

# **SOLUTIONS MANUAL FOR**

**Power Electronics: Advanced Conversion Technologies 2-nd edition**

by

Fang Lin Luo

Hong Ye



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*Power Electronics: Advanced Conversion Technologies 2-nd edition*

By Dr. Fang Lin **LUO** and Dr. Hong **YE**

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## Solutions for Chapter 1: Introduction

### 1.1:

From Equation (1.1), the impedance  $Z$  is

$$\begin{aligned} Z &= R + j\omega L - j\frac{1}{\omega C} = 10 + j120\pi \times 0.01 - j\frac{1}{120\pi \times 10^3} \\ &= 10 + j3.77 - j2.65 = 10 + j1.12 \Omega \end{aligned}$$

From Equation (1.2), the phase angle  $\phi$  is

$$\phi = \tan^{-1} \frac{\omega L - \frac{1}{\omega C}}{R} = \tan^{-1} \frac{1.12}{10} = 6.39^\circ$$

### 1.2:

From Equation (1.3), the impedance  $Z$  is

$$\begin{aligned} Z &= R + j\omega L = 10 + j120\pi \times 0.01 \\ &= 10 + j3.77 \Omega \end{aligned}$$

From Equation (1.4), the phase angle  $\phi$  is

$$\phi = \tan^{-1} \frac{\omega L}{R} = \tan^{-1} \frac{3.77}{10} = 20.66^\circ$$

From Equation (1.5), the time constant  $\phi$  is

$$\tau = \frac{L}{R} = \frac{0.01}{10} = 1 \text{ ms}$$

**1.3:**

From Equation (1.6), the impedance  $Z$  is

$$\begin{aligned} Z &= R + j \frac{1}{\omega C} = 10 - j \frac{1}{120\pi \times 10^3} \\ &= 10 - j2.65 \text{ } \Omega \end{aligned}$$

From Equation (1.7), the phase angle  $\phi$  is

$$\phi = -\tan^{-1} \frac{1}{\omega CR} = -\tan^{-1} \frac{1}{120\pi \times 0.01} = 14.86^\circ$$

From Equation (1.8), the time constant  $\phi$  is

$$\tau = RC = 10 \times 0.001 = 10 \text{ ms}$$

**1.4:**

The AC supply voltage is

$$v(t) = 240\sqrt{2} \sin 120\pi t$$

From the solution of Question 1.1, we have the impedance of the load. Therefore, by Ohm's law the circuit current is

$$i(t) = \frac{v(t)}{Z} = \frac{240\sqrt{2} \sin 120\pi t}{10 + j1.12} = 33.73 \sin(120\pi t - 6.39^\circ) \text{ A}$$

From equation (1.12), the apparent power S is

$$S = VI^* = 240 \times \frac{33.73}{\sqrt{2}} \angle 6.39^\circ = 5724.21 \angle 6.39^\circ \text{ VA}$$

From equations (1.15) and (1.16) the real power P and reactive power Q are

$$P = |S| \cos \phi = 5724.21 \cos 6.39 = 5695.4 \text{ W}$$

$$Q = |S| \sin \phi = 5724.21 \sin 6.39 = 573.6 \text{ VAR}$$

From Equation (1.11) the power factor PF is

$$PF = \cos \phi = \cos 6.39 = 0.994 \text{ lagging}$$

**1.5:**



Since the function is a central symmetrical function it is presented in Fourier transform

$$f(t) = E \sum_{n=1}^{\infty} b_n \sin n\omega t$$

where the Fourier coefficients are

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(n\omega t) d(\omega t) \quad n=1, 3, 5, 7, \dots \infty$$

Substituting the function in f(t)

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(n\omega t) d(\omega t) = \frac{2}{n\pi} \left[ \int_{\frac{7n\pi}{8}}^{\frac{7n\pi}{8}} \sin \theta d\theta + \int_{\frac{3n\pi}{8}}^{\frac{5n\pi}{8}} \sin \theta d\theta \right] \\ &= \frac{2}{n\pi} \left[ \left( \cos \frac{n\pi}{8} - \cos \frac{7n\pi}{8} \right) + \left( \cos \frac{3n\pi}{8} - \cos \frac{5n\pi}{8} \right) \right] = \frac{4}{n\pi} \left( \cos \frac{n\pi}{8} + \cos \frac{3n\pi}{8} \right) \end{aligned}$$

$$\text{or } b_n = \frac{4}{n\pi} \left( \cos \frac{n\pi}{8} + \cos \frac{3n\pi}{8} \right) \quad n=1, 3, 5, 7, \dots \infty$$

The amplitude of the fundamental harmonic is

$$b_1 = V_1 / E = \frac{4}{\pi} \left( \cos \frac{\pi}{8} + \cos \frac{3\pi}{8} \right) = \frac{4}{\pi} (0.9239 + 0.3827) = 1.664$$

From  $b_n$  we can get the coefficients of the higher-order harmonics

$$b_3 = \frac{4}{3\pi} \left( \cos \frac{3\pi}{8} + \cos \frac{9\pi}{8} \right) = \frac{4}{3\pi} (0.3827 - 0.9239) = -0.2297$$

$$b_5 = \frac{4}{5\pi} \left( \cos \frac{5\pi}{8} + \cos \frac{15\pi}{8} \right) = \frac{4}{5\pi} (-0.3827 + 0.9239) = 0.1378$$

$$b_7 = \frac{4}{7\pi} \left( \cos \frac{7\pi}{8} + \cos \frac{21\pi}{8} \right) = \frac{4}{7\pi} (-0.9239 - 0.3827) = -0.2377$$

The amplitudes of the harmonics are

$$V_1 = 1.664 \text{ E} \quad V_3 = 0.2297 \text{ E} \quad V_5 = 0.1378 \text{ E} \quad V_7 = 0.2377 \text{ E}$$

The HF's are

$$\text{HF}_3 = 0.138 \quad \text{HF}_5 = 0.083 \quad \text{HF}_7 = 0.143$$

The THD is

$$\text{THD} = \frac{\sqrt{\sum_{n=2}^{\infty} V_n^2}}{V_1} = \sqrt{\sum_{n=2}^{\infty} \text{HF}_n^2} \approx \sqrt{0.138^2 + 0.083^2 + 0.143^2} = \sqrt{0.046382} = 0.2153648$$

The WTHD is

$$\text{WTHD} = \frac{\sqrt{\sum_{n=2}^{\infty} \frac{V_n^2}{n}}}{V_1} = \sqrt{\sum_{n=2}^{\infty} \frac{\text{HF}_n^2}{n}} \approx \sqrt{\frac{0.138^2}{3} + \frac{0.083^2}{5} + \frac{0.143^2}{7}} = \sqrt{0.010647} = 0.1031847$$

## Solutions for Chapter 2: Uncontrolled AC/DC Converters

### 2.1:

From Equation (2.20)  $\phi = \tan^{-1} (\omega L/R) = 78.75^\circ$

Check Figure 2.4  $\beta = 272^\circ$

### 2.2:

From Equation (2.20)  $\phi = \tan^{-1} (\omega L/R) = 72.34^\circ$

Let  $\beta_1 = \pi + \phi = 252.34^\circ$

Step	$\beta$	$x = \sin (\beta - \phi)$	$y = e^{-\frac{R\beta}{\omega L}} \sin \phi$	Let $x = y$
1	$252.34^\circ$	0	0.234532	$x = 0.234532$
2 ↑	$265.907^\circ$	+0.234532	0.217508	$x = 0.217508$
3 ↓	$264.905^\circ$	+0.217508	0.218722	$x = 0.218722$
4 ↑	$264.979^\circ$	+0.218722	0.218635	$x = 0.218635$
5 ↓	$264.977^\circ$	+0.218635	0.218641	$x = 0.218641$
6 ↓	$264.972^\circ$	+0.218641	0.218641	$x = 0.218641$
7	$264.972^\circ$			

So, the extinction angle  $\beta = 264.972^\circ$  with high accuracy.

### 2.3:

For the parameters given

$$Z\sqrt{R^2 + \omega^2 L^2} = 106.9\Omega$$

$$\phi = \tan^{-1}(\omega L/R) = 0.361\text{rad} = 20.7^\circ$$

$$\omega L/R = 0.377$$

(a) From equation (2.25) for current

$$i = 0.936 \sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}$$

$\beta$  can be found numerically by equating  $i = 0$ .

$\beta$  is found to be 3.5 rad.

