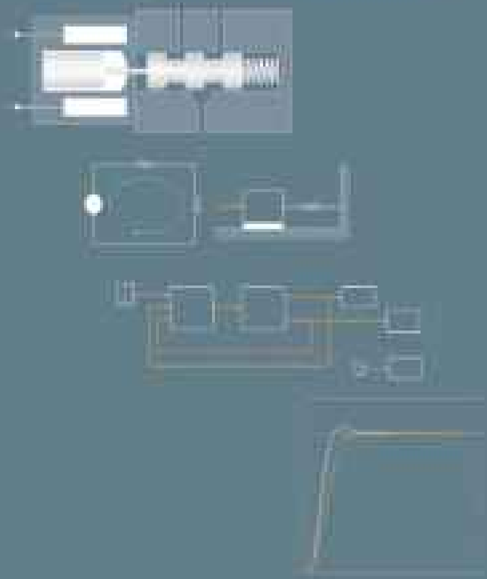
DYNAMIC SYSTEMS

Modeling, Simulation, and Control



CRAIG A. KLUEVER

WILEY

Dynamic Systems: Modeling, Simulation, and Control

Craig A. Kluever
University of Missouri-Columbia

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7.2 Analytical Solutions to Linear Differential Equations [illegible]
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11.4 Pneumatic Air Brake System [illegible]
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References [illegible]

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C.2 Building Simulink Models of Linear Systems [illegible]
C.3 Building Simulink Models of Nonlinear Systems [illegible]
C.4 Summary of Useful Simulink Blocks [illegible]

Index [illegible]

Preface

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[illegible]

[illegible]

[illegible] MATLAB [illegible]

[illegible]

[illegible]

1. [illegible]

2. [illegible]

3. [illegible]

are [illegible]. Therefore, my approach is to focus on the characteristic roots and the form of the system response in terms of [illegible]. Chapter 8 presents the Laplace transformation [illegible] to obtain the dynamic response. Therefore, instructors who wish to utilize Laplace transform methods may do so. However, Chapter 8 may be omitted if the instructor does not care to include the Laplace transform method.

4. Chapter 10 presents five engineering case studies in dynamic systems and control. The five case studies, inspired by research articles, are (1) vibration isolation in a vehicle suspension system, (2) an electromechanical [illegible] actuator, (3) a pneumatic air brake system, (4) hydraulic [illegible] and (5) [illegible]. These case studies [illegible] topics from the previous chapters: modeling, simulation, linearization, analytical methods, and [illegible]. [illegible] Again, I [illegible] the importance of presenting [illegible] and complex engineering systems to undergraduate students in order to motivate them, engage their interests, and illustrate key principles in dynamic systems and control.

Several people have contributed to the production of this textbook. I would like to thank Roger F. Fales, my colleague at the University of Missouri, for his [illegible] knowledge and comments on fluid systems. Two students at the University of Missouri provided [illegible] assistance: [illegible] the engineering [illegible] and helped with the solutions manual, and [illegible] the draft manuscript. Many reviewers provided valuable suggestions for improving this textbook and they are listed here:

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[illegible], [illegible] University

[illegible], Texas Tech University

[illegible], The University of [illegible]

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[illegible], University of [illegible]

Nejat Olgac, University of Connecticut

Andrew R. Plummer, University of Bath

[illegible], The University of [illegible]

[illegible], The University of Alabama

It was a [illegible] pleasure to work with the editorial and production staff at Wiley and I [illegible] [illegible]. I also thank [illegible] for her [illegible] work and [illegible] encouragement throughout this project. Finally, I acknowledge and thank the [illegible] who have had an [illegible] impact on my life: my advisor [illegible] of Iowa State University, [illegible], and my parents [illegible] Kluever.

[illegible], I thank my [illegible] to [illegible] along with my [illegible]. [illegible] all the makings of a [illegible] engineer, although [illegible] much more interested in becoming a Major League Baseball player.

Craig A. Kluever
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University of Missouri-Columbia
[illegible]

Chapter 1

Introduction to Dynamic Systems and Control

1.1 INTRODUCTION

Mathematical model: [illegible]

Simulation: [illegible]

System analysis: [illegible]

1.2 CLASSIFICATION OF DYNAMIC SYSTEMS

[illegible]

Distributed vs. Lumped Systems

[illegible]

Continuous-Time vs. Discrete-Time Systems

[illegible]

Time-Varying vs. Time-Invariant Systems

[illegible]

[illegible]

Linear vs. Nonlinear Systems

[illegible]

1. [illegible]
2. [illegible]

[illegible]

[illegible]

[illegible]

[illegible] (1.1)

[illegible] (1.2)

[illegible]

[illegible] (1.3)

[illegible]

[illegible]

1.2 MODELING DYNAMIC SYSTEMS

Simulation Tools

[illegible]

[illegible]

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1.4 OBJECTIVES AND TEXTBOOK OUTLINE

[illegible]

[illegible]

[illegible]

[illegible]

Chapter 10 [illegible]

Chapter 11 [illegible]

Appendix A [illegible] MATLAB [illegible]

REFERENCES

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

Chapter 2

Modeling Mechanical Systems

2.1 INTRODUCTION

[illegible]

[illegible]

2.2 MECHANICAL ELEMENT LAWS

[illegible]

Inertia Elements

[illegible]

[illegible]

[illegible] (2.1)

[illegible]

[illegible]

$$I = \frac{1}{2}mR^2 \qquad (2.2)$$

[illegible]

$$E_p = mgh \qquad (2.3)$$

[illegible]

$$E_k = \frac{1}{2}mv^2 \qquad (2.4)$$

[illegible]

$$E_k = \frac{1}{2}I\omega^2 \qquad (2.5)$$

[illegible]

Stiffness Elements

[illegible]

$$F = kx \qquad (2.6)$$

[illegible]

Figure 2.3 Force stretching the free end of a spring.

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

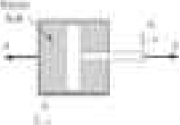
[illegible]

Friction Elements

[illegible]

$$[illegible]$$

[illegible]



Mechanical Transformers

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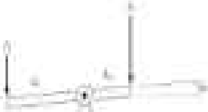


Figure 2.6 Ideal lever.

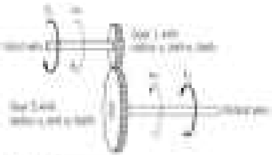


Figure 2.7 Ideal gear train.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible] TRANSLATIONAL MECHANICAL SYSTEMS

[illegible]

1. [illegible]
2. [illegible]

[illegible]

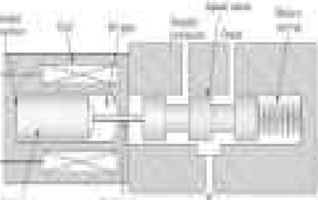
$$\sum F = ma \quad [illegible]$$

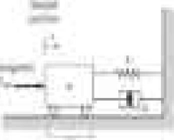
[illegible]

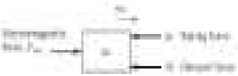
Example [illegible]

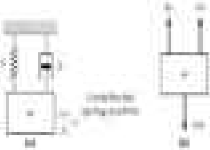
[illegible]

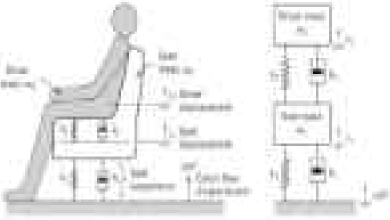
[illegible]

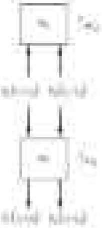












[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible] \quad (2.[illegible]a)$$

$$[illegible] \quad (2.[illegible]b)$$

[illegible]

Mechanical Systems with Nonlinearities

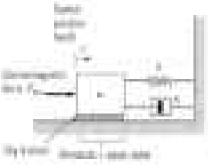
[illegible]

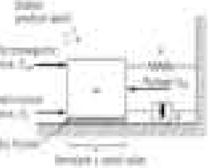
Example 2.[illegible]

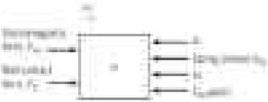
[illegible]

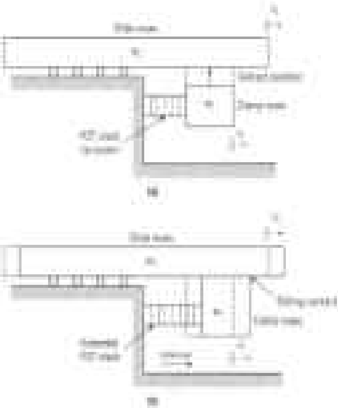
[illegible]

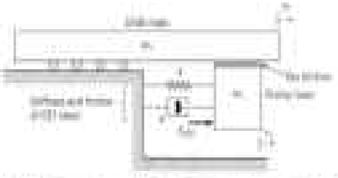
[illegible]

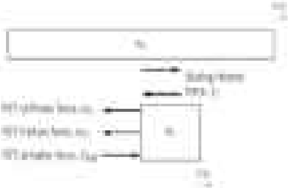












[illegible]

2.4 ROTATIONAL MECHANICAL SYSTEMS

[illegible]

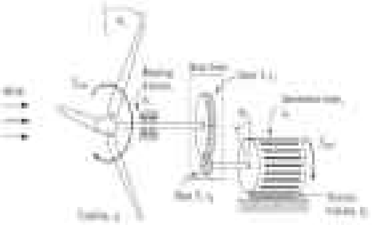
1. [illegible]
2. [illegible]

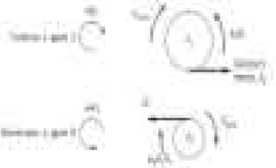
[illegible]

[illegible]

Example 2.7

[illegible]





[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

$$[illegible]$$

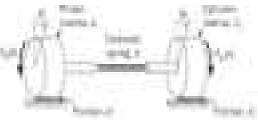
[illegible]

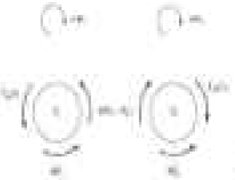
$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]





[illegible]

[illegible]

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$$[illegible]$$

[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

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SUMMARY

REFERENCES

PROBLEMS

Conceptual Problems

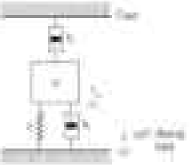


Figure P3.1

3.2 [illegible]

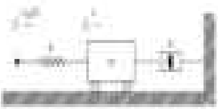


Figure P3.2

3.3 [illegible]

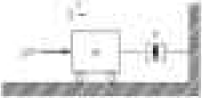


Figure P3.3

a. [illegible]

b. [illegible]

3.4 [illegible]

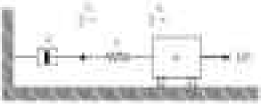


Figure P2.4

2.5 [illegible]

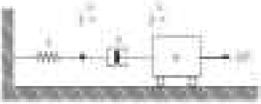
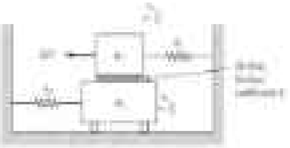


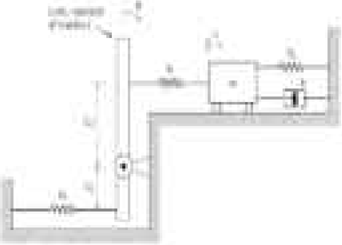
Figure P2.5

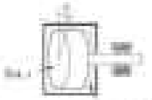
2.6 [illegible]

Figure P2.6

2.7 [illegible]







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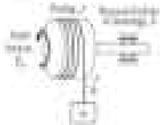


Figure P2.14

2.15 [illegible]

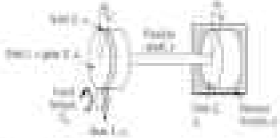


Figure P2.15

2.16 [illegible]

Figure P2.16

2.17 [illegible]

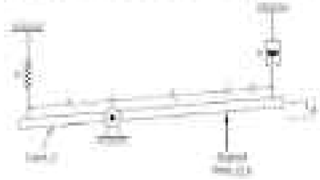


Figure P2.17

MATLAB Problems

2.18 [illegible]

Table P2.18

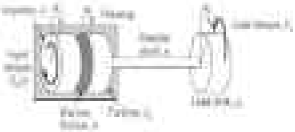
Load force (N)	Spring #1 deflection (mm)	Spring #2 deflection (mm)
0	0	0
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

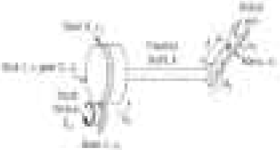
2.19 [illegible]

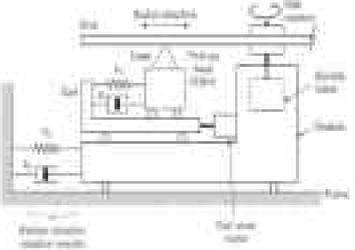
$$[illegible]$$

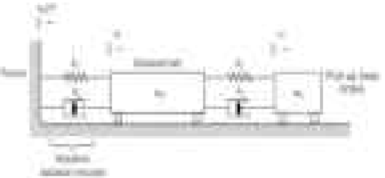
Engineering Applications

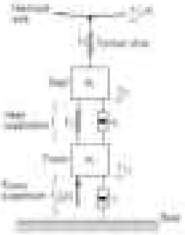
Figure P2.22

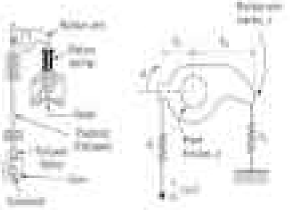


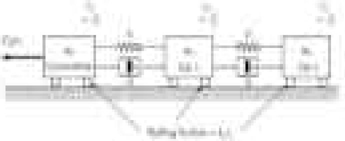


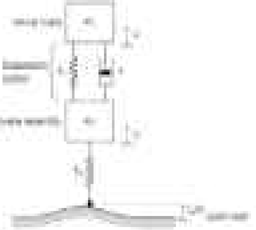


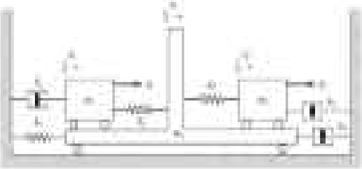












Chapter 3

Modeling Electrical and Electromechanical Systems

3.1 INTRODUCTION

[illegible]

[illegible]

3.2 ELECTRICAL ELEMENT LAWS

[illegible]

[illegible]

Figure [illegible] Resistor element.

Resistor

[illegible]

$$v_R = iR \quad (3.1)$$

[illegible]

[illegible] (3.2)

[illegible]

Capacitor

[illegible]

[illegible] (3.3)

[illegible]

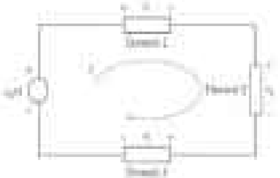
[illegible] (3.4)

Figure [illegible] Capacitor element.

Sources

3.3 ELECTRICAL SYSTEMS

Kirchhoff's Voltage Law



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Kirchhoff's Current Law

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Mathematical Models of Electrical Systems

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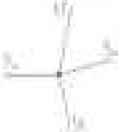
2. [illegible]



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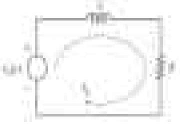

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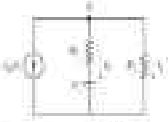

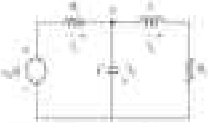

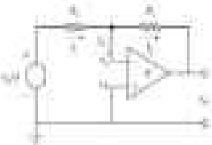

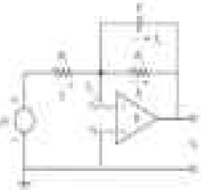

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3.5 ELECTROMECHANICAL SYSTEMS

[illegible]

Current–Magnetic Field Interaction

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1. [illegible]
2. [illegible]
3. [illegible]

[illegible]

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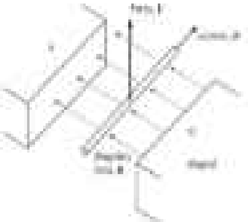

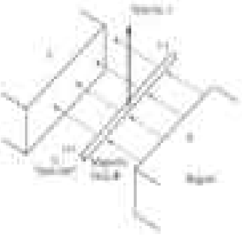

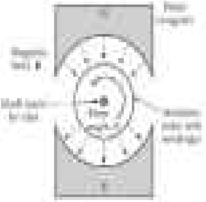

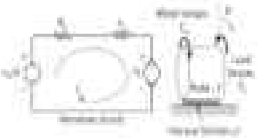

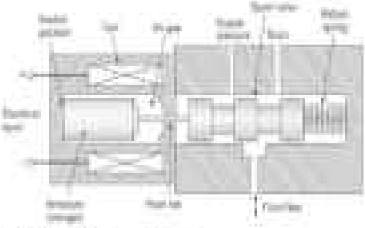

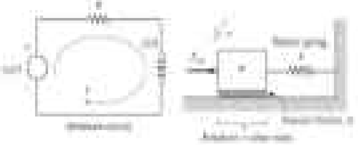

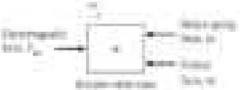

Electrostatic Microactuator

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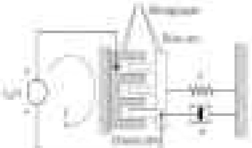

Figure 2.25 MEMS comb drive actuator.

