STUDY THE IMPACT OF LINEAR PROCESSING ON PERFORMANCE OF MASSIVE MULTIPLE INPUT MULTIPLE OUTPUT (MIMO) WITH IMPERFECT CSI AND INTER CELL INTERFERENCE CONDITIONS

By

Muhammad Usman Shah

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(Col Dr. Imran Rashid)

ABSTRACT

Massive Multiple Input Multiple Output (MIMO)also known as Multi user MIMO is a developing technology having the potential to meet the ever increasing demand of data throughput and E while remaining within the same bandwidth or power resources. This is done by using multiple antennas at the Base Station (BS) and simultaneously serving several user terminals using single/multiple antennas in the same time/frequency resource.

Massive MIMO systems make use of the channel reciprocity for the estimation of channel responses during pilot transmission when the uplink is carried out and then use the acquired CSI for both uplink receive combining and downlink transmit pre-coding of payload data. Pilot transmission is not done during downlink. In order to increase the throughput we need accurate Channel State Information (CSI) at the BS.

The performance of Linear processing schemes such as Zero Forcing (ZF) reaches almost the same level as that of Non linear processing schemes (DPC/SIC) when the number of antennas at the BS are increased in comparison to the user terminals. But Perfect CSI is assumed while doing this comparison. In this thesis, the impact of Linear processing on Massive MIMO is studied with Imperfect CSI.

Dedicated to,

My family,

My Teachers and my Colleagues

for their guidance , support and motivation all along

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ABBREVIATIONS

σ_t	RMS Delay Spread
T_c	Coherence Time
T_s	Symbol Time
f_m/B_d	Doppler Shift
B_s	Signal Bandwidth
B_c	Coherence Bandwidth
$\bar{\tau}$	Excess Delay
G_t/G_r	Gain of Transmitter / Receiver
X^H	Hermition
X^T	Transpose
$\hat{X}$	Estimate
X^*	Conjugate
[RxT]	Receive and Transmit Antennas
1G/2G/3G/4G	1st / 2nd / 3rd / 4th Generation
3GPP	3rd Generation Partnership Project
AWGN	Additive White Gaussian Noise
BC	Broadcast Channel
BER	Bit Error Rate
BS	Base Station
BW	Band Width
CCI	Co Channel Interference
CDMA	Code Division Multiple Access
CSI	Channel State Information
CSIR	CSI at Receiver
CSIT	CSI at Transmitter
DFT	Discrete Fourier Transform
DL	Down Link
DPC	Dirty paper Coding

EGC	Equal Gain Combining
FDD	Frequency Division Duplex
FFT	Fast Fourier Transform
GPS	Global Positioning System
GSM	Global System for Mobile Communication
ICI	Inter-Cellular Interference
ICIC	ICI Coordination
IDFT	Inverse Discrete Fourier Transform
LS	Least Squares
LTE	Long Term Evolution
MAC	Multiple Access Channel
MAI	Multiple Access Interference
MISO	Multiple Input Single Output
MIMO	Multiple Input Multiple Output
MMSE	Minimum Mean Squared Error
MS	Mobile Station
MSC	Mobile Switching Centre
MRC	Maximal Ratio Combining
MU MIMO	Multi-user MIMO
MUD	Multiple User Detection
NW	Network
OFDM	Orthogonal Frequency Division Multiplexing
OSTBC	Orthogonal Space Time Block Codes
PC	Pilot Contamination
P2P	Point to point
PSTN	Public switched Telephone Network
RBs	Resource Blocks
Rx	Receiver
SC	Selection Combining
SER	Symbol Error Rate

SISO	Single Input Single Output
SIMO	Single Input Multiple Output
SNR	Signal to Noise Ratio
SSC	Switch and Stay Combining
TC	Threshold Combining
TDD	Time Division Duplex
TDMA	Time Division Multiple Access
TPS	Triple Play Services
Tx	Transmitter
UAV	Unmanned Aerial Vehicle
VLSAS	Very Large Scale Antenna Systems
WLAN	Wireless Local Area Network
WLL	Wireless Local Loop
ZF	Zero Forcing

Chapter 1

INTRODUCTION

In today's world Wireless communications and Cellular Wireless communication including the internet services is advancing at the fastest pace when compared to any other industry [1]. The evolution of wireless communications for the last two decades is unprecedented; as growth in this area has been exponential. Indeed today, our dependence on cellular wireless communication services (including internet access) and wireless devices has increased so much, we cannot imagine a life without it. With every passing day this dependence seems to grow. Our needs for wireless services seem to surround every aspect of our lives. We need wireless connectivity for text, voice, video conferencing and we need wireless activated devices in our homes to make these 'smart homes'. We need them in our hospitals, classrooms, offices, on the road, on vacations and everywhere. We are looking for one device which we can carry anywhere in the world and with which we can connect and control everything, we own. In military, we need them for communication, identification, tracking of own and enemy movements, detection of nuclear, chemical and biological attacks, surveillance of airspace, control of unmanned robotic vehicles, UAVs, etc. Well, there is no limit to imagination, and with the present developments in the field of wireless communication, this all is becoming a reality.

Although our dreams are big, the varying requirements of various applications and services make it difficult to build a standard wireless system which can simultaneously satisfy all these requirements in an efficient manner. Although wired systems with huge data rates provide us such services, its availability on wireless systems is gradually becoming a reality. Although at present the wireless systems are fragmented.

With the arrival of 4G and beyond systems, the data rate hungry applications would be available to the users on the move, thus converging different wireless systems to one.

1.1 Current Wireless Systems

The present day wireless communication systems are enlisted; details of these can be obtained at [1,2,3,4], however the systems of interest to us are the Mobile Cellular Systems and Broadband Wireless Access.

1.2 Cellular Mobile Systems

The light weight, handheld and affordable devices seen in everyone’s hands today speak of the success of the Cellular Mobile Systems. These devices are not only used for voice, data and video communication, they are also used for mobile TV, FM radio, Gaming, watching movies, listening to music, GPS, Office work, and have numerous other applications. The systems initially designed for use within the buildings or inside moving vehicles with their antennas on their roofs are now being used by everyone through their mobile terminals. It has simply revolutionized the way people connect to each other and with other devices. Brief description of a cellular system is given below;

A Mobile cellular network consists of a large number of wireless mobile subscribers (MSs) who have handheld telephones which can be used conveniently anywhere. A number of base-stations (BSs) spread in a geographical area provide coverage to the MSs. The area in which a BS provides communication services is called a cell. Usually cells are shown as hexagonal region with the BS in the centre (see Figure 1a). This depiction is only to show that complete area is covered in a simplistic way, however in reality the cells are not hexagonal and the BSs are placed usually irregularly, depending on the location available where antenna could be placed easily, such as buildings or mountain tops that have good communication coverage (see Figure 1b). Similarly, MSs connected to BSs are chosen where communication is good rather than physical distance.

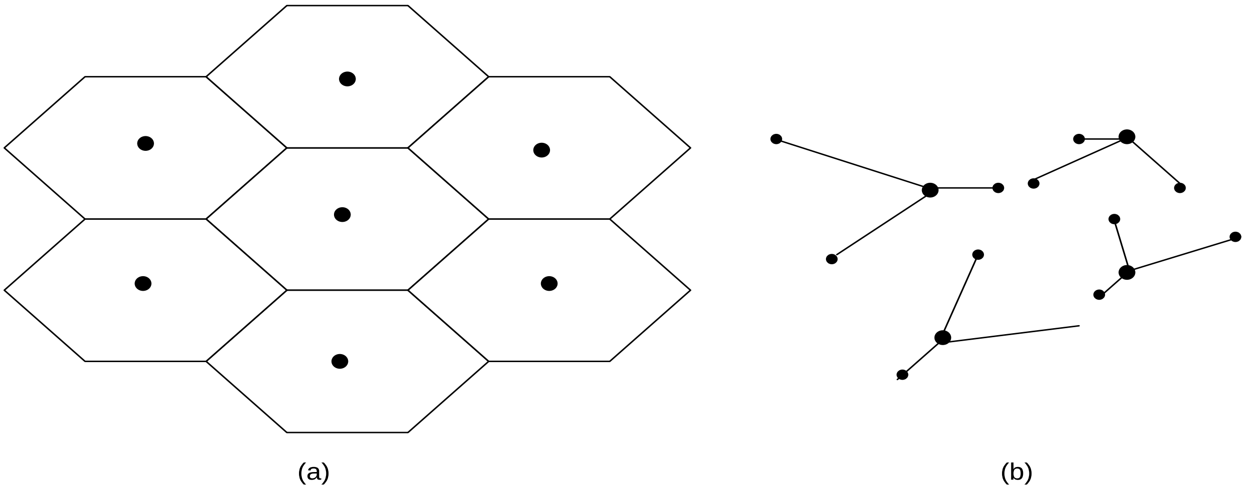

Fig-1.1 Cellular network with cells and BSs. (a) Cells depicted Hexagonal for simplicity. (b) A near realistic view of cells and BSs where MS chooses its nearest or most effective BS.

Call originated from a MS goes to BS and then to Mobile Station Switching Centre (MSC), which makes the decision for its destination. MSC is connected to Public Switched Telephone Network (PSTN), Internet and other Mobile Service providers. According to the destination, MSC switches the call to an ordinary wire line telephone, or another mobile subscriber. The MSC primarily decides which call to or from a user will be handled by which BS and when to handoff a user from one BS to another.

1.2.1 Mobile Cellular System (Generations)

First Generation (1G) older cellular systems, such as the Advanced Mobile Phone Service (AMPS)system developed in the eighties in the US, were analog. A voice signal was modulated on a carrier and transmitted without being converted into a digital stream.2nd Generation (2G) cellular systems were digital e.g. Global System for Mobile Communication (GSM) system standardized in Europe initially but is used worldwide now a days.IS-136and IS-95 are other systems based on Time-Division Multiple Access(TDMA) and Code Division Multiple Access(CDMA) developed in the USA. Since the above mentioned systems and corresponding standards, were developed keeping in mind the requirements of voice communication, the current data rates and delays faced in cellular systems are mainly because of voice requirements[5]. 3rd Generation (3G) cellular systems are designed to handle data and/or voice. Although some of the third-generation systems are the advanced or evolved form of second generation voice systems, others are designed from scratch to cater for the specific requirements of data.4th Generation (4G) LTE and LTE Advanced are designed to cater for triple play services (TPS) and can provide data rates exceeding 100 Mbps(for mobile users) and 1Gbps(for pedestrians). Third Generation Partnership Project (3GPP) Rel. 8 finalized the LTE standard, while 3GPP Rel. 9 onwards is about LTE-Advanced [6]. These systems will be using advanced features like Multiple Input Multiple Output (MIMO), Orthogonal Frequency Division Multiplexing (OFDM), and Virtual MIMO, micro / Pico / femto cells. In addition, work on Adhoc NWs/ Cooperative NWs, Cognitive radios, Spectrum sensing, Massive MIMO etc. is also underway in industry and academia for enhancing data rates, spectral efficiency and MS power efficiency.

1.2.2 Challenges needed to be addressed

Need of the hour is to address the challenges faced by the users so that they can enjoy the currently available wireless communications. A few are enumerated:

- MSs must have sufficient processing power to run various wireless applications.
- More signal and data Processing requires more power, which places much processing load on fixed BSs as compared to the MSs. However, in Adhoc NWs without infrastructure, which provide flexibility and robustness, the energy

efficiency at users becomes a challenge, as all the processing and control is performed by nodes (MSs).

