ANALYSIS OF EIGENVALUE BASED SPECTRUM SENSING TECHNIQUES IN COGNITIVE RADIO NETWORKS

MCS

By

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Abstract

Energy based spectrum sensing detection is optimal in terms of computational complexity but they have certain limitations of their dependence upon noise. In contrast Eigenvalue based algorithms do not depend upon noise uncertainty. Eigenvalue based algorithms are computationally complex as compared to energy detection method. Its complexity comes from two steps, the decomposition of the covariance matrix and the computation of Eigenvalue. The decomposition of covariance matrix does not offer enough room for complexity analysis as it has already been studied to its maxima while the computation of Eigenvalues is still an open field for research.

In this work we propose fast iterative algorithms to handle Eigenvalue problems for Eigenvalue based spectrum sensing detections. The proposed algorithm reduces the complexity of the Eigenvalue based spectrum sensing techniques to $\boldsymbol{O(L)}$**.** When the noise floor is high enough i.e. the signal is too weak its detection is challenging. The aim of the thesis is to detect weak signals in cognitive radios through varying the values for the received time sample (smoothing factor) L with minimal complexity. Simulations based on real-time GSM signals and the wireless microphone signals are presented to verify the proposed.

We have reduced the overall complexity of the Eigenvalue based spectrum sensing techniques which will be beneficial especially for cooperative spectrum field where we deal with multiple receiver and transmitter signals. Most importantly the significance of work is in the detection of weak signals in cognitive radios. As the signal becomes weak it will be smoothly detected using larger values for received time samples L. As L increase the complexity also increases where we can use the proposed work in Eigenvalue based spectrum sensing methods to obtain sensing results with reduced complexity.

Acknowledgement

First and Foremost I would like to thank Almighty Allah for all that He has bestowed upon me. Without His Grace, Mercy and Blessings I wouldn't have been able to start and complete this thesis.

During thesis I came across different problems and issues which were of great hindrance in its completion on time. Initially as a subject to was a newer one not have been targeted earlier, which caused many a problems in understanding the schematics of this subject but with extensive theoretical study and research work I was able to comprehend the gist of my area of interest in the said subject.

The completion of this project requires the mention of all the people who have helped, guided and motivated me in my endeavors. I would like to thank my supervisor Dr. Adnan Rashdi for his support, suggestions, guidance, unremitting discussions, meetings and vision he provided throughout the thesis. I would also extend my heartfelt gratitude to my friends and colleagues who with their arguments and suggestions gave me interesting insight to the elucidation of problems.

Finally, I want to acknowledge the contributions of my parents who with their unconditional love, moral support and guidance have greatly assisted all my academic pursuits and accomplishments.

List of Abbreviations

iid	=	Independent and identically distributed
MME	=	Maximum to Minimum Eigenvalue Detection
EME	=	Energy to Minimum Eigenvalue Detection
MED	=	Maximum Eigenvalue Detection
IRA	=	Implicitly restarted Arnoldi
IRL	=	Implicitly restarted Lanczos

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Chapter 1

Introduction

The emergence of new wireless technologies has paved the way for communications. This advancement in wireless communications leads to the spectrum efficiency [1, 2]. The radio spectrum for different technologies has been densely allocated. Heavily populated radio spectrum due to increased wireless communications has raised the spectrum scarcity problem. To come up with spectrum shortage, there is a need for efficient spectrum utilization. The radio spectrum can be efficiently utilized using Spectrum Sensing techniques. Spectrum sensing gives the opportunity to the licensed frequency band to be used by the unlicensed user during the time it is vacant.

In United States, Federal Communications Commission (FCC) is the organization responsible for the Radio spectrum licensing and management. The FCC has issued a radio spectrum chart for the frequencies allocated to various technologies that visibly highlights the electromagnetic spectrum scarcity problem.

Cognitive radio is a device that performs spectrum sensing. It comprises of a transmitter and a receiver that continuously senses the spectrum. It senses the licensed primary spectrum for any transmissions. It provides the unlicensed user with the opportunity to avail the licensed radio spectrum. Cognitive radio efficiently detects the licensed user, the time it is not utilized or underutilized and allows the unlicensed user to use that spectrum during the time it is available. When the licensed user again starts its transmission, the cognitive radio again detects it and guarantee's that no other unlicensed user can get hold of it. This spectrum sensing helps to manage the radio spectrum in an efficient way.

To have resourceful radio spectrum management, different spectrum sensing techniques have been proposed. These techniques mainly include Matched filter based detection,

Cyclostationary detection, Energy detection and the latest of all being proposed is the Eigenvalue based detection. Some of these techniques have least computationally complexity but are not accurate while the others are accurate but incorporate complexity. There is a compromise between accuracy and the complexity of these techniques. Energy detection is computationally the least complex method but it is not too accurate because of the fact that it compares the energy of the signal with the noise threshold which in reality is uncertain. Matched filter detection is accurate but has complexity on the higher side as it requires prior knowledge of the signal for filter implementation and its threshold also depends upon noise uncertainty. On the other hand cyclostationary detection exploits the cyclostationary features of the modulated signal for signal detection. The signals are generally modulated using sin waves that exhibits periodicity. Irrespective of the fact that the data is being random, this when modulated depicts as cyclostationary. These cyclostationary features are detected using spectral correlation task.

Eigenvalue based spectrum sensing techniques are the most accurate but are computationally complex. In short to summarize all spectrum sensing techniques Energy detection has the least complexity but lacks accuracy whereas Eigenvalue based spectrum sensing techniques are among the most accurate ones while they exhibits complexity. So to come up with a Spectrum sensing solution that would be feasible would surely be the one with minimal complexity and have accuracy. So we aim to reduce the complexity of Eigenvalue based spectrum sensing methods.

1.1 Problem Statement

Eigenvalue based algorithms are computationally complex as compared to energy detection method. Its complexity comes from two steps, the decomposition of the covariance matrix and the computation of eigenvalues. The decomposition of covariance matrix does not offer enough room for complexity analysis as it has already been studied to its maxima while the computation of eigenvalues is still an open field for research. In this work we propose fast iterative algorithms to handle eigenvalue problems for

eigenvalue based spectrum sensing detections. This reduces the complexity of the eigenvalue based spectrum sensing techniques to $\boldsymbol{O(L)}$ thus allowing the smooth detection of weak signals in cognitive radios.

1.2 Thesis Outline

This master's thesis consist of six chapters and begins with an Introduction of the spectrum sensing. Moreover objectives and motivations of this work have been presented like possible solution of the limited spectrum problem. The second chapter describes Spectrum Sensing techniques with their comparison. In the Chapter 3 the Eigenvalue based Spectrum Sensing methods are presented. Chapter 4 describes fast iterative Eigenvalue problem algorithms. Chapter 5 reports the computational complexity. Simulations and results are discussed in the sixth chapter. Conclusion is presented in the last section.

Chapter 2

Spectrum Sensing Techniques

2.1. Spectrum Sensing Techniques

Different spectrum sensing techniques have been proposed which have a compromise between accuracy and the complexity. Few of them require the prior signal information while others depends upon noise uncertainty [3,9]. These methods mainly include matched filter detection, Cyclostationary detection, Energy detection and the Eigenvalue based spectrum sensing which are discussed as follows;

2.1.1 Matched Filter Detection

It is a technique used for detecting the presence of the primary signal. The operation of the matched filter is based on the principle that it increases the signal to noise ratio of the input signal [5]. This is done by correlating the input signal with the unknown signal to detect the presence of the primary user. Matched filter based detection requires perfect knowledge of the primary users signaling features such as bandwidth, frequency, modulation etc. The prior signal information is used for the implementation of the coefficients of the FIR filter. The matched filter detecor is shown in figure 2.1.

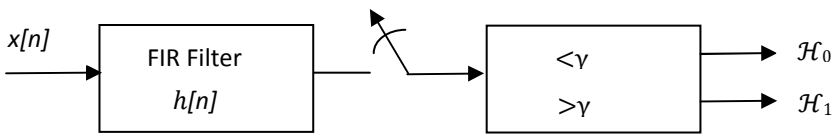

Figure 2.1 Match Filter Detector

The output of the FIR filter is compared with the threshold value γ which depends upon noise variance σ^2, probability of false alarm and the signal energy E. The signal is present when the output value is greater than the threshold γ which corresponds to hypothesis $\mathcal{H}_1$ and when this value is less than γ thus corresponds to hypothesis $\mathcal{H}_0$.

2.1.2 Cyclostationary Detection

The input signals are generally modulated using sine wave carriers. These modulated signals thus exhibit periodicities which are known as Cyclostationary features [5]. The Cyclostationary detector senses these features using spectral correlation. Cyclostationary detector is shown in figure 2.2.

