Channel Estimation of MIMO OFDM Systems

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DEDICATION

All our efforts are dedicated to our beloved parents and teachers who have been a constant source of encouragement for us throughout our lives. May Allah bless them with long lives and always provide us their loving and thorough guidance.

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All praises to the Almighty Allah, who enlightened us with the requisite knowledge to accomplish the project goals that we set for ourselves prior to the start of the project, and on a broader level in the completion of our degree.

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ABSTRACT

Wireless Communication Technology has developed many folds over the past few years. One of the techniques to enhance the data rates is called Multiple Input Multiple Output (MIMO) in which multiple antennas are employed both at the transmitter and the receiver. Multiple signals are transmitted from different antennas at the transmitter using the same frequency and separated in space. Various channel estimation techniques are employed in order to judge the physical effects of the medium present. In this project, we analyze and implement various estimation techniques for MIMO OFDM Systems such as Least Squares (LS), Minimum Mean Square Error (MMSE), Constant Modulus Algorithm (CMA) and linear Pre-coding. These techniques are therefore compared to effectively estimate the channel in MIMO OFDM Systems.

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CHAPTER 1: INTRODUCTION

It is a well-known fact that the amount of information transported over communication systems grows rapidly. Not only the file sizes increase, but also bandwidth-hungry applications such as video on demand and video conferencing require increasing data rates to transfer the information in a reasonable amount of time or to establish real-time connections. To support this kind of services, broadband communication systems are required.

Another well-known fact is that the mobility of communication systems is more and more demanded. One example is the enormous increase of mobile phone users worldwide. Another example is the trend of the work place becoming increasingly mobile. A term often used in this context is office hotelling. Hotelling is a form of alternative officing in which employees who work out of the office for significant periods of time can call ahead (just as they do in making hotel reservations) and reserve workspace. They select an office from a specially-designated block of workspaces when they come into the company's office facilities. This allows organizations to reduce space costs by relying on technology to create the appearance of a permanent office. Ideally, the employee should be completely mobile within the office, and still be able to connect his personal computer to the computer network. Thus, a wireless computer network is desirable.

Another place where wireless computer networking is in demand is at the university campus. A growing number of universities offer their students the possibility to buy a laptop from university. The way of teaching is adapted to the availability of laptops. To make this kind of studying efficient, students need to be able to connect to the computer network of the university in order to download the study material that the professor offers. However, it is undoable to adapt the computer network infrastructure in such a way that the students can plug-in in every (lecture-) room at the university. Here too, it seems that wireless network access is essential. As an example where a wireless network already is installed, the Carnegie Mellon University in Pittsburgh can be mentioned.

The above considerations justify research into new broadband wireless communication systems. Large-scale penetration of such systems into our daily lives will require significant reductions in cost and increases in bit rate and/or system capacity. Recent information theoretical studies have revealed that the multipath wireless channel is capable of huge capacities, provided that multipath scattering is sufficiently rich and is properly exploited through the use of the spatial dimension Appropriate solutions for exploiting the multipath properly, could be based on new techniques that recently appeared in literature, which are based on Multiple Input Multiple Output (MIMO) technology. Basically, these techniques transmit different data streams on different transmit antennas simultaneously. By designing an appropriate processing architecture to handle these parallel streams of data, the data rate and/or the Signal-to-Noise Ratio (SNR) performance can be increased.

Multiple Input Multiple Output (MIMO) systems are often combined with a spectrally efficient transmission technique called Orthogonal Frequency Division Multiplexing (OFDM) to avoid Inter Symbol Interference (ISI).

1.1 MIMO SYSTEMS

Various schemes that employ multiple antennas at the transmitter and receiver are being considered to improve the range and performance of communication systems. By far the most promising multiple antenna technology today happens to be the so called multiple-input multiple-output (MIMO) system. MIMO systems employ multiple antennas at both the transmitter and receiver.

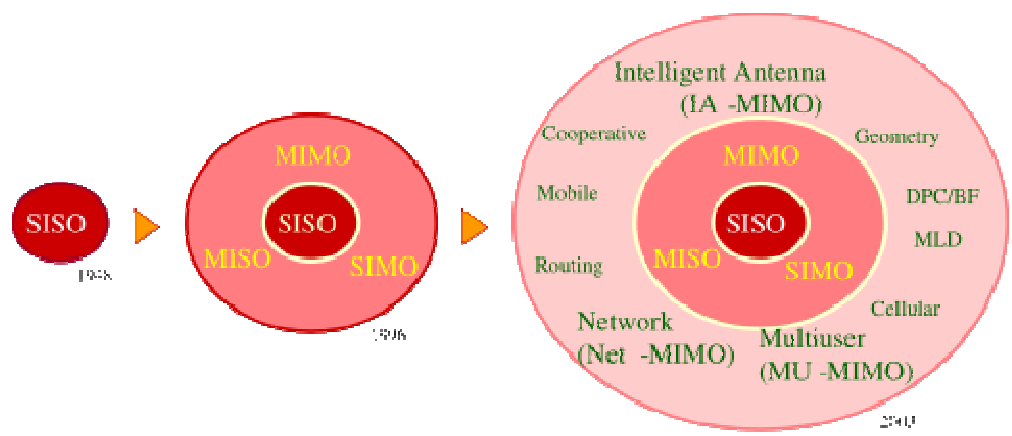

Figure 2.1 MIMO Evolution

1.2 SPACE DIVISION MULTIPLEXING

A promising solution for significant increase of the bandwidth efficiency is the exploitation of the spatial dimension. Recent information theory research has revealed that the multipath wireless channel is capable of enormous capacities, provided that the multipath scattering is sufficiently rich

.The multipath scattering can be properly exploited through the use of an appropriate processing architecture. The diagonally-layered space-time architecture proposed, known as diagonal BLAST (Bell Laboratories Layered Space Time) or D-BLAST, is such an approach. However, the diagonal approach is very complex and hard to implement. Therefore, a simplified version of BLAST, known as vertical BLAST or V-BLAST is proposed .A prototype has been built by which it is demonstrated that bandwidth efficiencies of 20 – 40 bps/Hz can be achieved in an indoor propagation environment at realistic SNRs and error rates.

These techniques can be captured under the more general term Space Division Multiplexing (SDM) or Space Division Multiple Access (SDMA). SDM techniques exploit the spatial dimension using multiple antennas at receiver as well as at the transmitter. Basically, these techniques transmit different signals on different transmit antennas simultaneously, with the goal of increasing the capacity and the SNR performance. At the receiver these different signals can be recovered by the Space Division Multiplexing techniques described in this report. For proper recovering of the transmitted signals, multiple antennas are required at the receiver as well. The difference between SDM and SDMA is that the latter allows different users to transmit simultaneously on a single antenna each), whereas in SDM a single user transmits simultaneously on multiple antennas. Hybrid schemes where several users transmit simultaneously on different antennas may also be possible.

One can naturally ask in which way SDM (A) techniques differ from traditional multiple access techniques. Some of these differences are worth pointing out: First, unlike code-division or other spread-spectrum multiple access techniques, the total channel bandwidth utilized by a SDM(A) system is only a small fraction in excess of the symbol rate, i.e. similar to the excess bandwidth required by a conventional transmission technique like Quadrature Amplitude Modulation (QAM). Second, unlike Frequency Division Multiple Access (FDMA), each transmitted signal occupies the entire system bandwidth. Finally, unlike Time Division Multiple Access (TDMA), the entire system bandwidth is used simultaneously by all of the transmitters all of the time. These differences together are precisely what give SDM (A) the potential to realize higher bandwidth efficiencies than the other multiple-access techniques.

1.3 ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM)

OFDM is a subset of frequency division multiplexing in which a single channel utilizes multiple sub-carriers on adjacent frequencies. In addition the sub-carriers in an OFDM system are overlapping to maximize spectral efficiency. Ordinarily, overlapping adjacent channels can interfere with one another. However, sub-carriers in an OFDM system are precisely orthogonal to one another. Thus, they are able to overlap without interfering. As a result, OFDM systems are able to maximize spectral efficiency without causing adjacent channel interference

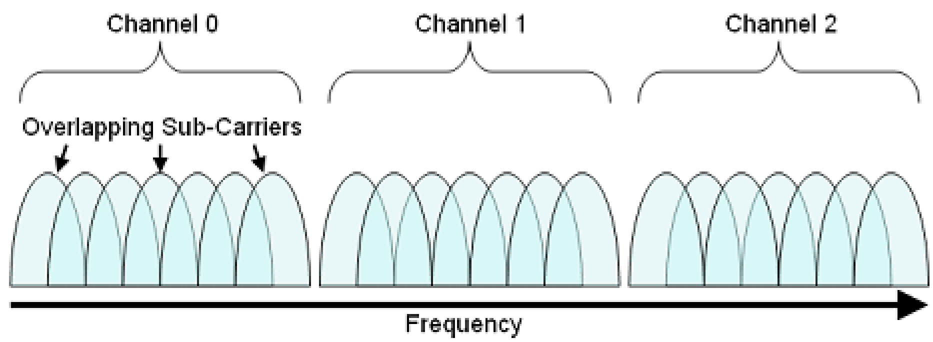

FIGURE 1.2 ORTHOGONAL SUBCARRIERS

Orthogonal frequency division multiplexing is commonly implemented in many emerging communications protocols because it provides several advantages over the traditional FDM approach to communications channels. More specifically, OFDM systems allow for greater spectral efficiency reduced inter-symbol interference (ISI), and resilience to multi-path distortion.

In mono-carrier systems, inter-symbol interference is often caused through the multi-path characteristics of a wireless communications channel. Note that when transmitting an electromagnetic wave over a long distance, the signal passes through a variety of physical mediums. As a result, the actual received signal contains the direct path signal overlaid with signal reflections of smaller amplitudes.
In wireless systems, this creates difficulty because the received signal can be slightly distorted. In this scenario, the direct path signal arrives as expected, but slightly

attenuated reflections arrive later in time. These reflections create a challenge because they interfere with subsequent symbols transmitted along the direct path. These signal reflections are typically mitigated through a pulse-shaping filter, which attenuates both the starting and ending sections of the symbol period. However, as the figure above illustrates, this problem becomes much more significant at high symbol rates. Because the reflections make up a significant percentage of the symbol period, ISI will also be substantial.

1.4 TARGET APPLICATIONS

One of the target areas we think will benefit from the research presented in this thesis is the area of the wireless local area networks (WLANs). Since the beginning of the nineties, WLANs for the900-MHz, 2.4-GHz and 5-GHz ISM (Industrial, Scientific and Medical) bands have been available, based on a range of proprietary techniques .In June 1997 the Institute of Electrical and Electronics Engineers (IEEE) approved an international interoperability standard, called IEEE 802.11 .This standard specifies a number of Medium Access Control (MAC) procedures and three different Physical Layers (PHY). Two of these PHYs are radio-based and use the 2.4-GHz band and the other PHY uses infrared light. All PHYs support a data rate of 1 Mbps and optionally 2 Mbps.

User demand for higher bit rates and the international availability of the 2.4-GHz band has spurred the development of a higher speed extension to the 802.11 standard. In July

1998, a proposal was selected for standardization, which describes a PHY providing a basic rate of 11 Mbps and a fall back rate of 5.5 Mbps. This PHY can be seen as a fourth option, to be used in conjunction with the MAC that is already standardized. Practical products, however, are expected to support both the high-speed 11 and 5.5 Mbit/s rate modes as well as the 1 and 2 Mbps modes. A product based on this IEEE 802.11 standard is WaveLAN™ of Lucent Technologies The modem that is implemented as PC card, is shown in Figure 1-5.

Since January 1997, in the United States 300 MHZ in the 5-GHz band came available for the use of a new category of unlicensed equipment called Unlicensed National Information Infrastructure (UNII) devices .In July 1998, the IEEE 802.11 standardization group selected Orthogonal Frequency Division Multiplexing (OFDM) as transmission technique for the recently available spectrum in the 5-GHz band. This adds a fifth PHY option to the 802.11 standard, with data rates ranging from 6 up to 54 Mbps.

Besides IEEE 802.11, two other standardization groups were formed. In Europe, the European Telecommunication Standards Institute formed the standardization group Broadband Radio Access Networks (ETSI BRAN) and in Japan equipment manufacturers, service providers and the Ministry of Post and Telecommunications are co-operating in the Multimedia Mobile Access Communication (MMAC) project .ETSI BRAN is now working on extensions of the ETSI HYPERLAN type 1 standard. Three extensions are under development: HIPERLAN/2, a wireless indoor LAN with a Quality of Service provision; Hyperlink, a wireless indoor backbone; and

HiperAccess, an outdoor, fixed wireless network providing access to a wired infrastructure. The MMAC project is started to define new wireless standards similar to those of IEEE 802.11 and ETSI BRAN. Additionally, MMAC is also looking into the possibility for ultra-high-speed wireless indoor LANs supporting large-volume data transmission at speeds up to 156 Mbps using frequencies in the 30- to 300-GHz band. Following the IEEE 802.11 selection of OFDM as basis for transmission in the 5-GHz band, ETSI BRAN and MMAC also adopted OFDM for their physical layer standards. The three standardization groups have worked in close co-operation since then to make sure that differences among the various standards are kept to a minimum, thereby enabling the manufacturing of equipment that can be used world-wide.So, since there is a lot of effort going on to adopt OFDM as a world-wide standard for Wireless LANs, the research to next generation WLANs with even higher data rates, but based on these OFDM standards, is a logical consequence. In this thesis, a system is proposed that combines Space Division Multiplexing with OFDM as a spectrum efficient transmission technique, to achieve higher data rates and to obtain sufficient delay-spread robustness.

FIGURE 1.3 A WLAN CARD

CHAPTER 2: ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING (OFDM)

One way to increase system capacity is by transmitting data parallel on different frequencies. In classical parallel data systems, the total signal frequency band is divided into N non-overlapping frequency sub channels. In such systems, there is sufficient guard space between adjacent sub channels to isolate them at the receiver with N demodulators, i.e., one per sub channel. This solution, however, is very bandwidth inefficient. A more efficient use of bandwidth can be obtained with a parallel system if the spectra of the individual sub channels are permitted to overlap, with specific orthogonally constraints imposed to facilitate separation of the sub channels at the receiver.

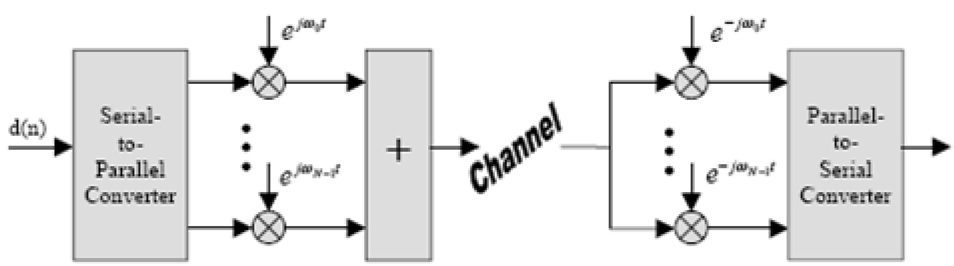

Figure 2.1 basic structure of a multicarrier system.

In the nearby future, more and more applications that operate on carrier frequencies in the order of several Giga-Hertz, like wireless LANs, will be based on multicarrier systems. Figure 2-1 shows the general structure of a multicarrier system. An example of a multicarrier technique that operates with specific orthogonally constraints is Orthogonal Frequency Division Multiplexing (OFDM).