- Finite Bandwidth (BW) and random channel behavior also require robust applications which can cope with such undesirable situations.
- Ever increasing requirements of data rates must also be catered. The requirement of TPS is increasing at a fast pace.
- Wireless communication is prone to Jamming, interference and interception. Adequate security measures are needed to be incorporated in wireless systems to address them.

1.3 Identification of the Problem Area

Maximizing throughput and reliability through MIMO by exploiting diversity is an established fact. More so, the availability of Channel State Information (CSI) at the Base Station (BS) and Mobile Station (MS) is mandatory when trying to maximize the system throughput. It is also important to understand that the availability of CSI in Frequency Division Duplex (FDD) and Time division Duplex (TDD) systems is through entirely different techniques. TDD systems exploit the property of reciprocity, where forward channel is used to estimate the reverse channel. Whereas in FDD systems, forward channel is used for pilots and Reverse channel is used for feedback.

To acquire CSI at the BS, ideal approach is that the BS transmits pilot symbols and MS performs channel estimation and feedback. Nonetheless with a limited coherence time (T_c), and in Massive MIMO this strategy fails as the number of BS antennas grows without limit [7] [8]. Thus the feasible way is the TDD system where channel reciprocity is made use of, for obtaining CSI. In this case the MS transmits the pilots and BS estimates the CSI.

Any signal / sequence (pilot) known to both transmitter and receiver when sent through a channel undergoes degradation. The degradation to the signal is measured and it is known as channel response. The information about the channel response is known as Channel State Information. Time division duplex (TDD) mode is used by Massive MIMO and the physical channels through which the signals propagate are reciprocal, i.e the responses of the channels are the same in both forward and reverse directions [9].

Massive MIMO systems make use of the channel reciprocity for the estimation of channel responses during pilot transmission when the uplink is carried out and then use the acquired CSI for both uplink receive combining and downlink transmit pre-coding of payload data. Pilot transmission is not done during downlink. To enhance throughput of Massive MIMO accurate CSI is needed.

Massive MIMO systems make use of the channel reciprocity for the estimation of channel responses during pilot transmission when the uplink is carried out and then use the acquired CSI for both uplink receive combining and downlink transmit pre-coding of payload data. But Perfect CSI is assumed while doing this comparison.

The payload in Massive MIMO is transmitted using linear processing at the BS. The BS applies linear processing to discriminate the signal transmitted by each terminal from the interfering signals (during uplink) and focus the transmitted signal from BS to its desired terminal (during downlink). One of the myths about Massive MIMO is that “Due to linear processing we face degradation in performance” [9].Recent study has shown that if the number of BS antennas are increased as compared to the number of users then linear processing schemes such as ZF processing can achieve almost the same performance as non-linear schemes(DPC/SIC). Perfect CSI has been assumed while carrying out the study which is unrealistic and cannot be used as design criterion. The effect of Imperfect CSI needs to be studied to get the realistic results.

1.3.1 Problem Statement

Study the Impact of Linear Processing on Performance of Massive Multiple Input Multiple Output (MIMO) with Imperfect CSI and Inter cell Interference conditions.

1.4 Thesis Goals

The presented thesis builds up the concept of impact of linear processing on massive MIMO when imperfect CSI and inter cell interference conditions are considered. In this study, we will focus on the effect of imperfect CSI conditions on spectral efficiency. The effect will be studied by increasing the number of antennas at the BS. It will be shown that how much decrease in spectral efficiency of a massive MIMO system occurs when the value of CSI is reduced due to the inter cell interference conditions. Realistic results of effect of imperfect CSI will be calculated and simulations will be carried out using MATLAB for the purpose of analysis.

1.5 Thesis Layout

This thesis is divided into six chapters. Second chapter covers basics of MIMO and Diversity. Wireless signal propagation and large and small scale fading are covered in chapter 3 in detail. Chapter 4 includes some major topics including MU MIMO, Massive MIMO and Channel estimation theory. Previous work on the subject is summarized in Chapter 5. Moreover, Simulation results and analysis is covered in chapter 5 and a few suggestions for future work are given in chapter 6.

Chapter 2

MIMO AND DIVERSITY GAIN

2.1 MIMO- An overview

The requirements for higher data rates are increasing day by day and to cater for these requirements systems are evolved using Multiple Antennas at Transmitter and Receiver, known generally as Multiple Input Multiple Output (MIMO). The astonishing BW efficiency of such systems seemed to be in violation of the Shannon Limit, but the diversity, multiplexing, array gains and interference reduction made it possible to transform a point to point (P2P) channel into multiple parallel or matrix channels, thus increasing the throughput [11]. Here we consider N_R x M_T number of Receivers(Rx) and Transmitters (Tx), respectively. The MIMO System model can be shown in Fig-2 below.

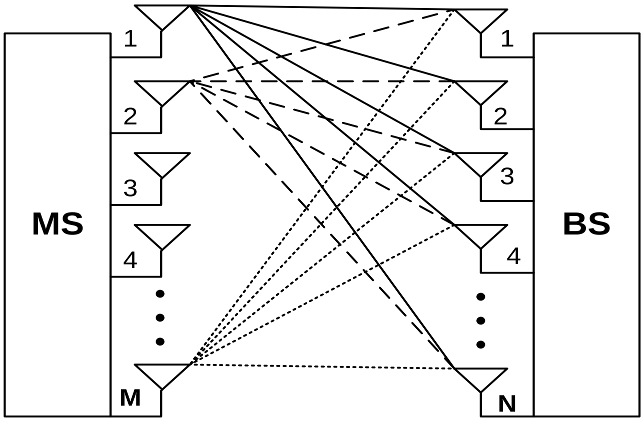

Fig-2.1.MIMO System Model with Transmit and Receive Antennas

Wireless channel matrix H for [NxM] MIMO system is given below. The subscript $i and j$ for h_{ij} mean the channel for the ith receive antenna to the jthtransmit antenna in a MIMO system.

$$H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1M} \\ h_{21} & h_{22} & \cdots & h_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N1} & h_{N2} & \cdots & h_{NM} \end{bmatrix}_{NxM} \tag{2.1}$$

The System model can be written as,

$$\bar{y} = H\bar{x} + \bar{n} \quad (2.2)$$

where $\bar{y}$ is received signal vector, $\bar{x}$ is transmitted signal vector, H is channel matrix and $\bar{n}$is Additive White Gaussian noise (AWGN).The advantages of a MIMO system are given below:

2.1.1 Array Gain

Having arrays of antennas at Tx and Rx provides array gain due to this array gain,the average received SNR is increased because of coherent combining effect. In order to have array gain at the transmitter/receiver the knowledge of channel conditions should be known at the transmitter and receiver [12].Channel State Information (CSI) is usually available at the receiver whereas at Tx, CSI(CSIT) is difficult to achieve. This is mainly because of difficulties in feedback of Channel State Information.

2.1.2 Diversity Gain

A wireless channel undergoes random fading / fluctuation and hence signal is also affected similarly. These techniques rely on transmitting the signal over several (ideally) independently fading paths (in time/frequency/space).

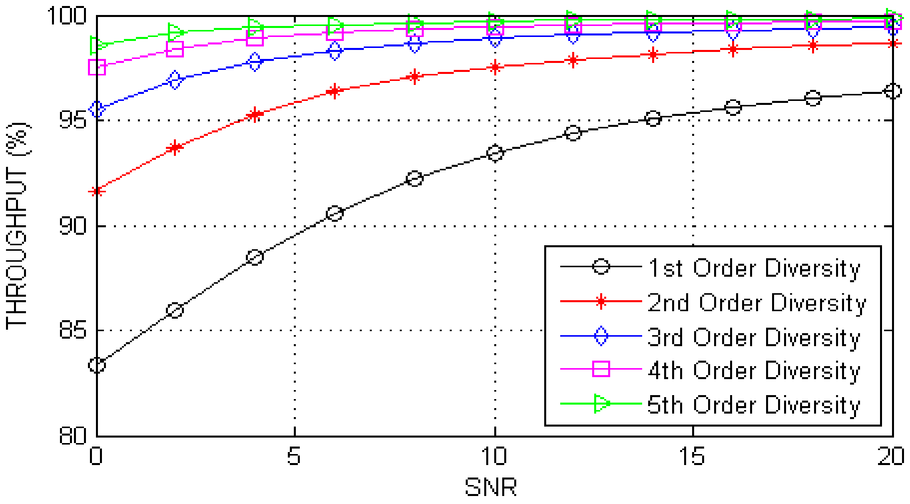

Fig-2.2. Diversity gain showing throughput increase for increasing antennas

With NxM independent links in a MIMO channel we have better chances of reconstructing the transmitted signal than the Single Input Single Output (SISO) channel and we get the min (NxM^{th}) order diversity. Space Time coding technique is used in case CSIT is not available at the Transmitter e.g. Sivash Alamouti, Vahid Tarokh work on Orthogonal Space Time Block Codes (OSTBC).

2.1.3 Spatial Multiplexing Gain

MIMO channels gives us increase without the expending additional power or bandwidth. Independent data signals from the individual antennas are transmitted and thus we achieve gain and the achieved again is called spatial multiplexing gain [12]. When we have channels which are uncorrelated, for example rich scattering, the different streams coming from transmitter are received at the receiver and are separated there, thus achieving a linear increase in capacity.

2.1.4 Interference Reduction

Co-channel interference (CCI) in wireless systems is faced because of reuse of frequency. When multiple antennas are used, interference is reduced by the exploitation of the differences between the spatial signatures of the desired signal and co-channel signals. Interference reduction allows frequency reuse and therefore increases system capacity.

It is however noteworthy that we cannot achieve all the gains simultaneously e.g. If array gain is to be achieved, we cannot have multiplexing gain.

2.2 System Models for Various Antenna Configurations

Depending upon the number of Tx and Rx antennas, a MIMO system can be sub classified into following various categories:

- Single Input Single Output (SISO), When M=N=1.

- Single Input Multiple Output (SIMO), When M=1 & N>M.
- Multiple Input Single Output (MISO), When N=1 & M>N.
- Multiple Input Multiple Output (MIMO), when M>1 & N>1.

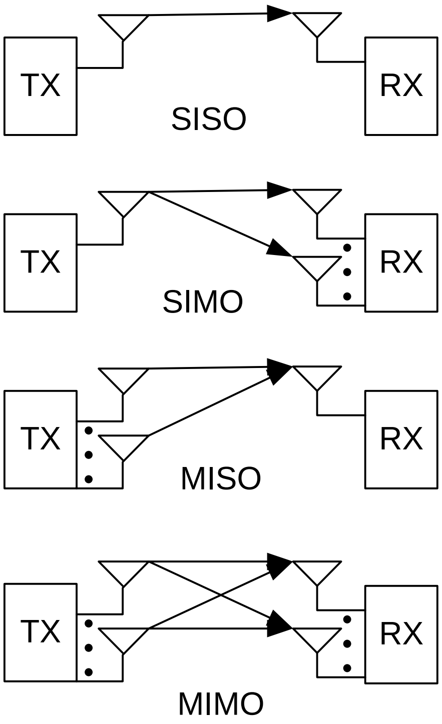

Fig-2.3.System models based on antenna configurations

2.3 **Diversity**

A wireless channel is characterized by fading due to path loss, multipath and/ or shadowing, which have a great penalty on the received signal power. One of the best ways to combat it is through Diversity. Diversity Technique that mitigate the effect of multipath fading is called *micro-diversity* and the technique which mitigates the effects of shadowing (due to buildings and objects) is called *macro-diversity*. Macro-diversity is implemented in infrastructure based cooperative wireless NWs, where signals received by various BSs are combined to achieve diversity.