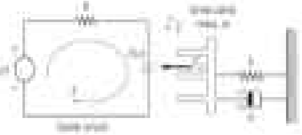

Figure 2.26 Schematic diagram of a MEMS comb drive actuator.

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SUMMARY

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REFERENCES

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PROBLEMS

Conceptual Problems

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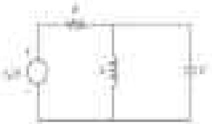

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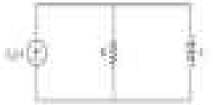


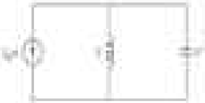


Figure P2.3

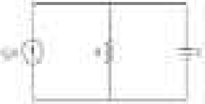


Figure P2.4

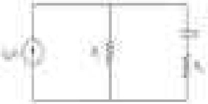


Figure P2.5

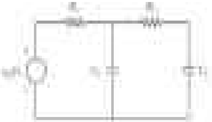


Figure P2.6

3.7 [illegible]

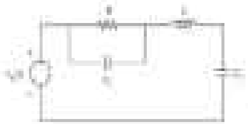


Figure P3.7

3.8 [illegible]

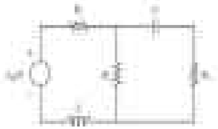


Figure P3.8

3.9 [illegible]

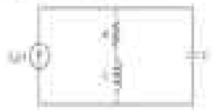
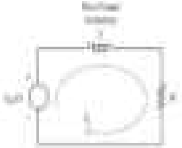
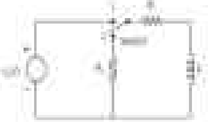


Figure P3.9

3.10 [illegible]

Figure P3.10





[illegible]

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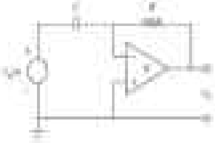


[illegible]

3.17 Figure P3.17 presents an op-amp circuit. Determine the relationship between input and output voltages.

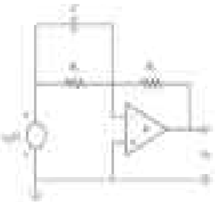


Figure P3.17

3.18 Figure P3.18 presents an op-amp circuit. Determine the relationship between input and output voltages.

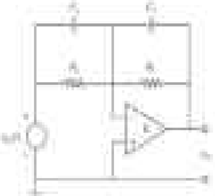


Figure P3.18

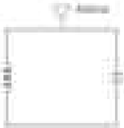
3.19 A microphone consists of a diaphragm with [illegible] connected to a [illegible] coil of [illegible] that [illegible] in a [illegible] magnetic field. Sound [illegible] cause [illegible] the diaphragm to [illegible] and the diaphragm [illegible] with [illegible] because of its motion. The [illegible]. Find the [illegible] variables and the input variable for this system.

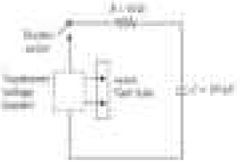
MATLAB Problems

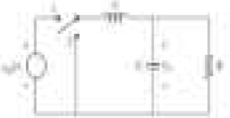
3.20 Consider again the RC circuit in Problem 3.[illegible]. Plot the energy stored in the capacitor vs. time during the discharge phase $t \ge 0$.

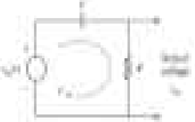
3.21 A series RL circuit is driven by a constant [illegible]-V voltage source (see Fig. [illegible] in Example [illegible]). The circuit parameters are $L =$ [illegible] H and $R =$ [illegible] Ω. As we saw in Chapter 2, the resulting current response for this system is

$$i(t) = [illegible](1 - e^{[illegible]t})\ A$$









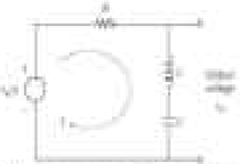


Figure [illegible]

a. [illegible]

b. [illegible]

[illegible]

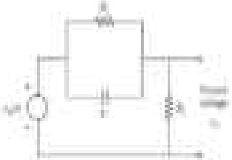


Figure [illegible]

a. [illegible]

b. [illegible]

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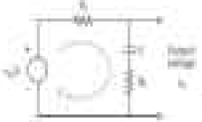


Figure [illegible]

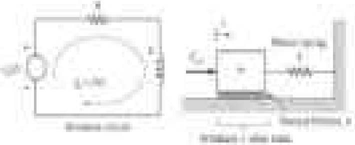


Figure P2.20

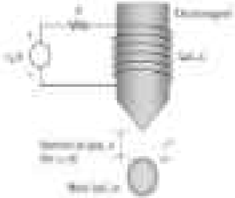


Figure P3.23

[illegible]

3.44 [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Chapter 4

Modeling Fluid and Thermal Systems

4.1 INTRODUCTION

[illegible]

[illegible]

4.2 HYDRAULIC SYSTEMS

[illegible]

[illegible]

Figure 4.2 Turbulent flow through an orifice.

current flow I is analogous to fluid flow Q. Laminar flow exists when the pressure difference ΔP is "small" and consequently the fluid flow Q is "small" (or low velocity). For laminar pipe flow where the pipe length L is significantly larger than the pipe diameter d (as in Fig. 4.1), the laminar fluid resistance can be computed using the Hagen–Poiseuille law

$$R_L = \frac{128\mu L}{\pi d^4} \tag{4.3}$$

where μ is the dynamic (or absolute) viscosity of the fluid, in Pa·s.

Figure 4.2 shows turbulent flow through a sharp-edged opening (or orifice) in a pipe. Turbulent flow is characterized by irregular, eddy-like, swirling flow, which the streamlines can not indicate or quantify. The turbulent fluid resistance exhibits a nonlinear relationship between the pressure drop ($\Delta P = P_1 - P_2$) across the orifice and the volumetric flow rate Q

$$\Delta P = R_T Q^2 \tag{4.4}$$

where R_T is the nonlinear (turbulent) fluid resistance (in Pa·s²/m⁶). Turbulent flow exists when the pressure difference ΔP across the orifice is "large" and consequently the fluid velocity is large. We can rewrite the turbulent flow equation (4.4) in terms of volumetric flow rate Q

$$Q = K_T \sqrt{\Delta P} \tag{4.5}$$

where $K_T = 1/\sqrt{R_T}$ is a turbulent flow coefficient.

Most industrial hydraulic systems involve high-pressure flow through valve openings or small passages, and hence the resulting flow is typically turbulent. Let us now develop an approximate model for turbulent hydraulic flow through an orifice or sharp-edged valve opening. Figure 4.3 shows hydraulic oil flowing through a sharp-edged orifice (such as a valve opening) with area A_0. The fluid has very high pressure P_1 at station 1, which is upstream from the orifice, and lower pressure P_2 at station 2, which is immediately downstream from the orifice. Let us assume that the velocity of the oil at station 1 is small and can be neglected.

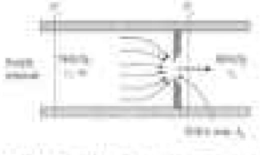


Figure 4.3 Hydraulic fluid flowing through a sharp-edged orifice.

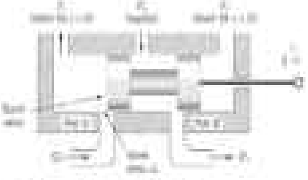


Figure 4.5 [illegible]

[illegible]

Fluid Capacitance

[illegible]

$$C = \frac{dV}{dP} \quad (4.10)$$

[illegible]

$$P = \rho g h + P_{atm} \quad (4.11)$$

[illegible]

$$dP = \rho g\, dh \quad (4.12)$$

[illegible]

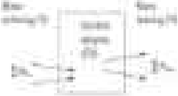
$$C = \frac{dV}{dP} = \frac{A\,dh}{\rho g\,dh} = \frac{A}{\rho g} \quad (4.13)$$

[illegible]

[illegible]

$$C\,dP = dV \quad (4.14)$$

Figure 4.6 Hydraulic reservoir.



Modeling Hydraulic Tank Systems

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[illegible]

[illegible]

[illegible]

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Modeling Hydromechanical Systems

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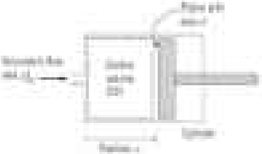
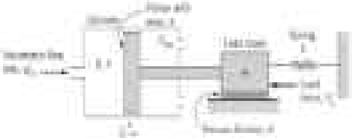
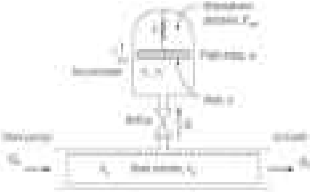






[illegible]

Example 4.4

[illegible]

$$[illegible]$$

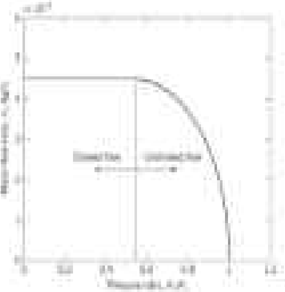
[illegible]

4.2 PNEUMATIC SYSTEMS

[illegible]

$$P = \rho R T \quad (4.76)$$

[illegible]



[illegible]

Pneumatic Capacitance

[illegible]

Modeling Pneumatic Systems

[illegible]

$$\dot{\rho} = \frac{1}{RT}\dot{P} \tag{4.72}$$

[illegible]

$$w_{net} = \frac{V}{RT}\dot{P} + \frac{P}{RT}\dot{V} = \sum w_{in} - \sum w_{out} \tag{4.73}$$

[illegible]

$$\dot{P} = \frac{RT}{V}\left(w_{net} - \frac{P}{RT}\dot{V}\right) \tag{4.74}$$

where $w_{net} = \sum w_{in} - \sum w_{out}$ [illegible]

Example 4.6

[illegible]

$$\dot{P} = \frac{RT}{V}\left(w_{in} - \frac{P}{RT}\dot{V}\right) \tag{4.75}$$

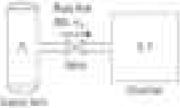


Figure 4.16 Pneumatic system for Example 4.6.

[illegible]

$$[illegible]$$

[illegible]

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[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

Example [illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

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$$[illegible]$$

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THERMAL SYSTEMS

[illegible]

[illegible]

[illegible]

[illegible]

Thermal Resistance

[illegible]

[illegible]

$$q = \frac{1}{R}\Delta T$$

[illegible]

[illegible]

$$\text{Conduction:} \quad R = \frac{L}{kA}$$

[illegible]

[illegible]

$$\text{Convection:} \quad R = \frac{1}{hA}$$

[illegible]

Thermal Capacitance

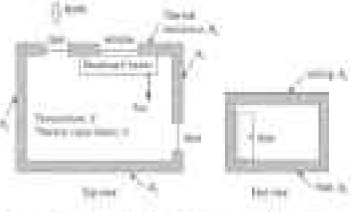
[illegible]

$$C = mc_p$$

[illegible]

Modeling Thermal Systems

[illegible]



[illegible]

$$[illegible]$$

[illegible]

Example 4.8

[illegible]

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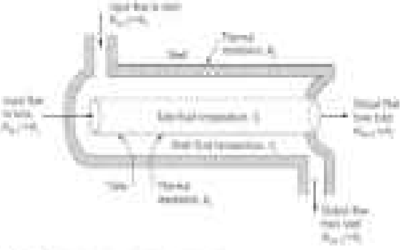
$$[illegible]$$

$$[illegible]$$



[illegible]

SUMMARY

[illegible]

REFERENCES

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

PROBLEMS

Conceptual Problems

4.1 [illegible]

$$[illegible]$$

[illegible]

4.2 [illegible]

Table P4.2

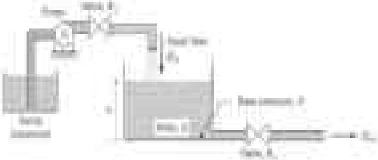
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

4.3 [illegible]

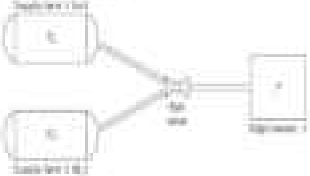
a. [illegible]

b. [illegible]



[illegible]

[illegible]



[illegible]

[illegible]

[illegible]

[illegible]

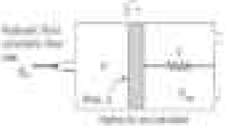


Figure P4.9

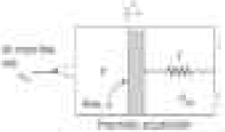


Figure P4.10

Engineering Applications

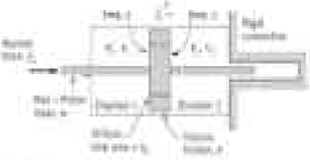


Figure P4.16

4.16 [illegible]

4.17 [illegible]

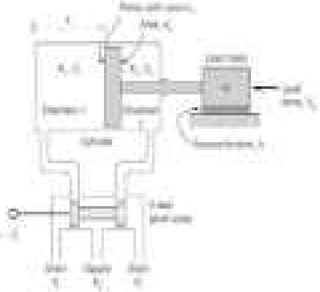
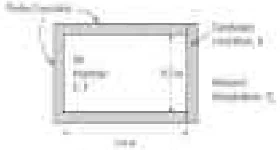
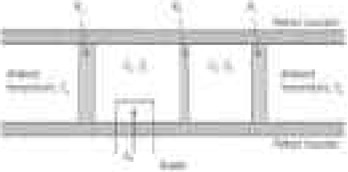
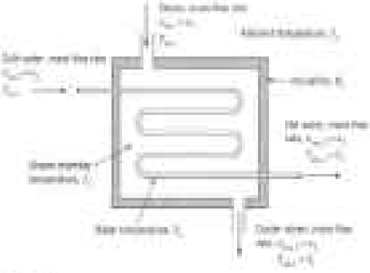


Figure P4.17







Chapter 5

Standard Models for Dynamic Systems

5.1 INTRODUCTION

[illegible]

[illegible]

5.2 STATE-VARIABLE EQUATIONS

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

$$[illegible]$$

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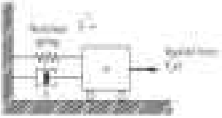
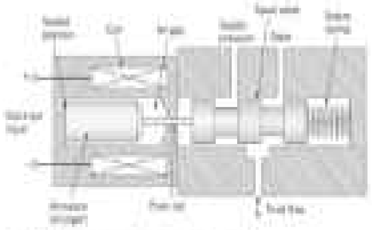
$$[illegible]$$

[illegible]





3.3 STATE-SPACE REPRESENTATION

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

[illegible]

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1r} \\ b_{21} & b_{22} & \cdots & b_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nr} \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1n} \\ c_{21} & c_{22} & \cdots & c_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ c_{m1} & c_{m2} & \cdots & c_{mn} \end{bmatrix} \qquad \mathbf{D} = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1r} \\ d_{21} & d_{22} & \cdots & d_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ d_{m1} & d_{m2} & \cdots & d_{mr} \end{bmatrix}$$

[illegible]

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{3.21}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{3.22}$$

[illegible]

[illegible]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ b_{41} & b_{42} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

Example [illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$\dot{x} = Ax + Bu$$
$$y = Cx + Du$$

[illegible]

$$A = \begin{bmatrix} 0 & 1 \\ [illegible] & [illegible] \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ [illegible] \end{bmatrix}$$

[illegible]

$$C = [\,1 \quad [illegible]\,], \quad D = 0$$

[illegible]

Example [illegible]

[illegible]

[illegible] (3.[illegible])

[illegible] (3.[illegible])

[illegible]

[illegible]

[illegible] (3.[illegible])

[illegible] (3.[illegible])

[illegible] (3.[illegible])

[illegible]

[illegible] (3.[illegible])

[illegible] (3.[illegible])

[illegible] (3.[illegible])

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

The complete SSR is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

where the state and input matrices are

$$[illegible]$$

and the output and direct-link matrices are

$$[illegible]$$

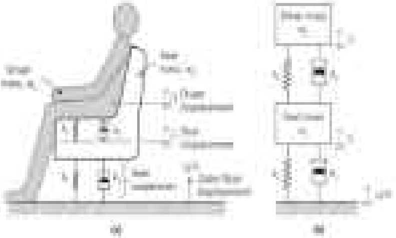
[illegible]

Example [illegible]

[illegible]

$$[illegible]$$

$$[illegible]$$



[illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

[illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

[illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

$$[illegible]$$ [illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible] LINEARIZATION

[illegible]

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]

[illegible]

[illegible]

$$[illegible]$$

Step 1. [illegible]

