DEVELOPMENT OF PASSIVITY-PRESERVING MODEL ORDER REDUCTION TECHNIQUES

By

Abid Khan

A thesis submitted to the faculty of Electrical Engineering Department,
Military College of Signals, National University of Sciences and Technology,
Islamabad, Pakistan, in partial fulfillment of the requirements for the degree of MS in
Electrical Engineering

May 2018

THESIS ACCEPTANCE CERTIFICATE

Certified that final copy of MS/MPhil thesis written by Mr/~~MS~~ **Abid Khan,** Registration No. **NUST201463805MMCS25014F** of **Military College of Signals** has been vetted by undersigned, found complete in all respect as per NUST Statutes/Regulations, is free of plagiarism, errors and mistakes and is accepted as partial, fulfillment for award of MS/MPhil degree. It is further certified that necessary amendments as pointed out by GEC members of the student have been also incorporated in the said thesis.

Signature: ______________________________

Name of Supervisor: Asst Prof Muhammad Imran, PhD

Date: ______________________________

Signature (HoD): ______________________________

Date: ______________________________

Signature (Dean): ______________________________

Date: ______________________________

ABSTRACT

This research examines the passivity preserving frequency weighted model order reduction (MOR) techniques for linear time invariant (LTI) systems. In this research different single and double sided passivity preserving techniques for MOR are presented. Different combinations of Lyapunov and arithmetic Riccati equations (ARE's) are used to deduce the controllability and observability Gramians from frequency weighted and un-weighted systems. First of all, augmented system (a system with both input and output frequency weights) is transformed into a new system using two different transformations. Then by using the same transformations, weighted controllability and observability Gramians (which are obtained from weighted ARE's) are transformed into new weighted controllability and observability Gramians.

For double sided passivity preserving, three schemes are presented in which both controllability and observability Gramians are weighted. In first scheme, an ARE based transformed weighted controllability Gramian and a Lyapunov based weighted observability Gramian and vice versa are used for balancing the system. In second scheme, an ARE based transformed weighted controllability Gramian and an ARE based weighted observability Gramian and vice versa are used to balance the system. In third scheme, an ARE based transformed weighted controllability Gramian and an ARE based transformed weighted observability Gramian are used for balancing purpose.

For single sided passivity preserving, five schemes are presented in which either a controllability Gramian is weighted and an observability Gramian is un-weighted or a controllability Gramian is un-weighted and an observability Gramian is weighted. In first scheme, an ARE based un-weighted controllability Gramian and a Lyapunov based weighted observability Gramian and vice versa are used to balance the system. In second scheme, an ARE based weighted controllability Gramian and an ARE based un-weighted observability Gramian and vice versa are used for balancing. In third scheme, an ARE based transformed weighted controllability Gramian and an ARE based un-weighted observability Gramian and vice versa are used for balancing purpose. In fourth scheme, an ARE based transformed weighted controllability Gramian and a Lyapunov based un-weighted observability Gramian and vice versa are used for balancing the system. In fifth scheme, an ARE based transformed weighted controllability Gramian and an ARE based un-weighted observability Gramian and vice versa are used for balancing. Several practical examples using different weighting functions are given to show the effectiveness of the proposed schemes.

DEDICATION

This thesis is committed to

MY FAMILY, FRIENDS AND TEACHERS

for their adoration, unending backing and support.

ACKNOWLEDGEMENTS

First of all, thanks to Allah Almighty for giving me strength and courage to undertake this task, and especially thanks to my supervisor Assistant Professor Dr. Muhammad Imran and also I would like to thank Col. Dr. Abdul Ghafoor who supported me in every step and guided me in a professional manner. Without their support, guidance and help this thesis would not have been possible. Also, I would like to thank my parents for their hardwork, patience and trust on me throughout my studies.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ACRONYMS

Model Order Reduction	MOR
Reduced Order Models	ROM
Linear Time Invariant	LTI
Arithmetic Riccati Equation	ARE
Balanced Truncation	BT
Ordinary Differential Equation	ODE
Partial Differential Equation	PDE
Singular Value Decomposition	SVD
Moment Matching	MM
Truncated Balanced Realization	TBR
Passive Reduced Order Interconnect Macromodeling Algorithm	PRIMA
Frequency Weighted Model Order Reduction	FWMOR
Passivity Preserving Frequency Weighted Model Order Reduction	PPFWMOR
Positive Real	PR
Positive Rreal Truncated Balanced Realization	PRTBR
Hankel Singular Value	HSV

Chapter 1

Introduction

1.1 Model Order Reduction Overview

Mathematical modeling of a physical system is considered to be an important tool for the design and analysis purpose in control engineering. Many physical systems have very large and complex mathematical models. Deriving a reasonable mathematical model is fundamental to the study and design of a dynamic system in control engineering. A large scale and a fairly complex model of a dynamic system can be obtained but its analysis and design can be difficult due to its complexity. However, analysis and design of a dynamic system is easier if a lower order model from a higher order model is derived. The process in which a lower order model is obtained from a higher order model is known as model order reduction (MOR). To approximate the large and complex mathematical models, different techniques for MOR are used. The main goal of MOR techniques is to approximate a reduced order model (ROM) which should have low approximation error and should also preserve stability and passivity like an original system. In the last four decades, different MOR techniques got considerable attention and have been widely used in control engineering [1]- [6], [17, 18], [22]- [24]

1.2 Summary of Model Order Reduction Techniques

In this section, a brief summary of some techniques for stable and passive ROM's in the presence of frequency weights is discussed.

1.2.1 Frequency Weighted Model Reduction

Balanced truncation (BT) [1] is the most famous and widely used MOR technique. BT [1] not only preserves stability of the ROM but also gives simple error bound formula. Enns' [2] modified the BT method [1] to include input and output frequency weights. The main reason behind the idea of using frequency weights in MOR techniques was to get a ROM with small error in a particular frequency range [2]. Enns' method [2] works well for the single sided weighting scenario but it may yield unstable ROMs when double sided weights are used. To address the stability issue of [2], many other techniques [3]- [6], [22] for MOR have been offered in literature.

In [3] Lin and Chiu have shown that a guaranteed stable ROM can be obtained for strictly proper two sided weights unlike the method of [2] which produces stable ROM in the presence of single sided weighting only. The technique of [2] was also modified by Wang *et al.* [4] which yields stable ROM even in the presence of double sided weights by making the indefinite matrices positive semi definite. An *a priori* error bound formula for frequency weighted MOR is also given in [4]. The concept of similar kind of an effect on all the eigenvalues of indefinite matrices and an error bound formula is also given in [5]. For a specific subclass of positive real (PR) systems, a modified PR balancing method along with an error bound formula is proposed in [6].

1.2.2 Passivity Preserving Model Order Reduction

Phillips *et al.* [7] presented a family of algorithms for passive ROM which are similar to the well-known BT method [1] for stable ROM. In [7] controllability and observability Gramians are deduced from Lur'e equations without using frequency weights. Although, the algorithms presented in [7] give passive ROM but these algorithms do not address the passivity issue in a particular frequency range. Muda *et al.* [8] extended the methods of [2]- [4] for RLC systems to ensure passivity while taking into account the effect of frequency weights, since [2]- [4] only yield stable ROM. Conditions for guaranteed passive ROM's are also discussed by [8] for the three extended techniques.

Heydari and Pedram claimed in [9] that their technique produces guaranteed passive ROM for double sided frequency weighted case, and the spectrally-weighted error bound is also available. The technique of [9] produces passive ROMs for the single sided weighting case similar to [2] which produces stable ROM for the case when only one sided weighting is used. In [10] it has been proved that the technique proposed in [9] may yields non-passive ROM for the passive original system in case of double sided frequency weighting. [10] also proved that the method of [9] can preserve passivity only when one sided weighting is present.

1.3 Problem Summary

Existing techniques [7]- [9] are studied so far and it has been observed that the problem of preserving passivity, in the presence of double sided frequency weights is yet to be an open challenge.

1.4 Summary of Contributions

Different passivity preserving algorithms are proposed in [7] which yield passive ROM without using frequency weighting. Heydari and Pedram [9] used frequency weights in their techniques but later on it was proved incorrect by [10], although, the technique presented in [9] preserves passivity for single sided frequency weighting and also gives an error bound formula.

1.5 Objective of the Research

The objective of this thesis (research) is summarized as,

- To produce such techniques which ensure the stability as well as passivity of the ROM.
- To explore such techniques which produce guaranteed passive ROM when both input and output frequency weights are used in MOR.

1.6 Outline of the Thesis

This thesis is split into five chapters:

- Chapter 1: In this chapter the summary of existing stability and passivity preserving MOR techniques is described.
- Chapter 2: This chapter includes existing stability and passivity preserving MOR techniques in detail.
- Chapter 3: This chapter incorporates different proposed schemes for computing a guaranteed passive ROM in case of single and double sided weights.
- Chapter 4: In this chapter numerical examples and simulation results are presented.
- Chapter 5: This chapter is about future work and conclusion.

Chapter 2

Frequency Weighted Model Reduction: A Review

2.1 Introduction

Control engineering mainly focuses on implementation of dynamic systems which are derived by mathematical modeling of a diverse range of physical systems. A large number of physical systems exist in the real world. Ordinary differential equations (ODEs) and partial differential equations (PDEs) are used to describe the dynamic behavior of a physical system. In most of the cases, the system architecture or the dimension of the system is too large and complex and we get a higher order mathematical model of a dynamic system. Different MOR techniques are used to reduce the size and complexity of these large and complex models for the ease of analysis, design and simulation (see Figure 2.1).

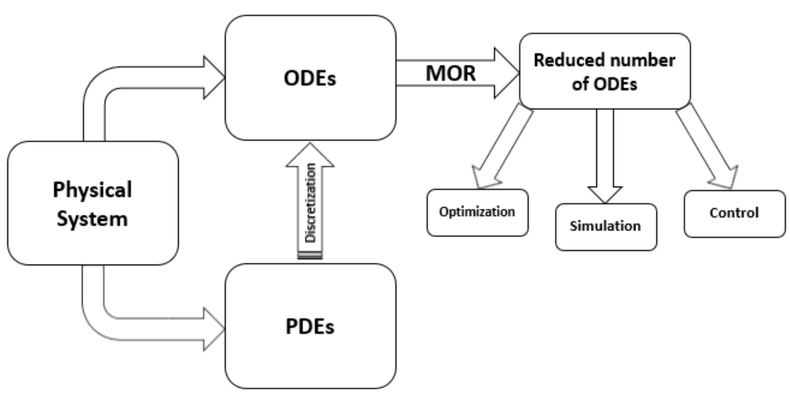

Figure 2.1: Importance of model order reduction

Several model order reduction (MOR) techniques have been proposed in literature since last four decades [1]- [9], [11, 15, 29]. These techniques can be classified into three broader categories known as Singular Value Decomposition (SVD), Moment Matching (MM), and the combination of SVD and MM. Methods based on SVD approximate the ROM according to their roots in the SVD. A famous technique which is known as balanced truncation (BT) or truncated balanced realization (TBR) [1] proposed by Moore was based on SVD approach. Moore transformed the original system into a balanced system in which every state was likewise controllable and observable. The ROM is obtained by directly truncating the low energy states from a balanced realization. The techniques which are based on SVD are very popular in control engineering because of the guaranteed stability of the ROM and easily computable error bounds [3, 4].

Techniques based on MM have the property of retaining some parameters of the original system in the ROM like time moments and Markov parameters. As the system performance depends upon the dominant poles so the main idea of MM is to remove those poles which do not play significant role in system performance. MM approximate a ROM by matching its poles to the moments of the original system. This method uses either explicit or implicit MM. Numerically the explicit MM is an unstable method, so implicit MM is widely studied in last few years [11,26]. MM is computationally efficient and it can be used for the reduction of a very large system but the main disadvantage of this method is if the order of a ROM is very small compared to the original system, the error between original system and the ROM will be very large.

To combine the advantages of SVD and MM, hybrid methods were established which were the outcome of the combination of the two methods discussed above. For example passive reduced-order interconnect macro modeling algorithm (PRIMA) [12] is used as a first stage of reduction and SVD is used as a second stage of reduction while preserving passivity as well as retaining error bound in the ROM [7]. For increasing the computational efficiency in solving the Lur'e and Lyapunov equations, another technique of combination [13] uses the SVD methods and projection based methods in these hybrid techniques.

In this chapter, we first review the BT method [1]. In BT method, the frequency response of the ROM follow the frequency response of the original system at infinite frequency. Then frequency weighted and passivity preserving MOR problem will be discussed. Next the review of frequency weighted MOR [2]- [5], [18, 22, 24] and passivity preserving frequency weighted MOR [7]- [9] will be explored.

2.2 Preliminaries

Since the proposed techniques in this thesis are mainly based on balanced realization [1], so first we discuss a brief summary of balanced realization/BT and its applications.

2.2.1 Balanced Truncation [1]

Let $G(s) = D + C(sI - A)^{-1}B$ be the n^{th} order original stable system, where the realization $\{A, B, C, D\}$ is a state space realization of $G(s)$, and $A \in \mathcal{R}^{n\times n}$, $B \in \mathcal{R}^{n\times m}$, $C \in \mathcal{R}^{p\times n}$ and $D \in \mathcal{R}^{p\times m}$. Controllability Gramian P_{BT} and observability Gramian Q_{BT} corresponding to $G(s)$ can be obtained by solving the following set of Lyapunov equations.

$$AP_{BT} + P_{BT}A^T + BB^T = 0 \tag{2.1a}$$

$$A^TQ_{BT} + Q_{BT}A + C^TC = 0 \tag{2.1b}$$

In (2.1a) P_{BT}, and in (2.1b) Q_{BT}, both matrices are symmetric and positive definite *i.e* $P_{BT} > 0$ and $Q_{BT} > 0$. Let T be the contragradient transformation which is attained by simultaneously diagonalizing the controllability Gramian P_{BT} and observability Gramian Q_{BT} such that:

$$T^{-1}P_{BT}T^{-T} = T^TQ_{BT}T = \text{diag}\{\sigma_1, \sigma_2, ..., \sigma_n\}$$

where σ_i are the Hankel Singular Values (HSV's) and $\sigma_i \geq \sigma_{i+1}$, for $i = 1, 2, ..., n-1$. The balanced realization $\{A_b, B_b, C_b, D_b\}$ is obtained as follows:

$$\left[\begin{array}{c|c} A_b & B_b \\ \hline C_b & D_b \end{array}\right] = \left[\begin{array}{c|c} TAT^{-1} & T^{-1}B \\ \hline CT & D \end{array}\right] = \left[\begin{array}{cc|c} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ \hline C_1 & C_2 & D \end{array}\right] \tag{2.2}$$

The ROM $\{A_{11}, B_1, C_1\}$ is obtained by truncating the low energy states in (2.2), where $A_{11} \in \mathcal{R}^{r\times r}$, $B_1 \in \mathcal{R}^{r\times m}$, $C_1 \in \mathcal{R}^{p\times r}$ and $D \in \mathcal{R}^{p\times m}$, and r is the order of the ROM $(r < n)$.

Remark 1 *The ROM obtained by BT method [1] does not address the stability issue in the presence of frequency weights.*

2.2.2 Properties of Balanced Truncation [1]

Following are the properties of BT [1]:

1. A state space realization $\{A, B, C, D\}$ can be transformed to a balanced realization $\{A_b, B_b, C_b, D_b\}$ if and only if it is minimal and asymptotically stable.

