Coalition Formation Game for Heterogeneous Spectrum Sharing in Cognitive Radio Networks

MCS

by

Muhammad Naveed Ul Haq

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Date: 5 / 9 / 2016 Professor Muhammad Faisal Amjad

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ABSTRACT

Coalition Formation Game for Heterogeneous Spectrum Sharing in Cognitive Radio Networks

By: Muhammad Naveed ul Haq

Chairperson of the Supervisory Committee: Professor Muhammad Faisal Amjad

Department of Information Security

Heterogeneity of available spectrum for secondary access by cognitive radio (CRN), with respect to its various quality of service parameters, leads to contention among collocated CRNs which results in interference and wastage of spectrum opportunity. Self-coexistence problem of CRNs approached in Coexistence Beacon Protocol (CBP) of IEEE 802.22 WRAN assumes that the contending networks do not have any preference over the set of available channels. Rationally all CRNs will contend for the best available channels and increased interference will occur resulting in inefficient utilization of available spectrum. In this paper, we analyze this situation from a game theoretic perspective and model coexistence of CRNs by proposing a novel form of coalition games called the C-Partition form. We model the situation of collocated CRN for heterogeneous spectrum using proposed algorithm and show that it is in every player interest to be a part of this game, form bigger coalitions ultimately leading to the formation of a grand coalition. Using proposed algorithm and its distributed implementation of rules, cooperation amongst different partitions can be enforced in contrast with coalition formation game in Partition form where no controlling mechanism exists between contending players. In this paper we proved that the final resulting C-Partition will be in Nash equilibrium (NE) and no player will have any incentive to deviate from strategy of c-partition.

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All praises to Allah Almighty who created the universe and knows whatever is in it, hidden or evident, and who bestowed upon me the intellectual ability and wisdom to search for its secrets. I am thankful to Almighty Allah who gave me an opportunity to do the Masters as a military course, the expenses of which I cannot bear if I wish doing it on my own.

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Last but certainly not least, deepest appreciations are extended to my beloved Parents, for my success is really the fruit of their devoted prayers and encouragement. May Almighty Allah bless them with good health and prosperous long lives and be a source of prayer for me.

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Glossary

Abbreviation	**Terminology**
CRN	Cognitive Radio Networks
WRAN	Wireless Regional Area Network
CBP	Coexistence Beacon Protocol
NE	Nash Equilibrium
FCC	Federal Communications Commission
DSA	Dynamic Spectrum Access
SNR	Signal To Noise Ratio
QoS	Quality Of Service
TVWS	TV White Space

Chapter-1

Introduction

Cognitive Radio networks (CRN) are intended to overcome the losses of the licensed spectrum bands. Various studies and measurements have shown that large portions of the licensed bands, under Federal Communications Commission (FCC), remain unused for about 80-90 percent of the time [1]. On the other hand, unlicensed bands like industrial scientific and medical (ISM) bands are getting populous due to proliferation of wireless access demand. Such underutilization of static spectrum allocation, coupled by congestion in unlicensed spectrum bands, is leading towards spectral crises which can only be solved by the dynamic use of cognitive radio techniques.

Efficient spectral utilization can be achieved by CRN once they adapt their own transmission to guarantee certain performance quality for the licensed users. Dynamic Spectrum Access (DSA) allows CRNs to ensure that their function does not cause interference to licensed users as well as ensures optimal utilization of spectrum opportunities [2]. Heterogeneity of available spectrum for secondary access by cognitive radio (CRN), with respect to its various quality of service parameters, leads to contention among collocated CRNs which results in interference and wastage of spectrum opportunity. Self-coexistence problem approached in Coexistence Beacon Protocol (CBP) of IEEE 802.22 WRAN, first wireless standard based on cognitive radios, assumes that the contending networks do not have any preference over the set of available channels [3]. Once confronted with heterogeneous channels, rationally all CRNs will contend for the best available channels and increased interference will occur resulting in inefficient utilization of available spectrum.

Cooperation among CRNs (Players) can maximize the spectrum utilization and their payoffs. While contending for optimized payoffs, players start by forming coalitions in partitions, followed by transitioning to coalitions with better payoffs i.e. bigger coalitions ultimately leading towards formation of the grand or canonical coalition. Coalitional game theory provides suitable modelling framework by which CRNs coexistence, in both dynamic and static spectrum access, can be presented [4]. However, from Game theoretic purpose, there is an absence of adequate type of coalition formation game, by which CRNs coexistence for heterogeneous spectrum access can be appropriately modelled.

In this paper, we analyze this situation from a game theoretic perspective [14] and model coexistence of CRNs by proposing a novel form of coalition games called the C-Partition form. Proposed form of coalition game provides an opportunity to model those scenarios in which CRNs coexistence needs both coalition formation games i.e. canonical and partition forms. Our proposed coalition game adequately represent the phenomena of CRNs coexistence under the conditions where spectral opportunity is heterogeneous by virtue of various quality parameters like bandwidth, channel availability and signal to noise ratio (SNR). We model the situation of collocated CRN for heterogeneous spectrum using proposed algorithm and presented that it is in every player's interest to form bigger coalitions ultimately leading to the formation of a grand coalition. Furthermore, by using proposed algorithm and distributed implementation of rules, cooperation amongst different coalitions or partitions can be enforced in contrast with coalition formation game in partition form where no controlling mechanism exists between contending players.

Nash Equilibrium strategy profile is self-enforcing as the players are searching for outcomes or solutions from which no player will have an incentive to deviate, then the only strategy profiles that satisfy such a requirement are the Nash equilibrium [17]. In this paper, we have shown that the c-partition strategy is the one which have the best prospects in terms of payoff for all the players. This enforce that no player will have any incentive to deviate from c-partition strategy and the final resultant c-partition will be in NE.

1.1 Contribution

In this paper, we have employed a cooperative strategy mechanism by which CRNs coexistence for heterogeneous spectrum access can be modelled appropriately and accurately. Specifically, we have made the following contributions:

Proposed a novel form of coalition formation game i.e. C-Partition form, a hybrid of canonical and partition form. Coalitional game in C-Partition form allows accurate modelling of CRNs coexistence for heterogeneous spectrum access using cooperative strategy. As against coalition game in partition or canonical form which only covers part of CRNs coexistence, under prevalent strategy, by not covering the transitional aspect of coalition formation process from partitions to a canonical form.

Internally CRNs or partitions are administered and controlled through base station, but there is absence of controlling entity by which cooperation amongst the coalitions or partitions can be handled. We proposed an effective rule based algorithm by which the resulting partitions i.e. CRNs can be administered effectively without the need of central controlling entity. Coalitions are benefited greatly, in terms of payoff, using the proposed algorithm as it allows cooperation among the coalition partitions and minimize the interference and spectrum wastage. Using the proposed algorithm, the payoffs of all the individual players can be maximized and system spectrum utilization is also guaranteed to be at maximum.

Chapter-2

Related Work

In this section we provide an overview of the work carried out in the domain of self-coexistence in CRNs and application of the game theoretic solution in the context of communication networks.

Authors of [5] have delineated upon heterogeneity of characteristics which are presented by spectrum band due to which various spectrum channels become more valuable for the CRNs than other spectrum channels. By deriving an evolutionary stable strategy, they derive equilibrium state for CRN spectrum sharing to neutralize the effect of greedy strategies employed by players for spectrum sharing.

Authors of [6] explores the possible impact of tradeoff between spectrum access and spectrum sharing between contending networks of users by modelling it in coalition formation game in partition form. Proposed algorithm allows the distributed functioning of contending networks to form disjoint coalitions, where coalition members cooperate to jointly optimize their sensing and access performance.

Authors of [7], [12], [13] states the importance of cooperative spectrum sensing as affective technique to improve CRNs spectrum sensing and access. Authors of [7] modelled a multi-channel spectrum sensing and access problem as hedonic coalition formation game, which allows users to join or leave the coalition using switch rule, and finally leads to network partition which is nash stable.

Authors in [8] studied the phenomenon of CRNs for a decentralized dynamic spectrum access. They employed multivariate global games theory and aimed at achieving Bayesian Nash equilibrium (BNE) for the resulting game with respect to spectrum hole occupancy, channel quality and observation noise.

Authors in [9] proposed an efficient scheduling algorithm by which adaptive spectrum sensing and data transmission can be achieved. They explored the scenario where CRN have regular and stochastic data transmission needs under conditions of time varying channels.

Authors in [10] focused on the problems of self-coexistence phenomena of CRNs which are caused due to lack of consensus of spectrum access policies and absence of controlling entity or channel. Utility graph coloring method is utilized for the allocation of spectrum for multiple overlapping CRNs by combination of aggregation, fragmentation of channel carriers, broadcast messages and contention resolution techniques.

Authors in [11] have described the existence of three temporal spectrum holes regions i.e. black region, grey region and white region with respect to time and space dimensions which can improve the spectrum utilization of cognitive radio networks (CRNs). The unlicensed users can opportunistically access the licensed spectrum in gray region, while in the white region users can exploit spatial spectrum opportunities and transmit at any time by taking advantage of their long distances from the licensed user.

Authors in [21] have described the overwhelming increase in the use of cloud resources primarily due to the introduction of data intensive applications. They described the necessity of resources to meet elastic demand which sometimes cannot be fulfilled by provider's limited resources. They dwell upon the cloud federation which is the prime technique considered for cloud elasticity to cater for the scarcity of resources. However a simple and clear solution is to form grand federation but there are situations when the grand coalition does not produce the optimal payoffs for the involved cloud platforms and need a more sophisticated handling.

Authors in [22] have described the method for the utility allocation which ensures the formation of Nash-stable partition. They proposed the definition of non-empty Nash-stable core and described that the use of relaxed efficiency in utility sharing allows its formation. They proposed an algorithm called Nash stability establisher for finding the Nash stability in a non-cooperative game in which each player determines its strategy using random round-robin scheduler.

Chapter-3

System Model and Assumptions

3.1 System Model

Consider a regional wireless area network where collocated CRNs, represented as $\mathcal{N}$ = {1, 2, 3, ….., n} contend for secondary access to licensed spectrum band. Licensed spectrum are heavily underutilized and present an opportunity for secondary access to unlicensed users on available channels, represented as $\mathcal{K}$ = {1, 2, 3, ….., k} while ensuring quality service threshold. The notations commonly used in this paper are shown in table I.

Notations & Acronyms	
Notation	**Definition**
a_k	CRN's action of selecting channel k
u_k	CRN's utility for gaining access to channel k
S	Coalitional structure of CRNs
u_S	CRN's utility gained by coalition structure S
I_S	CRN's Interference across the coalition structure S
$\mathcal{N}$	Set of contending CRNs
$\mathcal{K}$	Set of available channels
t	One time slot i.e. MAC super frame
$\mathcal{Z}$	Set of all possible S for given values of $\mathcal{N}$ and $\mathcal{K}$
$\mathcal{P}$	Partition or CRNs
$\mathcal{J}$	Jain's Fairness Index

Table 1: Notations and Acronyms

3.2 Assumptions

Following are the underlying assumptions for the work presented in this paper:

3.2.1 Heterogeneous Spectrum Band

The licensed spectrum band is heterogeneous by virtue of various quality of service (QoS) parameter, like spectrum availability w.r.t to licensed user inactivity, bandwidth, signal to noise ratio (SNR) etc. As shown in figure 1, channel 1 bear a total utility of 7 which includes utility of 2 with respect to availability of channel for secondary access, utility of 3 for bandwidth a channel offers and utility of 2 for SNR. As opposed to channel 1, channel 2 in figure 1 bear a better payoff i.e. utility of 8 which includes utility of 3 with respect to availability of channel for secondary access, utility of 3 for bandwidth a channel offers and utility of 2 for SNR. Spectrum contain channels which vary significantly in these QoS parameters, due to which some channels offer better payoff than the others and are more lucrative for the contending networks.