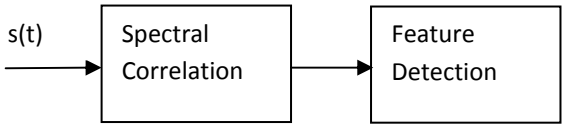

Figure 2.2 Cyclostationary Detector

2.1.3 Energy Detection

Energy detection unlike matched filter detection and the Cyclostationary methods does not require any prior knowledge of the signal. It compares the signal energy with the noise power to decide the signal presence. It is only optimal for independent and identically distributed (iid) signals and not the correlated signals.

$$E(N_S) = \frac{1}{N_S} \sum_{n=0}^{N_S-1} |x(n)| \tag{2.1}$$

2.1.4 Eigenvalue based Detection

Eigenvalue based spectrum sensing technique is optimal for both iid and correlated signals. Unlike other methods Eigenvalue based algorithms doesn't require any prior signal knowledge and are independent of noise uncertainty. Eigenvalue based methods are among the most accurate spectrum sensing techniques but involves complexity.

2.2. Comparison of Spectrum Sensing Techniques

Different spectrum sensing techniques have certain limitations and dependencies. They have a compromise between accuracy and the complexity. The comparison of various spectrum sensing techniques with respect to prior signal knowledge, dependency and computational complexity are summarized in Table 2.1.

Table 2.1 Spectrum Sensing Techniques Comparison

	Prior Signal Knowledge	Noise Dependence	Computational Complexity
Matched Filter Detection	Yes	Yes	Less
Cyclostationary Detection	Yes	Yes	High
Energy Detection	No	Yes	Less
Eigenvalue based Detection	No	No	High

Chapter 3

Eigenvalue based Spectrum Sensing Techniques

3.1 System Model

Signal detection model falls among the two hypothesis. $\mathcal{H}1$ represent the presence of the signal and $\mathcal{H}0$ represents its absence as shown in Figure 3.1.

$$\mathcal{H}0: \quad \text{No signal:} \quad x(n) = \eta(n) \tag{3.1}$$

$$\mathcal{H}1: \quad \text{Signal:} \quad x(n) = s(n) + \eta(n) \tag{3.2}$$

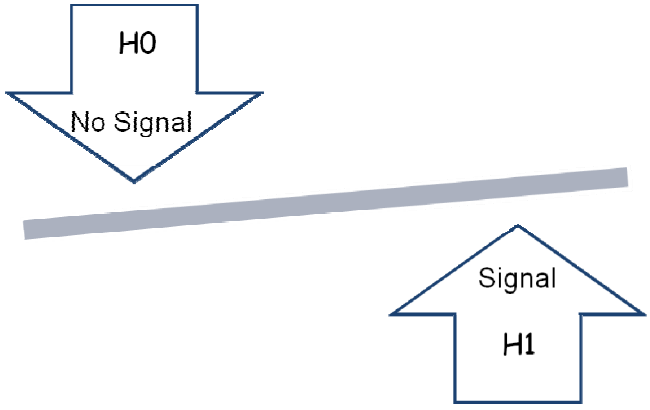

Figure 3.1 Signal Detection Hypothesis

Where $x(n)$ and $s(n)$ are the received signal and the primary signal respectively. Moreover $\eta(n)$ represents the noise with zero mean and variance σ_η^2. The two probabilities, probability of detection (P_d) and the probability of false alarm (P_{fa}) are of our interest. The probability of detection (P_d) is the probability that there is a signal and it is detected as a signal too. Whereas the probability of false alarm (P_{fa}) is the

probability that there is a signal and it is not detected as signal or the signal is not present and it is detected to be signal [6].

The presence of signal is represented by the probability of detection and corresponds to hypothesis $\mathcal{H}1$. The probability of false alarm represents the absence of signal and corresponds to hypothesis $\mathcal{H}0$. Let N_S be the total number of input samples, the received signal $x(n)$ is defined as

$$x(n) = [x_1(n), x_2(n), x_3(n), \dots, x_{Ns}(n)]^T \tag{3.3}$$

Let L be the received time samples (smoothing factor) then the estimated received signal is shown as

$$\hat{x}(n) = [x^T(n), x^T(n-1), \dots, x^T(n-L+1)]^T \tag{3.4}$$

For theoretical understanding an example for estimated received signal is summarized in Figure 3.2.

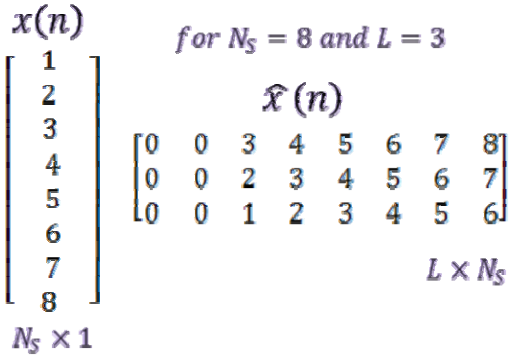

Figure 3.2 Received and Estimated Received Signal

The statistical covariance matrix of the received signal is defined as

$$R_x = E\left(\hat{x}(n)\hat{x}^T(n)\right) \tag{3.5}$$

Since the input samples are finite so the sample covariance matrix is as under:

$$R_x(N_S) = \frac{1}{N_S}\sum_{n=0}^{L-1} \hat{x}(n)\hat{x}^T(n) \tag{3.6}$$

The covariance matrix is Toeplitz in nature. Toeplitz matrix is a matrix whose all diagonal entries are same.

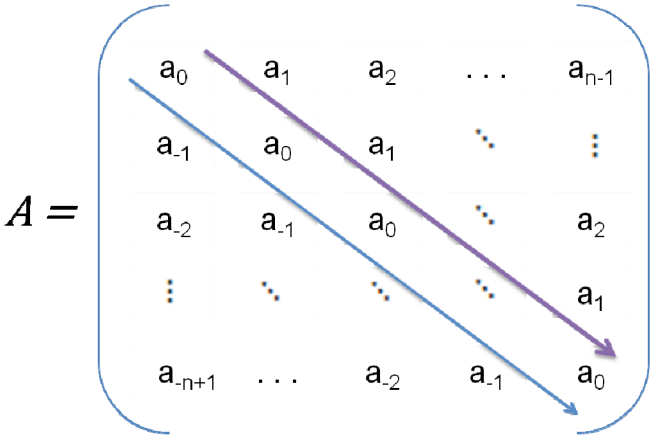

Figure 3.3 Toeplitz Matrix

3.2 General Algorithm

In general the Eigenvalue based Spectrum Sensing algorithms involve the steps shown in Figure 3.4.

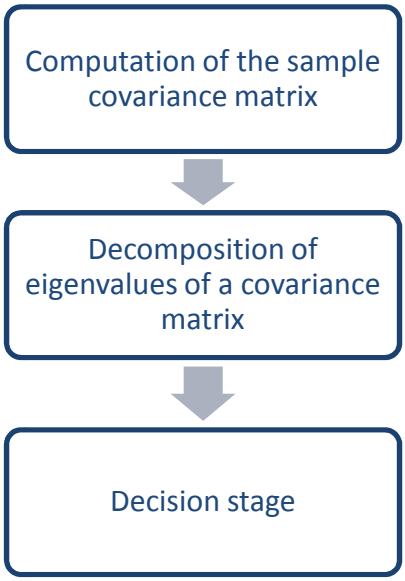

Figure 3.4 Steps for Eigenvalue based Spectrum Sensing Algorithms

3.3 Eigenvalue based Spectrum Sensing Algorithms

Different Eigenvalue based spectrum sensing algorithms have been proposed. These algorithms calculates the eigenvalues of the covariance matrix and uses them in various fashions for signal detection. These algorithms mainy includes maximum to minimum eigenvalue detection (MME), energy to minimum detection (EME), and maximum eigenvalue based detection (MED).

3.3.1 Maximum to Minimum Eigenvalue Detection

In maximum to minimum eigenvalue detection method the ratio of maximum to minimum eigenvalues are calculated and then compared with the threshold in the decision stage to determine signal presence [6].

Step 1 Compute the sample covariance matrix of the received input samples.

$$R_x(N_S) = \frac{1}{N_S}\sum_{n=0}^{L-1} \hat{x}\,(n)\hat{x}^T(n)$$

here N_S is the number of received samples and L is received input samples.

Step 2 Calculate the maximum eigenvalue λ_{max} of the covariance matrix R_X.