2. A subsystem $\{A_{ii}, B_i, C_i, D\}$ where, $i = 1, 2$, which is obtained from the original system $\{A, B, C, D\}$ is stable as well as internally balanced if no diagonal entries between two subsystems are common [14], *i.e* $\sigma_k \neq \sigma_l$ where, $k = 1, ..., r$, (where r is the order of the ROM) and $l = r + 1, ..., n$, (where n is the order of the original system).

3. The error bound can be obtained as follows

$$||G(s) - G_r(s)||_\infty \leqslant 2 \sum_{i=r+1}^{n} \sigma_i$$

where σ_i are the HSV's.

2.3 Motivation and Problem Formulation

Proposed techniques discussed in next chapter consider both stable as well as passive systems. So in this chapter, the problem of passivity preserving in MOR will be discussed, and then extension of the frequency weighting case will be elaborated.

2.3.1 Interconnect Network of RLC Circuit

Operating frequencies in communication systems are increasing with every day passing, as a result the size of circuit equations of high-frequency microwave subnetworks and interconnects are becoming large and large [15]. An interconnect network is modeled by a number of RLC elements which increases network complexity with increasing circuit elements.

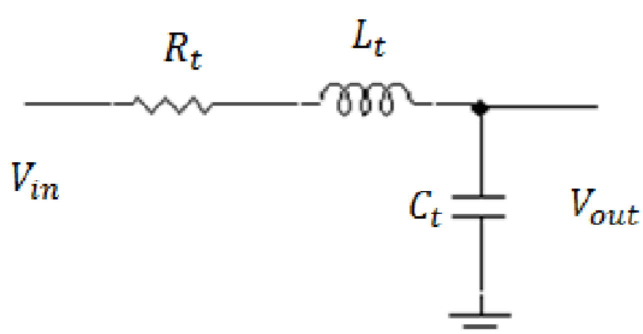

Figure 2.2: A simple lumped RLC circuit of an interconnect line

MOR can be one of the solutions to tackle large scale interconnect networks. The algorithm given in [12] generate guaranteed passive ROM for large scale interconnect models which are described by RLC type networks. A simple lumped RLC interconnect circuit is shown in Figure 2.2, and an RC interconnect line is shown in Figure 2.3.

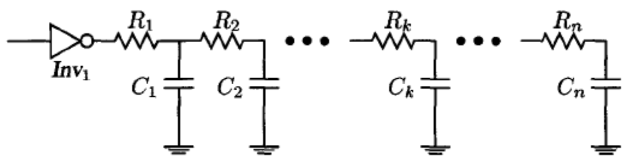

Figure 2.3: A resistive-capacitive interconnect line

2.3.2 Importance and Properties of Passivity Preserving Model Order Reduction

Passivity of a complex transfer function $G(s)$ is implied by its positive-realness for many electrical systems of interest [7]. $G(s)$ is called positive real (PR) if it is unable to generate energy internally, like an RLC circuit. A passive system always lies entirely in the right half of the complex plan called Nyquist plot, while non passive system lies in the left half of the complex plan (see Figure 2.4). For positive-realness, $G(s)$ must always satisfy the following PR condition

$$\mathcal{R}[G(s)] > 0 \quad if \quad \mathcal{R}(s) > 0$$

where $s = \sigma + j\omega$. So the PR condition can be written as

$$\mathcal{R}[G(s)] > 0 \quad if \quad \sigma > 0$$

and there is no constraint on $\mathcal{I}(s)$. Consider an n^{th} order PR system

$$G(s) = D + C(sI - A)^{-1}B \tag{2.3}$$

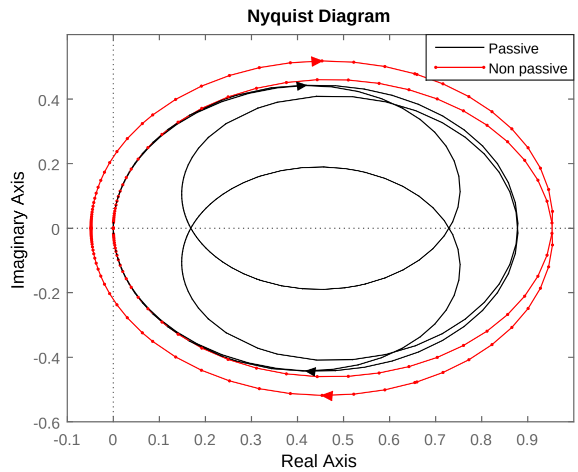

Figure 2.4: Passive and non passive systems - Nyquist diagram

where the state space realization $\{A, B, C, D\}$ is a minimal realization of $G(s)$. Because of the unique stability properties of PR systems, they are of special interest in the analysis and design of control system. Passivity is considered to be one of the very important properties of an RLC system. Since passive system is always stable but vice versa is not true [8]. So it is necessary for a ROM $G_r(s) = D + C_r(sI - A_r)^{-1}B_r$ to preserve passivity like an original system. A passive system always satisfies the following set of Lur'e equations

$$AP_{RE} + P_{RE}A^T = -K_iK_i^T \tag{2.4a}$$

$$P_{RE}C^T - B = -K_iW_i^T \tag{2.4b}$$

$$W_iW_i^T = D - D^T \tag{2.4c}$$

$$A^TQ_{RE} + Q_{RE}A = -K_o^TK_o \tag{2.5a}$$

$$Q_{RE}B - C^T = -K_o^TW_o \tag{2.5b}$$

$$W_o^TW_o = D - D^T \tag{2.5c}$$

where, $P_{RE} > 0$ is the controllability Gramian and $Q_{RE} > 0$ is the observability Gramian of the passive system, respectively. The above Lur'e equations can be solved for P_{RE} and Q_{RE} by using the following algebraic Riccati equations (AREs) [16].

$$AP_{RE} + P_{RE}A^T + (P_{RE}C^T - B)(D + D^T)^{-1}(CP_{RE} - B^T) = 0 \tag{2.6}$$

$$A^TQ_{RE} + Q_{RE}A + (Q_{RE}B - C^T)(D + D^T)^{-1}(B^TQ_{RE} - C) = 0 \tag{2.7}$$

2.3.3 A Review of Passivity Preserving Model Order Reduction

Phillips *et al.* [7] presented a family of algorithms for passive ROM's which are similar to the well-known BT method [1] for stable ROM. In [7] controllability and observability Gramians are obtained from Lur'e equations without using frequency weights. Muda *et al.* [8] extended the methods of [2]- [4] for RLC systems to ensure passivity, since [2]- [4] only yield stable ROMs. Conditions for guaranteed passivity are also given in [8] for the three extended techniques.

Heydari and Pedram claimed in [9] that their technique produces guaranteed passive ROM for the double sided frequency weighting case, and the spectrally-weighted error bound is also available. The technique of [9] produces passive ROM for the single sided weighting case similar to [2] which produces stable ROM for the case when only one sided weighting is used. In [10] it has been proved that the technique of [9] may yields non-passive ROM for the passive original system in case of double-sided frequency weighting. It is also proved in [10] that the method of [9] can preserves passivity only when one sided weighting is present.

2.4 Frequency Weighted Model Order Reduction

Let an n^{th} order stable original system $G(s)$ has state space realization as $\{A, B, C, D\}$. Let v^{th} order stable input weight $G_i(s)$ and w^{th} order stable output weight $G_o(s)$ have corresponding state space realizations as $\{A_i, B_i, C_i, D_i\}$ and $\{A_o, B_o, C_o, D_o\}$ respectively, where $A_i \in \mathcal{R}^{v\times v}$, $B_i \in \mathcal{R}^{v\times m}$, $C_i \in \mathcal{R}^{p\times v}$, and $D_i \in \mathcal{R}^{p\times m}$, and $A_o \in \mathcal{R}^{w\times w}$, $B_o \in \mathcal{R}^{w\times m}$, $C_o \in \mathcal{R}^{p\times w}$, and $D_o \in \mathcal{R}^{p\times m}$. v and w represent the number of states of input and output frequency weights respectively.

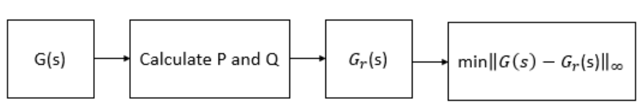

Figure 2.5: Un-weighted MOR problem

The objective of MOR is to find an r^{th} order stable ROM having state space minimal realization $\{A_r, B_r, C_r, D_r\}$, such that the error between $G(s)$ and $G_r(s)$ is made as small as possible, *i.e* min$||G_o(s)(G(s) - G_r(s))G_i(s)||_\infty$. This problem is so called the double sided frequency weighted MOR problem (see Figure 2.6).

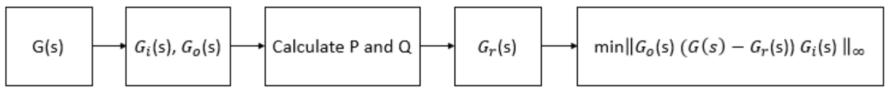

Figure 2.6: Double sided frequency weighted MOR problem

The problem is called single sided frequency weighted MOR problem if one of the either weights is identity. In this scenario, the objective is to minimize the error between $G(s)$ and $G_r(s)$, *i.e* in case of only input weight, min$||(G(s) - G_r(s))G_i(s)||_\infty$ (see Figure 2.7), and in case of only output weight, min$||G_o(s)(G(s) - G_r(s))||_\infty$ (see Figure 2.8).

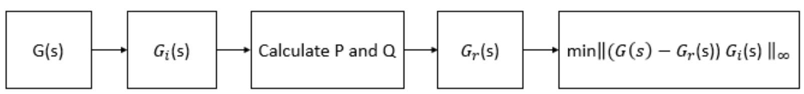

Figure 2.7: Input frequency weighted MOR problem

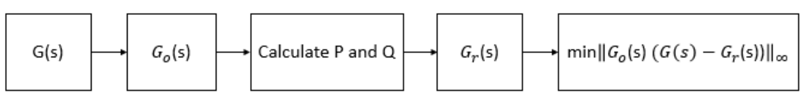

Figure 2.8: Output frequency weighted MOR problem

2.4.1 The Technique of Enns' [2]

Enns' [2] was the first to introduced frequency weights for MOR. This technique gives stable ROM for single sided frequency weighting case only. When double sided weights are used, this technique may produce unstable ROM [17]. Consider a stable original system $G(s)$ as given in (2.3). Let $G_i(s)$ and $G_o(s)$ be the input and output frequency weights

$$G_i(s) = D_i + C_i(sI - A_i)^{-1}B_i \tag{2.8a}$$

$$G_o(s) = D_o + C_o(sI - A_o)^{-1}B_o \tag{2.8b}$$

where, $\{A_i, B_i, C_i, D_i\}$ is the state space realization of input frequency weight and $\{A_o, B_o, C_o, D_o\}$ is the state space realization of output frequency weight. The input augmented system $G(s)G_i(s)$ and the output augmented system $G_o(s)G(s)$ are given as follows:

$$G(s)G_i(s) = \left[\begin{array}{c|c} \bar{A}_i & \bar{B}_i \\ \hline \bar{C}_i & \bar{D}_i \end{array}\right] = \left[\begin{array}{cc|c} A & BC_i & BD_i \\ 0 & A_i & B_i \\ \hline C & DC_i & DD_i \end{array}\right] \tag{2.9a}$$

$$G_o(s)G(s) = \left[\begin{array}{c|c} \bar{A}_o & \bar{B}_o \\ \hline \bar{C}_o & \bar{D}_o \end{array}\right] = \left[\begin{array}{cc|c} A & 0 & B \\ B_oC & A_o & B_oD \\ \hline D_oC & C_o & D_oD \end{array}\right] \tag{2.9b}$$

where, $\{\bar{A}_i, \bar{B}_i, \bar{C}_i, \bar{D}_i\}$ is the state space realization of input augmented system and $\{\bar{A}_o, \bar{B}_o, \bar{C}_o, \bar{D}_o\}$ is the state space realizations of output augmented system. Let $\bar{P}_E$ and $\bar{Q}_E$ satisfy the following Lyapunov equations.

$$\bar{A}_i\bar{P}_E + \bar{P}_E\bar{A}_i^T + \bar{B}_i\bar{B}_i^T = 0 \tag{2.10a}$$

$$\bar{A}_o^T\bar{Q}_E + \bar{Q}_E\bar{A}_o + \bar{C}_o^T\bar{C}_o = 0 \tag{2.10b}$$

where

$$\bar{P}_E = \begin{bmatrix} P_v & P_{12} \\ P_{12}^T & P_i \end{bmatrix} \tag{2.11a}$$

$$\bar{Q}_E = \begin{bmatrix} Q_o & Q_{12} \\ Q_{12}^T & Q_w \end{bmatrix} \tag{2.11b}$$

Remark 2 *$\bar{P}_E$ and $\bar{Q}_E$ obtained from above Lyapunov equations are symmetric and also $\bar{P}_E > 0$ and $\bar{Q}_E > 0$.*

Expanding the (1,1) block of (2.10a) and (2,2) block of (2.10b), we obtain:

$$AP_v + P_v A^T + X = 0 \tag{2.12a}$$

$$A^T Q_w + Q_w A + Y = 0 \tag{2.12b}$$

where

$$X = BC_i P_{12}^T + P_{12} C_i^T B^T + BD_i D_i^T B^T \tag{2.13a}$$

$$Y = C^T B_o^T Q_{12}^T + Q_{12} B_o C + C^T D_o^T D_o C \tag{2.13b}$$

The matrices X and Y are generally indefinite [4], and this is the main reason of instability of a ROM in case of double sided frequency weighting. Balancing transformation matrix T which is used to diagonalize the weighted Gramians P_v and Q_w such that $T^{-1} P_v T^{-T}$ = $T^T Q_w T$ = $diag\{\sigma_1, \sigma_2, ..., \sigma_n\}$ where σ_n are the HSV's and $\sigma_i \geq \sigma_{i+1}$ for $i = 1, 2, ..., n-1$. Then the ROM $\{A_r, B_r, C_r\} = \{A_{11}, B_1, C_1\}$ can be obtained as follows [2].

$$\left[\begin{array}{c|c} A_r & B_r \\ \hline C_r & D_r \end{array}\right] = \left[\begin{array}{c|c} TAT^{-1} & TB \\ \hline CT^{-1} & D \end{array}\right] = \left[\begin{array}{cc|c} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ \hline C_1 & C_2 & D \end{array}\right] \tag{2.14}$$

Remark 3 *The ROM obtained by [2] in the presence of double sided frequency weights may not stable but for single sided frequency weighting it will guaranteed to be stable [17].*

Wang *et al.* [4] , Varga and Anderson [18], Ghafoor and Sreeram [17], and Imran *et al.* [5] modified Enns' technique [2] to tackle the stability issue in case of double sided frequency weights.