2.1 THE PRINCIPLE

In OFDM the sub carrier pulse used for transmission is chosen to be rectangular. This has the advantage that the task of pulse forming and modulation can be performed by a simple Discrete Fourier Transform (DFT) which results in a remarkable reduction in equipment complexity (filters, modulators, etc.). In these references it is also shown that a multitone data signal is effectively the Fourier transform of the original data stream, and that a bank of coherent demodulators at the receiver is effectively an inverse Fourier transform. However, since definitions have been changes afterwards, it is now said, that a multitone data signal is effectively the inverse Fourier transform of the original data stream, and that a bank of demodulators is effectively a Fourier transform. Note that the (Inverse) Discrete Fourier Transform ((I)DFT) can be implemented very efficiently as a(n) (Inverse) Fast Fourier Transform ((I)FFT).

Consider the system shown in Figure 2-1. The input consists of a data symbol sequence dn that can be represented as $a_n + jb_n$ (where an and bn are real sequences representing the in-phase and quadrature components, respectively) and, using the Inverse Fourier Transform, the transmitted waveform can be represented as:

$$D(t) = \sum_{n=0}^{N-1} d_n e^{j\omega_n t} \qquad (2.1)$$

However, for digital systems it is useful to consider the discrete case. In this case an Inverse Discrete Fourier Transform (IDFT) is performed at the transmitter on a vector *d*

= ($d0, d1, \ldots, dN{-}1$), the result is a vector D = ($D0, D1, \ldots, DN{-}1$) with:

$$D_m = \sum_{n=0}^{N-1} d_n e^{j(2\pi nm/N)} = \sum_{n=0}^{N-1} d_n e^{j(2\pi f_n t_m)} \text{ with } m = 0,1,\ldots,N-1 \tag{2.2}$$

$$f_n = n\Delta f \;,\; \Delta f = \frac{1}{N\Delta t} \text{ and } t_m = m\Delta t \tag{2.3}$$

Where

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And s Δt =1/ fs (where fs is the symbol rate). Substituting t =mΔt in (2.1) and using (2.3) shows that the sampled sequence D (mΔt) is in fact the IDFT of the sequence d_n as shown in (2.2). Note that usually the whole OFDM symbol is up converted to a specific carrier frequency of the RF band. If we again look to the OFDM symbol time it can be noticed that the signaling interval T has been increased to NΔt, which makes the system less susceptible to delay spread impairments. Note that the act of truncating the rectangular signal to the interval (0, NΔt) in the time domain imposes a sin(x) / x frequency response on each sub channel with zeros at multiples of 1/T, which is the orthogonally principle of OFDM.

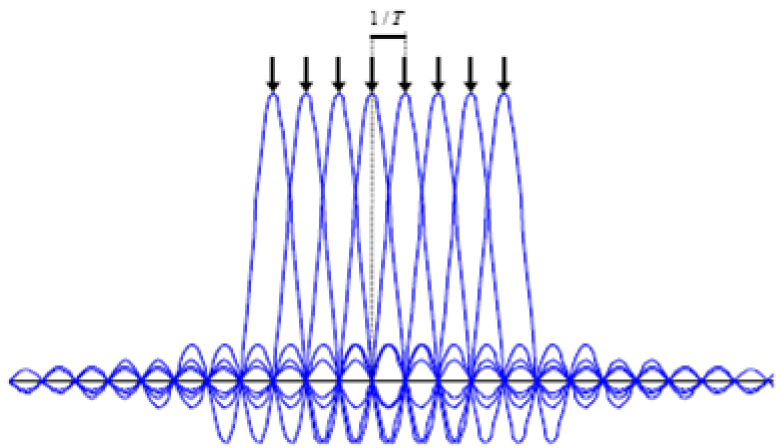


Figure 2.2 OFDM and the orthogonality principle.

At the receiver, the transmitted signals are recovered using the Discrete Fourier Transform. However, due to multipath distortion, the recovering of the transmitted signals brings some problems along.

2.2 MULTIPATH DISTORTION

The reason why the information transmitted over the carriers can still be separated at the receiver is the so-called orthogonally relation giving OFDM its name. By using an IFFT for modulation, the spacing of the sub carriers is implicitly chosen in such a way, that at the frequencies where the received signals are evaluated (indicated as arrows in Figure 2-2), all other signals are zero. In order for this orthogonally to be preserved the following must be true:

1. The receiver and the transmitter must be perfectly synchronized. This means they both must assume exactly the same modulation frequency and the same time-scale for transmission (which usually is not the case).

2. There should be no multipath channel.

3. The analogue components, part of transmitter and receiver, must be of very high quality.

Unfortunately multipath distortion is (almost) unavoidable in radio communication systems and, thus, the received signal is affected. It is shown that the truncated sub channel sinusoids are delayed by differing amounts (i.e. channel delays), however, the distortion is mainly concentrated at the on-off transmissions of these waveforms. Hence a guard time and cyclic prefix (consisting of a modest increase in the signal duration) that are chosen longer than the maximal delay spread, will eliminate most interference among channels (Inter Carrier Interference (ICI)) and between adjacent transmission blocks (Inter Symbol Interference (ISI)). The smoothing of the on-off transitions, to avoid out of band radiation, can be implemented, e.g., by windowing each OFDM symbol by a raised cosine window.

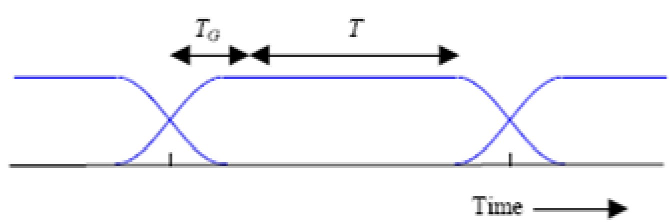


Figure 2.3 OFDM symbol structure showing guard time T_G and FFT interval T.

The validity of windowing each OFDM symbol by a raised cosine window can be explained by looking to the shapes of the sub channel sinusoids after the multipath-fading channel. An OFDM receiver uses only a part of this signal to calculate the FFT.

In the FFT interval T, every sub carrier has an integer number of cycles, which ensures orthogonally. In the multipath-fading channel, the receiver-input signal will be a sum of delayed and scaled replicas of the transmitted sub carriers. Note that a sum of scaled and delayed sinusoids is again a sinusoid. So, as long as the Guard Interval (GI) time TG is larger than the maximal channel delay there will still be an integer number of cycles within the FFT interval for each multipath component and all reflections of previous symbols are removed and the orthogonally is preserved. So, thanks to the cyclic prefix and guard time, the wideband multipath fading is experienced in OFDM as a set of narrowband fading sub carriers without ICI and ISI. The only remaining effect of multipath is a random phase and amplitude of each sub carrier. This effect can be minimized by adapting the sub carriers of the received signal with channel estimates per sub carrier, which can be obtained during a training phase. The only problem are the sub carriers in deep fades, in order to deal with these weak sub carriers that have a large probability to be decoded wrong, forward error correction across the sub carriers is applied.

2.3 BANDWIDTH EFFICIENCY

The spectrums of the sub carriers are not separated but overlap. Due to this overlapping, the bandwidth is much more efficiently used than in the classical multicarrier systems. Theoretically, M-ary digital modulation schemes using OFDM can achieve a bandwidth efficiency, defined as bit rate per unit bandwidth, of $\log_2 M$ bits/sHz .Given the symbol rate of the serial data stream is 1 /□, the bit rate for a corresponding *M*-ary system

$$R_b = \log_2 M / \Delta t$$

however, in case a guard time is added, the effective bit rate is equal to:

$$R_b = \frac{T}{T_G + T}\log_2 M / \Delta t = \frac{N\Delta t}{T_G + N\Delta t}\log_2 M / \Delta t \tag{2.4}$$

The Nyquist bandwidth of a sub channel is □*f*. Thus, the total bandwidth of the OFDM system is:

$$B = N\Delta f = \frac{1}{\Delta t} \tag{2.5}$$

Therefore, the bandwidth efficiency becomes:

$$\beta = \frac{R_b}{B} = \frac{T}{T_G + T}\log_2 M \tag{2.6}$$

For an optimal system without guard time (i.e., TG = 0), the bandwidth efficiency becomes log 2 M bits/s/Hz. However, the guard time is inserted in order to make the system more robust against multipath distortion, so a practical system will not achieve this optimal bandwidth efficiency. Furthermore, in reality, it is impossible to make an ideal band- pass filter, so in order to pass all sub carriers, the bandwidth B will be larger, which implies another decrease in bandwidth efficiency.

2.4 MAIN REASONS FOR USING OFDM

As mentioned before, one of the main reasons to use OFDM is its capability to deal with large delay spreads. The implementation complexity of an OFDM system is compared with a signal carrier system that is designed to deal with the same amount of delay spread. Here, a summary will be given.

The implementation complexity of a single carrier system is dominated by equalization, which is necessary when the delay spread is larger than about 10% of the symbol duration. In an OFDM system, however, equalization is not required, but instead, the complexity of an OFDM system is largely determined by the Fast Fourier Transform (FFT), which is used to demodulate the various sub carriers. It can be learned that at least 8 feed forward and 8 feedback taps are required to handle a delay spread of 100 ns for a Gaussian Minimum Shift Keying (GMSK) modem at a data rate of 24 Mbps. To get the same delay spread performance as the 24 Mbps mode of the new IEEE 802.11 OFDM standard, which equals 250 ns using a 64-point FFT, the number of equalizer taps has to be increased to 8*250/100 = 20. Fortunately, for GMSK, only the real outputs of the complex multiplications are used, so each multiplier has to perform 2 real multiplications per sample. Hence, the number of real multiplications per second becomes 2x20x24x106 = 960x106, where only the feed forward taps have been counted since the feedback taps have a much smaller implementation complexity. For the OFDM system, a 64-point FFT has to be processed every OFDM symbol duration which is 4s. With a radix-4 FFT algorithm, this requires 96 complex multiplications. One complex multiplication can be performed with four real multiplications and, thus, the processing load is equal to 4x̃96 / (4 xs) = 96x106 real multiplications per second. So, in terms of multiplications per second, the OFDM system has a processing advantage of a factor of 10!

The other advantage of OFDM over single carrier systems with equalizers is that for the latter systems, the performance degrades abruptly if the delay spread exceeds the value for which the equalizers are designed. Because of error propagation, the raw bit error probability increases so quickly that introducing lower rate coding or a lower constellation size does not significantly improve the delay spread robustness. For OFDM, however, there are no such non-linear effects as error propagation, and coding and lower constellation sizes can be employed to provide fallback rates that are significantly more robust against delay spread. This is an important consideration, as it enhances the coverage area and avoids the situation that users in bad spots cannot get any connection at all.

2.5 OFDM TRANSCEIVER

The general block diagram of the baseband processing of an OFDM transceiver is shown in Figure 2-4 the diagram will be briefly explained following the data-stream from transmitter to receiver.

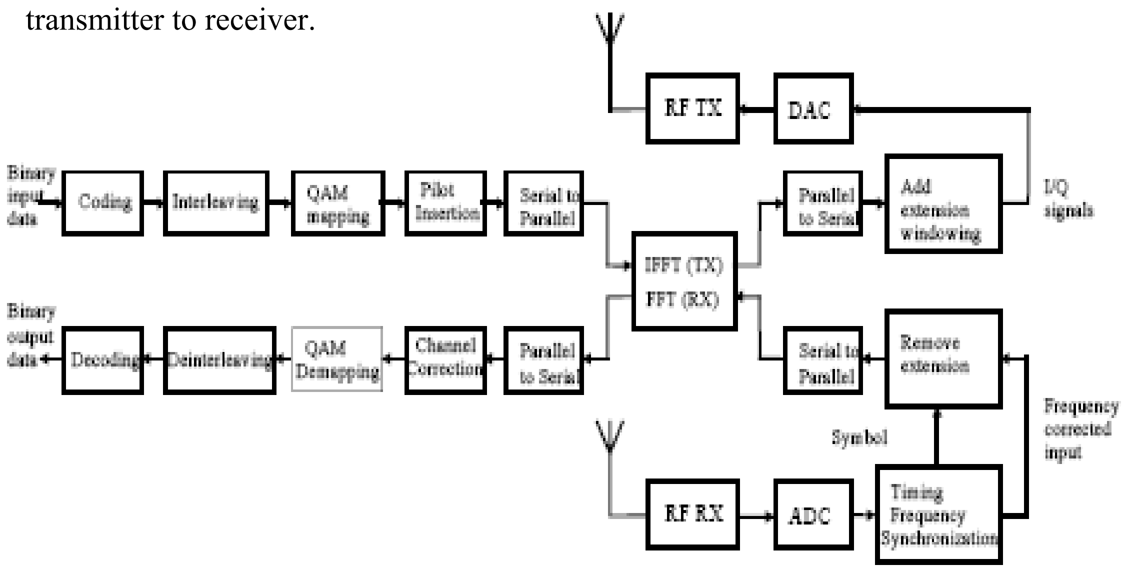


Figure 2.4: Block diagram of OFDM from transmitter to receiver

In the transmitter, a convolutional encoder first encodes the binary input data. After interleaving, the binary values are mapped on Quadrature Amplitude Modulation (QAM) values. In order to correct the signal in the receiver for a possible phase drift, pilot carriers can be introduced, but in this report we assume optimal timing and, thus, no pilot carriers will be used. In the Serial to Parallel block, the serial QAM input symbol-stream is converted to a parallel stream with width equal to the number of sub carriers.

These parallel symbols are modulated onto the sub carriers by applying the Inverse Fast Fourier Transform. Note that in order to get an output spectrum with a relative low out-of-band radiation, the size of the IFFT can be chosen larger than the number of sub carriers that is actually used to transmit the data.

After the IFFT block, the parallel output is converted back to serial. To make the system robust to multipath propagation, a cyclic prefix is added. Further, windowing is applied to get a narrower output spectrum. After this Step, the digital output signals are converted to analog signals. These analog signals are then up converted to the RF band, amplified and transmitted through an antenna.

The OFDM receiver basically performs the reverse operations of the transmitter, together with additional training tasks. First, the receiver has to estimate frequency offset and symbol timing, using special training symbols in a preamble. Then, it can do a Fast Fourier Transform for every symbol to recover the QAM values of all sub carriers. The training symbols and pilot sub carriers are used to correct for the channel response as well as the remaining phase drift. The QAM values are then demapped into binary values and de-interleaved, after which a Viterbi decoder can decode the information bits.

2.6. USES OF OFDM

In the 1 960s, the OFDM technique was used in several high-frequency military systems such as KINEPLEX [6], ANDEFI' [7], and KATHRYN [8]. For example, the variable-rate data modem in KATHRYN was built for the high-frequency band. It used up to 34 parallel low-rate phase-modulated channels with a spacing of 82 Hz.

In the l990s, OFDM was exploited for

- Wideband data communications over Mobile radio FM channels,
- High-bit-rate digital subscriber lines (HDSL; 1.6 Mbps),
- Asymmetric digital subscriber lines (ADSL; up to 6 Mbps),
- Very-high-speed digital subscriber lines (VDSL; 100 Mbps),
- Digital audio broadcasting (DAB) and
- High- definition television (HDTV) terrestrial broadcasting

2.7. ADVANTAGES

The OFDM transmission scheme has the following key advantages:

2.7.1 ROBUST AGAINST MULTIPATH

OFDM is an efficient way to deal with multipath; for a given delay spread, the implementation complexity is significantly lower than that of a single carrier system with an equalizer.

