Time Varying Equalization of Doubly Selective Channel

By

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MSEE-19

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ABSTRACT

Inter symbol interference and rapid time variations makes doubly selective channels difficult to equalize. The rapid time variations entail a receiver that also adapts itself to the channel variation. This increases the implementation complexity requirement for the receiver. Therefore faster techniques are much needed to establish real time communication over such severe channels. The focus of this research report is on faster equalization techniques for doubly selective channels. The block equalization technique was used as it emphases on low complexity equalization. MMSE equalizer is reformulated as the problem solving a system of linear equations. This allows the application of algorithms from linear algebra predominantly iterative methods for solution of system of linear equations. Jacobi, Gauss Seidel, Steepest Descent and Conjugate Gradient are used. Jacobi and Gauss Seidel are good for square matrices of lower order, but worse for higher order matrices. Steepest Descent takes more time and more number of iterations to converge than Conjugate Gradient. This report concludes by proposing the Conjugate Gradient method is suitable for low complexity block equalization problem from the perspective of run time and bit error rate.

DEDICATION

To my teachers, family and friends

ACKNOWLEDGEMENTS

Firstly, all praise is due to Allah (Glorified and Exalted is He), without his Immeasurable blessings and favors none of this could have been possible.

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List of Acronyms:

AWGN	Additive White Gaussian Noise
BLE	Block Linear Equalizer
CG	Conjugate Gradient
ISI	Inter Symbol Interference
LMS	Least Mean Square
MMSE	Minimum Mean Square Error
MLE	Maximum Likelihood Equalizer
SLE	Serial Linear Equalizer
ZF	Zero Forcing

Chapter 1: Introduction

1.1 Introduction:

Wireless channel fades in time and frequency due to high mobility in wireless broadband application. Such channels are named as doubly selective channel reflecting the fact that they exhibit both time and frequency selective fading. A very common problem occurs in transmitted signal known as ISI (inter symbol interference). To undo the effects of ISI and the noise introduced to the signal passed through the doubly selective channel different methodologies are used; Minimum Mean square error (MMSE) equalizer is one of them. This technique is used to increase the signal to noise ratio (SNR) and decrease bit error rate (BER).

1.2 Inter Symbol Interference (ISI):

Inter symbol interference is considered to be the main problem in telecommunication. It is a kind of interference in which a signal interferes with itself or in other words the signal interferes with its delayed versions. This phenomenon occurs due to the multipath propagation. In multipath propagation a transmitted signal follows more than one path to reach the destination and this creates error in that signal which results in ISI.

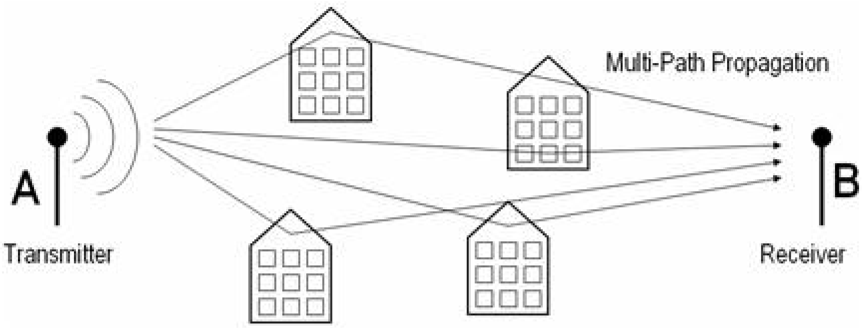

Figure 1.1: Multipath Propagation

1.3 Noise:

Noise is another factor which distorts the original signal. Wireless communication channels mostly deals with AWGN (Additive white Gaussian noise).

1.3.1 Additive White Gaussian Noise (AWGN):

The most basic model for thermal noise in communication systems have the following suppositions:-

- The received signal is the sum of transmitted signal and the noise hence the term additive.
- The power spectral density of the noise is unvaried. This is referred to the term white.
- Samples of the noise Gaussian distribution.

1.4 Fading:

Variation in time, of power in received signal due to variation in transmission path is known as fading. This is affected by changes in atmospheric conditions such as rainfall. The different types of fading that are popular are listed below:

Time selective fading happens when a channel varies with respect to time.

Frequency selective fading happens when a channel varies with respect to frequency.

1.5 Doubly Selective Channel:

Wireless channel fades in time and frequency both (Doubly Selective Channel) because of the wireless broadband applications and mobility. While the demand for increased data rates related with broadband uses results in frequency selective behavior, mobility in concurrence with multipath causes Doppler spread whereby the spectrum of the transmitted signal spreads as it passes over wireless medium. This Doppler spread results in time selective behavior of the channel response just as delay spread results in frequency selective behavior. With respect to the simple time selective or frequency selective channel, the doubly selective channel can provide more variety. But these channels are more challenging to equalize as the receiver has to deal with the selective nature of channel in both time and frequency.

This research report educates the equalization of Doubly Selective Channels (DSC) from the view of complexity. For a frequency selective channel, the equalizer (block/serial) needs only to be designed once and it should be used until the channel changes. DSC exhibits fast variations in the channel's impulse response so that the equalizer, block/serial, needs to be continually changed according to the channel. As opposed to slow fading channels where the equalizer may be designed once and then tracked using an adaptive filtering algorithm such as Least Mean Squares (LMS) in a decision directed mode, the doubly selective channel is rapidly changing and cannot be tracked by the slowly adapting LMS algorithm. The requirement of a continually redesigning the equalizer for a doubly selective fading channel places a huge signal processing burden on the receiver which must be alleviated to enable broadband wireless services to high mobile users.

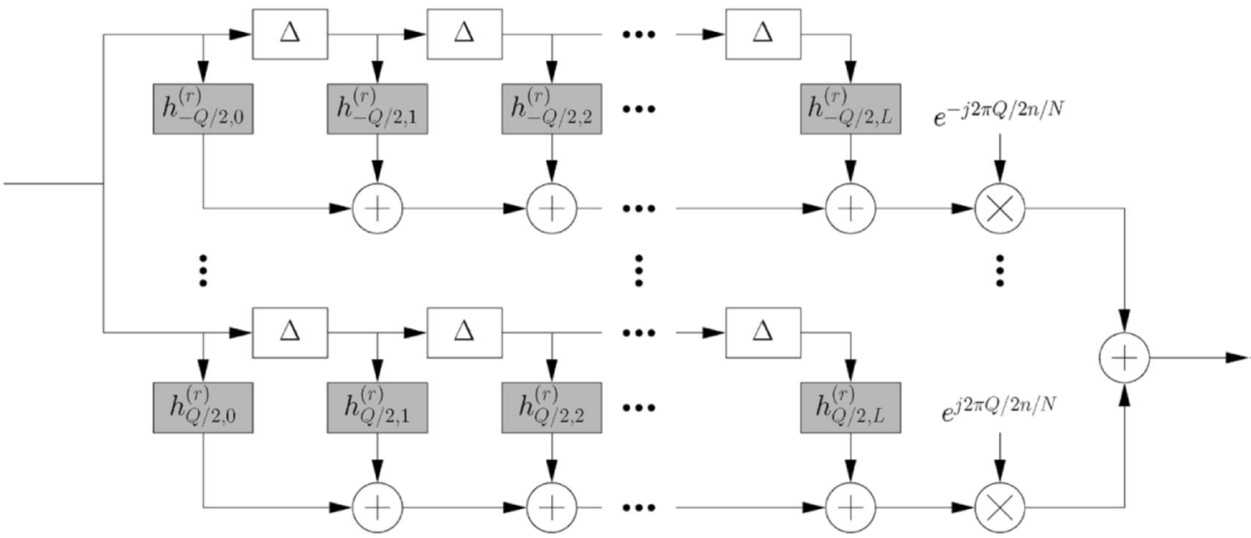

Figure 1.2: Block Diagram of a DSC

In figure 1.2 the block diagram of a (DSC) has been shown. The parameters 'Q' and 'L' are the Delay Spread and Doppler Spread respectively.