2.4.2 The Technique of Lin and Chiu [3]

Lin and Chiu [3] modified Enns' technique [2] to tackle the stability issue for the case when double sided frequency weights are used. Their technique first defines $\bar{X} = P_{12} P_i^{-1}$ and

$\bar{Y} = Q_o^{-1} Q_{12}^T$. Let

$$\tilde{T}_i = \begin{bmatrix} I & \bar{X} \\ 0 & I \end{bmatrix} \tag{2.15a}$$

$$\tilde{T}_o = \begin{bmatrix} I & -\bar{Y} \\ 0 & I \end{bmatrix} \tag{2.15b}$$

be the transformations applied to the input and output augmented realizations $\{\bar{A}_i, \bar{B}_i, \bar{C}_i, \bar{D}_i\}$ and $\{\bar{A}_o, \bar{B}_o, \bar{C}_o, \bar{D}_o\}$ respectively,

$$\left[\begin{array}{c|c} \tilde{A}_i & \tilde{B}_i \\ \hline \tilde{C}_i & \tilde{D}_i \end{array}\right] = \left[\begin{array}{c|c} \tilde{T}_i^{-1} \bar{A}_i \tilde{T}_i & \tilde{T}_i^{-1} \bar{B}_i \\ \hline \bar{C}_i \tilde{T}_i & \bar{D}_i \end{array}\right] = \left[\begin{array}{cc|c} A & A_{i12} & B_{i11} \\ 0 & A_i & B_i \\ \hline C & C_{i12} & DD_i \end{array}\right] \tag{2.16}$$

and

$$\left[\begin{array}{c|c} \tilde{A}_o & \tilde{B}_o \\ \hline \tilde{C}_o & \tilde{D}_o \end{array}\right] = \left[\begin{array}{c|c} \tilde{T}_o^{-1} \bar{A}_o \tilde{T}_o & \tilde{T}_o^{-1} \bar{B}_o \\ \hline \bar{C}_o \tilde{T}_o & \bar{D}_o \end{array}\right] = \left[\begin{array}{cc|c} A_o & A_{o12} & B \\ 0 & A & B_{o21} \\ \hline C_{o11} & C_o & D_o D \end{array}\right] \tag{2.17}$$

where

$$A_{i12} = AP_{12}P_i^{-1} + BC_i - P_{12}P_i^{-1}A_i \tag{2.18a}$$

$$B_{i11} = BD_i - P_{12}P_i^{-1}B_i \tag{2.18b}$$

$$C_{i12} = CP_{12}P_i + DC_i \tag{2.18c}$$

$$A_{o12} = Q_o^{-1}Q_{12}^T A + B_o C - A_o Q_o^{-1} Q_{12}^T \tag{2.18d}$$

$$B_{o21} = B_o D + Q_o^{-1} Q_{12}^T B \tag{2.18e}$$

$$C_{o11} = D_o C - C_o Q_o^{-1} Q_{12}^T \tag{2.18f}$$

Let the following Gramians be defined as

$$\tilde{P}_{LC} = \tilde{T}_i^{-1} P \tilde{T}_i^{-T} = \begin{bmatrix} P_n & 0 \\ 0 & P_i \end{bmatrix} \tag{2.19a}$$

$$\tilde{Q}_{LC} = \tilde{T}_o^T Q \tilde{T}_o = \begin{bmatrix} Q_o & 0 \\ 0 & Q_n \end{bmatrix} \tag{2.19b}$$

where $P_n = P_v - P_{12} P_i^{-1} P_{12}^T$ and $Q_n = Q_w - Q_{12} Q_o^{-1} Q_{12}^T$. Let $\tilde{P}_{LC}$ and $\tilde{Q}_{LC}$ be the solutions of the following Lyapunov equations

$$\tilde{A}_i \tilde{P}_{LC} + \tilde{P}_{LC} \tilde{A}_i^T + \tilde{B}_i \tilde{B}_i^T = 0 \tag{2.20a}$$

$$\tilde{A}_o^T \tilde{Q}_{LC} + \tilde{Q}_{LC} \tilde{A}_o + \tilde{C}_o^T \tilde{C}_o = 0 \tag{2.20b}$$

Expanding the (1,1) and (2,2) blocks of (2.20a) and (2.20b) respectively, we obtain

$$A P_n + P_n A^T + B_{i11} B_{i11}^T = 0 \tag{2.21a}$$

$$A^T Q_n + Q_n A + C_{o11}^T C_{o11} = 0 \tag{2.21b}$$

Simultaneously diagonalizing the weighted controllability and observability Gramians P_n and Q_n respectively,

$$T_{LC}^{-1} P_n T_{LC}^{-T} = T_{LC}^T Q_n T_{LC} = diag\{\sigma_1, \sigma_2, ..., \sigma_n\}$$

where σ_n are the HSV's and $\sigma_i \geq \sigma_{i+1}$, for $i = 1, 2, ..., n-1$.

Remark 4 *The ROM $\{A_r, B_r, C_r\} = \{A_{11}, B_1, C_1\}$ is obtained in a same way as obtained in Enns' method [2] by truncating the low energy states in (2.14).*

Remark 5 *The realization $\{A, B_{i11}, C_{o11}\}$ is minimal and the ROM $\{A_r, B_r, C_r\} = \{A_{11}, B_1, C_1\}$ is stable [17].*

2.4.3 The Technique of Wang et al. [4]

Wang *et al.* [4] tackle the stability issue of [2] by making indefinite matrices X and Y in (2.12a) and (2.12b) positive semi-definite using eigenvalue decomposition. As we know that the matrices X and Y are symmetric matrices so we can also write X and Y in the form of eigenvalue decomposition

$$X = USU^T$$
$$Y = VZV^T$$

where $S = diag\{s_1, s_2, ..., s_n\}$ and $Z = diag\{z_1, z_2, ..., z_n\}$. Symmetric matrices X and Y are replaced by X_W and Y_W such that

$$X_W = U|S|U^T$$
$$Y_W = V|Z|V^T$$

where $|S| = diag(|s_1|, |s_2|, ..., |s_n|)$, $|Z| = diag(|z_1|, |z_2|, ..., |z_n|)$, $|s_1| \geq |s_2| \geq ... \geq |s_n| \geq 0$ and $|z_1| \geq |z_2| \geq ... \geq |z_n| \geq 0$. The new controllability Gramian P_W and observability Gramian Q_W are obtained from the following Lyapunov equations

$$AP_W + P_W A^T + B_W B_W^T = 0 \tag{2.22a}$$
$$A^T Q_W + Q_W A + C_W^T C_W = 0 \tag{2.22b}$$

where the fictitious input and output matrices B_W and C_W, respectively, are defined as $B_W = U|S|^{\frac{1}{2}}$ and $C_W = |Z|^{\frac{1}{2}}V$. The new Gramians P_W and Q_W are diagonalized by T_W such that

$$T_W^{-1} P_W T_W^{-T} = T_W^T Q_W T_W = diag\{\sigma_1, \sigma_2, ..., \sigma_n\}$$

where σ_n are the HSV's and $\sigma_i \geq \sigma_{i+1}$, for $i = 1, 2, ..., n-1$.

Remark 6 *The realization $\{A, B_W, C_W\}$ is minimal and the ROM $\{A_r, B_r, C_r\} = \{A_{11}, B_1, C_1\}$ is obtained by truncating the low energy states in (2.14) and guaranteed to be stable.*

2.5 Passivity Preserving Model Order Reduction

Passivity preserving MOR is supposed to be an extension of balanced realization, which deals with both frequency weighted and un-weighted cases. Due to certain factors such as the importance of passivity preserving in MOR techniques and to limit the computational cost in SVD based methods, so for reasonable work has been done on passivity preserving MOR [8,10], [19]- [21]. In this section, a review some of the passivity preserving MOR techniques available in literature related to both frequency weighted and un-weighted scenarios will be presented.

2.5.1 The Technique of Phillips et al. [7]

Phillips *et al.* [7] presented a family of algorithms for passive ROMs which are similar to the well-known BT method [1] for stable ROMs. The main difference between two techniques is the way controllability Gramian and observability Gramian are computed. In BT [1] controllability Gramian and observability Gramian are computed by using the Lyapunov equations without using frequency weights, while Phillips *et al.* [7] computed the controllability and observability Gramians from the Lur'e equations without using frequency weights. By using BT [1] stable ROMs are obtained, while using the technique of Phillips *et al.* [7] passive ROMs are obtained. Positive real truncated balanced realization (PR-TBR) algorithm presented in [7] gives guaranteed passive ROM for the system $G(s) = D + C(sI - A)^{-1}B$. Controllability Gramian P_P and observability Gramian Q_P are the solutions of following Lure equations

$$AP_P + P_PA^T = -K_iK_i^T \tag{2.23a}$$

$$P_PC^T - B = -K_iW_i^T \tag{2.23b}$$

$$W_iW_i^T = D + D^T \tag{2.23c}$$

$$A^TQ_P + Q_PA = -K_o^TK_o \tag{2.24a}$$

$$Q_PB - C^T = -K_o^TW_o \tag{2.24b}$$

$$W_o^TW_o = D + D^T \tag{2.24c}$$

The above Lur'e equations can also be transformed into the following ARE's [16].

$$AP_P + P_PA^T + (P_PC^T - B)(D + D^T)^{-1}(CP_P - B^T) = 0 \quad (2.25a)$$

$$A^TQ_P + Q_PA + (Q_PB - C^T)(D + D^T)^{-1}(B^TQ_P - C) = 0 \quad (2.25b)$$

Algorithm 2 in [7] computes similarity transformation matrix T_P which is used to diagonalize P_P and Q_P such that $T_P^{-1}P_PT_P^{-T} = T_P^TQ_PT_P = diag\{\sigma_1, \sigma_2, ..., \sigma_n\}$ where σ_i are the HSV's and $\sigma_i \geq \sigma_{i+1}$ for $i = 1, 2, ..., n-1$. Desired ROM $\{A_{11}, B_1, C_1\}$ is obtained from the following balanced realization.

$$\left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C} & \hat{D} \end{array}\right] = \left[\begin{array}{c|c} T_P^{-1}AT_P & T_P^{-1}B \\ \hline CT_P & D \end{array}\right] = \left[\begin{array}{cc|c} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ \hline C_1 & C_2 & D \end{array}\right] \quad (2.26)$$

Remark 7 *When the system $G(s)$ is PR then the ROM obtained by using algorithm 3 of [7] is guaranteed to be passive.*

2.5.2 Techniques of Muda et al. [8]

In [8] Muda *et al.* extended three famous techniques [2]- [4] to preserve passivity of the ROM. Consider the input and output weighting functions in (2.8a) and (2.8b) respectively, and input and output augmented systems in (2.9a) and (2.9b), respectively. Let $\bar{P}_i$ and $\bar{Q}_o$ be the solution of the following Lur'e equations:

$$\bar{A}_i\bar{P}_i + \bar{P}_i\bar{A}_i^T = -\bar{K}_i\bar{K}_i^T \quad (2.27a)$$

$$\bar{P}_i\bar{C}_i^T - \bar{B}_i = -\bar{K}_i\bar{W}_i^T \quad (2.27b)$$

$$\bar{W}_i\bar{W}_i^T = \bar{D}_i + \bar{D}_i^T \quad (2.27c)$$

$$\bar{A}_o^T\bar{Q}_o + \bar{Q}_o\bar{A}_o = -\bar{K}_o^T\bar{K}_o \tag{2.28a}$$

$$\bar{Q}_o\bar{B}_o - \bar{C}_o^T = -\bar{K}_o^T\bar{W}_o \tag{2.28b}$$

$$\bar{W}_o^T\bar{W}_o = \bar{D}_o + \bar{D}_o^T \tag{2.28c}$$

where

$$\bar{W}_i = (\bar{D}_i + \bar{D}_i^T)^{\frac{1}{2}}V \tag{2.29a}$$

$$\bar{W}_o = U(\bar{D}_o + \bar{D}_o^T)^{\frac{1}{2}} \tag{2.29b}$$

$$\bar{K}_i = (\bar{B}_i - \bar{P}_i\bar{C}_i^T)(\bar{D}_i + \bar{D}_i^T)^{-\frac{1}{2}}V \tag{2.30a}$$

$$\bar{K}_o = U(\bar{D}_o + \bar{D}_o^T)^{-\frac{1}{2}}(\bar{C}_o - \bar{B}_o^T\bar{Q}_o) \tag{2.30b}$$

where V and U are arbitrary orthogonal matrices *i.e* $UU^T = VV^T = I$. $\bar{K}_i$ and $\bar{K}_o$ can be sub-divided as

$$\bar{K}_i = \begin{bmatrix} K_{i1} \\ K_{i2} \end{bmatrix}, \quad \bar{K}_o = \begin{bmatrix} K_{o1} & K_{o2} \end{bmatrix}$$

The Lur'e equations in (2.27a),(2.27b),(2.27c) and (2.28a),(2.28b),(2.28c) can also be written as the following ARE's

$$\bar{A}_i\bar{P}_i + \bar{P}_i\bar{A}_i^T + (\bar{P}_i\bar{C}_i^T - \bar{B}_i)(\bar{D}_i + \bar{D}_i^T)^{-1}(\bar{C}_i\bar{P}_i - \bar{B}_i^T) = 0 \tag{2.31a}$$

$$\bar{A}_o^T\bar{Q}_o + \bar{Q}_o\bar{A}_o + (\bar{Q}_o\bar{B}_o - \bar{C}_o^T)(\bar{D}_o + \bar{D}_o^T)^{-1}(\bar{B}_o^T\bar{Q}_o - \bar{C}_o) = 0 \tag{2.31b}$$

where

$$\bar{P}_i = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \tag{2.32a}$$

$$\bar{Q}_o = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \tag{2.32b}$$

Expanding (1,1) block of (2.27a),(2.27b),(2.27c) and (2.28a),(2.28b),(2.28c), we get

$$AP_{11} + P_{11}A^T = -\check{X} \tag{2.33a}$$

$$P_{11}C^T - \check{B} = -K_{i1}W_i^T \tag{2.33b}$$

$$W_iW_i^T = \bar{D}_i + \bar{D}_i^T \tag{2.33c}$$

$$A^TQ_{11} + Q_{11}A = -\check{Y} \tag{2.34a}$$

$$Q_{11}B - \check{C}^T = -K_{o1}^TW_o \tag{2.34b}$$

$$W_o^TW_o = \bar{D}_o + \bar{D}_o^T \tag{2.34c}$$

where

$$\check{X} = BC_iP_{12}^T + P_{12}C_i^TB^T + K_{i1}K_{i1}^T \tag{2.35a}$$

$$\check{Y} = C^TB_o^TQ_{12}^T + Q_{12}B_oC + K_{o1}^TK_{o1} \tag{2.35b}$$

$\check{B} = BD_i - P_{12}C_i^TD^T$ and $\check{C} = D_oC - D^TB_o^TQ_{12}^T$. The difference between Muda's extended techniques [8] is that the way frequency weighted controllability and observability Gramians are computed.

The Modified Enns' Technique [8]

Rewriting (2.35a) and (2.35b) as

$$\check{X} = \Psi + K_{i1}K_{i1}^T \tag{2.36a}$$

$$\check{Y} = \Phi + K_{o1}^TK_{o1} \tag{2.36b}$$

where $\Psi = BC_iP^T + P_{12}C_i^TB^T$ and $\Phi = C^TB_o^TQ_{12}^T + Q_{12}B_oC$. The modified Enns' technique [8] is very similar to the standard Enns' technique [2]. In standard Enns' technique

[2], the indefinite matrices X and Y in (2.12a) and (2.12b) are the reason of instability in case of double sided weighting. Similarly, in modified Enns' technique [8], the indefinite matrices Ψ and Φ in (2.36a) and (2.36b) are the reason of not ensuring passivity for double sided frequency weights.