There are many ways to achieve micro-diversity. Examples are Space, Polarization, Angular (directional antennas), Frequency and Time diversity.

2.3.1 Receive Diversity

In receive diversity, the fading paths are independent and are associated with multiple receive antennas. They are combined to get a signal that is then passed through a standard demodulator. Various ways are used for combining, most of which are linear, where the output is just a weighted sum of different fading paths or branches as shown in the Fig-2.4. It is important to know that combining more than one branch requires co-phasing the signals for coherent combining; otherwise the combining will not have desirable results due to constructive and destructive interference and will result in multipath fading as in SISO system. The average probability of error for some diversity systems can be expressed in the form

$$P = c\gamma^{-M} \tag{2.3}$$

where c is a constant which depends upon modulation, γ is average received SNR per branch and diversity order of the system is denoted by M. Signal Combining can be shown below, where$\alpha_i$are the weights used. As per above equation increasing M exponentially decreases the probability of error.

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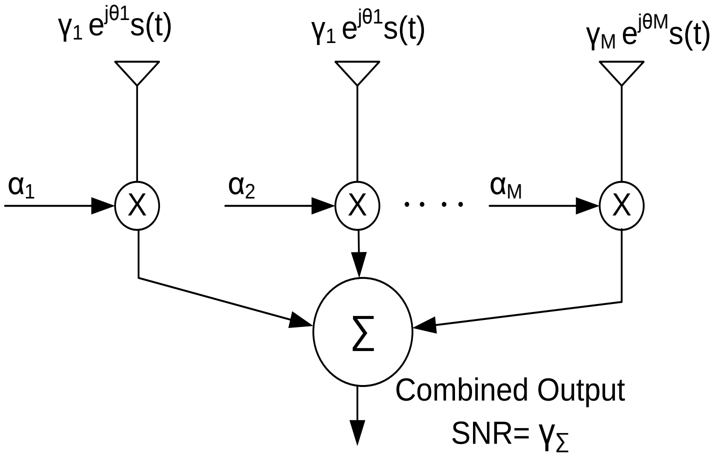


Fig-2.4.Linear Combiner

If diversity order of a system for M antennas is M, it is said to have achieved Maximum / Full Diversity order. The different combining techniques used for diversity are given below. In all subsequent examples we assume no correlation between the different branches received signals. For correlation below 0.5 the results are almost similar with little degradation.

2.3.1.1 Selection Combining(SC). All branches receiving signal are continuously monitored for the best SNR, but only one branch is selected which has the highest SNR. This technique can be employed with either coherent or differential modulation.

2.3.1.2 Threshold Combining (TC).This system does not require dedicated receiver on every branch like Selection Combining. It scans all the branches in sequential order for SNR above a threshold $\gamma_T$and selects the one which it finds first. It keeps this branch until the SNR on this branch drops the threshold level and again scans the branches and selects the one with required SNR. It also does not need co-phasing as only one branch is used at a time. This method is also called Switch and Stay Combining (SSC). The performance of TC lies in between no diversity and ideal SC.

2.3.1.3 Maximal Ratio Combining (MRC).MRC output is the weighted sum of all the branches. So the signals are co-phased prior to combining. In MRC the branches with high SNR are given more weights than the others thus maximizing the overall SNR. It is also called optimum combining scheme. Comparing the performance with SC, we see that the MRC gives better results [1]. Moreover at high SNR, MRC has diversity order M (No of Rx antennas), and so MRC achieves full diversity order. Few characteristics of MRC are given in succeeding paragraphs.

- **Beam-Forming in MRC**

When multiple receive antennas are available, we can have Beam-Forming. It involves selection of suitable weights at the receiver to maximize SNR. We consider a SIMO channel with single input and multiple receive antennas. The output for the system is given by following equation. Here $\bar{y}$, $\bar{h}$ and $\bar{n}$ n are vectors, while x is the input symbol.

$$\bar{y} = \bar{h}x + \bar{n} \tag{2.4}$$

$$\bar{h} = [h_1, h_2, \dots\dots..h_M]^T$$

$$\bar{n} = [n_1, n_2, \dots\dots..n_M]^T$$

And the output is given as;

$$\bar{y} = [y_1, y_2, \dots\dots..y_M]^T.$$

Although, we receive diversity due to different paths available to the input, the received output after MRC is given by,

$$r\ (t) = w^H y = w^H hx + w^H n. \tag{2.5}$$

Since the signal x(t) has unit average power, the instantaneous output SNRγ is;

$$\gamma = \frac{|w^H h|^2}{E\{|w^H n|^2\}} \tag{2.6}$$

The noise power is given as

$$P_n = E\{|w^H n|^2\} = E\{|w^H n n^H w|\} = w^H E\{n^H n\} w = \sigma_n^2 w^H I_M w$$

$$P_n = \sigma_n^2 w^H w = \sigma_n^2 ||w||^2 \tag{2.7}$$

whereI_M represents an MxM Identity matrix. Since constants do not matter, one could always scale w such that ||w|| = 1. Thus SNR for MRCgiven by equation 2.6is;

$$\gamma = \frac{|w^H h|^2}{E\left\{|w^H n|^2\right\}}$$

And if, $w = h$

Then, $\gamma = \frac{|h^H h|^2}{\sigma_n^2 h^H h} = \frac{h^H h}{\sigma_n^2} = \sum_i^M \frac{h_i}{\sigma_n^2} = \sum \gamma_i.$ (2.8)

It means that the SNR of MRC is the sum of the individual SNRs. And the expected value of SNR is M times the value of individual SNR

$$E\{\gamma\} = M.\gamma \tag{2.9}$$

- **BER with M Receive Antennas**

Let $\lambda = \sqrt{\frac{\gamma}{2+\gamma}}$ where, $\gamma = SNR,$ The BER for M antennas is given as under

$$P_e = \left(\frac{1-\lambda}{2}\right)^M \sum_{m=0}^{M-1} \binom{M-1+m}{m} \left(\frac{1+\lambda}{2}\right)^m \quad . \tag{2.10}$$

Whereas for single antenna, BER is given by;

$$BER = \frac{1}{2}(1-\lambda) = \frac{1}{2}(1-(\sqrt{\frac{\gamma}{2+\gamma}})).$$

At high SNR for multiple antennas;

$$P_e = \binom{2M-1}{M} \frac{1}{2^M} \left(\frac{1}{\gamma}\right)^M \tag{2.11}$$

i.e. $P_e \propto \left(\frac{1}{2\,\gamma}\right)^M$. (2.12)

Equation 2.12 shows that probability of error decreases with increasing diversity order [1].

- **Diversity Order:** Let BER of a wireless system be given as;

$$P_e(\mathrm{SNR})$$

The diversity order can be given as

$$d = -\lim_{SNR\to\infty} \frac{\log Pe(SNR)}{\log(SNR)} \tag{2.13}$$

For a single antenna at high SNR;

$$P_e(SNR) = {}^{1}\!/_{2SNR}$$

Thus the diversity order for this system can be found as;

$$d = -\lim_{SNR\to\infty} \frac{\log(\frac{1}{2SNR})}{\log(SNR)}$$

$$d = \lim_{SNR\to\infty} \frac{\log(SNR)+\log(2)}{\log(SNR)} \tag{2.14}$$

and at high SNR, $d = 1 + 0 = 1,$ thus diversity order is 1.

Now we consider the case of M antennas, BER for M antennas is given by;

$$P_e(\mathrm{SNR}) = \binom{2M-1}{M}\frac{1}{2^M}(\frac{1}{SNR})^M . \tag{2.15}$$

Thus the diversity order can be found as;

$$d = -\lim_{SNR\to\infty} \frac{\log(\binom{2M-1}{M}\frac{1}{2^M}(\frac{1}{SNR})^M)}{\log(SNR)}$$

$$d = \lim_{SNR\to\infty} \frac{\mathrm{M}\log(SNR) - \log(\binom{2M-1}{M}\frac{1}{2^M})}{\log(SNR)}$$

$$d = \lim_{SNR\to\infty} M + \frac{\log(\binom{2M-1}{M}\frac{1}{2^M})}{\log(SNR)}$$

and at high SNR, right term reduces to zero, thus, $d = M$. It means that the diversity order becomes M when there are M receive antennas.

- **Probability of Deep Fade**

For single antenna probability of deep fade P_{df} is given by

$$P_{df} = (\frac{1}{SNR}). \tag{2.16}$$

For M receive antennas Probability of deep fade falls exponentially on order of M for the same SNR

$$P_{df} = \frac{1}{M!}\left(\frac{1}{SNR}\right)^{M}. \tag{2.17}$$

2.3.1.4 Equal Gain combining (EGC). MRC needs knowledge of time varying SNR on each branch, which is hard to measure. A simpler way of obtaining diversity is to co-phase the signals on each branch and give equal weights to all irrespective of SNR. It is shown in [1] that the performance of EGC is quite closer to MRC, having less than 1dB penalty, which is the price paid for the reduced complexity.

2.3.2 Transmit Diversity

When we have more transmitters and the transmit power is divided among these antennas we can achieve transmit diversity [1]. Transmit diversity is difficult to achieve without the CSI at the Transmitter (CSIT), which is usually not accurately available at the transmitter because of feedback and quantization errors. Transmit diversity is broadly classified into two groups:

- When CSIT is available at Tx. It is also called Close Loop.
- When CSI is not available at the transmitter and only Receiver has CSI. It is also known as Open Loop.

First one feeds back the CSI acquired at receiver back to the Tx to be used for signal design while the 2nd does not require CSI at the transmitter. Due to the availability of (error free and instantaneous) CSI at the Tx, Closed Loop outperforms than Open Loop in terms of received SNR due to diversity Gain (array gain).

2.3.2.1 **CSI available at the Transmitter**. We take the first scenario where CSI is available at the Tx and using it we design our transmission symbols. System model and symbol preprocessing is given in Fig-2.6 and below:

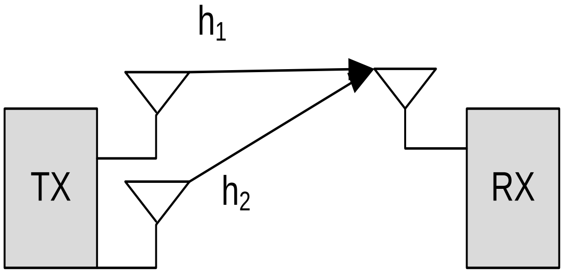


Fig-2.5.A simple MISO System

$$\begin{bmatrix} x_1 \\ x_1 \end{bmatrix} = \begin{bmatrix} \frac{h_1{}^*}{\|h\|} \\ \frac{h_2{}^*}{\|h\|} \end{bmatrix} x \qquad (2.18)$$

The input symbol x_1 is so designedto be known as Transmit Beam former.

The output can be given as, $$y = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} \frac{h_1{}^*}{\|h\|} \\ \frac{h_2{}^*}{\|h\|} \end{bmatrix} x + n \qquad (2.19)$$

or, $$y = \|h\| x + n \qquad (2.20)$$

and SNR is given as

$$SNR = \frac{\|h\|^2}{\sigma_n^2} P \qquad (2.21)$$

where P is the power of x. Above equation shows that Transmit beamforming has similar results as Receive beamforming with diversity order of 2 for two transmit antennas.