Following is the equation which illustrates the doubly selective channel

$$\boldsymbol{H^r} = \sum_{l=0}^{L} \sum_{q=-Q/2}^{Q/2} h_{q,l} \boldsymbol{D_q Z_l}$$

Where $\mathbf{D_q}$ is a diagonal matrix of the form

$$\boldsymbol{D_q} \coloneqq diag\{\left[1, \dots \dots, e^{\frac{j2\pi q(N-1)}{N}}\right]^T\}$$

And $\mathbf{Z_l}$ is the circular shifted matrix of order L+LXL+1, and $h_{q,l}$ is a scalar variable having random values.

1.6 MATLAB:

MATLAB is a high level language. It provides an environment for numerical computing. MATLAB includes a broad variety of applications, which includes following tools: Analog and Digital communication, Signal and Image processing, control systems design, testing and measuring, analysis and modeling. Some additional tools are also available in MATLAB that offers the solution for particular problems. MATLAB also gives the functionality of combining the code with other applications and languages e.g. C, C++, VB (Visual Basic) and JAVA. Because of all these benefits MATLAB is a best choice to process the complex data.

1.7 Overview

Over the period of time wireless communication industry has evolved rapidly. The digital cellular systems which are currently in use are designed to provide services like voice, internet access and video conferencing with high data rates and greater speed. These services demands data rates ranging from a some number of hundred Kbps for fast moving users to some Mbps for slow moving users. These high data rates introduce frequency-selective transmission, whereas speed of movement and carrier offsets give rise to time selectivity. This results in so-called doubly selective channel (DSC).

To battle against these DSC effects, equalizers have a vital role to play. In [1] time variant (TV) FIR equalizer was introduced, before that only time invariant (TIV) FIR equalizers were used. Basis Expansion Model (BEM) was used to approximate the DSC and serial liner equalizer (SLE) and block linear equalizer (BLE) with Minimum Mean Square Error (MMSE) and zero forcing (ZF) were studied. Generalized minimal residual (GMRES) and least squares (LSQR) were used in [2] to equalize BEM based DSC. The proposed 1-tap equalizer achieves results comparable to MMSE over Wi-MAX system. Multiple Input Multiple Output (MIMO) based Orthogonal Frequency Division Multiplexing (OFDM) channel was equalized by MMSE in [3], which results in improved BER performance with some Inter Symbol Interference (ISI) is still present. In [4] frequency domain representation of Linear TV MMSE equalizer was introduced and it made sure of a very adequate tradeoff among complexity, convergence speed, and performance. Conjugate Gradient (CG) method was used for channel estimation and equalization of DSC for OFDM in [5]. Linear MMSE and Decision Feedback Equalization (DFE) techniques were studied for equalization which concluded that LMMSE equalization provides better performance to simple DFE. This motivates us to use CG for BLE by using MMSE for DSC. In this paper we are proposing equalization of DSC with the help on MMSE-BLE-CG.

1.8 Thesis Compilation

This thesis is organized into 6 chapters. In chapter 2 brief description of equalizers are discussed. Then we moved on to the study of iterative methods in chapter 3. Chapter 4 comprises of implementation of conjugate gradient method over MMSE BLE. We compare through MATLAB simulations the BER of our proposed MMSE-BLE-CG with TV FIR equalization in chapter 5. Finally we draw our conclusion and future work in chapter 6.

Chapter 2: Equalization

2.1 Equalizer:

An equalizer is a filter that attempts to reverse the distortion occurred in a signal transmitted through a channel. Indeed the signal undergoes ISI while passing through the channel and white Gaussian noise is also added at the receiver. To equalize against the effects of channel, many equalization methods are used at the receiver end.

2.2 Zero forcing:

In wireless communication to overcome the problem of ISI different equalizers are used. Zero forcing equalizer is the simplest among them. It is a linear equalizer. The basic idea of zero forcing is to find the inverse convolution matrix of a channel and then multiply with the received symbols in order to cancel the effects of channel.

Zero forcing equalizer tries to suppress the ISI from the received symbols ŝ (n) to recover the transmitted symbols s (n). It provides good equalization result where the noise is not present or very low. The computation complexity is very less in this technique but it comes at the cost of performance. The block diagram of a simple zero forcing equalizer on a doubly selective channel is shown in figure 2.1.

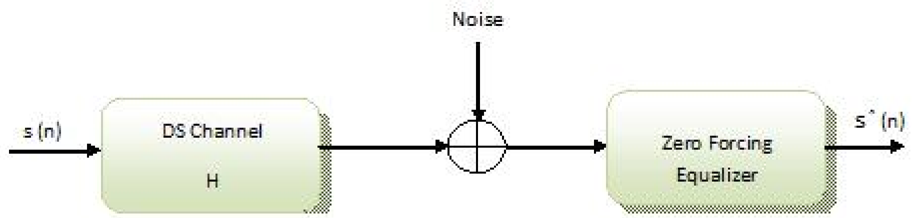

Figure 2.1: Block diagram of Zero Forcing Equalizer

At the cost of completely removing ISI zero forcing equalizer boosts up the noise due to inverse convolution matrix. If convolution matrix is of NxN order (a very high order) then it is almost impossible to take its inverse.

2.3 Maximum Likelihood Equalizer (MLE):

The maximum likelihood equalizer is the best among the linear and non-linear equalizers, but its computational complexity is very high. That’s why MLE is not used commonly. MLE is a non-linear equalizer. It uses Viterbi algorithm for making decision about the transmitted symbols. Viterbi algorithm finds the path metric and branch metric and then decides what is transmitted at every node. The path with the shortest metric is considered the best ML output, and the paths except these metric is skipped.

In figure 2.2 the block diagram of MLE is shown. It has the same block diagram as the zero forcing and MMSE equalizer. Only the equalization block is changed.