Remark 8 *The ROM obtained from modified Enns' technique [8] is passive only when the matrices Ψ and Φ are positive semi-definite. If the matrices Ψ and Φ in (2.36a) and (2.36b) are indefinite, then this technique may yields non passive ROM for double sided frequency weights.*

Remark 9 *The weighted Gramians P_{11} and Q_{11} are used to obtained transformation matrix T_E which is then used to balance the original system.*

The Modified Wang *et al.*'s Technique [8]

The matrices $\check{X}$ and $\check{Y}$ given in (2.36a) and (2.36b) are generally indefinite [4]. Inspiring from [4], the indefinite matrices $\check{X}$ and $\check{Y}$ can be made positive semi definite by taking the absolute of the eigenvalues of matrices $\check{X}$ and $\check{Y}$ by using eigenvalue decomposition such that $\check{X} = \Delta S \Delta^T$ and $\check{Y} = \Lambda Z \Lambda^T$ where, $S = diag\{s_1, s_2, ..., s_n\}$, and $Z = diag\{z_1, z_2, ..., z_n\}$. Now we shall replace $\check{X}$ and $\check{Y}$ by $\bar{K}_{i1}\bar{K}_{i1}^T$ and $\bar{K}_{o1}^T\bar{K}_{o1}$ such that, $\bar{K}_{i1}\bar{K}_{i1}^T = \Delta|S|\Delta^T$ and $\bar{K}_{o1}^T\bar{K}_{o1} = \Lambda|Z|\Lambda^T$ where, $|s_1| \geq |s_2| \geq ... \geq |s_n| \geq 0$ and $|z_1| \geq |z_2| \geq ... \geq |z_n| \geq 0$. Now we can write (2.33a), (2.33b), (2.33c) and (2.34a), (2.34b), (2.34c) as

$$AP_{w1} + P_{w1}A^T = -\bar{K}_{i1}\bar{K}_{i1}^T \tag{2.37a}$$

$$P_{w1}C^T - \hat{B} = -\bar{K}_{i1}\bar{W}_i^T \tag{2.37b}$$

$$\bar{W}_i\bar{W}_i^T = \bar{D}_i + \bar{D}_i^T \tag{2.37c}$$

$$A^TQ_{w1} + Q_{w1}A = -\bar{K}_{o1}^T\bar{K}_{o1} \tag{2.38a}$$

$$Q_{w1}B - \hat{C}^T = -\bar{K}_{o1}^T\bar{W}_o \tag{2.38b}$$

$$\bar{W}_o^T\bar{W}_o = \bar{D}_o + \bar{D}_o^T \tag{2.38c}$$

where $\hat{B} = P_{w1}C^T + \bar{K}_{i1}\bar{W}_i^T$ and $\hat{C} = B^TQ_{w1} + C^T + \bar{W}_o^T\bar{K}_{o1}$

Remark 10 *If the matrices $\check{X}$ and $\check{Y}$ in (2.33a) and (2.34a) respectively, are already positive semi-definite then, both modified Enns' and modified Wang et al. methods are same and guaranteed to be passive. In this case eigenvalue decomposition of $\check{X}$ and $\check{Y}$ is not required.*

Remark 11 *The state space realization $\{A, \hat{B}, \hat{C}\}$ is minimal and the ROM $\{A_r, B_r, C_r\} = \{A_{11}, B_1, C_1\}$ is obtained by truncating the low energy states as obtained from (2.14) and guaranteed to be passive.*

The Modified Lin and Chiu's Technique [8]

This modified technique is similar to Lin and Chiu's technique discussed in [3]. In this technique controllability and observability Gramians of input and output augmented systems respectively, are transformed into a block diagonal form to compute the frequency weighted controllability and observability Gramians of the original system using the following transformation matrices

$$T_i = \left[\begin{array}{c|c} I & \bar{X}_i \\ \hline 0 & I \end{array}\right] \tag{2.39a}$$

$$T_o = \left[\begin{array}{c|c} I & 0 \\ \hline -\bar{Y}_o & I \end{array}\right] \tag{2.39b}$$

where $\bar{X}_i = P_{12}P_{22}^{-1}$ and $\bar{Y}_o = Q_{22}^{-1}Q_{12}^T$. The Gramians P_i and Q_o are obtained by transforming $\bar{P}_i$ and $\bar{Q}_o$ as follows

$$P_i = T_i^{-1}\bar{P}_i T_i^{-T} = \left[\begin{array}{c|c} \hat{P} & 0 \\ \hline 0 & P_{22} \end{array}\right] \tag{2.40a}$$

$$Q_o = T_o^T\bar{Q}_o T_o = \left[\begin{array}{c|c} \hat{Q} & 0 \\ \hline 0 & Q_{22} \end{array}\right] \tag{2.40b}$$

where $\hat{P} = P_{11} - P_{12}P_{22}^{-1}P_{12}^T$ and $\hat{Q} = Q_{11} - Q_{12}Q_{22}^{-1}Q_{12}^T$. The transformed input and output augmented realizations $\{\tilde{A}_i, \tilde{B}_i, \tilde{C}_i, \tilde{D}_i\}$ and $\{\tilde{A}_o, \tilde{B}_o, \tilde{C}_o, \tilde{D}_o\}$ respectively, are as follows:

$$\left[\begin{array}{c|c}\tilde{A}_i & \tilde{B}_i \\ \hline \tilde{C}_i & \tilde{D}_i\end{array}\right] = \left[\begin{array}{c|c}T_i^{-1}\bar{A}_iT_i & T_i^{-1}\bar{B}_i \\ \hline \bar{C}_iT_i & \bar{D}_i\end{array}\right] = \left[\begin{array}{cc|c}A & A_{i12} & B_{i11} \\ 0 & A_i & B_i \\ \hline C & C_{i12} & \bar{D}_i\end{array}\right] \tag{2.41}$$

$$\left[\begin{array}{c|c}\tilde{A}_o & \tilde{B}_o \\ \hline \tilde{C}_o & \tilde{D}_o\end{array}\right] = \left[\begin{array}{c|c}T_o^{-1}\bar{A}_oT_o & T_o^{-1}\bar{B}_o \\ \hline \bar{C}_oT_o & \bar{D}_o\end{array}\right] = \left[\begin{array}{cc|c}A & 0 & B \\ A_{o21} & A_o & B_{o21} \\ \hline C_{o11} & C_o & \bar{D}_o\end{array}\right] \tag{2.42}$$

where

$$A_{i12} = A\bar{X}_i + BC_i - X_iA_i \tag{2.43a}$$

$$B_{i11} = BD_i - \bar{X}_iB_i \tag{2.43b}$$

$$C_{i12} = C\bar{X}_i + DC_i \tag{2.43c}$$

$$A_{o21} = \bar{Y}_oA + B_oC - A_o\bar{Y}_o \tag{2.43d}$$

$$B_{o21} = \bar{Y}_oB + B_oD \tag{2.43e}$$

$$C_{o11} = D_oC - C_o\bar{Y}_o \tag{2.43f}$$

The augmented realizations satisfy the following ARE's:

$$\tilde{A}_iP_i + P_i\tilde{A}_i^T + (P_i\tilde{C}_i^T - \tilde{B}_i)(\tilde{D}_i + \tilde{D}_i^T)^{-1}(\tilde{C}_iP_i - \tilde{B}_i^T) = 0 \tag{2.44a}$$

$$\tilde{A}_o^TQ_o + Q_o\tilde{A}_o + (Q_o\tilde{B}_o - \tilde{C}_o^T)(\tilde{D}_o + \tilde{D}_o^T)^{-1}(\tilde{B}_o^TQ_o - \tilde{C}_o) = 0 \tag{2.44b}$$

Expanding the (1,1) blocks of (2.44a) and (2.44b) yield

$$A\hat{P} + \hat{P}A^T + (\hat{P}C^T - B_{i11})(\bar{D}_i + \bar{D}_i^T)^{-1}(C\hat{P} - B_{i11}^T) = 0 \tag{2.45a}$$

$$A^T\hat{Q} + \hat{Q}A + (\hat{Q}B - C_{o11}^T)(\bar{D}_o + \bar{D}_o^T)^{-1}(B^T\hat{Q} - C_{o11}) = 0 \tag{2.45b}$$

Remark 12 *As the frequency weighted Gramians $\hat{P}$ and $\hat{Q}$ satisfy the above ARE's, these Gramians also satisfy the corresponding Lur'e equation so, the ROM obtained from modified*

Lin and Chiu's technique [8] is guaranteed to be passive.

Remark 13 *The realization* $\{A, B_{i11}, C_{o11}\}$ *is minimal, and the ROM is obtained by balancing and partitioning this minimal realization.*

2.5.3 The Technique of Heydari and Pedram [9]

The technique of Heydari and Pedram [9] is the extension of Phillips *et al.*'s technique [7] to include the effect of frequency weights in MOR. Let $G_i(s)$ and $G_o(s)$ be the PR input and output frequency weights respectively, as given in (2.8a) and (2.8b) with state space realizations $\{A_i, B_i, C_i, D_i\}$ and $\{A_o, B_o, C_o, D_o\}$. Let the augmented systems $G(s)G_i(s)$ and $G_o(s)G(s)$ as defined in (2.9a) and (2.9b) respectively. Let $\bar{P}$ and $\bar{Q}$ are obtained from the following AREs

$$\bar{A}_i\bar{P} + \bar{P}\bar{A}_i^{\,T} + (\bar{P}\bar{C}_i^{\,T} - \bar{B}_i)(\bar{D}_i + \bar{D}_i^{\,T})^{-1}(\bar{C}_i\bar{P} - \bar{B}_i^{\,T}) = 0 \tag{2.46a}$$

$$\bar{A}_o^{\,T}\bar{Q} + \bar{Q}\bar{A}_o + (\bar{Q}\bar{B}_o - \bar{C}_o^{\,T})(\bar{D}_o + \bar{D}_o^{\,T})^{-1}(\bar{B}_o^{\,T}\bar{Q} - \bar{C}_o) = 0 \tag{2.46b}$$

where

$$\bar{P} = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \tag{2.47a}$$

$$\bar{Q} = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \tag{2.47b}$$

Expanding the (1,1) block of (2.46a) and (2.46b) yields

$$AP_{11} + P_{11}A^T + \underbrace{BC_iP_{12}^T + P_{12}C_i^TB^T} + (P_{11}C^T + \underbrace{P_{12}C_i^TD^T} - BD_i)(DD_i + D_i^TD^T)^{-1}$$
$$(CP_{11} + \underbrace{DC_iP_{12}^T} - D_i^TB^T) = 0 \tag{2.48a}$$

$$A^TQ_{11} + Q_{11}A + \underbrace{C^TB_o^TQ_{12}^T + Q_{12}B_oC} + (Q_{11}B + \underbrace{Q_{12}B_oD} - C^TD_o^T)(D_oD + D^TD_o^T)^{-1}$$
$$(B^TQ_{11} + \underbrace{D^TB_o^TQ_{12}^T} - D_oC) = 0 \tag{2.48b}$$

In (2.48a) and (2.48b), we define the combined effect of under braced terms as $\bar{X}$ and $\bar{Y}$ respectively, where

$$\bar{X} = BC_iP_{12}^T + P_{12}C_i^TB^T + P_{11}C^T(DD_i + D_i^TD^T)^{-1}DC_iP_{12}^T + P_{12}C_i^TD^T(DD_i + D_i^TD^T)^{-1}CP_{11} + P_{12}C_i^TD^T(DD_i + D_i^TD^T)^{-1}DC_iP_{12}^T - P_{12}C_i^TD^T(DD_i + D_i^TD^T)^{-1}D_i^TB^T - BD_i(DD_i + D_i^TD^T)^{-1}DC_iP_{12}^T \tag{2.49a}$$

$$\bar{Y} = C^TB_o^TQ_{12}^T + Q_{12}B_oC + Q_{11}B(D_oD + D^TD_o^T)^{-1}D^TB_o^TQ_{12}^T + Q_{12}B_oD(D_oD + D^TD_o^T)^{-1}B^TQ_{11} + Q_{12}B_oD(D_oD + D^TD_o^T)^{-1}D^TB_o^TQ_{12}^T - Q_{12}B_oD(D_oD + D^TD_o^T)^{-1}D_oC - C^TD_o^T(D_oD + D^TD_o^T)^{-1}D^TB_o^TQ_{12}^T \tag{2.49b}$$

Similar to the technique of [4] for stable ROMs, Heydari and Pedram [9] made generally indefinite symmetric matrices $\bar{X}$ and $\bar{Y}$ in (2.49a) and (2.49b) positive semi definite by taking absolute of the eigenvalues of $\bar{X}$ and $\bar{Y}$ using eigenvalue decomposition such that $\bar{X} = USU^T$ and $\bar{Y} = VZV^T$, where $S = diag\{s_1, s_2, ..., s_n\}$, $Z = diag\{z_1, z_2, ..., z_n\}$, $|s_1| \geq |s_2| \geq ... \geq |s_n| \geq 0$ and $|z_1| \geq |z_2| \geq ... \geq |z_n| \geq 0$.

Balancing transformation matrix T_H which is used to diagonalize the weighted Gramians $P_H = P_{11}$ and $Q_H = Q_{11}$ such that $T_H^{-1}P_HT_H^{-T} = T_H^TQ_HT_H = diag\{\sigma_{h1}, \sigma_{h2}, ..., \sigma_{hn}\}$ where σ_{hi} are the HSV's and $\sigma_{hi} \geq \sigma_{hi+1}$ for $i = 1, 2, ..., n-1$. Then the ROM $\{A_r, B_r, C_r\} = \{A_{11}, B_1, C_1\}$ can be obtained as follows [9].

$$\left[\begin{array}{c|c} A_b & B_b \\ \hline C_b & D_b \end{array}\right] = \left[\begin{array}{c|c} T_HAT_H^{-1} & T_HB \\ \hline CT_H^{-1} & D \end{array}\right] = \left[\begin{array}{cc|c} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ \hline C_1 & C_2 & D \end{array}\right] \tag{2.50}$$

2.6 Muda *et al.*'s Comment on Heydari and Pedram's Technique [10]

Muda *et al.* [10] proved that the technique of Heydari and Pedram [9] neither yields passive nor stable ROM in the case of double sided frequency weights. And passivity is guaranteed only for the case of single sided weighting. Let $G_i(s)$ and $G_o(s)$ be the input and output frequency weights as given in (2.8a) and (2.8b). Let the input and output augmented systems

$G(s)G_i(s)$ and $G_o(s)G(s)$ respectively, as defined in (2.9a) and (2.9b).

