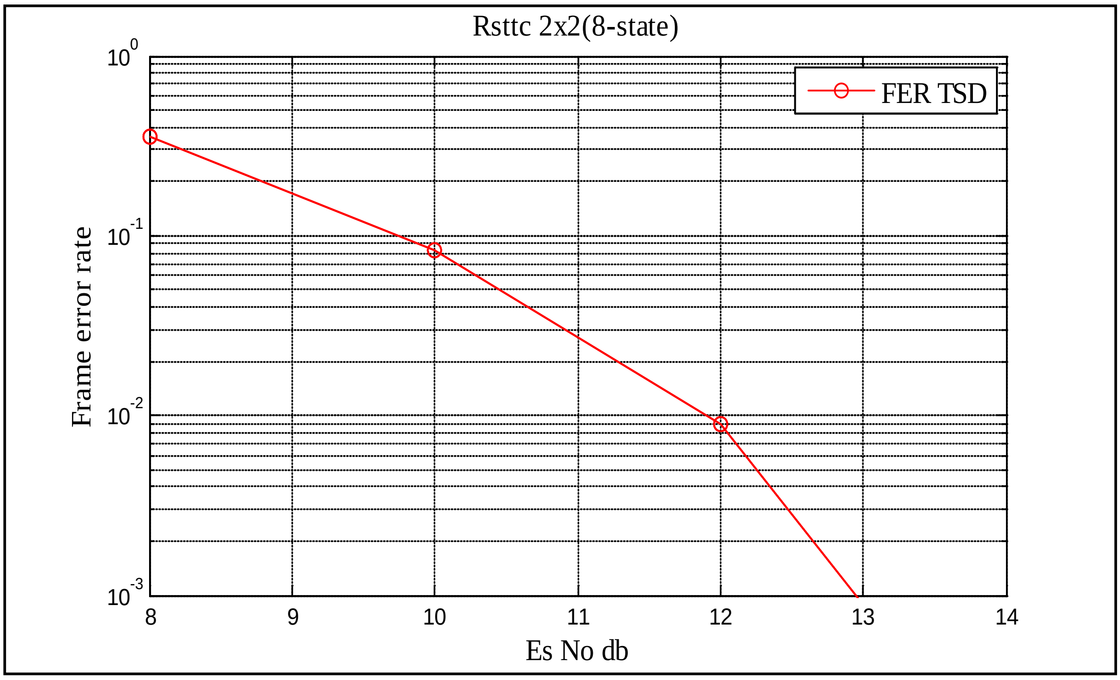

Figure 4.8: Space Time Turbo Trellis Codes 2x2 with 8 States by using HSD

The increased number of states also plays an important role in the delayed decoding of the symbols because the lookup table becomes very populated. As we can see the curve is still ahead the 1db gain as compared to the existing random interleaver which fails to increase the de-correlation of the incoming symbols.

The Figure 4.9 shows the performance curve of Space time turbo trellis codes with HSD interleaver in a slow fading MIMO channel with 16 states. This shows a cutoff at 10-3 around 13 db which is far less than the random interleaver. The degraded performance as compared to 8 states is quiet evident.