Step 3 Decision:

Signal exists, if $\lambda_{max}/\lambda_{min} > \gamma_1$ else signal does not exist. Here γ_1 is the threshold

3.3.2 Energy to Minimum Eigenvalue Detection

In energy to minimum eigenvalue detection method the ratio of energy to minimum eigenvalues are calculated and then compared with the threshold in the decision stage to determine signal presence [6].

Step 1 Compute the sample covariance matrix of the received input samples.

$$R_x(N_S) = \frac{1}{N_S}\sum_{n=0}^{L-1} \hat{x}\,(n)\hat{x}^T(n)$$

here N_S is the number of received samples and L is received input samples.

Step 2 Calculate the minimum eigenvalue $\lambda_{min}$of the covariance matrix R_X. Also compute the Energy $T(N_S)$ as defined by,

$$T(N_S) = \frac{1}{MN_S}\sum_{i=1}^{M}\sum_{n=0}^{N_s-1} |x_i(n)|^2$$

Where M and N_S represents the number of transmitted signals (primary users) and the total number of received samples respectively.

Step 3 Decision:

Signal exists, if $T(N_S)/\lambda_{min} > \gamma_2$ else signal does not exist. Here γ_2 is the threshold

3.3.3 Maximum Eigenvalue Detection

The sample auto correlations of the received signal is defined as,

$$\lambda(l) = \frac{1}{N_s}\sum_{m=0}^{N_s-1} x(m)x(m-l), \qquad l = 0,1,2,\dots,L-1 \tag{3.7}$$

Here N_s are the number of received input samples. The statistical covariance matrix R_X can be approximated by sample autocorrelation samples as [7],

$$R_x(N_s) = \begin{bmatrix} \lambda(0) & \lambda(1) & \cdots\lambda(L-1) \\ \lambda(1) & \lambda(0) & \cdots\lambda(L-2) \\ \vdots & \vdots & \vdots \quad \vdots \\ \lambda(L-1) & \lambda(L-2) & \cdots \quad \lambda(0) \end{bmatrix} \tag{3.8}$$

Note that the sample covariance matrix is symmetric and Toeplitz (matrix property).

Step 1 Compute the sample autocorrelations and form the covariance matrix R_x as defined above

Step 2 Calculate the maximum eigenvalue $\lambda_{max}$of the covariance matrix R_x.

Step 3 Decision:

Signal exists, if $\lambda_{max} > \gamma\ \sigma_\eta^2$ else signal does not exist. Here γ is the threshold and σ_η^2 is the noise variance

3.4 Threshold Calculation

The thresholds for the Eigenvalue based spectrum sensing methods have been calculated using different statistical distributions [6]. These thresholds includes Tracy Widom distribution for the largest Eigenvalues and the Gaussian distribution for the smallest Eigenvalues.

3.4.1 Tracy Widom Distribution

The distribution for the largest Eigenvalue has recently proposed by I. M. Johnstone and K. Johansson [6]. The Tracy Widom distribution plot is shown in Figure 3.5.

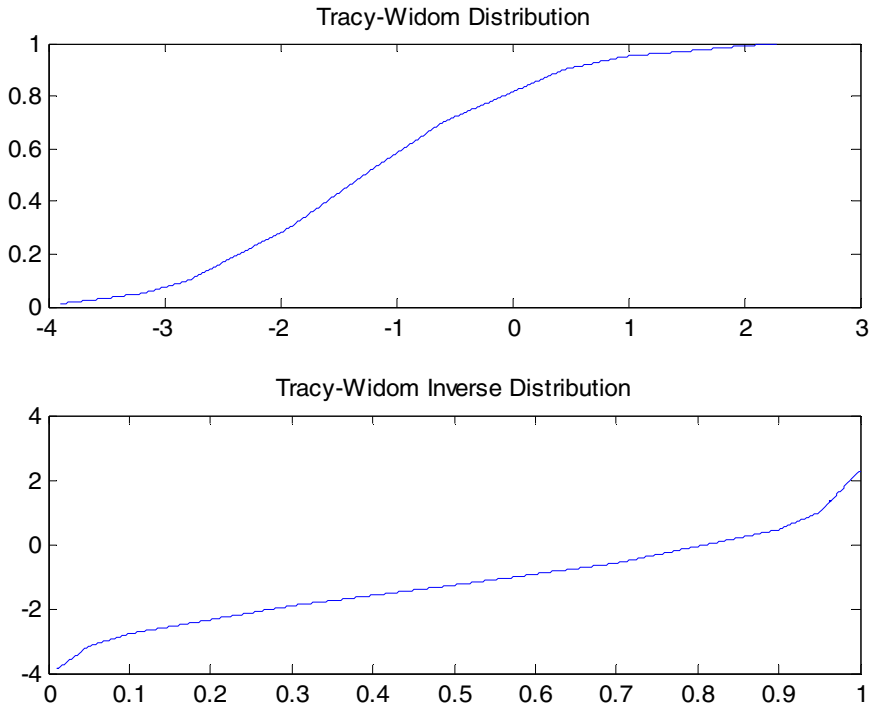

Figure 3.5 Tracy Widom Disribution of Order 1

The threshold γ_1 for the maximum to minimum Eigenvalue (MME) detection is given by [6],

$$\gamma_1 = \frac{\left(\sqrt{N_S}+\sqrt{ML}\right)^2}{\left(\sqrt{N_S}-\sqrt{ML}\right)^2}\left(1+\frac{\left(\sqrt{N_S}+\sqrt{ML}\right)^{-2/3}}{(N_S ML)^{1/6}}F^{-1}\left(1-P_{fa}\right)\right) \tag{3.9}$$

The threshold γ for the maximum Eigenvalue detection (MED) is given by [7],

$$\gamma = \frac{\left(\sqrt{N_S}+\sqrt{L}\right)^2}{N_S}\left(1+\frac{\left(\sqrt{N_S}+\sqrt{L}\right)^{-2/3}}{(N_S L)^{1/6}}F^{-1}\left(1-P_{fa}\right)\right) \tag{3.10}$$

The thresholds does not depend upon noise any prior signal knowledge which is the key for Eigenvalue based spectrum sensing technique. The threshold only depends upon number of samples N_S, received time samples L, number of received signal M and the probability of false alarm P_{fa}. Here F_1 shows the inverse Tracy Widom distribution.

3.4.2 Gaussian Distribution

The threshold γ_2 for the Energy with Minimum Eigenvalue algorithm (EME) is approximated by Gaussian distribution and is represented by [6],

$$\gamma_2 = \left(\sqrt{\frac{2}{MN_S}}Q^{-1}\left(P_{fa}\right)+1\right)\frac{N_S}{\left(\sqrt{N_S}-\sqrt{ML}\right)^2} \tag{3.11}$$

In the above equation Q shows the Q-function. Unlike classical spectrum sensing techniques, the eigenvalue based spectrum sensing methods does not require any prior knowledge of the signal and also are independent of noise unncertainity. These methods depends upon the number of samples N_S, received time samples L and the number of received signals M.

Chapter 4

Fast Iterative Eigenvalue Problem Algorithms

The majority of the eigenvalue based detection methods either want the maximum eigenvalue or the minimum eigenvalue [6, 7]. If a certain frequency band is of interest, one might like to get few eigenvalues that are of our interest. If the matrix size is really large, the computation of its complete spectrum is a tough ask. Even if not the case if we only interested in few extreme eigenvalues then to go for the complete spectrum is not an efficient way. Eigenvalue algorithms for large problems are meant to deal with these issues and they carry out this by computing the target subspace associated with the desired eigenvalues [13, 16].

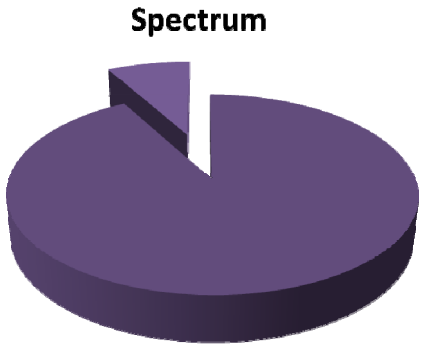

Figure 4.1 Target Subspace of Desired Eigenvalues in Spectrum

Eigenvalue solving problem basically fall in two categories, the direct methods and the iterative methods.

4.1 Direct Methods

Direct methods are used to compute all the eigenvalues when the matrix size is small. When the matrix size is large these methods are either impossible or too slow hence they

are practically not feasible. The direct methods expands the entire subspace to find the eigenvalues and involves matrix matrix multiplication that invokes computational complexity.

4.2 Iterative Methods

Iterative methods are used for calculating the extreme eigenvalues. These methods are approximated methods that are fast and only require $O(N^2)$ operations unlike direct methods which require $O(N^3)$ operations. The solutions have some errors which can easily be tolerated.