Step 2. [illegible]

$$[illegible]$$

Step 3. [illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

Example [illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible] MATLAB [illegible]

```
>> [illegible]
```

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

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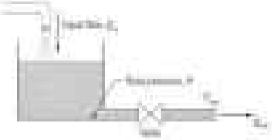
$$[illegible]$$

[illegible]

$$[illegible]$$



[illegible]

(3.74)

[illegible]

(3.75)

[illegible]

(3.76)

[illegible]

[illegible]

Example 3.17

[illegible]

(3.77)

(3.78)

(3.79)

[illegible]

(3.80)

or

(3.81)

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

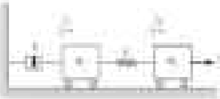
$$[illegible]$$

[illegible]

[illegible] INPUT–OUTPUT EQUATIONS

[illegible]

Differential Operator



[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

5.3 TRANSFER FUNCTIONS

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

Example [illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

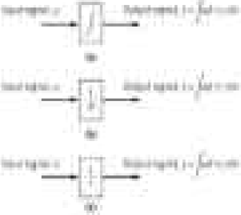
[illegible]

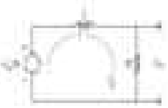
Example [illegible]

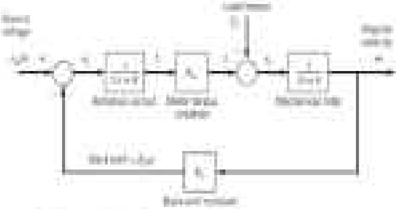
[illegible]

$$[illegible]$$

[illegible]







STANDARD INPUT FUNCTIONS

Step Input

Ramp Input

Ramped Step Input

[illegible]

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[illegible]

Pulse Input

[illegible]

$$[illegible]$$

[illegible]

Impulse Input

[illegible]

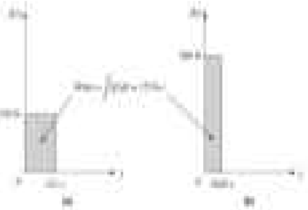


Figure [illegible]

Sinusoidal Input

SUMMARY

[illegible]

PROBLEMS

Conceptual Problems

3.1 [illegible]

$$[illegible]$$

3.2 [illegible]

$$[illegible]$$

a. [illegible]
b. [illegible]
c. [illegible]

3.3 [illegible]

$$[illegible]$$

3.4 [illegible]

$$[illegible]$$

[illegible]

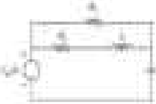
3.5 [illegible]

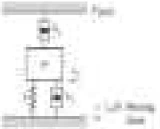
$$[illegible]$$

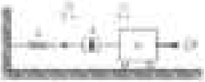
[illegible]

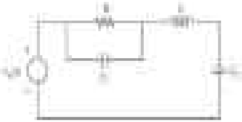
3.6 [illegible]

$$[illegible]$$







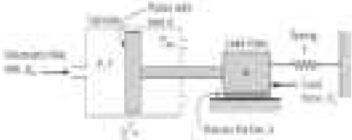


MATLAB Problems

[illegible]

Engineering Applications

[illegible]



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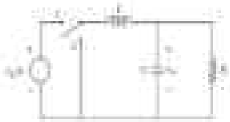


Figure P3.24

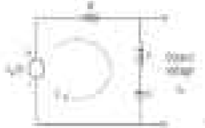


Figure P3.25

[illegible]

[illegible]

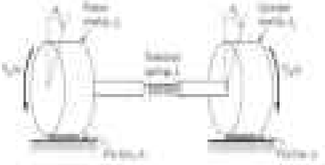


Figure P3.20

A simplified model of a solenoid plunger is: [illegible]

$$[illegible]$$

[illegible]

[illegible]

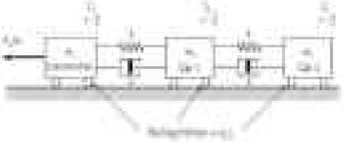


Figure P3.21

[illegible]

[illegible]

Figure [illegible]

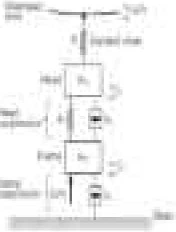
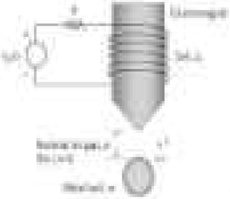
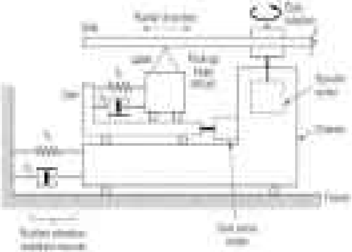


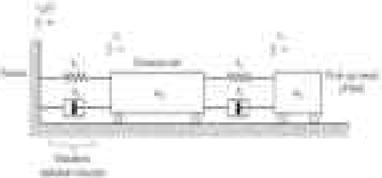
Figure [illegible]

[illegible]

[illegible]







Chapter 6

Numerical Simulation of Dynamic Systems

6.1 INTRODUCTION

8.2 SYSTEM RESPONSE USING MATLAB COMMANDS

[illegible]

[illegible]