2.3.2.2 CSI not available at the transmitter. Now we consider the case when CSI is not available at the transmitter, rather available only at receiver. We can achieve transmit diversity or delayed diversity by exploiting space and time. Simplest examples are that of Alamouti and Tarokh Orthogonal Space Time Block Codes (OSTBC) [1, 10, 13,14]. For understanding, Alamouti scheme is given below:

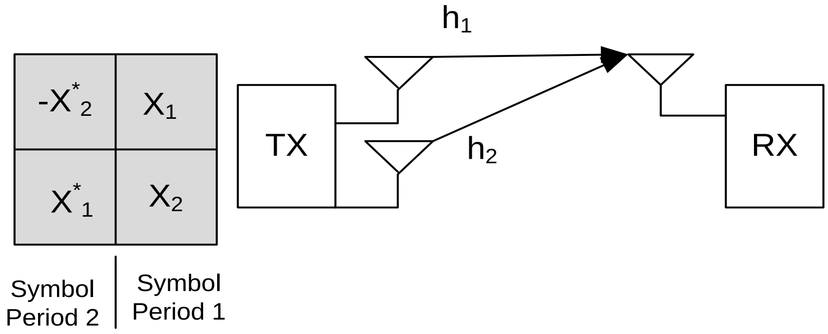


Fig-2.6.Symbol transmission design in Alamouti scheme

Fig-2.6 illustrates Alamouti Scheme with two transmit antennas and one receive antenna. In the first symbol period, $x_1$and x_2 are transmitted from 1st and 2nd antenna respectively. In the second symbol period,$-x_2^*$ and $x_1^*$are transmitted from 1st and 2nd antenna respectively. For this scheme to take effect, we assume that CSI remains constant for the two symbol periods, i.e. Flat Fading for two time intervals. The received symbols at time instances 1 and 2, are given as;

$$y(1) = [h_1 \quad h_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n(1) \tag{2.22}$$

$$y(2) = [h_1 \quad h_2] \begin{bmatrix} -x^*{}_2 \\ x^*{}_1 \end{bmatrix} + n(2)^*. \tag{2.23}$$

Combining both and after little manipulation we get,

$$\begin{bmatrix} y(1) \\ y^*(2) \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n(1) \\ n(2)^* \end{bmatrix}. \tag{2.24}$$

It can also be written as,

$$\begin{bmatrix} y(1) \\ y^*(2) \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \end{bmatrix} \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix} + \begin{bmatrix} n(1) \\ n(2)^* \end{bmatrix}. \tag{2.25}$$

To decouple the two symbols we multiply the output with some weights,

$$w_1 = \begin{bmatrix} \frac{h_1}{\|h\|} \\ \frac{h_2^*}{\|h\|} \end{bmatrix} \tag{1.26}$$

$$w_1^H y = \hat{x}_1 = \|h\| x_1 + n \tag{2.27}$$

Similarly,

$$w_2 = \begin{bmatrix} \frac{h_2}{\|h\|} \\ \frac{-h_1^*}{\|h\|} \end{bmatrix} \tag{2.28}$$

$$w_2^H y = \hat{x}_2 = \|h\| x_2 + n \tag{2.29}$$

Since total transmit power P is distributed between two antennas,$P1 = P2 = P/2$, The SNR we get is

$$SNR = \frac{P}{2} \frac{\|h\|^2}{\sigma_n^2} \tag{2.30}$$

It means that SNR in this case is half of that in Rx Beamforming. Thus 3dB penalty is imposed in Alamouti. Similarly since two symbols are received in two time intervals, Alamouti is also known as Rate 1 or Full Rate code.

2.4 Summary

In this chapter we started with the basics of MIMO wireless communication. Various benefits of the MIMO system including Array, Diversity, Multiplexing gains and Interference reduction have been covered with focus on Receive and Transmit Diversity. Ways to maximize both Transmit and Receive Diversity have been presented which help improved data rates and coverage and reduce the power requirements at the mobile handset.

Chapter 3

RADIO CHANNEL, SIGNAL PROPAGATION AND FADING

3.1 Introduction

Wireless communication is dependent on wireless channel. Unlike wired communication, wireless channel is probabilistic / random in nature. The signal transmitted in air to reach the destination, follows a straight line (Line of Sight (LOS)) and/or many multiple paths due to reflection, scattering and diffraction. As a result at the receiver we have a direct LOS copy, and many delayed copies of the signal due to above mentioned phenomenon, which is also known as multipath. The time between multiple copies of the signal at the receiver define the Delay Spread (σ_t) and thus Coherence BW (B_c). On top of it the movement of the transmitter and /or Receiver and /or surrounding objects causes a drift in the frequency called Doppler Spread (B_d/f_m) which defines the Coherence Time (T_c). Symbol period (T_s), T_c and $\sigma_t$characterize the channel to be Flat / Frequency Selective, Fast or Slow Fading in 'time domain', while signal BW (B_s), B_c and B_d are used equivalently for the 'frequency domain'.

Variation in signal amplitude over frequency and time is called 'fading' which is a particular phenomenon in wireless communications. Multi-path fading and shadowing due to obstructions (buildings, hills etc.) are causes of fading. Broad classification of fading is:

- Large Scale Propagation Fading.
- Small Scale Propagation Fading.

3.2 Large Scale Propagation Fading

Those propagation models which predict the mean signal strength for an arbitrary Tx-Rx separation distance $d$are known as Large Scale Propagation Models. They are useful in estimating the wireless coverage area of a transmitter [1, 2].

3.2.1 Free-Space Path Loss Model

In systems with clear LOS between Tx and Rx we make use of this propagation model. It provides an estimate of received power at a Tx-Rx separation of$d$. We consider such a system with antenna gains at Tx and Rx as $G_t \ and \ G_r$. The received power $P_r(d)$ when transmitter power is P_t(watts) can be given by following expression[2, 16]:

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L} \tag{3.1}$$

Where λ is the wavelength of radiation (in meters) and L is system loss factor, independent of propagation environment. In a lossless system L=1. Loss in actual system hardware is represented by system loss factor. It can be seen that the received power deteriorates exponentially with increase in distance d.

Similarly Path Loss$PL_F(d)$in a system can also be determined, which is simply a ratio of transmit power to the received power [10]. It can be given as:

$$PL_F(d)[dB] = 10log_{10}\left(\frac{P_t}{P_r}\right) \tag{3.2}$$

or, $$PL_F(dB) = -10log_{10}\left(\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}\right). \tag{3.3}$$

In case there are no antennas gains,$G_t = G_r = 1$, Equation 3.3 reduces to;

$$PL_F(d)[dB] = 10log_{10}\left(\frac{P_t}{P_r}\right) = 20log_{10}(4\pi d/\lambda). \tag{3.4}$$

Fig-3.1 shows the Path Loss for various antenna gains. It is observed that with increasing separation between Tx and Rx the Path Loss increases.

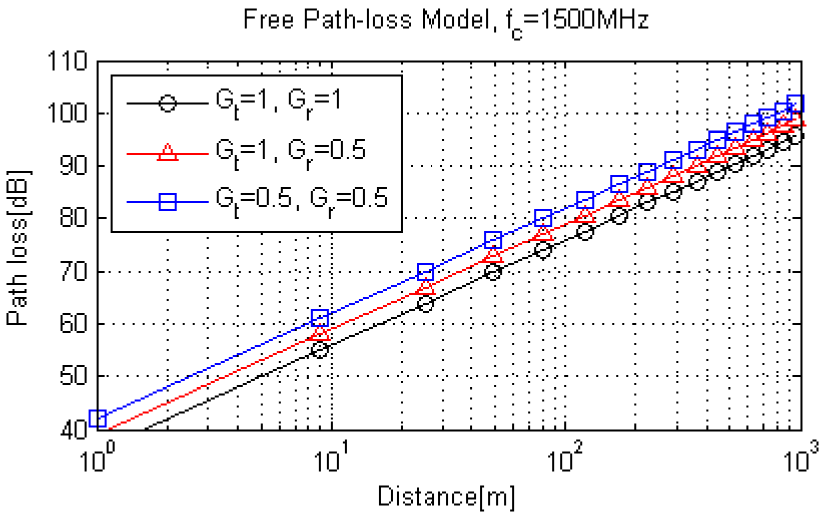


Fig-3.1. Free Space Path Loss Model [10]

3.2.2 Log-Distance Path Loss Model

Theoretical and measurement based models indicate that in outdoor or indoors, as the distance increases the average received power decreases logarithmically. The average large scale Path loss for Tx-Rx separation d can be given as

$$PL(dB) = PL(d_o) + 10nlog(\frac{d}{d_o})$$

(3.5)where$n$ is the path loss exponent, which indicates the rate at which the path loss increases with the distance d. $n$is 2 for free space. The value of n increases with increase in number and size of obstruction between transmitter and receiver as shown in Table 3.1.

3.2.3 Other Models

The measurement data for particular area and communication systems further validate the empirical models. Over time a number of models have emerged, both for indoors and outdoors [2, 10]. A few are enumerated here:-

- Okumura / Hata Model. It is used mainly for urban areas and has been obtained by extensive experiments for obtaining antenna heights and coverage area for mobile communication systems.
- IEEE 802.16d Model. It is based on Log Normal Shadowing Path Loss Model. It has three different types (A, B, C) depending upon the extent of scatterers between the transmitter and receiver.

In a similar manner various indoor models have also been discussed in literature [1, 2, 10]. Indoor radio propagation is effected similarly as outdoor propagation, but conditions are more variable. Various objects in an indoor environment also play a big role, e.g. opened doors, or windows, or partitions, antennas mounted at table level or at ceiling level, etc.

3.3 Small Scale Propagation Fading

Over a short period of time, the rapid fluctuations in the amplitudes, phase, or multipath delays of a radio signal, are known as Small scale fading. It is mainly due to the multipath waves arriving at the receiver and also because of the speed of the mobile or surrounding objects, which results in a combined signal which varies widely in amplitude, frequency and phase. The factors which influence Small Scale Fading are:

- Multipath propagation of transmitted signal due to the various reflectors / scatterers / diffractors along the travel path.
- The speed of the mobile, surrounding objects which result in Doppler Shift f_m in frequency.
- The transmission BW, which makes the channel Flat or Frequency Selective Fading.

3.3.1 Parameters of Small Scale Fading

3.3.1.1 Doppler Shift (f_m) and Coherence Time (T_C). When a mobile moves towards or away from the BS, the frequency received by the mobile varies slightly due to this motion. The change in frequency is termed Doppler Spread / Shift f_m, and can be calculated by;

$$f_m = \frac{v}{\lambda} cos\theta \qquad (3.6)$$

where, v is the speed of the mobile, λ is the wavelength, and θ is the wave arrival angle. If a communication system has a carrier frequency f_c, when MS moves towards BS the received frequency is $f_c + f_m$ and when MS moves away from BS we receive $f_c - f_m$. Similarly it is also known that Coherence Time is inversely related to the Doppler Spread.

$$T_C \propto \frac{1}{f_m}. \qquad (3.7)$$

And more precisely, for correlation above 0.5,

$$T_C = \frac{9}{16\pi f_m}. \qquad (3.8)$$

Another way to represent Coherence time is the Geometric Mean of above two equations.

Thus , $$T_C = \sqrt{\frac{9}{16\pi f_m}} = \frac{0.423}{f_m}. \qquad (3.9)$$

Coherence Time means that two signals arriving with a time difference greater than T_C will be affected differently from each other. If $T_s > T_C$ or$B_s < B_d$, then transmit signal is subjected to Fast Fading. Otherwise it will experience Slow Fading [2,10].