4.2.1 Krylov Subspace

Krylov methods are one important type of iterative methods. Let we have a large n x n matrix **A** and a vector **x**. we can find the Krylov sequence as [12],

$$x, Ax, A^2x, A^3x, \ldots \tag{4.1}$$

For a given matrix **A** and nonzero vector **x**, the Krylov subspace is defined by

$$\mathrm{K}_m(A, x) = span\,\{x, Ax, A^2x, A^3x, \ldots, A^{m-1}x\} \tag{4.2}$$

referred to as the m[th] Krylov subspace. It is associated with the pair (A, x) and denoted by $\mathrm{K}_m(A, x)$ or simply K_m. In order to build up the Krylov subspace, we need a starting vector x. In the beginning we don't have any idea about the invariant subspace. Considering all that we choose vector x at random to start and we will be able to get better approximation for x later after some iterations.

Krylov methods unlike direct methods employ matrix vector multiplication thus avoiding matrix-matrix multiplication that involves complexity. The Arnoldi and Lanczos

algorithms are methods to compute an orthonormal basis of the Krylov subspace. The matrix vector multiplication is shown in Figure 4.2.

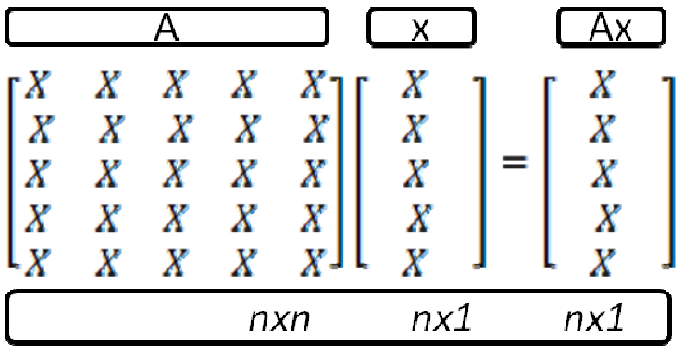

Figure 4.2 Krylov Sequence: A.x Matrix Vector Multiplication

4.3 Fast Eigenvalue Computation Algorithms

Fast Eigenvalue computation method includes Arnoldi algorithm, Lanczos algorithm and Jacobi-Davidson algorithm [13]. An unfortunate side of the Arnoldi and the Lanczos methods is that the number of iterations can be high to determine the eigenvalues. The number of iterations k increases until convergence is reached thus the cost of operation becomes $O(L^2)$. The number of iterations depends upon the properties of the matrix, particularly the starting initial vector and the distribution of its eigenvalues in the spectrum. Large number of iterations requires huge memory to store Arnoldi / Lanczos vectors and thus increases computational complexity.

Implicit restart deals with this increasing cost by limiting the search space [16]. The idea behind the implicitly restarted Arnoldi (IRA) and the implicitly restarted Lanczos (IRL) algorithms is that the number of iteration is stopped (if the convergence is not reached) after certain number of steps and the initial staring vector is replaced with a new improved starting vector and resumes the Arnoldi / Lanczos iteration. So the three fast eigenvalue computation methods are the Implicit Restart Arnoldi Algorithm, Implicit Restart Lanczos Algorithm and Jacobi-Davidson Algorithm.

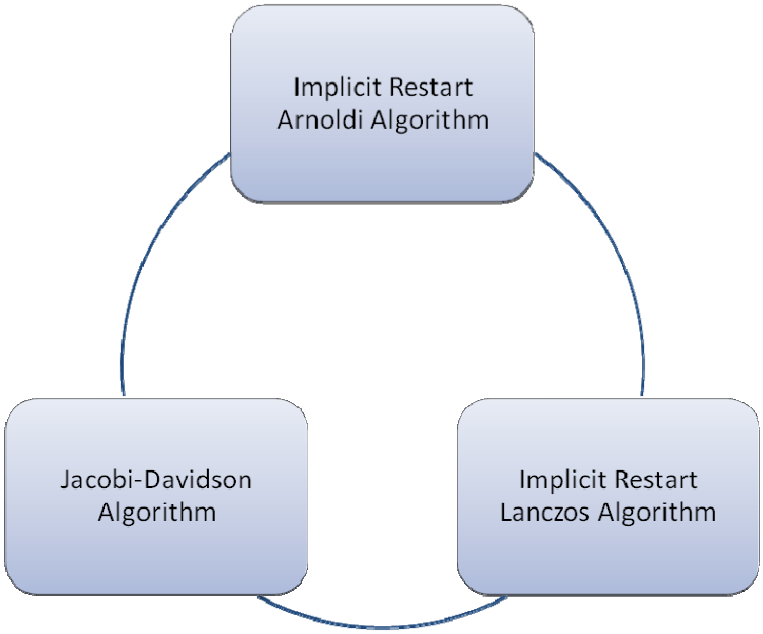

Figure 4.3 Fast Eigenvalue Computation Methods

When the eigenvalues are well separated in the spectrum then the implicit restarted Arnoldi and Implicit restarted Lanczos algorithms are an efficient choice. But when the eigenvalues are close enough to each other i.e. they are not separated from each other the Jacobi-Davidson algorithm is used. These methods are very efficient when the matrix size is large.

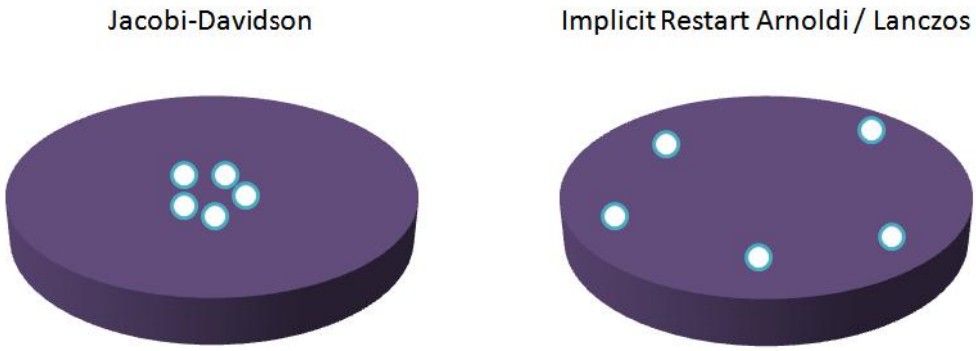

Figure 4.4 Orientation of Eigenvalues in Spectrum

4.3.1 Implicit Restart Arnoldi Algorithm

The implicit restart Arnoldi method is for non hermition matrices [14]. The algorithm is given Table 4.1.

Table 4.1 Implicit Restart Arnoldi Algorithm

Let the Arnoldi relation $AQ_m = Q_m H_m + r_m e_m^*$ be given

repeat

Determine *k* shifts $\mu_1, \dots, \mu_k$;

$v^* := e_m^*$;

for $i = 1, \dots, k$ **do**

$H_m - \mu_i I = V_i R_i$; /* QR Factorization */

$H_m := V_i^* H_m V_i$; $Q_m := Q_m V_i$;

$v^* := v^* V_i$;

end for

$r_P := q_{P+1}^+ \beta_P^+ + r_m v_{m,P}^+$;

$Q_P := Q_m(:, 1:p)$; $H_p := H_m(1:p, 1:p)$;

Starting with

$$AQ_p = Q_p H_p + r_p e_p^*$$

execute *k* additional steps of the Arnoldi algorithm until

$$AQ_m = Q_m H_m + r_m e_m^*$$

until convergence

In the Implicitlty restarted Arnoldi algorithm, A is the input matrix, Q is the unitary matrix and H represents the upper Hessenberg matrix. The QR factorization is done for k fixed number of iterations and then the starting vector is replaced with new one and performed the iterations until convergence is reached. The algorithm reduces the matrix into upper Hessenberg form. Upper Hessenberg matrix is defined as,

$$H(i,j) = 0 \quad for \; i > (j+1)\,; \tag{4.3}$$

4.3.2 Implicit Restart Lanczos Algorithm

The Lanczos algorithm is used for finding the extreme eigenvalues of symmetric matrices [15]. The algorithm for the Implicitly Restarted Lanczos algorithm is shown in Table 4.2.