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[illegible]
```

[illegible]

$$[illegible]$$

[illegible]

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[illegible]
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[illegible]

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[illegible]
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[illegible]

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[illegible]
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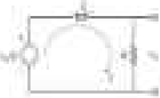
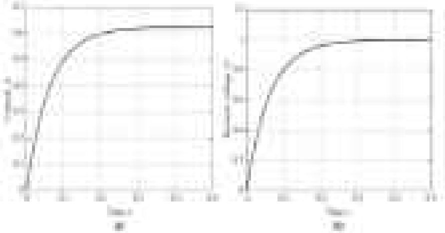
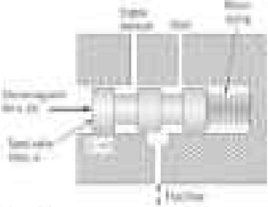
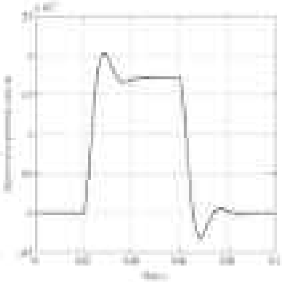
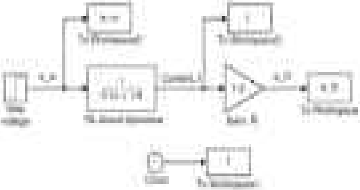
[illegible]

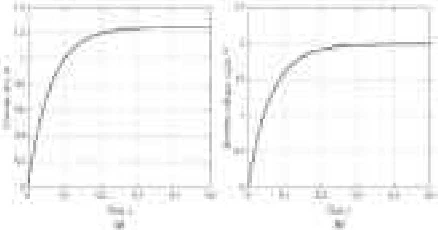












[illegible]

[illegible]

6.4 SIMULATING LINEAR SYSTEMS USING SIMULINK

[illegible]

1. Transfer functions
2. State-space representation
3. [illegible]

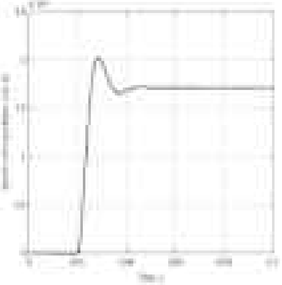
[illegible]

Example 6.4

[illegible]

[illegible]

$$[illegible] \tag{6.12}$$



[illegible]

[illegible]

[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

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[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

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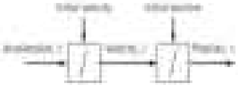
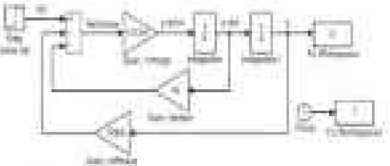
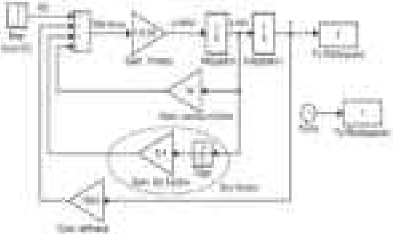
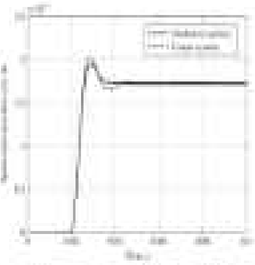
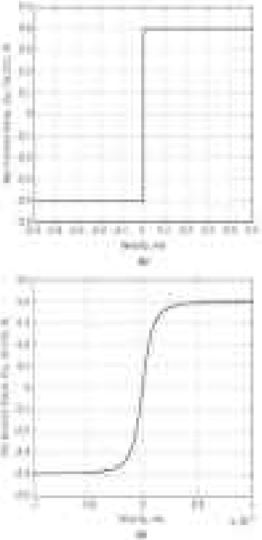
[illegible]











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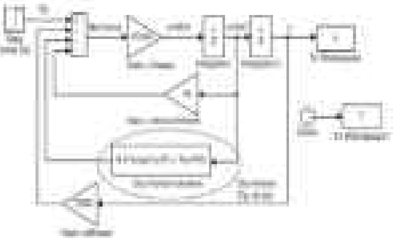

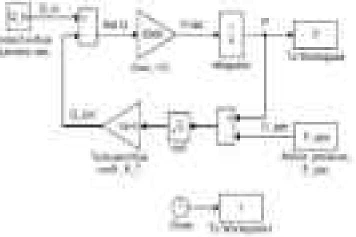

[illegible]

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[illegible]
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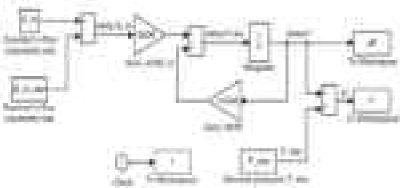
[illegible]

Linear system simulation

[illegible]



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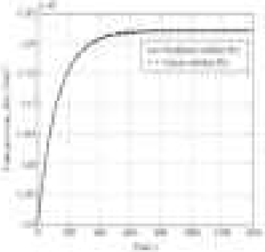

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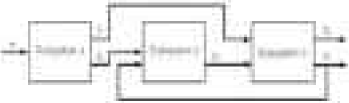

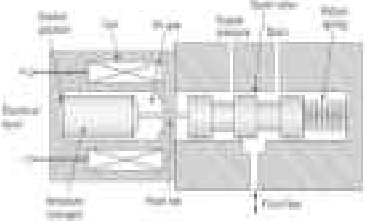

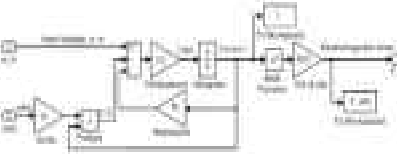

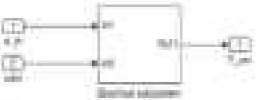

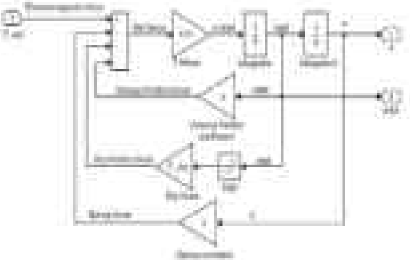

Figure [illegible] [illegible]

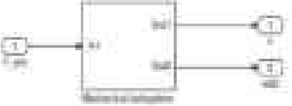

Figure [illegible] [illegible]

Figure [illegible] [illegible]

[illegible]

Table 8.1 [illegible]

System Parameter	Value
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

MATLAB M-file [illegible]

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[illegible]
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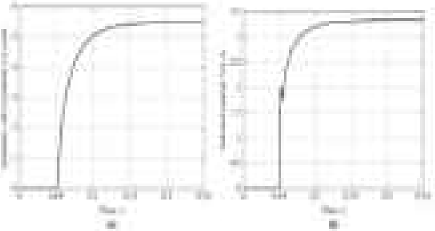
[illegible]

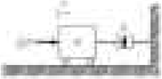
[illegible]

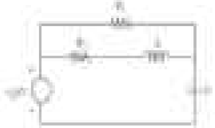


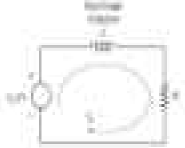
REFERENCES

PROBLEMS

Conceptual Problems







[illegible]

Figure P6.11

[illegible]

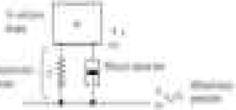
[illegible]

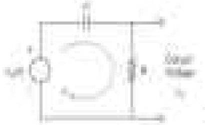
Engineering Applications

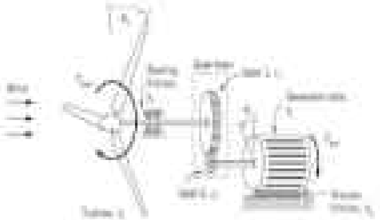
[illegible]

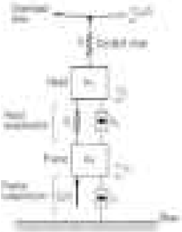
[illegible]

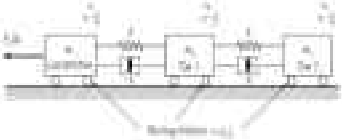
[illegible]

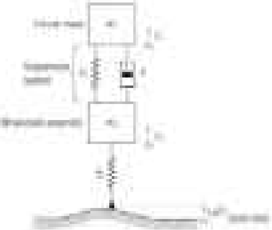


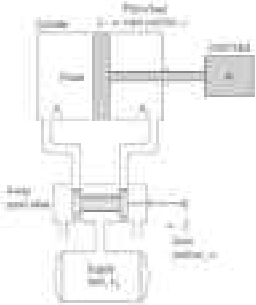


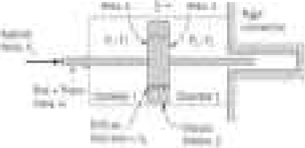












Chapter 7

Analytical Solution of Linear Dynamic Systems

7.1 INTRODUCTION

[illegible]

7.2 ANALYTICAL SOLUTIONS TO LINEAR DIFFERENTIAL EQUATIONS

[illegible]

[illegible] (7.1)

[illegible]

[illegible] (7.2)

[illegible]

[illegible] (7.3)

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

Example [illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

Example [illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

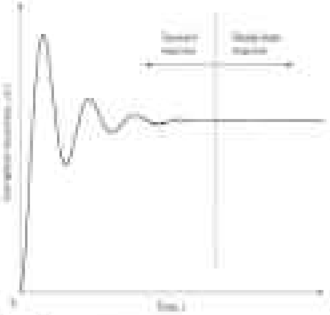
$$[illegible]$$

$$[illegible]$$

[illegible]

The Complete Response

[illegible]



[illegible]

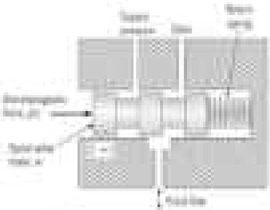
Characteristic Roots and the Transfer Function

[illegible]

[illegible]

[illegible]

[illegible]



[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

DC Gain

The DC gain [illegible] "direct current" [illegible] "alternating current" or AC. [illegible]

[illegible]

[illegible] (7.31)

[illegible]

[illegible] (7.32)

[illegible]

[illegible] (7.33)

[illegible]

$$[illegible]$$

[illegible]

7.3 FIRST-ORDER SYSTEM RESPONSE

[illegible]

[illegible]

$$[illegible] \quad (7.25)$$

[illegible]

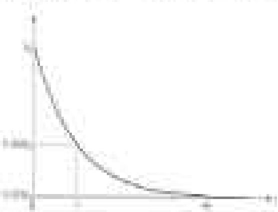
Table 7.1 [illegible]

Physical System	Input Variable	Output Variable	Modeling Equation
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

Table 7.2 [illegible]

Physical System	Output Variable	Constant τ
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

First-Order Response with Zero Input



Step Response of a First-Order System

[illegible]

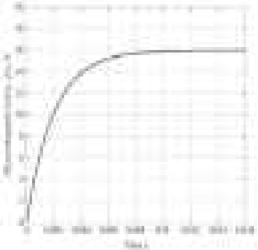


Figure 7.8 [illegible]

[illegible]

$$[illegible] \qquad [illegible]$$

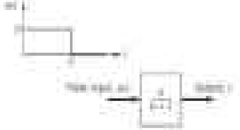
[illegible]

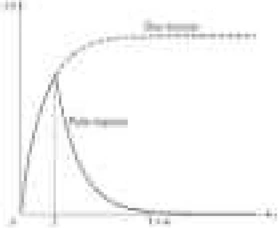
Pulse Response of a First-Order System

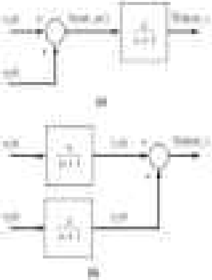
[illegible]

$$u(t) = \begin{cases} 0 & \text{for } t \le 0 \\ P & \text{for } 0 < t \le T \\ 0 & \text{for } t > T \end{cases} \qquad [illegible]$$

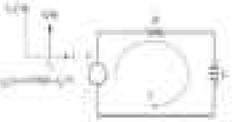
[illegible]

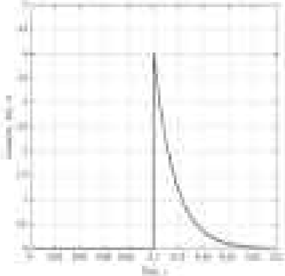






Impulse Response of a First-Order System





[illegible]

7.4 SECOND-ORDER SYSTEM RESPONSE

[illegible]

[illegible]

[illegible]

Physical System	Input Variable	Output Variable	Modeling Equation
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]

[illegible]

Second-Order Response with Zero Input

[illegible]

[illegible] (7.29)

[illegible]

[illegible] (7.30)

[illegible]

[illegible] (7.31)

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

Case 1: two real, distinct roots

[illegible]

[illegible] (7.32)

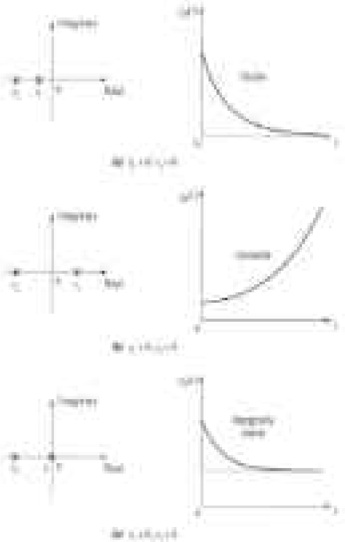
[illegible]

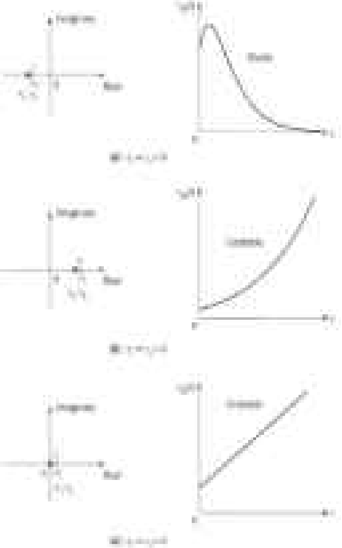
Case 2: two real, repeated roots

[illegible]

[illegible] (7.33)

[illegible]





[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

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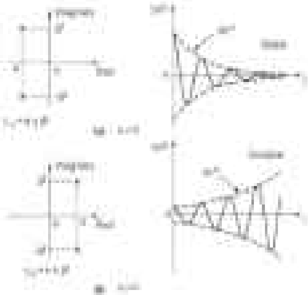
[illegible]

[illegible]

[illegible]



- [illegible]
- [illegible]

Example [illegible]

[illegible]

(a) [illegible]

[illegible]

$$[illegible]$$

or

$$[illegible]$$

Therefore, [illegible]

$$[illegible]$$

[illegible]

(b) [illegible]

The characteristic equation is

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

The characteristic equation is

[illegible]

or

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The characteristic equation is

[illegible]

[illegible]

[illegible]

[illegible]

Damping Ratio and Undamped Natural Frequency

[illegible]

[illegible]

[illegible]

$$(\text{Characteristic roots}) \quad [illegible]$$

[illegible]

$$\text{Damping ratio:} \quad \zeta = [illegible]$$

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]

[illegible]

[illegible]

[illegible]

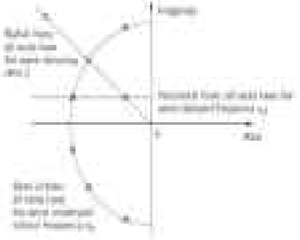
[illegible]

[illegible]

[illegible]

[illegible]

$$\text{Undamped natural frequency:} \quad \omega_n = [illegible]$$



[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]

Step Response of an Underdamped Second-Order System

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible] \quad (7.70)$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible] \quad (7.71)$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible] \quad (7.72)$$

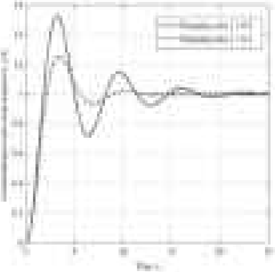
[illegible]

[illegible]

$$[illegible] \quad (7.73)$$

[illegible]

$$[illegible] \quad (7.74)$$



[illegible]

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

Peak time: [illegible]

Maximum overshoot: [illegible]

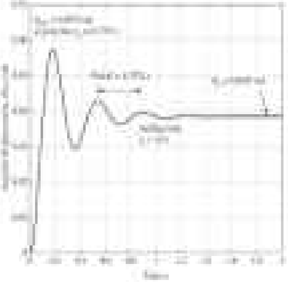
Settling time: [illegible]

Period of oscillation: [illegible]

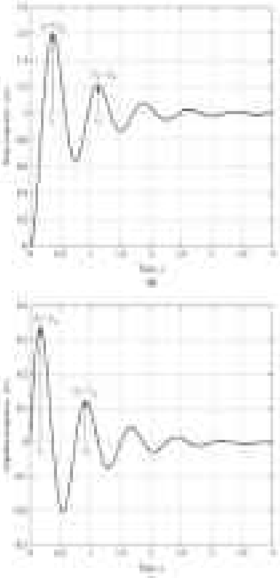
Number of cycles to settling time: [illegible]

[illegible]

$$[illegible]$$



Log Decrement and the Damping Ratio



[illegible]

$$[illegible] \quad (7.[illegible])$$

[illegible]

$$[illegible] \quad (7.[illegible])$$

[illegible]

$$[illegible] \quad (7.[illegible])$$

Finally, [illegible]

$$\zeta = [illegible] \quad (7.[illegible])$$

[illegible]

$$[illegible]$$

[illegible]

7.3 HIGHER-ORDER SYSTEMS

[illegible]

$$[illegible] \quad (7.[illegible])$$

The characteristic equation is

$$[illegible] = 0 \quad (7.[illegible])$$

7.8 STATE-SPACE REPRESENTATION AND EIGENVALUES

[illegible]

$$[illegible] \quad (1.98)$$

[illegible]

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ [illegible] & [illegible] & [illegible] \end{bmatrix} \quad (1.99)$$

The matrix $s\mathbf{I} - \mathbf{A}$ is:

$$[illegible] \quad (1.100)$$

The determinant of Eq. (1.100) is:

$$[illegible] \quad (1.101)$$

[illegible]

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]

[illegible]

[illegible]

[illegible]

Example [illegible]

[illegible]

$$[illegible]$$ ([illegible])

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$ ([illegible])

MATLAB [illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$G(s) = [illegible]$$

The poles of the transfer function [illegible]

$$[illegible]$$ ([illegible])

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$\dot{x} = [illegible]$$

The eigenvalues are computed from the determinant [illegible]

$$[illegible]$$ ([illegible])

[illegible]

7.7 APPROXIMATE MODELS

[illegible]

[illegible]

[illegible]

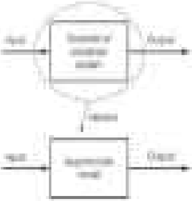
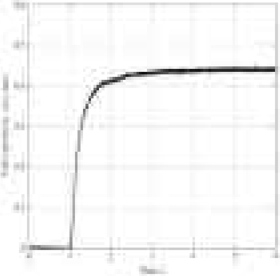
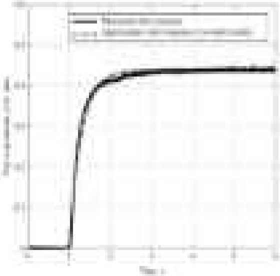


Figure 7.34 Replacing a complex or uncertain system with an approximate model.

Example 7.11

[illegible]





REFERENCE

1. [illegible]

PROBLEMS

Conceptual Problems

7.1 [illegible]

$$[illegible]$$

[illegible]

7.2 [illegible]

$$[illegible]$$

a. [illegible]
b. [illegible]
c. [illegible]

7.3 [illegible]

$$[illegible]$$

a. [illegible]
b. [illegible]
c. [illegible]

7.4 [illegible]

a. [illegible]
b. [illegible]
c. [illegible]
d. [illegible]

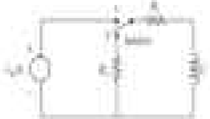
7.5 [illegible]

$$[illegible]$$

[illegible]

a. [illegible]
b. [illegible]

7.6 [illegible]



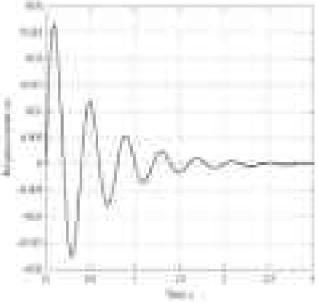


Figure P7.11

7.34 Given the system PE equation [illegible]

$$[illegible]$$

a. Compute the characteristic roots.
b. Compute the poles of the transfer function.
c. Compute the eigenvalues of the system matrix from an SSR.
d. [illegible]

7.35 Figure P7.3[illegible] [illegible]

Figure P7.3[illegible]

MATLAB Problems

7.36 Given the SSR

$$[illegible]$$

a. Compute the eigenvalues "by hand".
b. Use MATLAB to verify your answer in part (a).
c. [illegible]
d. Use MATLAB or Simulink to verify your answer to part (c). [illegible]

7.37 Given the SSR

$$[illegible]$$

a. Use MATLAB to determine the eigenvalues.
b. [illegible]
c. Use MATLAB or Simulink to verify your answer to part (b). [illegible]

Figure P7.19

Figure P7.20

Engineering Applications

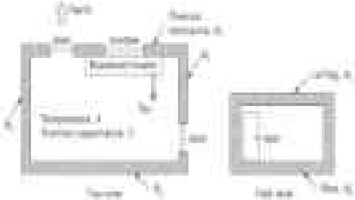
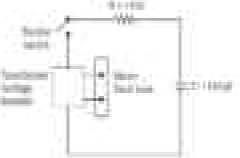
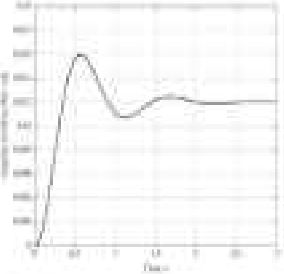


Figure P7.26

Figure P7.27





Chapter 8

System Analysis Using Laplace Transforms

8.1 INTRODUCTION

[illegible]

[illegible]

8.2 LAPLACE TRANSFORMATION

[illegible]

[illegible]

[illegible]

[illegible]

Laplace Transforms of Common Time Functions

[illegible]

Example 8.1

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

Example 8.2

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

Example 8.3

[illegible]

Using Eq. (8.1):

$$\mathcal{L}\{A\sin\omega t\} = \int_0^{\infty} A\sin\omega t\, e^{-st}\,dt$$

We can use Euler's formula to write sinusoidal functions as a complex exponential function:

$$e^{j\omega t} = \cos\omega t + j\sin\omega t$$

or $\sin\omega t = \text{Im}(e^{j\omega t})$. Using the complex conjugate of Euler's formula

$$e^{-j\omega t} = \cos\omega t - j\sin\omega t$$

we can substitute for $\sin\omega t$; solving the two equations to yield

$$\sin\omega t = \frac{1}{2j}\left(e^{j\omega t} - e^{-j\omega t}\right)$$

Using this result the Laplace transform integral becomes:

$$\mathcal{L}\{A\sin\omega t\} = \frac{A}{2j}\int_0^{\infty}\left(e^{j\omega t} - e^{-j\omega t}\right)e^{-st}\,dt$$

$$= \frac{A}{2j}\int_0^{\infty}\left(e^{-(s-j\omega)t} - e^{-(s+j\omega)t}\right)dt$$

$$= \frac{A}{2j}\left[\left.\frac{-e^{-(s-j\omega)t}}{s-j\omega}\right|_0^{\infty} + \left.\frac{e^{-(s+j\omega)t}}{s+j\omega}\right|_0^{\infty}\right]$$

$$= \frac{A}{2j}\left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega}\right]$$

Multiplying both terms by the appropriate complex conjugates ($s + j\omega$ or $s - j\omega$) will produce a common denominator and the result is

$$F(s) = \frac{A\omega}{s^2+\omega^2}$$

which is the Laplace transform of the sine function $f(t) = A\sin\omega t$.

In summary, the examples show that computing the Laplace transform of a time function $f(t)$ requires integrating the product $f(t)$ and e^{-st} from $t = 0$ to $t = \infty$. Once we have computed the Laplace transform of a common time function, we do not need to recalculate it every time it is needed; instead we can consult a table of precomputed Laplace transforms. Table 8.1 summarizes the Laplace transforms of common time functions, including the results from Examples 8.1, 8.2, and 8.3.

Laplace Transform Using MATLAB

MATLAB's Symbolic Math Toolbox can be used to compute the Laplace transform of a given time function $f(t)$. To do so, the user must define the time function $f(t)$ as the symbolic object [illegible] and the corresponding

Table 8.1 [illegible]

Number	Time Function, f(t)	Laplace Transform, [illegible] = F(s)
1	[illegible]	1
2	[illegible]	[illegible]
3	[illegible]	[illegible]
4	[illegible]	[illegible]
5	[illegible]	[illegible]
6	[illegible]	[illegible]
7	[illegible]	[illegible]
8	[illegible]	[illegible]
9	[illegible]	[illegible]
10	[illegible]	[illegible]
11	[illegible]	[illegible]

[illegible]

```
>> [illegible]        % [illegible]
>> [illegible]        % [illegible]
>> [illegible]        % [illegible]
>> [illegible]        % [illegible]
```

[illegible]

[illegible]

[illegible]

Poles and Zeros of Laplace Transforms

[illegible]

$$F(s) = [illegible]$$

[illegible]

$$\mathcal{L}[f(t)] = \frac{1}{s+3}$$

[illegible]

[illegible]

[illegible]

Laplace Transform Properties and Theorems

[illegible]

Superposition (Linearity)

a. $\mathcal{L}[af(t)] = aF(s)$ [illegible]

b. $\mathcal{L}[a_1 f_1(t) + a_2 f_2(t)] = a_1 F_1(s) + a_2 F_2(s)$ [illegible]

Differentiation

a. [illegible]

b. [illegible]

c. [illegible]

Integration

Multiplication $f(t)$ by e^{-at} in the time domain

Final-value theorem

Initial-value theorem

Example [illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

Example 5.2

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

(b)

[illegible]

(c)

[illegible]

8.3 INVERSE LAPLACE TRANSFORMATION

[illegible]

Example 8.8

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

Partial Fraction Expansion Method

[illegible]

[illegible]

[illegible]

Partial fraction expansion with distinct poles

[illegible]

$$[illegible] \qquad [illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible] \qquad [illegible]$$

[illegible]

[illegible]

[illegible]

Example 8.7

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

```
[illegible]
```

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

Partial fraction expansion with repeated poles

[illegible]

$$[illegible] \quad [illegible]$$

[illegible]

$$[illegible] \quad [illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

Therefore, the partial fraction expansion is

[illegible]

and the inverse Laplace transform is

[illegible]

This result can be verified using the following MATLAB commands:

```
[illegible]
```

[illegible]

Partial fraction expansion with complex poles

[illegible]

Example 8.[illegible]

Compute the inverse Laplace transform of

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Inverse Laplace Transform Using MATLAB

[illegible]

```
[illegible]
```

[illegible]

[illegible]

[illegible]

8.4 ANALYSIS OF DYNAMIC SYSTEMS USING LAPLACE TRANSFORMS

[illegible]

[illegible]

Laplace Transform of the Input-Output Equation

[illegible]

[illegible]

[illegible]

[illegible]

Left-hand side: [illegible]

Right-hand side [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The residues are:

[illegible]

[illegible]

Therefore, the Laplace transform [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Initial value: [illegible]

Final value: [illegible]

[illegible]

```
>> [illegible]        % [illegible]
>> [illegible]        % [illegible]
>> [illegible]        % [illegible]
>> [illegible]        % [illegible]
```

The result is

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Example [illegible]

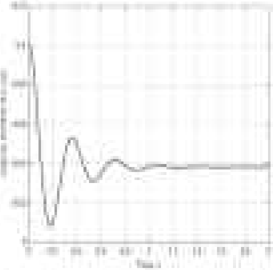
[illegible]

[illegible]

[illegible]

Left-hand side: [illegible]

Right-hand side (component): [illegible]



[illegible]