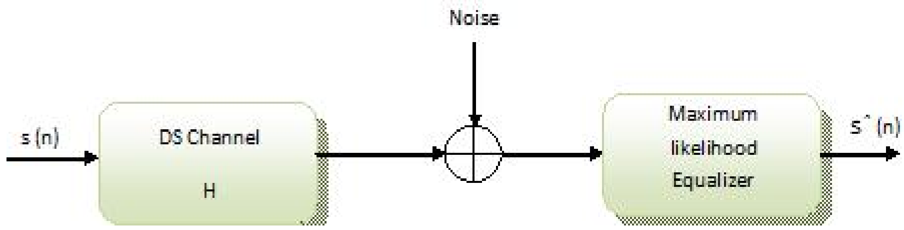

Figure 2.2: Block Diagram of Maximum Likelihood Equalizer (MLE)

2.4 MMSE (minimum mean square error) Equalizer:

To deal with ISI and noise at the same time MMSE Equalizer is used. It minimizes the difference between original transmitted signal and the estimated received signal. It acts as a balancing bridge between ISI and noise. It is mostly used in MIMO (multiple input multiple output).

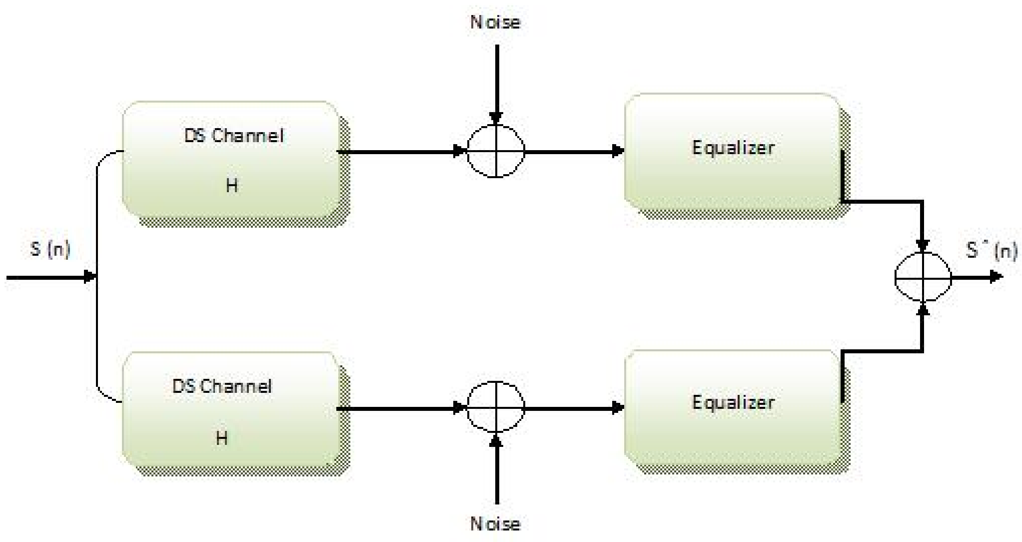

Figure 2.3: Block Diagram of MMSE Equalizer

Figure 2.3 shows the complete model for the two receiver antenna MMSE equalizer. This model is implemented in MATLAB.

2.4.1 MMSE Linear Equalizer:

MMSE-LE stands for minimum mean square error linear equalizer. It is a balanced equalizer, which tries to reduce the mean square error. The problem of zero forcing equalizer has been removed in this technique. It does not boost up the noise as in the case of zero forcing, it tries to remove the ISI and noise at the same time by balancing between them.

The MMSE-LE tries to remove the ISI and only permits equalized symbols to pass. In reality, it does not totally remove the ISI. It passes some remaining ISI. If this remaining ISI is removed by force then the noise will increase by default. Due to this remaining ISI, the performance of the equalizer suffers but still the results of MMSE-LE are better than ZF equalizer.

2.4.2 Derivation of MMSE-LE:

To find out the predefined equation weight a cost function is used. The cost function minimizes the Mean Square Error (MSE). The entities used in derivation of MMSE-LE are discussed below.

J = Cost Function

h = Channel

η = AWGN noise

Y = Distorted Received symbols

w = MMSE Linear Equalizer

S_n = Transmitted Symbols

$\hat{S}_n$ = Output Estimated Symbols

E = Estimation

Leq = Length of Equalizer

$$J = E\{|s_n - \hat{s}_n|^2\} \tag{2.1}$$

$$\boldsymbol{Y} = \boldsymbol{Hs} + \boldsymbol{\eta}$$

Where **Y** is the received symbols:

$$\begin{bmatrix} \boldsymbol{y(n)} \\ \boldsymbol{y(n-1)} \\ \boldsymbol{y(n-2)} \\ \vdots \\ \boldsymbol{y(n-Leq)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{h(n,0)} & \dots & \boldsymbol{h(n,l)} & \boldsymbol{0} \dots & \boldsymbol{0} \\ \boldsymbol{0} & \ddots & \cdot & \ddots & \vdots \\ \vdots & \boldsymbol{0} & . & \cdot & \cdot \\ \vdots & \vdots & \ddots & \cdot & . \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{h(n,0)} & \dots & \boldsymbol{h(n,l)} \end{bmatrix} \begin{bmatrix} \boldsymbol{s(n)} \\ \boldsymbol{s(n-1)} \\ \boldsymbol{s(n-2)} \\ \vdots \\ \boldsymbol{s(n-Leq)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\eta(n)} \\ \boldsymbol{\eta(n-1)} \\ \boldsymbol{\eta(n-2)} \\ \vdots \\ \boldsymbol{\eta(n-Leq)} \end{bmatrix}$$

The equalization symbols $\hat{S}_n$ are produced by passing received symbols **Y** from the equalizer.

$$\hat{s}_n = \boldsymbol{w^H y} = \boldsymbol{w^H}(\boldsymbol{Hs} + \boldsymbol{\eta}) \tag{2.2}$$

By substituting 2.2 in 2.1, the resultant is

$$J = E\{|s_n - \boldsymbol{w^H}(\boldsymbol{Hs} + \boldsymbol{\eta})|^2\}$$

Where

$$|s_n - \boldsymbol{w^H}(\boldsymbol{Hs} + \boldsymbol{\eta})|^2 = (s_n - \boldsymbol{w^H Hs} - \boldsymbol{w^H \eta})\,(s_n^* - \boldsymbol{s^H H^H w} - \boldsymbol{\eta^H w})$$

So, the equation becomes

$$J = E\{(s_n - \boldsymbol{w^H Hs} - \boldsymbol{w^H \eta})\,(s_n^* - \boldsymbol{s^H H^H w} - \boldsymbol{\eta^H w})\}$$

$$J = E\{s_n s_n^* - s_n \boldsymbol{s^H H^H w} - s_n \boldsymbol{\eta^H w} - \boldsymbol{w^H Hs} s_n^* + \boldsymbol{w^H Hss^H H^H w} + \boldsymbol{w^H Hs\eta^H w} - \boldsymbol{w^H \eta} s_n^* + \boldsymbol{w^H \eta s^H H^H w} + \boldsymbol{w^H \eta\eta^H w}\}$$