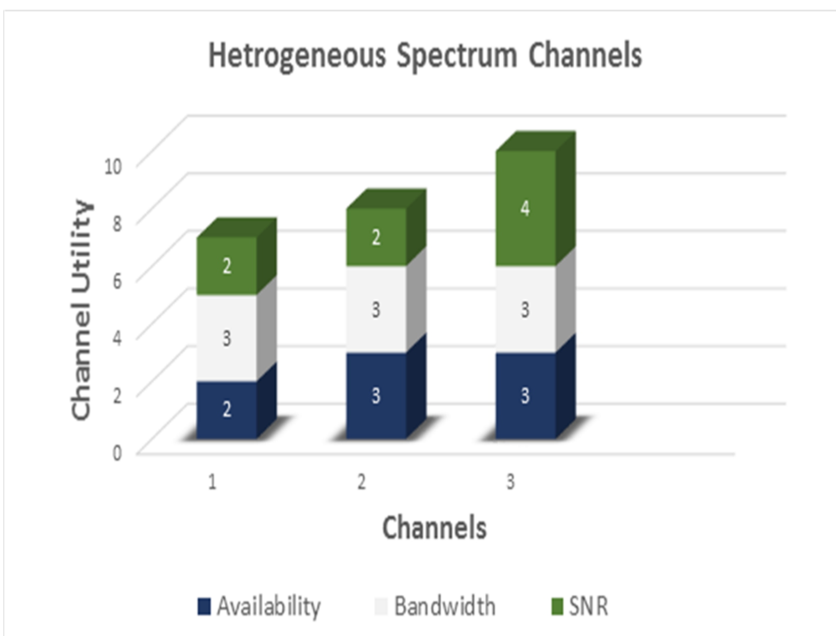

Figure 1: Heterogeneous Spectrum Channels

3.2.2. Wireless Regional Area Network: Coexistence beacon protocol

IEEE 802.22 WRAN for CRNs is considered to model wireless regional area of collocated CRNs which contend for secondary access to licensed heterogeneous spectrum band, as shown in figure 2. Spectrum opportunity, payoff and wastage: Spectrum opportunity arises once licensed users is idle in its timeslot. Such opportunity is often called as TVWS (TV white space) [15] which can be utilized for secondary access by unlicensed users. Payoff of secondary access is determined by the channel utility to which secondary access is made. If more than one unlicensed users access the same available channel in a timeslot, then it results in interference and opportunity is wasted.

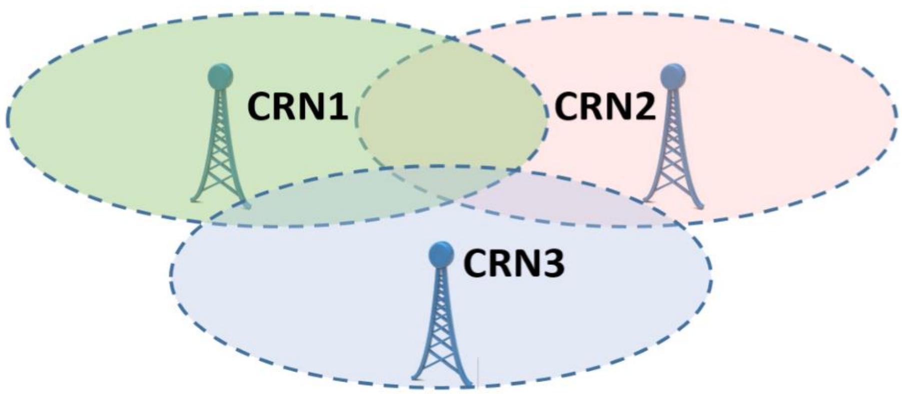

Figure 2: Wireless Regional Area Network (WRAN)

3.2.3. Coalition decision time frame

Single MAC super frame is considered as one timeslot for decision making. Decision to make new coalition has to be controlled by allowing only one decision per time slot i.e.'t'.

Chapter-4

Coalition Formation Game in C-Partition Form

C-Partition form is hybrid of coalition games in partition form and canonical form. It is proposed in order to represent the coexistence phenomena of CRNs in heterogeneous spectrum access. Coalitional games in partition form allows player to form coalitions called partitions, where structure of forming partitions affects the outcomes of other partitions or coalitions. Coalitional game in Canonical form, which is also referred as grand coalition, allows all players to make single coalition and cooperate with each other with a view to increase the payoffs of the players and the coalition. The scenario of CRN coexistence under heterogeneous spectrum access where players employ greedy strategy to gain access to channels will result in contention, interference and wastage. This can be overcome if players start cooperating with each other by forming coalitions i.e. partitions or grand coalition. Many studies [4], [6] have modelled the CRN in one of the both form, but to the best of our knowledge no study has yet focused on the transitional aspect of this coalition formation process from partitions into a grand coalition. Thus true representation of coexistence phenomenon of CRN, once players intend employing cooperative strategy, cannot be truly demonstrated by coalition games in partition and canonical form. Therefore C-Partition form of coalition games is presented to overcome this presented barrier to truly model the CRN coexistence phenomena.

4.1. C-Partition From

In C-Partition from, players will start forming smaller coalitions called partitions and then ultimately lead towards the bigger coalitions with better payoffs. This process will continue and new coalitions will form ensuring to must possess greater payoff than the previous partitions. The coalition formation process will be governed by the use of Rule for coalition formation given in equation (2). This process will transit across three types of coalitions or partitions:

a. Singleton partitions, with single unlicensed user.
b. Intermediate partitions, with more than one unlicensed user but short of canonical partition.
c. Canonical partition, with all users in single coalition.

Case of 4 unlicensed users applying C-Partition form including their partitions are depicted in table II. Coalition formation process in c-partition form for $\mathcal{N}$=3 and $\mathcal{K}$=3 is shown in figure 3.

Coalition Formation Game in C-Partition Form: Representation of possible Coalitional structures i.e. $\mathcal{S}$.					
Players	Partitions				Structure
	1	2	3	4	
1,1,1,1	1	1	1	1	4 Singleton partitions: (1,1,1,1)
1,1,1,1	1,1	1	1	-	3 Intermediate partitions: 1 coalition of two users and 2 singleton coalitions : (2,1,1)
1,1,1,1	1,1	1,1	-	-	2 Intermediate partitions: Each containing 2 users: (2,2)
1,1,1,1	1,1,1	1	-	-	2 Intermediate partitions: 1 coalition of 3 users and 1 singleton coalition: (3,1)
1,1,1,1	1,1,1,1	-	-	-	1 Canonical or Grand partition: (4,0)

Table 2: Representation of possible coalition structures in C-Partition Form for N=4

4.2. Structure of C-Partitions

For any $\mathcal{S} \in \mathcal{Z}$, let $\mathcal{P}_{\mathcal{S}} = \{\mathcal{P}1, \mathcal{P}2, \ldots, \mathcal{P}n\}$ be a partitions [16] of $\mathcal{S}$, such that,

$$\sum_{i=1}^{r} \mathcal{P}i = S \,, \forall i : \mathcal{P}i \neq 0, \forall k : \mathcal{P}i \cap \mathcal{P}k = \emptyset, if\ k \neq i \qquad (1)$$

Above equation depicts that union of all the partitions that belongs to $\mathcal{Z}$ will be the coalition structure in C-Partition form i.e. $\mathcal{S}$, where every partition must contain $\mathcal{N}\geq 1$ and any player cannot be a member of two partitions.

4.3. Rule for Coalition Formation in C-Partition Form

Rule for coalition formation for any $\mathcal{S} \in \mathcal{Z}$, form $\mathcal{S}$ if and only if,

$$\mathcal{U}x > \mathcal{U}y \qquad (2)$$

Where $\mathcal{U}x$ is the utility of new coalition strucutre i.e. $\mathcal{S}x$ and $\mathcal{U}y$ is the utility of previous coalition structure i.e. $\mathcal{S}y$.

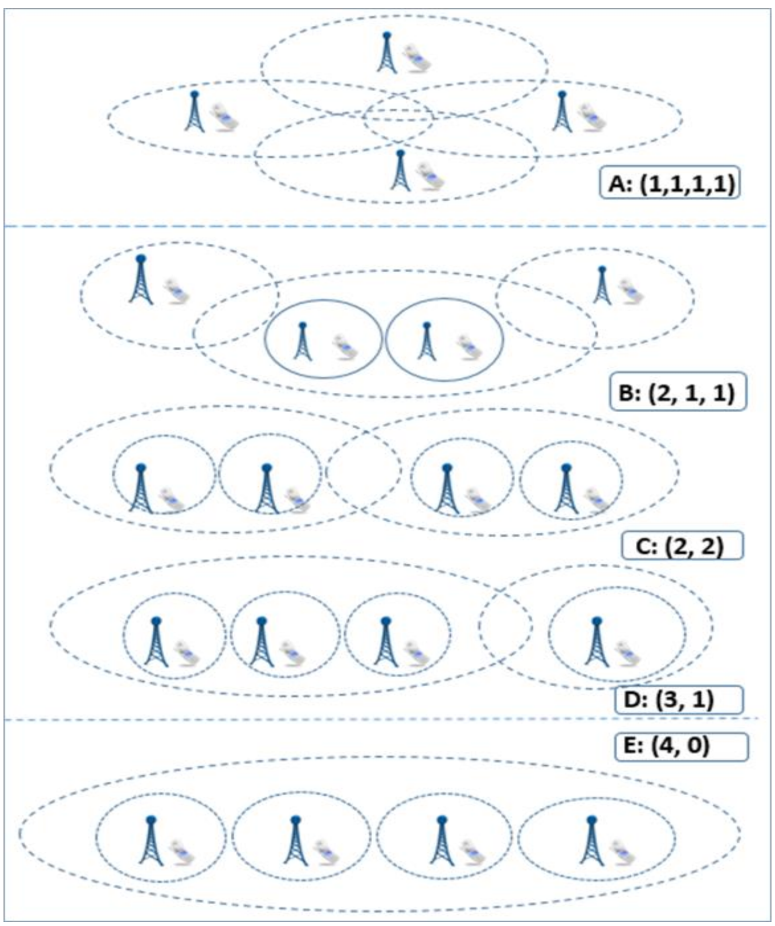

Figure 3: Coalition Formation Process for N=4

Chapter-5

Game Formulation

5.1. Game Design for Simulation

Coalitional game in C-Partition form comprises three distinct element for the purpose of simulation:

a. CRNs considered as players and represented by $\mathcal{N}$.

b. Choice of selecting spectrum channel or action strategy represented as a_k.

c. Payoff achieved by action strategy represented as u_s.

Mathematically;

$$\mathcal{G} = [\mathcal{N}, a_k, u_s] \qquad (3)$$

If two or more i.e. $\mathcal{N} \geq 2$ contending players seek to access same channel then gains or payoff of all players will be zero. However if players employ a strategy to access different channels then their corresponding payoff will be as per the utility of the accessed channels. Mathematically it is represented below for a game of three contending players;

$$u_s = u_s(a_i, a_j, a_k) = \begin{cases} (u_i, u_j, u_k), & i \neq j \neq k \\ (u_i), & j = k, i \neq j, i \neq k \\ (u_j), & i = k, j \neq i, j \neq k \\ (u_k), & i = j, k \neq i, k \neq j \\ 0, & i = j = k \end{cases} \qquad (4)$$

Where the first element of $u(a_i,a_j,a_k)$ represents the payoff for player which selects channel i and the second element for player which selects channel j and so on. If all players selects different channels then payoff will be sum of all three channels utility i.e. Case:1 of equation (4). If more than one player selects the same channel then it will result in wastage of that channel utility, as no one will be able to access the channel i.e. Case: 2-4 of equation (4). If all players access the same channel then the payoff of entire coalition structure will be zero i.e. Case: 5 of equation (4). Method of calculating the average utility of coalition structure is explained in Algorithm-1.

ALGORITHM 1. Algorithm for Calculation of coalition Structure Payoff

Data: u_k, a_k, S, N, K.
Output: u_s, I_s.

1: **for** All channel combinations of given Coalitional structure **do**
2: Add the payoffs of all channels K which are accessed by the players of coalition structure combination : $\sum_{i=1}^{|K|} u_i$.
3: Determine the interference across coalition strucutre combination : $\sum_{n=1}^{|m|} I_m$.
4: Calculate the resultant payoff of the coalition combination by : $u_s = \sum_{i=1}^{|K|} u_i - \sum_{n=1}^{|m|} I_m$.
5: **end for**..
6: Calculate average payoff and interference of the coalition structure combinations.

Algorithm 1: Calculation of Coalition Structure Payoff

5.2. Analysis

We analyze spectrum sharing mechanism as follows:

a. Analysis of the payoff for coalition structures S, when $\mathcal{N}$ and $\mathcal{K}$ are same.
b. Analysis of the payoff for coalition structures S, when $\mathcal{N}$ is kept same and values of $\mathcal{K}$ are varied.

c. Analysis of the payoff for coalition structures S, when $\mathcal{K}$ is kept same and values of $\mathcal{N}$ are varied.

d. Analysis of interference across coalition structures S, when $\mathcal{N}$ and $\mathcal{K}$ are same.

e. Comparison between payoffs gained by CRNs while using C-Partition form and the payoffs gained by players which employ non cooperative strategy for heterogeneous spectrum sharing.

f. The number of combination becomes intractable when $\mathcal{K}$, $\mathcal{N} > 6$. Therefore analysis is restricted to $\mathcal{K}$, $\mathcal{N} = 6$.