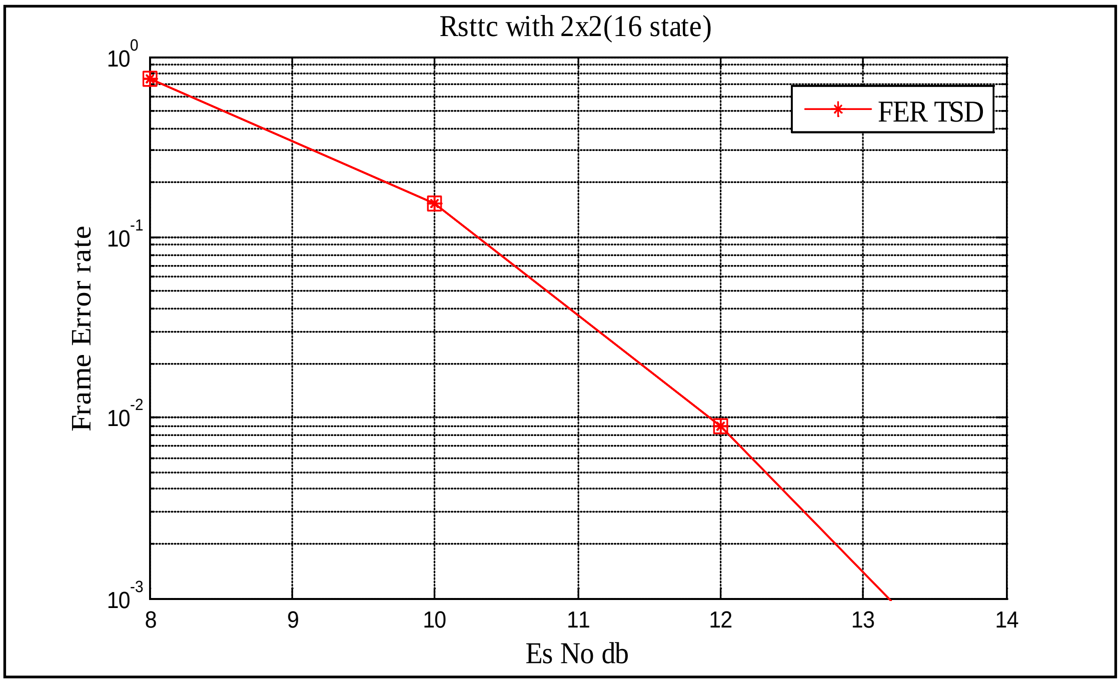

Figure 4.9: Space Time Turbo Trellis Codes 2x2 with 16 States by using HSD

4.5.4 Comparison of Interleavers

The both interleaver works almost in the same fashion as a two step interleaver, they both works on even and odd symbols but the basic difference behind them is the pattern they follow. The random interleaver interleaves the symbols randomly increases the probability of error by the inherited effect of randomness that can lead to place two neighboring symbols adjacently .Whereas the HSD interleaver works in a deterministic fashion which decreases the probability of error. The below section illustrates the comparison of both interleavers in the same environment, in which a slow fading Raleigh model is used for the practical realization of the system the decoder is set on 5 iterations which is very less number of for the decoders to converge. This shows an enhanced performance with HSD interleaver as compared to random interleaver. The first comparison curve is shown in Figure 4.10, which shows the frame error rate curve with both random and HSD interleaver.