(b) Average output current

$$I_d = \int_0^{3.5} \frac{1}{2\pi} (0.936 \sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}) d(\omega t)$$

or

$$I_d = \frac{\sqrt{2V}}{2\pi R}(1 - \cos \beta) = 0.308A$$

(c) Average output voltage is

$$V_d = \frac{\sqrt{2V}}{2\pi}(1 - \cos \beta) = 30.8V$$

**2.4:**

a) From (2.25), the angle  $\alpha$  at which D starts to conduct is

$$\alpha = \sin^{-1} m = \sin^{-1} \frac{V_c}{\sqrt{2V}} = \sin^{-1} \frac{100}{110\sqrt{2}} = 40^\circ$$

b) From equation (2.42),  $\gamma$  is

$$\gamma = \pi - 2\alpha = 180 - 80 = 100 \text{ deg}$$

c) From equation (2.43), the average rectified current is

$$I_0 = \frac{1}{2\pi} \int_{40^\circ}^{140^\circ} 110\sqrt{2} \left( \sin 120\pi t - \frac{100}{110\sqrt{2}} \right) d(\omega t) = 10.2A$$

d) The rms value of the rectified current is

$$I_R = \left[ \frac{1}{2\pi} \int_{40^\circ}^{140^\circ} [110\sqrt{2}]^2 \left( \sin 120\pi t - \frac{100}{100\sqrt{2}} \right)^2 d(\omega t) \right]^{1/2} = 21.2 A$$

e) The power delivered by the ac source is

$$P = RI_R^2 + V_c I_0 = 1 \times 21.2^2 + 100 \times 10.2 = 1469 W$$

f) The power factor is

$$PF = \frac{\text{Power delivered}}{VI_R} = \frac{1469}{110 \times 21.2} = 0.63.$$

## 2.5:

Calculation of the angle  $\theta$ . At  $\omega t = \theta$ , the slopes of the voltage functions are equal to each other as shown in (2.59),

$$\sqrt{2V} \cos \theta = \frac{\sqrt{2V} \sin \theta}{-\omega RC} e^{-(\theta-\theta)/\omega RC}$$

$$\therefore \frac{1}{\tan \theta} = \frac{-1}{\omega RC}$$

$$\text{Thus } \theta = \pi - \tan^{-1}(\omega RC)$$

Therefore,  $\theta = \pi - \tan^{-1}(100\pi * 100 * 0.0001) = 180 - 72.34 = 107.66^\circ$

Calculation of the angle  $\alpha$  using **Iterative method 1**.

Since  $RC = 0.01$  sec, and  $\omega RC = \pi$ , we obtain the equation below. At  $\omega t = \alpha$ , the input voltage is equal to the output voltage,

$$\sin \alpha = (\sin \theta) e^{-(2\pi + \alpha - \theta)/\omega RC}$$

or

$$\sin \alpha = (\sin \theta) e^{-(2\pi + \alpha - \theta)/\omega RC} = 0.953 e^{-(252.34 + \alpha)/\pi}$$

Using the iterative method 1, define:

$$x = \sin \alpha$$

$$y = 0.953 e^{-(252.34 + \alpha)/\pi}$$

Make a table with initial  $\alpha = 30^\circ$ :

$\alpha$ ( $^\circ$ )	x	y	x : y
30	0.5	0.1777	>
20	0.34	0.21	>
12	0.2079	0.2194	<
13	0.225	0.21822	>
12.6	0.21814	0.2187	<

12.7	0.21985	0.21858	>
12.63	0.21865	0.21867	≈

We can choose  $\alpha = 12.63^\circ$ .

## 2.6:

Calculation of the angle  $\theta$ . At  $\omega t = \theta$ , the slopes of the voltage functions are equal to each other.

$$\sqrt{2V} \cos \theta = \frac{\sqrt{2V} \sin \theta}{-\omega RC} e^{-(\theta-\theta)/\omega RC}$$

$$\therefore \frac{1}{\tan \theta} = \frac{-1}{\omega RC}$$

$$\text{Thus } \theta = \pi - \tan^{-1}(\omega RC)$$

$$\theta = \pi - \tan^{-1} / (100\pi * 100 * 0.0001) = 180 - 72.34 = 107.66^\circ$$

With comparison with the discharging angle  $\theta$  in **Problem 2.5**, it can be seen that both values are same since the R, C and  $\omega$  have the same value.

Calculation of the angle  $\alpha$  using Iterative method 1.

$$RC = 0.01 \text{ sec, and } \omega RC = \pi.$$

We obtain the equation below. At  $\omega t = \alpha$ , the input voltage is equal to the output voltage,



$$\sin \alpha = (\sin \theta)e^{-(\pi + a - \theta)/\omega RC}$$

$$\sin \alpha = (\sin \theta)e^{-(\pi + a - \theta)/\omega RC} = 0.953e^{-(72.34 + \alpha)/\pi}$$

Using the iterative method 1, define:

$$x = \sin \alpha$$

$$y = 0.953e^{-(72.34 + \alpha)/\pi}$$

Make a table with initial  $\alpha = 45^\circ$ :

$\alpha$ ( $^\circ$ )	x	y	$ x  >, =, < y$ ?
45	0.7071	0.4965	>
35	0.5736	0.5249	>
30	0.5	0.5397	<
33	0.5446	0.5308	>
32.2	0.5328	0.5332	<
32.3	0.5344	0.5329	>
32.22	0.53317	0.53311	$\approx$

We can choose  $\alpha = 32.22^\circ$ .

The average output voltage  $V_d$  is

$$\begin{aligned}
V_d &= \frac{\sqrt{2}V}{\pi} \left[ \int_{\alpha}^{\theta} \sin(\omega t) d\omega t + \int_{\theta}^{\pi+\alpha} 0.953e^{-(\omega t - \theta)/\pi} d\omega t \right] \\
&= \frac{\sqrt{2}V}{\pi} \left[ (\cos \alpha - \cos \theta) + 0.953\pi(1 - e^{-(\pi+\alpha-\theta)/\pi}) \right] \\
&= \frac{240\sqrt{2}}{\pi} [(0.846 + 0.3034) + 0.953\pi(1 - 0.4597)] \\
V_d &= 108.038[(1.1494) + 0.953\pi(0.5403)] = 298.94 \text{ V}
\end{aligned}$$

The average capacitor current is zero  $I_C = 0$

The average resistor and diode current is

$$I_R = I_d = \frac{V_d}{R} = \frac{298.49}{100} = 2.9894 \text{ A}$$

## Solutions for Chapter 3: Controlled AC/DC Converters

### 3.1:

From the parameters given,

$$V_m = 120\sqrt{2} = 169.7 \text{ V}$$

$$Z = \sqrt{R^2 + \omega^2 L^2} = 12.5 \Omega$$

$$\phi = \tan^{-1}(\omega L / R) = 0.646 \text{ rad}, \quad \omega L / R = 0.754, \quad \alpha = \pi / 3$$

As  $\alpha > \phi$ , the load current is discontinuous.

(a) Substituting the data into equation (3.17)

$$i = 13.6 \sin(\omega t - 0.646) - 21.2e^{-\omega t/0.754} \quad \text{for } \alpha \leq \omega t \leq \beta.$$

Solving  $i(\beta) = 0$  numerically for  $\beta$ . We obtain  $\beta = 3.78 \text{ rad} = 216^\circ$ .

(b) The average load current is,

$$I_0 = \frac{1}{\pi} \int_{\alpha}^{\beta} i d(\omega t) = 7.05 \text{ A}$$

(c) The average output voltage is given by,

$$v_0 = \frac{\sqrt{2}V}{\pi}(\cos \alpha - \cos \beta) = \frac{\sqrt{2} \times 120}{\pi}(\cos 60^\circ - \cos 216^\circ) = 70.71V$$

### 3.2:

(a) The firing angle  $\alpha = 15^\circ$ , the output voltage and current are continuous. Referring to the formulae (3.24) (3.25) and (3.26), the output voltage and current are

$$V_0 = 1.17V_{in} \cos \alpha = 1.17 \times 240 \times \cos 15^\circ = 271.2 V$$

$$I_0 = \frac{V_0}{R} = \frac{271.2}{100} = 2.712 A$$

(b) The firing angle  $\alpha = 75^\circ$ , which is greater than  $\pi/6 = 30^\circ$ . The output voltage and current are discontinuous. Referring to the formulae (3.27) and (3.28), we have the output voltage and current to be

$$\begin{aligned} V_0 &= 0.675V \left( \frac{\sqrt{3}}{2} \cos \alpha - \frac{\sin \alpha}{2} + 1 \right) \\ &= 0.675 \times 240 (0.224 - 0.483 + 1) = 120V \end{aligned}$$

$$I_0 = \frac{V_0}{R} = \frac{120}{100} = 1.2 A$$

### 3.3:

(a) The firing angle  $\alpha = 20^\circ$ , the output voltage and current are continuous. Referring to the

formula (3.36), we have the output voltage and current to be

$$V_o = 2.34V \cos \alpha = 2.34 \times 240 \cos 20^\circ = 527.73 \text{ V}$$

$$I_o = \frac{V_o}{R} = \frac{527.73}{100} = 5.28 \text{ A}$$

(b) The firing angle  $\alpha = 100^\circ$ , the output voltage and current are continuous and negative values. Referring to the formula (3.36), we have the output voltage and current to be

$$V_o = 2.34V \cos \alpha = 2.34 \times 240 \cos 100^\circ = -97.5 \text{ V}$$

$$I_o = \frac{V_o}{R} = \frac{-97.5}{100} = -0.975 \text{ A}$$

## Solutions for Chapter 4: Power Factor Correction

### Implementing in AC/DC Converters

#### 4.1:

Since supply frequency is 60 Hz and switching frequency is 2.4 kHz, there are 20 switching periods in a half supply period (8.33 ms). The voltage transfer gain of the P/O self-lift Luo-Converter is

$$V_o = \frac{1}{1-k} V_{in}$$
$$k = \frac{V_o - V_{in}}{V_o} = \frac{400 - 200\sqrt{2} \sin \omega t}{400} = 1 - \frac{\sin \omega t}{\sqrt{2}}$$

Duty-cycle k is listed in following Table 4.7.