```
[illegible]
```

[illegible]

[illegible]

[illegible]

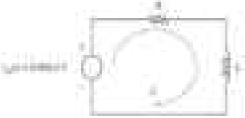
[illegible]

[illegible]

Transfer Function Analysis

[illegible]

[illegible]



[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

$$[illegible]$$

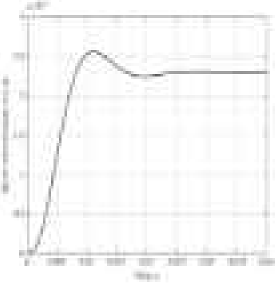
and

$$[illegible]$$

[illegible]

$$[illegible]$$

$$[illegible]$$



SUMMARY

[illegible]

REFERENCES

[illegible]

PROBLEMS

Conceptual Problems

[illegible]

i. [illegible]

j. [illegible]

[illegible] [illegible]

a. [illegible]

b. [illegible]

c. [illegible]

d. [illegible]

e. [illegible]

f. [illegible]

g. [illegible]

h. [illegible]

i. [illegible]

j. [illegible]

[illegible] [illegible]

[illegible] [illegible]

a. [illegible]

b. [illegible]

c. [illegible]

d. [illegible]

e. [illegible]

f. [illegible]

g. [illegible]

h. [illegible]

i. [illegible]

[illegible] [illegible]

a. [illegible]

b. [illegible]

Figure P8.6

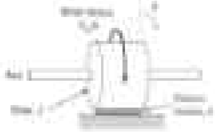


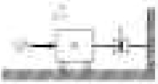
Figure P8.7

[illegible]

[illegible]

MATLAB Problems

[illegible]



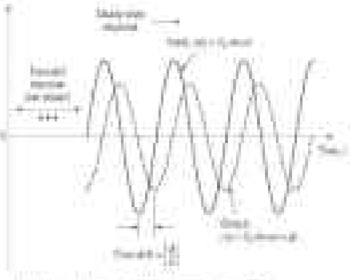
Engineering Applications

Chapter 9

Frequency-Response Analysis

9.1 INTRODUCTION

9.2 FREQUENCY RESPONSE



[illegible]

Sinusoidal Transfer Function

[illegible]

[illegible]

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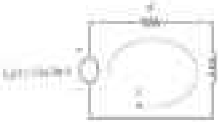
[illegible]

[illegible]

Derivation of the Frequency Response



[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

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[illegible]

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[illegible]
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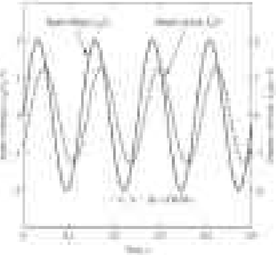


[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

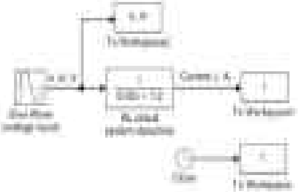
[illegible]

[illegible]

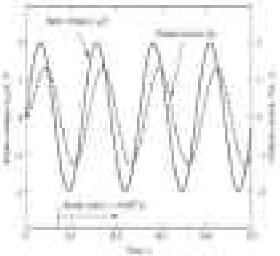
[illegible]

$$[illegible]$$

[illegible]



[illegible]



[illegible]

[illegible]

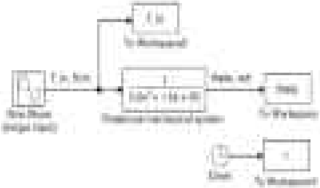
[illegible]

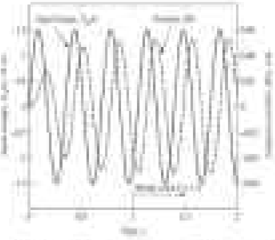
[illegible]

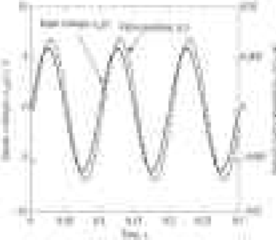
$$[illegible]$$

[illegible]

$$[illegible]$$







[illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]

[illegible]

$$[illegible] \quad (9.[illegible])$$

[illegible]

$$[illegible] \quad (9.[illegible])$$

[illegible]

Magnitude: [illegible] (9.[illegible])

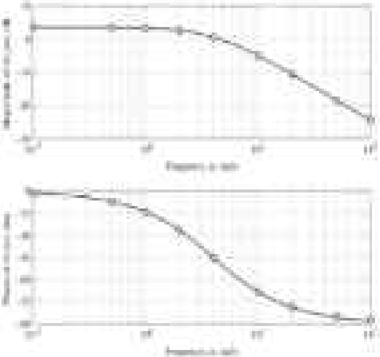
Phase: [illegible] (9.[illegible])

[illegible]

[illegible]

Table 9.1 [illegible]

Input Frequency ω (rad/s)	Magnitude $\lvert G(j\omega)\rvert$	Magnitude $20\log_{10}\lvert G(j\omega)\rvert$ (dB)	Phase $\angle G(j\omega)$
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]



Constructing the Bode Diagram Using MATLAB

The required MATLAB commands are:

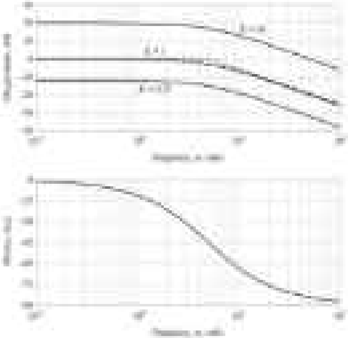
[illegible]

Bode Diagrams of First-Order Systems

[illegible]

$$G(s) = \frac{K}{\tau s + 1}$$

[illegible]

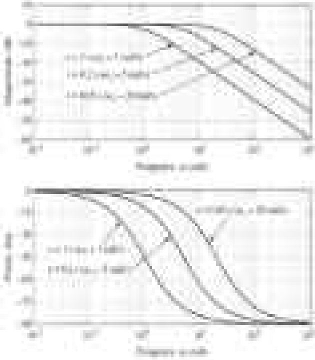


[illegible]

[illegible]

[illegible]

[illegible]



$$\tau = 0.05\ \text{s} : \quad \omega_c = 20\ \text{rad/s}$$

$$\tau = 0.2\ \text{s} : \quad \omega_c = 5\ \text{rad/s}$$

$$\tau = 1\ \text{s} : \quad \omega_c = 1\ \text{rad/s}$$

[illegible]

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]

[illegible]

$$[illegible]$$

[illegible]

Example [illegible]

[illegible]

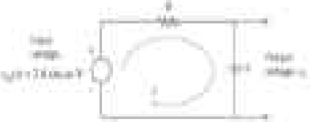
[illegible]

$$[illegible]$$

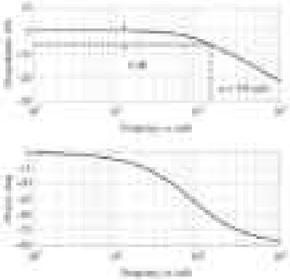
[illegible]

$$[illegible]$$

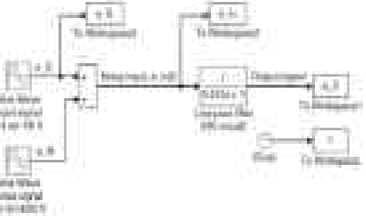
[illegible]



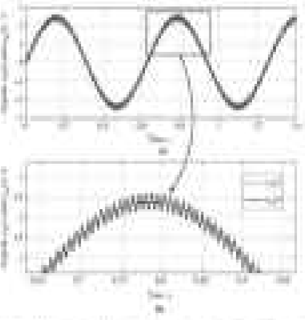
[illegible]



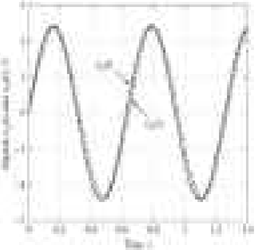
[illegible]



[illegible]



[illegible]

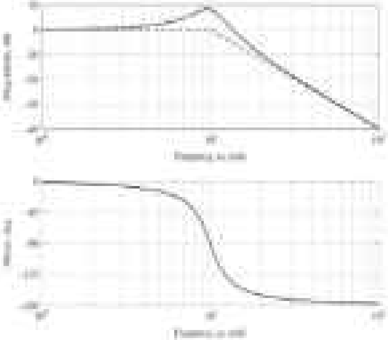


[illegible]

[illegible]

Bode Diagram of Second-Order Systems

[illegible]



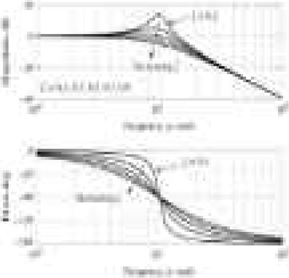


Figure 9.20 Bode diagram of second-order system [illegible]

4. The peak magnitude increases as damping ratio ζ is decreased.
5. The phase angle ϕ starts at 0° (at low frequencies) and asymptotically approaches −180° at high frequencies.
6. The phase angle shows a sharper transition from 0° to −180° as damping ratio ζ is decreased.
7. The phase angle $\phi = -90°$ when the input frequency is the corner frequency ω_n.

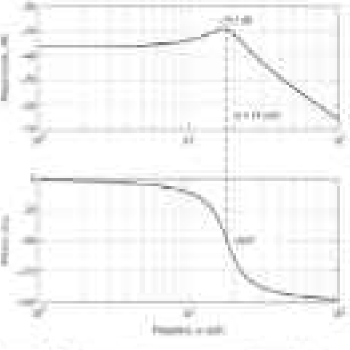
The above summary tells us that the Bode diagram of a second-order system is essentially the "doubling" of the Bode diagram of a first-order system [illegible]

$$\omega_r = \omega_n\sqrt{1-2\zeta^2}$$

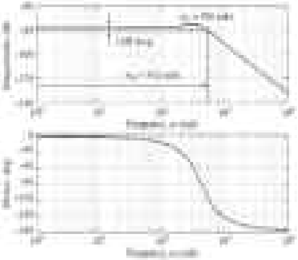
[illegible]

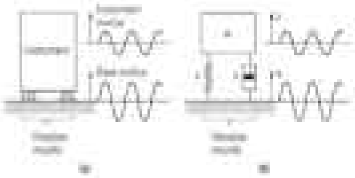
Example 9.8

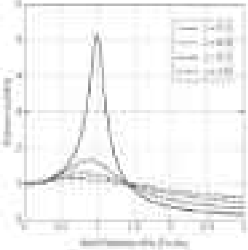
[illegible]



Bandwidth







[illegible]

3. [illegible]

4. [illegible]

5. [illegible]

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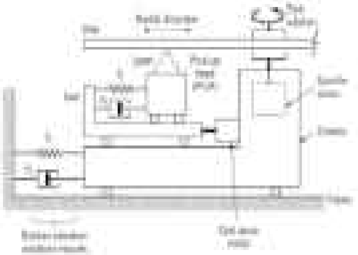

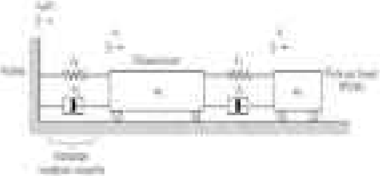

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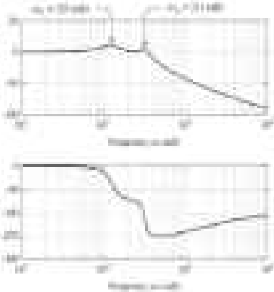
$$[illegible]$$

[illegible]

$$[illegible]$$

Table 8.2 [illegible]

Optical Disk Drive System Parameter	Numerical Value
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]



SUMMARY

[illegible]

REFERENCES

1. [illegible]
2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]

PROBLEMS

Conceptual Problems

9.1 [illegible]

[illegible]

9.2 [illegible]

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

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$$[illegible]$$

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$$[illegible]$$

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a. [illegible]

b. [illegible]

c. [illegible]

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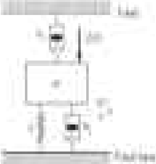
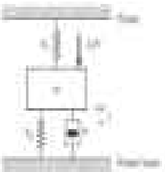


MATLAB Problems



Engineering Applications

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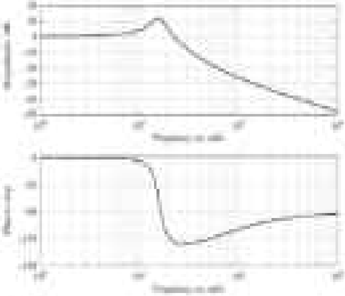

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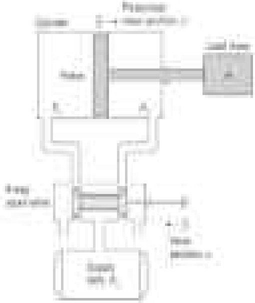

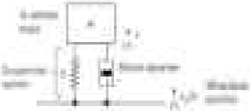

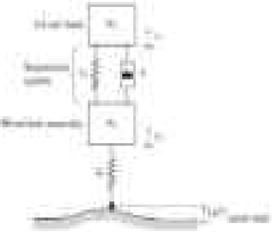

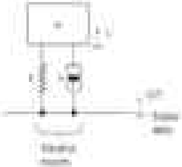

Chapter 10

Introduction to Control Systems

10.1 INTRODUCTION

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10.2 FEEDBACK CONTROL SYSTEMS

[illegible]

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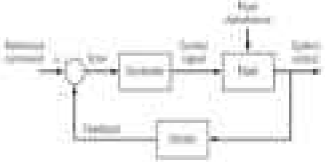

[illegible]

Control System Transfer Functions

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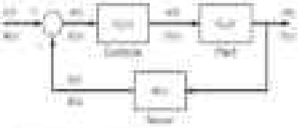
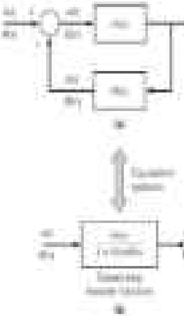
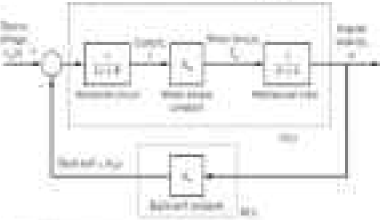


Figure 10.2 Closed-loop feedback control system.





[illegible]

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[illegible]

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[illegible]

Example 10.3

[illegible]

$$[illegible]$$

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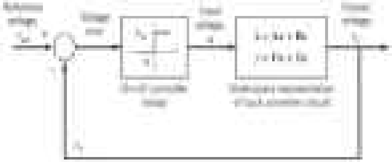
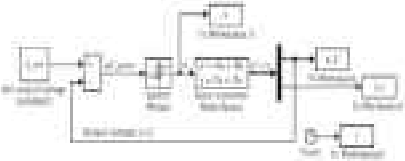
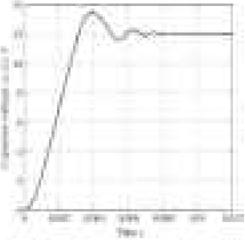






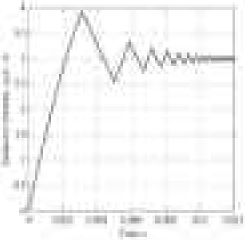


Figure 10.19 [illegible] Example 10.4

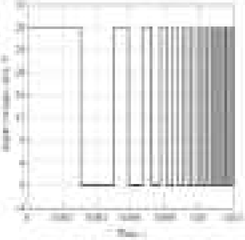
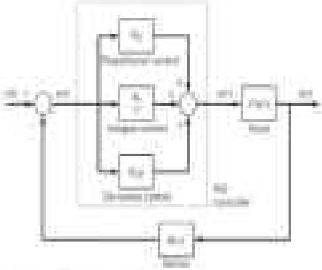


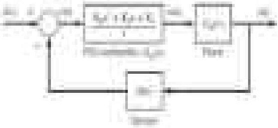
Figure 10.20 [illegible] Example 10.4

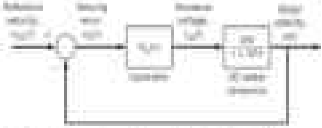
PID Controllers

It is likely that PID controllers are the most common controllers used in industry. Their applications (to name a few) include vehicle control, hydraulic control systems, chemical processing plants, fuel pumps and control of aerospace vehicles. Figure 10.21 shows a PID controller in a feedback control system. We see that the PID controller takes the feedback error signal $e(t)$ and produces three control signals that are summed together to create the control input signal $u(t)$ to the plant. The PID control law is

$$u(t) = K_P e(t) + K_I \int e(t)\,dt + K_D \dot{e}(t) \quad (10.11)$$







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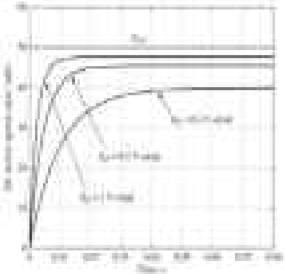
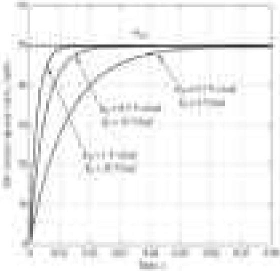
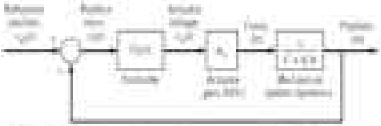
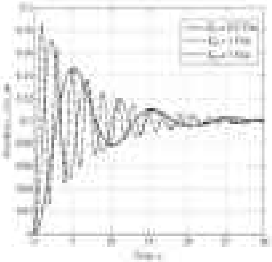
$$[illegible]$$

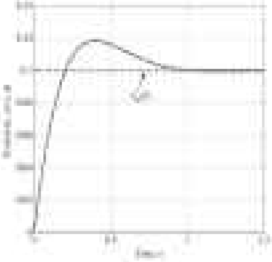
[illegible]











[illegible]

PID Tuning Rules

[illegible]

[illegible]

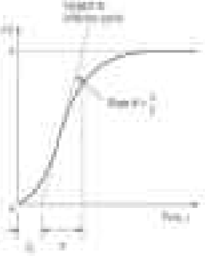


Figure 10.21 Reaction curve from an open-loop step test.

Table [illegible]

Controller Type	Gains
P	[illegible]
PI	[illegible]
PID	[illegible]

Table [illegible]

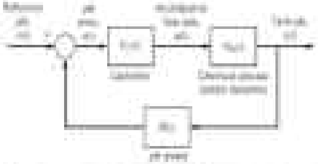
Controller Type	Gains
P	[illegible]
PI	[illegible]
PID	[illegible]

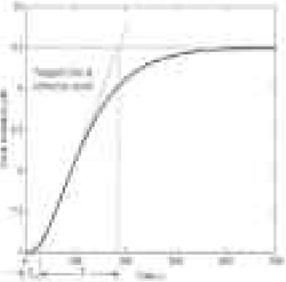
[illegible]

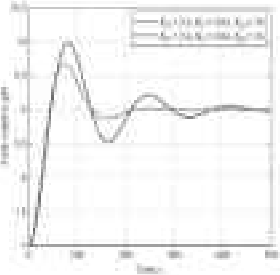
[illegible]

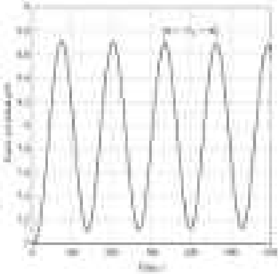
Example [illegible]

[illegible]









[illegible]

10.4 STEADY-STATE ACCURACY

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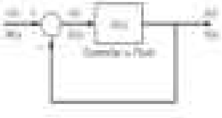

[illegible]

Substituting the closed-loop transfer function $Y(s)/R(s) = G(s)/[1 + G(s)]$ yields

$$\frac{E(s)}{R(s)} = 1 - \frac{G(s)}{1 + G(s)} = \frac{1 + G(s)}{1 + G(s)} - \frac{G(s)}{1 + G(s)} \tag{10.26}$$

Finally, the transfer function relating tracking error to the reference command is

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)} \tag{10.27}$$

Equation (10.27) can be used to compute the Laplace transform of the tracking error

$$E(s) = \frac{1}{1 + G(s)} R(s) \tag{10.28}$$

Recall that the final value theorem can be used to compute the final (or steady-state) tracking error

$$e(\infty) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \tag{10.29}$$

Or, using Eq. (10.28)

$$e(\infty) = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)} \tag{10.30}$$

Equation (10.30) shows that the steady-state tracking error depends on (1) the nature of the input $R(s)$, and (2) the forward transfer function $G(s)$.

System type is a designation of the forward transfer function $G(s)$ that is useful for analyzing the steady-state error. It represents the number of "free integrators" that exist in the forward transfer function. [illegible] this, [illegible] the forward transfer function using the following form:

$$G(s) = \frac{P(s)}{s^N Q(s)} \tag{10.31}$$

where $P(s)$ and $Q(s)$ are polynomials in s that do not contain a root at $s = 0$. The index N is called the system type and it is the number of "free integrators" (1/s terms) that can be divided out of $G(s)$. As a quick example, consider the forward transfer function

$$G(s) = \frac{8(s+2)}{s^2 + [illegible]} = \frac{P(s)}{s^N Q(s)}$$

Hence, $N = 0$ and $P(s)$ and $Q(s)$ are the numerator and denominator polynomials of $G(s)$. Because $N = 0$, this forward transfer function $G(s)$ is called a "type 0 system." As a second example, consider the forward transfer function

$$G(s) = \frac{[illegible]}{s^2 + 8s} = \frac{[illegible]}{s(s + 8)} = \frac{P(s)}{s^N Q(s)}$$

Clearly, $N = 1$ and this is a "type 1 system." In other words, we can "divide out" a free integrator $1/s$ from $G(s)$ in this example. The reader should recall that the forward transfer function $G(s)$ is the product of the controller and plant transfer functions, and therefore a PI or PID controller will introduce a "free integrator"

[illegible]

[illegible]

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[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Table [illegible] [illegible]

System Type, N	Unit Step Input r(t) = 1	Unit Ramp Input r(t) = t	Unit Parabolic Input r(t) = [illegible]
0	$\frac{1}{1+K_p}$	∞	∞
1	0	$\frac{1}{K_v}$	∞
2	0	0	$\frac{1}{K_a}$

Static position error constant K_p = [illegible]

Static velocity error constant K_v = [illegible]

Static acceleration error constant K_a = [illegible]

Proportional controller

Proportional-integral controller

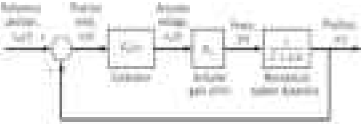
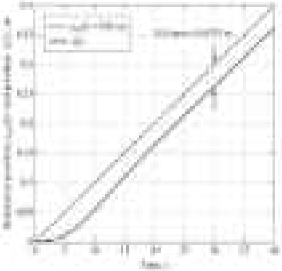


Figure 10.27 Closed-loop position control of a mechanical system



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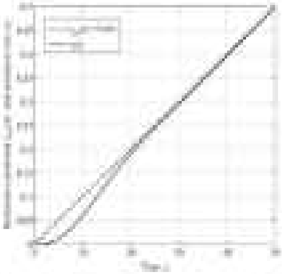

[illegible]

10.5 CLOSED-LOOP STABILITY

[illegible]

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[illegible] MATLAB [illegible]

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[illegible] MATLAB [illegible]

Example [illegible]

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[illegible] MATLAB [illegible]

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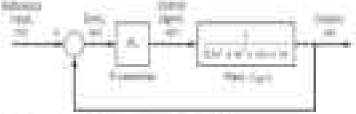

[illegible]

[illegible]

Control Gain, K_p	Closed-Loop Poles	Stability Status
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]

[illegible]

10.8 ROOT LOCUS METHOD

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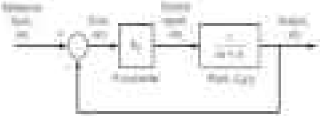

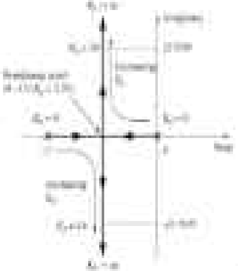

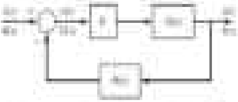

5. [illegible]

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1. [illegible]
2. [illegible]
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4. [illegible]
5. [illegible]

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Constructing the Root Locus Using MATLAB

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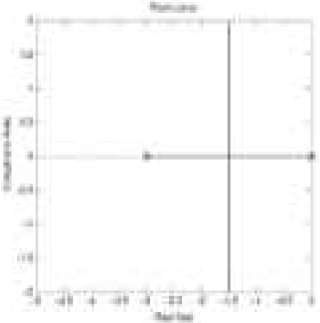
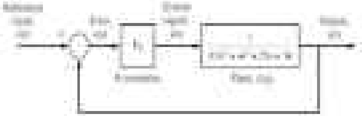
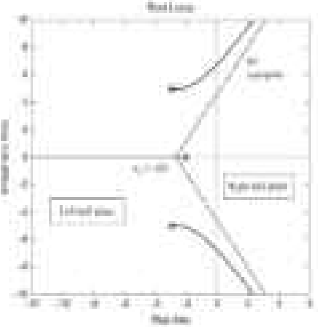
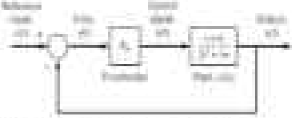
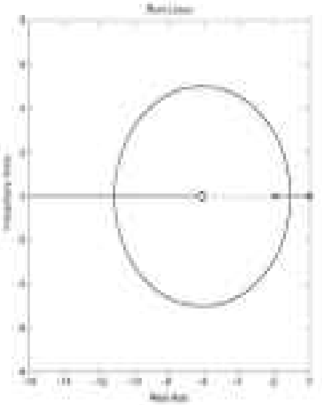
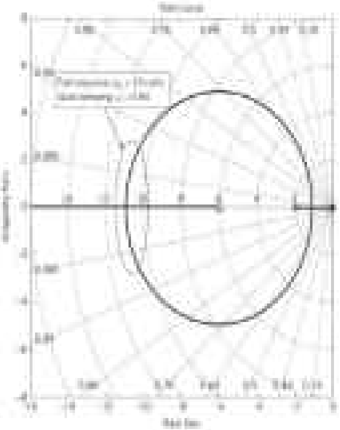








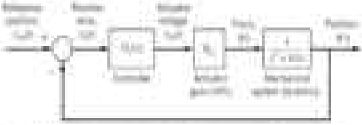




[illegible]

Controller Design Using Root Locus

[illegible]



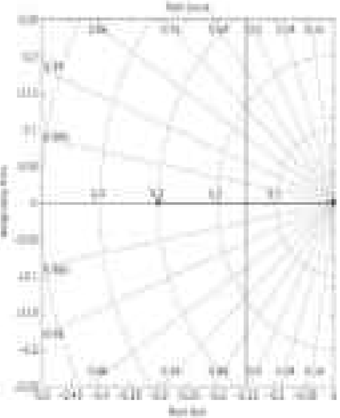


Figure 10.4[illegible] [illegible]

[illegible]

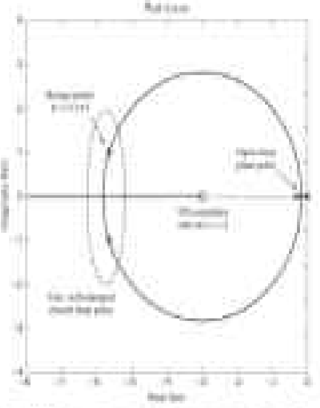
Proportional-derivative controller

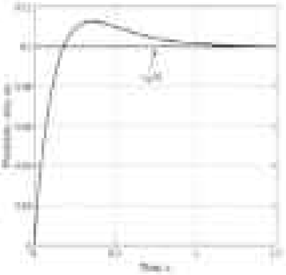
[illegible]

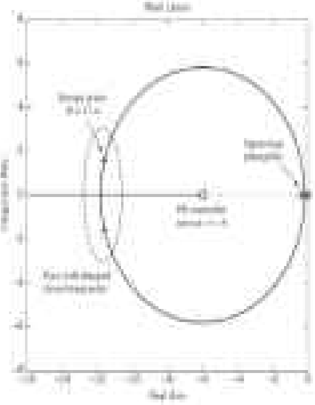
$$[illegible] \quad (10.[illegible])$$

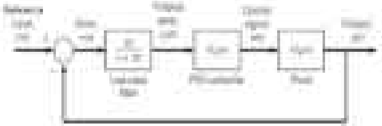
[illegible]

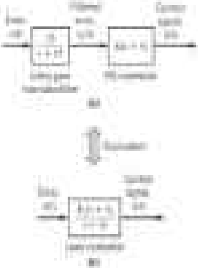
$$[illegible] \quad (10.[illegible])$$

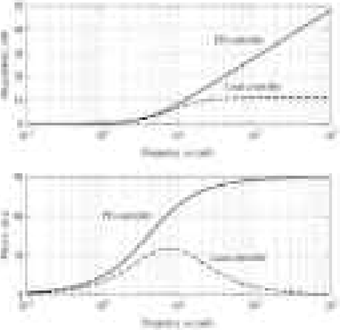


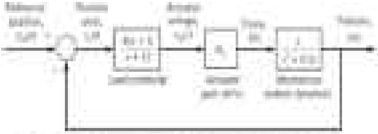


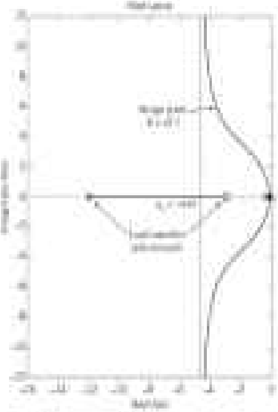


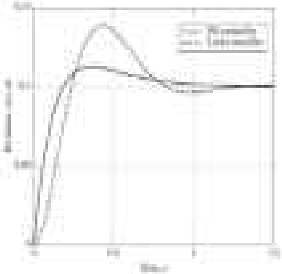












[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

10.7 STABILITY MARGINS

[illegible]

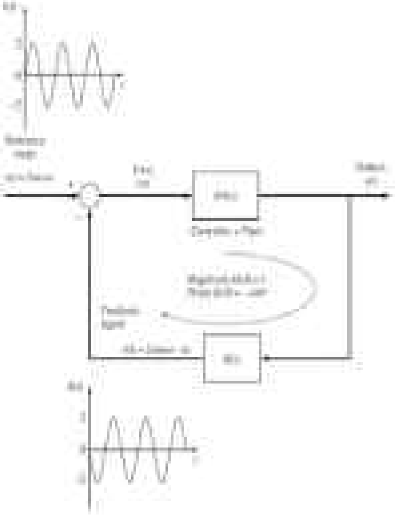
[illegible]

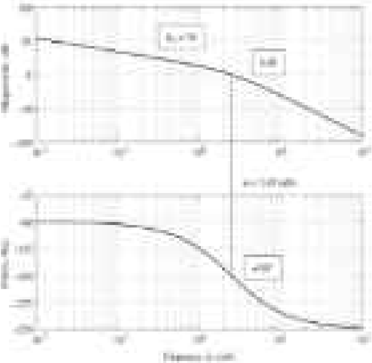
[illegible]

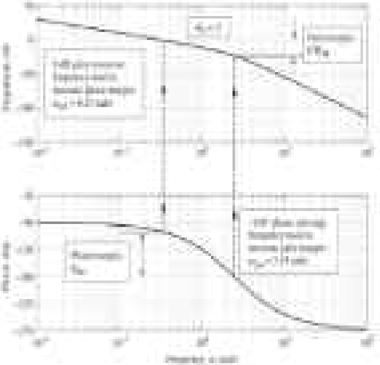
[illegible]

[illegible]

[illegible]







[illegible]

$$\text{Gain margin} = 10^{[illegible]}$$

[illegible]

[illegible]

[illegible]

[illegible]

Gain and Phase Margins Using MATLAB

[illegible]