Noise (**η**) and Transmitted Symbols (**s**) are random variables. Following properties are being used for the estimation of correlation

$$E\{\boldsymbol{ss^H}\} = I$$

$$E\{\boldsymbol{ss}^*\} = 1_\delta$$

$$E\{\boldsymbol{\eta\eta^H}\} = \sigma_n{}^2$$

$$E\{\boldsymbol{s\eta}\} = 0$$

Where σ_n^2 is noise power. Now the equation becomes

$$J = \boldsymbol{w^H H}\, E\{\boldsymbol{ss^H}\}\boldsymbol{H^H w} - \boldsymbol{w^H H} E\{\boldsymbol{ss_n^*}\} + \boldsymbol{w^H} E\{\boldsymbol{\eta\eta^H}\}\boldsymbol{w} - E\{s_n \boldsymbol{s^H}\}\boldsymbol{H^H w} + E\{s_n s_n^*\}$$

$$J = \boldsymbol{w^H HH^H w} - \boldsymbol{w^H H 1_\delta} + \sigma_n{}^2 \boldsymbol{w^H w} - \boldsymbol{1_\delta H^H w} + 1 \qquad (2.3)$$

The purpose of this derivation is to design an MMSE-LE '**w**' and to reduce the cost function *J* of the equalizer with respect to equalizer '**w**'. To minimize the cost of equalizer apply the derivative on equation 2.3 w.r.t. '**w***'.

So

$$\frac{\partial J}{\partial w*} = \boldsymbol{HH^H w} - \boldsymbol{H 1_\delta} + \sigma_n{}^2 \boldsymbol{w} + 0 + 0 \qquad (2.4)$$

Put equation 2.4 equals to zero to find the equation of the equalizer

$$\boldsymbol{HH^H w} - \boldsymbol{H 1_\delta} + \sigma_n{}^2 \boldsymbol{w} = \boldsymbol{0}$$

$$\boldsymbol{HH^H w} + \sigma_n{}^2 \boldsymbol{w} = \boldsymbol{H 1_\delta}$$

$$\boldsymbol{w} = \left(\boldsymbol{HH^H} + \sigma_n^2 \boldsymbol{I}\right)^{-1} \boldsymbol{H 1_\delta} \qquad (2.5)$$

The equation 2.5 is desired equation for MMSE-LE '**w'**, which sets the weights of equalizer. In the absence of noise it will work as a zero forcing equalizer. In the worst noise condition the results will not be so good but the equalizer will try to minimize the MSE.

2.5 MMSE Serial Linear Equalizer:

In MMSE Serial Linear Equalizer (SLE), equalization is done by taking chunks of the convolution matrix one by one and then those chunks is passed through the equalizer. SLE operates on a small window of received symbols around the desired symbol to be detected. It is often preferred because of it lower complexity requirements. The problem of MMSE equalization is to find an equalizer '**w**' such that the cost function '*J*' is minimized which has already been done.

The Equation for MMSE-SLE is as follows

$$\boldsymbol{w} = \left(\boldsymbol{H}\boldsymbol{H}^{\boldsymbol{H}} + \sigma_n^2\boldsymbol{I}\right)^{-\mathbf{1}}\boldsymbol{H}\mathbf{1}_{\boldsymbol{\delta}}$$

2.6 MMSE Block Linear Equalizer:

MMSE Block Linear Equalizer (BLE) takes the whole convolution matrix, inverts it and then by inverting it performs equalization. BLEs are quite better than SLEs. They have better bit error rate (BER) results. But problem occurs when there is a huge data block involved. The equalization of that huge block itself is a tough job to do. Whole matrix is passed through the equalizer and then the equalizer gets the original signal from that matrix.

The equation for MMSE-BLE is as follows:

$$\boldsymbol{w} = \left(\boldsymbol{H}\boldsymbol{H}^{\boldsymbol{H}} + \sigma_n^2\boldsymbol{I}\right)^{-\mathbf{1}}\boldsymbol{H}^{\boldsymbol{H}}$$

Chapter 3: Iterative Methods

3.1 Jacobi Method:

3.1.1 Theory:

In linear algebra, Jacobi is probably the simplest of all the iterative methods. The matrix formed by such equations is such that the largest absolute values in each row horizontally and column vertically is dominated by the diagonal element. Jacobi takes advantage of this characteristic of the matrix, and each diagonal element is solved for, and an approximated value plugged in. Then the process is iterated, approaching the solution.

Let the system of linear equations be

$$\mathbf{Ax} = \mathbf{b}$$

Where **A** is a matrix under consideration, It should be clear that for this system to converge **A** has to be positive definite and symmetric.

Now **A** is easily split into its diagonal and off-diagonal component matrices. i.e

$$\mathbf{A} = \mathbf{D} + \mathbf{R}$$

Where the elements of **D** are diagonal components and elements of **R** are the off-diagonal components of **A**. So the original system can be written as:

$$(\mathbf{D} + \mathbf{R})\mathbf{x} = \mathbf{b}$$
$$\mathbf{Dx} + \mathbf{Rx} = \mathbf{b}$$
$$\mathbf{Dx} = \mathbf{b} - \mathbf{Rx}$$
$$\mathbf{x}_{l+1} = \mathbf{D}^{-1}(\mathbf{b} - \mathbf{Rx}_l)$$

Since Jacobi tried to solve for each element individually, so the element based formula turns out to be

$$\mathbf{x}_j^{(l+1)} = \frac{\mathbf{1}}{\boldsymbol{a_{jj}}}\left(\boldsymbol{b_j} - \sum_{k\neq j} a_{jj}\mathbf{x}_k^{(l)}\right) \quad , j = 1, 2, 3, \ldots\ldots\ldots\ldots\ldots, n.$$

3.1.2 Jacobi Method:

Start with the initial guess $\boldsymbol{x}_0$.

for *j := 1*: n do

$\sigma = 0$

for *k := 1:* n do

if *k != I* then

$\sigma = \sigma + \alpha_{jk}\mathbf{x}_k^{(l-1)}$

End if

End

$$\mathbf{x}_j^{(l)} = \frac{\mathbf{b}_j - \sigma}{a_{jj}}$$

End

Check for convergence

End (while divergence-holds loop)

3.2 Gauss Seidel Method:

3.2.1 Theory:

Gauss Seidal is another technique which tries to determine a result to a system of linear equations iteratively. It works like the Jacobi Method; mentioned in section 3.1, with the exception that it is functional with the matrices having non-zero diagonal elements. If a matrix is either dominant diagonally or symmetric and positive definite then convergence is guaranteed. Square structure of 'n' linear equations with anonymous **x**:

$$\mathbf{Ax} = \mathbf{b}$$

Since:

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{a}_{11} & \boldsymbol{a}_{12} & \cdots & \boldsymbol{a}_{1n} \\ \boldsymbol{a}_{21} & \boldsymbol{a}_{22} & \cdots & \boldsymbol{a}_{n2} \\ \vdots & \vdots & \ddots & \boldsymbol{a}_{n3} \\ \boldsymbol{a}_{n1} & \boldsymbol{a}_{n2} & \cdots & \boldsymbol{a}_{nn} \end{bmatrix}, \qquad \boldsymbol{X} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_n \end{bmatrix}, \qquad \boldsymbol{b} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \\ \vdots \\ \boldsymbol{b}_n \end{bmatrix}$$