Table 4.7. Duty ratio k in the 20 chopping-periods in half-cycle (8.33 ms)

$\omega t$ (deg)	Input current = $200\sqrt{2} \sin(\omega t)$ (V)	k
9	44.2	0.889
18	87.4	0.781
27	128.4	0.679
36	166.3	0.584
45	200	0.5
54	288.8	0.428

63	252	0.37
72	269	0.328
81	279.4	0.302
90	282.8	0.293
99	279.4	0.302
108	269	0.328
117	252	0.37
126	288.8	0.428
135	200	0.5
144	166.3	0.584
153	128.4	0.679
162	87.4	0.781
171	44.2	0.889
180	0	$\infty$

#### 4.2:

Since supply frequency is 60 Hz and switching frequency is 3.6 kHz, there are 30 switching periods in a half supply period (8.33 ms). The voltage transfer gain of the P/O super-lift Luo-Converter is

$$V_o = \frac{2-k}{1-k} V_{in}$$

$$k = \frac{V_o - 2V_{in}}{V_o - V_{in}} = \frac{600 - 400\sqrt{2} \sin \omega t}{600 - 200\sqrt{2} \sin \omega t}$$

Duty-cycle k is listed in following Table 4.8.

Table 4.8. Duty ratio k in the 20 chopping-periods in half-cycle (10 ms)

$\omega t$ (deg)	Input current = $200\sqrt{2} \sin(\omega t)$ (V)	k
6	29.6	0.948
12	58.8	0.891
18	87.4	0.829
24	115	0.763
30	141.4	0.692
36	166.3	0.617
42	189.3	0.539
48	210	0.462
54	228.8	0.383
60	244.9	0.31
66	258.4	0.244
72	269	0.187
78	276.7	0.144
84	281.3	0.117
90	282.8	0.108
96	281.3	0.117
102	276.7	0.144
108	269	0.187
114	258.4	0.244



120	244.9	0.31
126	228.8	0.383
132	210	0.462
138	189.3	0.539
144	166.3	0.617
150	141.4	0.692
156	115	0.763
162	87.4	0.829
168	58.8	0.891
174	29.6	0.948
180	0	$\infty$

## Solutions for Chapter 5: Ordinary DC/DC Converters

### 5.1.

1) From (5.22), the output voltage is  $V_2 = \frac{1}{1-k} V_1 = \frac{1}{0.4} 20 = 50 \text{ V}$

2) From (5.29), the output voltage ripple is

$$\Delta v_2 = \Delta v_c = \frac{kV_2}{fRC} = \frac{0.6 \times 50}{50k \times 20 \times 20\mu} = 1.5 \text{ V}$$

3) From (5.28), the inductor  $L = 10 \text{ mH} >$

$$L_{\min} = \frac{k(1-k)^2}{2f} R = \frac{0.6(0.4)^2}{2 \times 50k} 20 = 0.0192 \text{ mH}$$

This boost converter works in CCM.

### 5.2.

In steady state, the output current is  $I_o$ . The current of the capacitor C during switching-on is

$$I_{C-on} = I_o$$

The current of the capacitor C during switching-off is

$$I_{C-off} = \frac{k}{1-k} I_O$$

The input current  $I_{in}$  and inductor current  $I_L$  during switching-off is

$$I_{in} = I_L = I_{L-off} = I_{C-off} + I_O = \frac{1}{1-k} I_O$$

The power relation is  $P_{in} = V_{in} I_{in} = P_O + P_{loss} = V_O I_O + I_L^2 r_L$

i.e. 
$$V_{in} I_{in} = V_O I_O + I_L^2 r_L = V_O I_O + \left(\frac{I_O}{1-k}\right)^2 r_L = V_O I_O \left[1 + \left(\frac{1}{1-k}\right)^2 \frac{r_L}{R}\right]$$

Hence 
$$M = \frac{V_O}{V_{in}} = \frac{I_{in}}{I_O \left[1 + \left(\frac{1}{1-k}\right)^2 \frac{r_L}{R}\right]} = \frac{k}{(1-k) \left[1 + \left(\frac{1}{1-k}\right)^2 \frac{r_L}{R}\right]} = \frac{k}{(1-k) + \frac{r_L}{(1-k)R}}$$

Substituting the data into the above Equation, the voltage transfer gain  $M$  is,

$$M = \frac{V_O}{V_{in}} = \frac{k}{(1-k) + \frac{r_L}{(1-k)R}} = \frac{0.8}{(1-0.8) + \frac{0.5}{(1-0.8)20}} = \frac{0.8}{0.2 + \frac{0.5}{4}} = \frac{0.8}{0.325} = 2.4615385 \approx 2.46$$

i.e. 
$$I_O = \frac{V_O}{R} = \frac{MV_{in}}{R} = \frac{2.46 \times 30}{20} \approx 3.69 \text{ A} \quad \text{and} \quad P_O = I_O^2 R = 3.69^2 \times 20 = 272.322 \text{ W}$$

then 
$$I_L = \frac{I_O}{1-k} = \frac{3.69}{0.2} = 18.45 \text{ A} \quad \text{and} \quad I_{in} = \frac{k}{1-k} I_O = \frac{0.8}{0.2} I_O = 4 \times 3.69 = 14.76 \text{ A}$$

Finally 
$$P_{in} = V_{in} I_{in} = 30 \times 14.76 = 442.8 \text{ W} \quad \text{and} \quad \eta = \frac{P_O}{P_{in}} = \frac{272.322}{442.8} = 61.5 \%$$

### 5.3.

1) From (5.45), the output voltage is 
$$V_2 = \frac{1}{1-k} V_1 = \frac{0.6}{0.4} 20 = 50 \text{ V}$$

2) From (5.41), the output voltage ripple is

$$\Delta v_2 = \Delta v_c = \frac{kV_2}{fRC} = \frac{0.6 \times 20}{50k \times 20 \times 20\mu} = 0.6 \text{ V}$$

3) From (5.40), the inductor  $L = 10 \text{ mH} > L_{\min} = \frac{1-k}{2f} R = \frac{0.4}{2 \times 20k} 20 = 0.2 \text{ mH}$

This boost converter works in CCM.

#### 5.4.

Refer to the formulae (5.51a), we obtain

$$n_1 = \frac{V_{o1}}{kV_1} = \frac{6}{0.5 \times 20} = 0.6$$

$$n_2 = \frac{V_{o2}}{kV_1} = \frac{9}{0.5 \times 20} = 0.9$$

$$n_3 = \frac{V_{o3}}{kV_1} = \frac{12}{0.5 \times 20} = 1.2$$

The particular winding turn's number are (with primary winding turn's  $N_1 = 600$ )

$$N_{2-1} = N_1 n_1 = 600 \times 0.6 = 360$$

$$N_{2-2} = N_1 n_2 = 600 \times 0.9 = 540$$

$$N_{2-3} = N_1 n_3 = 600 \times 1.2 = 720$$

The first secondary winding has 360 turns, the second secondary winding has 540 turns and

the third secondary winding has 720 turns.

### 5.5.

From the output voltage calculation formula, the turn's ratio is

$$n = \frac{1-k}{kV_{in}} V_o = \frac{0.2}{0.8 \times 50} 1500 = 7.5$$

The particular turn's number of the secondary winding is

$$V_2 = N_1 n = 200 \times 7.5 = 1500$$

The secondary winding has 1500 turns.