4.3. Calculation of utility / payoff

Utility gained by the CRNs which employ coalition game in C-Partition form for secondary access can be calculated using following equation;

$$u_s = \sum_{i=1}^{k} u_s - \sum_{n=1}^{m} I_m \tag{5}$$

Where u_i represents the utility of CRN for selecting channel i and I_m represents the interference across particular CRN. Calculation of payoff of a coalition structure using C-Partition form is described in Algorithm-1. This involves calculation of average payoff of all possible coalition structures under given values of $\mathcal{N}$ and $\mathcal{K}$. For each combination, sum of utilities for all accessed channels will be calculated. Secondly sum of all interferences across coalition structures and partitions will be added. Finally total utility will be subtracted by the interference to determine the payoff that coalition structure.

Chapter-6

Simulation and Results

6.1. Simulation Setup

Brief explanation on symbols and notations used for representation of coalition structures and partitions is given in table II. For simplicity and ease of understanding, we assumed that channel 1 i.e. $k1$ holds a utility of 5 i.e. $u1 = 5$, $k2$ offers $u2 = 6$, $k3$ offers $u3 = 7$, $k4$ offers $u4 = 8$, $k5$ offers $u5 = 9$ and $k6$ offers $u6 = 10$.

6.2. Results

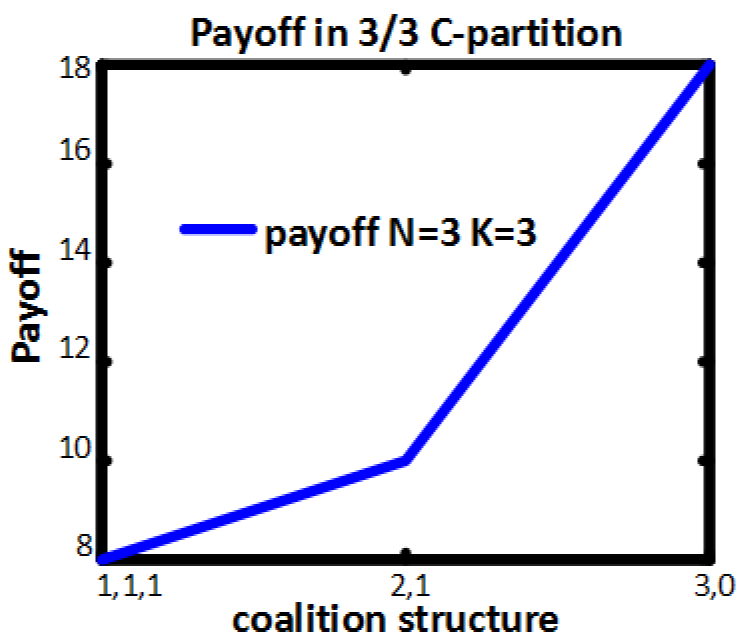

Figure 4: Payoff in 3/3 C-Partition

Figure 4 depicts the payoff trend when number of users $\mathcal{N}$ and channels $\mathcal{K}$ are same i.e. the payoff trend of a coalition structures when $\mathcal{N}=3$ $\mathcal{K}=3$. It is shown that players achieve least utility when they form singleton coalitions like 1,1,1 for $\mathcal{N}=3$. However payoff starts increasing as coalition structure is formed by making bigger coalitions like 2,1 and 3,0. Payoff is maximized once canonical coalition structure is formed i.e. 3,0 for $\mathcal{N}=3$. Payoff increases as bigger coalition are formed like 2,1 3,0. On x-axis; 1,1,1 means coalition structure of 3 singleton coalitions with average payoff of $\mathcal{U}x$ = 8. Whereas 2,1 means structure with 1 coalition of 2 users and 1 singleton coalition with average payoff of $\mathcal{U}y$ = 11 i.e. $\mathcal{U}y > \mathcal{U}x$. On x-axis; 3,0 means 1 canonical coalition of 3 user with average payoff of $\mathcal{U}z$ = 18 i.e. $\mathcal{U}z > \mathcal{U}y$ and $\mathcal{U}z > \mathcal{U}x$. Thus it is proved that it is every player's interest to cooperate and form coalitions. Once players will follow C-Partition structure, it will maximize their average individual payoffs as well as payoff of entire coalition structure is also assured to be at maximum.

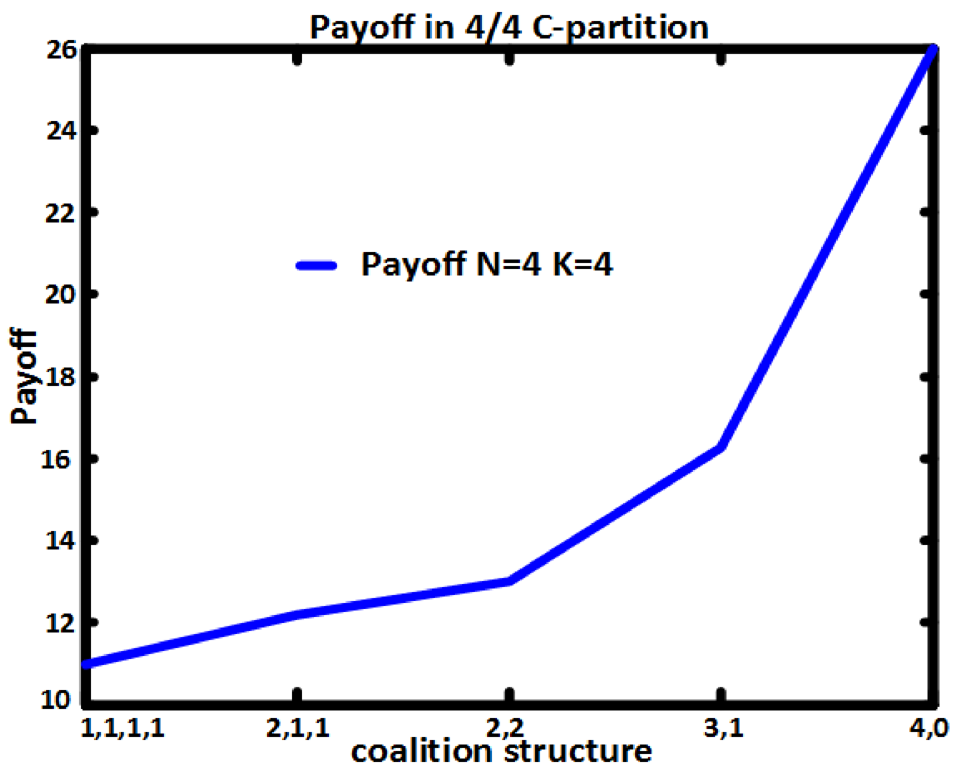

Figure 5: Payoff in 4/4 C-Partition

Figure 5 depicts the payoff trend when number of users $\mathcal{N}$ and channels $\mathcal{K}$ are same i.e $\mathcal{N}$ = 4, $\mathcal{K}$ = 4 means 4 users and 4 channels. It is shown that players achieve least utility when they form singleton coalitions like 1,1,1,1 for $\mathcal{N}$=4. However payoff starts increasing as intermediate coalition structures i.e. bigger coalitions like 2,1,1 2,2 3,1 are formed. Payoff is at maximum once canonical coalition structure i.e. 4,0 is formed. Payoff in singleton coalition structure is $\mathcal{U}x$ =11 while in intermediate coalitions payoffs are $\mathcal{U}y$= 12, 13 and 16. Finally in grand or canonical partition structure payoff is at maximum i.e. $\mathcal{U}z$= 26. On x-axis; 1,1,1,1 means coalition structure of 4 singleton coalitions with average payoff of $\mathcal{U}x$ = 11. Whereas 2,1,1 means structure with 1 coalition of 2 users and 2 singleton coalition with average payoff of $\mathcal{U}y$ = 12 i.e. $\mathcal{U}y > \mathcal{U}x$. Similarly; 3,1 means 1 coalition of 3 user and 1 singleton coalition with average payoff of $\mathcal{U}x$ = 16 i.e. $\mathcal{U}z > \mathcal{U}y$ and $\mathcal{U}z > \mathcal{U}x$.

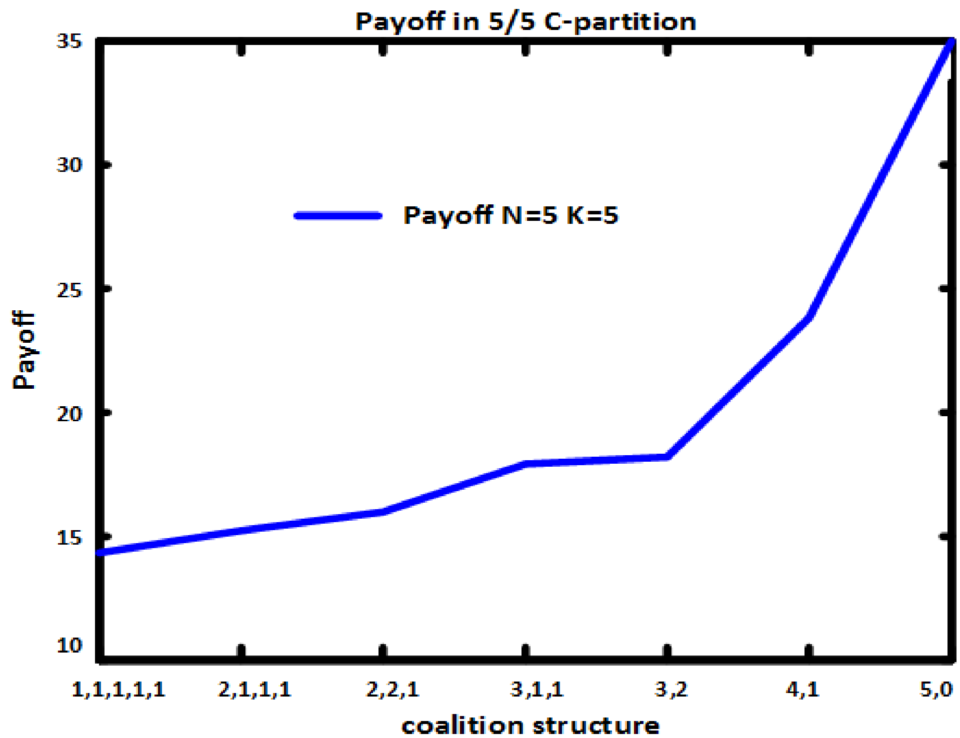

Figure 6: Payoff in 5/5 C-Partition

Figure 6 depicts the payoff trend when number of users $\mathcal{N}$ and channels $\mathcal{K}$ are same i.e. $\mathcal{N}$ = 5, $\mathcal{K}$ = 5. It is shown that players achieve least utility when they form singleton coalitions like 1,1,1,1,1 for $\mathcal{N}$=4. However payoff increases with bigger coalition formation like 2,1,1,1 2,2,1 3,1,1 3,2 4,1 and payoff is at maximum once canonical coalition structure i.e. 5,0 is formed. On x-axis; 1,1,1,1,1 means coalition structure of 5 singleton coalitions with average payoff of $\mathcal{U}x$ = 14. Whereas 2,1,1,1 2,2,1 3,1,1 3,2 and 4,1 are intermediate coalitions partition structure with increasing trend in their achieved payoffs i.e. $\mathcal{U}x$, $\mathcal{U}y$=15, 16, 17, 18 and 24 respectively. Similarly 5,0 on x-axis depicts the grand coalition structure by achieving the maximum payoff i.e $\mathcal{U}z$=35 .