Table 4.2 Implicit Restart Lanczos Algorithm

start with $v_1 = v/\|v\|$ with starting vector v

compute an *m-step* Lanczos factorization

$$AV_m = V_m T_m + r_m e_m^*$$

repeat until convergence ($T_k = D_k\ diagonal$)

compute $\sigma(T_m)$ and select p shifts $\mu_1, \mu_2, \dots, \mu_p$

initialize $Q = I_m$

for $j = 1, 2, \dots, p$,

QR-factorize $Q_j R_j = T_m - \mu_j I$

update $T_m = Q_j^* T_m Q_j,\ Q = QQ_j$

end for

$r_k = v_{k+1}\beta_k + r_m\sigma_k, with\ \beta_k = T_m(k+1, k)$ and $\sigma_k = Q(m, k)$

$V_k = V_m Q(:, 1:k);\ \ T_k = T_m(1:k, 1:k)$

beginning with the *k-step* Lanczos factorization

$$AV_k = V_k T_k + r_k e_k^*,$$

Apply p additional steps of the Lanczos process to obtain new m-step Lanczos factorization,

$$AV_m = V_m T_m + r_m e_m^*,$$

end repeat

Ideally for eigenvalue computations the initial starting vector v should be the one that is in the direction of the required subspace. In the absence of any prior knowledge of the subspace, the random starting vector is a reasonable choice. The shifts μ_j are selected with respect to user's desired sets of eigenvalues. Here k is the number of eigenvalues to

be computed and *r* represents the residual. The desired sets specification includes the k algebraically smallest eigenvalues, the k algebraically largest eigenvalues.

4.3.3 Jacobi-Davidson Algorithm

The Arnoldi and Lanczos methods are very effective to compute the eigenvalues when the eigenvalues are well separated from the spectrum [17]. When this is not the case then Jacobi-Davison is an attractive algorithm to compute the extreme eigenvalues.

Let $v_1, v_2, \dots, v_m$ be the set of orthogonal vectors, spanning the search space with $V_m = [v_1, v_2, \dots, v_m]$. Let u_j be the approximation to the eigenvector x. Jacobi proposed this correction by a vector t such that

$$A(u_j + t) = \lambda(u_j + t), \qquad u_j \perp t \tag{4.4}$$

for eigenvalue problem $Ax = \lambda x$. The Jacobi-Davidson correction equation is given as

$$(I - u_j u_j^*)(A - \vartheta_j I)(I - u_j u_j^*)t = -r_j = -(A - \vartheta_j I)u_j, \tag{4.5}$$

Where in the above equation ϑ_j represents the Ritz value. The residual is denoted by r_j and shown as $r_j = Au_j - \vartheta_j u_j$. The algorithm for the Jacobi-Davidson method is shown in Table 4.3.

Table 4.3 Jacobi-Davidson Algorithm

initialize search subspace and Rayleigh-Ritz procedure

$\mathrm{v}_0 = \mathrm{v}_0/\|\mathrm{v}_0\|\,; \quad \mathrm{v}_A = A\mathrm{v}_0;$

$V = \mathrm{v}_0; \quad j = 1;$

$G = \mathrm{v}_0^T \mathrm{v}_A;$

$\mathrm{u} = V;\ \theta = G;$

$r = \mathrm{v}_A - \theta \mathrm{u};$

$while(1)$

$-$ *expanding the search space* –

Solving Jacobi-Davidson correction equation for z

$$(I - \mathrm{u}\mathrm{u}^T)(A - \theta I)(I - \mathrm{u}\mathrm{u}^T)z = -\mathrm{r}; \quad \mathrm{z}^{\mathrm{T}}\mathrm{u} = 0;$$

$orhonormalize\ z\ against\ V\ using\ modified\ Gram - Schmidt\ method$

$V = [V, \mathrm{z}]; \quad \mathrm{v}_A = A\mathrm{z};$

$-$ *compute Ritz pairs* $-$

$G = [G,\ V^T \mathrm{v}_A;\ \mathrm{v}_A^T V, \mathrm{v}_A^T z]; \quad update\ G$

$[W, S] = \mathrm{eig}(G); \qquad solve\ projected\ eigenproblem$

$[\theta, imax] = \max\big(diag(S)\big);\ select\ largest\ Ritz\ value$

$\mathrm{u} = VW(:, imax); \qquad associated\ Ritz\ vector$

$r = A\mathrm{u} - \theta\mathrm{u}; \qquad compute\ residual$

$j = j + 1;$

$if\ \|\mathrm{r}\| < \varepsilon, \quad stopping\ criterian$

$break;$

$end\ if$

$end\ while$

$\lambda = \theta;$

4.4 Computational Basics

Iterative algorithms shown in last section exploits the advantage of matrix types and properties to get rid of unnecessary computations thus reducing the computational complexity. The implicitly restarted Arnoldi algorithm converts the covariance matrix first in upper Hessenberg form. Hessenberg matrix is a matrix which has all zero entries below the first sub diagonal. These zero entries hence reduces the computations for these entries. Likewise implicitly restarted Lanczos algorithm uses the tridaigonal form of matrix to reduce operations. Tridiagonal matrix has all zero entries below the first sub diagonal and above the first sub diagonal.

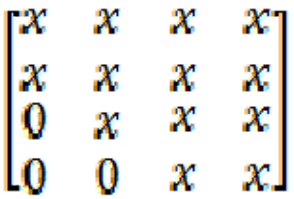

Figure 4.5 Upper Hessenberg Matrix

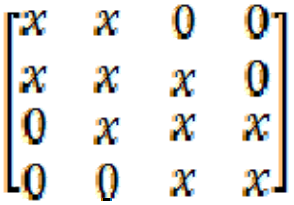

Figure 4.6 Tridiagonal Matrix

Hessenberg and tridiagonal matrices are the starting points for fast eigenvalue algorithms because the zero entries reduce the complexity of the problem.

Chapter 5

Computational Complexity

Eigenvalue based spectrum sensing methods perform well in noise as compared to energy detection, which is susceptible to noise. When the noise floor is high enough the signal is too weak to be identified in noise. When the SNR is as low as -20 dB, the detection of signal is challenging [11]. The matlab simulations are performed for signals under different noise floors as shown in Figure 5.1

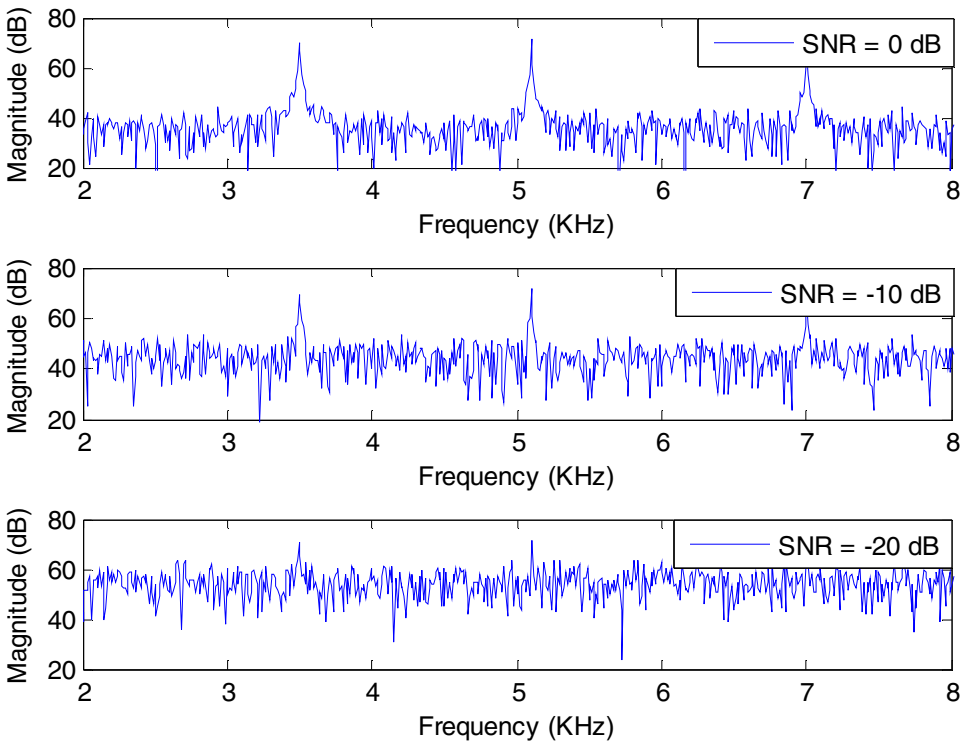

Figure 5.1 Signal with Different Noise Floors

The weak signals are smoothly detected by varying received time samples L. This follows the same concept assampling in which increasing the sampling can get better reconstruction of the signal. The weak signals can be detected using the increased values for received time samples L. As the complexity of eigenvalue based spectrum sensing techniques is directly proportional to the covariance matrix size, hence complexity increases with increased values for received time samples L. The fast iterative algorithms are incorporated in eigenvalue based spectrum sensing methods to get the eigenvalues with reduced complexity. Hence the proposed thesis work can be used to detect the presence of weak signals smoothly in cognitive radios.