3.3.1.2 Delay Spread (σ_t) and Coherence BW (B_C). For Delay Spread we need to first find $excess\ delay\ , \bar{\tau}$, and then we find $ms\ delay\ spread\ \sigma_t$, which are given below;

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}. \qquad (3.10)$$

The$rms\ delay\ spread\ \sigma_t$, is given by [1, 12];

$$\sigma_t = \sqrt{(\overline{\tau^2} - (\overline{\tau})^2)} \qquad (3.11)$$

where,

$$\overline{\tau^2} = \frac{\sum_k a_k^2 {\tau^2}_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)} \qquad (3.12)$$

where$a\ and\ P$ are symbol amplitude and power respectively. Subscript k is for the kth time instant. The relationship between $rms\ delay\ spread\ \sigma_t$ and B_C is also inversely proportional and can be given as under:

For above 0.5 correlation, $$B_c \approx \frac{1}{5\ \sigma_t}. \qquad (3.13)$$

For above 0.9 correlation,

$$B_c \approx \frac{1}{50\ \sigma_t}. \qquad (3.14)$$

If $B_s > B_c\ \ or\ \ T_s < \sigma_t$, channel is Frequency Selective. For converse, channel will be Flat Fading. For further clarity see Table 3.2 and Fig 3.2.

Domain	**Flat Fading**	**Frequency Selective Fading**
Time	$T_s \gg \sigma_t$	$T_s < \sigma_t$
Frequency	$B_s \ll B_c$	$B_s > B_c$
	Slow Fading	**Fast Fading**
Time	$T_s \ll T_c$	$T_s > T_c$
Frequency	$B_s \gg B_d$	$B_s < B_d$

Table 3.1 Relationship between Fading and various channel and signal parameters

Flat Fading can be described as the constant gain and linear phase response of a channel whose BW is greater than the BW of the signal. When the Signal BW is more than the Channel Bandwidth, then the excess Bandwidth experiences a different phase and gain. This is called *Frequency Selective*. Thus channel induces Intersymbol Interference (ISI). Similarly in *Fast Fading* within the duration of the symbol, there is a rapid change in channel impulse response [2]. If Channel response remains constant within a symbol period we experience *Slow Fading*.

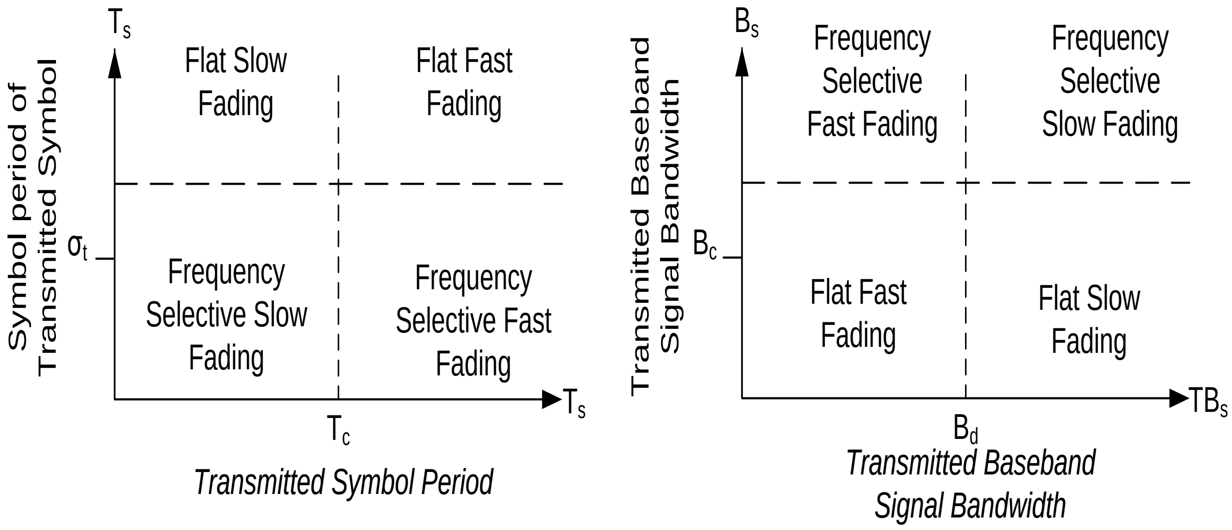


Fig-3.2.Type of fading experienced by a channel in various channel parameters [2]

3.4 Summary

Large Scale and Small Scale Fading have been briefly discussed in this chapter. Having seen the characteristics of the channel and its effects on the signal propagating through it, achieving high data rates is a challenging task. Different technologies are being used to make the probabilistic nature of wireless channel near deterministic so that wireless communication could be made reliable and efficient. With this we mean better BER and maximum throughput. MIMO and its advancements, Massive MIMO are a way forward. In the next chapter we shall look at these techniques and also see how we can obtain the Channel State Information.

Chapter 4

MULTI USER MIMO, MASSIVE MIMO AND CHANNEL ESTIMATION

4.1 Introduction to Multiple User MIMO

Earlier we have seen MIMO systems with M_T and N_R Transmit and Receive systems with single user only. In this chapter we will see multiple users being served by a BS having N_B antennas and each user having N_M antennas. Subscript $_B$ and $_M$are for Base station and Mobile Station respectively. Thus multiple users share the same radio spectrum. Fig-4.1 below illustrates such a system, where single BS provides services to multiple users in a cellular system.

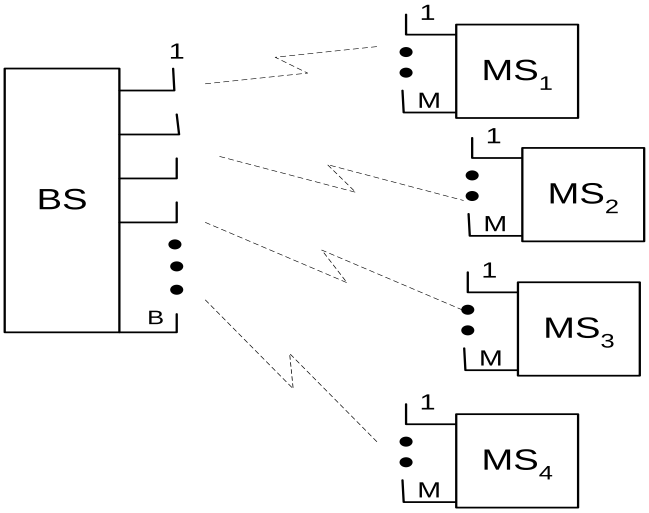
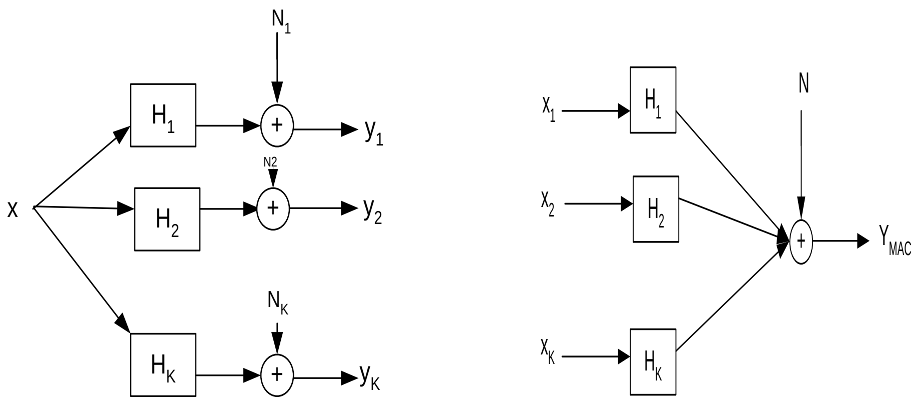


Fig-4.1.Multiple User MIMO System with K=4 [16]

Let K users are scheduled at first time interval and are allocated resource blocks (frequency, time and antennas). In this scenario as K independent users form a virtual set of $K.N_M$ antennas, which communicate with the BS having N_B number of antennas, on the DL we get a $[K.N_M \text{ x } N_B]$ MIMO system and on the UL we have $[N_B \text{ x } K.N_M]$ system.These are depicted in the Fig-4.2 (a) and (b) respectively.



(a) **(b)**

Fig-4.2. DL and UL Channel model for Multi user MIMO

In the multi user MIMO system such a scheme with multiple users communicating with the BS is known as Multiple Access Channel (MAC) and BS communicating with multiple users is called Broadcast channel (BC) [5, 10]. Mathematically we can show the BC and MAC with the help of equation 4.1 and 4.2 respectively:

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_K \end{bmatrix} = \begin{bmatrix} H_1^{DL} \\ H_2^{DL} \\ \vdots \\ H_K^{DL} \end{bmatrix} x + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_K \end{bmatrix} \tag{4.1}$$

$$y_{MAC} = [H_1^{UL} H_2^{UL} \; .... \; H_K^{UL}] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_K \end{bmatrix} + \text{n} \tag{4.2}$$

Where H_u represents the channel gains in the uplink and downlink.

4.1.2 Benefits of MU MIMO

Multiple antennas in MU MIMO can significantly improve the overall performance of the system. We get following benefits using this system [1], but it must be noted that these are only general benefits, not all can be obtained simultaneously.

- Multiple antennas can be used for provision of diversity gain thus improving BER performance. More distinct paths to the receiver help achieve reliability.
- Multiplexing gain due to multiple antennas at Tx and Rx help improve the system capacity.
- Multiple antennas can help provide directivity to geographically separated users thus reducing interference.
- Spectral efficiency by using same channel for multiple users.

4.1.3 Challenges in the MU-MIMO

Although in MU MIMO we achieved Space Division Multiple Access (SDMA), there are certain challenges which need to be addressed. In the UL the users transmit to the BS over the same channel. The challenge here is to separate the user signals at the BS by array processing, multi user detection (MUD) or by other methods. This interference of the user's signals is due to their uncoordinated transmissions.

Similarly in the DL, BS simultaneously broadcasts to multiple users over the same channel. With MUD it may be possible for the users to overcome the multiple access interference (MAI), but since these techniques are too costly for use at the Rx, the interference is ideally mitigated at the Tx by intelligently designing the Tx signals, i.e. by intelligent beam forming or the use of dirty paper coding (DPC). [22] Shows that DPC outperforms linear processing techniques.

MU MIMO systems have the potential to meet the capacity and spectral efficiency requirements set for the newer Next Generation Networks, yet the challenges it faces are quite enormous and need to be addressed, making this area widely researched. An informative account has been given in [15].

4.2 Massive MIMO

Massive MIMO is a technology which is developing at a fast pace, and it scales up MIMO by possibly orders of magnitude, compared to the present systems. With massive, we mean arrays of hundreds of antennas at the BS serving many tens of MSs. Same frequency, time resource may be used. When the ratio between BS antennas and MS antennas is high, system is said to be Massive MIMO, Large scale MIMO, Very Large MIMO, Hyper MIMO, Full dimension MIMO and ARGOS [16,17]. It will enable all the benefits of MIMO systems but on a much larger scale. Thus enabling the dream of achieving smart homes, cloud of remotely activated devices, mobile internet etc.Fig-4.3 below shows a simple Massive MIMO system.