2.7.2 MAXIMIZES THROUGHPUT

In relatively slow time-varying channels, it is possible to significantly enhance the capacity by adapting the data rate per subcarrier according to the signal-to- noise ratio of that particular subcarrier.

2.7.3 ROBUST AGAINST NARROWBAND INTERFERENCE.

OFDM is robust against narrowband interference, because such interference affects only a small percentage of the subcarriers.

2.7.4 ESTABLISHES SINGLE-FREQUENCY NETWORK

OFDM makes single-frequency networks possible, which is especially attractive for broadcasting applications.

2.7.5 FREQUENCY DIVERSITY.

OFDM is the best place to employ Frequency Diversity. In fact, in a combination of OFDM and CDMA called the MC-CDMA transmission technique, frequency diversity is inherently present in the system. (i.e., it is available for free). Even though, OFDM provides a lot advantages for Wireless Transmission, it has a few serious disadvantages that must be overcome for this technology to become a success.

2.8. DISADVANTAGES

On the other hand, OFDM also has some drawbacks compared with single- carrier modulation:

2.8.1 THE PEAK POWER PROBLEM IN OFDM

One of the most serious problems with OFDM transmission is that, it exhibits a high peak-to-average ratio. In other words, there is a problem of extreme amplitude excursions of the transmitted signal. The OFDM signal is basically a sum of N complex random variables, each of which can be considered as a complex modulated signal at different frequencies. In some cases, all the signal components can add up in phase and produce a large output and in some cases, they may cancel each other producing zero output. Thus the peak-to-average ratio (PAR) of the OFDM system is very large. The problem of Peak-To-Average Ratio is more serious in the transmitter. In order to avoid clipping of the transmitted waveform, the power-amplifier at the transmitter front end must have a wide linear range to include the peaks in the transmitted waveform. Building power amplifiers with such wide linear ranges is a costly affair. Further, this also results in high power consumption.

The DAC's and the ADC's must also have a wide range to avoid clipping. There has been a lot of research put into the study of overcoming the PAR problem in OFDM. The following sections discuss some of the most common and important of those techniques as well as other issues.

2.8.2 FREQUENCY OFFSET

Frequency offset is introduced when there is a mismatch between frequencies at the transmitter end and at the receiver end. The difference in frequencies is translated into Intercarrier Interference (ICI), causing the carriers to interfere with each other and thus losing their orthogonality.

2.8.3 TIME OFFSET.

Time offset is caused by delay spread. Time offset results in Inter-symbol Interference (ISI) i.e. symbols interfere with each other which eventually results in the loss of data.

CHAPTER 3: MIMO OFDM

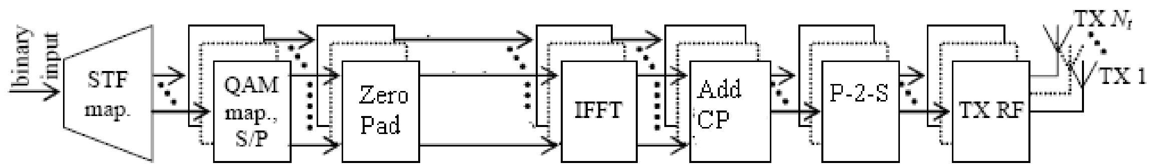
In this chapter, first the combination of MIMO and OFDM is described in Section 5.3. Section 5.4 describes a system model for MIMO OFDM which shows that the MIMO OFDM processing transfers the wideband frequency-selective MIMO channel into a number of parallel flat-fading MIMO sub channels.

3.1 ADVANTAGE OF OFDM

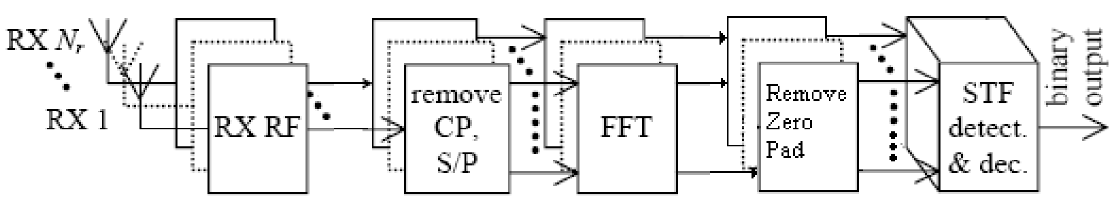
The MIMO algorithms are narrowband algorithms. In order to deal with the frequency-selective nature of wideband wireless channels, MIMO can be combined with OFDM. Effectively, OFDM transforms a frequency-selective channel into parallel flat-fading sub channels, i.e., the signals on the subcarriers undergo narrowband fading. In this way MIMO and OFDM are complement each other. Hence, by performing MIMO transmission and detection per subcarrier, MIMO algorithms can be applied in broadband communication.

3.2 SYSTEM DESCRIPTION

Consider a MIMO OFDM system with *Nt* transmit (TX) and *Nr* receive (RX) antennas. In addition to the spatial and temporal dimension of MIMO, OFDM adds one extra dimension to exploit, namely, the frequency dimension. In general, the incoming bit stream is first encoded by a one-dimensional encoder after which the encoded bits are mapped onto the three available dimensions by the Space-Time-Frequency (STF) mapper. After the STF mapper, each TX branch consists of almost an entire OFDM transmitter.



At receiver side, the CP is removed and the *FFT* is done per receiver branch. In the context of the unified view, at this point, overall STF detection and decoding must be performed to recover the binary data stream. In general, however, because the MIMO algorithms are single carrier algorithms, MIMO detection is performed per OFDM subcarrier. To that end, the received signals of subcarrier *i* are routed to the *i*-th MIMO detector to recover the *Nt* QAM symbols transmitted on that subcarrier. Next, the symbols per TX stream are combined and, finally, STF demapping and decoding are performed on these *Nt* parallel streams and the resulting data are combined to obtain the binary output data.



Finally, note that OFDM has as advantage that it introduces a certain amount of parallelism by means of its *Nc* subcarriers. This fact can be exploited by MIMO OFDM.

Namely, if MIMO detection is performed per subcarrier, then a given detector is allowed to work Nc times slower than the MIMO detector of an equivalent single carrier system with comparable data rate. Although in the case of MIMO OFDM Nc of such detectors are required, they can work in parallel, which might ease the implementation.

Chapter 4: Pilot Channel Estimation Techniques

Channel Estimation is the process of characterizing the effect of the physical medium on the input sequence. It is an important and necessary function for wireless systems. Even with a limited knowledge of the wireless channel properties, a receiver can gain insight into the data sent over by the transmitter.

The main goal of Channel Estimation is to measure the effects of the channel on known or partially known set of transmissions. Orthogonal Frequency division multiplexing (OFDM) Systems are especially suited for channel estimation. The sub carriers are closely spaced. While the system is generally used in high speed applications that are capable of computing channel estimates with minimum delay.

Two types of channel estimation techniques are :

- Pilot based channel Estimation
- Blind Channel Estimation

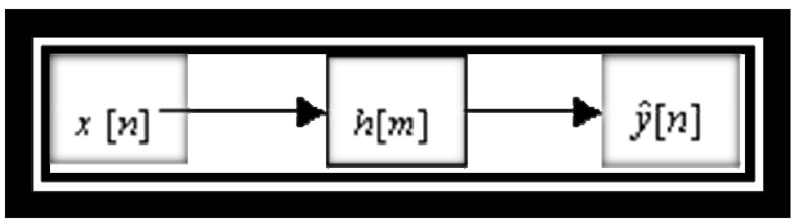
4.1 Training Based Channel Estimation:

Channel is estimated based on the training sequence which is known to both transmitter and receiver. The receiver can utilize the known training bits and the corresponding received samples for estimating the Channel.

- Least Squares (LS)
- Minimum Mean Squares (MMSE)

4.1.2 Least Squares (LS)

The Least Squares Error (LSE) estimation method can be used to estimate the system $h[m]$ by minimizing the squared error between estimation and detection



In matrix form, it can be written as

$$y = Xh$$

So the error ‘e’ can be defined as

$$e = y'' - y$$

Where y” is the expected output.

The squared error (S) can be defined as

$$S = | e |^2$$

$$S = (y'' - y)^2$$

$$S = (y'' - y)*(y'' - y)^t$$

Where superscript ‘t’ stands for complex transpose of a matrix.

$$S = (y'' - Xh)*(y'' - Xh)^t$$

This equation can be minimized by taking its derivative w.r.t ‘h’ and equating it equal to zero. The final equation we get is:

$$h'' = (X^t X)^{-1} X^t y$$

which can be written as

$$h'' = X^{-1} y$$

$$hls = X^{-1} y$$

This equation can be implemented on SISO as well as MIMO systems.

4.1.3 Problem in implementing MIMO:

When inputs are transmitted from Tx antennas, they are affected by the channel. Each Rx antenna is receiving signals from each Tx antenna. Now the received signal at Rx antenna is not a product of single channel response and single input signal but it is combination of signals from each Tx antenna multiplied with their respective channel responses.

4.1.4 Solution

- To overcome this problem matrix properties were exploited
- Instead of using one pilot, two pilots were used
- The use of two pilots were to make the matrices square and the number of equations must be equal to the number of unknown variables
- Then equating the equations channel response for each channel is calculated
- Use of multiple pilots is a drawback in multiple antenna system

4.2 Minimum Mean Square Error (MMSE)

The MMSE estimator minimizes the mean-square error.

If 'X' is transmitted over a channel 'h' such that

$$y = X\,h$$

Error is given as

$$e = y'' - y$$

Where y" is expected output.

Mean Square Error = mean$\{(y''- y)^{\wedge 2}\}$ =

$$E\,\{(y''- y)^{\wedge 2}\}$$

where 'E' is operator for expected value

Concept of expected value and correlation can be used to derive the equations for finding the channel response.

R_{gg} = auto covariance matrix of ‘g’

R_{YY} = auto covariance matrix of ‘Y’

R_{gY} = cross covariance matrix of ‘g’ and ‘Y’

The estimated channel H_{mmse} can be found out by the equation

$$H_{mmse} = F * (R_{gY} * R_{YY}^{-1} * Y)$$

where F is a noise matrix

$$R_{gy} = R_{gg} * F' * X'$$

$$R_{YY} = X * F * R_{gg} * F' *X' + \text{variance of noise} * \text{Identity Matrix}$$

The equation can be used for both ISO as well as MIMO systems.

4.2.1 Problem with MIMO Implementation:

When inputs are transmitted from Tx antennas, they are affected by the channel. Each Rx antenna is receiving signals from each Tx antenna. Now the received signal at Rx antenna is not a product of single channel response and single input signal but it is combination of signals from each Tx antenna multiplied with their respective channel responses.

Received signal is combination of all the transmitted signals following their own paths.
In MMSE matrix properties cannot be utilized as was the case in LS
The algorithm don’t include simple matrix operations

4.2.2 Solution

- First we train our receiver for all channels
- The pilot is transmitted from each transmitter separately
- At a time all the inputs except for one Tx are zero.
- Using the MMSE technique and set of equations, channel response will be calculated for each channel.

Chapter 5: Blind Channel Estimation Techniques

Although ISI can be avoided, via the use of cyclic prefix in OFDM modulation, the phase and gain of each subchannel is needed for coherent symbol detection. An estimate of these parameters can be obtained with pilot/training symbols, at the expense of bandwidth. Blind channel estimation methods avoid the use of pilot symbols, which makes them good candidates for achieving high spectral-efficiency. Existing blind channel estimation methods for OFDM systems can be classified as:

1. Statistical
2. Deterministic

The statistical methods explore the cyclo-stationarity that the cyclic prefix induces to the transmitted signal. They recover the channel using cyclic statistics of the received signal, or subspace decomposition of the correlation matrix of the pre-DFT received blocks. The deterministic methods process the post DFT received blocks, and exploit the finite-alphabet property of the information bearing symbols. Maximum likelihood and iterative Bayesian methods are two examples. Taking into account, specific properties of M-PSK or QAM signals, while utilizing an exhaustive search. In comparison to the statistical methods, the deterministic ones converge much faster, however, they involve high complexity, which becomes even higher as the constellation order increases.

5.1 Equalization:

The technique employed is from the deterministic class of blind channel estimation. It involves the use of equalizers. An equalizer removes the channel effects on a transmitted signal and reduces the Intersymbol Interference (ISI). The equalizer transfer function needed to compensate the channel distortion is the inverse of the channel transfer function, given by equation 5-1

$$H_e(f) = (H_{ch}(f))^{-1} \qquad 5.1$$

5.1.2 Adaptive Equalization:

The type of equalization, capable of tracking a slowly time-varying channel response is known as adaptive equalization. It can be implemented to perform tap-weight adjustments periodically or continually. Periodic adjustments are accomplished by periodically transmitting a preamble or short training sequence of digital data that is known to the receiver in advance. The receiver also uses the preamble to detect start of transmission, to set the automatic gain control (AGC) level. Continual adjustments are accomplished by replacing the known training sequence with a sequence of data. When performed continually, the adaptive procedure is referred to as decision directed. Decision directed only addresses how filter taps weights are adjusted. It should not be confused with a decision feedback (DFE), it refers to the fact that there exists an additional filter that operates on the detector output and recursively feeds back a signal to the detector input. Thus, with DFE there are two filters, a feed-forward filter and a feedback filter that process the data and help mitigate the ISI. Adaptive Equalization, particularly decision-directed adaptive equalization, successfully cancels ISI when the initial probability of error doesn't exceeds one percent. If the probability of error exceeds one percent, the decision directed equalizer might not converge**.** A common solution to the problem is to initialize the equalizer with an alternate process, such as a preamble, to provide good channel error performance, and the switch to the decision directed mode. To avoid the overhead represented by a preamble, many systems designed to operate in a continuous mode use blind equalization algorithms to form initial channel estimates. These algorithms adjust filter co-efficients in response to sample statistics rather than in response to sample decisions

5.2 Blind Equalization:

5.2.1 Bussgang Algorithm:

Blind channel equalizer, where the channel input is reconstructed as accurately as possible by using an adaptive filter to cancel the adverse effects of the channel, particularly the presence of Intersymbol Interference (ISI) and additive noise.

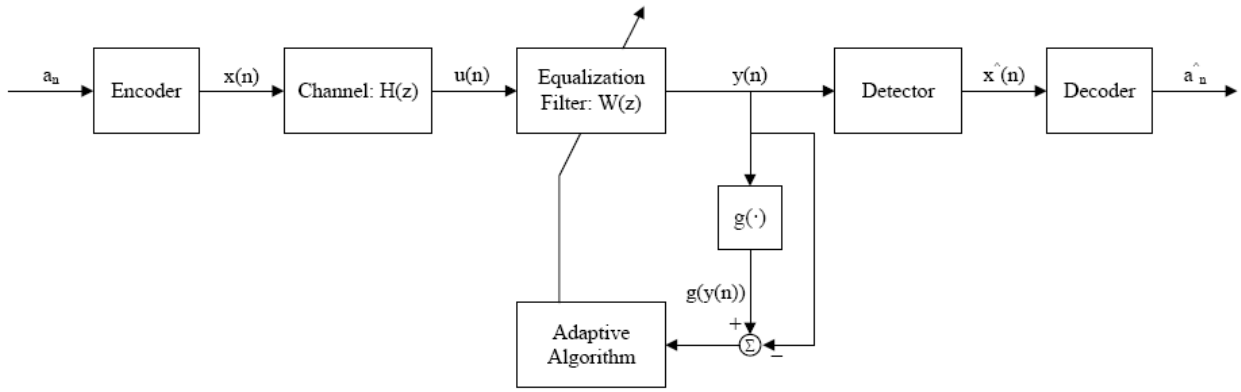


Figure 5.1: Block diagram of a blind channel equalizer.