```
[illegible]
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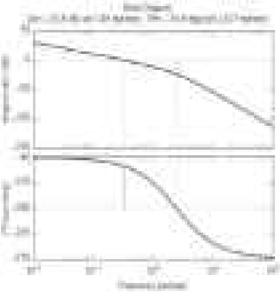


Controller Design in the Frequency Domain

[illegible]

$$\text{Lead controller: } [illegible]$$

[illegible]

Example [illegible]

[illegible]

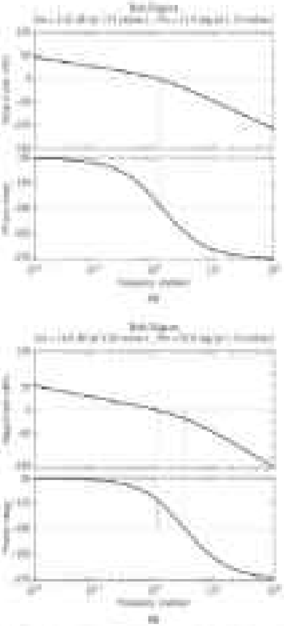
$$\text{Plant: } [illegible]$$

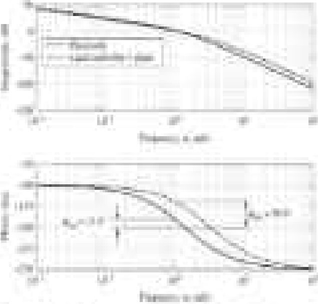
[illegible]

[illegible]

$$\text{Lead controller: } [illegible]$$

[illegible]





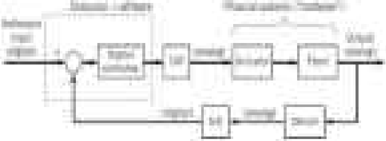
[illegible]

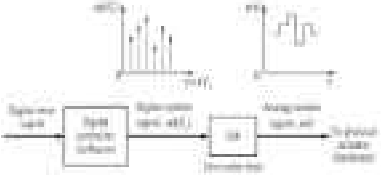
IMPLEMENTING CONTROL SYSTEMS

[illegible]

Digital Control Systems

[illegible]