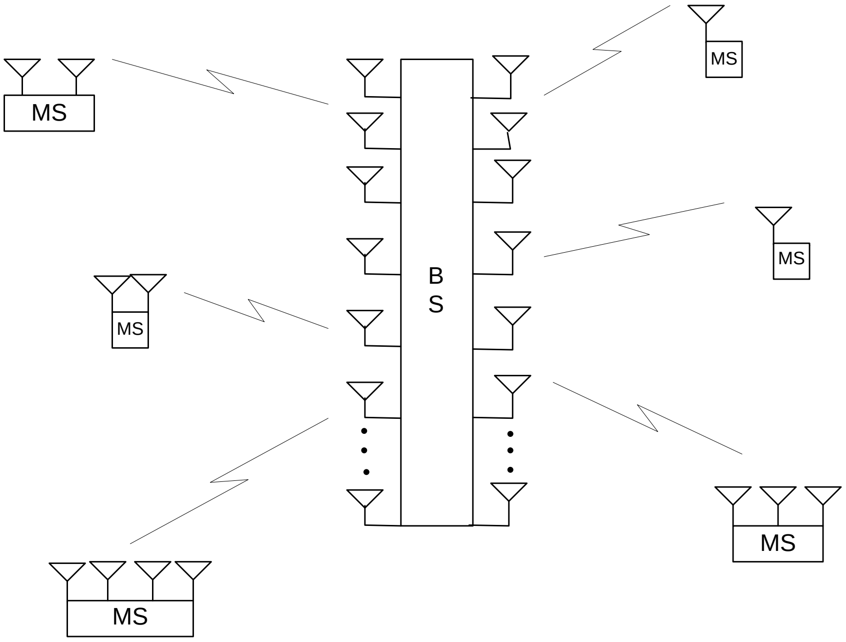


Fig-4.3. A simplified Massive MIMO System

Massive MIMO relies on CSI at Tx and Rx for exploiting multiplexing gain in DL and UL channel. In the UL CSI is easier to obtain when MSs send pilots and BS estimates channel, but in the DL channel estimation is quite difficult due to interference between user’s pilots and need to have orthogonal pilots for the users for each channel. Massive

MIMO system requires hundreds of more resources than a normal MIMO system would require. Similarly since the number of channel estimates required to be calculated by each terminal are equal to the number of antennas at the BS, therefore for the MSs to inform BS CSI, UL resources needed would be very high [16]. Therefore generally, TDD systems are used in Massive MIMO where channel reciprocity works. Pilots are sent by MSs and BS estimates the channels and may feedback.

4.2.1 Potentials of Massive MIMO

Some potential advantages of Massive MIMO are enumerated:

- Massive MIMO can improve capacity to more than ten times whereas, efficiency in terms of power radiated is on order of almost the number of BS antennas. This is due to coherent beamforming achieved by multiples of BS antennas.
- Spectral efficiency on order of ten times is achieved than in a conventional MIMO system mainly because multiple users are simultaneously served in the same frequency time resource.
- Components used for Massive MIMO are inexpensive and have low power consumption. Many arrays of antenna using low power and devoid of power amplifiers are used in Massive MIMO systems.
- Massive MIMO provide robustness against jamming and interference due to large scale antennas used.

4.2.2 Limiting Factors of Massive MIMO

A few limitations of Massive MIMO are also enumerated below [16,17]:

- **Channel reciprocity**. Although in TDD system channel is said to be reciprocal, however due to the equipment calibration errors there may be some errors or deviation in the UL and DL channel states, which would need to be addressed.

- **Pilot Contamination**. In Massive MIMO systems there is a requirement of orthogonal pilot sequence to be sent by the MSs to the BS. Due to limited Coherence Time $T_{c,}$there may be situations where pilots are fully exhausted and reuse of pilots in the neighboring cells results in Pilot contamination.
- **Orthogonal Channel Responses**. It is very important in Massive MIMO for all the channel responses to be independent and orthogonal to each other. Correlation in channels can prove to be very harmful.

4.3 Channel Estimation

Any signal / sequence (pilot) known to both transmitter and receiver when sent through a channel undergoes degradation. The degradation to the signal is measured and it is known as channel response (usually represented by h).Fig-4.4 below shows the [5x1] SIMO channel.

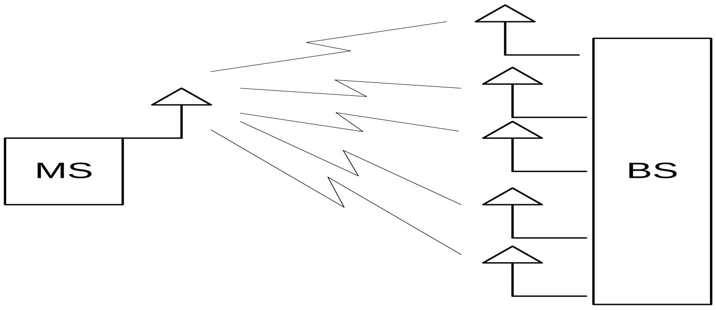


Fig-4.4.A SIMO System having five unknown channels

In OFDM system, multiple carriers are used. Pilots can be sent at all the carriers and channel is estimated at all carriers. Another way is to use pilots at few carriers and interpolation may be used to estimate channel at other carriers. Fig-4.5 below shows the estimated (shown by dashes) and actual channels (shown by lines)for above given SIMO system using OFDM with 32 carriers.

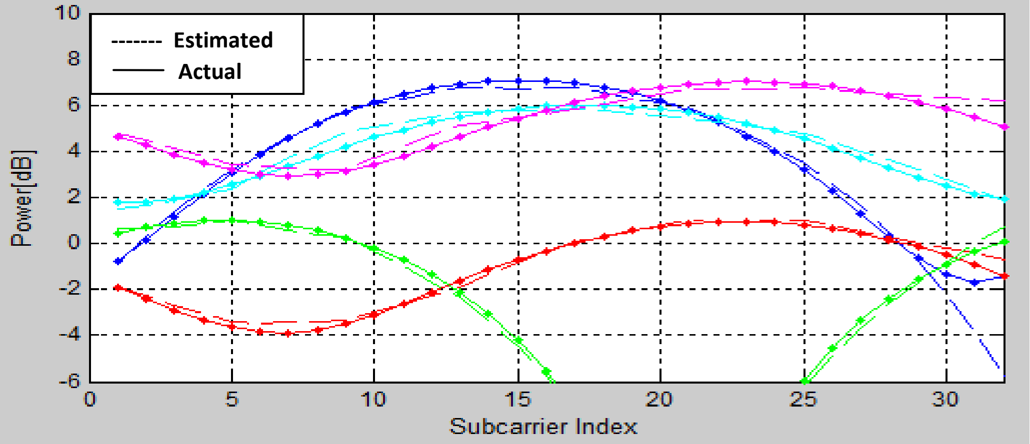


Fig-4.5. Original and estimated Channels for 5x1 OFDM system with 32 carriers

4.3.1 Channel Estimation using Training Symbols

Channel estimation based on training sequence or symbols is a simplest technique for channel estimation. The overhead of these training symbols however affects its performance. Least Squares (Zero Forcing) and Minimum Mean Squared Error (MMSE) schemes are mostly used for channel estimation in this scenario. Let X be a diagonal pilot matrix having orthogonal pilots for N carriers. When such pilots are sent through the air, the channels $\bar{h} = [h_1 h_2 h_3 \ldots\ldots h_{N-1}]^T$effect them and at the receiver we get an output vector $\bar{y}$= $[y_1 y_2 y_3 \ldots\ldots y_{N-1}]^T$. It can be mathematically expressed as follows:

$$\bar{y} = \bar{h}X + \bar{n}$$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_{N-1} \end{bmatrix} \begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & 0 & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & X_{N-1} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{N-1} \end{bmatrix} . \qquad (4.5)$$

4.3.2 Least Squares Channel Estimation

Due to its simplicity this method is widely used for channel estimation. On the lines similar to the LS estimation for the symbol detection, we can obtain the LS estimate for the channel H minimizingthe following cost function [16]:

$$\min J(H) = \|Y - HX\|^2$$

$$= (Y - HX)^H (Y - HX)$$

$$= Y^H Y - Y^H HX - H^H X^H Y + H^H X^H X H^H$$

and by setting the derivative of above w.r.t H equal to zero, we get

$$H_{LS} = (X^H X)^{-1} X^H Y. \tag{4.6}$$

Let us denote each component of LS channel estimate by $H_{LS}[k], k = 0, 1, 2, \ldots.N-1$ for N subcarriers and represent channel estimate at each carrier by the following;

$$H[k] = \frac{Y[k]}{X[k]}, \quad \text{k=0, 1, 2,.......,N-1.} \tag{4.7}$$

Similarly the Mean Squared Error (MSE) of this LS estimate can be given as:

$$MSE_{LS} = E[(H - H_{LS})^H (H - H_{LS})]$$

$$= E[(H - X^{-1}Y)^H (H - X^{-1}Y)]$$

$$= E[(X^{-1}n)^H (X^{-1}n)]$$

$$= E\left(\frac{n^H n}{X^H X}\right) = \frac{\sigma_n^2}{\sigma_x^2}\,. \tag{4.8}$$

Looking at equation 4.8 we see that MSE is inversely proportional to the SNR, which reflects that in case of more SNR we have less MSE and in case of deep fade MSE is subjected to noise enhancement.

4.3.3 MMSE Channel Estimation

Let the channel estimate made above be given by the equation below:

$$\widehat{H_{LS}} = X^{-1} Y \triangleq \tilde{H}$$

In MMSE we define the MMSE estimate as $\widehat{H} \triangleq W\widetilde{H}$, where W is a weight matrix. The MSE of channel estimate $\widehat{H}$ is given as:

$$J(\widehat{H}) = E\{\|e\|^2\} = E\{\|H - \widehat{H}\|\} \tag{4.9}$$

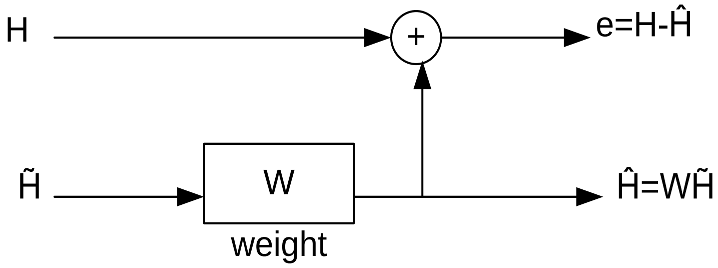


Fig-4.6. MMSE Channel Estimation

TheMMSE channel estimate minimizes the MSE and thus finds a better channel estimate. Also orthogonality principle states that estimation error vector $e = H - \widehat{H}$ is orthogonal to $\widetilde{H}$ such that;

$$E\{e\widetilde{H}^H\} = E\{(H - \widehat{H})\widetilde{H}^H\}\}$$

$$= E\{(H - W\widetilde{H})\widetilde{H}^H\}$$

$$= E\{H\widetilde{H}^H\} - E\{W\widetilde{H}\widetilde{H}^H\}$$

$$= R_{H\widetilde{H}} - WR_{\widetilde{H}\widetilde{H}}=0\ . \tag{4.10}$$

Thus $$W = \frac{R_{H\widetilde{H}}}{R_{\widetilde{H}\widetilde{H}}}$$

and since, $$\widetilde{H} = X^{-1}Y = H + X^{-1}n\ .$$

We first need to find$R_{\widetilde{H}\widetilde{H}}$, which is autocorrelation of LS channel estimates. Whereas,$R_{H\widetilde{H}}$ is the cross correlation between the true and estimated channel.

$$R_{\widetilde{H}\widetilde{H}} = E\{\widetilde{H}\widetilde{H}^H\}$$

$$= E\{(X^{-1}Y)(X^{-1}Y)^H\}$$

$$= E\{(H + X^{-1}n)(H + X^{-1}n)^H\}$$

$$= E\{HH^H + X^{-1}nH^H + Hn^H(X^{-1})^H + X^{-1}nn^H(X^{-1})^H)\}$$

$$= E\{HH^H\} + E\{X^{-1}nn^H(X^{-1})^H\}$$

$$= E\{HH^H\} + \tfrac{\sigma_n^2}{\sigma_x^2}I = \ R_{HH} + \tfrac{\sigma_n^2}{\sigma_x^2}I. \tag{4.11}$$

Now using the equation $\widehat{H} = W\widetilde{H}$ we find the MMSE channel estimate;

$$\widehat{H} = W\widetilde{H} = \frac{R_{H\widetilde{H}}}{R_{\widetilde{H}\widetilde{H}}}\widetilde{H} = R_{H\widetilde{H}}(R_{\widetilde{H}\widetilde{H}})^{-1}\widetilde{H}$$

$$= R_{H\widetilde{H}}(R_{HH} + \tfrac{\sigma_n^2}{\sigma_x^2}I)^{-1}\widetilde{H}\ . \tag{4.12}$$

Further details can be obtained at [16].