The operation of the blind channel equalizer is illustrated in Figure 5.1. Suppose we are given a signal u(n) which is the output from an LTI channel. The equalizer output is:

$$y(n) = w(n) * u(n) = \sum_i w_i u(n-i)$$

We then apply the filter output y(n) to a memoryless nonlinear estimator g(y(n)) that acts as the desired response to an adaptive algorithm, such as the LMS algorithm. Clearly, we cannot use the channel input x(n) as our desired signal since it is unknown, so instead we must use some estimate of x(n) based upon observation of y(n). After computing the error e(n) = g(y(n)) − y(n), we update the tap weights. This general procedure is known as the Bussgang algorithm. Steps of Bussgang algorithm are:

1. Filtering: $y(n) = w^H(n)\, u(n)$
2. Error computation: $e(n) = g(y(n)) - y(n)$
3. Updating: $w(n+1) = w(n) + \mu\, u(n)\, e^*(n)$

In the LMS algorithm, the cost function $J(n) = E[|e(n)|^2]$ is a convex quadratic function of the tap weights and therefore has a unique minimum point. However, the cost function in the Bussgang algorithm is generally not a quadratic function due to the nonlinearity in $g(\cdot)$. Therefore, the iterative deconvolution procedure may have several local minima in addition to global minima. For this reason, it is essential that the tap weight vector w(n) is initialized properly in order for w(n) to converge to the desired equalization filter.

For this algorithm to converge, we require the following mean-value convergence condition:

$$E[y(n)y(n-k)] = E[y(n)g(y(n-k))]$$

Any stochastic process y(n) that satisfies this condition is known as a Bussgang process. Clearly, we find that the validity of the algorithm depends on the proper nonlinear estimator $g(\cdot)$. For example, in a simple binary PAM system where $x(n) \in \{-1, +1\}$, we can use the following estimator:

$$g(y(n)) = \alpha \operatorname{sgn}(y(n))$$

where the constant α is a data-dependent gain, given as:

$$\alpha = \frac{E[x^2(n)]}{E[|x(n)|]}$$

This choice of estimator results in a special case of the Bussgang algorithm, known as the Sato algorithm.

5.2.2 Constant-Modulus Algorithm

In the Sato algorithm, it can easily be seen that the gradient of the cost function reaches zero when perfect equalization is attained. However, if we use the same cost function for some M-ary system such as M-ary QAM or M-ary PSK, convergence may not be achieved. Instead, we can use a cost function that is suited for multi-dimensional communication systems, such as the following:

$$J(n) = E\left[(|y(n)|^p - R_p)^2\right]$$

where the constant R_p ensures convergence of the algorithm:

$$R_p = \frac{E[|x(n)|^{2p}]}{E[|x(n)|^p]}$$

and the error term is defined as follows:

$$e(n) = y(n)|y(n)|^{p-2}\left(R_p - |y(n)|^p\right)$$

The Bussgang algorithm that uses this choice of cost function is known as the Godard algorithm.

The Godard algorithm is more robust to phase perturbations than other Bussgang algorithms because it only relies on the magnitude of the received signal. The Sato algorithm is a special case of the Godard algorithm where p = 1. In the case of p = 2, we have the following values:

$$J(n) = E\left[(|y(n)|^2 - R_2)^2\right]$$
$$R_2 = \frac{E[|x(n)|^4]}{E[|x(n)|^2]}$$
$$e(n) = y(n)(R_2 - |y(n)|^2)$$

The algorithm that uses this cost function is known as the constant-modulus algorithm (CMA). Steps of the CMA algorithm are:

1. Initialization:
$$w_i(0) = \begin{cases} 1 & i = \lceil M/2 \rceil \\ 0 & \text{otherwise} \end{cases}$$
2. Filtering: $y(n) = w^H(n)\, u(n)$
3. Estimation: $g(y(n)) = y(n)(1 + R2 - |y(n)|^2)$
4. Error computation: $e(n) = g(y(n)) - y(n)$

5. Updating: $w(n + 1) = w(n) + \mu\, u(n)\, e^*(n)$

As stated before, the initial equalization is very important for the convergence of the Bussgang algorithm. So, we initialize the tap weight to 1 and all others to zero.

5.3 Linear Pre-coding :

It belongs to the statistical class. It consists of a simple linear transformation applied to blocks of symbols before they enter the OFDM system, which enables blind channel estimation at the output of the OFDM system via cross-correlation operations. It does not introduce redundancy to the block, nor does it change the block length. The method is computationally simpler than deterministic methods, and its performance is comparable to that of subspace approaches at significantly lower complexity

Let us consider an N-block OFDM system, and apply at its input a linear pre-coding block that performs the following task. It transforms the *i*-th OFDM block of *N* information symbols, $[d_{i,k},\ k = 0,1\ldots\ldots N\text{-}1]$ according to:

$$s_{i,k} = \frac{1}{\sqrt{1+|A|^2}}(d_{i,k} + (-1)^k A d_{i,T}),\ \ k = 0,\ldots,N-1$$

where T and A are both predefined numbers, assumed to be known to the transmitter and receiver; T is an integer in [0, N-1], and A is a pure imaginary number with $|A| < 1$. This pre-coding has several properties:

1. It introduces no redundancy to the transmitted data, which makes the approach bandwidth efficient.
2. It preserves the transmission power on each subcarrier. We should note here that the requirement for A to be pure imaginary is necessary in order to maintain the power of the T-th carrier.
3. It maintains the zero-mean of the signal transmitted on each subcarrier.
4. It maintains zero DC offset in each block.
5. It introduces a correlation structure in signals transmitted over different subcarriers, which can be explored for channel estimation.

The coded block [$s_{i,k}$, k = 0,1…..N-1] goes through the regular OFDM transmission steps. The *i*-th received OFDM block after removal of the CP and DFT is:

$$y_{i,k} = H(k)s_{i,k} + v_{i,k} = \frac{1}{\sqrt{1+|A|^2}}H(k)(d_{i,k} + (-1)^k A d_{i,T}) + v_{i,k}, \;\; k = 0, \ldots, N-1$$

where H(k) is the complex gain of the k-th subcarrier, and $v_{i;k}$ models the noise. It is assumes that:

(i) The baud-rate channel can be modeled as an FIR filter of length L with tap coefficients $h(l)$, l = 0,1…..L-1, and hence,

$$H(k) = \sum_{l=0}^{L-1} h(l) e^{-j\frac{2\pi}{N}kl},\ k = 0, \ldots, N-1;$$

(ii) The channel stays the same for at least the duration of the block, and is quasi-stationary between blocks;

(iii) The noise $v_{i;k}$ is complex, circular Gaussian, zero-mean, white across subcarriers and blocks, and independent of the information symbols.

The channel frequency response *H*(*k*), k = 0,1….N-1, or equivalently, the channel impulse response *h*(*l*), l = 0,1…..L-1, is needed in order to recover the transmitted signal $s_{i;k}$ and consequently the information symbols $d_{i;k}$, k = 0,1…..N-1.

Consider the correlation of the signals on the *k*-th and *T*-th subcarriers,

$$z_{k,T} \triangleq \mathrm{E}[y_{i,k} y_{i,T}^*]$$

Based on $z_{k,T}$, an estimate of the channel *H*(*k*) can be obtained as:

$$\hat{H}(k) \triangleq \begin{cases} \frac{1+|A|^2}{(-1)^k A + (-1)^{k+T}|A|^2} z_{k,T}, & k = 0, \ldots, N-1,\ k \neq T, \\ z_{k,T}, & k = T. \end{cases}$$

The channel estimate can be further improved, (if the length of the channel is known) noting that Ĥ (k), k = 0,1….N-1, should be the DFT of the channel impulse response *h*(*l*), l = 0,1…..L-1, where *L* < *N*. The latter length constraint can be enforced by performing

IDFT on ˆH (k), setting to zero the last N - L samples of the IDFT output, and then performing an N-point DFT on the result. This procedure is referred to as *denoising.*

A potential problem with the estimate might arise when the T-th carrier is in deep fade, in which case $z_{k;T}$ is close to zero for all k's. However, it is interesting to note that, at the receiver, any subcarrier can play the same role as the T-th one. Let R be an integer in [0,1….N-1] with R ≠ T. Then it holds:

$$z_{k,R} \triangleq \mathrm{E}[y_{i,k} y_{i,R}^*]$$

The channel response $H(k)$ can be estimated as:

$$\hat{H}(k) \triangleq \begin{cases} \frac{1+|A|^2}{(-1)^{k+R}|A|^2} z_{k,R}, & k \neq R, T \\ z_{k,R}, & k = R, \\ \frac{1+|A|^2}{(-1)^R A^* + (-1)^{T+R}|A|^2} z_{k,R}, & k = T \ . \end{cases}$$

A criterion for selecting R can still be implemented at the receiver. It can be selected as:

$$R_o = \arg\max_R \sum_{k=0}^{N-1} |z_{k,R}|^2.$$

This step would require the estimation of the entire correlation matrix of the received blocks, thus introducing a small increase in complexity.

5.3.1 Scalar ambiguity

Scalar ambiguity is common to all blind channel estimation methods. One way to resolve it is using a pilot symbol. The pilot could be inserted in the block before or after precoding. In the former case, a good place for inserting the pilot would be the T-th location within the block. Then, the $d_{i;T}$ would be the known pilot. An important property of $d_{i;T}$ is that, it varies from block to block, thus maintaining a zero-mean for the symbol transmitted over each carrier. Thus, if $d_{i;T}$ were to be set to some known value, this value would have to be assigned for each i by a pseudo-random number generator. Perfect cooperation between transmitter and receiver would be needed in this case for the synchronization of the pilot sequence. The pilot could also be placed in the block after precoding. Assuming that one pilot is placed at location P in the precoded block, then, we can obtain an estimate of H(P) based on J received blocks as:

$$\hat{H}(P) = \frac{1}{J}\sum_{i=1}^{J} y_{i,P}/s_{i,P}$$

where $s_{i;P}$ is the pilot symbol used in block i. Subsequently, the channel estimate obtained before can be normalized with respect to $\hat{H}$ (P).

Linear block precoding (LBP) has been used extensively for increasing throughput and diversity gains in MIMO OFDM systems. The design of codes to achieve these goals has recently become an area of huge interest and vast potential. For instance, algebraic number theory, coding theory , and iterative greedy optimization have being used to design optimal codes. However, very little work has been done on developing LBP schemes that allow channel estimation.

Consider a MR × MT , (MR≥MT) MIMO OFDM system with 'MT' users and 'MR' receivers. A non redundant linear block precoding is applied at the inputs before they enter the OFDM system, which increases multipath diversity and allows for blind channel estimation at the receiver. The channel estimation approach employs computationally simple cross-correlation operations and yields the channel up to a diagonal ambiguity. It does not require channel length information, and is not sensitive to additive stationary noise. The precoding does not increase transmission power and maintains even distribution of power between OFDM blocks. In a MR ×MT multi-user OFDM system the received symbol at the m-th receive antenna consists of contributions from all MT users, given by:

$$y^i_m(k) = \sum_{p=0}^{M_T-1} H_{mp}(k) s^i_p(k) + n_m(k),\ \ k = 0, ..., N-1,\ \ m = 0, ..., M_R - 1$$

where 'i' is the block index, 'k' is the carrier index 'p' is the user index, '$s_p(k)$' is the transmitted symbol over the k-th carrier, which, is derived from the source symbols $d_p(k)$ for k = 0, ...,N − 1; $H_{mp}(k)$ is the gain of the k-th carrier between the m-th receive antenna and the p-th user; $n_m(k)$ denotes noise. 'N' is the block size.

Let $d_{i,p}=[d_{i,p(0)} \ldots d_{i,p(N-1)}]^T$ denote the i-th block of symbols corresponding to the p-th user before any precoding. During the i-th block, the symbols of the p-th user to be transmitted over carrier k are generated based on an $N \times 1$ code vector, $w_p(k; i)$, as:

$$s_{i,p}(k) = w^H{}_p(k; i)\, d_{i.p}$$

Equivalently, the i-th block of the p-th user is precoded according to:

$$s_{i,p} = [s_{ip(0)} \ldots s_{ip(N-1)}]^T = [\, w_p(0; i)\, ,\, w_p(1; i), \ldots, w_p(N-1; i)\,]^H dip \; = W_{i,p}\, d_{i,p}$$

where $W_{i,p}$ is a matrix whose k-th row equals $w^H{}_p(k; i)$.

The purpose of the precoding matrix is to introduce some correlation structure in the transmitted blocks. Also, by changing the coding matrix between subsequent blocks we will create diversity that will allow us to estimate the channel matrix. Since we will need to obtain autocorrelation estimates, the coding matrices have to stay the same for a number of blocks. Thus, take them to be periodic in i with period M, $M \geq MT$. i.e.,

$$W_{i,p} = W_{i+M,\, p}$$

For the i-the received block, the symbols of all users that were received on carrier k in vector $y_i(k)$. It holds:

$$y_i(k)=[y_{i,\,0}(k) \ldots y_{i,MR-1}(k)]^T = H(k)\, s_i(k) + n(k)$$

where $s_i(k)=[s_{i,0}(k) \ldots . s_{i,MT-1}(k)]^T$.