### 5.6.

1) Since the normalized impedance  $Z_N = \frac{R}{2fL} = \frac{3000}{2 \times 20k \times 10m} = 7.5$  and  $\frac{1}{1-k} = 2.5$ , the

converter from here we work in DCM. From (5.94) and the output voltage is

$V_o = k(1-k) \frac{R}{2fL} V_i = 36 V$ . From here we can see that the output voltage is higher than it in

CCM ( $M_E = 1.5$ ).

2) From (5.91), the variation ratio of  $v_o$  is:

$$\varepsilon = \frac{k}{128} \frac{1}{f^3 C C_o L_o R} = \frac{0.6}{128} \frac{1}{(20k)^3 (20\mu)^2 10m \times 3000} = 4.9 \times 10^{-8}$$

## Solutions for Chapter 6: Voltage Lift Converters

### 6.1.

1) From (6.47), the output voltage is  $V_o = V_i/(1-k) = 20/0.5 = 40$  V, i.e.  $M = 2$ .

2) From the formulae we can get the ratios:

$$\xi_1 = \frac{1}{2M^2} \frac{2}{fL} = \frac{1}{2 \times 2^2} \frac{40}{50k \times 1m} = 0.1$$

$$\xi_2 = \frac{k}{2M} \frac{R}{fL_o} = \frac{1}{2 \times 2^2} \frac{40}{50k \times 1m} = 0.1$$

$$\rho = \frac{k}{2} \frac{1}{fCR} = \frac{0.5}{2} \frac{1}{50k \times 20\mu \times 40} = 0.00625$$

$$\sigma_1 = \frac{M}{2} \frac{1}{fC_1R} = \frac{2}{2} \frac{1}{50k \times 20\mu \times 40} = 0.025$$

$$\varepsilon = \frac{k}{128} \frac{1}{f^3 L_o C C_o R} = \frac{0.5}{128} \frac{1}{(50k)^3 \frac{1m \times (20\mu)^2}{40}} = \frac{0.5}{128 \times 2000} \approx 2 \times 10^{-6}$$

### 6.2.

The boundary of CCM and DCM of a self-lift negative output Luo-Converter is shown in equation (6.49).

The condition is  $m = \frac{1}{\xi} = \frac{M^2}{k \frac{R}{2fL}} = \frac{2^2}{0.5 \frac{1000}{2 \times 50k \times 1m}} = \frac{4}{5} < 1$ , i.e. the converter works in DCM.

Therefore the output voltage is

$$V_o = [1 + k^2 (1 - k) \frac{R}{2fL}] V_i = [1 + 0.5^2 (1 - 0.5) \frac{1000}{2 \times 50k \times 1m}] \times 20 = 45 \text{ V}$$

### 6.3.

Since the enhanced self-lift positive output Luo-Converter has higher voltage transfer gain to be calculated by (6.62), its output voltage is

$$V_o = (1 + \frac{1}{1 - k}) V_i = (1 + \frac{1}{1 - 0.5}) 20 = 60 \text{ V}$$

### 6.4.

From (6.123) we obtain the output voltage is  $V_o = \frac{3}{1 - k} V_i = \frac{3}{1 - 0.5} 20 = 120 \text{ V}$

The variation ratios:



$$\xi = \frac{k}{M_T^2} \frac{3R}{2fL} = \frac{0.5}{6^2} \frac{3 \times 300}{2 \times 50k \times 1m} = 0.125$$

$$\xi = \frac{k}{16} \frac{1}{f^2 C L_o} = \frac{0.5}{16} \frac{1}{(50k)^2 \times 20\mu \times 1m} = 6.25 \times 10^{-4}$$

$$\chi_1 = \frac{k(1-k)}{2M_T} \frac{R}{fL_1} = \frac{0.5(1-0.5)}{2 \times 6} \frac{300}{50k \times 1m} = 0.125$$

$$\chi_2 = \frac{k(1-k)}{2M_T} \frac{R}{fL_2} = \frac{0.5(1-0.5)}{2 \times 6} \frac{300}{50k \times 1m} = 0.125$$

$$\rho = \frac{k}{2} \frac{1}{fCR} = \frac{0.5}{2} \frac{1}{50k \times 20\mu \times 300} = 8.33 \times 10^{-4}$$

$$\sigma_1 = \frac{M_T}{2} \frac{1}{fC_1R} = \frac{6}{2} \frac{1}{50k \times 20\mu \times 300} = 0.01$$

$$\sigma_2 = \frac{M_T}{2} \frac{1}{fC_2R} = \frac{6}{2} \frac{1}{50k \times 20\mu \times 300} = 0.01$$

$$\sigma_3 = \frac{M_T}{2} \frac{1}{fC_3R} = \frac{6}{2} \frac{1}{50k \times 20\mu \times 300} = 0.01$$

$$\varepsilon = \frac{k}{128} \frac{1}{f^3 C C_o L_o R} = \frac{0.5}{128} \frac{1}{(50k)^3 \times (20\mu)^2 \times 1m \times 300} = 2.6 \times 10^{-7}$$

Therefore, the variations are small.

## 6.5.

From (6.209) the voltage transfer gains of this enhanced double output self-lift DC-DC converters are

$$\begin{cases} M_{boost^1-S^+} = \frac{V_{O^+}}{V_{in}} = \frac{1}{(1-k)^2} = \frac{1}{(1-0.5)^2} = 4 \\ M_{boost^1-S^-} = \frac{V_{O^-}}{V_{in}} = -\frac{1}{(1-k)^2} = -\frac{1}{(1-0.5)^2} = -4 \end{cases}$$

Therefore the output voltages are

$$\begin{cases} V_{O^+} = 4V_{in} = 80 \text{ V} \\ V_{O^-} = -4V_{in} = -80 \text{ V} \end{cases}$$

## 6.6.

From Equation (6.220), the output voltage is  $V_o = \frac{4k}{1-k} V_{in} = \frac{k}{1-k} \times 80 \text{ V}$ .

The output voltages corresponding various k are shown in the following table:

Duty cycle k	Output voltage (V)
0.1	8.89
0.2	20
0.3	34.29
0.4	53.33
0.5	80
0.6	120
0.7	186.67

0.8	320
0.9	720

## Solutions for Chapter 7: Super-Lift and Ultra-Lift Converters

### 7.1.

From formula (7.10), we can get the variation ratio of current  $i_{L1}$ ,

$$\xi_1 = \frac{k(1-k)^4}{2(2-k)^3} \frac{R}{fL_1} = \frac{0.6(1-0.6)^4}{2(2-0.6)^3} \frac{100}{50k \times 10m} = 0.0069$$

$$\xi_1 = \frac{k(1-k)^2}{2(2-k)} \frac{R}{fL_1} = \frac{0.6(1-0.6)^2}{2(2-0.6)} \frac{100}{50k \times 10m} = 0.00686$$

From formula (7.8), we can get the output voltage  $V_o = \left(\frac{2-k}{1-k}\right)^2 V_{in} = \left(\frac{2-0.6}{1-0.6}\right)^2 20 = 245 \text{ V}$ .

From (7.12), its variation ratio is  $\varepsilon = \frac{k}{2RfC_4} = \frac{0.6}{2 \times 100 \times 50k \times 20\mu} = 0.003$ .

### 7.2.

a) The output current is  $I_o$ . The current of the capacitor  $C_2$  during switching-on is

$$I_{C2-on} = I_o$$

The current of the capacitor  $C_2$  during switching-off is

$$I_{C2-off} = \frac{k}{1-k} I_o$$

The current of the inductor during switching-off is

$$I_L = I_{L-off} = I_{C2-off} + I_o = \frac{1}{1-k} I_o$$

The current of the capacitor  $C_1$  during switching-off is

$$I_{C1-off} = I_L$$

The current of the capacitor  $C_1$  during switching-on is

$$I_{C1-on} = \frac{1-k}{k} I_L$$

The input current is  $I_{in} = k(I_L + I_{C1-on}) + (1+k)I_L = (2-k)I_L$

The power relation is  $P_{in} = V_{in} I_{in} = P_O + P_{loss} = V_O I_O + I_L^2 r_L$

i.e. 
$$V_{in} I_{in} = V_O I_O + I_L^2 r_L = V_O I_O + \left(\frac{I_O}{1-k}\right)^2 r_L = V_O I_O \left[1 + \left(\frac{1}{1-k}\right)^2 \frac{r_L}{R}\right]$$

Hence

$$M = \frac{V_O}{V_{in}} = \frac{I_{in}}{I_O \left[1 + \left(\frac{1}{1-k}\right)^2 \frac{r_L}{R}\right]} = \frac{2-k}{(1-k) \left[1 + \left(\frac{1}{1-k}\right)^2 \frac{r_L}{R}\right]} = \frac{2-k}{(1-k) + \frac{r_L}{(1-k)R}}$$

b) From above Equation, the voltage transfer gain M is,

$$M = \frac{V_O}{V_{in}} = \frac{2-k}{(1-k) + \frac{r_L}{(1-k)R}} = \frac{2-0.6}{(1-0.6) + \frac{0.4}{(1-0.6)10}} = \frac{1.4}{0.4 + \frac{0.4}{4}} = \frac{1.4}{0.5} = 2.8$$