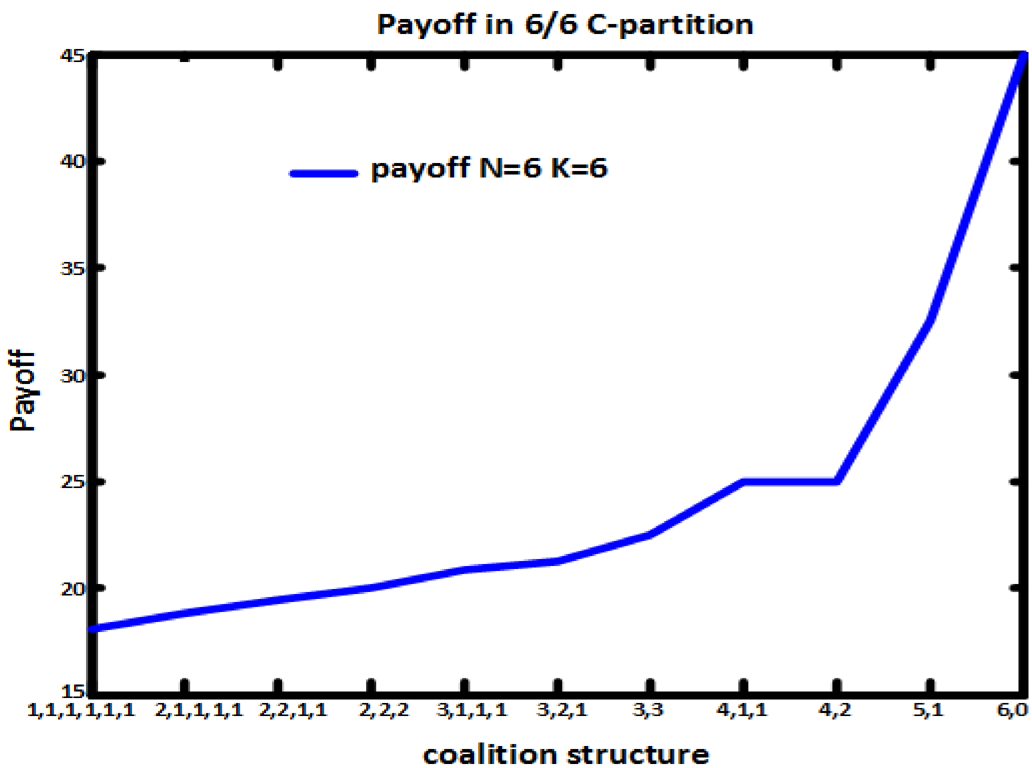

Figure 7: Payoff in 6/6 C-Partition

Figure 7 depicts the payoff trend when number of users $\mathcal{N}$ and channels $\mathcal{K}$ are same i.e. $\mathcal{N}$= 6, $\mathcal{K}$ = 6. It is shown that when players form singleton coalitions like 1,1,1,1,1,1 they achieve less utility. Increase in payoff of coalitions is observed as intermediate coalition structures are formed i.e. bigger coalitions like 2,1,1,1,1 2,2,1,1 2,2,2 3,1,1,1 3,2,1 3,3 4,1,1 4,2 and 5,1. The payoff is at maximum once canonical coalition structure i.e. 6,0 is formed. On x-axis, 1,1,1,1,1,1 means coalition structure of 6 singleton coalitions with average payoff of $\mathcal{U}x$ = 17. Whereas 2,1,1,1,1 2,2,1,1 2,2,2 3,1,1,1 3,2,1 3,3 4,1,1 4,2 and 5,1 are intermediate coalitions partition structure with increasing trend in their achieved payoffs i.e. $\mathcal{U}y$=18, 19, 20, 21, 22, 23, 24, 25 and 33 respectively. Similarly 6,0 on x-axis depicts the grand coalition structure by achieving the maximum payoff i.e $\mathcal{U}z$=45.

So far we have analyzed the payoff trends in channel user's combination where $\mathcal{N} = \mathcal{K}$ and found out that cooperation among contending players is useful to maximize their payoffs as smaller coalitions get lesser utility than the bigger and grand coalitions. We now extend our analysis a more depth by exploring payoff trends in varied channels user's combination where $\mathcal{N} \neq \mathcal{K}$. First we analyze the payoff trend once $\mathcal{N}$ is kept constant and $\mathcal{K}$ is varied.

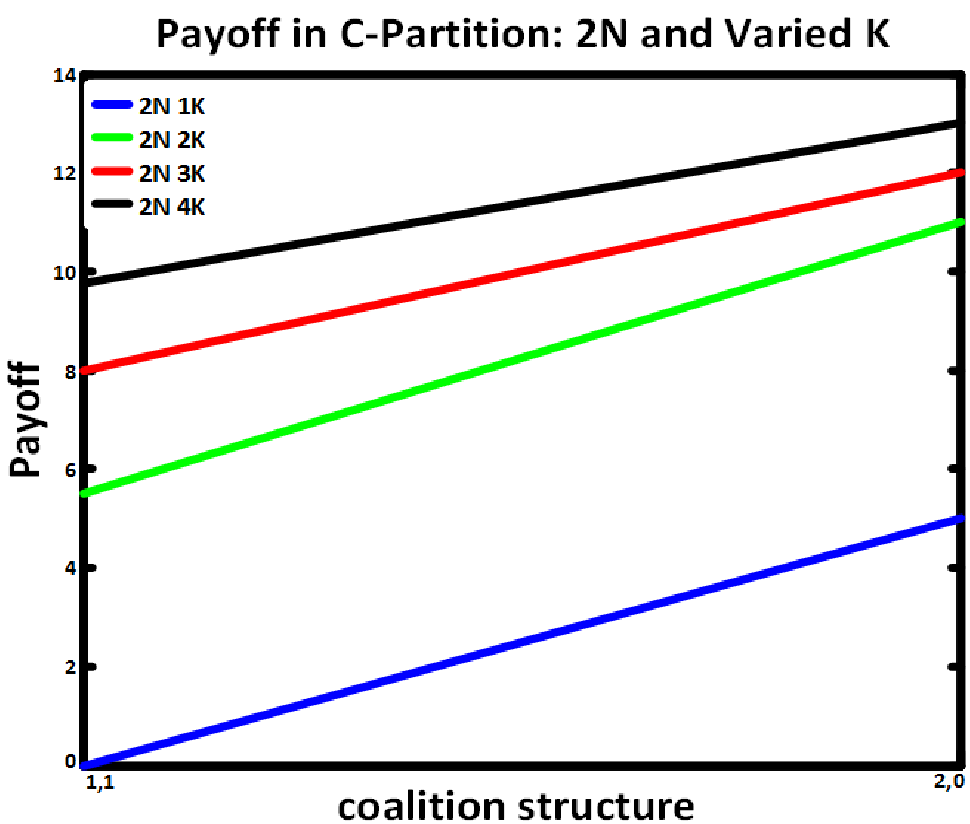

Figure 8: Payoff in 2N and Varied K C-Partition

Figure 8 depicts payoff trend when number of users $\mathcal{N}$ are kept constant while number of channels $\mathcal{K}$ are varied i.e $\mathcal{N}$=2 and $\mathcal{K}$=1,2,3 and 4. Payoff increases as bigger coalition are formed. As the number of channels are increased, resulting payoffs in singleton coalitions rises from 0 to 9 and for canonical coalitions from 5 to 13. It is shown that players will achieve lesser utility when the number of channels are lesser i.e. $\mathcal{N} > \mathcal{K}$. As number of channels are increased i.e. $\mathcal{N} \leq \mathcal{K}$, it will result in increased payoff and lesser interference. However the previous findings also hold true here i.e. lesser payoff with smaller coalition structures like singleton coalitions and increased payoff will be achieved with bigger coalition structures.

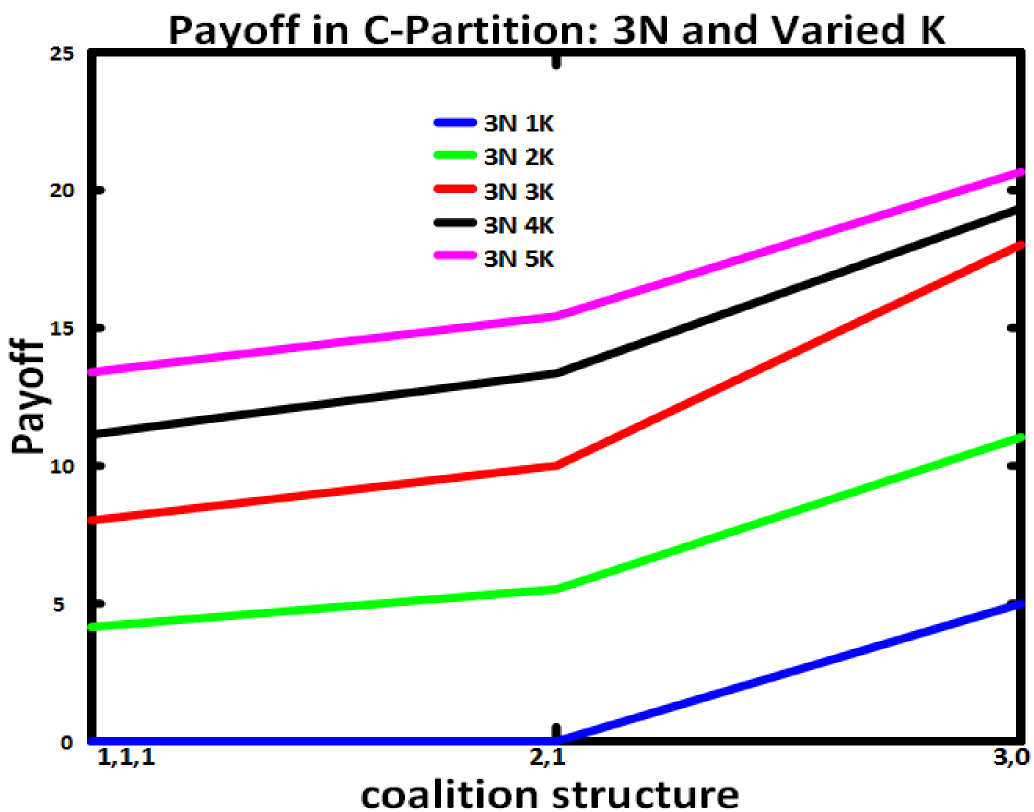

Figure 9: Payoff in 3N and Varied K C-Partition

Fig. 9: Payoff trend when number of users $\mathcal{N}$ are kept constant while number of channels $\mathcal{K}$ are varied i.e. $\mathcal{N}$=3 and $\mathcal{K}$=1,2,3,4 and 5. Payoff increases as bigger coalition are

formed. As the number of channels are increased, resulting payoffs in singleton coalitions rises from 0 to 14, for the intermediate coalitions payoff rises from 0 to 16 and for canonical coalitions from 5 to 21. It is shown that with lesser number of channels i.e. $\mathcal{N} > \mathcal{K}$, players will achieve less utility. With increase in the numbers of channels i.e. $\mathcal{N} \leq \mathcal{K}$, will result in increased payoff and lesser interference. However the previous findings also hold true here i.e. lesser payoff with smaller coalition structures like singleton coalitions and increased payoff will be achieved with bigger coalition structures, except in a case when $\mathcal{N}$=3 and $\mathcal{K}$=1 where intermediate coalition i.e. 2,1 also bears same payoff as of singleton structure i.e. 1,1,1. The reason behind this is availability of only one channel which will be accessed by both the coalitions of coalition structure 2,1 i.e. intermediate coalition of 2 users and 1 singleton coalition.

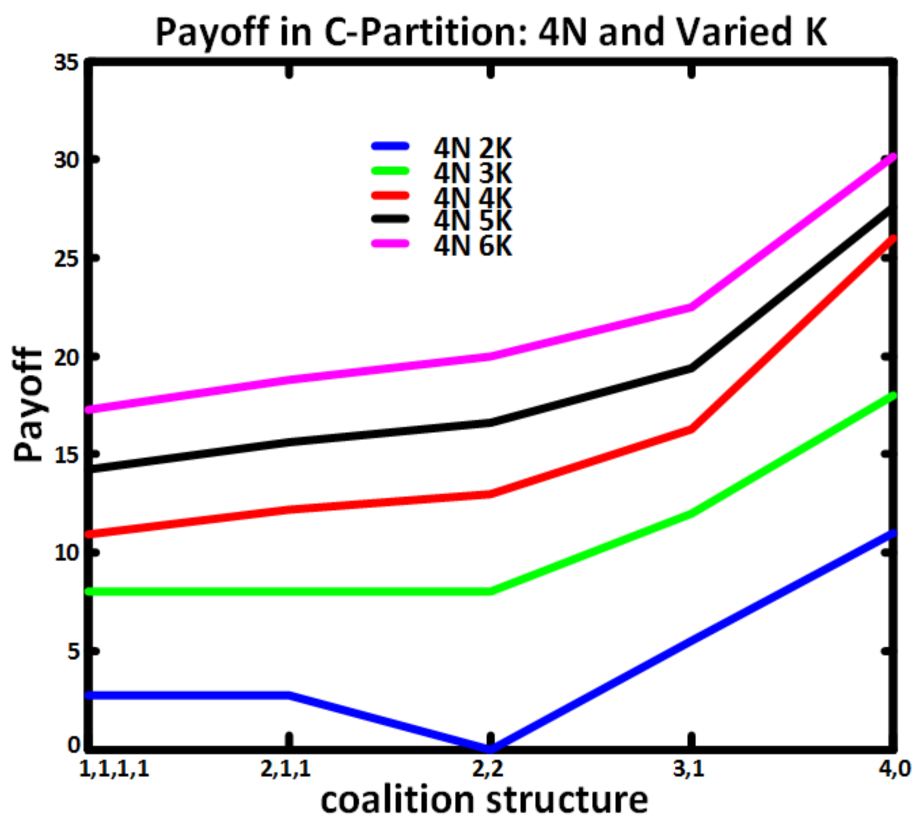

Figure 10: Payoff in 4N and Varied K C-Partition

Fig. 10: Payoff trend when number of users $\mathcal{N}$ are kept constant while number of channels $\mathcal{K}$ are varied i.e. $\mathcal{N}$=4 and $\mathcal{K}$=2,3,4,5 and 6. Payoff increases as bigger coalition are

formed. As the number of channels are increased, resulting payoffs in singleton coalitions rises from 3 to 17, for the intermediate coalitions payoff rises from 4 to 23 and for canonical coalitions from 11 to 31. It is shown that players will achieve lesser utility when the number of channels are lesser i.e. $\mathcal{N} > \mathcal{K}$. As number of channels are increased i.e. $\mathcal{N} \leq \mathcal{K}$, it will result in increased payoff and lesser interference. However the previous findings also hold true here i.e. lesser payoff with smaller coalition structures like singleton coalitions and increased payoff will be achieved with bigger coalition structures, except in a case when $\mathcal{N}$=4 and $\mathcal{K}$=2 where intermediate coalition i.e. 2,1,1 also bears same payoff as of singleton structure i.e. 1,1,1,1. The reason behind this is availability of only two channel which will be accessed by all the coalitions of coalition structure 2,1,1 i.e. intermediate coalition of 2 users and 2 singleton coalitions. So it will cause same interference as in case of coalition structure 1,1,1,1 resulting in same payoff for both 1,1,1,1 and 2,1,1.