It can be written as:

$$\mathbf{y}^i(k) = \mathbf{H}(k)\mathbf{\Psi}^i(k)\mathbf{d}^i + \mathbf{n}(k)$$

Where

$$\mathbf{\Psi}^i(k) \triangleq \begin{bmatrix} \mathbf{w}_0^H(k;i) & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{w}_1^H(k;i) & \ldots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{w}_{M_T-1}^H(k;i) \end{bmatrix}$$

Consider a sequence of received symbols that start at block i and are spaced apart by M blocks:

$$\tilde{\mathbf{y}}^i(k) = \{\mathbf{y}^i(k), \mathbf{y}^{i+M}(k), \mathbf{y}^{i+2M}(k), \ldots\}$$

The correlation matrix of ˜yi(k), is given by:

$$\mathbf{R}_{kl}^i \triangleq E\{\tilde{\mathbf{y}}^i(k)\tilde{\mathbf{y}}^i(l)^H\}, \quad i = 0, ..., M-1$$

If U_{kl} is a full row rank matrix,

$$\mathbf{U}_{kl} \triangleq \begin{pmatrix} \mathbf{w}_0^H(k;0)\mathbf{w}_0(l;0) & \cdots & \mathbf{w}_0^H(k;M-1)\mathbf{w}_0(l;M-1) \\ \vdots & \ddots & \vdots \\ \mathbf{w}_{M_T-1}^H(k;0)\mathbf{w}_{M_T-1}(l;0) & \cdots & \mathbf{w}_{M_T-1}^H(k;M-1)\mathbf{w}_{M_T-1}(l;M-1) \end{pmatrix}$$

And

$$\mathbf{C}_{ukl} \triangleq \frac{1}{\sigma^2}[\mathbf{R}_{kl}^0\mathbf{e}_u,\ \mathbf{R}_{kl}^1\mathbf{e}_u,\ ...,\ \mathbf{R}_{kl}^{M-1}\mathbf{e}_u]$$

Then to find the estimate:

$$\hat{\mathbf{H}}_{lu}(k) \triangleq \mathbf{C}_{ukl}\mathbf{U}_{kl}^{\dagger}$$

SIMULATION RESULTS

The following simulation results are displayed:

- Pilot Estimation (LS,MMSE)
- Blind Estimation (CMA, Pre-coding)

PILOT BASED ESTIMATION
Least Squares (SISO)

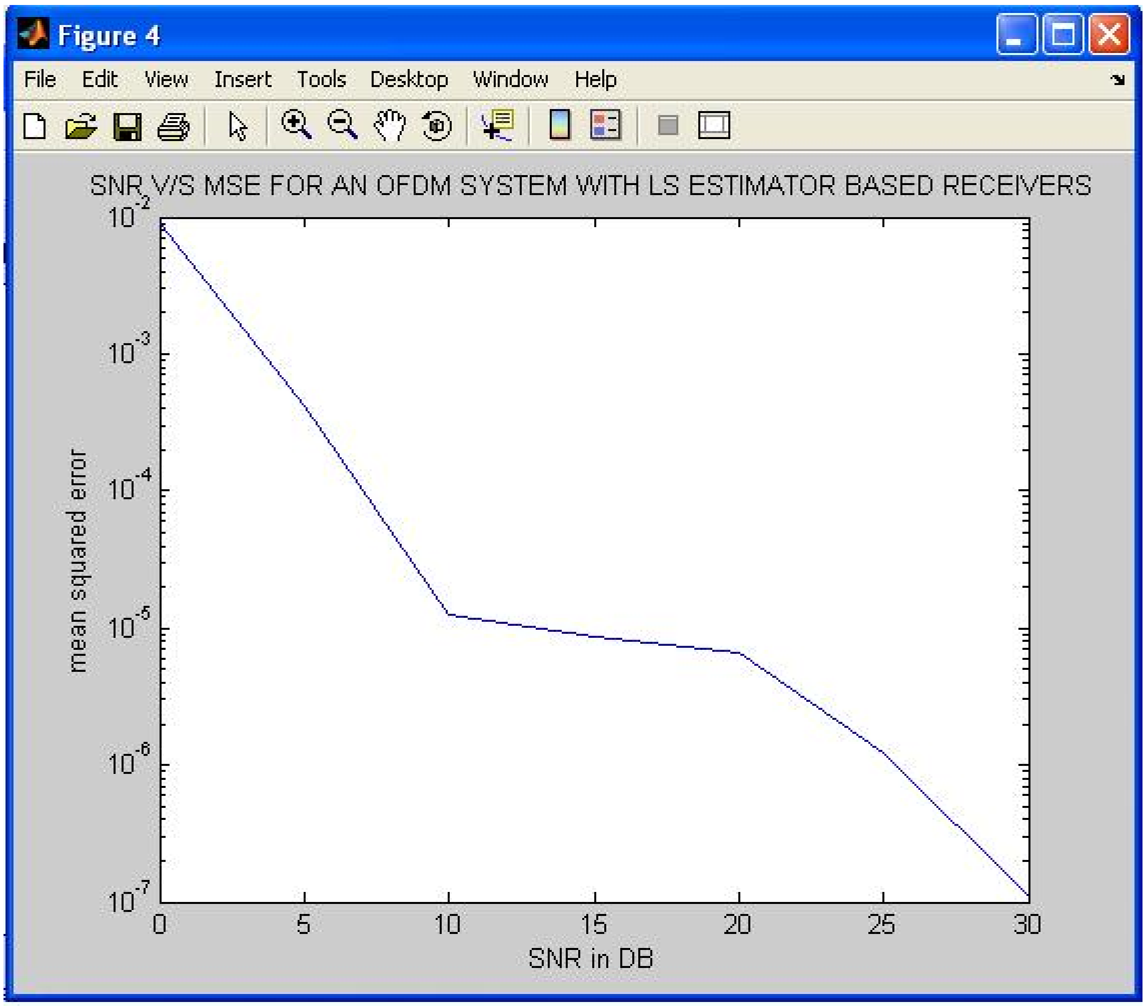


Figure 3 SNR v/s MSE LS SISO PLOT

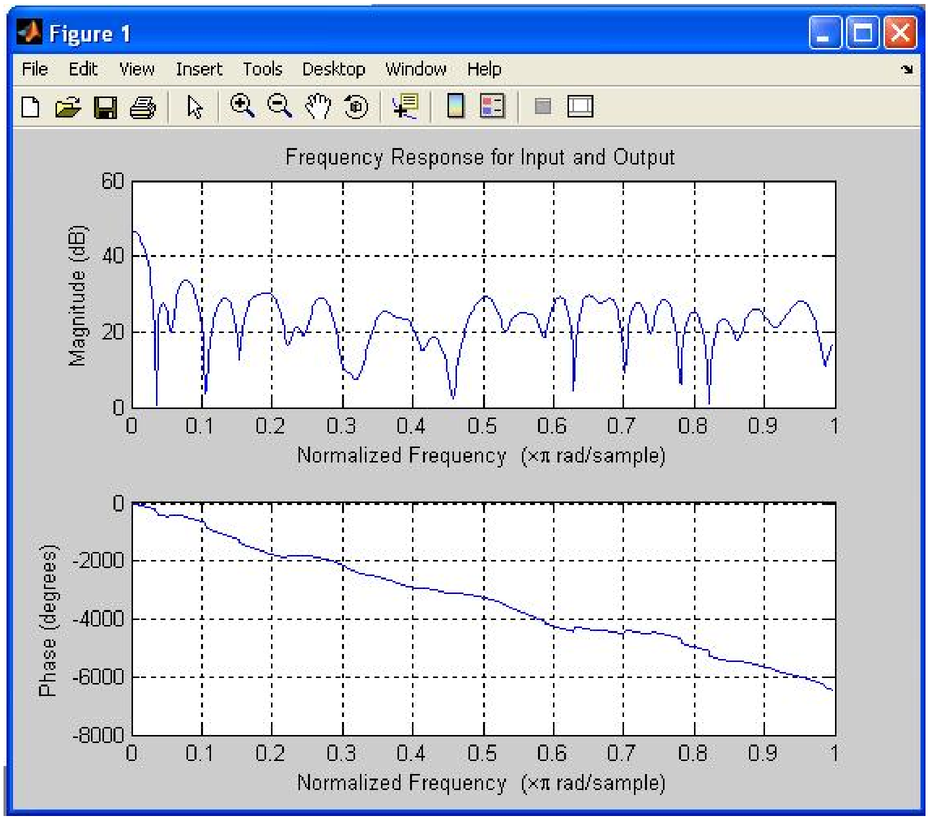


Figure 4 Frequency Response of LS SISO

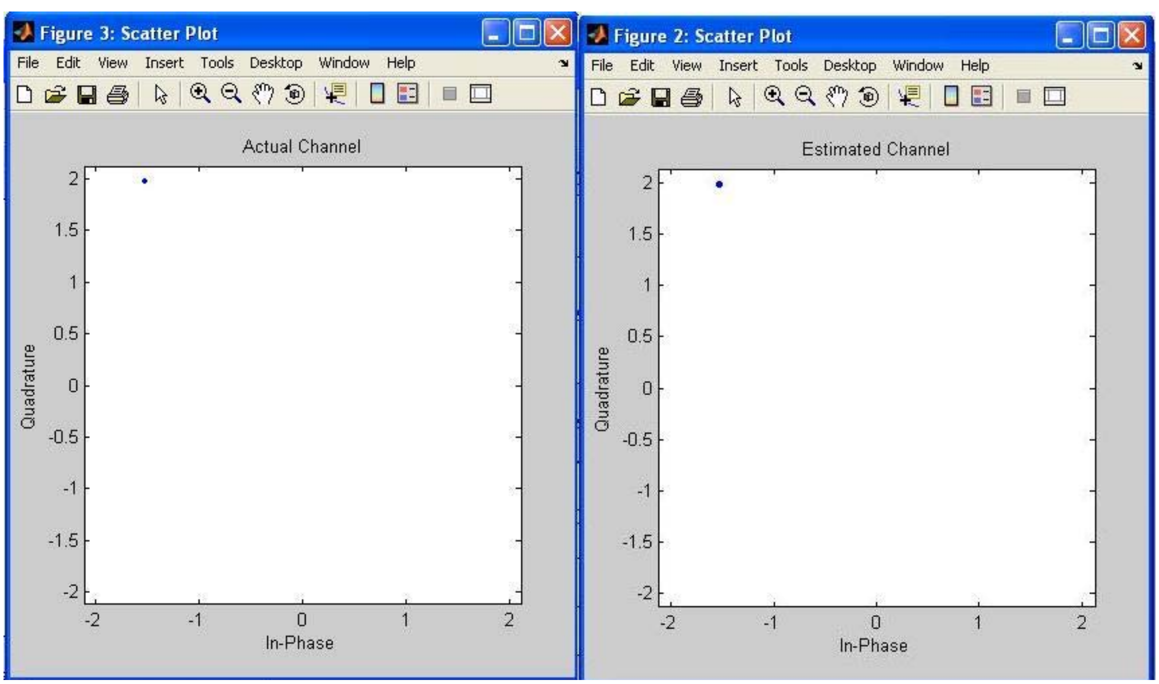


Figure 5 Actual and Estimated Channel Scatter Plots LS SISO

Least Squares (MIMO)

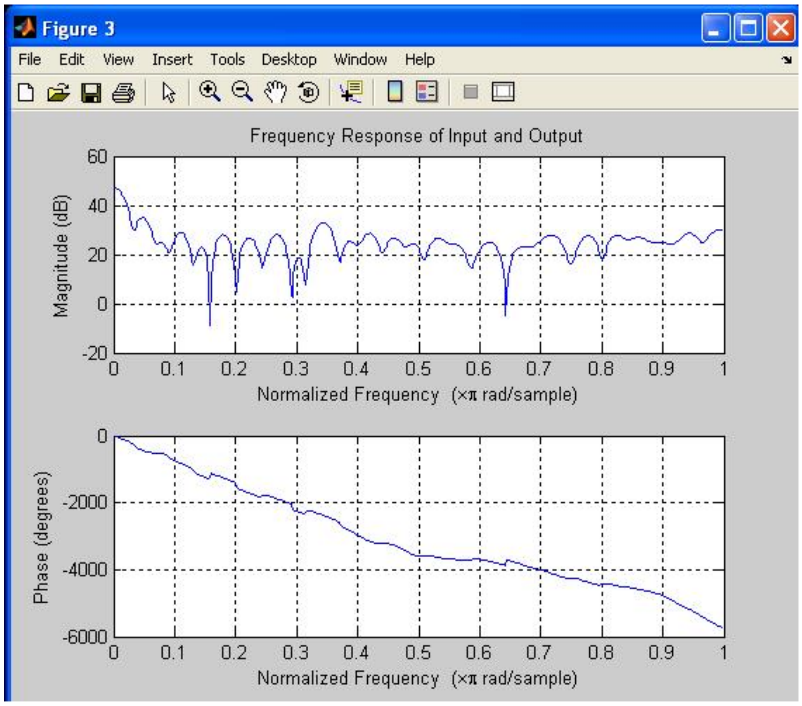


Figure 6 Frequency Response LS MIMO

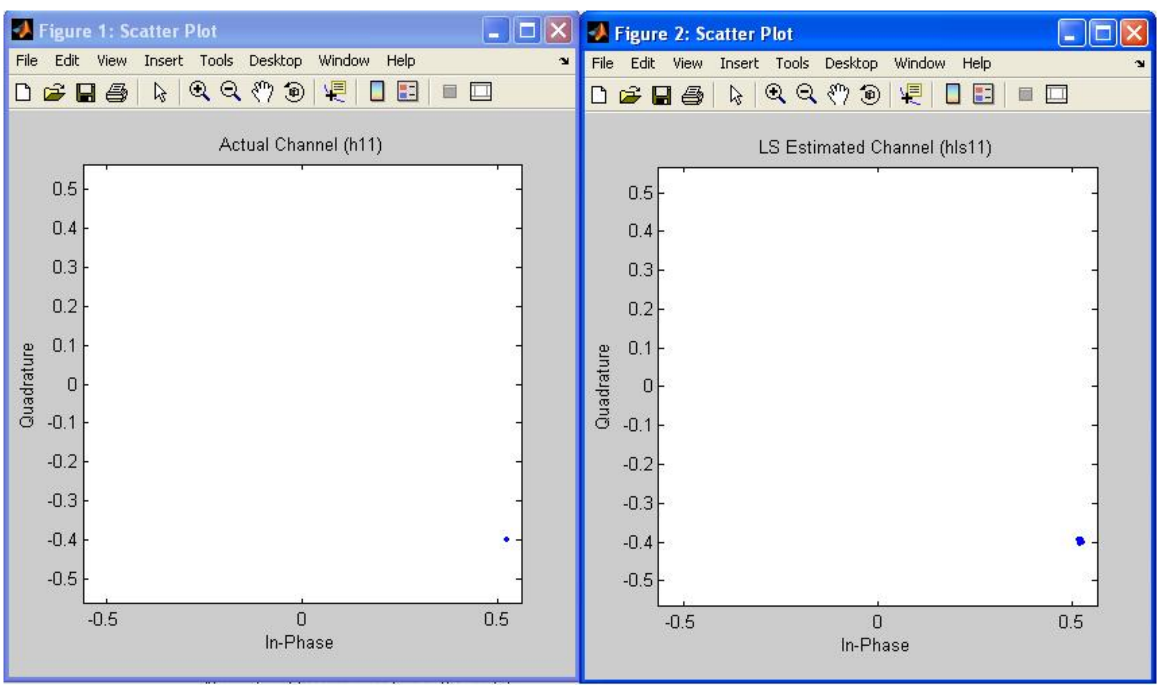


Figure 7 LS MIMO Actual and Estimated Channel Scatter Plots

Minimum Mean Square Error (SISO)