Let the controllability Gramian $\bar{P}_i$ and observability Gramian $\bar{Q}_o$ be the solutions of the Lur'e equations in (2.27a), (2.27b), (2.27c) and (2.28a), (2.28b), (2.28c), respectively. If $D = 0$, then the Lur'e equations in (2.27a), (2.27b), (2.27c) and (2.28a), (2.28b), (2.28c) reduces to

$$\bar{A}_i\bar{P}_i + \bar{P}_i\bar{A}_i^T = -\bar{K}_i\bar{K}_i^T \tag{2.51a}$$

$$\bar{P}_i\bar{C}_i^T = \bar{B}_i \tag{2.51b}$$

$$\bar{A}_o^T\bar{Q}_o + \bar{Q}_o\bar{A}_o = -\bar{K}_o^T\bar{K}_o \tag{2.52a}$$

$$\bar{Q}_o\bar{B}_o = \bar{C}_o^T \tag{2.52b}$$

where

$$\bar{P}_i = \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \tag{2.53a}$$

$$\bar{Q}_o = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12}^T & Q_{22} \end{bmatrix} \tag{2.53b}$$

and

$$\bar{K}_i = \begin{bmatrix} K_{i1} \\ K_{i2} \end{bmatrix}, \quad \bar{K}_o = \begin{bmatrix} K_{o1} & K_{o2} \end{bmatrix} \tag{2.54}$$

Remark 14 *For $D = 0$ the solution of the Lur'e equations is not as simple as when $D \neq 0$. For $D = 0$, Lur'e equations can be solved by the method of [16].*

Expanding the (1,1) block of (2.51a), (2.51b) and (2.52a), (2.52b) yield

$$AP_{11} + P_{11}A^T = -\check{X} \tag{2.55a}$$
$$P_{11}C^T = BD_i \tag{2.55b}$$

$$A^TQ_{11} + Q_{11}A = -\check{Y} \tag{2.56a}$$
$$Q_{11}B = C^TD_o^T \tag{2.56b}$$

where $\check{X}$ and $\check{Y}$ are same as defined in (2.35a) and (2.35b). The matrices $\check{X}$ and $\check{Y}$ are generally indefinite. To ensure passivity, these matrices should be positive semi-definite. This can be accomplished by eigenvalue decomposition such that $\check{X} = \Delta S \Delta^T$ and $\check{Y} = \Lambda Z \Lambda^T$ where, $S = diag\{s_1, s_2, ..., s_n\}$, and $Z = diag\{z_1, z_2, ..., z_n\}$. Now we shall replace $\check{X}$ and $\check{Y}$ by $\bar{K}_{i1}\bar{K}_{i1}^T$ and $\bar{K}_{o1}^T\bar{K}_{o1}$ such that, $\bar{K}_{i1}\bar{K}_{i1}^T = \Delta|S|\Delta^T$ and $\bar{K}_{o1}^T\bar{K}_{o1} = \Lambda|Z|\Lambda^T$ where, $|s_1| \geq |s_2| \geq ... \geq |s_n| \geq 0$ and $|z_1| \geq |z_2| \geq ... \geq |z_n| \geq 0$.
Now we can write (2.55a), (2.55b) and (2.56a), (2.56b) as

$$AP_h + P_hA^T = -\bar{K}_{i1}\bar{K}_{i1}^T \tag{2.57a}$$
$$P_hC^T = \hat{B} \tag{2.57b}$$

$$A^TQ_h + Q_hA = -\bar{K}_{o1}^T\bar{K}_{o1} \tag{2.58a}$$
$$Q_hB = \hat{C}^T \tag{2.58b}$$

Transformation matrix $\hat{T}$ is used for diagonalizing the Gramians P_h and Q_h, *i.e*; $\hat{T}^{-1}P_h\hat{T}^{-T} = \hat{T}^TQ_h\hat{T} = diag\{\sigma_1, \sigma_2, ..., \sigma_n\}$ where σ_i are the HSV's and $\sigma_i \geq \sigma_{i+1}$ for $i = 1, 2, ..., n-1$. Then the ROM $\{A_{11}, B_1, C_1\}$ can be obtained from the following balanced realization.

$$
\left[\begin{array}{c|c} A_b & B_b \\ \hline C_b & D_b \end{array}\right] = \left[\begin{array}{c|c} \hat{T}A\hat{T}^{-1} & \hat{T}B \\ \hline C\hat{T}^{-1} & D \end{array}\right] = \left[\begin{array}{cc|c} A_{11} & A_{12} & B_1 \\ A_{21} & A_{22} & B_2 \\ \hline C_1 & C_2 & D \end{array}\right] \tag{2.59}
$$

Remark 15 *The ROM obtained from Heydari and Pedram [9] is not passive because (2.57a), (2.57b) and (2.58a), (2.58b do not correspond to the same system but two different systems* $\{A, \hat{B}, C\}$ *and* $\{A, B, \hat{C}\}$*, respectively.*

Remark 16 *The transformation matrix* $\hat{T}$*, which is obtained by diagonalizing the Gramians of two different systems* $\{A, \hat{B}, C\}$ *and* $\{A, B, \hat{C}\}$*, applied to the original system* $\{A, B, C\}$ *does not produce passive ROM.*

2.7 Summary

The existing techniques discussed in this chapter are summarized as:

The ROM obtained from BT [1] is guaranteed stable but without the presence of frequency weights. Enn's technique [2] does not yield guaranteed stable ROM in case of double sided frequency weights. The techniques of Lin and Chiu [3] and Wang *et al.* [4] produce guaranteed stable ROM's when double sided frequency weights are used. Although, The Lin and Chiu's method [3] produces guaranteed stable ROM but due to pole-zero cancellation of the controller with the frequency weights, this technique can not be used in the controller reduction applications. Wang *et al.*'s method [4] gives frequency response error bound as well.

The technique of Phillips *et al.* [7] gives passive ROM without the presence of frequency weights while the technique of Muda *et al.* [8] gives passive ROM in the presence of frequency weights. Heydari and Pedram's [9] technique does not guarantee passivity in case of double sided weights [10].

2.8 Conclusion

First of all we discussed the important properties of BT and frequency weighted MOR. Then we formulated together frequency weighted and passivity preserving MOR problem and also discussed the important properties of frequency weighted passivity preserving MOR. Several frequency weighted and un-weighted MOR techniques were discussed and their important characteristics were also presented. We can conclude that, like un-weighted MOR, preserving stability and passivity of the ROM as well as low approximation error are also desirable in the frequency weighted MOR.

Chapter 3

Development of New Passivity-Preserving Model Order Reduction Schemes

3.1 Introduction

Phillips *et al.* [7] presented a family of algorithms for passive ROM which are similar to the well-known BT method [1] for stable ROMs. In [7] controllability and observability Gramians are obtained by solving the Lur'e equations without using frequency weights. Muda *et al.* [8] extended the methods of [2]- [4] for RLC systems to ensure passivity, since [2]- [4] yield only stable ROMs. Conditions for guaranteed passivity are also given in [8] for the three extended techniques. A relationship between the Lur'e equations and the algebraic Riccati equations (ARE's) is also given in [8]. For $D = 0$ in a state space realization $\{A, B, C, D\}$, the solution of an ARE is not straight forward. Both cases, for $D = 0$ and $D \neq 0$, are discussed in detail in [16]. Some frequency weighted passivity preserving MOR techniques appear in [19], [20], [21].

Heydari and Pedram claimed in [9] that their technique produces guaranteed passive ROM for the double sided frequency weighting case, and the spectrally-weighted error bounds are also available. The technique of [9] produces passive ROM for the single sided weighting case similar to [2] which produces stable ROM for the case when only one sided weighting is used. In [10] it has been proved that the technique of [9] may yields non-passive ROM for the passive original system in case of double-sided weighting. [10] also proved that the method of [9] can preserve passivity only when one sided weighting is present. In this chapter, a family of frequency weighted passivity preserving MOR algorithms are proposed which ensure stability and passivity of the ROM for both single and double sided weighting case.

3.2 Preliminaries

Let a passive system $G(s)$ has state space realization $\{A,B,C,D\}$. Let the input weight $G_i(s)$ and output weight $G_o(s)$ have corresponding state space realizations as $\{A_i,B_i,C_i,D_i\}$

and $\{A_o,B_o,C_o,D_o\}$ respectively. Let the augmented system $G_o(s)G(s)G_i(s)$ has the corresponding state space realization $\{\bar{A},\bar{B},\bar{C},\bar{D}\}$, where

$$G_o(s)G(s)G_i(s) = \left[\begin{array}{c|c} \bar{A} & \bar{B} \\ \hline \bar{C} & \bar{D} \end{array}\right] = \left[\begin{array}{ccc|c} A_o & B_oC & B_oDC_i & B_oDD_i \\ 0 & A & BC_i & BD_i \\ 0 & 0 & A_i & B_i \\ \hline C_o & D_oC & D_oDC_i & D_oDD_i \end{array}\right] \tag{3.1}$$

Let the transformations T_i and T_o be defined as

$$\bar{T}_i = \begin{bmatrix} I & X_i & \frac{1}{2}X_iY_i \\ 0 & I & Y_i \\ 0 & 0 & I \end{bmatrix} \tag{3.2a}$$

$$\bar{T}_o = \begin{bmatrix} I & X_o & X_oY_o \\ 0 & I & Y_o \\ 0 & 0 & I \end{bmatrix} \tag{3.2b}$$

where

$$Y_i = P_{23}P_{33}^{-1} \tag{3.3a}$$

$$X_i = (P_{12} - P_{13}Y_i^T)(P_{22} - \frac{1}{2}Y_iP_{23}^T - P_{23}Y_i^T + \frac{1}{2}Y_iP_{33}Y_i^T)^{-1} \tag{3.3b}$$

$$X_o = -Q_{11}^{-1}Q_{12} \tag{3.3c}$$

$$Y_o = -(\frac{1}{2}X_o^TQ_{11}X_o + \frac{1}{2}Q_{12}^TX_o + X_o^TQ_{12} + Q_{22})^{-1}(X_o^TQ_{13} + Q_{23}) \tag{3.3d}$$

The transformed input and output augmented realizations $\{\bar{A}_i,\bar{B}_i,\bar{C}_i,\bar{D}_i\}$ and $\{\bar{A}_o,\bar{B}_o,\bar{C}_o,\bar{D}_o\}$ respectively, are as follows:

$$\left[\begin{array}{c|c}\bar{A}_i & \bar{B}_i \\ \hline \bar{C}_i & \bar{D}_i\end{array}\right] = \left[\begin{array}{c|c}\bar{T}_i^{-1}\bar{A}\bar{T}_i & \bar{T}_i^{-1}\bar{B} \\ \hline \bar{C}\bar{T}_i & \bar{D}\end{array}\right] = \left[\begin{array}{ccc|c} A_o & A_{i12} & A_{i13} & B_{i11} \\ 0 & A & A_{i23} & B_{i21} \\ 0 & 0 & A_i & B_i \\ \hline C_o & C_{i12} & C_{i13} & D_i \end{array}\right] \tag{3.4}$$

$$\left[\begin{array}{c|c}\bar{A}_o & \bar{B}_o \\ \hline \bar{C}_o & \bar{D}_o\end{array}\right] = \left[\begin{array}{c|c}\bar{T}_o^{-1}\bar{A}\bar{T}_o & \bar{T}_o^{-1}\bar{B} \\ \hline \bar{C}\bar{T}_o & \bar{D}\end{array}\right] = \left[\begin{array}{ccc|c} A_o & A_{o12} & A_{o13} & B_{o11} \\ 0 & A & A_{o23} & B_{o21} \\ 0 & 0 & A_i & B_o \\ \hline C_o & C_{o12} & C_{o13} & D_o \end{array}\right] \tag{3.5}$$

where

$$A_{i12} = A_oX_i + B_oC - X_iA \tag{3.6a}$$

$$A_{i13} = \frac{1}{2}A_oX_iY_i + B_oCY_i - X_iAY_i + B_oDC_i - X_iBC_i + \frac{1}{2}X_iY_iA_i \tag{3.6b}$$

$$A_{i23} = AY_i + BC_i - Y_iA_i \tag{3.6c}$$

$$B_{i11} = B_oDD_i - X_iBD_i + \frac{1}{2}X_iY_iB_i \tag{3.6d}$$

$$B_{i21} = BD_i - Y_iB_i \tag{3.6e}$$

$$C_{i12} = C_oX_i + D_oC \tag{3.6f}$$

$$C_{i13} = \frac{1}{2}C_oX_iY_i + D_oCY_i + D_oDC_i \tag{3.6g}$$

$$A_{o12} = A_oX_o + B_oC - X_oA \tag{3.6h}$$

$$A_{o13} = A_oX_oY_o + B_oCY_o - X_oAY_o + B_oDC_i - X_oBC_i \tag{3.6i}$$

$$A_{o23} = AY_o + BC_i - Y_oA_i \tag{3.6j}$$

$$B_{o11} = B_oDD_i - X_oBD_i \tag{3.6k}$$

$$B_{o21} = BD_i - Y_oB_i \tag{3.6l}$$

$$C_{o12} = C_oX_o + D_oC \tag{3.6m}$$

$$C_{o13} = C_oX_oY_o + D_oCY_o + D_oDC_i \tag{3.6n}$$

Let the un-weighted Lyapunov based controllability and observability Gramians P_{UL} and

Q_{UL} respectively, corresponding to $G(s)$, be defined as:

$$AP_{UL} + P_{UL}A^T + BB^T = 0 \tag{3.7a}$$

$$A^TQ_{UL} + Q_{UL}A + C^TC = 0 \tag{3.7b}$$

while the weighted Lyapunov based controllability and observability Gramians $\bar{P}_{WL}$ and $\bar{Q}_{WL}$ respectively, corresponding to (3.1), be defined as:

$$\bar{A}\bar{P}_{WL} + \bar{P}_{WL}\bar{A}^T + \bar{B}\bar{B}^T = 0 \tag{3.8a}$$

$$\bar{A}^T\bar{Q}_{WL} + \bar{Q}_{WL}\bar{A} + \bar{C}^T\bar{C} = 0 \tag{3.8b}$$

where

$$\bar{P}_{WL} = \begin{bmatrix} P_{L11} & P_{L12} & P_{L13} \\ P_{L12}^T & P_{L22} & P_{L23} \\ P_{L13}^T & P_{L23}^T & P_{L33} \end{bmatrix} \tag{3.9a}$$

$$\bar{Q}_{WL} = \begin{bmatrix} Q_{L11} & Q_{L12} & Q_{L13} \\ Q_{L12}^T & Q_{L22} & Q_{L23} \\ Q_{L13}^T & Q_{L23}^T & Q_{L33} \end{bmatrix} \tag{3.9b}$$

3.3 Main Result

Expanding the (2,2) blocks of (3.8a) and (3.8b) yield

$$AP_{L22} + P_{L22}A^T + BD_iD_i^TB^T + \underbrace{BC_iP_{L23}^T + P_{L23}C_i^TB^T} = 0 \tag{3.10a}$$

$$A^TQ_{L22} + Q_{L22}A + C^TD_o^TD_oC + \underbrace{C^TB_o^TQ_{L12} + Q_{L12}^TB_oC} = 0 \tag{3.10b}$$

Moreover, let the un-weighted ARE based controllability and observability Gramians P_{UA}

and Q_{UA} respectively, corresponding to $G(s)$, be defined as:

$$AP_{UA} + P_{UA}A^T + (P_{UA}C^T - B)(D + D^T)^{-1}(CP_{UA} - B^T) = 0 \tag{3.11a}$$

$$A^TQ_{UA} + Q_{UA}A + (Q_{UA}B - C^T)(D + D^T)^{-1}(B^TQ_{UA} - C) = 0 \tag{3.11b}$$

while the weighted ARE based controllability and observability Gramians $\bar{P}_{WA}$ and $\bar{Q}_{WA}$ respectively, corresponding to (3.1), be defined as:

$$\bar{A}\bar{P}_{WA} + \bar{P}_{WA}\bar{A}^T + (\bar{P}_{WA}\bar{C}^T - \bar{B})(\bar{D} + \bar{D}^T)^{-1}(\bar{C}\bar{P}_{WA} - \bar{B}^T) = 0 \tag{3.12a}$$

$$\bar{A}^T\bar{Q}_{WA} + \bar{Q}_{WA}\bar{A} + (\bar{Q}_{WA}\bar{B} - \bar{C}^T)(\bar{D} + \bar{D}^T)^{-1}(\bar{B}^T\bar{Q}_{WA} - \bar{C}) = 0 \tag{3.12b}$$

where

$$\bar{P}_{WA} = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12}^T & P_{22} & P_{23} \\ P_{13}^T & P_{23}^T & P_{33} \end{bmatrix} \tag{3.13a}$$

$$\bar{Q}_{WA} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12}^T & Q_{22} & Q_{23} \\ Q_{13}^T & Q_{23}^T & Q_{33} \end{bmatrix} \tag{3.13b}$$