The output current is,

$$I_O = \frac{V_O}{R} = \frac{M V_{in}}{R} = \frac{2.8 \times 20}{10} = 5.6 \text{ A}$$

Therefore, the output power is,

$$P_O = I_O^2 R = 5.6^2 \times 10 = 313.6 \text{ W}$$

The inductor current is,  $I_L = \frac{I_O}{1-k} = \frac{5.6}{0.4} = 14 \text{ A}$

The input current is,  $I_{in} = (2-k)I_L = 1.4 \times 14 = 19.6 \text{ A}$

Therefore, the input power is,  $P_{in} = V_{in} I_{in} = 20 \times 19.6 = 392 \text{ W}$

Therefore, the efficiency is,  $\eta = \frac{P_O}{P_{in}} = \frac{313.6}{392} = 0.8$

### 7.3.

From formula (7.27), we can get the variation ratio of current  $i_{L1}$ ,

$$\xi_1 = \frac{k(1-k)^2}{4(3-k)} \frac{R}{fL_1} = \frac{0.6(1-0.6)^2}{2(3-0.6)} \frac{1000}{50k \times 10m} = 0.04$$

From formula (7.25), we can get the output voltage  $V_o = \left(\frac{3-k}{1-k}\right)V_{in} = \left(\frac{3-0.6}{1-0.6}\right)20 = 120V$ .

From (7.19), its variation ratio is  $\varepsilon = \frac{k}{2RfC_{12}} = \frac{0.6}{2 \times 1000 \times 50k \times 20\mu} = 0.0003$ .

### 7.4.

From formula (7.128), we can get the variation ratio of current  $i_{L1}$ , ( $G_3 = 41.875$ )

$$\xi_1 = \frac{k(1-k)^3}{(2-k)^2 G_3} \frac{R}{2fL_1} = \frac{0.6(1-0.6)^3}{(2-0.6)^2 \times 41.875} \frac{200}{2 \times 50k \times 10m} = 0.00094$$

From formula (7.127), we can get the output voltage

$$V_o = \left[\left(\frac{2-k}{1-k}\right)^3 - 1\right]V_{in} = \left[\left(\frac{2-0.6}{1-0.6}\right)^3 - 1\right] \times 20 = 837.5V$$

From (7.131), its variation ratio is  $\varepsilon = \frac{k}{2RfC_6} = \frac{0.6}{2 \times 200 \times 50k \times 20\mu} = 0.0015$ .

## 7.5.

From formula (7.239), we can get the variation ratio of current  $i_{L1}$ ,

$$\xi_{s_1} = \frac{k(1-k)^2}{8} \frac{R}{fL_1} = \frac{0.6(1-0.6)^2}{8} \frac{400}{50k \times 10m} = 0.0096$$

From formula (7.236), we can get the output voltage

$$V_o = \frac{2}{1-k} V_{in} = \frac{2}{1-0.6} \times 20 = 100 \text{ V}$$

From (7.240), its variation ratio is  $\varepsilon = \frac{k}{2RfC_{12}} = \frac{0.6}{2 \times 400 \times 50k \times 20\mu} = 0.00075$ .

## 7.6.

From formula (7.409), we can get the output voltage

$$V_o = \left[ \left( \frac{j+1}{1-k} \right)^3 - 1 \right] V_{in} = \left[ \left( \frac{5+1}{1-0.6} \right)^3 - 1 \right] \times 20 = 67480 \text{ V}$$

From (7.407), its variation ratio is  $\varepsilon = \frac{k}{2RfC_{310}} = \frac{0.6}{2 \times 10000 \times 50k \times 20\mu} = 0.00003$ .

## 7.7.

From formula (7.436), we can get the value of  $\xi_1$ ,

$$\xi_1 = \frac{(1-k)^4 R}{2(2-k) fL_1} = \frac{(1-0.6)^4 \times 10000}{2(2-0.6) \times 50k \times 1m} = 1.83 \geq 1$$

Therefore, this converter works in DCM. From formula (7.443), we can get the voltage transfer gain  $G_{DCM}$

$$G_{DCM} = \frac{k(1-k)^2}{2} Z_N = \frac{0.6(1-0.6)^2}{2} \frac{10000}{50k \times 1m} = 9.6$$

The output voltage is  $V_2 = G_{DCM} \times V_{in} = 9.6 \times 20 = 192 V$ .



## Solutions for Chapter 8: Pulse-Width-Modulated DC/AC Inverters

### 8.1.

From Equation (8.3), we have the general rms values

$$(V_O)_h = \frac{V_d}{\sqrt{2}} \frac{(\hat{V}_{AO})_h}{V_d/2} = \frac{200}{\sqrt{2}} \frac{(\hat{V}_{AO})_h}{V_d/2} = 282.84 \frac{(\hat{V}_{AO})_h}{V_d/2} V$$

Checking the data from Table 8.1 we can get the rms values as follows:

Fundamental: $(V_O)_1 = 282.84 \times 0.8 = 226.27 V$	at 50 Hz
$(V_O)_{33} = 282.84 \times 0.22 = 62.23 V$	at 1650 Hz
$(V_O)_{35} = 282.84 \times 0.818 = 231.37 V$	at 1750 Hz
$(V_O)_{37} = 282.84 \times 0.22 = 62.23 V$	at 1850 Hz
$(V_O)_{67} = 282.84 \times 0.139 = 39.32 V$	at 3350 Hz
$(V_O)_{69} = 282.84 \times 0.314 = 88.81 V$	at 3450 Hz
$(V_O)_{71} = 282.84 \times 0.314 = 88.81 V$	at 3550 Hz
$(V_O)_{73} = 282.84 \times 0.139 = 39.32 V$	at 3650 Hz

etc.

### 8.2.

Refer to the Figure 8.9, it can be see that the first pulses applied to the switches  $S_{1+}/S_{1-}$  and  $S_{2+}/S_{2-}$ . Assume that the amplitude of the triangle wave is 1, the amplitude of the sin-wave is

0.8. The leading age of the first pulses is at  $t = 0$ .

The equation to determine the trailer age of the first pulse to turn-off  $S_{1+}$  is:

$$0.8\sin 100\pi t = 1600t - 2$$

Using iterative method, let  $x = 0.8\sin 100\pi t$  and  $y = 1600t - 2$ , and initial  $t_0 = 1.6667 \text{ ms} = 30^\circ$

T (ms/degree)	x	y	x :y	remarks
1.6667/30 <sup>o</sup>	0.4	0.6667	<	decrease t
1.5/27 <sup>o</sup>	0.3632	0.4	<	decrease t
1.47222/26.5 <sup>o</sup>	0.357	0.3555	≈	

The width of the first pulse to switch-on and off the switches  $S_{1+}$  is 1.47222 ms (or 26.5<sup>o</sup>).

The equation to determine the trailer age of the first pulse to turn-off  $S_{2+}$  is:

$$-0.8\sin 100\pi t = 1600t - 2$$

Using iterative method, let  $x = -0.8\sin 100\pi t$  and  $y = 1600t - 2$ , and initial  $t_0 = 1.111 \text{ ms} = 20^\circ$

T (ms/degree)	x	y	x :y	remarks
1.11/20 <sup>o</sup>	-0.2736	-0.2224	>	decrease t
1.0555/19 <sup>o</sup>	-0.26	-0.3111	<	increase t
1.08333/19.5 <sup>o</sup>	-0.267	-0.2666	≈	

The width of the first pulse to switch-on and off the switches  $s_{2+}$  is 1.08333 ms (or 19.5°).