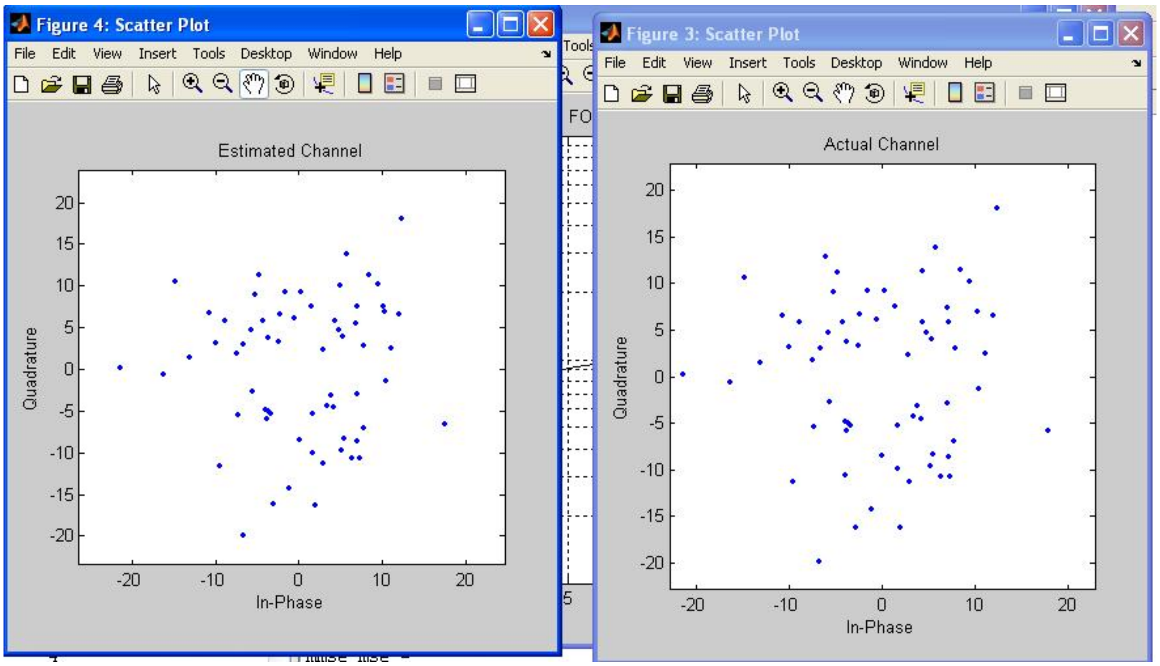


Figure 8 MMSE MIMO Channel Scatter Plots

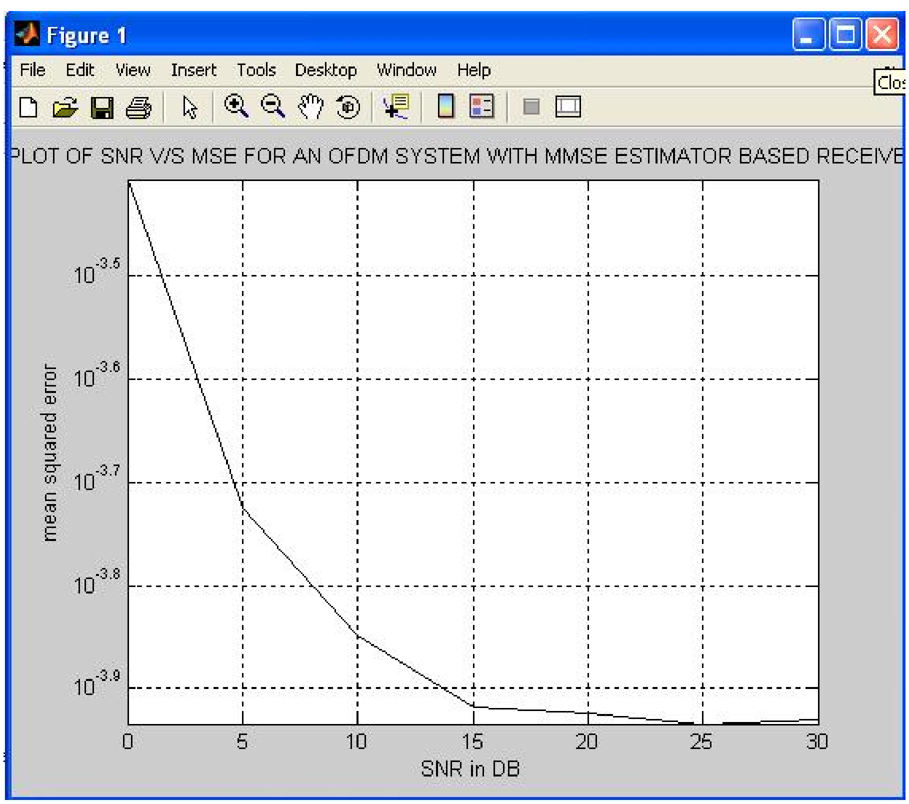


Figure 9 SNR v/s MSE MMSE MIMO Plot

Minimum Mean Square Error (MIMO)

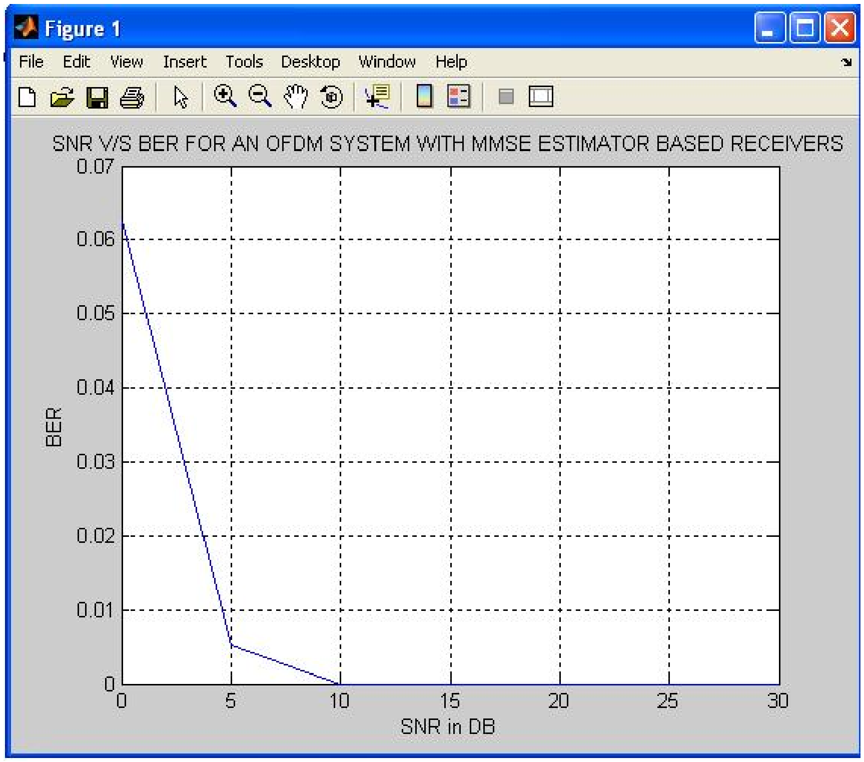


Figure 10 Bit error Rate MMSE MIMO

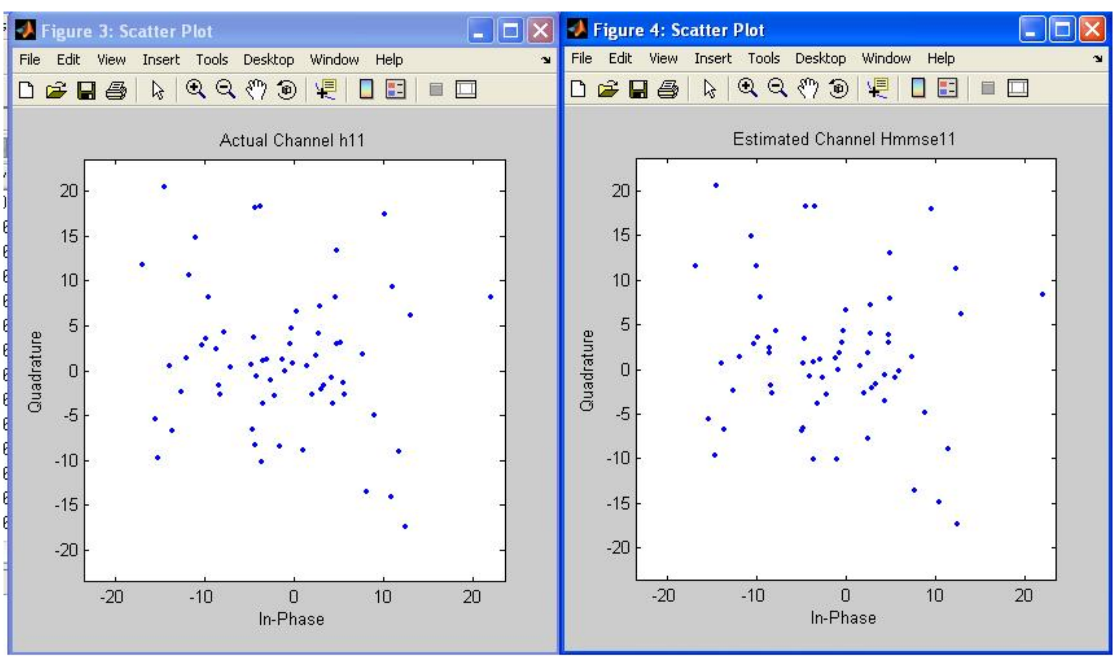


Figure 11 MMSE MIMO channel Scatter Plots

BLIND ESTIMATION

Constant Modulus Algorithm (CMA)

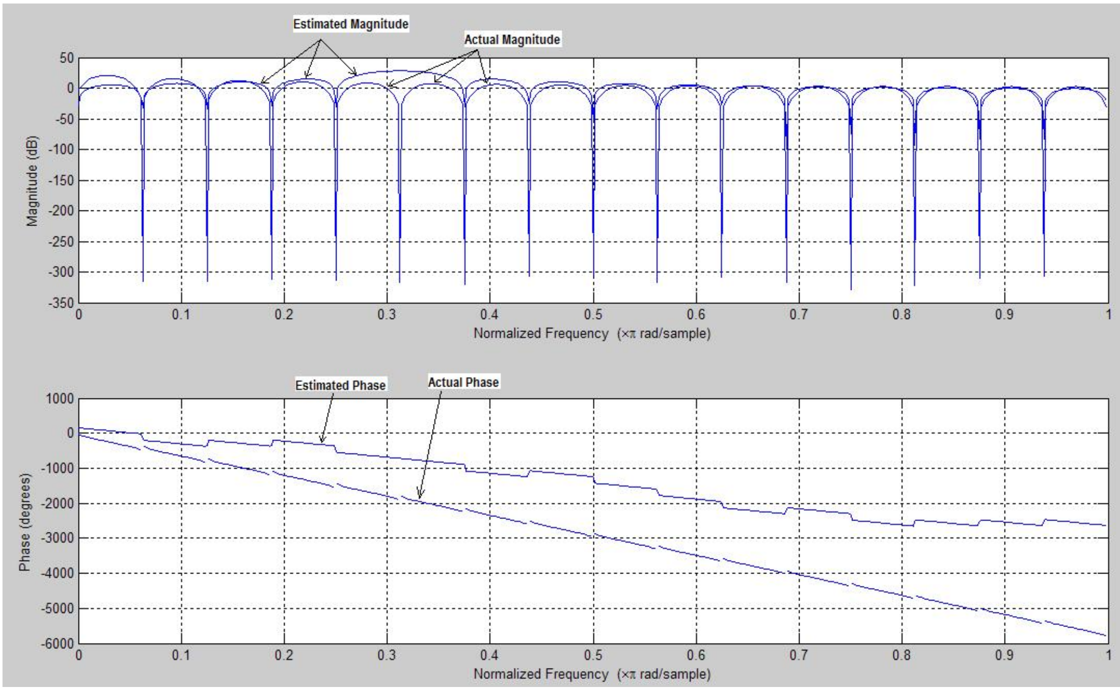


Figure 12 CMA QAM based frequency response

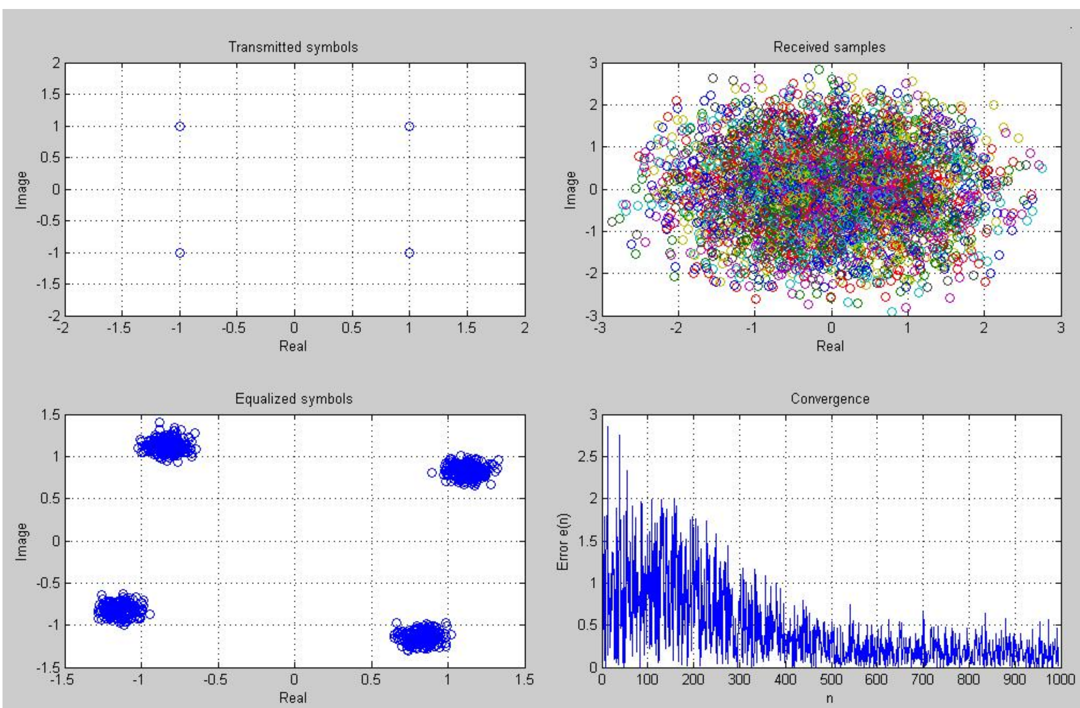


Figure 13 CMA Equalization

LINEAR PRECODING

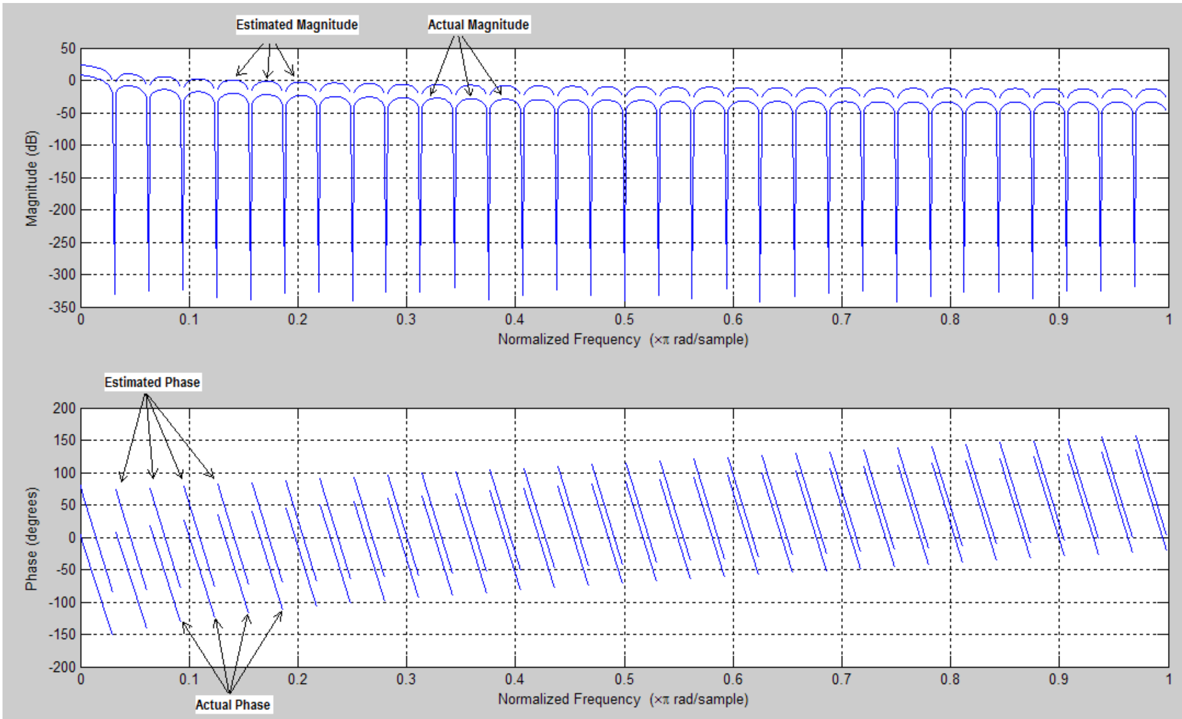


Figure 14 Frequency response of the channel

References:

[1] Coleri, S., Ergen, M., Puri, A., and Bahai, A., "Channel Estimation Techniques Based on Pilot Arrangement in OFDM Systems," *IEEE Transactions on Broadcasting*, vol. 48, pp. 223–229, Sept. 2002.