Figure 10 also highlights another important behavior of varied channels user combination. Here a bigger coalition like 2,2 holds payoff of zero $\mathcal{U}x$ =0 while smaller coalition like 2,1,1 still hold a better payoff $\mathcal{U}y$=3 when $\mathcal{N}$=4 and $\mathcal{K}$=2. This highlights absolute interference phenomena that can occur in varied channel user combinations. Since there are two coalitions of two users and there are only two channels which results in both channels being accessed by both coalitions. Resultantly absolute interference will occur and no coalition will get any payoff.

So after analyzing the payoff trends in varied channel user's combination where $\mathcal{N}$ is kept constant and $\mathcal{K}$ is varied, we found out that cooperation among contending players is useful to maximize their payoffs as smaller coalitions get lesser utility than the bigger and grand coalitions. We also found out the phenomena of absolute interference as shown in figure 10. We now extend our analysis a more depth by exploring payoff trend in varied channels user's combination where $\mathcal{N}$ 6= $\mathcal{K}$ from another perspective i.e. $\mathcal{N}$ is varied and $\mathcal{K}$ is kept constant.

Figure 11 depicts the payoff trend when number of users $\mathcal{N}$ are varied while number of channels $\mathcal{K}$ are constant. For $\mathcal{N}$=2,3 and 4 and $\mathcal{K}$=2, bigger coalition yield more payoffs.

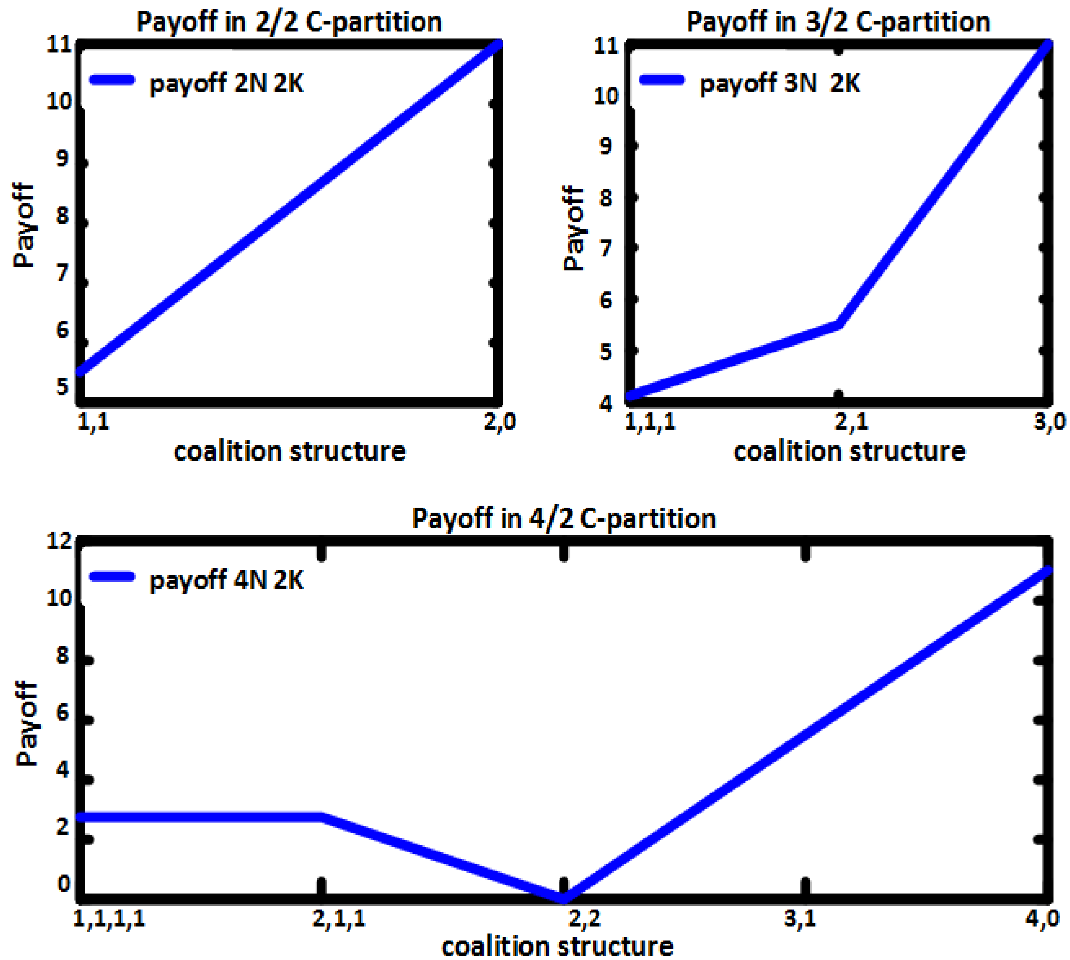

Figure 11: Payoff in Varied N and 2K C-Partition

However when value of $\mathcal{N}$=2 and $\mathcal{K}$=2, coalition achieve more payoff i.e. $\mathcal{U}x$=5.5 to 11. Whereas when values of $\mathcal{N}$=3 and 4, the payoff of coalitions ranges between $\mathcal{U}y$= 4 to 11 and 3 to 11. This shows smaller values of $\mathcal{N}$ allows achieving higher payoff in singleton coalitions. Also as the number of users are increased, payoffs trend shifts from a straight line pattern to elliptical curve. It is shown that players will achieve more utility in earlier timeframe, when the number of users are lesser than number of channels i.e. $\mathcal{N} < \mathcal{K}$. As number of users are increased, it will result in increase of payoff and lesser interference but the payoff curve will become elliptical. The reason behind this is that with increase in number of $\mathcal{N}$ there will be increase in number of possible coalition structures that can be formed. However the previous findings also hold true here i.e. lesser payoff with smaller coalition structure like singleton coalitions and increased payoff will be achieved with bigger coalition structures.

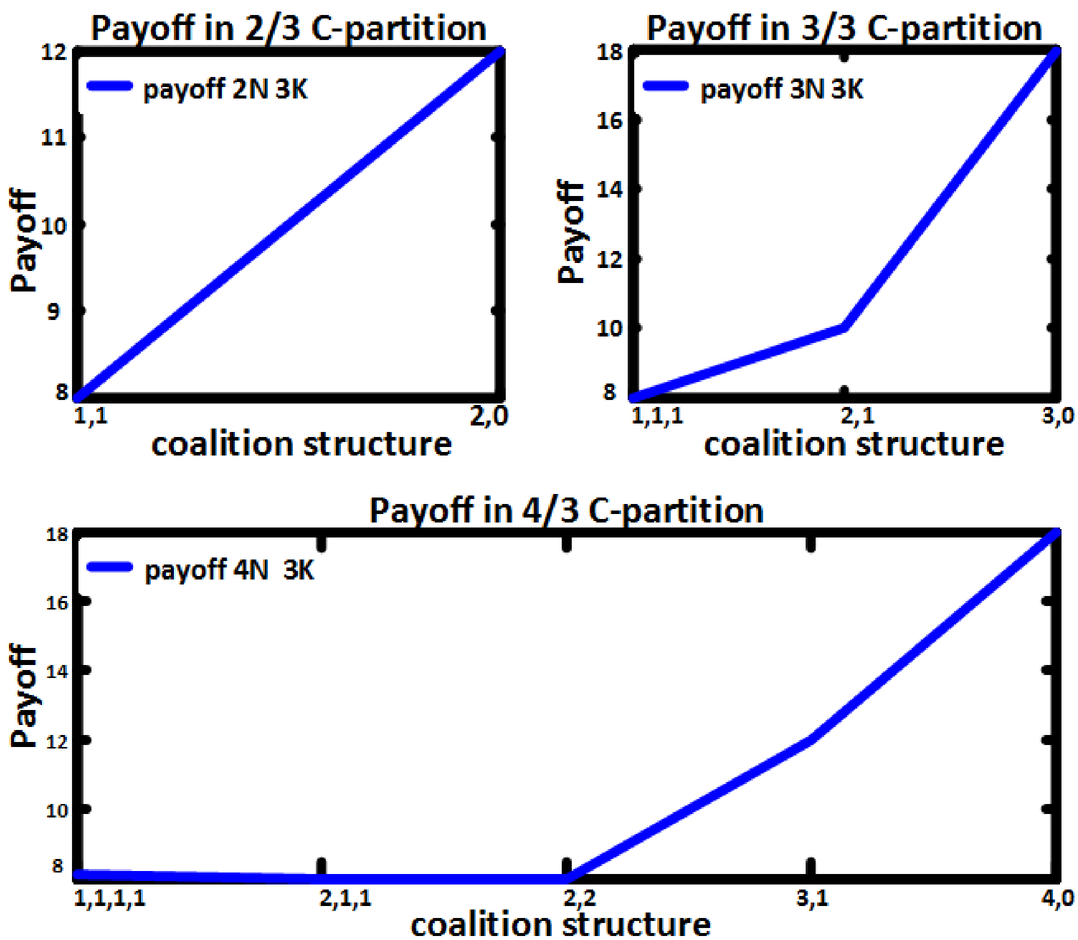

Figure 12: Payoff in Varied N and 3K C-Partition

Figure 12 depicts the Payoff trend when number of users $\mathcal{N}$ are varied while number of channels $\mathcal{K}$ are constant. For $\mathcal{N}$=2,3 and 4 and $\mathcal{K}$=3, bigger coalition yield more payoffs. However when value of $\mathcal{N}$=2 and $\mathcal{K}$=3, coalition achieves payoff of $\mathcal{U}x$=8 to 12. Whereas when values of $\mathcal{N}$=3 and 4, the payoff of coalitions ranges between $\mathcal{U}y$= 8 to 18. This shows smaller values of $\mathcal{N}$ allows achieving higher payoff in singleton coalitions. Also as the number of users are increased, payoffs trend shifts from a straight line pattern to elliptical curve. It is shown that players will achieve more utility in earlier timeframe, when the number of users are lesser than number of channels i.e. $\mathcal{N} < \mathcal{K}$. As number of users are increased, it will result in increase of payoff and lesser interference but the payoff curve will become elliptical. The reason behind this is that with increase in number of $\mathcal{N}$ there will be increase in number of possible coalition structures that can be formed. However the previous findings also hold true here i.e. lesser payoff with smaller coalition structure like singleton coalitions and increased payoff will be achieved with bigger coalition structures.