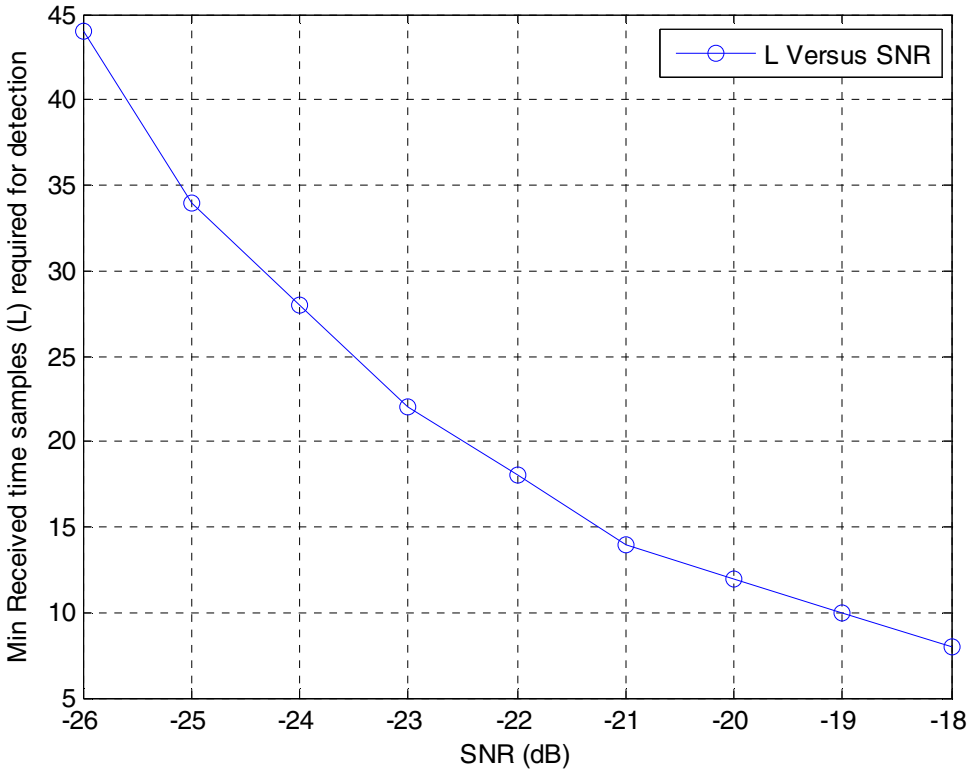

Figure 5.2 Minimum Received Time Samples L vs. SNR

The above figure shows the plot for the received time samples L versus the SNR. The decreasing trend of the graph shows that when the SNR is as low as -26 dB, the weak signal is detected using the high values for L. Similarly for the increased SNR signals can

be easily detected using lower values for the L. The simulations are performed on different test vectors using ensemble averages. The results are obtained using ensemble average over 100 iterations for different values for L under different noise floors to get optimum values for L for smooth detection of the weak signals. The computational complexity of the eigenvalue based Spectrum Sensing techniques is given as [6],

$$M^2LN_S + \mathrm{O}(M^3L^3) \tag{5.1}$$

The first part of equation 5.1 shows the complexity of covariance matrix formation whereas the latter part of the equation shows the complexity of the eigenvalue decomposition. The fast iterative eigenvalue problem algorithms reduce the complexity of the eigenvalue decomposition part to $O(L)$. The improved overall complexity of the eigenvaluebased spectrum sensing algorithm is

$$M^2LN_S + \mathrm{O}(M^3L) \tag{5.2}$$

Thus the complexity of the eigenvalue based spectrum sensing techniques reduces by using fast iterative eigenvalue problem algorithms.

Chapter 6

Simulations and Results

6.1 Wireless Microphone Signal

The wireless microphone test vectors are generated according to IEEE 802.22 Wireless RANS standard [19, 20]. The FM modulated wireless microphone signal is generated, sample rate 6 MHz at the receiver. For simulations, the number of samples N_S is taken as 100,000. The fast iterative algorithms are incorporated in Eigenvalue based spectrum sensing for smooth signal detection. Simulation results as depicted in Figure 6.1.

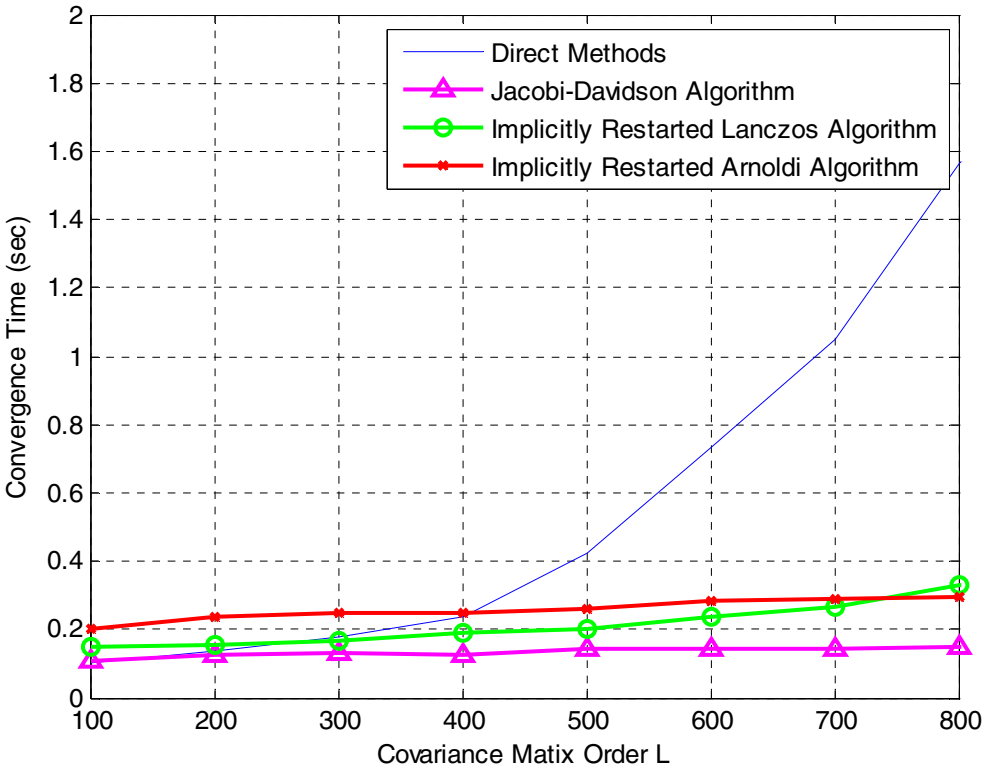

Fig 6.1 Convergence Time vs. Covariance Matrix Size

The above graph shows that the proposed algorithms become more efficient as the covariance matrix size increases unlike direct methods whose complexity is directly proportional to the covariance matrix size. Among the proposed three fast iterative algorithms, simulation illustrates that implicitly restarted Lanczos algorithm has the least convergence time for all covariance matrix sizes.

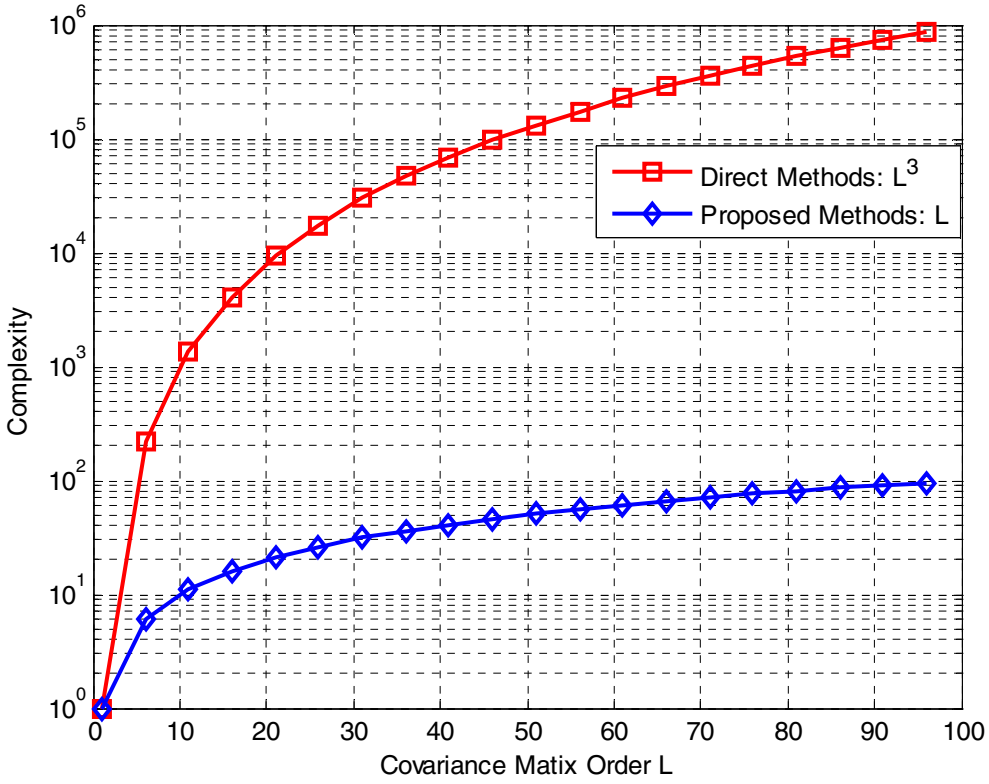

Fig 6.2 Log Normal Plot of Complexity vs Covariance Matrix

Figure 6.2 shows the complexity of the conventional Eigenvalue computation algorithm versus the proposed algorithms. The proposed methods have complexity of $O(L)$ as compared to the complexity $O(L^3)$. These proposed methods are used in Eigenvalue based spectrum sensing techniques to perform spectrum sensning with reduced complexity. The results for the proposed fast Eigenvalue algorithms are peformed on

both real and the simulated sample to verify the results.