Then **A** is disintegrated into a lower triangular components **L**, and a strictly upper triangular components **U**:

$$\mathbf{A} = \mathbf{L} + \mathbf{U}$$

Where

$$\mathbf{L} = \begin{bmatrix} a_{11} & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}, \qquad \boldsymbol{U} = \begin{bmatrix} 0 & a_{12} & \dots & a_{1n} \\ 0 & 0 & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}$$

The system of equations is modified as:

$$\mathbf{Lx} = \mathbf{b} - \mathbf{Ux}$$

Now this method tries to solve for the LHS equation for unknown value i.e x, using the past value for x on the RHS. Systematically, this can be expressed as

$$\mathbf{x}^{(l+1)} = \mathbf{L}^{-1}\,(\mathbf{b} - \mathbf{U}\mathbf{x}^{l})$$

3.2.2 Gauss Seidel Method:

Input variables: $\mathbf{C}, \mathbf{d}$

Output variable: $\boldsymbol{\alpha}$

Initial guess to $\boldsymbol{\alpha}^{(0)}$ to the solution

Repeat till convergence

For $j = 1{:}n$

$$\sigma \leftarrow 0$$

For $k = 1{:}j - 1$

$$\sigma \leftarrow \sigma + c_{jk}\,\boldsymbol{\alpha}^{(l+1)}$$

End ($k - loop$)

For $k = j + 1{:}n$

$$\sigma \leftarrow \sigma + c_{jk}\,(\boldsymbol{\alpha})^{l}$$

End ($k - loop$)

$$\boldsymbol{\alpha}_j^{(l+1)} \leftarrow \frac{1}{c_{jj}}\,(d_j - \sigma)$$

End ($j - loop$)

Check for convergence

End (repeat)

3.3 Method of Steepest Descent:

3.3.1 Theory

The method of steepest descent (SD) is an old optimization technique. Basically, it is used to understand the concept of gradient based adaptation which is implemented in various ways. This method uses some initial values for the weight vectors. It improves itself with the increased number of iterations. The final computed value for the tap-weight vector converges to the Wiener solution. The important point to note that is the method of steepest descent is descriptive of deterministic feedback system that finds the minimum point of ensemble averaged error performing surface without having knowledge of surface itself.

Assume that it is essential to find the least of a function $f(x), x \in Rn$ and $f: Rn \rightarrow R$. The gradient of f is denoted by $gk = g(xk) = \nabla f(xk)$.The calculation of step vector in any search path is the general hint for most of the minimization techniques. $\mathbf{d_k}$, for example,

$$\boldsymbol{x_{k+1}} = \boldsymbol{x_k} + \alpha k \boldsymbol{dk}, k = 0, 1, \ldots$$

α_k is the step size and it is calculated as follows:-

$$\alpha k = argmin\ f(xk + \alpha dk)$$

argmin represents the argument of the minimum for the identified function in above equation. $\alpha k = -\nabla f(xk)$ is the search path for SD method. The SD algorithm can be written as follows:

The two main computational benefits of the SD algorithm are:-

1. Algorithm can be implemented easily.
2. Requires low storage.

3.3.2 Steepest Descent

Initialize $\mathbf{x_0}, \mathbf{d}_{0,} = -\mathbf{g}_0$, tolerance *tol*

For $k = 0$ to max-iteration do

Set $\boldsymbol{\alpha}_k = argmin\ \varphi\ (\alpha) = f\ (\mathbf{x}_k) - \boldsymbol{\alpha}_k \mathbf{g}_k$

$\mathbf{x}_{k+1} = \mathbf{x}_k - \alpha_k \mathbf{g}_k$

Compute $\mathbf{g}_{k+1} = \nabla f(\mathbf{x}_k + 1)$

If $||\mathbf{g}_{k+1}||2 \leq tol$ then converged

End if

End for

3.4 Conjugate Gradient (CG) Method:

3.4.1 Theory

CG method is used to solve system of linear equations having symmetric and positive definite matrix especially. It is an iterative method, so it can be applied to sparse structures that are extremely large for the direct techniques to be deal with. These systems often arise when differential equations are solved mathematically.

Let there is a system of linear equations:-

$$\mathbf{Ax} = \mathbf{b}$$

Where the $n\ x\ n$ matrix $\mathbf{A}$ is symmetric (i.e., $\mathbf{A}^T = \mathbf{A}$), positive definite (i.e., $\mathbf{x}^T\mathbf{Ax} > 0$ for all non-zero vectors $\mathbf{x}$ in Rn), and real. It indicates the exclusive solution of this system by $\mathbf{x}*$.

By indicating the preliminary guess for $\mathbf{x}^*$ by $\mathbf{x}_0$, it can be expected that deprived of loss of generalization that $\mathbf{x_0} = 0$ (or else, consider the system $\mathbf{Az} = \mathbf{b} - \mathbf{Ax_0}$ in its place). Beginning with initial guess $\mathbf{x_0}$ this algorithm searches for the result and it needs a metric to check if it is nearer to the result $\mathbf{x}^*$ in every iteration or not. The metric is made on the point that the result $\mathbf{x}^*$ is the distinctive minimizer of $f(x)$ given below; so in order to make the algorithm closer to $\mathbf{x}^*$, $f(x)$ should be minimized.

$$f(x) = \frac{\mathbf{1}}{\mathbf{2x}^T\mathbf{Ax}} - \mathbf{x}^T\mathbf{b}, x\boldsymbol{\epsilon}\boldsymbol{R^n}$$

This proposes to take initial basis vector $\mathbf{p}_1$ as the opposite of the gradient of f at $\mathbf{x} = \mathbf{x_o}$. This gradient matches $\mathbf{Ax_o{-}b{=}0}$.

Since $\mathbf{x_o} = 0$, hence $\mathbf{P_1{=}b}$. Rest of the basis vectors shall be conjugating to the gradient, therefore it is called CG (Conjugate Gradient) method.

Let $\mathbf{r}_l$ be the left over by the l^{th} stage:

$$\mathbf{r}_l = \mathbf{b} - \mathbf{Ax}_l$$

Note that $\mathbf{r}_l$ is the negative gradient of f at $\mathbf{x} = \mathbf{x}_l$, so the gradient descent method would be to move in the direction $\mathbf{r}_l$. Here the directions $\boldsymbol{p_l}$ are conjugate to each other.