[2] Yang, B., Letaief, K. B., Cheng, R. S., and Cao, Z., "Channel Estimation for OFDM Transmission in Multipath Fading channels Based on Parametric Channel Modeling," IEEE Transactions on Communications, vol. 49, pp. 467–479, March 2001.

[3] Sanzi, F., Sven, J., and Speidel, J., "A Comparative Study of Iterative Channel Estimators for Mobile OFDM Systems," IEEE Transactions on Wireless Communications, vol.2, pp. 849–859, Sept. 2003.

[4]] Li, Y., "Simplified Channel Estimation for OFDM Systems with Multiple Transmit Antennas," IEEE Transactions on Communications, vol. 1, pp. 67-75, January 2002.

[5] Y. Li, Z. Ding, "Convergence analysis of finite length blind adaptive equalizers," IEEE Transactions on Signal Processing, vol. 43, pp. 2120 – 2129, Sept. 1995.

[6] S. Haykin, Ed., Blind Deconvolution. PTR Prentice Hall, Inc., 1994.

[7] Z. Ding and Y. G. Li, Blind Equalization and Identification, ser. Signal Processing and Communication Series. Marcel Dekker, Inc., 2001.

[8] H. Bolcskei, R.W. Heath, and A.J. Paulraj, "Blind channel identification and equalization in OFDM based multiantenna systems," IEEE Trans. on Signal Processing, vol. 50(1), pp. 96-109, Jan. 2002.

[9] C.Y. Wong, R.S. Cheng, K.B. Letaief, and R.S. Murch, "Multiuser OFDM with adaptive subcarrier, bit, and power allocation," IEEE Journal on Sel. Areas in Commun., vol. 17, no. 10, pp. 1747-1758, Oct. 1999.

[10] A. P. Petropulu, R. Zhang and R. Lin, "Blind OFDM Channel Estimation through Simple Linear Precoding," IEEE Trans. on Wireless Communications, vol. 3, no. 2, pp. 647-655, Mar. 2004.

[11] S. Yatawatta and Athina Petropulu, "Blind channel estimation in multiuser OFDM systems," in Proc. 36th Asilomar Conf. on Signals Systems and Computers, vol. 2, pp. 1709-1713, Nov. 2002.

[12] M. Debbah, W. Hachem, P. Loubaton, and M. de Courville, "MMSE analysis of certain large isometric random precoded systems," IEEE Trans. Inform. Theory, vol. 49, no. 5, pp. 1293 - 1311, May 2003.

[13] S. M. Alamouti, "A simple transmit diversity scheme for wireless communications,"IEEE J. Select. Areas Commun., vol. 16, pp. 1451–1458, Oct. 1998.

[14] A. Paulraj and C. B. Papadias, "Space–time processing for wireless communications," IEEE Signal Processing Mag., vol. 14, pp. 49–83, Nov. 1997.

[15] H. Ge, K. D. Wong, and J. C. Liberti, "Characterization of multiple-input multiple-output (MIMO) channel capacity," in Proc. IEEE Wireless Communications and Networking Conf. (WCNC), Orlando, FL, 2002.

[16] Y. Lin and S. Phoong, "BER minimized OFDM systems with channel independent precoders," IEEE Trans. on Signal Processing, vol. 51, no. 9, pp. 2369-2380, Sep. 2003

[17] Xia-Gen Xia, "Precoded and vector OFDM robust to channel spectral nulls and with reduced cyclic prefix length in single transmit antenna systems," IEEE Trans. on Communications, vol. 49, no. 8, pp. 1363-1374, Aug. 2001.

APPENDIX "A"

PROJECT CODES

Least Squares

LS SISO

```
function ls = LS_siso()
clear all

%%%%%%%%%% System Parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%
T=1;
R=1;
mod_scheme = input('Set Modulation Scheme 2(BPSK) 4(QAM) 8(8QAM)  =   ');


%%%%% OFDM   Parameters  %%%%%%%%%%%%%

zero_pad =0;
block_size = 64;
data_size =  block_size -zero_pad ;
prefix_size = 0  ;
total_block_size = block_size +  prefix_size ;

E_db =0: 5 :30;
err_rate_Eb_LS = [ ones(1,length(E_db) ) ] ;

 tic
 it =7;
 ber = [];
for l =1:length(E_db)
for k=1:it

   for h=1:T
   x(h,:) = randint(1,block_size,mod_scheme);
end


switch mod_scheme
   case 2
      X = 2*x-1;

   case 4
```

```
        X = qammod(x,4);
    case 8
        X = qammod(x,8);

end

X_ofdm = fft(X,64);


h     = randn (R,T) + j*randn (R,T);
H = fft(h,64);

Y = X_ofdm.*H;

Y=awgn(Y,E_db(l));


hls_est = Y./X_ofdm;;

ms_error=(((abs(hls_est-H))/abs(H)).^2);

ls_mse(it ,l) = ms_error;

rec_x_ofdm = Y./hls_est;
rec_x_mod = ifft(rec_x_ofdm,64);
rec_x = qamdemod(rec_x_mod,mod_scheme);

end
[num_bit_LS , ratio_bit_LS] = biterr(x,rec_x);
ber = [ ber ratio_bit_LS];
end
ber
ms_avg = mean(ls_mse);

freqz(x)
hold on
title('Frequency Response for Input and Output');
pause
freqz(rec_x)


scatterplot(hls_est)
title('Estimated Channel');
scatterplot(H)
title('Actual Channel');
figure
```

```
semilogy(E_db,ms_avg,'b-');
xlabel('SNR in DB');
ylabel('mean squared error');
title('SNR V/S MSE FOR AN OFDM SYSTEM WITH LS ESTIMATOR BASED
RECEIVERS');
figure
plot(E_db,ber,'b');
xlabel('SNR in DB');
ylabel('Bit Error Rate');
title('Bit error Rate Plot At different values of SNR');
```

LS (MIMO)

```
function ls = LS_mimo()
clear all

%%%%%%%%%% System Parameters
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

T=2;
R=2;
mod_scheme = input('Set Modulation Scheme 2(BPSK) 4(QAM) 8(8QAM)  =   ');

%%%%%%%%%%%%%%%%%%%%%%%% OFDM   Parameters
%%%%%%%%%%%%%%%
zero_pad =0;
block_size = 64;
data_size =  block_size -zero_pad ;
prefix_size = 0  ;
total_block_size = block_size +  prefix_size ;

E_db =0: 5 :30;
err_rate_Eb_LS = [ ones(1,length(E_db) ) ] ;
it=5;
ber = [];

for l =1:length(E_db)
for k=1:it

%%%%%%%%%%%%%%%%%%%%% Data generation
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
   x1 = randint(1,block_size,mod_scheme);
   x2 = randint(1,block_size,mod_scheme);
   x3 = randint(1,block_size,mod_scheme);
   x4 = randint(1,block_size,mod_scheme);
```

```
x_org = [x1 x2 ; x3 x4];

X1 =qammod(x1,mod_scheme);
X2 =qammod(x2,mod_scheme);
X3 =qammod(x3,mod_scheme);
X4 =qammod(x4,mod_scheme);

X1_ofdm = fft(X1,64);
X2_ofdm = fft(X2,64);
X3_ofdm = fft(X3,64);
X4_ofdm = fft(X4,64);

X_ofdm = [X1_ofdm X3_ofdm ; X2_ofdm X4_ofdm];

%%%%%%%%%%%%%%%%%%%%%%%%%    Channel Effect
%%%%%%%%%%%%%%%%%%%%%%%%%%
 for u=1:T
    for o=1:R
        h(u,o) = randn  + j*randn ;
    end
 end

 for w=1:R*T
 H(w,:) = fft(h(w),block_size);
 end

 h11 = H(1,:);
 h12 = H(3,:);
 h21 = H(2,:);
 h22 = H(4,:);

 H_act = [h11 h12 ; h21 h22];

 Y1 = X1_ofdm.*h11 + X3_ofdm.*h21;
 Y2 = X4_ofdm.*h21 + X2_ofdm.*h11;
 Y3 = X1_ofdm.*h12 + X3_ofdm.*h22;
 Y4 = X2_ofdm.*h12 + X4_ofdm.*h22;

 Y = [Y1 Y3;Y2 Y4];

%%%%%%%%%%%%%%%%
NOISE%%%%%%%%%%%%%%%%%
Y = awgn(Y,E_db(l));
y1 = Y(1,1:64);
y2 = Y(2,1:64);
```

```
y3 = Y(1,65:128);
y4 = Y(1,65:128);
%%%%%%%%%%%%% Estimationg H %%%%%%%%%%%
tx1 = X1_ofdm ;
tx2 = X2_ofdm ;
tx3 = X3_ofdm ;
tx4 = X4_ofdm ;

hls11=(tx4.*y1-tx3.*y2)./(tx1.*tx4-tx2.*tx3);
hls12=(tx4.*y3-y4.*tx3)./(tx1.*tx4-tx2.*tx3);
hls21=(tx1.*y2-tx2.*y1)./(tx1.*tx4-tx2.*tx3);
hls22=(tx1.*y4-tx2.*y3)./(tx1.*tx4-tx2.*tx3);

Hls=[hls11 hls12;hls21 hls22];

ms_error = ((abs(Hls)-abs(H_act))/abs(H_act)).^2;

ls_error(it,l) = ms_error(1);

rec_x1 = (y1.*hls22-y3.*hls21)./(hls11.*hls22-hls12.*hls21);
rec_x3 =(-1*y1.*hls12+y3.*hls11)./(hls11.*hls22-hls12.*hls21);
rec_x2 = (y2.*hls22-y4.*hls21)./(hls11.*hls22-hls12.*hls21);
rec_x4 =(-1*y2.*hls12+y4.*hls11)./(hls11.*hls22-hls12.*hls21);

rec_x1_mod=ifft(rec_x1,block_size);
rec_x3_mod=ifft(rec_x3,block_size);
rec_x2_mod=ifft(rec_x2,block_size);
rec_x4_mod=ifft(rec_x4,block_size);

rec1 = qamdemod(rec_x1_mod,mod_scheme);
rec3 = qamdemod(rec_x2_mod,mod_scheme);
rec2 = qamdemod(rec_x3_mod,mod_scheme);
rec4 = qamdemod(rec_x4_mod,mod_scheme);

x_act = [rec1 rec2 ; rec3 rec4];
end
[num , ratio] = biterr(rec1,x1);
ber = [ber ratio];
end

ls_error_avg = mean(ls_error);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Plots
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

scatterplot(h11)
```

```
pause
scatterplot(hls11)

figure

freqz(x1)
hold on
freqz(rec1)


ber
% pause
%
% figure
%
% err_rate_Eb_mmse
%
% err_rate_Eb_mmse = err_rate_Eb_mmse;
% length(E_db)
%
% semilogy(E_db,err_rate_Eb_mmse,'--r+')
% %
% % grid
```

MMSE (SISO)

```
clc;
clear all;

T=1;
R=1;
mod_scheme = input('Set Modulation Scheme 2(BPSK) 4(QAM) 8(8QAM)  =   ');

%%%%% OFDM   Parameters  %%%%%%%%%%%%%
zero_pad =0;
block_size = 64;
data_size =  block_size -zero_pad ;
prefix_size = 0  ;
total_block_size = block_size +  prefix_size ;

E_db =0: 5 :30;

X=zeros(block_size,block_size);
x=randint(1,block_size,mod_scheme);
d=qammod(x,mod_scheme);
```

```
d=fft(d);

 for i=1:64
     X(i,i)=d(i);
 end

p=[];

for i=1:64
h(i) = randn +j*randn;
end

G=h';
H=fft(G);
%%%%%%%%%%%%%%%%%Evaluation of the autocovariance matrix of
G-Rgg%%%%%%%%%
co_cal=zeros(64,64);
for i=1:64
    co_cal(i,i)=G(i);
end
co_myu = sum(co_cal, 1)/64;
co_mid = co_cal - co_myu(ones(64,1),:);
sum_co_mid= sum(co_mid, 1);
Rgg = (co_mid' * co_mid- (sum_co_mid'  * sum_co_mid) / 64) / (64 - 1);


%%%%Running for a dozen trials to try and average out the
results%%%%%%%%



for n=1:length(E_db)
    for m=1:12
SNR_send=E_db(n);
XFG=X*H;
n1=ones(64,1);
n1=n1*0.0000000000000000001i;
noise=awgn(n1,SNR_send);
vari=var(noise);
N=fft(noise);
Y=XFG+N;

[mean_squared_error_mmse Hmmse]=mmse_calc(X,H,Y,Rgg,vari);

SNR(n)=SNR_send;

mmse_mse(m,n)=mean_squared_error_mmse;
```

```
X_rec_ofdm = Y./Hmmse;
X_rec_mod = ifft(X_rec_ofdm);
X_rec = qamdemod(X_rec_mod,mod_scheme);

end;
[num, ratio] = biterr(X_rec',x);
p = [p ratio];
end;
mmse_mse

mmse_mse_ave=mean(mmse_mse);

%Now just the display part.....
semilogy(SNR,mmse_mse_ave,'k-');
grid on;
xlabel('SNR in DB');
ylabel('mean squared error');
title('PLOT OF SNR V/S MSE FOR AN OFDM SYSTEM WITH MMSE
ESTIMATOR BASED RECEIVERS');

figure
semilogy(SNR,p,'b-');
grid on;
xlabel('SNR in DB');
ylabel('BER');
title('SNR V/S BER FOR AN OFDM SYSTEM WITH MMSE ESTIMATOR
BASED RECEIVERS');

scatterplot(H)
title('Actual Channel');
scatterplot(Hmmse)
title('Estimated Channel');
```