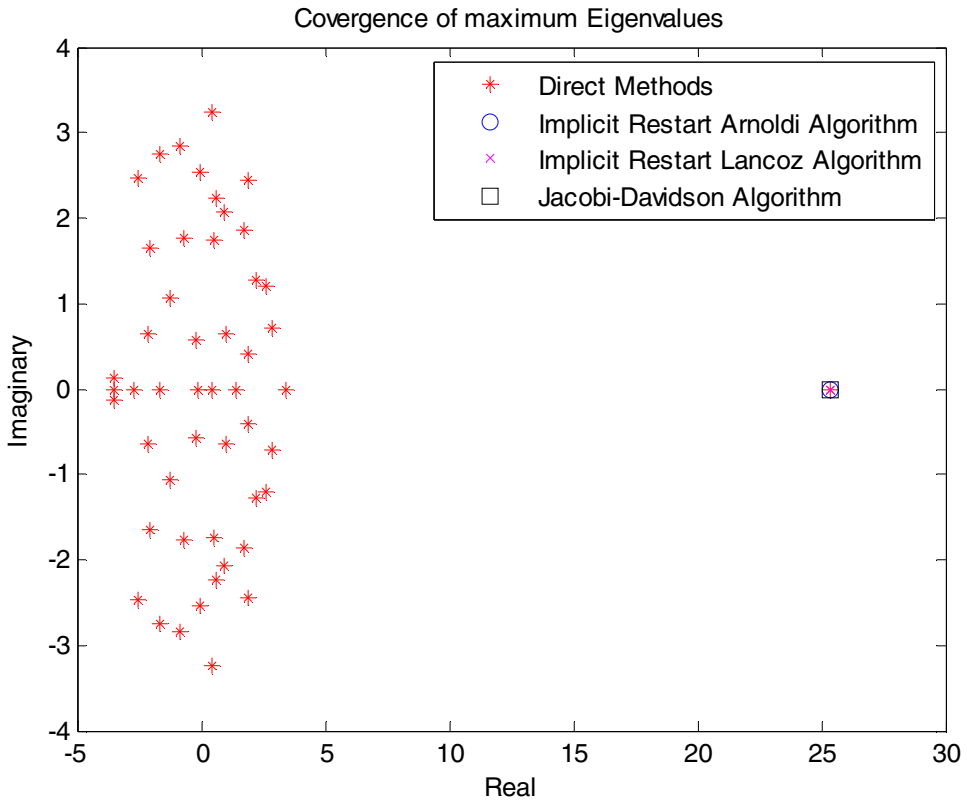

Fig 6.3 Convergence of Eigenvalues

Figure 6.3 shows the convergence of maximum eigenvalue computed using direct and different iterative methods. The plot shows that the maximum eigenvalue computed through various eigenvalue computation algorithms converge to same values.

6.2 Real-time GSM Signal

The downlink GSM band is from 925 MHz to 960 MHz. The 5 MHz GSM band is recorded using the National Instruments (NI) equipment that comprises of digitizer and down converter, centered at 929.5 MHz and has sampling frequency 12 MHz. Figure 6.4 shows the GSM band at baseband level centered at zero from 2.50 MHz to 2.50 MHz.

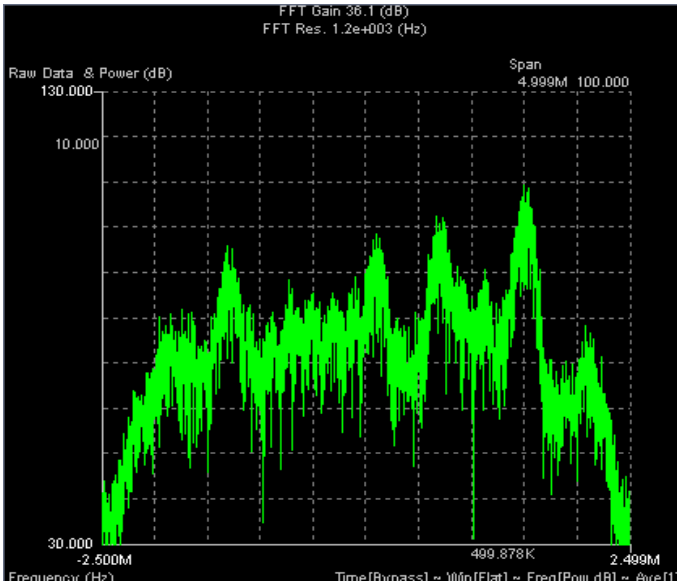

Fig 6.4 GSM 5MHz Band at Baseband Level.

The entire 5 MHz GSM band is filtered to 200 kHz fractions starting from 927 MHz. Low pass and band pass filters are used to filter 200 kHz fraction parts of the recorded band.

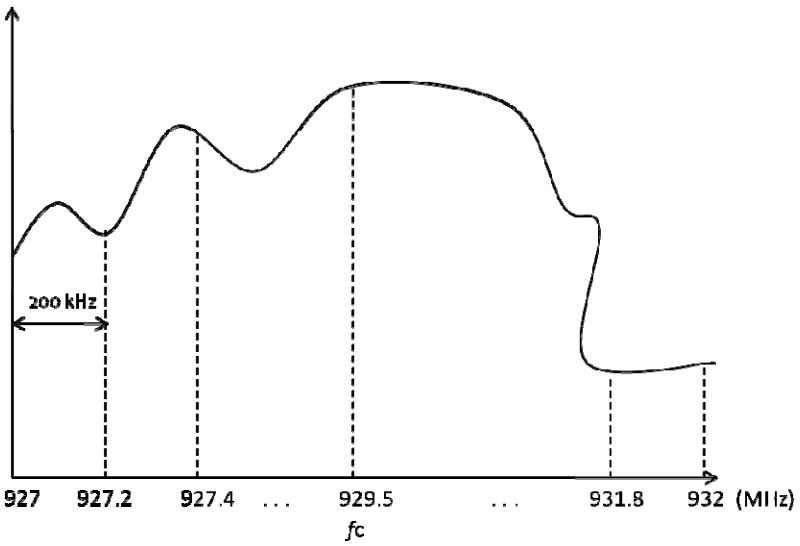

Fig 6.5 GSM Recorded Band Fragmention

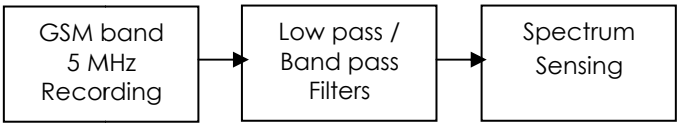

Fig 6.6 Flow Diagram for GSM Simulation

Each portion GSM recoded band is then passed through the proposed eigenvalue based spectrum sensing to detect signal presence as shown in Figure 6.5 and 6.6. The real time GSM signal is used to verify the results for the proposed work.

The probability of detection curves are shown in Figure 6.7. The probability of detection curves for different value of L are shows the improvement in probability of signal detection by increasing the value for smoothing factor L.