### 8.3.

Since the output voltages are supplied between legs, we still use the Equation (8.13) to calculate the general rms values

$$(V_o)_h = \frac{2V_d}{\sqrt{2}} \frac{(\hat{V}_{AO})_h}{V_d/2} = \frac{1000}{\sqrt{2}} \frac{(\hat{V}_{AO})_h}{V_d/2} = 707.1 \frac{(\hat{V}_{AO})_h}{V_d/2} V$$

Checking the data from Table 8.1 we can get the rms values as follows:

Fundamental: $(V_o)_1 = 707.1 \times 1.0 = 707.1 V$	at 50 Hz
$(V_o)_{37} = 707.1 \times 0.018 = 12.73 V$	at 1850 Hz
$(V_o)_{39} = 707.1 \times 0.318 = 224.86 V$	at 1950 Hz
$(V_o)_{41} = 707.1 \times 0.601 = 424.97 V$	at 2050 Hz
$(V_o)_{43} = 707.1 \times 0.318 = 224.86 V$	at 2150 Hz
$(V_o)_{45} = 707.1 \times 0.018 = 12.73 V$	at 2250 Hz
$(V_o)_{77} = 707.1 \times 0.033 = 23.33 V$	at 3850 Hz
$(V_o)_{79} = 707.1 \times 0.212 = 149.91 V$	at 3950 Hz
$(V_o)_{81} = 707.1 \times 0.181 = 127.99 V$	at 4050 Hz
$(V_o)_{83} = 707.1 \times 0.181 = 127.99 V$	at 4150 Hz
$(V_o)_{85} = 707.1 \times 0.212 = 149.91 V$	at 4250 Hz
$(V_o)_{87} = 707.1 \times 0.033 = 23.33 V$	at 4350 Hz
etc.	

## Solutions for Chapter 9: Multi-level and Soft-Switching

### DC/AC Inverters

#### 9.1.

Refer to Table 9.2, the best switching angles in positive half-cycle are

$$\alpha_1 = 0.2242 \text{ rad} = 12.85^\circ$$

$$\alpha_2 = 0.7301 \text{ rad} = 41.83^\circ$$

$$\alpha_3 = \pi - \alpha_2 = 180^\circ - 41.83^\circ = 138.17^\circ$$

$$\alpha_4 = \pi - \alpha_1 = 180^\circ - 12.85^\circ = 167.15^\circ$$

The best switching angles in negative half-cycle are

$$\alpha_5 = \pi + \alpha_1 = 180^\circ + 12.85^\circ = 192.85^\circ$$

$$\alpha_6 = \pi + \alpha_2 = 180^\circ + 41.83^\circ = 221.83^\circ$$

$$\alpha_7 = \pi + \alpha_3 = 180^\circ + 138.17^\circ = 318.17^\circ$$

$$\alpha_8 = \pi + \alpha_4 = 180^\circ + 167.15^\circ = 347.15^\circ$$

The switches referring to Figure 9.3 (b) operate in a cycle ( $0^\circ - 360^\circ$ ) as below:

Turn on two upper switches  $S_3$  and  $S_4$  and two lower switches  $S_{1'}$  and  $S_{2'}$  in  $0^\circ - \alpha_1$ .

Turn on three upper switches  $S_2 \sim S_4$  and one lower switch  $S_{1'}$  in  $\alpha_1 - \alpha_2$ .

Turn on all upper switches  $S_1 \sim S_4$  in  $\alpha_2 - \alpha_3$ .

Turn on three upper switches  $S_2 \sim S_4$  and one lower switch  $S_{1'}$  in  $\alpha_3 - \alpha_4$ .

Turn on two upper switches  $S_3$  and  $S_4$  and two lower switches  $S_{1'}$  and  $S_{2'}$  in  $\alpha_4 - \alpha_5$ .

Turn on one upper switch  $S_4$  and three lower switches  $S_{1'} \sim S_{3'}$  in  $\alpha_5 - \alpha_6$ .

Turn on all lower switches  $S_{1'} \sim S_{4'}$  in  $\alpha_6 - \alpha_7$ .

Turn on one upper switch  $S_4$  and three lower switches  $S_{1'} \sim S_{3'}$  in  $\alpha_7 - \alpha_8$ .

Turn on two upper switches  $S_3$  and  $S_4$  and two lower switches  $S_{1'}$  and  $S_{2'}$  in  $\alpha_8 - 360^\circ$ .

The best THD = 16.42 %.

## 9.2.

Refer to Figure 1.11, the switching angles in a cycle are

$$\alpha_1 = 22.5^\circ$$

$$\alpha_2 = 67.5^\circ$$

$$\alpha_3 = 112.5^\circ$$

$$\alpha_4 = 157.5^\circ$$

$$\alpha_5 = 202.5^\circ$$

$$\alpha_6 = 247.5^\circ$$

$$\alpha_7 = 292.5^\circ$$

$$\alpha_8 = 337.5^\circ$$

The switches referring to Figure 9.8 (b) operate in a cycle ( $0^\circ - 360^\circ$ ) as below (only one):

Turn on the switches  $S_1, S_2, S_{1'}, S_{4'}$ :  $v_{an} = 2E$  (upper C4)  $- 2E$  (C2); in  $0^\circ - \alpha_1$ .

Turn on the switches  $S_1, S_2, S_3, S_{1'}$ :  $v_{an} = 2E$  (upper C4)  $- E$  (C1); in  $\alpha_1 - \alpha_2$ .

Turn on all upper switches  $S_1 \sim S_4$  in  $\alpha_2 - \alpha_3$ .

Turn on the switches  $S_1, S_2, S_3, S_{1'}$ :  $v_{an} = 2E$  (upper C4)  $- E$  (C1); in  $\alpha_3 - \alpha_4$ .

Turn on the switches  $S_1, S_2, S_{1'}, S_{4'}$ :  $v_{an} = 2E$  (upper C4)  $- 2E$  (C2); in  $\alpha_4 - \alpha_5$ .

Turn on the switches  $S_1, S_{1'}, S_{2'}, S_{3'}$ :  $v_{an} = 2E$  (upper C4)  $- 3E$  (C3); in  $\alpha_5 - \alpha_6$ .

Turn on all lower switches,  $S_{1'} \sim S_{4'}$  in  $\alpha_6 - \alpha_7$ .

Turn on the switches  $S_1, S_{1'}, S_{2'}, S_{3'}$ :  $v_{an} = 2E$  (upper C4)  $- 3E$  (C3); in  $\alpha_7 - \alpha_8$ .

Turn on the switches  $S_1, S_2, S_{1'}, S_{4'}$ :  $v_{an} = 2E$  (upper C4)  $- 2E$  (C2); in  $\alpha_8 - 360^\circ$ .

The function  $f(t)$  is in the period  $0 - 2\pi$ :

$$f(t) = \begin{cases} 2 & \frac{3\pi}{8} \leq \omega t < \frac{5\pi}{8} \\ 1 & \frac{\pi}{8} \leq \omega t < \frac{3\pi}{8}, \frac{5\pi}{8} \leq \omega t < \frac{7\pi}{8} \\ 0 & \text{other} \\ -1 & \frac{9\pi}{8} \leq \omega t < \frac{11\pi}{8}, \frac{13\pi}{8} \leq \omega t < \frac{15\pi}{8} \\ -2 & \frac{11\pi}{8} \leq \omega t < \frac{13\pi}{8} \end{cases}$$

The Fourier coefficients are

$$\begin{aligned} b_n &= \frac{1}{\pi} \int_0^{2\pi} f(t) \sin(n\omega t) d(\omega t) = \frac{2}{n\pi} \left[ \int_{\frac{n\pi}{8}}^{\frac{7n\pi}{8}} \sin \theta d\theta + \int_{\frac{3n\pi}{8}}^{\frac{5n\pi}{8}} \sin \theta d\theta \right] \\ &= \frac{2}{n\pi} \left[ \left( \cos \frac{n\pi}{8} - \cos \frac{7n\pi}{8} \right) + \left( \cos \frac{3n\pi}{8} - \cos \frac{5n\pi}{8} \right) \right] = \frac{4}{n\pi} \left( \cos \frac{n\pi}{8} + \cos \frac{3n\pi}{8} \right) \end{aligned}$$

With  $n = 1, 3, 5, \dots, \infty$

Finally, we obtain

$$F(t) = \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\sin(n\omega t)}{n} \left( \cos \frac{n\pi}{8} + \cos \frac{3n\pi}{8} \right) \quad n = 1, 3, 5, \dots, \infty$$

The fundamental harmonic has the amplitude  $\frac{4}{\pi}(1.306563)=1.6636$ . If we consider the higher order harmonics until 7<sup>th</sup>-order, i.e.  $n = 3, 5, 7$ . The HF's are

$$HF_3 = \frac{0.689}{3 \times 1.6636} = 0.138; \quad HF_5 = \frac{0.689}{5 \times 1.6636} = 0.0828; \quad HF_7 = \frac{1.6636}{7 \times 1.6636} = 0.143$$

The values of the HF's should be absolute values. The THD is

$$THD = \frac{\sqrt{\sum_{n=2}^{\infty} v_n^2}}{V_1} = \sqrt{\sum_{n=2}^{\infty} HF_n^2} = \sqrt{0.138^2 + 0.0828^2 + 0.143^2} = 0.2153$$