This gives the following expression:

$$\mathbf{P}_{l+1} = \mathbf{r}_l - \sum_{i\leq l} \frac{\mathbf{P}_i^T\mathbf{Ar}_l}{\mathbf{P}_i^T\mathbf{AP}_i}\mathbf{P_i}$$

$$\mathbf{x}_{l+1} = \mathbf{x}_l + \boldsymbol{\alpha}_{l+1}\mathbf{P}_{l+1}$$

$$\boldsymbol{\alpha}_{l+1} = \mathbf{P}_{l+1}^T\mathbf{r}_l/\mathbf{P}_{l+1}^T\mathbf{AP}_{l+1}$$

3.4.2 Conjugate Gradient:

$$\mathbf{r}_0 \coloneqq \mathbf{b} - \mathbf{A}\mathbf{x}_\mathbf{0}$$

$$\mathbf{P}_\mathbf{0} \coloneqq \mathbf{r}_\mathbf{0}$$

$$l \coloneqq 0$$

Repeat

$$\alpha_l \coloneqq \mathbf{r}_l^T \mathbf{r}_l / \mathbf{P}_l^T \mathbf{A}\mathbf{P}_l$$

$$\mathbf{x}_{l+1} \coloneqq \mathbf{x}_l + \alpha_l \mathbf{P}_l$$

$$\mathbf{r}_{l+1} \coloneqq \mathbf{r}_l - \alpha_l \mathbf{A}\mathbf{P}_l$$

If $\mathbf{r}_{l+1}$ is very very small then exit loop end if

$$\beta_l \coloneqq \mathbf{r}_{l+1}^T \mathbf{r}_{l+1} / \mathbf{r}_l^T \mathbf{r}_l$$

$$\mathbf{P}_{l+1} \coloneqq \mathbf{r}_{l+1} + \beta_l \mathbf{P}_l$$

$$l = l + 1$$

End repeat

Chapter 4: Implementation of Conjugate Gradient Algorithm

4.1 Basics of Complexity Reduction:

To recover the transmit block, various equalization techniques have been suggested in literature such as zero forcing equalization, linear MMSE equalization, maximum likelihood detection, decision feedback equalization and iterative techniques. The zero forcing (ZF) solution can be obtained as

$$\hat{\mathbf{x}}_{ZF} = \mathbf{H}^{\dagger}\mathbf{y}$$

Where $\mathbf{H}^{\dagger}$ is the Pseudo inverse of the convolution matrix H. Although ZF solution completely eliminates the ISI, it tends to enhance noise. The MMSE solution which provides a compromise between noise and ISI is obtained as

$$\hat{\mathbf{x}}_{MMSE} = (\mathbf{H}^{H}\mathbf{H} + \mathbf{I}\sigma_{\omega}^{2})^{-1}\mathbf{H}^{H}\mathbf{y}$$

It has been suggested in [3] that the computational complexity associated with these equalizers is $\mathcal{O}(\mathrm{N}^3)$. One approach to obtaining the block equalization solution directly (without evaluating the block equalizer itself because the interest lies in the transmit data) is to use the CG method.

4.1 Conjugate Gradient (CG) Method:

Consider the equation for the MMSE solution which is rewritten as

$$\underbrace{(\mathbf{H}^{H}\mathbf{H} + \sigma^{2}\mathbf{I})}_{\mathrm{A}}\ \underbrace{\mathbf{x}}_{\mathrm{x}} = \underbrace{\mathbf{H}^{H}\mathbf{y}}_{\mathrm{b}} \quad (4.1)$$

To solve such systems, CG method [6, 7], iteratively searches for a Krylov sequence, i.e. a set of points $\mathbf{x_q}$ in a sequence of Krylov subspace κ_q

$$\mathbf{x}_q = \arg\min_{\mathbf{x} \in \boldsymbol{\kappa}_q} f(x) \quad (4.2)$$

Where $f(x) = \mathbf{x}^{H}\mathbf{A}\mathbf{x} - \mathbf{b}^{H}\mathbf{x} - \mathbf{x}^{H}\mathbf{b}$ and the *qth* Krylov subspace is the space spanned by the columns of the matrix

$$[\mathbf{b} \ \ \mathbf{Ab} \ \ \dots \ \ \mathbf{A}^{q-1}\mathbf{b}]$$

It is known by the **Cayley Hamilton theorem** that the solution to the system of equations must lie in the Krylov subspace of order *N*, even if it does not span R^N. Define the residual at iteration *q* as $\mathbf{r}_q = \mathbf{A}\mathbf{x}_q - \mathbf{b}$ and the normalized conjugate directions as

$$\mathbf{p}_q = \frac{\|\mathbf{r}_{q-1}\|^2}{(\mathbf{x}_q - \mathbf{x}_{q-1})^H \mathbf{r}_{q-1}} (\mathbf{x}_q - \mathbf{x}_{q-1}) \quad (4.3)$$

4.2 Algorithm for Complexity Reduction:

Conjugate Gradient (CG) method takes only 50% of the iterations than Steepest Descent (SD) Method [30]. SD searches in the pattern of "Zig Zag" in each iteration whereas CG searches for the lowest possible solution in each iteration. Following figure shows the difference between both of these methods.

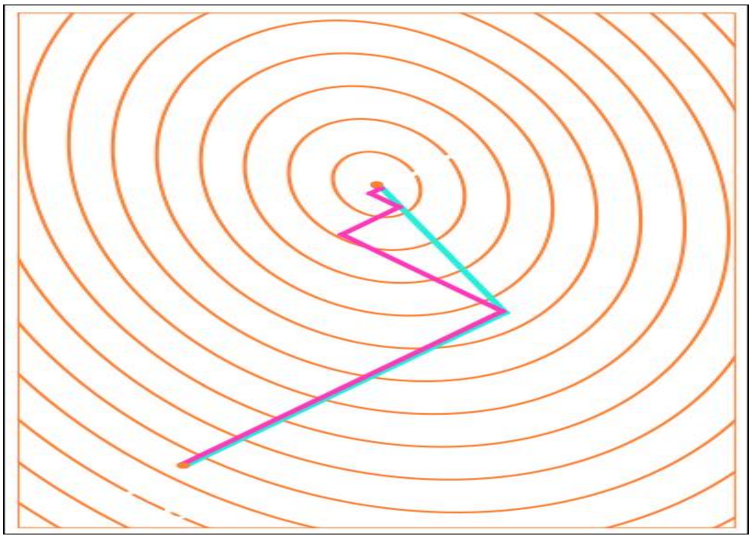

Figure 4.1 Conjugate Gradient vs Steepest Descent

Magenta line shows the iterations for SD and Cyan shows the no of iteration for CG. Starting from the center which is the initial guess SD takes four to five iterations to get to its optimum solution whereas CG took only two iterations which conclude that CG is less complex and more efficient method to solve system of linear equations than SD.

Chapter 5: Simulation Results and Discussions

5.1 MMSE-Serial Linear Equalizer (MMSE-SLE)

5.1.1 Discussion

To observe the results of MMSE-SLE SISO system is used. BEM coefficients were used to model the doubly selective channel. A block of size N=100,000 was transmitted over the channel and equalized by the equalizer having length of equalizer (Leq)=10. BER vs Eb/No curve of MMSE-SLE is as under.