Expanding the (2,2) blocks of (3.12a) and (3.12b) yield

$$\begin{aligned} AP_{22}+P_{22}A^T+\underbrace{BC_iP_{23}^T+P_{23}C_i^TB^T}+(P_{22}C^TD_o^T-BD_i+\underbrace{P_{12}^TC_o^T+P_{23}C_i^TD^TD_o^T})\\ (D_oDD_i+D_i^TD^TD_o^T)^{-1}(D_oCP_{22}-D_i^TB+\underbrace{C_oP_{12}+D_oDC_iP_{23}^T})=0 \end{aligned} \tag{3.14a}$$

$$\begin{aligned} A^TQ_{22}+Q_{22}A+\underbrace{C^TB_o^TQ_{12}+Q_{12}^TB_oC}+(Q_{22}BD_i-C^TD_o^T+\underbrace{Q_{12}^TB_oDD_i+Q_{23}B_i})\\ (D_oDD_i+D_i^TD^TD_o^T)^{-1}(D_i^TBQ_{22}-D_oC+\underbrace{D_i^TD^TB_o^TQ_{12}+B_iQ_{23}^T})=0 \end{aligned} \tag{3.14b}$$

Let the Gramians $\bar{P}_i$ and $\bar{Q}_o$, corresponding to the transformed augmented realizations in

(3.4) and (3.5), respectively. satisfy the following ARE's

$$\bar{A}_i\bar{P}_i + \bar{P}_i\bar{A}_i^T + (\bar{P}_i\bar{C}_i^T - \bar{B}_i)(\bar{D}_i + \bar{D}_i^T)^{-1}(\bar{C}_i\bar{P}_i - \bar{B}_i^T) = 0 \tag{3.15a}$$

$$\bar{A}_o^T\bar{Q}_o + \bar{Q}_o\bar{A}_o + (\bar{Q}_o\bar{B}_o - \bar{C}_o^T)(\bar{D}_o + \bar{D}_o^T)^{-1}(\bar{B}_o^T\bar{Q}_o - \bar{C}_o) = 0 \tag{3.15b}$$

where $\bar{P}_i$ and $\bar{Q}_o$ are obtained by transforming $\bar{P}$ and $\bar{Q}$ using transformation matrices $\bar{T}_i$ and $\bar{T}_o$ respectively, as follows

$$\bar{P}_i = \bar{T}_i^{-1}\bar{P}\bar{T}_i^{-T}$$

$$= \begin{bmatrix} I & -X_i & \frac{1}{2}X_iY_i \\ 0 & I & -Y_i \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{12}^T & P_{22} & P_{23} \\ P_{13}^T & P_{23}^T & P_{33} \end{bmatrix} \begin{bmatrix} I & -X_i & \frac{1}{2}X_iY_i \\ 0 & I & -Y_i \\ 0 & 0 & I \end{bmatrix}^T = \begin{bmatrix} P_{i11} & 0 & P_{i13} \\ 0 & P_{i22} & 0 \\ P_{i13}^T & 0 & P_{i33} \end{bmatrix} \tag{3.16}$$

$$\bar{Q}_o = \bar{T}_o^T\bar{Q}\bar{T}_o$$

$$= \begin{bmatrix} I & X_o & X_oY_o \\ 0 & I & Y_o \\ 0 & 0 & I \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{12}^T & Q_{22} & Q_{23} \\ Q_{13}^T & Q_{23}^T & Q_{33} \end{bmatrix} \begin{bmatrix} I & X_o & X_oY_o \\ 0 & I & Y_o \\ 0 & 0 & I \end{bmatrix} = \begin{bmatrix} Q_{o11} & 0 & Q_{o13} \\ 0 & Q_{o22} & 0 \\ Q_{o13}^T & 0 & Q_{o33} \end{bmatrix} \tag{3.17}$$

where

$$P_{i11} = P_{11} - X_iP_{12}^T + \frac{1}{2}X_iY_iP_{13}^T + (P_{12} - X_iP_{22} + \frac{1}{2}X_iY_iP_{23}^T)(-X_i^T) + (P_{13} - X_iP_{23} + \frac{1}{2}X_iY_iP_{33})(\frac{1}{2}Y_i^TX_i^T) \tag{3.18a}$$

$$P_{i12} = P_{12} - X_iP_{22} + \frac{1}{2}X_iY_iP_{23}^T - (P_{13} - X_iP_{23} + \frac{1}{2}X_iY_iP_{33})(Y_i^T) \tag{3.18b}$$

$$P_{i13} = P_{13} - X_iP_{23} + \frac{1}{2}X_iY_iP_{33} \tag{3.18c}$$

$$P_{i21} = P_{12}^T - Y_iP_{13}^T + (P_{22} - Y_i(P_{23}^T)(-X_i^T) + (P_{23} - Y_iP_{33})(\frac{1}{2}Y_i^TX_i^T) \tag{3.18d}$$

$$P_{i22} = P_{22} - Y_iP_{23}^T - (P_{23} - Y_iP_{33})Y_i^T \tag{3.18e}$$

$$P_{i23} = P_{23} - Y_iP_{33} \tag{3.18f}$$

$$P_{i31} = P_{13}^T - P_{23}^TX_i^T + P_{33}(\frac{1}{2}Y_i^TX_i^T) \tag{3.18g}$$

$$P_{i32} = P_{23}^T - P_{33}Y_i^T \tag{3.18h}$$

$$P_{i33} = P_{33} \tag{3.18i}$$

$$Q_{o11} = Q_{11} \tag{3.18j}$$

$$Q_{o12} = Q_{11}X_o + Q_{12} \tag{3.18k}$$

$$Q_{o13} = Q_{11}X_oY_o + Q_{12}Y_o + Q_{13} \tag{3.18l}$$

$$Q_{o21} = X_o^TQ_{11} + Q_{12}^T \tag{3.18m}$$

$$Q_{o22} = (X_o^TQ_{11} + Q_{12}^T)(X_o) + X_o^TQ_{12} + Q_{22} \tag{3.18n}$$

$$Q_{o23} = (X_o^TQ_{11} + Q_{12}^T)(X_oY_o) + (X_o^TQ_{12} + Q_{22})(Y_o) + (X_o^TQ_{13} + Q_{23}) \tag{3.18o}$$

$$Q_{o31} = Y_o^TX_o^TQ_{11} + Y_o^TQ_{12}^T + Q_{13}^T \tag{3.18p}$$

$$Q_{o32} = (Y_o^TX_o^TQ_{11} + Y_o^TQ_{12}^T + Q_{13}^T)(X_o) + Y_o^TX_o^TQ_{12} + Y_o^TQ_{22} + Q_{23}^T \tag{3.18q}$$

$$Q_{o33} = (Y_o^TX_o^TQ_{11} + Y_o^TQ_{12}^T + Q_{13}^T)(X_oY_o) + (Y_o^TX_o^TQ_{12} + Y_o^TQ_{22} + Q_{23}^T)(Y_o) + Y_o^TX_o^TQ_{13} + Y_o^TQ_{23} + Q_{33}^T \tag{3.18r}$$

Expanding the (2,2) block of (3.15a) and (3.15b) yield

$$AP_{i22} + P_{i22}A^T + (P_{i22}C_{i12}^T - B_{i21})(D_i + D_i^T)^{-1}(C_{i12}P_{i22} - B_{i21}^T) = 0 \tag{3.19a}$$

$$A^TQ_{o22} + Q_{o22}A + (Q_{o22}B_{o21} - C_{o12}^T)(D_o + D_o^T)^{-1}(B_{o21}^TQ_{o22} - C_{o12}) = 0 \tag{3.19b}$$

Let $P_i \in \{P_1 = P_{UL}, P_2 = P_{L22}, P_3 = P_{UA}, P_4 = P_{22}, P_5 = P_{i22}\}$ and $Q_j \in \{Q_1 = Q_{UL}, Q_2 = Q_{L22}, Q_3 = Q_{UA}, Q_4 = Q_{22}, Q_5 = Q_{o22}\}$, where, $i = 1, 2, ..., 5$, and $j = 1, 2, ..., 5$, and $T_{i,j}$ be a balancing transformation, such that:

$$T_{i,j}^{-1} P_i T_{i,j}^{-T} = T_{i,j}^T Q_j T_{i,j} = \text{diag}\{\sigma_1, ..., \sigma_r, \sigma_{\text{r+1}}, ..., \sigma_n\}$$

where r is the order of the ROM and n is the order of the original system. Let $B_l \in \{B_1 = B, B_2 = B_{i21}\}$, and $C_m \in \{C_1 = C, C_2 = C_{o12}\}$ where, $l = 1, 2$ and $m = 1, 2$. Balanced realization $\{A_b, B_b, C_b, D_b\}$ is obtained using $T_{i,j}$ as follows:

$$\left[\begin{array}{c|c} A_b & B_b \\ \hline C_b & D_b \end{array}\right] = \left[\begin{array}{c|c} T_{i,j} A T_{i,j}^{-1} & T_{i,j}^{-1} B_l \\ \hline C_m T_{i,j} & D \end{array}\right] = \left[\begin{array}{cc|c} A_{11} & A_{12} & B_{11} \\ A_{21} & A_{22} & B_{21} \\ \hline C_{11} & C_{12} & D \end{array}\right] \tag{3.20}$$

3.4 Discussion

The ROM $\{A_{11}, B_{11}, C_{11}\}$ is obtained by truncating the low energy states in (3.20). The properties of ROM are summarized in Tabel 3.1, which shows three schemes (five arrangements) for double sided passivity preserving, five schemes (ten arrangements) for single sided passivity preserving and two schemes (three arrangements) for un-weighted passivity preserving schemes. The arrangement (5,5), is a Phillips *et al.* technique [7] of un-weighted passivity preserving.

For double sided passivity preserving, three schemes are presented in which both controllability and observability Gramians are weighted. In first scheme, an ARE based transformed weighted controllability Gramian and a Lyapunov based weighted observability Gramian (arrangement (10,3)) and vice versa (arrangement (3,10)) are used for balancing the system. In second scheme, an ARE based transformed weighted controllability Gramian and an ARE based weighted observability Gramian (arrangement (10,7)) and vice versa (arrangement (7,10)) are used to balance the system. In third scheme, an ARE based transformed weighted controllability Gramian and an ARE based transformed weighted observability Gramian (arrangement (10,10)) are used for balancing purpose.

For single sided passivity preserving, five schemes are presented in which either a controllability Gramian is weighted and observability Gramian is un-weighted or a controllability Gramian is un-weighted and observability Gramian is weighted. In first scheme, an ARE based un-weighted controllability Gramian and a Lyapunov based weighted observability Gramian (arrangement (5,3)) and vice versa (arrangement (3,5)) are used to balance the system. In second scheme, an ARE based weighted controllability Gramian and an ARE based un-weighted observability Gramian (arrangement (7,5)) and vice versa (arrangement (5,7)) are used for balancing. In third scheme, an ARE based transformed weighted controllability Gramian and an ARE based un-weighted observability Gramian (arrangement (9,5)) and vice versa (arrangement (5,9)) are used for balancing purpose. In fourth scheme, an ARE based transformed weighted controllability Gramian and a Lyapunov based un-weighted observability Gramian (arrangement (10,1)) and vice versa (arrangement (1,10)) are used for balancing the system. In fifth scheme, an ARE based transformed weighted controllability Gramian and an ARE based un-weighted observability Gramian (arrangement (10,5)) and vice versa (arrangement (5,10)) are used for balancing.

Also, there are eleven schemes (twenty two arrangements) for single sided stability preserving and five schemes (ten arrangements) for un-weighted stability preserving. Out of these arrangements, the (1,1) arrangement is Moore's technique [1], (1,3) and (3,1) arrangements are Enns' techniques [2] for single sided weighting, and (3,3) arrangement is Enns' technique [2] for double sided weights.

Table 3.1: Summary of Single and Double Sided Frequency Weighted Passivity Preserving MOR: S $\rightarrow$ Stable, P $\rightarrow$ Passive, S? $\rightarrow$ Stability not always guaranteed

$T_{i,j} \in$		Q_1		Q_2		Q_3		Q_4		Q_5		
$\{P_i, Q_j\}$		$m=1$	$m=2$	$m=1$	$m=2$	$m=1$	$m=2$	$m=1$	$m=2$	$m=1$	$m=2$	index
P_1	$l=1$	S	S	S	S	S,P	S	S	S	S	S,P	1
	$l=2$	S	S	S	S	S	S	S	S	S	S	2
P_2	$l=1$	S	S	S?	S?	S,P	S?	S?	S?	S?	S,P	3
	$l=2$	S	S	S?	S?	S?	S?	S?	S?	S?	S?	4
P_3	$l=1$	S,P	S	S,P	S?	S,P	S?	S,P	S?	S,P	S,P	5
	$l=2$	S	S	S?	S?	S?	S?	S?	S?	S?	S?	6
P_4	$l=1$	S	S	S?	S?	S,P	S?	S?	S?	S?	S,P	7
	$l=2$	S	S	S?	S?	S?	S?	S?	S?	S?	S?	8
P_5	$l=1$	S	S	S?	S?	S,P	S?	S?	S?	S?	S?	9
	$l=2$	S,P	S	S,P	S?	S,P	S?	S,P	S?	S?	S,P	10
	index	1	2	3	4	5	6	7	8	9	10	

3.5 Conclusion

First of all an augmented system and transformation matrices are defined. Then using the same transformations, input and output augmented system are transformed. Different Gramians are computed using different combinations of un-weighted Lyapunov equations and ARE's and weighted Lyapunov equations and ARE's. Same transformation matrices are used to compute a specific form of controllability and observability Gramians. These Gramians are the main contribution to obtain a passive ROM in case of single and double sided frequency weights. All passivity preserving techniques are then summarized in Table 3.1.

Chapter 4

Numerical Examples

4.1 Introduction

In this chapter, different single and double sided frequency weighted numerical examples are presented to show the effectiveness of the schemes proposed in chapter 3. Simulations are performed using MATLAB 2016 on the system intel core $i3$, having 4 GB RAM and 2.20 GHz processor. Comparison between proposed schemes and existing frequency weighted technique [9] is also presented.

Example 1: Consider a two-port lumped RLC circuit (see Figure.3 of [15]), with parameters $R_i = 0.2\Omega$, $C_i = 0.02F$, and $L_i = 0.09H$. Let

$$G_i(s) = \frac{s+5}{s+2}, \quad G_o(s) = \frac{s+3}{s+6}$$

be the input and output weighting functions, respectively. The original system of order $n = 41$ is reduced to the order $r = 1$. In Figure 4.1 and Figure 4.2 the Nyquist and the eigenvalue plots of original system, Heydari and Pedram's technique [9], and the proposed schemes for double sided weights are shown. It can be seen in Figure 4.1 that the 1^{st} order ROM obtained from Heydari and Pedram's technique [9] is non-passive, because the Nyquist plot of Heydari and Pedram's technique [9] extends to the left half of complex plane, while the Nyquist plot of the proposed schemes lies completely in the right half of the complex plane. In Figure 4.2, when the frequency is less than 10 rad/s, the eigenvalues of the 1^{st} order ROM of Heydari and Pedram's technique [9] are negative while the the eigenvalues of the proposed schemes of same ROM are positive, which indicate that the proposed schemes are passive.