4.3.4 Other Estimation Methods

4.3.4.1 DFT Based Channel estimation. In this technique we take the MMSE estimates and improve its performance by eliminating the effect of noise outside the maximum channel delay L. It is important that Lis known in advance. Let $\widehat{H}$[k] denote the channel gains obtained by LS or MMSE method. We first take the IDFT of these estimates $\{\widehat{H}[k]\}_{k=0}^{N-1}$ as under [10, 18];

$$IDFT\{\widehat{H}[k]\} = h[n] + z[n] \triangleq \hat{h}[n] \qquad n\text{=0,1, … N-1} \tag{4.13}$$

Where $z[n]$ denotes the noise component in the time domain. Ignoring the coefficients that contain noise only, we define the coefficients for maximum channel delay L as;

$$\hat{h}_{DFT}[n] = \begin{cases} h[n] + z[n], & n = 0,1,2, \dots . L - 1 \\ 0, & otherwise \end{cases} \tag{4.14}$$

And again converting these L coefficients into frequency domain by taking DFT as under;

$$H_{DFT}[k] = DFT\{\hat{h}_{DFT}(n)\} \tag{4.15}$$

4.3.4.2. **Other Methods**.

Various other methods have been mentioned in the literature, which are listed below. Details can be obtained at [16, 26].

- Decision Directed Channel Estimation
- Advanced Channel Estimation Techniques;
    - Blind Estimation.
    - Using superimposed signal estimation.
    - Fast Time Varying Channels Estimate.
    - EM Algorithm- Channel Estimate.

4.4 Summary

In this chapter we covered some major topics including MU-MIMO, Massive MIMO and Channel Estimation. Channel estimation plays a very important role in MIMO for achieving the desired benefits. Correct and accurate channel estimation in Massive MIMO and its error free feedback are the topic of research these days. Various methods are being studied for accurate and low overhead feedback of channel estimates.

Chapter 5

SIMULATION AND RESULTS

5.1 Literature Review

Multi user MIMO or Massive MIMO is one of the most researched area of communication, specially explored as a promising candidate for the upcoming 5th Generation mobile communications, commonly known as 5G. A lot of effort has been put in by researchers in recent years to explore and harness the potential of massive MIMO, deliberating upon its various aspects.

It has been shown in [19] that the size of the cell does not affect the throughput and number of terminals, and spectral efficiency is not dependant on bandwidth. Coherence time is the limiting factor which decides that how many terminals are to be served by a BS.

The effect of number of antennas at the BS and the number of user terminals on the spectral efficiency have been studied in [20] and it has been shown that its not necessary that the ratio of antennas at BS to the user terminals is more than 10. Rather a lesser number of BS antennas can also achieve optimal spectral efficiency.

The antenna arrays at the BS extracts the individual data streams transmitted by the user terminals using the Channel State Information gained through the pilots already transmitted by the user terminals. In [21] the trade-off between power and spectral efficiency has been discussed for MRC, ZF and MMSE schemes.

The effect of linear processing schemes like Maximum Ratio (MR), Zero Forcing and Minimum Mean Square Error (MMSE) has been studied in [9]. In this paper the authors have dispelled the misconception that by using linear processing schemes, performance degradation is too much. It has been shown that this is not the case. The performance of linear processing schemes reaches almost to that of non linear processing schemes if the number of antennas at the BS are increased as compared to the user terminals. But this is done assuming perfect CSI i.e CSI=1, which is unrealistic.

In this thesis, imperfect CSI is used to investigate the effect of linear processing schemes on Massive MIMO systems.

5.2 Simulation Parameters

A quantitative comparison between Favorable Propagation (FP), non-linear processing schemes such as DPC/SIC (denoted by 'sum capacity' in the simulation figures) and the linear processing schemes (MR and ZF) is carried out using K=20 user terminals and the values of power per user / SNR, corresponding CSI and number of service antennas at the base station (M) are varied. The channels are i.i.d Rayleigh fading and the results are true for both the uplink and downlink due to channel reciprocity.

The spectral efficiency is calculated based on the following equation:

$$Spectral\ Efficiency = \log_2\left(1 + \frac{C_{CSI}.M.SNR_{u/d}}{K.SNR_{u/d} + 1}\right)$$

CSI is calculated by the formula:

$$CSI = \frac{1}{1 + \left(\frac{1}{K.SNR_{u/d}}\right)}$$

The values of spectral efficiency are calculated as the number of service antennas are varied. The values of power per user, SNR and CSI are are given in Table 6.1.

Power per user (PdB)	$SNR = 10^{\frac{PdB}{10}}$	**CSI=1/(1+1/K.SNR)**
-1 dB	0.794	0.94
-2 dB	0.6309	0.92
-3 dB	0.5011	0.91
-4 dB	0.398	0.89
-5 dB	0.3162	0.86
-7 dB	0.1995	0.80
-10 dB	0.1	0.66
-15 dB	0.031	0.38
-20 dB	0.01	0.16

Table 5.1 CSI values corresponding to SNR values

5.3 Simulation Results

5.3.1 Evaluation of Impact of Imperfect CSI on Linear processing

The effect of decreased CSI values is evaluated using the simulations. We see that as the value of CSI is decreased the spectral efficiency is also decreased.

5.3.1.1 Perfect CSI (CSI = 1)

Simulation output is as under:-

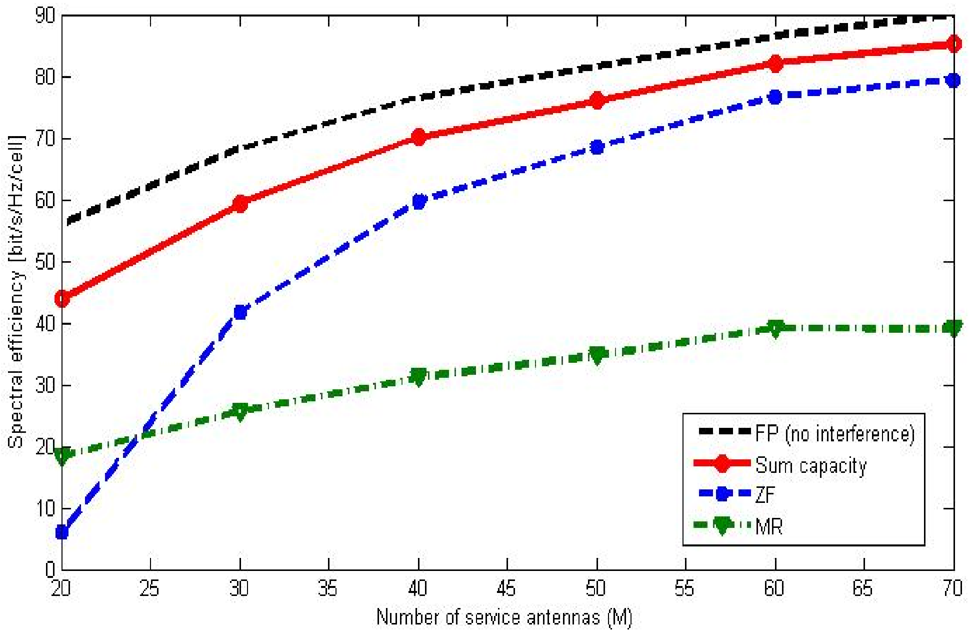


Fig-5.1. Spectral Efficiency for perfect CSI

Analysis

In the simulation we see that as the number of service antennas are increased the performance of linear processing schemes specially the ZF processing reaches almost the same level as that of the optimal DPC/SIC. When the number of M and K are approximately same, then the performance of linear processing is very low as compared to the non-linear processing but as the number of antennas are increased the performance of linear processing reaches almost the same level as that of non-linear processing.

5.3.1.2 CSI = 0.86

Simulation output is as under:-

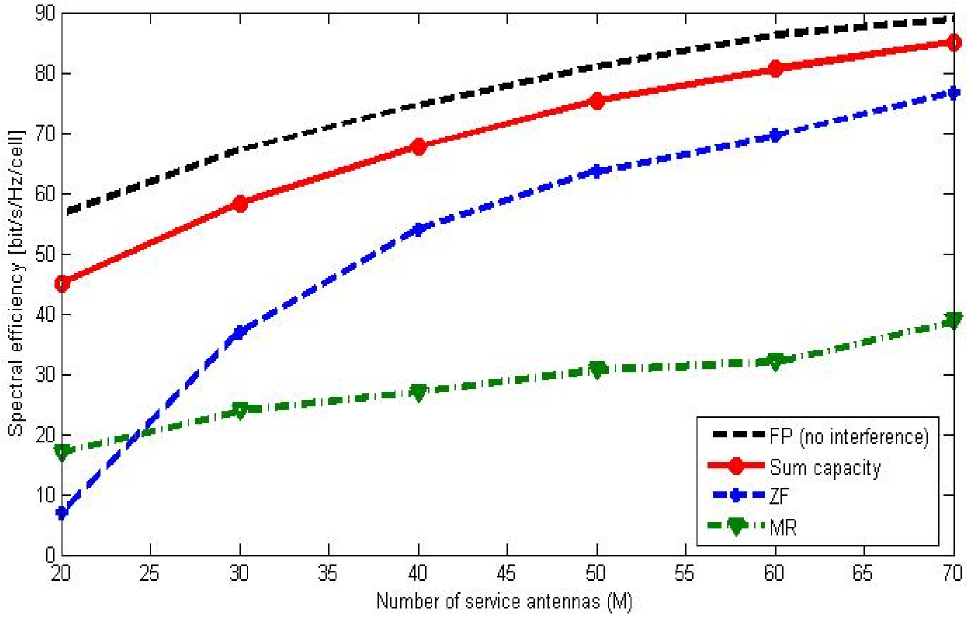


Fig-5.2. Spectral Efficiency for CSI=0.86

Analysis

We can see the impact of decrease of CSI value from 1 to 0.86 on spectral efficiency of Massive MIMO systems using linear processing schemes. At M=70 the spectral efficiency of ZF has reduced from 82 to 78 bits/Hz/cell. There is about 4.9 % decrease in the performance of ZF processing scheme when Imperfect CSI (CSI = 0.86) is used instead of Perfect CSI (CSI=1). A slight reduction can also be noticed in the performance of non-linear processing schemes.