5.1.2 Simulation result

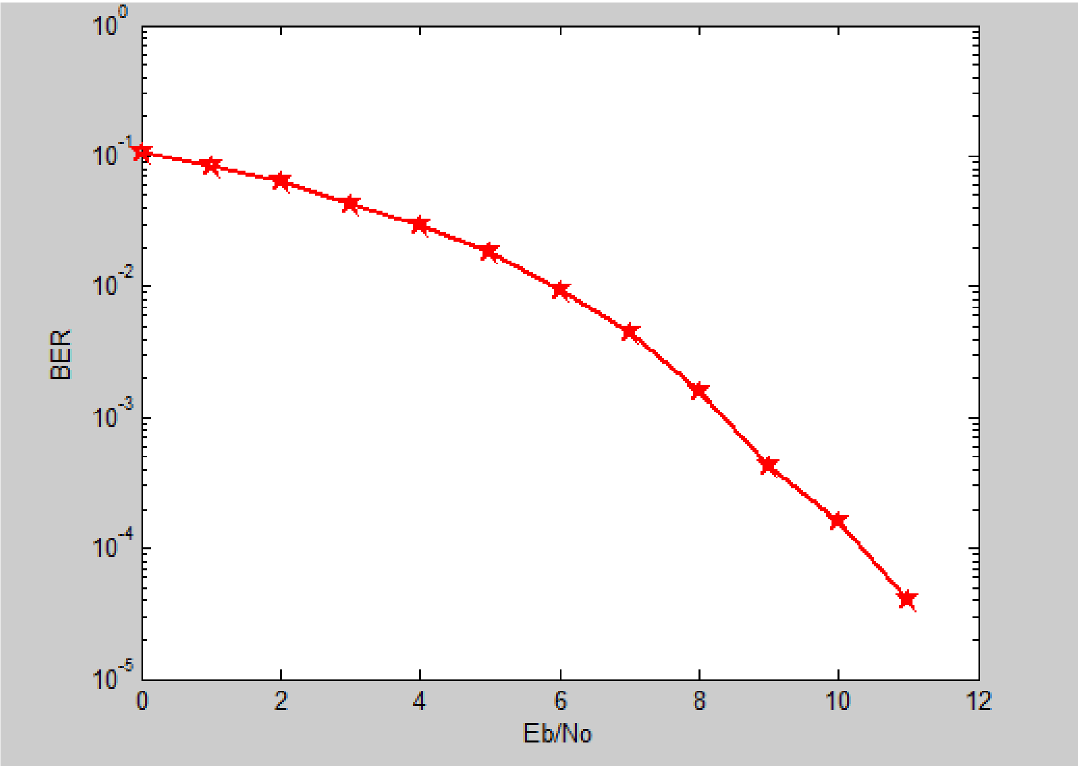

Figure 5.1: BER vs Eb/No Curve of MMSE-SLE

5.2 MMSE-Block Linear Equalizer

5.2.1 Discussion

Same channel was modeled for MMSE-BLE as it was in MMSE-SLE. Same block is transmitted over the channel with the same length of equalizer. BER vs Eb/No curve of MMSE-SLE is as under.

5.2.2 Simulation Result

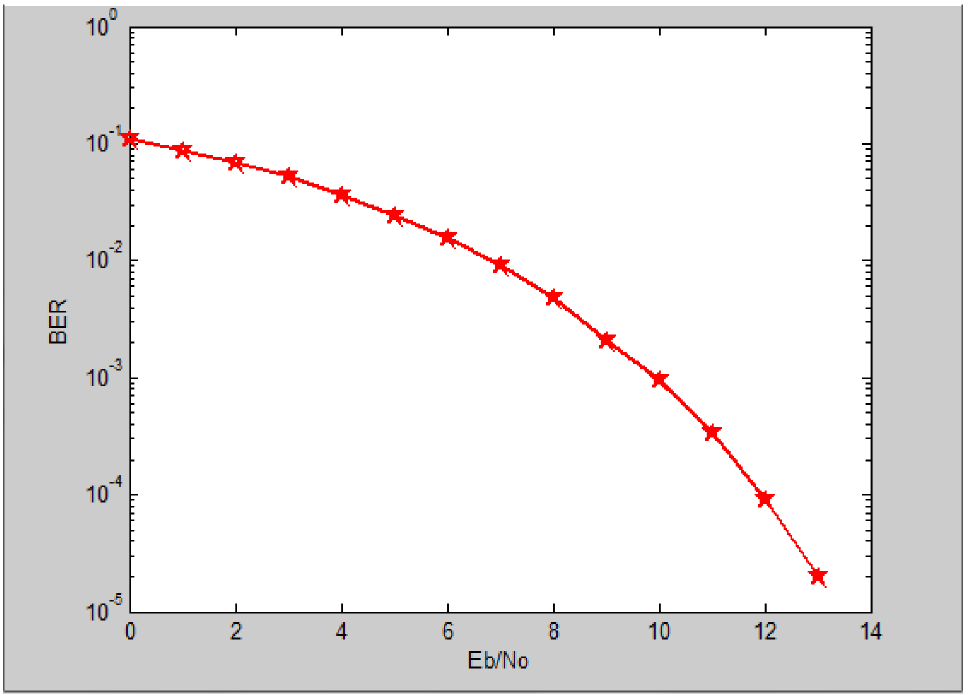

Figure 5.2: BER vs Eb/No Curve of MMSE-BLE

5.3 Equalization using Conjugate Gradient Method:

5.3.1 Discussion

In figure 5.3, a doubly selective channel has been equalized by conjugate gradient method. As it is an iterative method, so the performance of each iteration has been shown as a separate BER curve. 15 iterations were considered here. It is clear from the graph that after every iteration the solution is converging to the original signal that was transmitted. In comparison to the results of [1] in fig 5.4 time varying FIR equalization using MMSE BLE, BEM equalizer for one receive antenna results in BER curve starts getting smooth at 32dB whereas using MMSE BLE, CG using BEM with conjugate gradient approach results in BER curve between 15dB to 20dB. Note that the maximum Doppler spread of 100 Hz corresponds to a vehicle speed of 120 km/h and a carrier frequency of 900 MHz .Following parameters were used

- Doppler spread f_{max}= 100Hz
- Delay spread $\mathcal{T}_{max}$ =75µs
- Block size N=1024
- Symbol/sample period T= 25µs;
- Discrete Doppler spread Q/2= $[f_{maxNT}]$ =2
- Discrete delay spread L = $[\mathcal{T}_{max}/T]$ = 3