### 9.3.

The dc link voltages of HB $i$  (the  $i^{\text{th}}$  HB),  $V_{dci}$ , is  $2^{i-1} E$ . In a 3-HB one phase leg,

$$V_{dc1} = E, V_{dc2} = 3E, V_{dc3} = 9E$$

The operation listed below:

$$+0: v_{H1} = 0, v_{H2} = 0, v_{H3} = 0,$$

$$+1E: v_{H1} = E, v_{H2} = 0, v_{H3} = 0,$$

$$+2E: v_{H1} = -E, v_{H2} = 3E, v_{H3} = 0,$$

$$+3E: v_{H1} = 0, v_{H2} = 3E, v_{H3} = 0,$$

$$+4E: v_{H1} = E, v_{H2} = 3E, v_{H3} = 0,$$



+5E:  $v_{H1} = -E, v_{H2} = -3E, v_{H3} = 9E,$   
 +6E:  $v_{H1} = 0, v_{H2} = -3E, v_{H3} = 9E,$   
 +7E:  $v_{H1} = E, v_{H2} = -3E, v_{H3} = 9E,$   
 +8E:  $v_{H1} = -E, v_{H2} = 0, v_{H3} = 9E,$   
 +9E:  $v_{H1} = 0, v_{H2} = 0, v_{H3} = 9E,$   
 +10E:  $v_{H1} = E, v_{H2} = 0, v_{H3} = 9E,$   
 +11E:  $v_{H1} = -E, v_{H2} = 3E, v_{H3} = 9E,$   
 +12E:  $v_{H1} = 0, v_{H2} = 3E, v_{H3} = 9E,$   
 +13E:  $v_{H1} = E, v_{H2} = 3E, v_{H3} = 9E,$   
 -E:  $v_{H1} = -E, v_{H2} = 0, v_{H3} = 0,$   
 -2E:  $v_{H1} = E, v_{H2} = -3E, v_{H3} = 0,$   
 -3E:  $v_{H1} = 0, v_{H2} = -3E, v_{H3} = 0,$   
 -4E:  $v_{H1} = -E, v_{H2} = -3E, v_{H3} = 0,$   
 -5E:  $v_{H1} = E, v_{H2} = 3E, v_{H3} = -9E,$   
 -6E:  $v_{H1} = 0, v_{H2} = 3E, v_{H3} = -9E,$   
 -7E:  $v_{H1} = -E, v_{H2} = 3E, v_{H3} = -9E,$   
 -8E:  $v_{H1} = E, v_{H2} = 0, v_{H3} = -9E,$   
 -9E:  $v_{H1} = 0, v_{H2} = 0, v_{H3} = -9E,$   
 -10E:  $v_{H1} = -E, v_{H2} = 0, v_{H3} = -9E,$   
 -11E:  $v_{H1} = E, v_{H2} = -3E, v_{H3} = -9E,$   
 -12E:  $v_{H1} = 0, v_{H2} = -3E, v_{H3} = -9E,$   
 -13E:  $v_{H1} = -E, v_{H2} = -3E, v_{H3} = -9E,$

[Insert Figure 9.99 here]

[caption]Figure 9.99. Waveforms of trinary hybrid multilevel inverter

As shown in above figure, the output waveform,  $v_{an}$ , has 27 levels. This circuit has the greatest level number for a given number of HBs among existing multilevel inverters. The HB with higher dc link voltage has lower number of commutation and thereby reducing the associated switching losses. The higher switching frequency components, e.g. IGBT, are used to construct the HB with lower dc link voltage.

**Solutions for Chapter 10: Best Switching Angles to Obtain Lowest THD for Multilevel DC/AC Inverters**

**10.1.**

From Equation (10.1), if  $m = 5$ , the main switching angles in the area  $0 - \pi$  are,

$$\alpha_1 = 36^\circ$$

$$\alpha_2 = 72^\circ$$

$$\alpha_3 = 108^\circ$$

$$\alpha_4 = 144^\circ$$

**10.2.**

From Equation (10.3), if  $m = 5$ , the main switching angles in the area  $0 - \pi$  are,

$$\alpha_i = \sin^{-1}\left(\frac{2i-1}{m-1}\right)$$

$$\alpha_1 = \sin^{-1}\frac{1}{4} = 14.477512^\circ \approx 14.5^\circ$$

$$\alpha_2 = \sin^{-1}\frac{3}{4} = 48.590378^\circ \approx 48.6^\circ$$

$$\alpha_3 = 180^\circ - \alpha_2 = 180^\circ - 48.6^\circ = 131.4^\circ$$

$$\alpha_4 = 180^\circ - \alpha_1 = 180^\circ - 14.5^\circ = 165.5^\circ$$

**10.3.**

From Equation (10.5), if  $m = 83$ , the lowest THD is,

$$\begin{aligned} THD_{Lowest} &= 72.42e^{-0.4503m} + 11.86e^{-0.08739m} = 72.42e^{-37.3749} + 11.86e^{-7.2531154} \\ &= 72.42 * 5.8652586 * 10^{-17} + 11.86 * 7.0796532 * 10^{-4} \\ &= 4.2476203 * 10^{-15} + 8.396468 * 10^{-3} \approx 0.0084 \text{ OR } 0.84\% \end{aligned}$$

## Solutions for Chapter 11: Traditional AC/AC Converters

### 11.1.

From Equation (11.1), the output rms voltage is

$$\begin{aligned}V_o &= V_s \left(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right)^{1/2} = 220 \left(1 - \frac{75}{180} + \frac{0.5}{2\pi}\right)^{1/2} \\ &= 220(1 - 0.41666 + 0.079577)^{1/2} = 179.12 \text{ V}\end{aligned}$$

The output rms current is

$$I_o = \frac{V_o}{R} = \frac{179.12}{200} = 0.8956 \text{ A}$$

The fundamental harmonic wave delayed to the supply voltage by the firing angle  $\alpha = 60^\circ$ .

Therefore, the DPF is

$$DPF = \cos \alpha = 0.259$$

### 11.2.

From Equation (11.1), the output rms voltage is

$$V_o = V_s \left(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right)^{1/2}$$

$$\text{i.e.} \quad \left(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi}\right)^{1/2} = \frac{V_o}{V_s} = 0.7071 \quad 1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi} = 0.5$$

This is a transcendental equation. After determination, the firing angle  $\alpha = 90^\circ$ .

The output rms current is

$$I_o = \frac{V_o}{R} = \frac{155.56}{100} = 1.5556 \text{ A}$$

The fundamental harmonic wave delayed to the supply voltage by the firing angle  $\alpha = 90^\circ$ .

Therefore, the

$$DPF = \cos \alpha = 0.$$

### 11.3.

Since the input rms voltage is 120 V and the duty cycle  $k = 0.6$ , the output rms voltage is

$$V_o = V_s \sqrt{k} = 120 \times \sqrt{0.6} = 92.95 \text{ V}$$

The power factor is

$$PF = \sqrt{k} = \sqrt{0.6} = 0.775$$

### 11.4.

Since the input rms voltage is 120 V and the *modulation index*  $k = 0.6$ , the output rms voltage is

$$V_o = V_s \sqrt{k} = 120 \times \sqrt{0.6} = 92.95 \text{ V}$$

The power factor is

$$PF \sqrt{k} = \sqrt{0.6} = 0.775.$$

### 11.5.

The table is shown below (the blank means no-firing pulse applied): Positive rectifier

Half Cycle	1	2	3	4	5	6	7	8	9	10
No. in fo										
SCR	P <sub>1</sub> P <sub>4</sub>	P <sub>2</sub> P <sub>3</sub>	P <sub>1</sub> P <sub>4</sub>	P <sub>2</sub> P <sub>3</sub>	P <sub>1</sub> P <sub>4</sub>	P <sub>2</sub> P <sub>3</sub>	P <sub>1</sub> P <sub>4</sub>	P <sub>2</sub> P <sub>3</sub>	P <sub>1</sub> P <sub>4</sub>	P <sub>2</sub> P <sub>3</sub>
$\alpha_\pi$	$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_2$	$\alpha_1$					

Negative rectifier

Half Cycle	1	2	3	4	5	6	7	8	9	10
No. in fo										
SCR	N <sub>1</sub> N <sub>4</sub>	N <sub>2</sub> N <sub>3</sub>	N <sub>1</sub> N <sub>4</sub>	N <sub>2</sub> N <sub>3</sub>	N <sub>1</sub> N <sub>4</sub>	N <sub>2</sub> N <sub>3</sub>	N <sub>1</sub> N <sub>4</sub>	N <sub>2</sub> N <sub>3</sub>	N <sub>1</sub> N <sub>4</sub>	N <sub>2</sub> N <sub>3</sub>
$\alpha_v$						$\alpha_1$	$\alpha_2$	$\alpha_3$	$\alpha_2$	$\alpha_1$