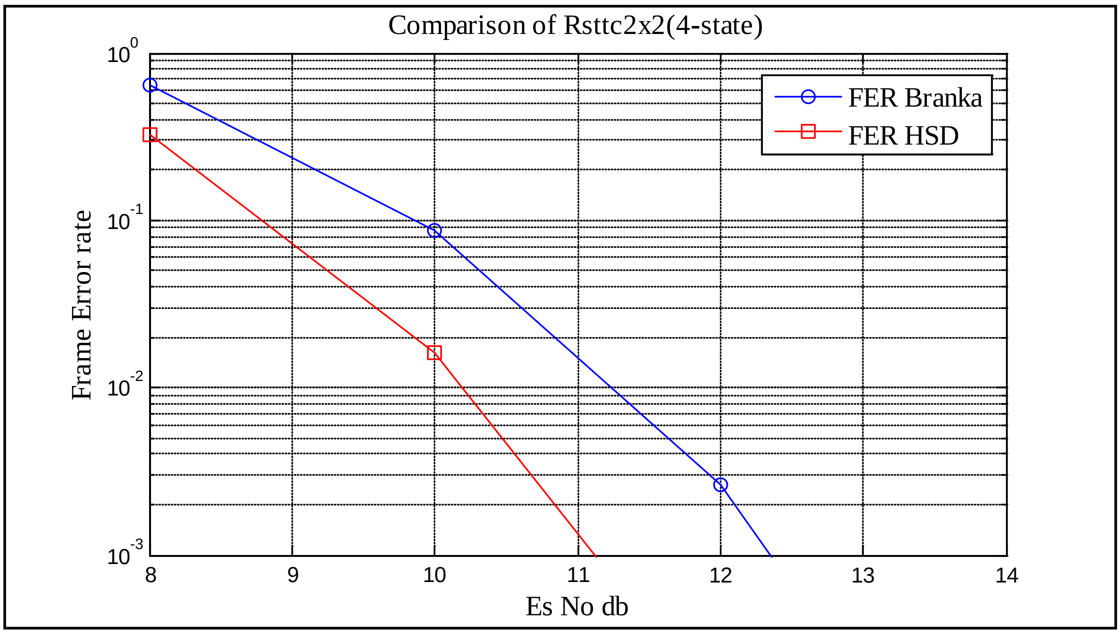

Figure 4.10: Comparison of ST Turbo TC with both Interleaver Design

This graph shows as prominent gain in terms of SNR of almost 1db as compared to the previously used random interleaver for a 4 state trellis with 2 transmit and 2 receive antenna. Second comparison is shown in Figure 4.11 in 2x2 MIMO system with 8 states of trellis having random and HSD as constituent interleaver.

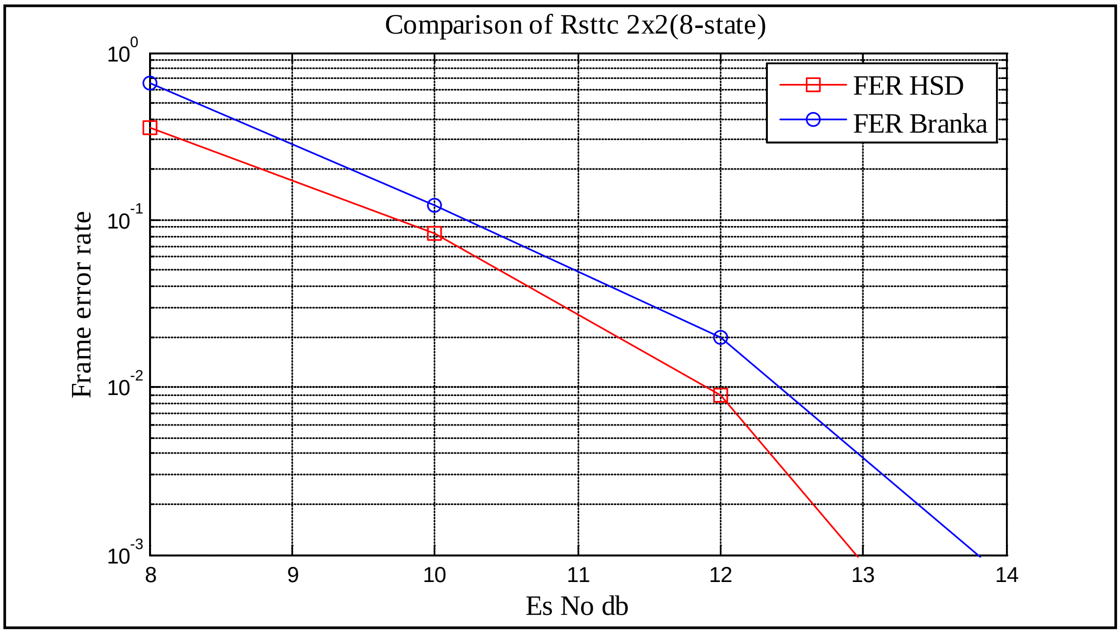

Figure 4.11: Comparison of ST Turbo TC with both Interleaver Design

Results of 16 state trellis space time turbo coded system are compared in Figure 4.12, this comparison clearly shows a almost 1 db gain in terms of SNR by using HSD interleaver.