5.3.2 Simulation result:

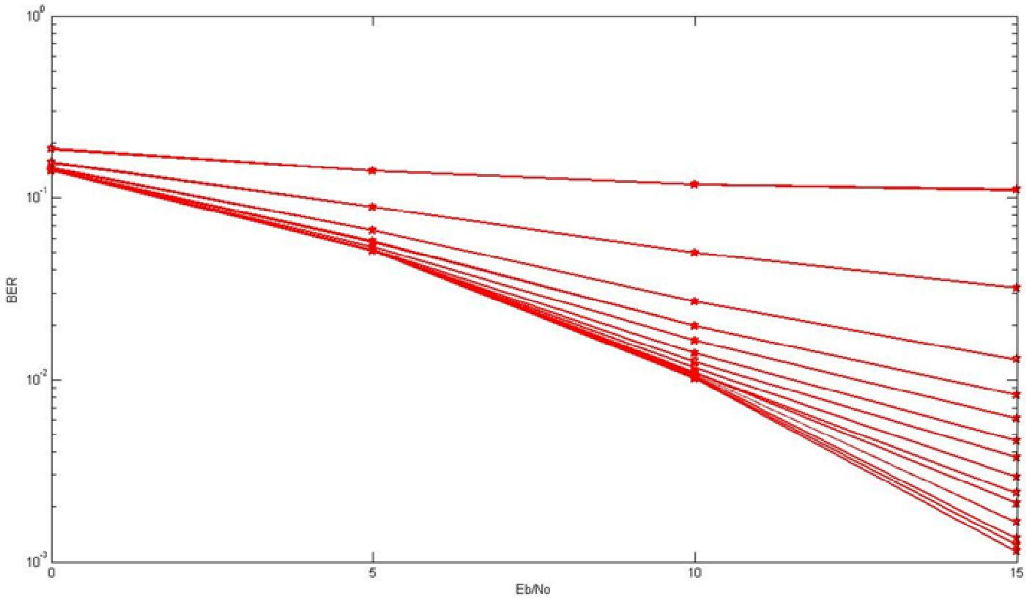

Figure 5.3: BER vs SNR for MMSE BLE CG

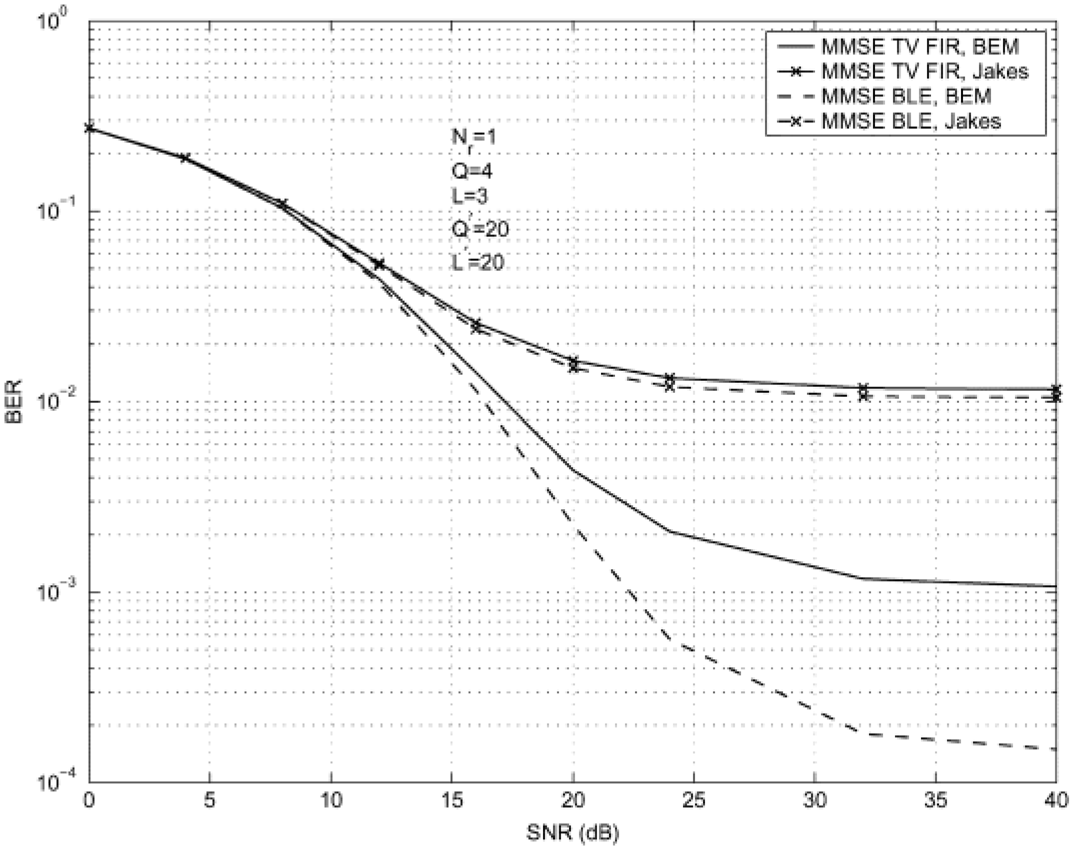

Figure 5.4: BER vs SNR for MMSE BLE, BEM

5.4 Complexity Analysis:

To implement the MMSE-BLE-CG, we require $\mathcal{O}(N^2)$ flops for DSC. The implementation complexity associated with the BLE requires N^2 Multiplication and addition operations. Comparing the complexity of proposed solution MMSE-BLE-CG with the complexity of MMSE-BLE-BEM of [1] it is noted that complexity of both the techniques is $\mathcal{O}(N^2)$ but there is difference in run time and BER vs SNR results. Runtime of MMSE-BLE-BEM in [1] is 352800 whereas runtime of MMSE-BLE-CG is 16506. This is a significant reduction is run time. These complexities are shown in Table I for MMSE-TV-FIR- and MMSE-BLE-CG criterion.

TABLE 1 : EQUALIZERS COMPLEXITY TABLE

MMSE-BLE-BEM		MMSE-BLE-CG	
Complexity $\mathcal{O}(N^2)$	CT	Complexity $\mathcal{O}(N^2)$	CT
640,000	352800	640,000	16506

Chapter 6: Conclusions

6.1 Overview

In communication systems, a channel through which the information flows is represented by a matrix, and the received information is the result of the transmitted information manipulated by that channel. Mathematics has a best tool for studying such a scenario, namely system of linear equations. This research is based on a comparative study of traditional iterative methods of solving a system of linear equations. The research was started off by studying how these methods differ from each other, and which method would be most suitable in a specific scenario. Once the different factors involved were studied, it was realized that there was room for improvement in the MMSE equalization technique. Efforts were put in to reduce its complexity and were successful. MMSE is generally considered the best solution to linear equalization problems. Its attractiveness can be more improved by decreasing its complexity by bringing in a new technique i.e. method of conjugate gradient.

6.2 Limitations:

The scope of this research is limited to the receiver end of a communication system. Also, all the findings are theoretical and computer simulations were used to find their validity. Since there was no real-time data available so the proposed solution in this research could yield results slightly different than expected.

6.3 Future Work:

There is always room for improvement, and so is the case with this research. It is intended to expand the approach (reducing complexity of existing solutions instead of trying to finding out new solutions to engineering problems) to other areas of communication engineering, specifically those involved with estimating a channel and blind equalization. It is hoped that by doing so, the vendors will have a chance to manufacture less expensive hardware used in communications, because they will not have to redesign their products from scratch

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