In Figure 4.3, the Nyquist plot of single sided frequency weighted proposed schemes (only output weight $G_o(s) = \frac{s+5}{s+4}$ is used in this case) as well as Heydari and Pedram's technique [9] is shown, which indicates that 1^{st} order ROM of the proposed schemes and

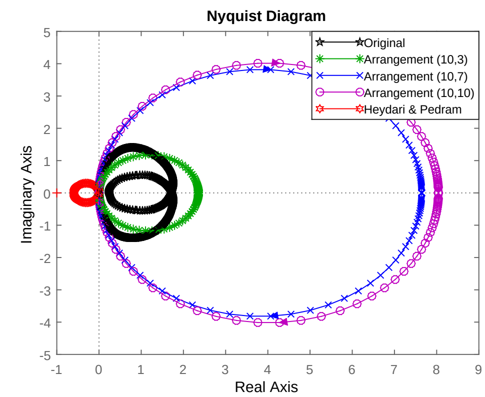

Figure 4.1: Passivity behaviour via Nyquist plot for double sided weights

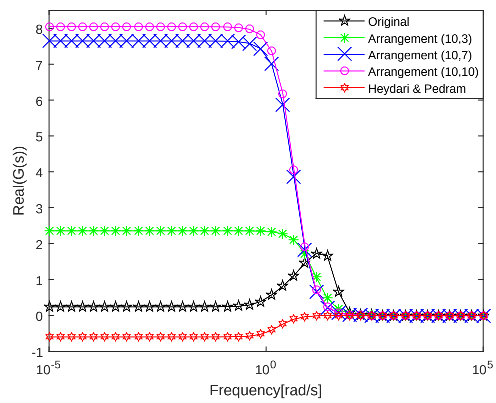

Figure 4.2: Passivity behaviour via eigenvalue plot for double sided weights

Heydari and Pedram's technique [9] are passive because their Nyquist plot lies completely in the right half of complex plane in case of single sided weighting. Also, it can be seen in Figure 4.4, the eigenvalues of the 1^{st} order ROM of Heydari and Pedram's technique [9] and the proposed schemes are positive for the given frequency interval which confirms the passivity of the Heydari and Pedram's technique [9] and the proposed schemes in case of single sided weighting.

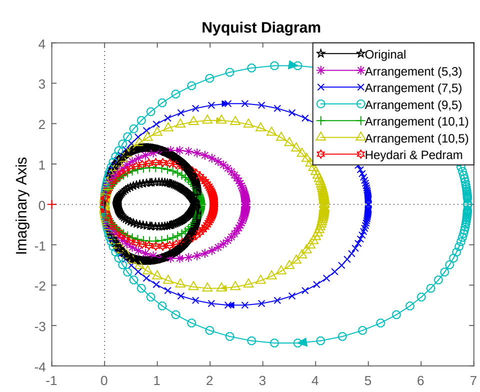

Figure 4.3: Passivity behaviour via Nyquist plot for single sided weight

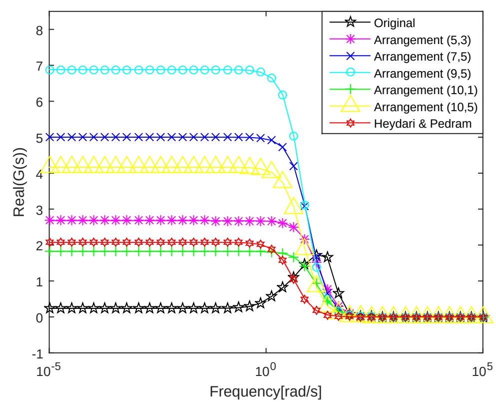

Figure 4.4: Passivity behaviour via eigenvalue plot for single sided weight

Example 2: Consider a 6^{th} order passive system

$$A = \begin{bmatrix} 0 & 0 & 0 & 100 & -100 & 0 \\ 0 & 0 & 0 & 0 & 100 & 0 \\ 0 & 0 & -110 & 0 & 0 & -100 \\ -100 & 0 & 0 & -10 & 0 & 100 \\ 100 & -100 & 0 & 0 & -10 & 0 \\ 0 & 0 & 100 & -100 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 100 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 10 & 0 & 0 & 0 \end{bmatrix}, \quad D = 1$$

Let

$$G_i(s) = \frac{s+0.1}{s+3}, \quad G_o(s) = \frac{s+0.36}{s+2}$$

be the input and output weighting functions. 1^{st} order ROM obtained from Heydari and Pedram's technique [9] in case of double sided frequency weights is non passive, while the same ROM obtained from proposed schemes is passive. The Nyquist and the eigenvalue plots of 1^{st} order ROM verify the results in Figure 4.5 and in Figure 4.6, respectively.

In Figure 4.7 and in Figure 4.8, the Nyquist and eigenvalue plots respectively, of proposed schemes as well as Heydari and Pedram's technique [9] for single sided weighting (only input weight is used in this case) are shown, which clearly indicate that 1^{st} order ROM obtained from proposed schemes and Heydari and Pedram's technique [9] is passive in case of single sided weighting.

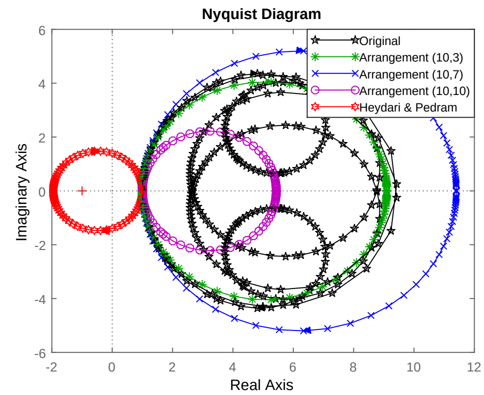

Figure 4.5: Passivity behaviour via Nyquist plot for double sided weights

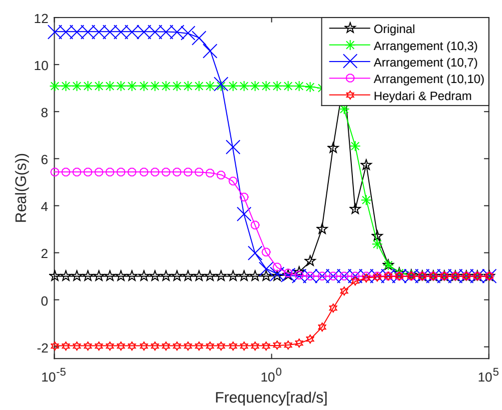

Figure 4.6: Passivity behaviour via eigenvalue plot for double sided weights

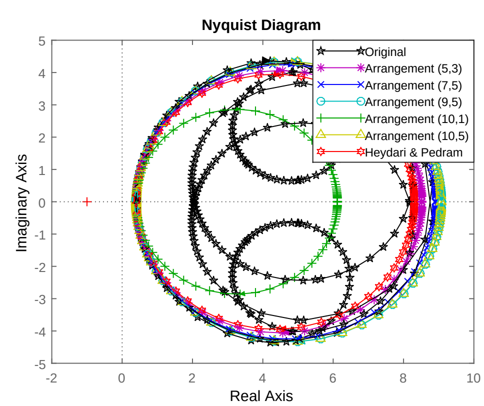

Figure 4.7: Passivity behaviour via Nyquist plot for single sided weight

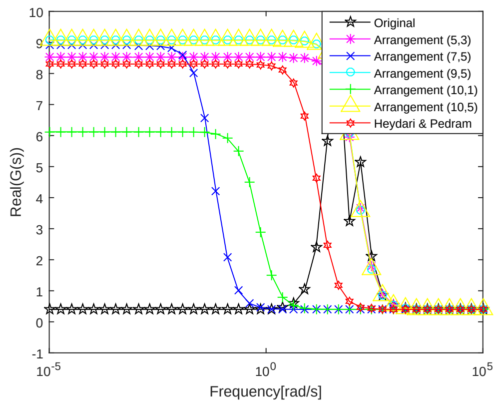

Figure 4.8: Passivity behaviour via eigenvalue plot for single sided weight

Example 3: Consider a 4^{th} order PR system given in [10]

$$A = \begin{bmatrix} -110 & 0 & -100 & 0 \\ 0 & -10 & 100 & -100 \\ 100 & -100 & 0 & 0 \\ 0 & 100 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 100 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, \quad D = 0$$

Let

$$G_i(s) = \frac{s+5}{s+0.1}, \quad G_o(s) = \frac{s+0.2}{s+4}$$

be the input and output weighting functions, respectively. In Figure 4.9 and Figure 4.10 the Nyquist and the eigenvalue plots of the original system, Heydari and Pedram's technique [9], and the proposed schemes are shown, which show that the 2^{nd} order ROM obtained from Heydari and Pedram's technique [9] is non-passive, while the same 2^{nd} order ROM obtained from proposed schemes is passive.

Example 4: Consider a 7^{th} order passive system

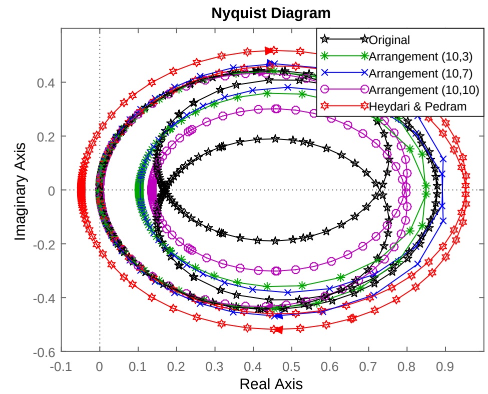

Figure 4.9: Passivity behaviour via Nyquist plot for double sided weights

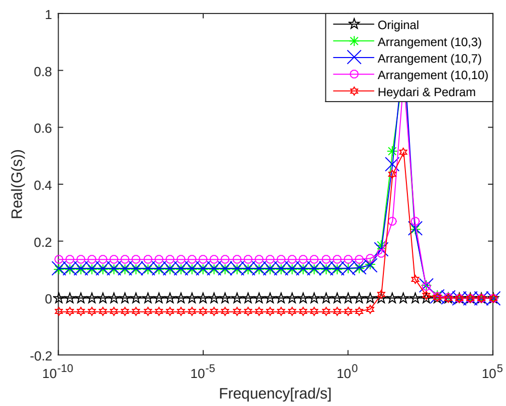

Figure 4.10: Passivity behaviour via eigenvalue plot for double sided weights

$$A = \begin{bmatrix} 0 & -0.05 & 0 & 0 & 0 & 0.05 & 0 \\ 0.076923 & -0.0076923 & -0.076923 & 0 & 0 & 0 & 0 \\ 0 & 0.083333 & 0 & 0 & -0.083333 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -0.10000 & 0.10000 \\ 0 & 0 & 0.25000 & 0 & -1.2500 & 0 & 0 \\ -1.2500 & 0 & 0 & 1.2500 & 0 & -0.25000 & 0 \\ 0 & 0 & 0 & -5.0000 & 0 & 0 & -50 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}^T, \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = 0$$

Let

$$G_i(s) = G_o(s) = \frac{s+4}{s+0.7}$$

be the input and output weighting functions. 2^{nd} order ROM obtained from Heydari and Pedram's technique [9] in case of double sided frequency weights is non passive, while the same ROM obtained from proposed schemes is passive. The Nyquist and the eigenvalue plots of 2^{nd} order ROM verify the results in Figure 4.11 and in Figure 4.12, respectively.

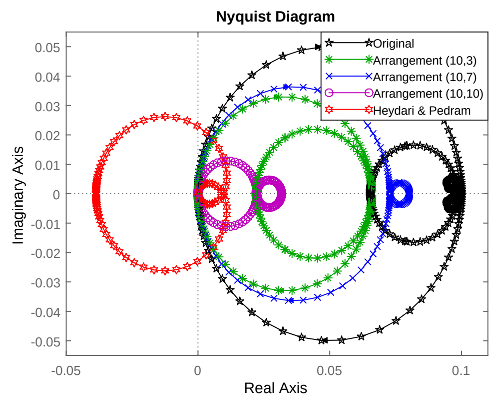

Figure 4.11: Passivity behaviour via Nyquist plot for double sided weights

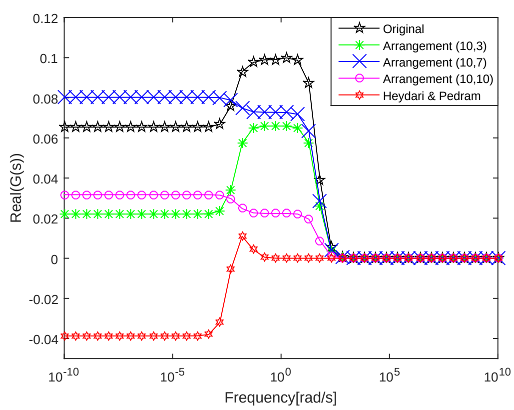

Figure 4.12: Passivity behaviour via eigenvalue plot for double sided weights

In Figure 4.13 and in Figure 4.14, the Nyquist and eigenvalue plots respectively, of proposed schemes as well as Heydari and Pedram's technique [9] for single sided weighting (only input weight is used in this case) are shown, which clearly indicate that 2^{nd} order ROM of Heydari and Pedram's technique [9] as well as proposed schemes is passive in case of single sided weighting.

Example 5: Consider a two port network, with parameters $R = 2\Omega$, $C_i = 0.1F$, and $L_i = 0.1H$.

Let

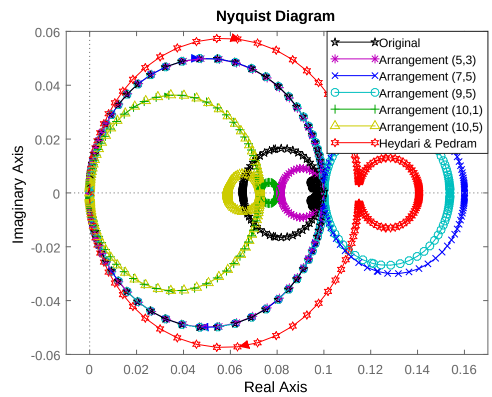

Figure 4.13: Passivity behaviour via Nyquist plot for single sided weight

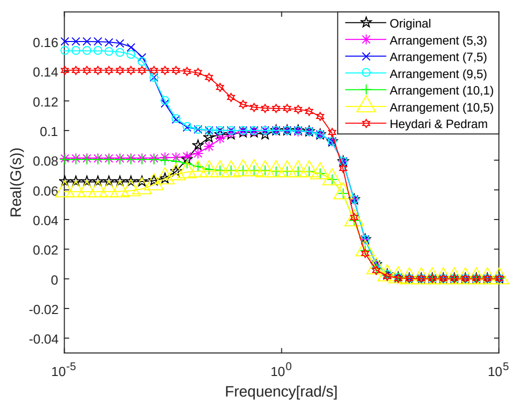

Figure 4.14: Passivity behaviour via eigenvalue plot for single sided weight

$$G_i(s) = G_o(s) = \frac{s + 0.1}{s + 2}$$

be the input and output weighting functions. 4^{th} order ROM obtained from Heydari and Pedram's technique [9] in case of double sided frequency weights is non passive, while the same ROM obtained from proposed schemes is passive. The Nyquist and the eigenvalue plots of 4^{th} order ROM verify the results in Figure 4.15 and Figure 4.16, respectively.

In Figure 4.17 and in Figure 4.18, the Nyquist and eigenvalue plots respectively, of proposed schemes as well as Heydari and Pedram's technique [9] for single sided weighting (only input weight is used in this case) are shown, which clearly indicate that 2^{nd} order ROM of Heydari and Pedram's technique [9] as well as proposed schemes is passive in case of single sided weighting.