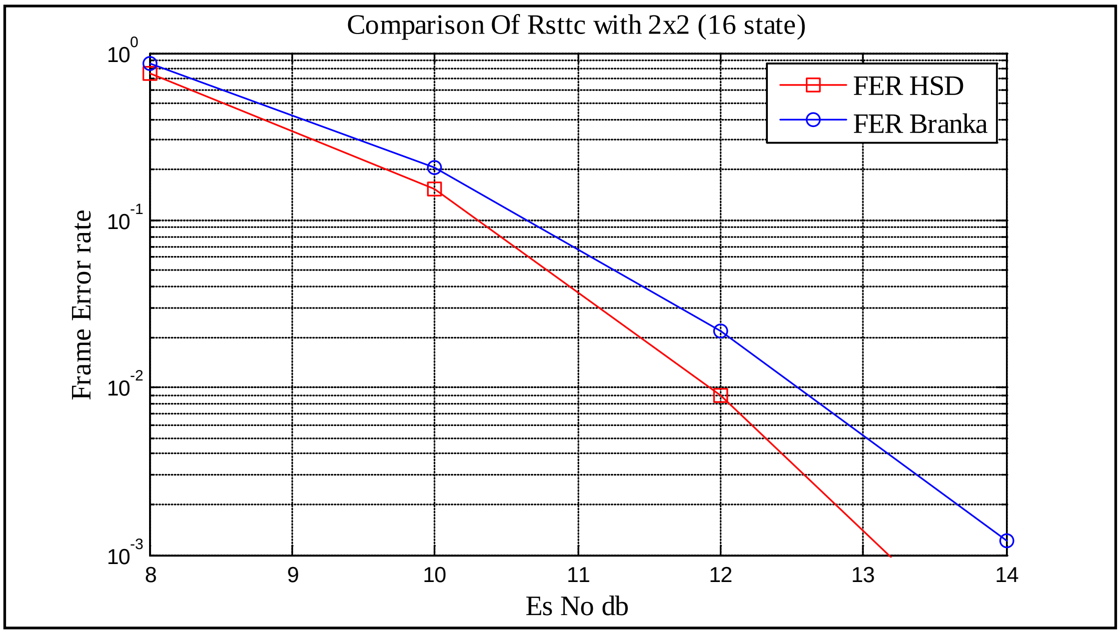

Figure 4.12: Comparison of ST Turbo TC with both Interleaver Design

4.5.5 ST Turbo TC with 4 Transmit and 4 Receive Antennas

To take the research upto the next level, the four transmit and four reciveve antenna system for the performance evaluation of ST Turbo TC with random and deterministic interlaevers has been implemented in Matlab with practice realization of the system. The implementation of four transmit and four receive antenna environment is a challenging task because of the systems increased complexity, but the expected gain in performance compelled me to go for it.

The behavior of the system shows an expected consistency in terms of the effect of interleaver on the performance of turbo codes. As the number of antennas increased the array gain is claimed which shows almost 5 to 6 db gain as compared to the 2x2 antenna is quiet evident. This gain shows that we can increase the gain with the help of array gain, but this increased performance is due to the increased spatially diversity and the time diversity achieved due to the space time coding employed in Figure 4.13. This increased gain is also significant as compared to the Almouti's space time coded system.

The result in the figure 4.13 shows the frame error rate of Space Time Turbo Trellis coded system with the maximum number of transmit and receive antennas ever proposed shows a cutoff at around 5.1 db by increasing the number of antennas a total gain of around 5 to 6 db in general is observed.