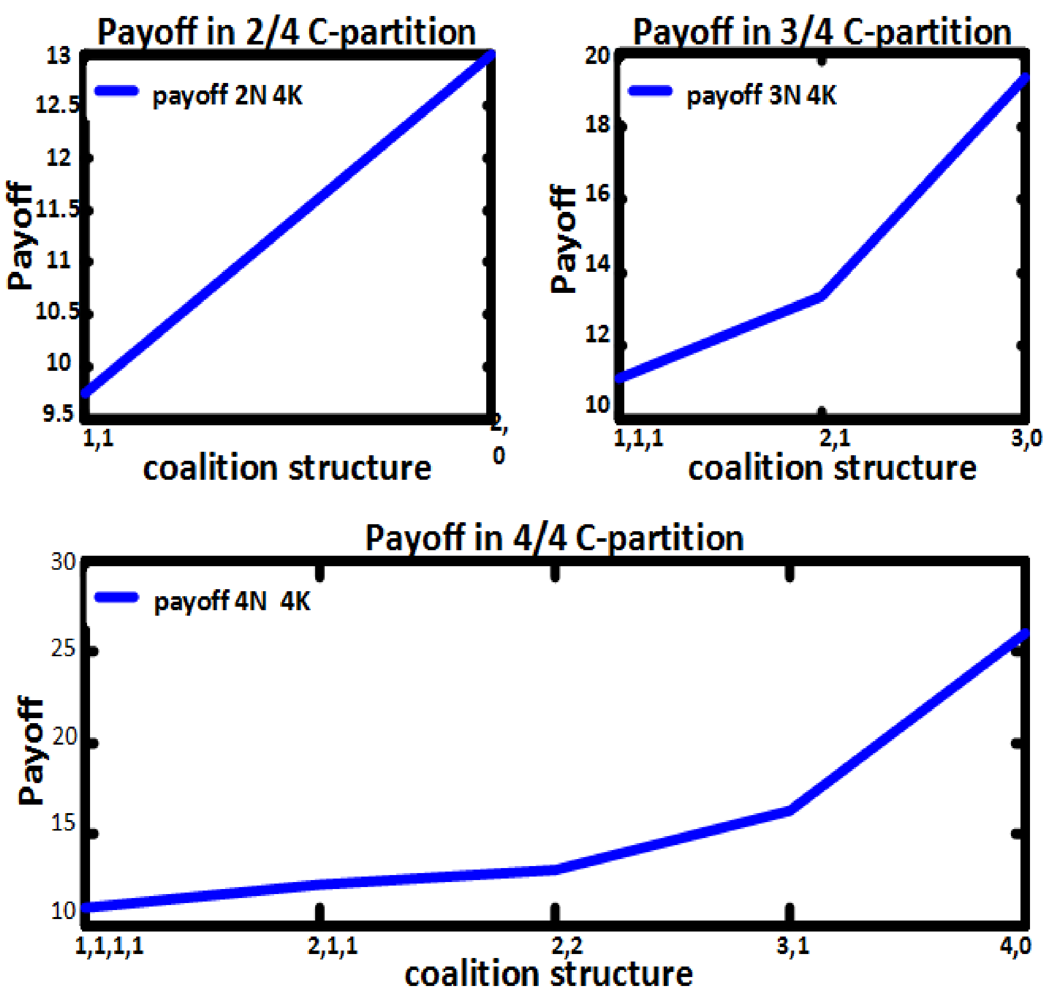

Figure 13: Payoff in Varied N and 4K C-Partition

Figure 13 depicts the payoff trend when number of users $\mathcal{N}$ are varied while number of channels $\mathcal{K}$ are constant. For $\mathcal{N}$=2,3 and 4 and $\mathcal{K}$=4, bigger coalition yield more payoffs. However when value of $\mathcal{N}$=2 and $\mathcal{K}$=4, coalition achieves payoff of $\mathcal{U}x$ =9.75 to 13. Whereas when values of $\mathcal{N}$=3 and 4, the payoff of coalitions ranges between $\mathcal{U}y$= 11 to 19 and 11 to 26. This shows smaller values of $\mathcal{N}$ allows achieving higher payoff in singleton coalitions. Also as the number of users are increased, payoffs trend shifts from a straight line pattern to elliptical curve. It is shown that players will achieve more utility in earlier timeframe, when the number of users are lesser than number of channels i.e. $\mathcal{N} < \mathcal{K}$. As number of users are increased, it will result in increase of payoff and lesser interference but the payoff curve will become elliptical. The reason behind this is that with increase in number of $\mathcal{N}$ there will be increase in number of possible coalition structures that can be formed. However the previous findings also hold true here i.e. lesser payoff with smaller coalition structure like singleton coalitions and increased payoff will be achieved with bigger coalition structures.

In analysis of the payoff trends in varied channel user's combination where $\mathcal{N}$ is varied and $\mathcal{K}$ is kept constant, we found out that it confirms our previous findings that cooperation among contending players is useful to maximize their payoffs as bigger and grand coalitions fetch maximum utility. We now extend our analysis to explore the interference trend in both same and varied channels user's combination i.e. $\mathcal{N} = \mathcal{K}$ and $\mathcal{N} \neq \mathcal{K}$.

Figure 14 depicts the interference phenomena across coalition structure in C-Partition form. This analysis proves that there is a decrease in interference due to cooperative strategy employed by players $\mathcal{N}$ while selection of heterogeneous spectrum channels $\mathcal{K}$ and distributed implementation of C-Partition rules for coalition formation. It can be seen that smaller coalition structure like 1,1,1,1 when $\mathcal{N}$ = 4 and $\mathcal{K}$=4 face larger interference i.e. 15. As the bigger coalition structures like 2,1,1 2,2 and 3,1 are formed it results in lesser interference i.e. 14-11 , which leads to even better situation and interference becomes zero once canonical coalitions like 4,0 is formed. Bigger coalition have lesser interference with canonical coalition having no interference at all. As the coalition size is minimized the interference tends to rise due to contention for selecting best channels and also results in rise in opportunity wastage.

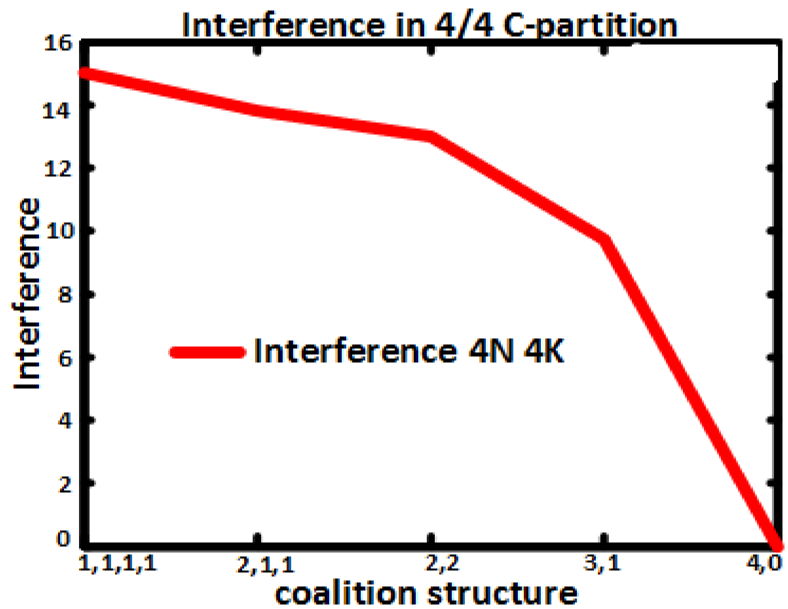

Figure 14: Interference in 4/4 C-Partition

Figure 15 depicts the interference trend across coalition structure, when number of users $\mathcal{N}$ and number of channels $\mathcal{K}$ are same. Bigger coalition have lesser interference with canonical coalition having no interference at all. As the coalition size is minimized the interference tends to rise due to contention for selecting best channels and also results in rise in opportunity wastage. This analysis provides light on another positive aspect of Coalitional game in C-Partition form, which is of decrease in interference due to cooperative strategy employed by players $\mathcal{N}$ while selection of heterogeneous spectrum channels $\mathcal{K}$ and distributed implementation of C-Partition rules for coalition formation. It can be seen that smaller coalition structure like 1,1,1,1,1 when $\mathcal{N}$ = 5 and $\mathcal{K}$=5 face larger interference i.e. 21. As the bigger coalition structures like 2,1,1,1 2,2,1 3,1,1 3,2 and 4,1 are formed it results in lesser interference i.e. 20-10 , which leads to even better situation and interference becomes zero once canonical coalitions like 5,0 is formed.

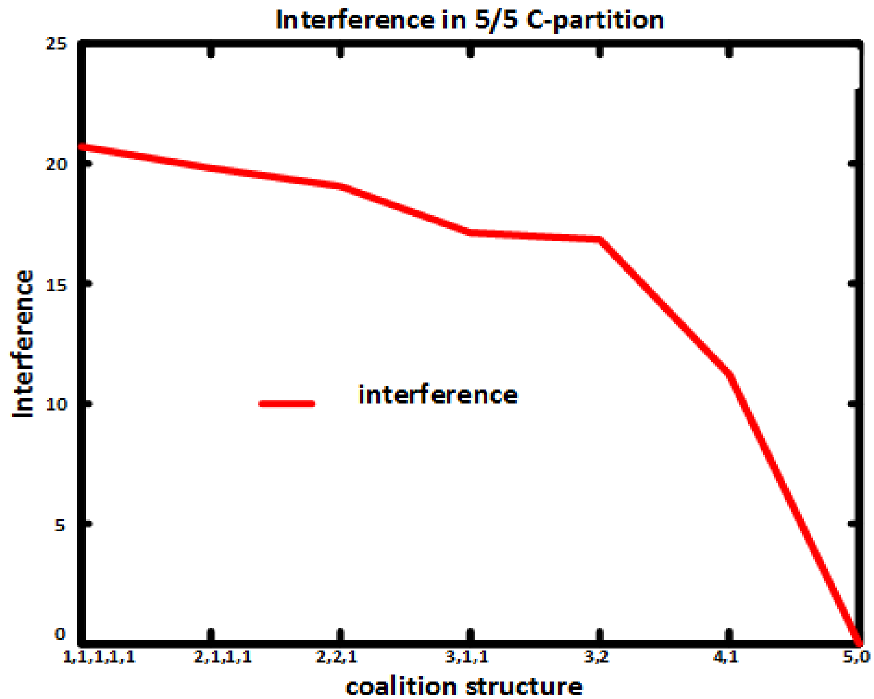

Figure 15: Interference in 5/5 C-Partition

Figure 16 depicts the interference trend across coalition structure, when number of users $\mathcal{N}$ and number of channels $\mathcal{K}$ are same. Bigger coalition have lesser interference with canonical coalition having no interference at all. As the coalition size is minimized the interference tends to rise due to contention for selecting best channels and also results in rise in opportunity wastage. This analysis supports that there is of decrease in interference due to cooperative strategy employed by players $\mathcal{N}$ while selection of heterogeneous spectrum channels $\mathcal{K}$ and distributed implementation of C-Partition rules for coalition formation. It can be seen that smaller coalition structure like 1,1,1,1,1 when $\mathcal{N}$ = 6 and $\mathcal{K}$=6 face larger interference i.e. 27. As the bigger coalition structures like 2,1,1,1,1 2,2,1,1 2,2,2 3,1,1,1 3,2,1 3,3 4,1,1 4,2 and 5,1 are formed it results in lesser interference i.e. 25-11 , which leads to even better situation and interference becomes zero once canonical coalitions like 6,0 is formed.

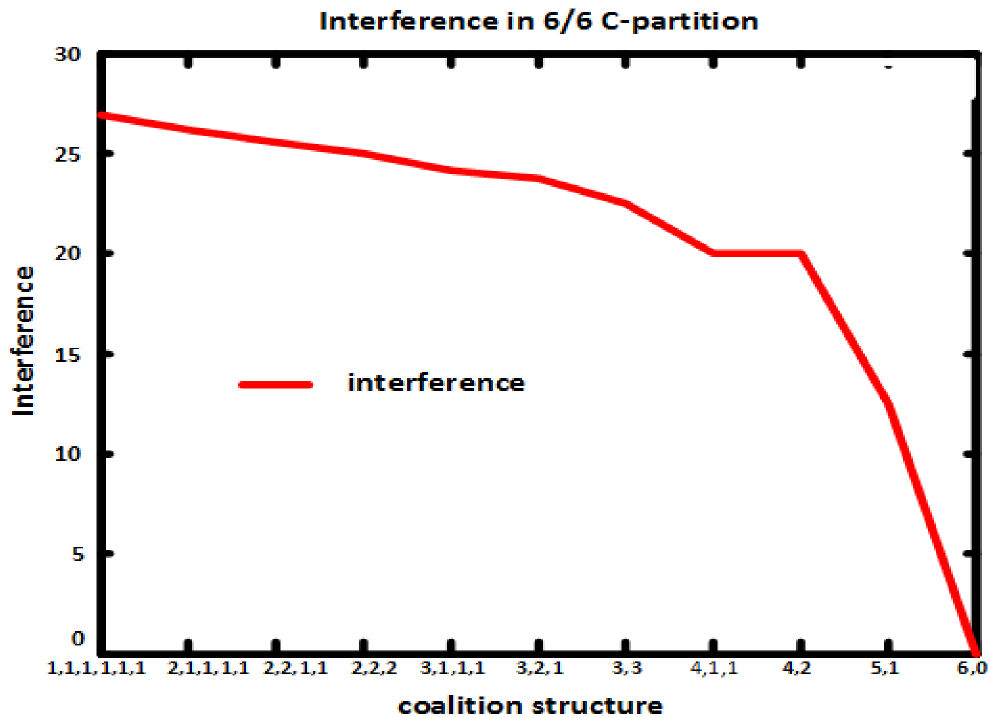

Figure 16: Interference in 6/6 C-Partition

Figure 17 depicts the interference trend across coalition structure, when number of users $\mathcal{N}$ is kept same and number of channels $\mathcal{K}$ are varied. Bigger coalition have lesser interference with canonical coalition having no interference at all. As the coalition size is minimized the interference tends to rise due to contention for selecting best channels and also results in rise in opportunity wastage. It can be seen that smaller coalition structure like 1,1 when $\mathcal{N}$ = 2 and $\mathcal{K}$=1 face larger interference i.e. 10. As the number of K is increased, interference starts decreasing i.e. for $\mathcal{K}$=2,3,4 values of interference is I= 5.5-3 Interference becomes zero once canonical coalitions like 2,0 is formed.