Example 6: Consider a 5^{th} order PR system represented by

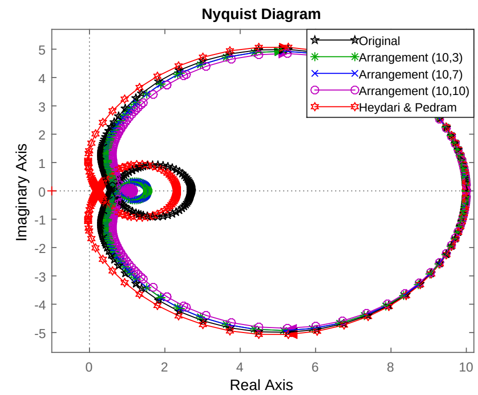

Figure 4.15: Passivity behaviour via Nyquist plot for double sided weights

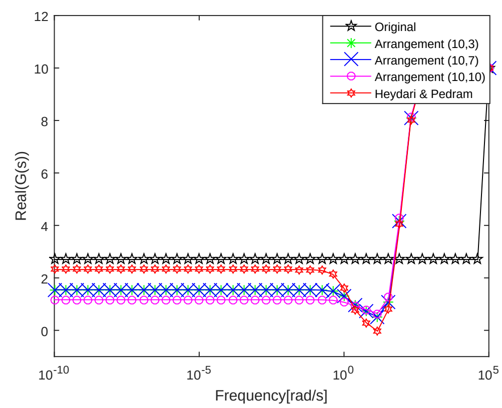

Figure 4.16: Passivity behaviour via eigenvalue plot for double sided weights

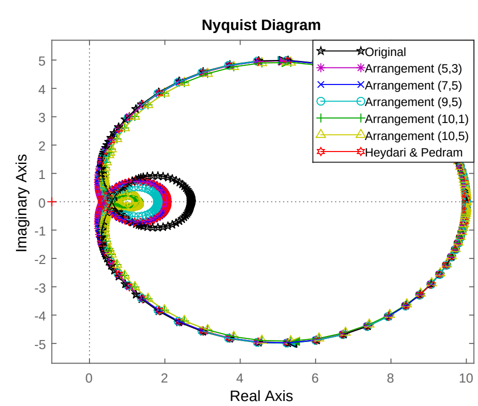

Figure 4.17: Passivity behaviour via Nyquist plot for single sided weight

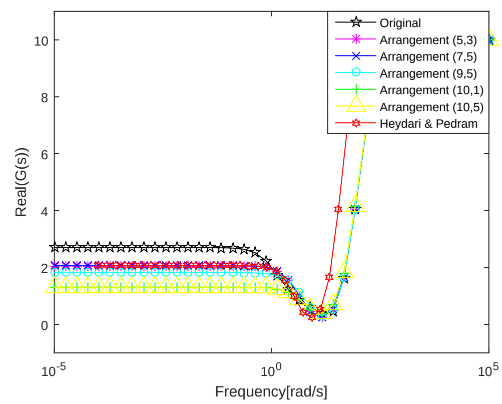

Figure 4.18: Passivity behaviour via eigenvalue plot for single sided weight

$$A = \begin{bmatrix} -20 & -10 & 0 & 0 & 0 \\ 10 & 0 & -10 & 0 & 0 \\ 0 & 10 & 0 & -10 & 0 \\ 0 & 0 & 10 & 0 & -10 \\ 0 & 0 & 0 & 10 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 20 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 5 \end{bmatrix}, \quad D = 1$$

Let

$$G_i(s) = \frac{s + 0.01}{s + 3}, \quad G_o(s) = \frac{s + 0.36}{s + 2}$$

be the input and output frequency weights, respectively. In Figure 4.19 and Figure 4.20 the Nyquist and the eigenvalue plots respectively, of the original system, Heydari and Pedram's technique [9], and the proposed techniques are shown. It is shown in Figure 4.19 and in Figure 4.20 that the 1^{st} order ROM obtained from Heydari and Pedram's technique [9] is non-passive, while the same ROM obtained from proposed schemes is passive.

In Figure 4.21 and Figure 4.22, the Nyquist and eigenvalue plots of proposed schemes as well as Heydari and Pedram's technique [9] for single sided weighting (only input weight is used in this case) are shown, which clearly indicate that 1^{st} order ROM of Heydari and Pedram's technique [9] is also passive in case of single sided weighting.

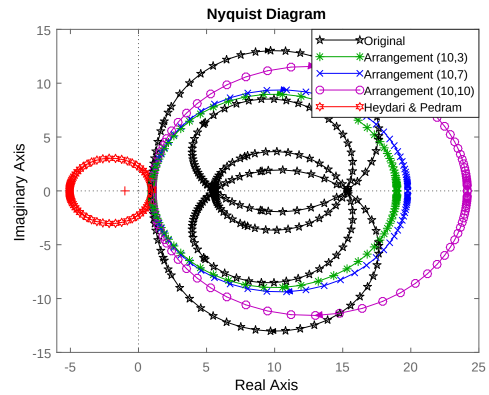

Figure 4.19: Passivity behaviour via Nyquist plot for double sided weights

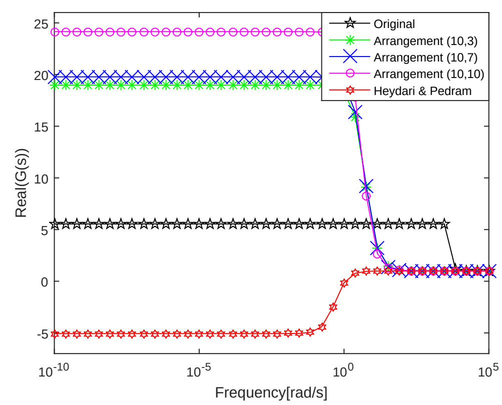

Figure 4.20: Passivity behaviour via Eigenvalue plot for double sided weights

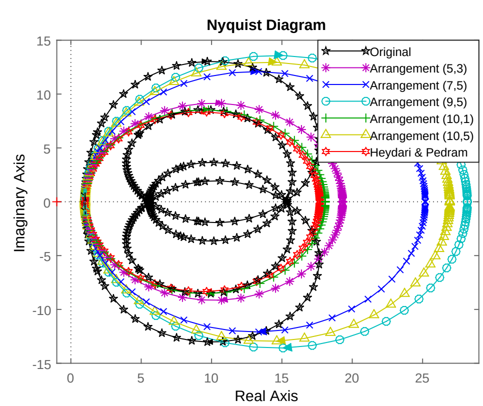

Figure 4.21: Passivity behaviour via Nyquist plot for single sided weight

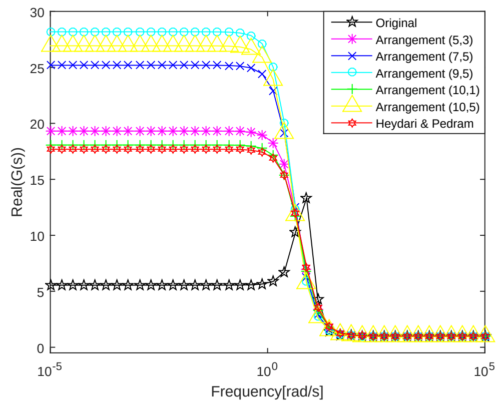

Figure 4.22: Passivity behaviour via eigenvalue plot for single sided weight

4.2 Conclusion

In this chapter, both single and double sided passivity preserving frequency weighted MOR techniques are presented and compared with the existing passivity preserving technique [9]. Simulation results show that some of the proposed techniques preserve passivity of a ROM in the desired frequency range in case of single sided frequency weight and some of the proposed techniques preserve passivity of a ROM in the desired frequency range in case of double sided frequency weights.

Chapter 5

Conclusion and Future Prospects

5.1 Overview of the Thesis

This thesis has explored the problem of passivity preserving for frequency weighted MOR. Both stability and passivity preserving techniques were studied.

In chapter 2, a brief analysis of existing MOR techniques was presented which takes account of both stability and passivity preserving problems for frequency weighted and unweighted cases. Several valuable remarks about the techniques discussed in this chapter were also given.

In chapter 3, a family of techniques were proposed which preserve passivity in the presence of double sided weights. Some techniques preserve only stability while some techniques do not preserve stability as well as passivity. All the proposed techniques were critically examined and several remarks were presented about their behavior whether they preserve or do not preserve basic properties of a system like stability and passivity etc.

In chapter 4, numerical examples with double sided frequency weights were presented to show the usefulness of the proposed schemes discussed in chapter 3. Simulation results and mathematical equations/derivations show that the proposed techniques serve the purpose. Simulation results were also compared with one of the existing techniques which clearly indicated the proposed techniques have an edge over the existing technique in terms of preserving passivity.

5.2 Conclusion

In this thesis, a family of MOR techniques based on passivity preserving for single and double sided frequency weights are presented and compared with the existing technique [9]. Simulation results show that some of the proposed techniques preserve passivity of a ROM in the desired frequency range in case of single sided frequency weight and some of the proposed techniques preserve passivity of a ROM in the desired frequency range in case of double sided frequency weights.

5.3 Future Work

In this section, we suggest/recommend that this research can be further enhanced to a level where one can desire. For future directions it is recommended:

- The derivation of an error bound is still an open area for passivity preserving MOR. Proposed techniques as well as the techniques discussed in [8] do not give an error bound formula, although the technique of [7] gives error bound but this valid only for un-weighted MOR. So further research about an error bound expression for double sided passivity preserving frequency weighted techniques may be conducted in future.
- Lyapunov stability criteria and Lur'e/ARE passivity criteria are not necessary to yield stable and passive ROM's, respectively. This area is also open to yield efficient stable and passive ROM's in case of double sided frequency weights.
- The computational cost/memory of the proposed techniques in terms of Lyapunov equations and ARE's/Lure equations can be improved by using new efficient algorithms.
- As the proposed techniques are realization dependent and it is unknown that which new realization produces less approximation error, so it is also an open question and needs further research.
- Selection of frequency weights for least approximation error need more investigation because different frequency weights yield different approximation errors/results.

The Bibliography

[1] B. C. Moore, “Principal component analysis in linear systems: Controllability, observability, and model reduction”, *IEEE Transactions on Automatic Control*, vol. 26, no. 1, pp. 1732, 1981.

[2] D. F. Enns, “Model reduction with balanced realizations: an error bound and a frequency weighted generalization”, *Proceedings of Conference on Decision and Control*, Las Vegas, pp. 127-132, 1984.

[3] C. A. Lin, and T. Y. Chiu, “Model reduction via frequency weighted balanced realization”, *Control-Theory and Advanced Technology*, vol. 8, no. 2, pp. 341-351, 1992.

[4] G. Wang, V. Sreeram, and W. Q. Liu, “A new frequency-weighted balanced truncation method and an error bound”, *IEEE Transactions on Automatic Control*, vol. 44, no. 9, pp. 1734-1737, 1999.

[5] M. Imran, A. Ghafoor, and V. Sreeram, “A frequency weighted model order reduction technique and error bounds”, *Automatica*, vol. 50, no. 12, pp. 3304-3309, 2014.

[6] S. Gugercin, and A. C. Antoulas, “A survey of model reduction by balanced truncation and some new results.”, *International Journal of Control*, vol. 77, no. 8, pp. 748-766, 2004.

[7] J. R. Phillips, L. Daniel, and L. M. Silveira, “Guaranteed passive balancing transformations for model order reduction”, *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 22, no. 8, pp. 1027-1041, 2003.

[8] W. M. W. Muda, V. Sreeram, and H. H. C. Lu, “Passivity-preserving frequency weighted model order reduction techniques for general large-scale RLC sytems”, *IEEE Conference on Control Automation Robotics & Vision*, pp. 1310-1315, 2010.

[9] P. Heydari, and M. Pedram., “Model-order reduction using variational balanced truncation with spectral shaping”, *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 53, no. 4, pp. 879-891, 2006.

[10] W. M. W. Muda, V. Sreeram, M. B. Ha, and A. Ghafoor, “Comments on Model-Order Reduction Using Variational Balanced Truncation With Spectral Shaping”, *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 62, no. 1, pp. 333-335, 2015.

[11] A. C. Antoulas, “Approximation of large-scale dynamical systems”, *Advances in design and control, SIAM*, 2005.

[12] A. Odabasioglu, M. Celik, and L. T. Pileggi, “PRIMA: Passive reduced order interconnect macromodeling algorithm”, *IEEE/ACM international conference on Computer-aided design*, pp. 58-65, 1997.

[13] T. Reis, and T. Stykel, “Passivity-preserving balanced truncation for electrical circuits”, *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 29, no. 9, pp. 1354-1367, 2010.

[14] L. Pernebo, and L. M. Silverman, “Model reduction via balanced state space repre-santation”, *IEEE Transactions on Automatic Control*, vol. 27, pp. 382-387, 1982.

[15] D. Saraswat, R. Achar, and M. S. Nakhla, “Passive reduction algorithm for RLC interconnect circuits with embedded state-space systems (PRESS)”, *IEEE Transactions on Microwave Theory and Techniques*, vol. 52, no. 9, pp. 2215-2226, 2004.

[16] N. Sadegh, J. D. Finney, and B. S. Heck, “An explicit method for computing the positive real lemma matrices. In Decision and Control”, *Proceedings of the 33rd IEEE Conference on Decision and Control*, vol. 2, pp. 1464-1469, 1994.

[17] A. Ghafoor, and V. Sreeram, “A survey/review of frequency-weighted balanced model reduction techniques”, *Journal of Dynamic Systems, Measurement, and Control*, vol. 130, no. 6, pp. 061004, 2008.

[18] A. Varga, and B. D. O. Anderson, “Accuracy enhancing methods for the frequency-weighted balancing related model reduction”, *Proceedings of Conference on Decision and Control*, Orlando, pp. 3659-3664, 2001.

[19] U. Zulfiqar, W. Tariq, L. Li, and M. Liaquat, “A passivity-preserving frequency-weighted model order reduction technique”, *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 64, no. 11, pp. 1327-1331, 2017.

[20] M. Imran, and A. Ghafoor, “Frequency weighted passivity preserving model reduction technique”, *IMA Journal of Mathematical Control and Information*, vol. 2017.

[21] A. Ghafoor, and M. Imran, “Passivity preserving frequency weighted model order reduction technique”, *Circuits, Systems, and Signal Processing*, vol. 36, no. 11, pp. 4388-4400, 2017.

[22] V. Sreeram, B. D. O. Anderson, and A. G. Madievski, “New results on frequency weighted balanced reduction technique”, *Proceedings of the American Control Conference*, vol. 6, pp. 4004-4009, 1995.

[23] M. Imran, and A. Ghafoor, “A frequency limited interval Gramians-based model reduction technique with error bounds”, *Circuits, Systems, and Signal Processing*, vol. 34, no. 11, pp. 3505-3519, 2015.

[24] M. Imran, A. Ghafoor, and V. Sreeram, “Frequency weighted model order reduction technique and error bounds for discrete time systems”, *Mathematical Problems in Engineering*, 2014.

[25] L. M. Silveira, I. M. Elfadel, and J. K. White, “A guaranteed stable order reduction algorithm for packaging and interconnect simulation”, *Electrical Performance of Electronic Packaging*, pp. 165-168, 1993.

[26] D. C. Sorensen, “Passivity preserving model reduction via interpolation of spectral zeros”, *Systems and Control Letters*, vol. 54, no. 4, pp. 347-360, 2005.

[27] W. F. Arnold, and A. J. Laub, “Generalized eigenproblem algorithms and software for algebraic Riccati equations”, *Proceedings of the IEEE*, vol. 72, no. 12, pp. 1746-1754, 1984.

[28] R. Ober, “Balanced parametrization of classes of linear systems”, *SIAM Journal on Control and Optimization*, vol. 29, no. 6, pp. 1251-1287, 1991.

[29] N. Wong, V. Balakrishnan, C. K. Koh, and T. S. Ng, “Two algorithms for fast and accurate passivity-preserving model order reduction”, *IEEE Transactions on Computer-Aided Design of Integrated Circuits and Systems*, vol. 25, no. 10, pp. 2062-2075, 2006.