MMSE (MIMO)

```
clc;
clear all;

T=2;
R=2;
mod_scheme = input('Set Modulation Scheme 2(BPSK) 4(QAM) 8(8QAM)  =   ');
block_size = 64;
E_db =0: 5 :30;
```

```
X1=zeros(block_size,block_size);
X1=zeros(block_size,block_size);


% x1=randint(1,block_size,mod_scheme);
% x2=randint(1,block_size,mod_scheme);

% d1=qammod(x1,mod_scheme);
% d1=fft(d1);
% d2=qammod(x2,mod_scheme);
% d2=fft(d2);
%
%
%  for i=1:64
%      X1(i,i)=d1(i);
%      X2(i,i)=d2(i);
%
%  end

p1=[];

for i=1:64
h1(i) = randn +j*randn;
h2(i) = randn +j*randn;
h3(i) = randn +j*randn;
h4(i) = randn +j*randn;
end
G1=h1';
H1=fft(G1);
G2=h2';
H2=fft(G2);
G3=h3';
H3=fft(G3);
G4=h4';
H4=fft(G4);

Rgg1 = co_cal(G1);
Rgg2 = co_cal(G2);
Rgg3 = co_cal(G3);
Rgg4 = co_cal(G4);

%%%%Running for a dozen trials to try and average out the
results%%%%%%%%
```

```
for n=1:length(E_db)

    x1=randint(1,block_size,mod_scheme);
    x2=randint(1,block_size,mod_scheme);
    d1=qammod(x1,mod_scheme);
    d1=fft(d1);
    d2=qammod(x2,mod_scheme);
    d2=fft(d2);


 for i=1:64
     X11(i,i)=d1(i);
     X22(i,i)=d2(i);

 end


 for m=1:12

if m==1
    X2=0;
    X1=X11;
end

if m==2
    X1=0;
    X2=X22;
end
if m>2
    X1=X11;
    X2=X22;
end

SNR_send=E_db(n);
XFG1=X1*H1;
XFG2=X2*H2;
XFG3=X1*H3;
XFG4=X2*H4;
n1=ones(64,1);
n1=n1*0.0000000000000000001i;
noise=awgn(n1,SNR_send);
vari=var(noise);
N=fft(noise);
Y1=XFG1+N;
Y2=XFG2+N;
Y3=XFG3+N;
```

```
Y4=XFG4+N;

y1 = Y1+Y2;
y2 = Y3+Y4;


if m==1
[mean_squared_error_mmse1 Hmmse1]=mmse_calc(X1,H1,y1,Rgg1,vari);
[mean_squared_error_mmse3 Hmmse3]=mmse_calc(X1,H3,y2,Rgg3,vari);
end

if m==2
[mean_squared_error_mmse2 Hmmse2]=mmse_calc(X2,H2,y1,Rgg2,vari);
[mean_squared_error_mmse4 Hmmse4]=mmse_calc(X2,H4,y2,Rgg4,vari);

end

SNR(n)=SNR_send;

mmse_mse(m,n)=mean_squared_error_mmse1;

if m>2
X_rec_ofdm1 = (Hmmse4.*y1-Hmmse2.*y2)./(Hmmse1.*Hmmse4-
Hmmse2.*Hmmse3);
X_rec_mod1 = ifft(X_rec_ofdm1);
X_rec1 = qamdemod(X_rec_mod1,mod_scheme);
end
 end
if m>2
[num1 , ratio1] = biterr(X_rec1',x1);
p1 = [p1 ratio1];
end
end

% mmse_mse
% mmse_mse_ave=mean(mmse_mse);

%Now just the display part.....
% semilogy(SNR,mmse_mse_ave,'k-');
% grid on;
% xlabel('SNR in DB');
% ylabel('mean squared error');
% title('PLOT OF SNR V/S MSE FOR AN OFDM SYSTEM WITH MMSE
ESTIMATOR BASED RECEIVERS');
%
figure
```

```
plot(SNR,p1,'b-');
grid on;
xlabel('SNR in DB');
ylabel('BER');
title('SNR V/S BER FOR AN OFDM SYSTEM WITH MMSE ESTIMATOR
BASED RECEIVERS');

scatterplot(H1)
title('Actual Channel');

scatterplot(Hmmse1)
title('Estimated Channel');

scatterplot(H2)
title('Actual Channel');

scatterplot(Hmmse2)
title('Estimated Channel');
scatterplot(H3)
title('Actual Channel');

scatterplot(Hmmse3)
title('Estimated Channel');
scatterplot(H4)
title('Actual Channel');

scatterplot(Hmmse4)
title('Estimated Channel');

p1
```

LS v/s MMSE Compare

```
clc;
clear all;
T=1;
R=1;
mod_scheme = input('Set Modulation Scheme 2(BPSK) 4(QAM) 8(8QAM)  =   ');

%%%%% OFDM   Parameters  %%%%%%%%%%%%%

zero_pad =0;
block_size = 64;
data_size =  block_size -zero_pad ;
prefix_size = 0  ;
```

```
total_block_size = block_size +  prefix_size ;

E_db =0: 5 :30;

X=zeros(block_size,block_size);
x=randint(1,block_size,mod_scheme);
d=qammod(x,mod_scheme);
d=fft(d);
ber=[];

 for i=1:64
     X(i,i)=d(i);
 end

X_ofdm=d;

p=[];

for i=1:64
h(i) = randn +j*randn;
end

G=h';
H=fft(G);

%%%%%%%%%%%%%%%%Evaluation of the autocovariance matrix of G-
Rgg%%%%%%%%%
co_cal=zeros(64,64);
for i=1:64
   co_cal(i,i)=G(i);
end
co_myu = sum(co_cal, 1)/64;
co_mid = co_cal - co_myu(ones(64,1),:);
sum_co_mid= sum(co_mid, 1);
Rgg = (co_mid' * co_mid- (sum_co_mid'  * sum_co_mid) / 64) / (64 - 1);

%%%%Running for a dozen trials to try and average out the
results%%%%%%%
for n=1:length(E_db)
   for m=1:12
SNR_send=E_db(n);
XFG=X*H;

n1=ones(64,1);
n1=n1*0.000000000000000001i;
noise=awgn(n1,SNR_send);
```

```
vari=var(noise);
N=fft(noise);
Y=XFG+N;

[mean_squared_error_mmse Hmmse]=mmse_calc(X,H,Y,Rgg,vari);

SNR(n)=SNR_send;

mmse_mse(m,n)=mean_squared_error_mmse;


X_rec_ofdm = Y./Hmmse;
X_rec_mod = ifft(X_rec_ofdm);
X_rec = qamdemod(X_rec_mod,mod_scheme);

Hl = H';
Y_ls = X_ofdm.*Hl;
Y_ls=awgn(Y_ls,E_db(n));

hls_est = Y_ls./X_ofdm;;

ms_error=(((abs(hls_est-Hl))/abs(Hl)).^2);

ls_mse(m ,n) = ms_error;

rec_x_ofdm = Y_ls./hls_est;
rec_x_mod = ifft(rec_x_ofdm,64);
rec_x = qamdemod(rec_x_mod,mod_scheme);

end;
[num, ratio] = biterr(X_rec',x);
p = [p ratio];
[num_bit_LS , ratio_bit_LS] = biterr(x,rec_x);
ber = [ ber ratio_bit_LS];
end;
mmse_mse

mmse_mse_ave=mean(mmse_mse);
ls_ms=mean(ls_mse);

% p
% ber

semilogy(SNR,mmse_mse_ave,'k-');
grid on;
hold on
```

```
semilogy(SNR,ls_ms,'b-');
legend('MMSE','LS')
xlabel('SNR in DB');
ylabel('mean squared error');
title('SNR V/S MSE FOR MMSE and LS ESTIMATIONS');

figure
plot(SNR,p,'b-');
grid on;
xlabel('SNR in DB');
ylabel('BER');
title('Bit error Rates');

% scatterplot(H')
% title('Actual Channel')

scatterplot(Hl)
title('Actual Channel');

scatterplot(Hmmse')
title('Estimated Channel using MMSE');

scatterplot(hls_est)
title('Estimated Channel Using LS');

p
ber
```

Constant Modulus algorithm (CMA)

```
clc
clear all;
close all;

D=8000;                 % data size
x1=2*randint(1,D)-1 ;     %input 1
x2=2*randint(1,D)-1;      %input 2

% eyediagram(x1,2)
% eyediagram(x2,2)

X=[x1;x2];      %input marix

%assume channel is a square matrix and real
h=[-0.05 0.07; -0.5 0.72];
% h12=[-0.005 0.009 -0.024 0.854];
```

```
% h21=[-0.05 0.07 -0.5 0.72 ];
% h22=[-0.005 0.009 -0.024 0.854];
% h=[h11 h12; h21 h22];
% h=[1.4511+1.1575i   1.2404+1.2053i;   0.9965+1.2136i   0.7400+1.1820i];

y1=(conv(x1,h(1,1)))+(conv(x2,h(2,1)));
y2=(conv(x1,h(1,2)))+(conv(x2,h(2,2)));

y1=awgn(y1,30);
y2=awgn(y2,30);

y1(1:3)=[];                %first three and last three bits removed
y1(end:-1:end-2)=[];
y2(1:3)=[];
y2(end:-1:end-2)=[];

Y=[y1;y2];  %combined received data with awg noise

% % % % % % % % % %  factors
R=1;              %dispersion constant for CMA
N=11;              %filter order
mu=0.0001;           %algorithm stepsize
w=zeros(N,1);      %filter coefficients
w((N+1)/2)=1;       %initial initialization

regressor=zeros(1,N);   %initial regressor
N1=N-1;

%%%%%%% Firts loops for different receivers second for performing the time
%%%%%%% domain equalization of receviers
for k=1:2
   convdata=Y(k,:);
   x=X(2,:);
   for i=4:length(convdata)
      regressor=[convdata(i) regressor(1:N1)];
      y=regressor*w;
      w=w + mu*y*(R-y^2)*regressor';
      symbol_or_bit_error(i) = sign(y)-x(i-3);
      e(i) = y*(R-y^2);
      fil_out(i-3)=y;
      symbol_or_bit_error(i) = abs(x(i))-abs(y);
   end
   stem(symbol_or_bit_error),title('Symbol or Bit Error')
   eyediagram(fil_out,2)
   figure, plot(abs(e)),title('ERROR PLOT')
```

```
    %%%%%% Mse plot
    for s = 1:length(e)
       MSE(s) = (sum((e(1:s)).^2))/s;
    end

    t = 1:length(e);
    figure, plot(t,10*log(MSE),'b'), title('MSE PLOT')
    pause
    W=fft(w',32)
    A=1./W
    H=fft(h,32)
    freqz(H)
    hold on
    pause
    freqz(A)
    pause
    hold off
end
```

Linear Precoding

```
clc
clear all
N=64;
data1=randint(1,N,4);
data2=randint(1,N,4);
d1=qammod(data1,4);
d2=qammod(data2,4);
var1=var(d1);
var2=var(d2);
D=[d1 d2]';
j=sqrt(-1);
l=29;
j=sqrt(-1);
beta=0.9;
alpha=j*0.432;
gamma=1.1;
Sci=ones([N,N]);
W1=eye([N,N]);
W0=zeros([N,N]);
for i=1:N
    if i~=l
        W0(i,:)=beta.*(Sci(l,:))+alpha.*(Sci(i,:));
    end
    if i==l
```

```
        W0(i,:)=gamma.*(Sci(l,:));
    end
end
Z=zeros([1,N]);

for k=1:N
        fhi=[W0(k,:) Z ; Z W1(k,:) ];
        s(k,:)=fhi*D;
end
s1=s(:,1);
s2=s(:,2);
mod_data1=s1;
extra=0;
ifft_size = 64;
if rem(N,ifft_size)~=0
extra =ifft_size-rem(N,ifft_size);
mod_data1 = [s zeros(1,extra)];
end
s2p = reshape(mod_data1,ifft_size,[]);
data_ifft = ifft(s2p,ifft_size);
p2s = reshape(data_ifft,1,[]);
cp=8;
prefix=p2s(end-(cp-1):1:end);
data_tx1 =[prefix p2s];

mod_data2=s2;
extra=0;
ifft_size = 64;
if rem(N,ifft_size)~=0
extra =ifft_size-rem(N,ifft_size);
mod_data2 = [s zeros(1,extra)];
end
s2p = reshape(mod_data2,ifft_size,[]);
data_ifft = ifft(s2p,ifft_size);
p2s = reshape(data_ifft,1,[]);
cp=8;
prefix=p2s(end-(cp-1):1:end);
data_tx2 =[prefix p2s];

h11 = [0.2710 + 0.1843i];%, -0.4851, 0.7276];
h12 = [-0.0009 + 0.0718i];%, 0.9387, 0.1280];
h21 = [0.2710 + 0.1843i];%, 0.8823, 0.2941];
h22 = [0.2356 - 0.0318i]; %, 0.8729, -0.4364];
H=[h11 h12; h21 h22];

y1=conv(data_tx1,h11)+conv(data_tx2,h12);
```

```
y2=conv(data_tx1,h21)+conv(data_tx2,h22);
y1=awgn(y1,50);
y2=awgn(y2,50);

fft_size=ifft_size;

y1(1:8)=[];
data_rx1 = y1;
s2p_rec = reshape(data_rx1,fft_size,[]);
data_fft = fft(s2p_rec,fft_size);
p2s_rec = reshape(data_fft,1,[]);
rec_data1 = p2s_rec(1:length(p2s_rec)-extra);

y2(1:8)=[];
data_rx2 = y2;
s2p_rec = reshape(data_rx2,fft_size,[]);
data_fft = fft(s2p_rec,fft_size);
p2s_rec = reshape(data_fft,1,[]);
rec_data2 = p2s_rec(1:length(p2s_rec)-extra);

Y1=rec_data1;
Y2=rec_data2;

for k=1:N
   R1(k)=mean ( Y1(k)*conj(Y1(l)'));
end
for k=1:N
   R2(k)=mean ( Y2(k)*conj(Y2(l)'));
end


for k=1:N
   if k~=l
      U=[beta*conj(gamma) 0 ; 0 beta*conj(gamma)];
   end
   if k==l
      U=[abs(gamma)^2 1 ; 1 abs(gamma)^2 ];
   end
   g=(1/var1)*R1(k).*(inv(U));
   g=diag(g);
   Hhat1(:,k)=g;

end


for k=1:N
```

```
    if k~=l
        U=[beta*conj(gamma) 0 ; 0 beta*conj(gamma)];
    end
    if k==l
        U=[abs(gamma)^2 1 ; 1 abs(gamma)^2 ];
    end
    g=(1/var1)*R2(k).*(inv(U));
    g=diag(g);
    Hhat2(:,k)=g;
end
for k=1:2
    HHhat(k,:)=Hhat1(k,:);
    HHhat(k+2,:)=Hhat2(k,:);
end
for k=1:4
    hhat(k,:)=ifft(HHhat(k,:));
    hhat(k,end)=0;
end

for k=1:4
    o=hhat(k,:).*0.02;
    O=fft(o,64);
    freqz(O)
    hold on
    if k==1
        oh=h11;
    end
    if k==2
        oh=h12;
    end
    if k==3
        oh=h21;
    end
    if k==4
        oh=h22;
    end
    aH=fft(oh,64);
    freqz(aH)
    hold off
    pause
    end
```