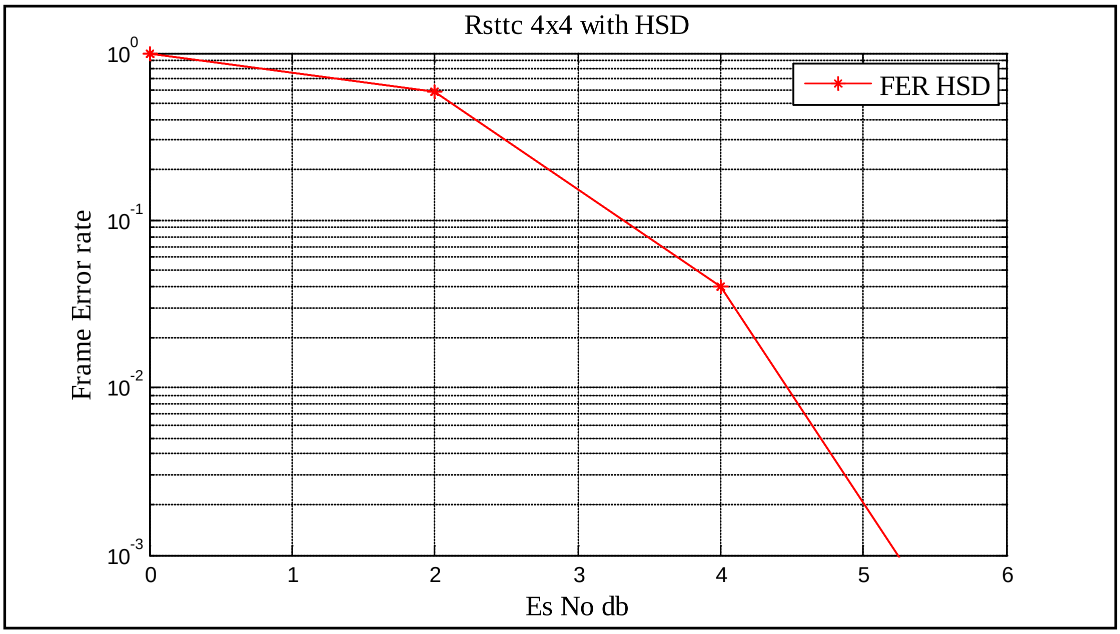

Figure 4.13:FER Perfomance of ST Turbo TC with 4Tx&4Rx Antenna

Whereas Figure 4.14 displays the result of a space time coded system with four transmit and four receive antennas, random type of interleaver previously used for 2 transmit and 2 receive antenna is employed. Which failed to deliver the performance at this short block size even on 4x4 system where the redundancy of the system is quite enhance as compared to a 2x2 system.

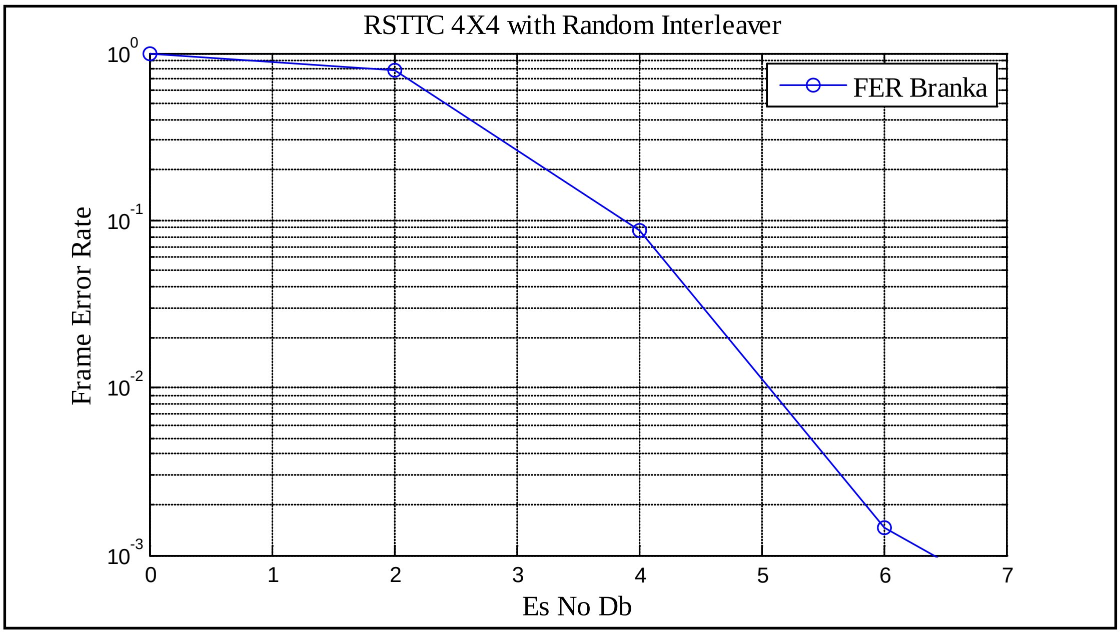

Figure 4.14: FER Perfomance of ST Turbo TC with 4Tx&4Rx Antenna with HSD

The comparison of both interleaver gain has been analyzed in Figure 4.15 .The results show almost difference of 1 db in Frame error rate which is almost consistent with the previous behaviors.