5.3.1.3 CSI = 0.66

Simulation output is as under:-

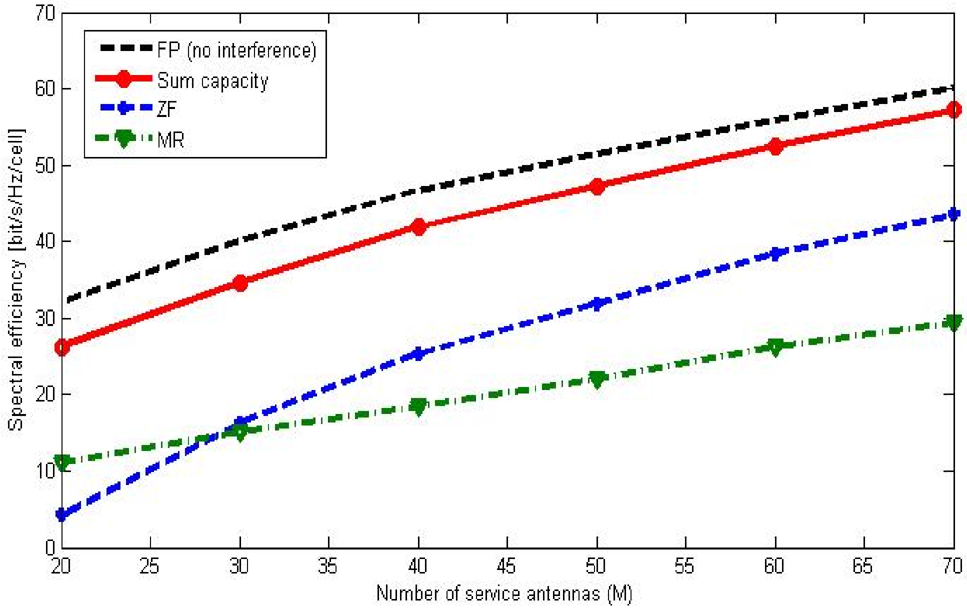


Fig-5.3. Spectral Efficiency for CSI=0.66

Analysis

We can see the impact of decrease of CSI value from 1 to 0.66 on spectral efficiency of Massive MIMO systems using linear processing schemes. At M=70 the spectral efficiency of ZF has reduced from 82 to 43 bits/Hz/cell. There is about 47 % decrease in the performance of ZF processing scheme when Imperfect CSI (CSI = 0.66) is used instead of Perfect CSI (CSI=1). 33 % can also be noticed in the performance of non-linear processing schemes.

5.3.1.4 CSI =0.38

Simulation output is as under:-

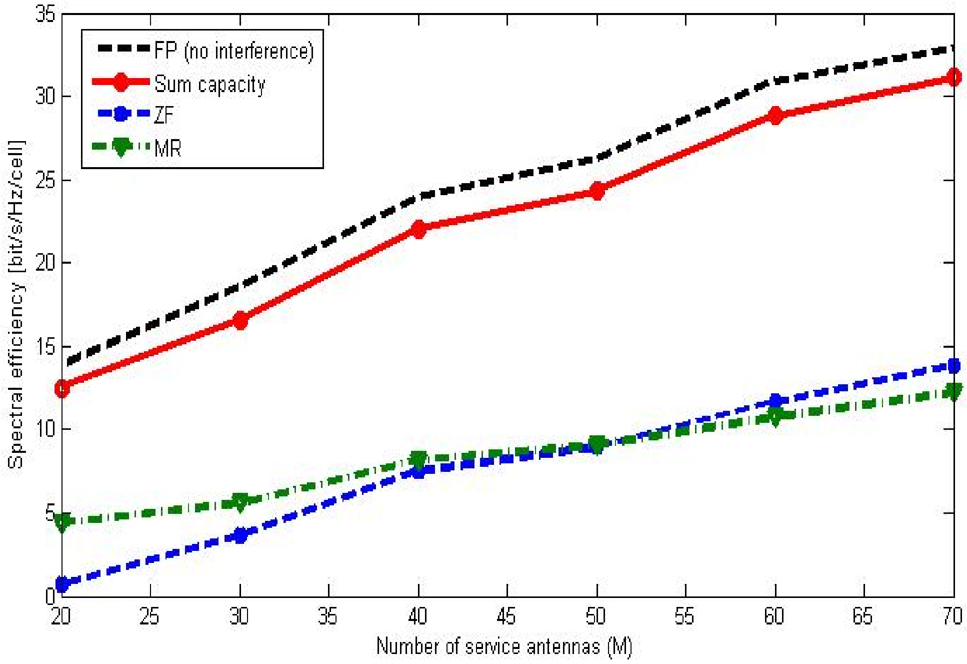


Fig-5.4. Spectral Efficiency for CSI=0.38

Analysis

We can see the impact of decrease of CSI value from 1 to 0.38 on spectral efficiency of Massive MIMO systems using linear processing schemes. At M=70 the spectral efficiency of ZF has reduced from 82 to 14 bits/Hz/cell. There is about 82 % decrease in the performance of ZF processing scheme when Imperfect CSI (CSI = 0.38) is used instead of Perfect CSI (CSI=1). 61% decrease can also be noticed in the performance of non-linear processing schemes.

5.3.1.5 CSI =0.16

Simulation output is as under:-

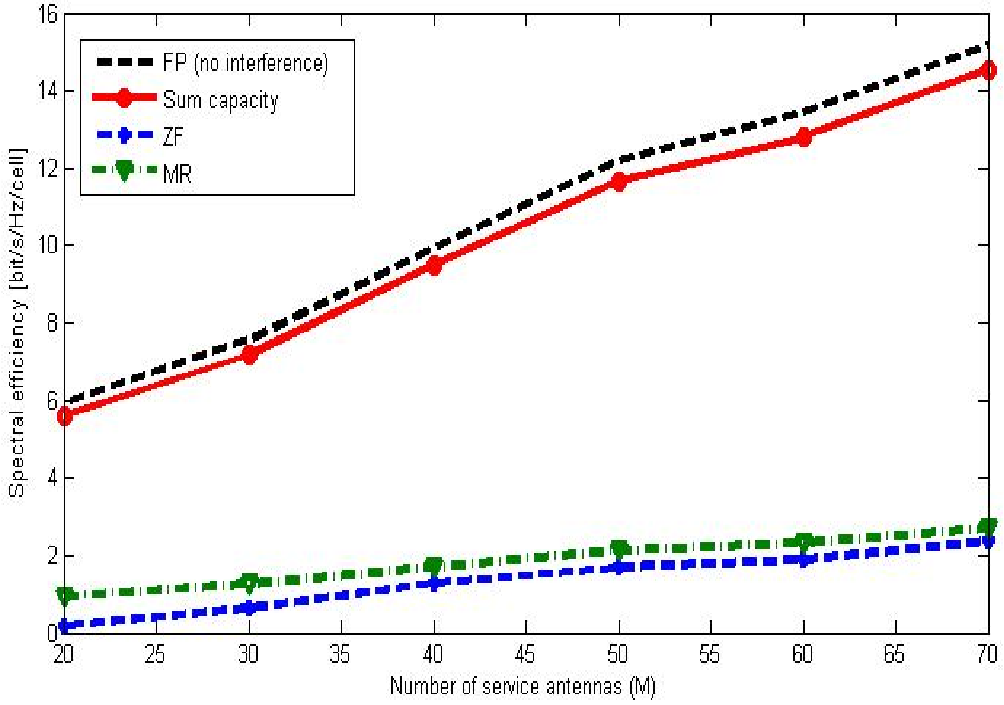


Fig-5.5. Spectral Efficiency for CSI=0.16

Analysis

We can see the impact of decrease of CSI value from 1 to 0.16 on spectral efficiency of Massive MIMO systems using linear processing schemes. At M=70 the spectral efficiency of ZF has reduced from 82 to 14 bits/Hz/cell. There is about 96 % decrease in the performance of ZF processing scheme when Imperfect CSI (CSI = 0.16) is used instead of Perfect CSI (CSI=1). 82% decrease can also be noticed in the performance of non-linear processing schemes.

5.3.2 Evaluation of Impact of Imperfect CSI on Non Linear processing (Sum Capacity)

The effect of Imperfect CSI on Non linear processing is simulated using various values of Imperfect CSI used in section 2.1. Simulation output is as under:-

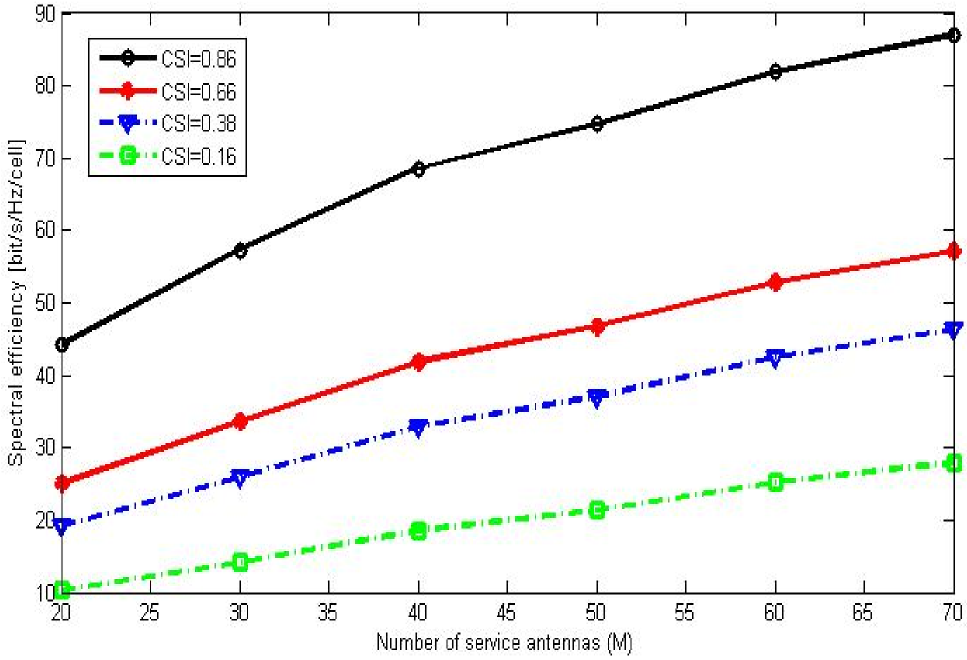


Fig-5.6 Impact of Imperfect CSI on Non Linear processing (Sum Capacity)

Analysis

We can see the impact of decrease of CSI value on spectral efficiency of Massive MIMO systems using Non linear processing schemes. As the value of CSI is reduced the performance of Non linear processing also decreases correspondingly.

5.3 Conclusion

When Perfect CSI is assumed while studying the impact of Linear processing schemes on Massive MIMO systems the results obtained are unrealistic. Therefore we have used various values of Imperfect CSI to calculate the reduction in performance of Massive MIMO. It clearly shows that there is indeed a substantial decrease in the performance of Massive MIMO when Linear processing schemes are used with decreased CSI.

Chapter 6

FUTURE WORK

Massive MIMO technology is one of the most researched areas in communications field nowadays and it is being explored heavily for its potential to meet the demand for increased mobility and higher data rates. Since it is a relatively new field of research, many aspects are still not fully explored and need further elaboration and deliberation. In this work we focused on the impact of linear processing on Massive MIMO technology under imperfect CSI conditions. Future work may be carried out as following

- Future work may include the counter measures to overcome the effect of imperfect CSI conditions on the spectral efficiency of Massive MIMO systems.

- The impact of imperfect CSI conditions in linear processing may may be extended to power efficiency vis-à-vis spectral efficiency.

- The effect of linear processing and CSI conditions on Frequency Division Duplex (FDD) systems may be explored.

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