The full regulation condition is

$$\frac{\sqrt{2}V_o}{\pi/5} \int_{\frac{2\pi}{5}}^{\frac{3\pi}{5}} \sin \alpha d\alpha \leq \sqrt{2}V_s \frac{1}{\pi} \int_0^\pi \sin \alpha d\alpha$$

$$V_s \geq 5V_o \cos \frac{2\pi}{5} = 1.54V_o$$

i.e.

$$V_s = 140 \geq 5V_o \cos \frac{2\pi}{5} = 1.54V_o = 139.06 \text{ V}$$

$$\sqrt{2}V_o \frac{5}{\pi} \int_0^{\frac{\pi}{5}} \sin \theta d\theta = \sqrt{2}V_s \frac{1}{\pi} \int_{\alpha_1}^\pi \sin \theta d\theta$$

$$5(1 - \cos \frac{\pi}{5})V_o = (1 + \cos \alpha_1)V_s$$



$$\alpha_1 = \cos^{-1} \left( \frac{0.955V_o}{V_s} - 1 \right) = \cos^{-1} (-0.3861) = 112.7^\circ$$

$$\sqrt{2}V_o \frac{5}{\pi} \int_{\frac{\pi}{5}}^{\frac{2\pi}{5}} \sin \theta d\theta = \sqrt{2}V_s \frac{1}{\pi} \int_{\alpha_2}^2 \sin \theta dg\theta$$

$$5 \left( \cos \frac{\pi}{5} - \cos \frac{2\pi}{5} \right) V_o = (1 + \cos \alpha_2) V_s$$

$$\alpha_2 = \cos^{-1} \left( \frac{2.5V_o}{V_s} - 1 \right) = \cos^{-1} (0.6071) = 52.6^\circ$$

$$\sqrt{2}V_o \frac{5}{\pi} \int_{\frac{2\pi}{5}}^{\frac{3\pi}{5}} \sin \theta d\theta = \sqrt{2}V_s \frac{1}{\pi} \int_{\alpha_3}^{\pi} \sin \theta d\theta$$

$$5 \left( \cos \frac{2\pi}{5} - \cos \frac{3\pi}{5} \right) V_o = (1 + \cos \alpha_3) V_s$$

$$\alpha_3 = \cos^{-1} \left( \frac{3.09V_o}{V_s} - 1 \right) = \cos^{-1} (0.9865) = 9.4^\circ$$

The phase-angle shift  $\sigma$  in the input voltage over the period  $T_s = 1/f_s$  is

$$\sigma = \frac{\alpha_1}{2} = \frac{1}{2} 112.7 = 56.35^\circ$$

## Solutions for Chapter 12: Improved AC/AC Converters

### 12.1.

We obtain the data below:

$$v_o = \frac{v_s}{1-k}$$

$$i_s = \frac{i_o}{1-k}$$

$$v_o = Ri_o$$

$$P_{in} = v_s i_s = v_o i_o = P_o \text{ with } \eta = 1.$$

$$PE = \int_0^{T_m} V_s i_s(t) dt = V_s \int_0^{T_m} i_s(t) dt = V_s I_s T_m$$

$$W_L = \frac{1}{2} L I_L^2 = \frac{1}{2} L i_s^2 = \frac{1}{2(1-k)^2} L i_o^2$$

$$W_C = \frac{1}{2} C V_C^2 = \frac{1}{2} C v_o^2$$

$$SE = \frac{1}{2} \left[ \frac{L i_o^2}{(1-k)^2} + C v_o^2 \right] = \frac{1}{2} \left[ \frac{L}{(1-k)^2} + CR^2 \right] i_o^2$$

$$EF = \frac{SE}{PE} = \frac{\frac{L}{(1-k)^2} + CR^2}{2v_o i_o T_m} i_o^2 = \frac{\frac{L}{R(1-k)^2} + CR}{2T_m}$$

$$CIR = \frac{0.5 \frac{L}{(1-k)^2} i_o^2}{0.5 C v_o^2} = \frac{L}{CR^2 (1-k)^2} = \frac{1}{4}$$

$$\tau = \frac{2T_m \times EF}{1 \times CIR} = \frac{\frac{L}{R(1-k)^2} + CR}{1 + CIR} = 40 \mu s$$

$$\tau_d = \frac{2T_m \times EF}{1 + 1/CIR} = \frac{\frac{L}{R(1-k)^2} + CR}{1 + 1/CIR} = 10 \mu s$$

$$G(s) = \frac{k}{1 + s\tau + 0.25s^2\tau^2} = \frac{M}{(1 + 0.00002s)^2}$$

This transfer function is in the critical condition with two folded poles. The corresponding step-response in the time domain has fast response without overshoot and oscillation.

$$g(t) = k \left[ 1 - \left( 1 + \frac{2t}{\tau} \right) e^{-\frac{2t}{\tau}} \right] = k \left[ 1 - \left( 1 + \frac{2t}{0.00004} \right) e^{-\frac{2t}{0.00004}} \right]$$

## 12.2.

Since the output voltage is

$$v_o = \frac{v_s}{1-k} = \frac{240}{1-k}$$

the duty cycle is calculated as

$$k = \frac{v_o - v_s}{v_o} \left\{ \begin{array}{l} \frac{500 - 240}{500} = 0.52 \\ \frac{1000 - 240}{1000} = 0.76 \end{array} \right\}$$

The range of the duty-cycle  $k$  is 0.52 – 0.76.

The output rms current is 5 – 10 A, and the output power is 2500 – 10000 W.

## 12.3.

Since the output voltage is

$$v_o = \left( \frac{1}{1-k} \right)^2 v_s = \left( \frac{1}{1-k} \right)^2 240$$

the duty cycle is calculated as

$$k = 1 - \sqrt{\frac{v_s}{v_o}} = \begin{cases} 1 - \sqrt{\frac{240}{1000}} = 0.51 \\ 1 - \sqrt{\frac{240}{1000}} = 0.742 \end{cases}$$

The range of the duty-cycle  $k$  is 0.51 – 0.742.

The output rms current is 10 – 36 A, and the output power is 10000 – 129600 W.

## Solutions for Chapter 13: AC/DC/AC and DC/AC/DC Converters

### 13.1.

Use a three level diode-clamped AC/DC/AC converter in Figure 13.4 to rectify the input AC voltage to an unstable output DC voltage with the efficiency  $\eta$  can be 92 ~ 97%. Since the wind turbine single-phase output voltage is 300 V  $\pm$  25 % independent from the frequency 50 Hz  $\pm$  15 %. The rectified output half DC link voltage ( $v_d/2$ ) can be 135 V  $\pm$  25 %, i.e. 101 ~ 169 VDC.

The three-level diode-clamped DC/AC converter has maximum output rms voltage is 0.71 half DC link voltage for each leg. For lowest half DC link voltage 101 V, each leg rms voltage can be 72 V. The line-to-line voltage can be  $72\sqrt{3} = 124$  V. This output 3-phase voltage is satisfactory.

### 13.2.

The output voltage  $V_2$  is

$$V_2 = \frac{1}{1-k} V_1 = 2 \times 40 = 80 \text{ V}$$

Therefore, the power is

$$P = \frac{V_2^2}{R} = \frac{80^2}{10} = 640 \text{ W}$$

From the known data  $T = 1/f = 50 \mu s$ , use the formulae (5.26) and (5.27), we have

$$I_{\max} = \frac{V_1}{R(1-k)^2} \frac{V_1}{2L} kT = \frac{40}{10(1-0.5)^2} + \frac{40}{2 \times 10m} 0.5 \times 50 \mu = 16.1 \text{ A}$$

$$I_{\min} = \frac{V_1}{R(1-k)^2} - \frac{V_1}{2L} kT = \frac{40}{10(1-0.5)^2} - \frac{40}{2 \times 10m} 0.5 \times 50 \mu = 15.9 \text{ A}$$

Substitute the values into (13.10), the output power is

$$P = \frac{1}{2} fL(I_{\max}^2 - I_{\min}^2) = \frac{20k \times 10m}{2} (16.1^2 - 15.9^2) = 640 \text{ W}$$

It is verified, but the power transferred is restricted by the inductor L.

### 13.3.

From Equation (13.14), the output voltage  $V_O$  is

$$v_O = n(k_1 + k_2)v_i = 5(0.4 + 0.4) 40 = 160 \text{ V}$$

Therefore the power is

$$P_O = \frac{v_O^2}{R} = \frac{160^2}{R} = 2560 \text{ W}$$

### 13.4.

From Equation (13.15), the output voltage  $V_o$  is

$$v_o = 2 (k_1 + k_2)v_i = 2(0.4 + 0.4) 40 = 64 V$$

Therefore, the power is

$$P_o = \frac{v_o^2}{R} = \frac{64^2}{10} = 409.6 W$$