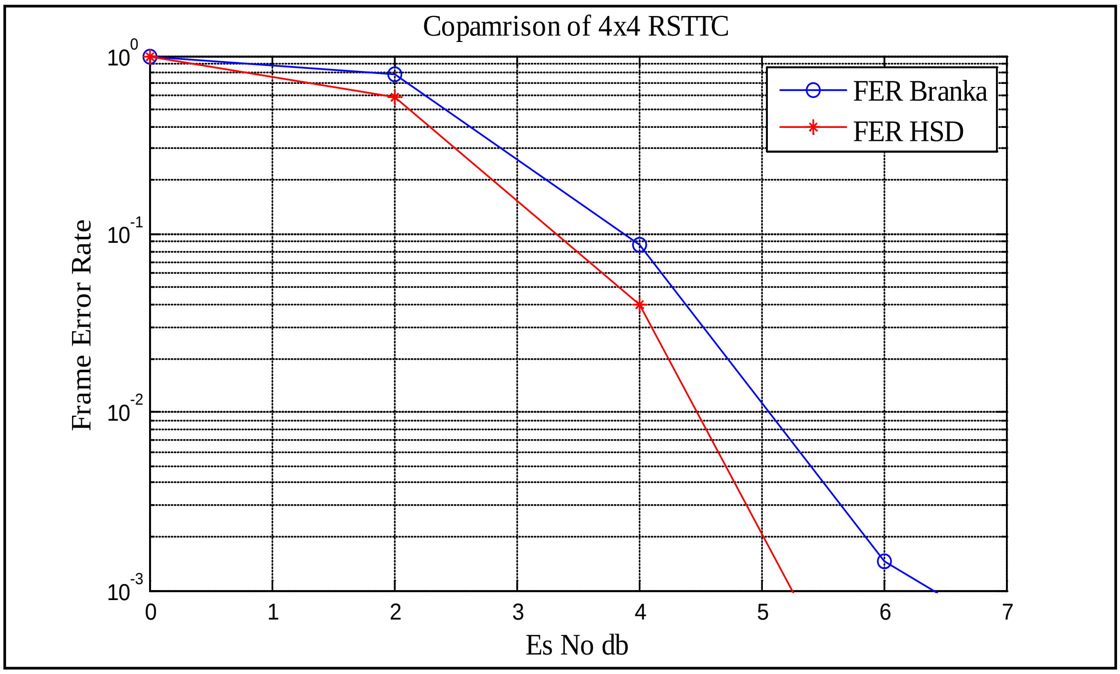

Figure 4.15: FER Performance Comparisons of ST Turbo TC with 4x4 Antenna

4.6 CONCLUSION

The design of full rate codes were performed in previous years but to get a maximum output of the system these designs were not optimized on the basis of interleaver design. It has been showed here that the results with a deterministic interleaver which had introduced the deterministic behavior can outperform the random interleavers used previously. The deterministic nature is not only the key of success but the incorporation of circular shift and displacement to increase the minimum distance between the adjacent symbols are the major players of the mechanism.

The turbo codes with recursive trellis codes as constituent codes haven't showed this much improvement as were expected from them when used in a MIMO channels, although there convergence was considered good. But with the use of random interleavres the increased D_{min} by the help of encoder failed to show any improvement because the random interleaver failed to cater short length error events and also short block length.

MAP decoder is used for the iterative decoder which needs the maximum un correlated symbols for the decoding process. This is done only with the help of HSD interleaver which reduce the probability of short length error events and also enhance the power of decoder to mitigate single error events, which usually occurs at high SNR.