Digital Controller Algorithms

[illegible] consider the PID control law:

$$u(t) = K_P e(t) + K_I \int_0^t e(t)\,dt + K_D \dot{e}(t) \quad [illegible]$$

[illegible] difference equation

$$[illegible] = \frac{[illegible]}{T_s} \quad [illegible]$$

[illegible] digital PID control algorithm is

$$u(k) = K_P e(k) + K_I [illegible] + K_D [illegible] \quad [illegible]$$

[illegible]

[illegible] PD controller [illegible] lead controller. [illegible]

PD control: [illegible]

The PD controller transfer function is

PD controller: [illegible]

The PD controller [illegible] lead controller. [illegible]

Lead controller: [illegible]

The lead controller [illegible] MATLAB command c2d [illegible]

$$[illegible]$$

[illegible]

[illegible]

SUMMARY

[illegible]

REFERENCES

1. [illegible]
2. [illegible]
3. [illegible]

4. [illegible]

5. [illegible]

6. [illegible]

7. [illegible]

8. [illegible]

PROBLEMS

Conceptual Problems

P9.1 [illegible]

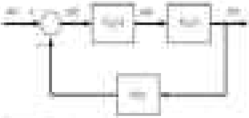


Figure P9.1

a. [illegible]

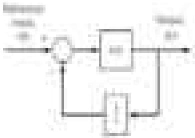
b. [illegible]

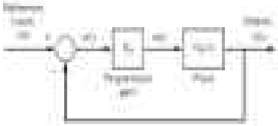
c. [illegible]

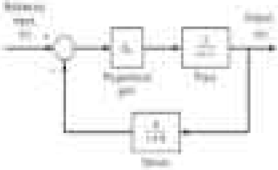
d. [illegible]

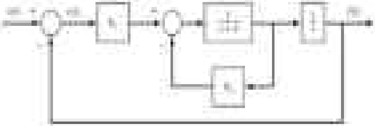
e. [illegible]

f. [illegible]









[illegible]

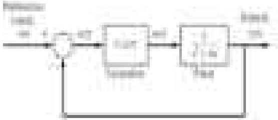


Figure P9.6

a. [illegible]

b. [illegible]

[illegible]

Figure P9.7

a. [illegible]

b. [illegible]

MATLAB Problems

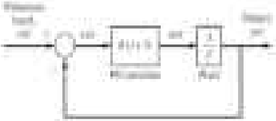
[illegible]

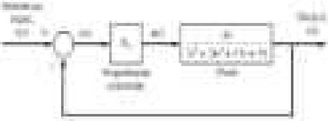
$$[illegible]$$

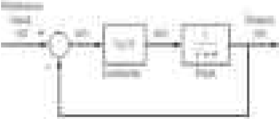
a. [illegible]

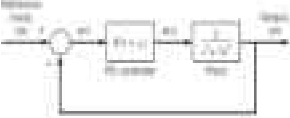
b. [illegible]

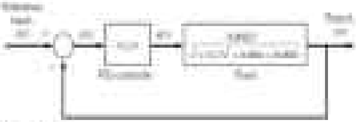
c. [illegible]

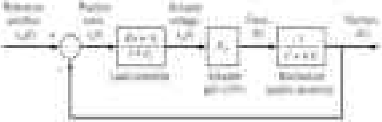


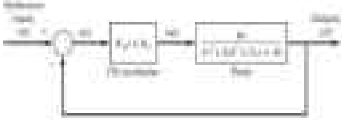


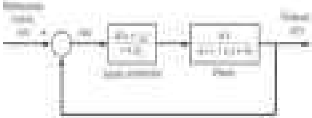












[illegible]

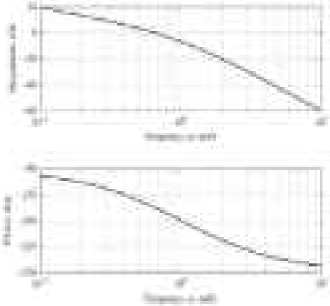


Figure P10.23

Engineering Applications

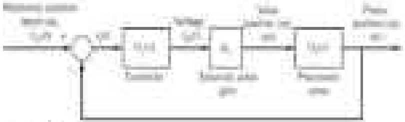
[illegible]

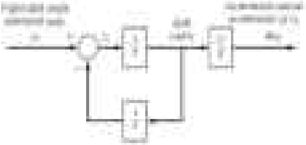
$$[illegible]$$

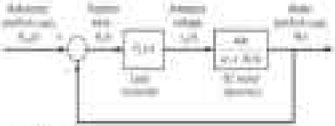
[illegible]

a. [illegible]

b. [illegible]







Chapter 11

Case Studies in Dynamic Systems and Control

11.1 INTRODUCTION

[illegible]

11.2 VIBRATION ISOLATION SYSTEM FOR A COMMERCIAL VEHICLE

[illegible]

Mathematical Model

[illegible]

[illegible] (11.1a)

[illegible] (11.1b)

[illegible]

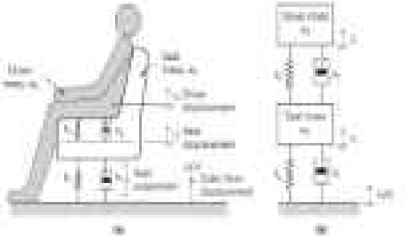


Table 11.1 [illegible]

System Parameter	Value
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible] **A**. [illegible]

[illegible] Table 11.1 [illegible]

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ [illegible] & [illegible] & [illegible] & [illegible] \\ 0 & 0 & 0 & 1 \\ [illegible] & [illegible] & [illegible] & [illegible] \end{bmatrix}$$

The system eigenvalues are computed using the MATLAB command

[illegible]

The four eigenvalues (or characteristic roots) are

[illegible]

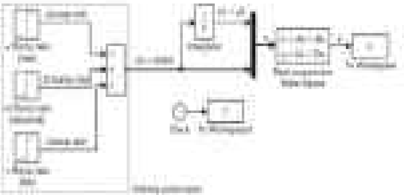
[illegible]

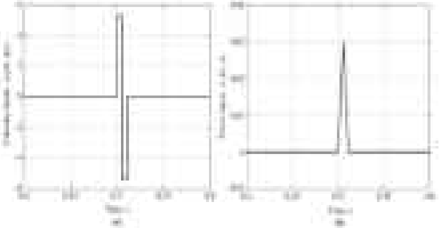
$$[illegible] \qquad (11.4)$$

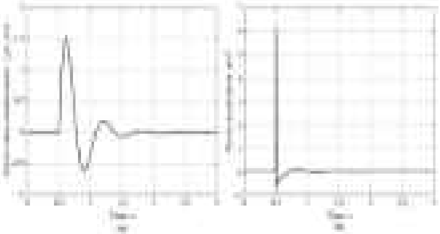
[illegible]

[illegible]

$$[illegible] \qquad (11.5)$$







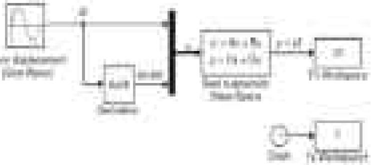
[illegible]

Frequency Response

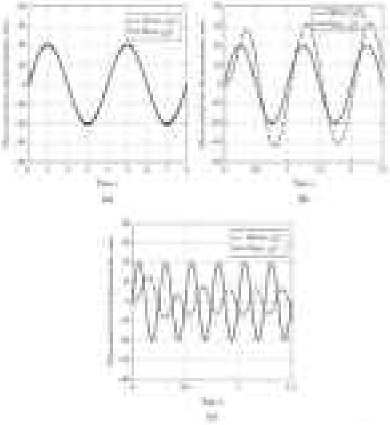
[illegible]

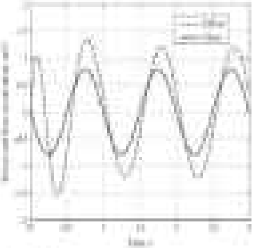
[illegible]

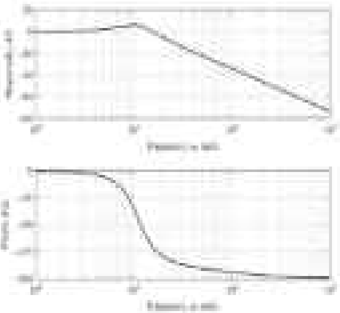
[illegible]

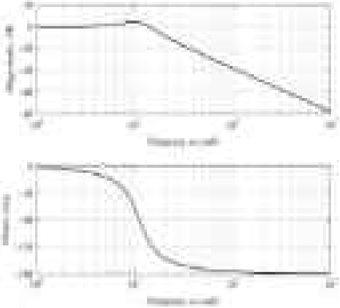


[illegible]









[illegible]

[illegible]

$$[illegible]$$

[illegible]

Parametric Sensitivity Analysis

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

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[illegible]
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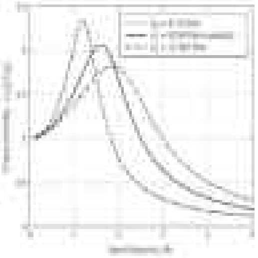
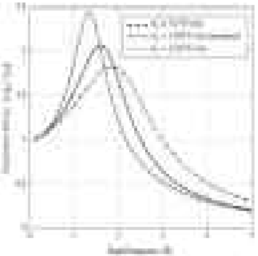
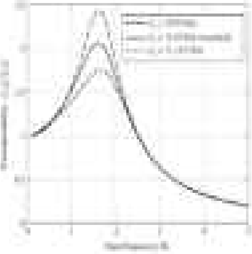
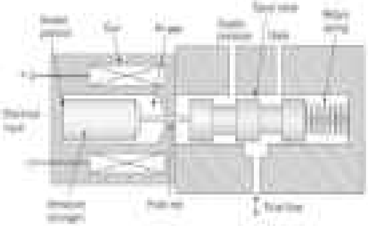






11.3. SOLENOID ACTUATOR–VALVE SYSTEM



Mathematical Model

Table 11.2 [illegible]

System Parameter	Value
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

$$[illegible]$$

[illegible]

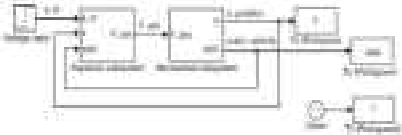
$$[illegible]$$

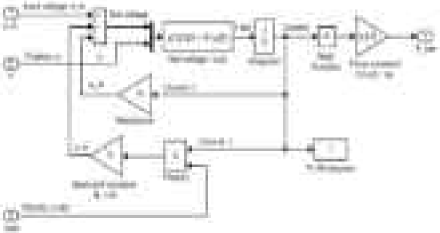
[illegible]

[illegible]

Simulink Model

[illegible]





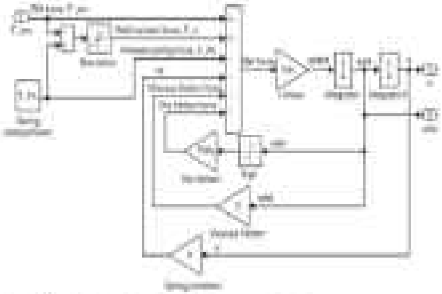


Figure 11.14 [illegible]

Solenoid-Actuator Design

[illegible]

[illegible]

$$[illegible]$$

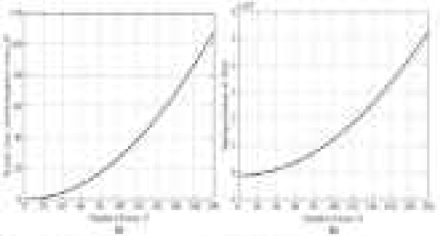
[illegible]

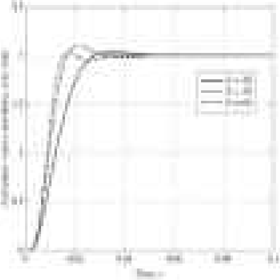
$$[illegible]$$

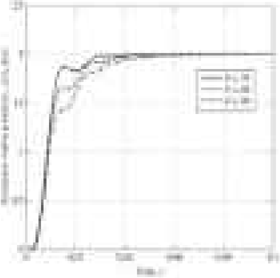
[illegible]

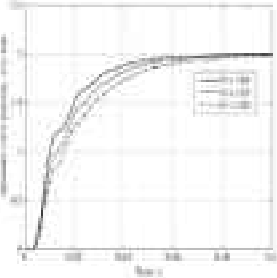
$$[illegible]$$

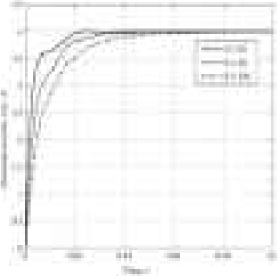
[illegible]











[illegible]

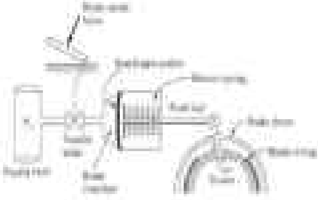
[illegible]

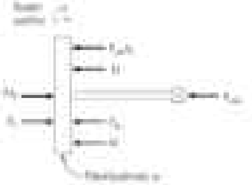
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

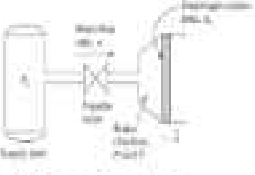
11.4 PNEUMATIC AIR-BRAKE SYSTEM

[illegible]

[illegible]







$$[illegible]$$

[illegible]

$$[illegible]$$

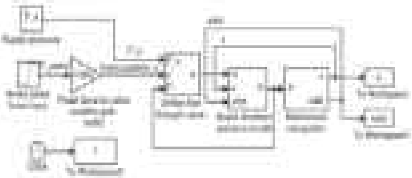
[illegible]

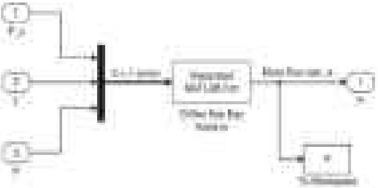
Simulink Model

[illegible]

Table 11.4 [illegible]

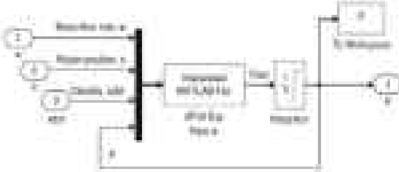
System Parameter	Value
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]





MATLAB M-file 17.2

```
% [illegible]
%
% [illegible]
% [illegible]
% [illegible]
%
% [illegible]
%              [illegible]
%              [illegible]
%              [illegible]
%
% [illegible]
%

function [illegible]

% [illegible]
[illegible]

% [illegible]
[illegible]
[illegible]
[illegible]
[illegible]
[illegible]

% [illegible]
[illegible]
[illegible]
[illegible]

% [illegible]
[illegible]

% [illegible]
% [illegible]
if [illegible]
   [illegible]
else
   [illegible]
end

% [illegible]
[illegible]
[illegible]

% [illegible]
[illegible]

% [illegible]
[illegible]

% [illegible]
if [illegible]
   % [illegible]
   [illegible]
      [illegible]
else
   % [illegible]
   [illegible]
end
```



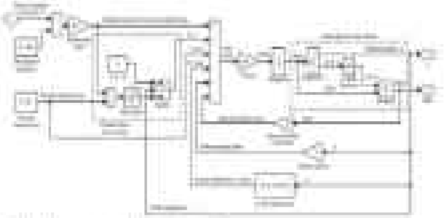
[illegible]

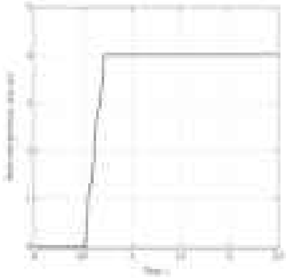
[illegible]

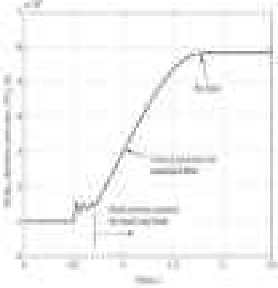
[illegible]

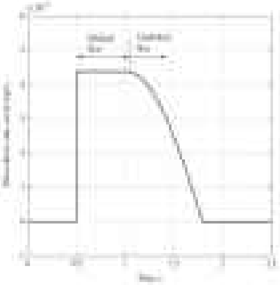
Air-Brake System Analysis

[illegible]









[illegible]

[illegible]

[illegible]

$$[illegible]$$

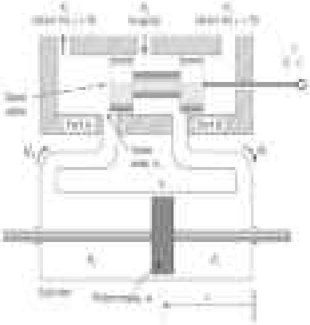
[illegible]

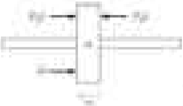
$$[illegible]$$

[illegible]

11.2 HYDRAULIC SERVOMECHANISM CONTROL

[illegible]





[illegible]