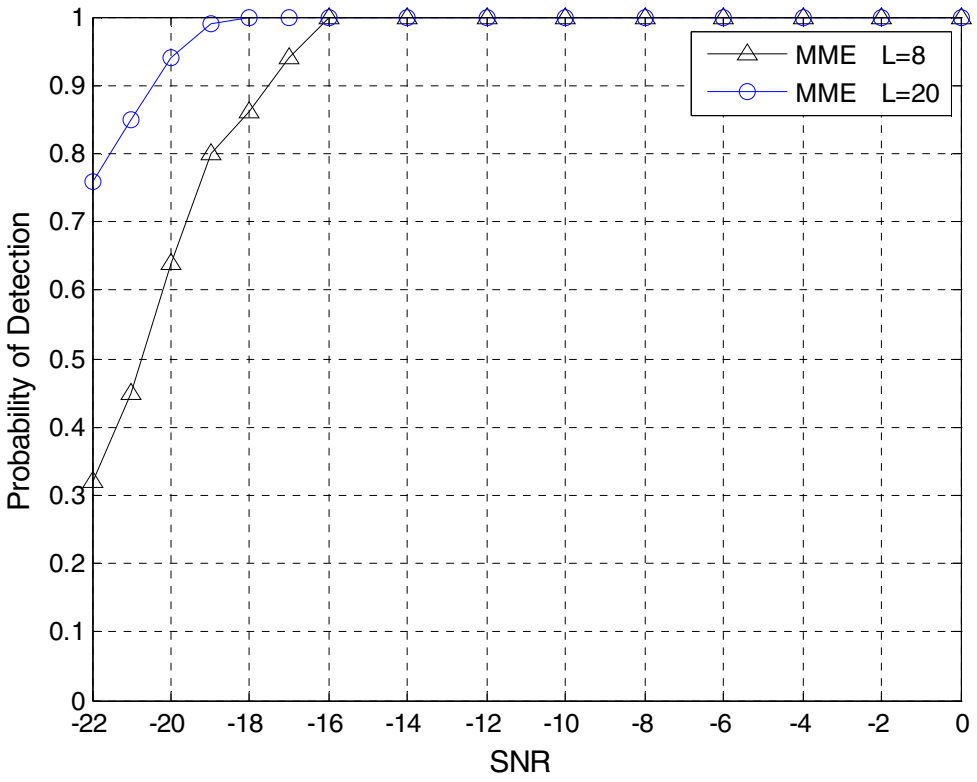

Fig 6.7 Probability of Detection

The probability of detection plot is shown for different value of L. As the increased value for L improves the results for signal detection, thus it improves the probability of detection for the spectrum sensing as depicted in Figure 6.7, that for L = 20 the probability of detection increased as compared the probability of detection results for value L = 8.

Conclusion and Future Work

Conclusion

Conventional spectrum sensing methods depends upon prior signal information and the noise uncertainity. Eigenvalues based spectrum sensing techniques unlike these methods does not require any prior signal knowledge and are independent of noise uncertainity. Eigenvalue based spectrum sensing methods are computationally complex but are very accurate. Their complexity mainly comes from two parts, the covariance matrix formation and the eigenvalues decomposition. Received time samples (smoothing factor) L are directly proportional to covariance matrix size that is directly related to computational complexity. As the value of L increases the complexity of eigenvalue based spectrum sensing techniques increases. The direct methods for the computation of the eigenvalues invlolves the complexity $\boldsymbol{O(L^3)}$**.** The proposed eigenvalue based spectrum sensing techniques reduces the complexity to $\boldsymbol{O(L)}$**.**

Eigenvalue based spectrum sensing methods behave well in noise unlike classical spectrum sensing methods particularly energy detection and matched filter detection due to their dependence upon noise. But when the noise level is high enough i.e. the signal is too weak to detect, the detection of such weak signal is difficult ask. The eigenvalue based spectrum sensing methods with reduced complexity can be used to smoothly detect the presence of signal by using the increased value of received time samples L. Thus as the noise level increases we can smoothly detect the presence of the primary signal by increasing the values for the received time samples L.

Future work

The complexity analysis of eigenvalue based spectrum sensing techniques opens up a new eve for research in that domain. The implementation of the proposed eigenvalue based spectrum sensing methods in hardware platform for the smooth detection of weak signals in cognitive radios. On the other, hand the extension of the work in a cooperative spectrum sensing domain can also be very benificail. In cooperative spectrum sensing the covariance matrix size is large due to multiple transmitter and receiver antennas. As the covariance matrix size increases the complexity of the algorithm also increases. The proposed methods will help to reduce the complexity of the eigenvalue based spectrum sensing techniques to be used in cooperative spectrum sensing.

References

[1] A. Sahai and D. Cabric, "Spectrum sensing: fundamental limits and practical challenges," in *Proc. IEEE International Symposium on New Frontiers in Dynamic Spectrum Access Networks (DySPAN)*, (Baltimore, MD), Nov. 2005.

[2] Simon Haykin, Davis J. Thomsan and Jeffrey H. Reed, "Spectrum Sensing for Cognitive Radio," *Proceedings of the IEEE, Special Issue on "Cognitive Radio - II, vol. 97, no. 5,* May 2009

[3] H.-S. Chen, W. Gao, and D. G. Daut, "Signature based spectrum sensing algorithms for IEEE 802.22WRAN," in *IEEE Intern. Conf. Comm. (ICC)*, Glascow, United Kingdom, June 2007.

[4] Yonghong Zeng, Ying-Chang Liang, Anh Tuan Hoang, and Rui Zhang, "A Review on SpectrumSensing for Cognitive Radio: Challenges and Solutions," *EURASIP Journal on Advances in Signal Processing,* Jan 2010

[5] Shahzad A. Malik, Madad Ali Shah, Shahid A. Khan, "Comparative Analysis of Primary Transmitter Detection Based Spectrum Sensing Techniques in Cognitive Radio Systems,'' *Australian Journal of Basic and Applied Sciences*, 4(9), 2010, pp: 4522- 531, INSInet Publication.

[6] Yonghong Zeng and Ying-Chang Liang, "Eigenvalue based Spectrum Sensing Algorithms for Cognitive Radio," *IEEE PIMRC*, Athens, Greece, Sep. 2007

[7] Yonghong Zeng, Choo Leng Koh and Ying-Chang Liang, "Maximum Eigenvalue Detection: Theory and Applications," *IEEE International Conference on Communications (ICC'08)*, Beijing, China, May 2008

[8] Yonghong Zeng, Ying-Chang Liang and Rui Zhang, "Blindly Combined Energy Detection for Spectrum Sensing in Cognitive Radio," *IEEE Signal Processing Letters*, vol. 15, Oct 2008

[9] H. Urkowitz, "Energy detection of unkown deterministic signals," *Proceedings of the IEEE on Engineering Profession;General Topics for Engineers (Math, Science & Engineering)*, vol. 55, April 1967

[10] Bassem Zayen and Aawatif Hayar, "Cooperative Spectrum Sensing Technique Based on Sub Space Analysis for Cognitive Radio Networks," *COGIS'09, COGnitive systems with Interactive Sensors Conference*, Nov 2009, Paris, France

[11] R. Tandra and A. Sahai, "Fundamental limits on detection in low SNR under noise uncertainty," in *WirelessCom 2005*, Maui, HI, June 2005.

[12] David S. Watkins, "The Matrix Eigenvalue Problem: GR and Krylov Subspace Methods," Society for Industrial and Applied Mathematics (SIAM), 2007

[13] Y. Saad, "*Numeric Methods for Large Eigenvalue Problems*," Manchester University Press, Manchester, 1994

[14] R. B. Lehoucq, D. C. Sorensen and C. Yang, "ARPACK Users Guide: Solution of Large-Scale Eigenvalue Problems by Implicitly Restarted Arnoldi Methods," SIAM, Philadelphia, PA, USA, 1998

[15] D. Calvetti, L. Reichel and D.C. Sorensen, "An Implicitly Restarted Lanczos Method for Large Symmetric Eigenvalue Problems," Electronic Transactions on Numerical Analysis, vol 2, March 1994

[16] Zhaojun Bai, James Demmel, Jack Dongarra, Axel Ruhe and Henk van der Vorst, *Templates for the Solution of Algebric Eigenvalue Problems: A practical Guide,* Society for Industrial and Applied Mathematics (SIAM), 2000

[17] G.L.G Sleijpen and H.A Van der Vorst, "A Jacobi-Davidson iteration method for linear eigenvalue problems," *SIAM J. Matrix Anal. Appl*., 1996

[18] Abhijeet Mate, Kuo-Hao Lee, and I-Tai Lu, "Spectrum Sensing Based on Time Covariance Matrix Using GNU Radio and USRP for Cognitive Radio", *IEEE Long Island Systems, Applications and Technology Conference (LISAT),* Farmingdale, NY, USA, May 2011

[19] 802.22 Working Group, *IEEE P802.22/D0.1 Draft Standard for Wireless Regional Area Networks*. http://grouper.ieee.org/groups/802/22/, May 2006.

[20] V. Tawil, *51 captured DTV signal*. http://grouper.ieee.org/groups/802/22/, May 2006