Hence our results showed that the use of HSD inteleaver in the ST Turbo TC becomes a source of increased performance which is measured in terms of Frame error rate, while considering error floor occurring at 10^{-03}. The use of HSD interleaver enabled us to get a performance gain of about almost 1db in every scenario, which was expected.

Chapter 5

CONCLUSION AND FUTURE WORK

5.1 CONCLUSION

As discussed in the previous chapters the different ways to exploit the channel capacity, one of them is MIMO. Whenever the system goes to high data rates the channel induced impairments in the data causes errors. In proposed model emphasis is on to improve the performance up to the promised marks of the ST Turbo TC codes with the help of High spread deterministic interleaver. Which has never been used in ST Turbo TC the results discussed in chapter 4 shows a clear improvement in the performance of ST Turbo TC.

The combination of parallel concatenation and recursive decoding allows these codes to achieve much improved performance. The FER of the system has been dropped further down to the range of 10e-4 by the application of HSD interleaver. The proposed system has been passed through AWGN as well as slow fading channel and the performance of the system has been tested individually with and without the HSD interleaver. Along with this, the decoder has been implemented using MAP decoding algorithm based on the exchange of soft information between two component MAP decoders. As the numbers of iterations are increased the LLR of the decoder output diverges from its mean 0 towards +ive or –ive side to depict a 1 or 0 decoded bit.

Thus it can be concluded that a new dimension has been introduced to research in the field ST Turbo TC that the by just optimizing the interleaver can affect the performance of the system. This new side has never been discussed because the research in the field of ST Turbo Trellis codes has been left due to the increased complexity of the system and reduced performance so by proposed work this area of research should get again some attention in this area by other researchers.

5.2 FUTURE WORK

This thesis is focused on the importance of interleaving pattern in turbo codes and this thing still persists when turbo codes introduced in the MIMO environments. The system considered in this thesis has been presented as the most bandwidth efficient system due to the presence of trellis codes as RSC constituent codes but the results shown in previous works shows a lack of fully exploited capacity. The idea of introducing a new interleaver shows a drastic increase in the results.

Yet there is space to improve the system capacity with higher order modulation schemes like 64-QAM with HSD as interleaver. While using a higher order modulation scheme the main problem is that in the same space the symbols are more correlated than a lower order modulation scheme, so by using an HSD interleaver one can get the gain of interleaver whose inherent property is to increase the decorrealtion of the symbols. The increased complexity is in itself a challenge for this interleaver to be tested.

Space time turbo trellis codes can be implemented with OFDM(Orthogonal frequency division multiplexing), which helps to exploit the multipath and channel in a different way and by using enhanced channel estimation and equalization techniques this system can be refined for the practical implementation. But as the MIMO itself enchases the channel and multipath effects of channel.

The computational complexity of MAP algorithm is widely know, to decrease the computational complexity of the system a different decoding algorithm can be used like SOVA and LOG-MAP. SOVA can be good candidate for reduced complexity but on the other hand you have to compromise on the performance. As the complexity of the system depends on the number of multiplications and number of maximizations used, on the other hand decoder is always an iterative decoder for turbo codes and in competition of iterative decoder outperforms all of other decoder in terms of performance gain but increases the complexity of the system.

Adaptive modulation technique is used to meet user requirements and fighting channel conditions by changing the modulation scheme accordingly. As bandwidth becomes a prime concern for researchers so by using an adaptive modulation model in a space time turbo trellis coded system, user requirements can be meet in terms of efficient bandwidth utilization. Generator polynomials plays an important role in the performance of space time turbo trellis codes as they help to increase the Euclidian distance of the symbols [28]. In this thesis generator polynomials of Table 4.1 and Table 4.1 have been used and they performed well for QPSK modulation scheme but there is more need investigate theses generators for 8-PSK and 64-QAM, as the higher order modulation schemes have very correlated symbols this correlation can be catered with good generators and interleaver.

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