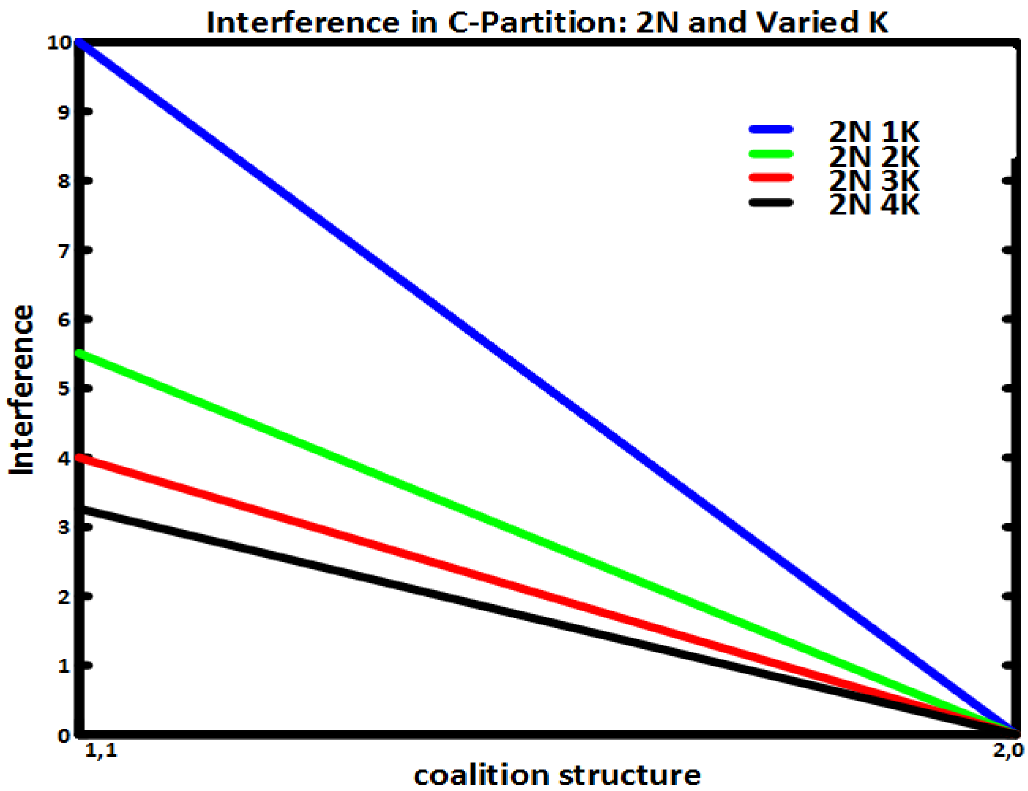

Figure 17: Interference in 2N and Varied K C-Partition

Figure 18 depicts the interference phenomena across coalition structure in C-Partition form when value of $\mathcal{N}$ is kept same and value of K is varied. It can be seen that smaller coalition structure like 1,1,1 when $\mathcal{N}$ = 3 and $\mathcal{K}$=1 face larger interference i.e. 15. As the number of $\mathcal{K}$ is increased, interference starts decreasing i.e. for $\mathcal{K}$=2,3,4 values of interference is $\mathcal{I}$= 12-8. Interference becomes zero once canonical coalitions like 2,0 is formed. In this figure interference for $\mathcal{N}$=3 and $\mathcal{K}$=1 when coalition structure 2,1 is formed, is lesser than values of interference when $\mathcal{N}$=3 and $\mathcal{K}$=2. These both cases are of absolute interference phenomena but since we assumed that channel 1 i.e. k1 holds a utility of 5 i.e. $\mathcal{U}$1 = 5 and k2 offers $\mathcal{U}$2 = 6, so coalition structure of $\mathcal{N}$=3 and k=2 bears larger values of interference. Bigger coalition have lesser interference with canonical coalition having no interference at all. As the coalition size is minimized the interference tends to rise due to contention for selecting best channels and also results in rise in opportunity wastage.

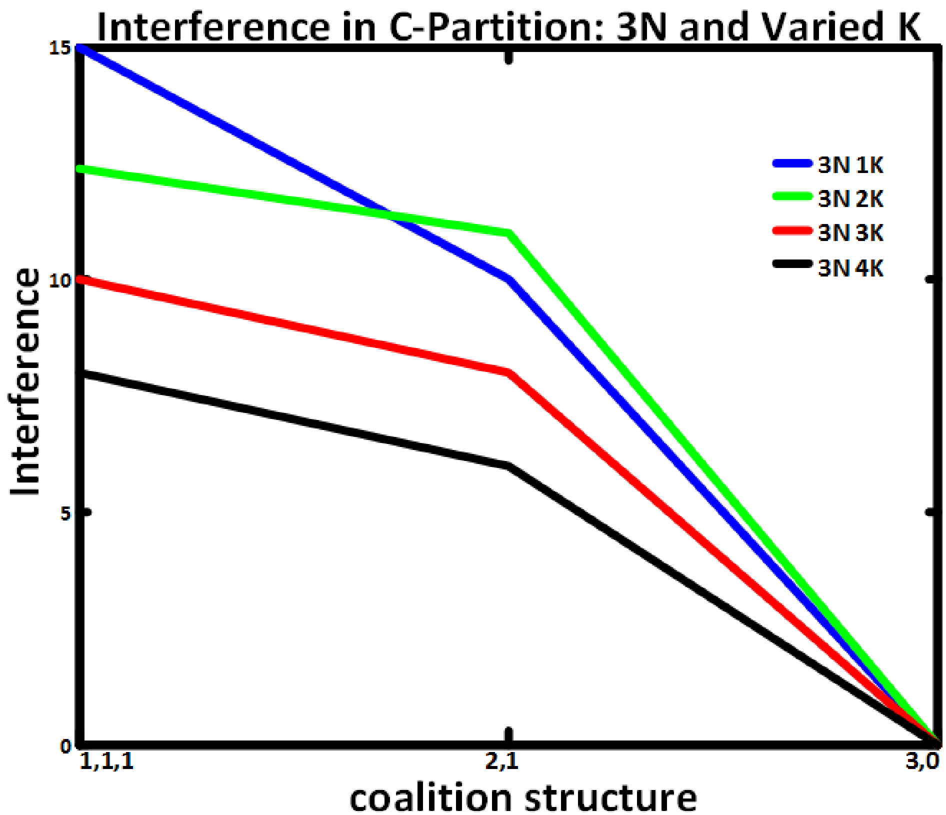

Figure 18: Interference in 3N and Varied K C-Partition

Figure 19 depicts the interference phenomena across coalition structure in C-Partition form when value of $\mathcal{N}$ is kept same and value of $\mathcal{K}$ is varied. It can be seen that smaller coalition structure like 1,1,1,1 when $\mathcal{N}$ = 4 and $\mathcal{K}$=2 face larger interference i.e. 19. As the number of $\mathcal{K}$ is increased, interference starts decreasing i.e. for $\mathcal{K}$=3,4,5 and 6 values of interference is $\mathcal{I}$= 17-14. Interference becomes zero once canonical coalitions like 2,0 is formed. In this figure interference for $\mathcal{N}$=4 and $\mathcal{K}$=2 when coalition structure 2,1,1 is formed, it is facing lesser interference than the bigger coalition structure of 2,2. This is another view of the phenomena already discussed with respect to payoff in figure 10 where absolute interference is discussed. Since there are two coalitions of two users and there are only two channels which results in both channels being accessed by both coalitions. Resultantly absolute interference will occur and no coalition will get any payoff resulting in complete interference which is larger than the interference of a smaller coalition like 2,1,1.

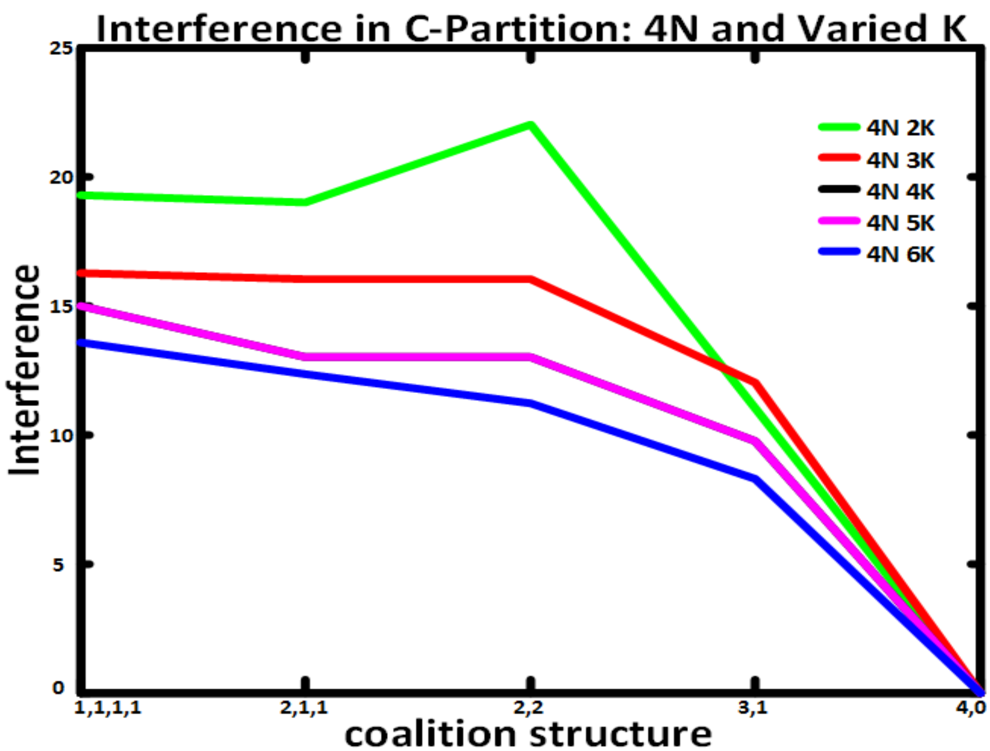

Figure 19: Interference in 4N and Varied K C-Partition

We have analyzed the interference trends in channel user's combination where N = K as well as in N 6= and found out that cooperation among contending players is useful to minimize their interference as smaller coalitions get larger interference than the bigger coalitions. We see that grand coalitions result in absolutely interference free partition structure which is the best trend for players who seek maximization in their utilities.