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

[illegible]

$$[illegible]$$

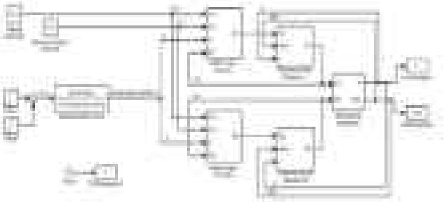
[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

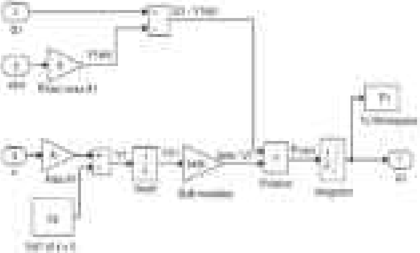


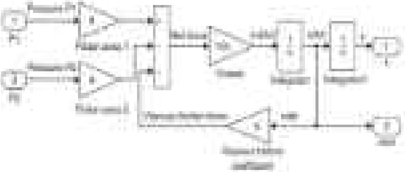
[illegible]

[illegible]

[illegible]

```
[illegible]
```





Pulse Response of the EHA

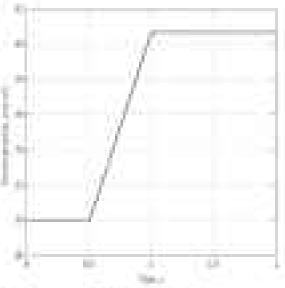
[illegible]

[illegible]

[illegible]

Linear EHA Model

[illegible]



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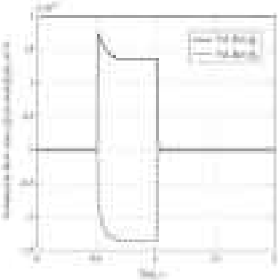

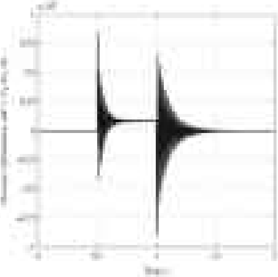

[illegible]

In flow to chamber 1: [illegible]

Out flow from chamber 2: [illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

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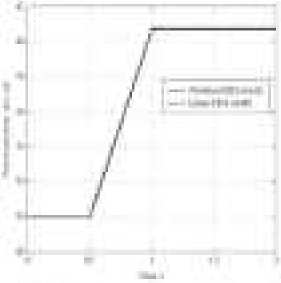
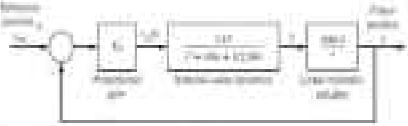
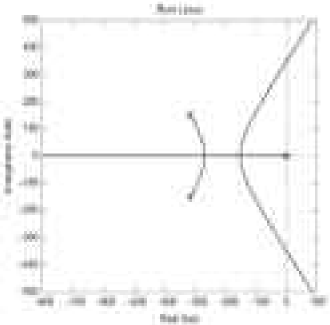
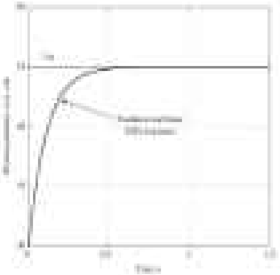
$$[illegible]$$

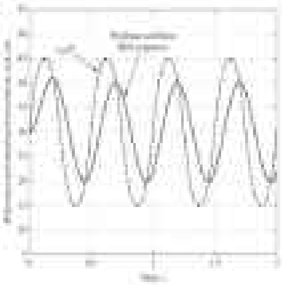
[illegible]

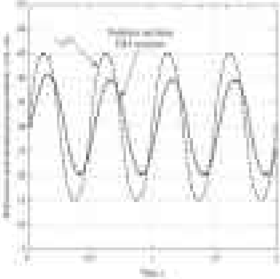


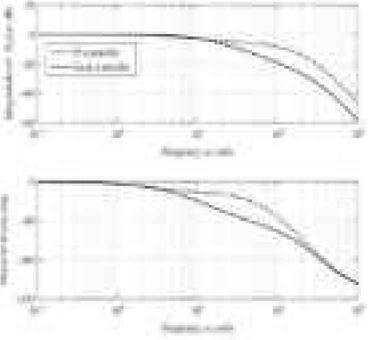








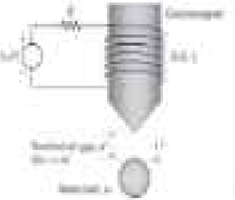




[illegible]

11.8 FEEDBACK CONTROL OF A MAGNETIC LEVITATION SYSTEM

[illegible]



[illegible]

Mathematical Model

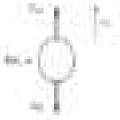


Table [illegible]

System Parameter	Value
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

[illegible]

Linear Maglev Model

[illegible]

[illegible]

$$[illegible]$$

$$[illegible]$$

$$[illegible]$$

[illegible]

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

$$[illegible]$$

[illegible]

First-state equation: $[illegible]$

Third-state equation: $[illegible]$

[illegible]

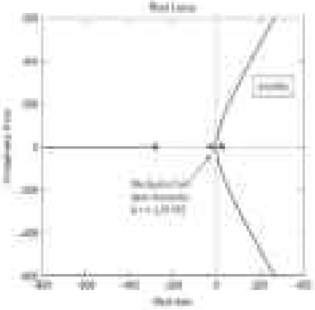
$$[illegible] \quad \text{and} \quad [illegible]$$

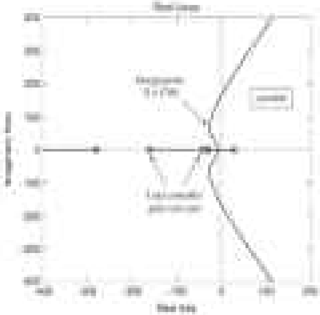
[illegible]

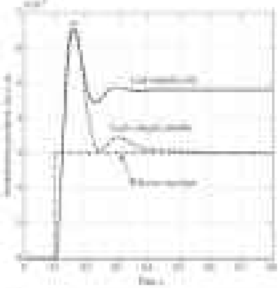
$$[illegible]$$

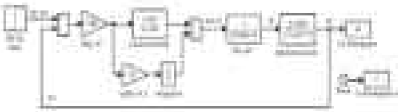
$$[illegible]$$

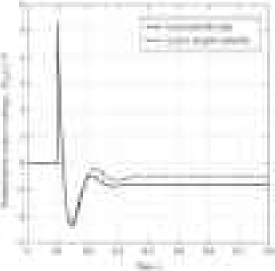
Maglev Control System Design

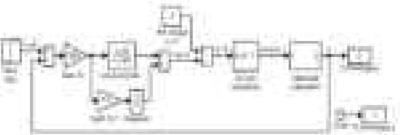


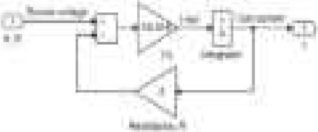


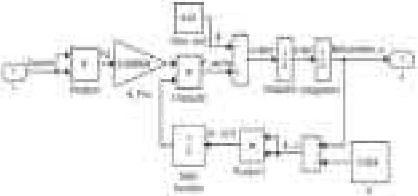


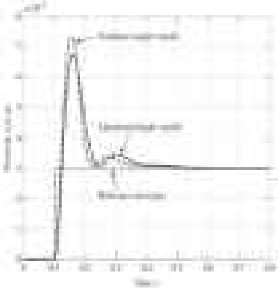


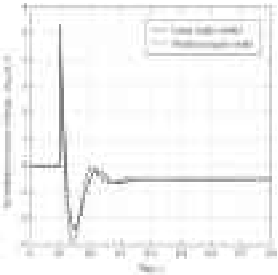












[illegible]

SUMMARY

[illegible]

REFERENCES

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2. [illegible]
3. [illegible]
4. [illegible]
5. [illegible]
6. [illegible]
7. [illegible]
8. [illegible]
9. [illegible]

Appendix A

Units

[illegible]

[illegible]

Table A.1 [illegible]

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electrical current	ampere	A
Temperature	kelvin	K

Table A.2 [illegible]

Quantity	Unit	Symbol	Dimensions
Force	newton	N	[illegible]
Energy	joule	J	[illegible]
Power	watt	W	[illegible]
Pressure	pascal	Pa	[illegible]
Electrical charge	coulomb	C	[illegible]
Voltage	volt	V	[illegible]
Electrical resistance	ohm	Ω	[illegible]
Electrical capacitance	farad	F	[illegible]
Magnetic flux	weber	Wb	[illegible]
Inductance	henry	H	[illegible]

Appendix B

MATLAB Primer for Analyzing Dynamic Systems

B.1 INTRODUCTION

[illegible]

[illegible]

B.2 BASIC MATLAB COMPUTATIONS

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

```
[illegible]
```

[illegible]

```
[illegible]
```

[illegible]

Vectors and Matrices

[illegible]

```
[illegible]
```

[illegible]

```
[illegible]
```

[illegible]

[illegible]

$$[illegible]$$

[illegible]

```
[illegible]
```

which gives us the answer:

```
B =
    [illegible]
    [illegible]
```

The command size(A) displays the size (dimensions) of matrix A (m by n).

A vector of equispaced data can be generated by defining the starting value, an incremental step size, and the final value:

```
>> x = [illegible]                    % [illegible]
```

We can also use the command linspace to generate N linearly spaced data points from a desired starting value to a desired final value:

```
>> x = linspace([illegible])          % [illegible]
```

The command logspace will generate N logarithmically spaced data points between the decades [illegible] starting and ending values:

```
>> x = logspace([illegible])          % [illegible]
```

MATLAB has built-in functions to construct the identity matrix, and arrays whose elements are either all ones or zeros:

```
>> I = eye([illegible])               % [illegible]
>> B = ones([illegible])              % [illegible]
>> B = zeros([illegible])             % [illegible]
```

Complex Variables

Complex variables can be entered by simply typing their real and imaginary parts. MATLAB assigns both i and j to the imaginary number $\sqrt{-1}$. Complex variables can be defined, added, and multiplied as shown below:

```
>> [illegible]                        % [illegible]
>> [illegible]                        % [illegible]
>> [illegible]                        % [illegible]
>> [illegible]                        % [illegible]
```

The MATLAB commands abs and angle can be used to compute the absolute value (magnitude) and phase angle of a complex variable:

```
>> [illegible]                        % [illegible]
>> [illegible]                        % [illegible]
>> [illegible]                        % [illegible]
```

Table B.1 summarizes the basic MATLAB built-in functions described in this section as well as other common functions, such as exponential and logarithmic functions.

Table B.1 [illegible]

MATLAB Command	Description
abs(x)	Absolute value of x, magnitude if x is complex
angle(x)	Phase angle [illegible] of complex variable x
cos(x)	Cosine of x [illegible]
sin(x)	Sine of x [illegible]
tan(x)	Tangent of x [illegible]
acos(x)	Inverse cosine of x [illegible]
asin(x)	Inverse sine of x [illegible]
atan(x)	Inverse tangent of x [illegible]
exp(x)	Exponential of x [illegible]
log(x)	Natural logarithm of x
log10(x)	Base 10 logarithm of x
max(x)	Maximum element of vector x
min(x)	Minimum element of vector x
size(x)	Size (dimensions) of vector or matrix x
sqrt(x)	Square root of x
sum(x)	Sum of the elements of vector x

B.[illegible] PLOTTING WITH MATLAB

[illegible]

Example B.1

[illegible]

```
[illegible]
```

[illegible]

```
[illegible]
```

[illegible]

B.4 CONSTRUCTING BASIC M-FILES

MATLAB M-file B.1

```
[illegible]
```

B.5 COMMANDS FOR LINEAR SYSTEM ANALYSIS

[illegible]

$$[illegible]$$

Defining transfer function object sysG

```
>> [illegible]
>> [illegible]
>> [illegible]
```

[illegible]

[illegible]

[illegible]

Roots of polynomial [illegible]

[illegible]

Poles of transfer function [illegible]

[illegible]

Undamped natural frequency and damping ratio

[illegible]

Plot of unit-step response of [illegible]

[illegible]

Plot of [illegible] **response to an arbitrary input**

[illegible]

[illegible]

B.8 COMMANDS FOR LAPLACE TRANSFORM ANALYSIS

[illegible]

Partial-fraction expansion

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

```
[illegible]
```

[illegible]

$$[illegible]$$

Laplace transform using Symbolic Math Toolbox

MATLAB's Symbolic Math Toolbox can be used to compute the Laplace transform of a given time function [illegible]

[illegible]

Inverse Laplace transform using Symbolic Math Toolbox

[illegible]

[illegible]

8.7 COMMANDS FOR CONTROL SYSTEM ANALYSIS

[illegible]

Table 8.x [illegible]

MATLAB Command	Description
[illegible]	[illegible]
F = laplace(f)	[illegible]
f = ilaplace(F)	[illegible]

[illegible]

$$[illegible]$$

[illegible]

[illegible]

[illegible]

Table B.4 [illegible]

MATLAB Command	Description
[illegible] = feedback([illegible])	[illegible]
rlocus([illegible])	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]
[illegible]	[illegible]

Appendix C

Simulink Primer

C.1 INTRODUCTION

[illegible]

C.2 BUILDING SIMULINK MODELS OF LINEAR SYSTEMS

[illegible]

[illegible]

```
[illegible]
```

[illegible]

[illegible]

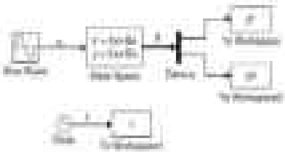
$$[illegible]$$

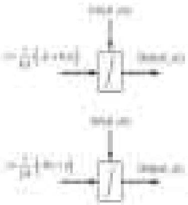
[illegible]

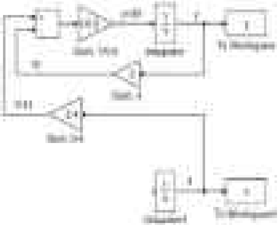
$$[illegible]$$

[illegible]

[illegible]







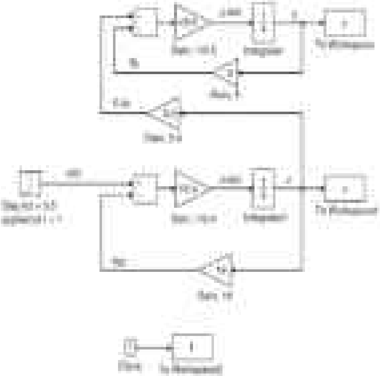


Figure E.9 [illegible]

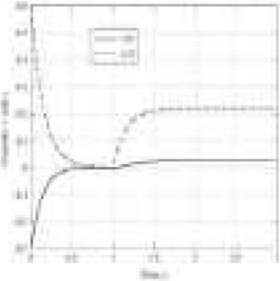
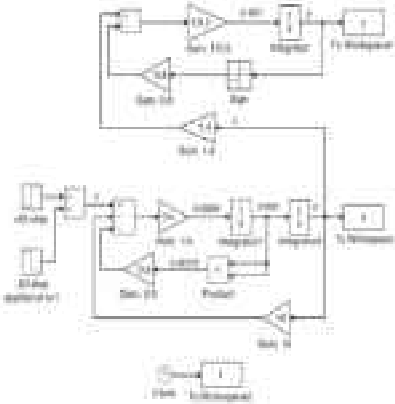
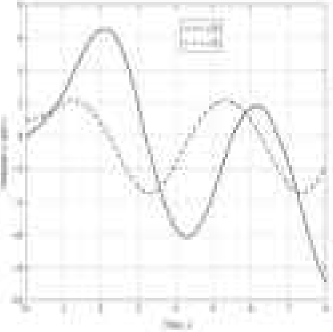


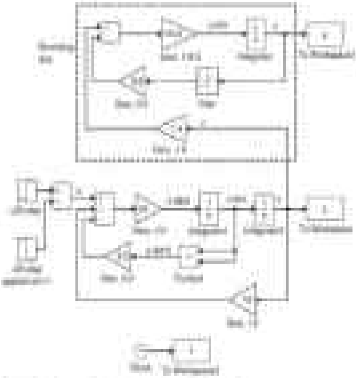
Figure E.10 [illegible]

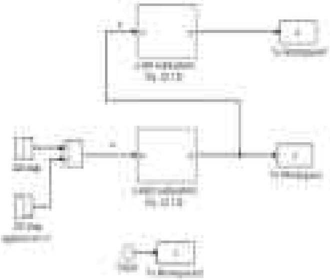
C.5 BUILDING SIMULINK MODELS OF NONLINEAR SYSTEMS

Example C.4









[illegible]

[illegible]

C.4 SUMMARY OF USEFUL SIMULINK BLOCKS

[illegible]

Continuous Library

[illegible]

Discontinuities Library

[illegible]

[illegible]

Math Operations Library

[illegible]

Ports and Subsystems Library

[illegible]

Signal Routing Library

[illegible]

Sinks Library

[illegible]

Sources Library

User-Defined Functions Library

Index

[illegible]

[illegible]

[illegible]