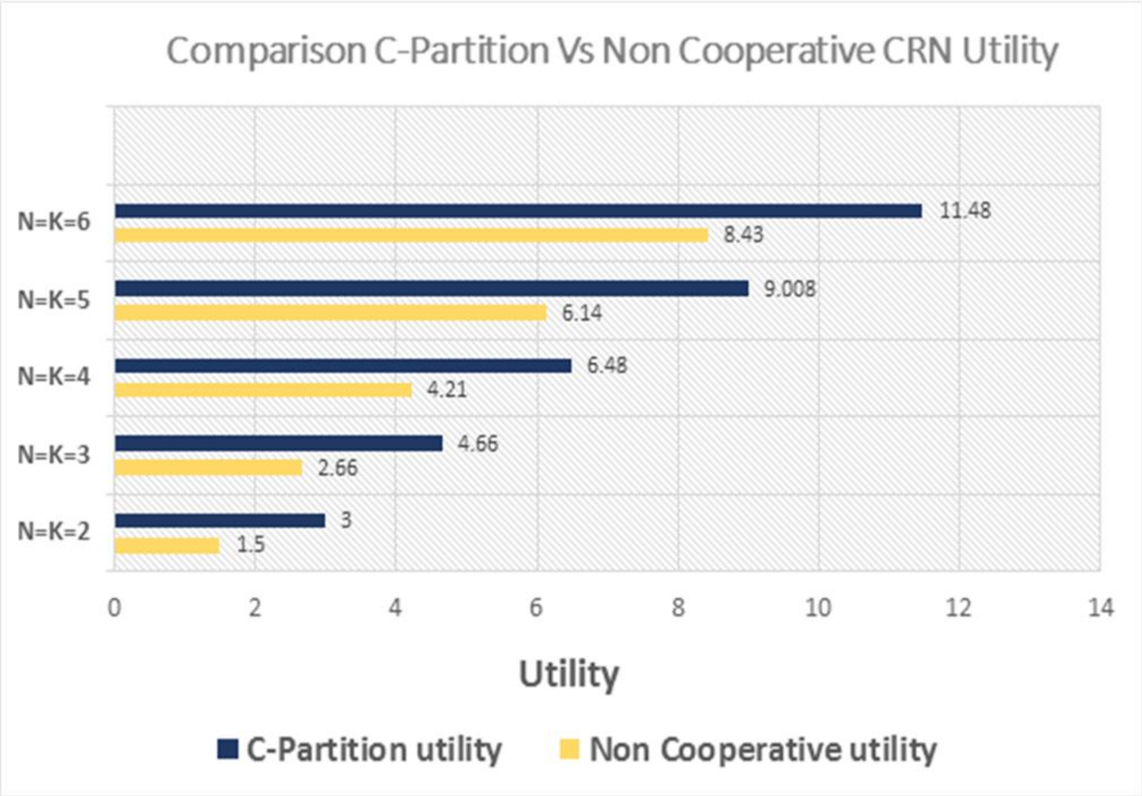

Figure 20: Comparison C-Partition Vs Non Cooperative C-Partition

Figure 20 draws a comparison between payoffs gained by CRNs while using coalition game in C-Partition form for heterogeneous spectrum sharing with the utilities gained by players which employ non cooperative strategy for spectrum sharing. When $\mathcal{N} = \mathcal{K} = 2$ the payoffs of players in C-Partition form (i.e. $\mathcal{U}s = 3$) is twice than those with non-cooperative strategy (i.e. $\mathcal{U}s = 1.5$). Similarly when $\mathcal{N} = \mathcal{K} = 3$ the payoffs in C-Partition form (i.e. $\mathcal{U}s = 4.66$) as against players with non-cooperative strategy (i.e. $\mathcal{U}s = 2.66$) and once $\mathcal{N} = \mathcal{K} = 6$ the C-Partition players gets $\mathcal{U}s = 11.68$ as against players with non-cooperative strategy payoff of $\mathcal{U}s = 8.43$. This comparison highlights the importance of cooperative strategy implementation which players can employ for improvement in available spectrum sharing for secondary access under self-coexistence by CRNs. Secondly it ensures the optimization of spectrum utilization and increased payoffs for the players. As shown in figure 7, payoff gained by the CRNs with C-Partition form is always higher than those which employ non cooperative strategy.

From the analysis it is clear that cooperation among contending players in CRN coexistence can maximize their utilities where spectral opportunities are heterogeneous. Coalitional games in C-Partition form and its distributed implementation of rules allows maximum spectrum utilization for secondary access by reducing the losses and interference. Using the proposed algorithm the average payoff gained by the entire coalition structure is also maximized.

Chapter-7

Nash Equilibrium in C-Partition form

Finally resulting partition structure of our proposed coalition formation game in C-Partition form will be in Nash equilibrium (NE). In game theory, the Nash equilibrium is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of the other players, and no player has anything to gain by changing only his or her own strategy [18]. If a Nash equilibrium is common knowledge, then every player would indeed play the Nash equilibrium strategy, thereby resulting in the Nash equilibrium being played. In other words, a NE strategy profile is self-enforcing as the players are searching for outcomes or solutions from which no player will have an incentive to deviate, then the only strategy profiles that satisfy such a requirement are the Nash equilibrium [19]. Rule for coalition formation shown in equation (2), i.e. for any $S \in \mathcal{Z}$, form S if and only if, $u_x > u_y$, ensures that every CRN will yield better payoff than the last coalitional structure. This strategy binds all the players to stick to this strategy as it ensure better payoff. Thus finally resultant c-partition structure i.e. canonical or grand coalition will be in NE since no player will have any incentive to deviate.

7.1. Definition 1:

The Nash Equilibrium [18], [19] of the spectrum sharing game is an action profile $a^* \in A$ of actions, such that:

$$u\,(a_i{}^*,\ a_{\_i}{}^*) \geq u\,(a_i\,,\ a_{\_i}{}^*)\ ;\forall i \in \mathcal{N} \qquad (6)$$

Whereas ≥ is a relation showing preference over payoffs of strategies $a_i{}^*$ and $a_{\_i}{}^*$.This entails that for $a_i{}^*$ to be a NE, it must fulfil the conditions, that no player i has another strategy that yields a higher payoff than the one for playing $a_i{}^*$given that every other player plays their equilibrium strategy $a_{\_i}{}^*$.

7.2. NE for C-Partition game when N=3 and K=3

7.2.1. Lemma 1:

Strategy pairs **f** i.e. (X-a_i, Y-a_j , W-a_k), **h** i.e. (X-a_i, Y-a_k, W-a_j) , **l** i.e. (X-a_j, Y-a_i , W-a_k), **p** i.e. (X-a_j, Y-a_k , W-a_i) , **t** i.e. (X-a_k, Y-a_i , W-a_j) and **v** i.e. (X-a_k, Y-a_j , W-a_i) are NE strategies of the c-partition game of table III.

7.2.2. Proof:

Assume that first player is the column 2 player i.e. X, the second player to be the column 3 player i.e. Y and third player to be the column 4 player i.e. W in table III. Action strategies are represented by rows i.e. action strategy **a** is (X-a_i, Y-a_i, W-a_i) and strategy **b** is (X-a_i, Y-a_i , W-a_j). Also u_i and u_j and u_k are positive values and therefore the payoffs for strategy pairs **f**, **h**, **l**, **p**, **t** and **v** are greater than the payoffs for all other strategy pairs.

Consider the payoff for strategy pair **f** i.e. (X-a_i, Y-a_j , W-a_k) from table 1 row **f**, the first player playing strategy X-a_i continues to play this strategy, then from definition 1 for NE, it follows that the second player playing strategy Y-a_j does not have any incentive to change its choice to Y-a_i or Y-a_k, and the third player playing strategy W-a_k does not have any incentive to change its choice to W-a_i or W-a_j. Therefore, (X-ai, Y-aj , W-ak) is a NE.

Similarly it can be proved that the strategy pair **h** i.e. (X-a_i, Y-a_k, W-a_j) from table III row **h**, is the second NE of this game. Other four NE strategies pairs are **l** i.e. (X-a_j, Y-a_i , W-a_k), **p** i.e. (X-a_j, Y-a_k , W-a_i), **t** i.e. (X-a_k, Y-a_i , W-a_j) and **v** i.e. (X-a_k, Y-a_j , W-a_i) from table III row **p**, **t** and **v**.

NE strategies for C-Partition game when N=3 and K=3				
#	**X**	**Y**	**W**	**Utility** u_S
a.	X-a_i	Y-a_i	W-a_i	(0,0,0)
b.	X-a_i	Y-a_i	W-a_j	(0,0,u_j)
c.	X-a_i	Y-a_i	W-a_k	(0,0,u_k)
d.	X-a_i	Y-a_j	W-a_i	(0,u_j,0)
e.	X-a_i	Y-a_j	W-a_j	(u_i,0,0,)
f.	X-a_i	Y-a_j	W-a_k	(u_i, u_j, u_k) - **NE**
g.	X-a_i	Y-a_k	W-a_i	(0,u_k,0)
h.	X-a_i	Y-a_k	W-a_j	(u_i, u_k, u_j) - **NE**
i.	X-a_i	Y-a_k	W-a_k	(u_i,0,0,)
j.	X-a_j	Y-a_i	W-a_i	(u_j,0,0,)
k.	X-a_j	Y-a_i	W-a_j	(0,u_i,0)
l.	X-a_j	Y-a_i	W-a_k	(u_j, u_i, u_k) - **NE**
m.	X-a_j	Y-a_j	W-a_i	(0,0,u_i)
n.	X-a_j	Y-a_j	W-a_j	(0,0,0)
o.	X-a_j	Y-a_j	W-a_k	(0,0,u_k)
p.	X-a_j	Y-a_k	W-a_i	(u_k, u_j, u_i) - **NE**
q.	X-a_j	Y-a_k	W-a_j	(0,u_k,0)
r.	X-a_j	Y-a_k	W-a_k	(u_j,0,0,)
s.	X-a_k	Y-a_i	W-a_i	(u_k,0,0,)
t.	X-a_k	Y-a_i	W-a_j	(u_k, u_i, u_j) - **NE**
u.	X-a_k	Y-a_i	W-a_k	(0,u_i,0)
v.	X-a_k	Y-a_j	W-a_i	(u_k, u_j, u_i) - **NE**
w.	X-a_k	Y-a_j	W-a_j	(u_k,0,0,)
x.	X-a_k	Y-a_j	W-a_k	(0,u_j,0)
y.	X-a_k	Y-a_k	W-a_i	(0,0,u_i)
z.	X-a_k	Y-a_k	W-a_j	(0,0,u_j)
aa.	X-a_k	Y-a_k	W-a_k	(0,0,0)

Table 3: NE strategies for C-Partition game

Since now it is proved that these strategies of three players i.e. $\mathcal{N}$=3 c-partition game are in NE, we can see that these strategies can only be played once canonical or grand coalition c-partition structure is formed. Since all the players are rational and contends for best available channel where spectrum offers heterogeneous channels i.e. all having different payoff, so these strategies can only be played if players form a grand coalition being agreed to cooperate with each.

Final resulting partition of C-Partition form is Canonical / Grand coalition partition structure, the one which offers the best payoff for every player as well as for entire coalitional structure as shown in figures 4 to 13. It also guarantees the optimum utilization of available spectrum opportunity with zero interference in the final resulting partitions as shown in figures 14 to 19. Due to this players in C-Partition form will have no incentive to deviate and thus final resulting coalitional structure of partition will be in NE.

Chapter-8

Fairness Analysis of Derived NE in C-Partition form

In the previous section we proved that final resulting partition structure of coalition formation game in c-partition form is canonical coalition and is in NE. Now in this section we need to assess the fairness of the derived NE. Jains fairness index [20] is one of the most popular method to calculate the fairness mechanism. In our model the utility / payoff gained by each player i.e. CRN is represented by, u_i. Thus fairness of derived NE can be represented as:

$$J(ui, uj, \ldots., un) = \frac{\left(\sum_{i=1}^{n} ui\right)^2}{N * \sum_{i=1}^{n} ui^2} \qquad (7)$$

If Jains index would be equal to 1 i.e., the maximum, distribution of payoff in derived NE will be considered fair while for an unequal distribution of payoffs it would be smaller than 1. For the sake of clarity and ease of understanding, we consider the case of $\mathcal{N}$=3 and k=3 i.e. 3 users and 3 channel heterogeneous spectrum sharing game in c-partition form. The same arguments can be applied for analyzing $\mathcal{N}$-user and $\mathcal{K}$-channel scenario. We consider the following two Nash equilibrium (out of total 4 NE as explained in section VII) of the derived solutions which are (X-a_i, Y-a_j , W-a_k) and (X-a_i, Y-a_k, W-a_j).

As explained earlier that spectrum channels are heterogeneous which means each channel bears different payoff, so we assume channel i is of higher quality than channel j therefore $u_i \succ u_j$ and so on . Then from the payoff matrix of table III, gaining access to channel i brings a larger payoff to a CRN whereas being of comparatively lower quality, channel j and k brings a smaller payoff. Both considered NE strategies seems to be unfair because one player always gets a smaller payoff than the others in that particular channel user combination. However our finally resulting partition i.e. canonical or grand coalition structure will traverse across all possible channels users' combination as shown table III, which ultimately results in equal access to all channels for each user. This can be confirmed with equation 7 as the value of Jain fairness index is equal to 1 in both considered NE strategies. Thus it is proved that the resource distribution for final partition structure in coalition formation game in c-partition form is fair.

Chapter-9

Conclusion

Spectrum available for secondary access by CRNs will be heterogeneous by virtue of various quality parameters. All CRNs can employ cooperative strategy to maximize their individual and coalition utility. In this paper, we have proposed a novel form of coalition formation game i.e. C-Partition form, a hybrid of canonical and partition form, by which CRNs coexistence for heterogeneous spectrum access can be modelled accurately. Using proposed algorithm and its distributed implementation of rules, cooperation amongst different partitions can be administered effectively without the need of centralized controlling entity and the payoffs of all the individual players can be maximized as well as the system spectrum utilization is also guaranteed to be at maximum. Finally the resulting C-Partition will be in Nash equilibrium (NE) and no player will have any incentive to deviate from strategy of c-partition.

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