BANDWIDTH EFFICIENT OFDM COMMUNICATION SYSTEM

By

GC-HASEEB ZAHOOR
NC – TALHA YASIN
NC-ASAD JAVED
PC – RAFAY ANWAR

(TCC- 14)

PROJECT ADVISOR

ASST. PROF MAJ(R) FAZAL

MILITARY COLLEGE OF SIGNALS
NATIONAL UNIVERSITY OF SCIENCES AND TECHNOLOGY
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PROJECT SUPERVISOR

Asst. Prof. Maj.(R) Fazal
Electrical Engineering Department
Military College of Signals
NUST

____

HEAD OF DEPARTMENT

Col Mohsin Bukhari
Electrical Engineering Department
Military College of Signals
NUST

DEPARTMENT OF ELECTRICAL ENGINERING
MILITARY COLLEGE OF SIGNALS, RAWALPINDI

NATIONAL UNIVERSITY OF SCIENCES AND TECHNOLOGY
RAWALPINDI
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Dedicated To Our Parents, Without Their Love And Support We Could Not Have Gone This Far.

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Last but not the least we are also very thankful to our Parents, who helped us both financially and morally in achieving this goal of ours.

ABSTRACT

Due to ever-increasing bandwidth demands in future wireless service, the radio frequency band becomes more and more invaluable. In conventional OFDM there is a lot of wastage of bandwidth in the Guard Interval. We aim to reduce or eliminate the Guard Interval completely to increase bandwidth efficiency. This was suggested by Yi Sun in an IEEE journal paper titled, ‘Bandwidth Efficient Wireless OFDM’. We consider the OFDM that achieves high efficiency of bandwidth usage, allows ISI and ICI to exist, and uses sufficient statistics in symbol demodulation. A one-tap decision feedback equalizer (DFE) is proposed for equalization of the bandwidth-efficient OFDM system Though occupying a narrower bandwidth, the one-tap DFE-based bandwidth-efficient OFDM system achieves lower symbol error rate and higher mutual information than the conventional DFT-based OFDM system.The proposed OFDM system presents a monotonically increasing symbol error rate.

Orthogonal Frequency Division Multiplexing (OFDM) an efficient modulation scheme for frequency selective channels, e.g. how Intersymbol Interference (ISI) can be avoided using orthogonal subcarriers. In order to transfer data over the channel, data is compressed in frames of desired length. At the receiver the signal is sampled. A key issue is to synchronize the system in order to distinguish what frame the samples belong to. Another issue is to avoid interference, which may deteriorate the performance of the modem.

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CHAPTER 1

INTRODUCTION

Communication involves the transmission of information from one point to another through a succession of processes. The source originates a message which is converted by an in put transducer into electrical waveform referred to as base band or message signal. The transmitter modifies the base band signal for efficient transmission. The channel is a medium through which the transmitter output is sent. The signal is contaminated on the channel by undesirable signals usually called noise. The receiver responses the signal received from the channel by undoing the signal modification made at the transmitter and the channel. An output transducer converts the waveform to its original waveform.

1.1 Digital Communication

Digital messages are constructed with a finite number of symbols. For example printed language consists of 26 alphabets 10 letters a spacebar and a number of punctuation marks. Thus text message consists of about 50 symbols. Human speech is also a digital message, because it is made up of finite vocabulary in language. A digital symbol constructed with M symbols is called an M-ary message.

Digital messages are transmitted using a finite set of electrical waveforms. The transmitter according to its input transmits the appropriate waveform. The task of the receiver is to extract the message from a distorted or noisy signal at the channel output. The principle feature of digital communication system is that during a finite interval of time, it sends a waveform from a finite set of possible waveforms, in contrast to an analog communication which sends a waveform from an infinite variety of waveform shapes with theoretically infinite resolution. In a digital communication system, the objective of the receiver is not to reproduce a transmitted waveform with precision, it is instead to determine from a noise perturbed signal, which waveform from a finite set of waveform had been sent by the transmitter.

1.1.1 Baseband Signals

In context of communication system, a signal of primary interest is the message signal delivered by a source of information. This signal is also referred to as a baseband signal, with the term baseband used to designate the band of frequencies representing the message signal. Baseband signal can be of analog or digital type. In analog time signal time takes on values in continuum, and so does the amplitude of the signal. Analog baseband signals arise when a physical waveform such as light or acoustic is converted into an electrical signal. In digital signal on the other hand, both time and signal's amplitude takes on discrete values. The output of a digital computer is an example of baseband signal of digital type. In this particular example only two values commonly represented by 0 and 1 are used so the signal is called a binary signal. Analog signal of primary interest is the transmitted signal, the characterization of which is determined by the type of channel used in the communication system. In the context we speak of baseband or passband. In baseband transmission, as the name implies the band of transmission frequencies supported by the channel closely matches the band of frequencies occupied by the message signal.

1.1.2 Passband Signals

In a passband transmission, the transmission band of the channel is centered at a frequency much higher than the maximum frequency component of the message signal. In the latter case, the transmitted signal is said to be a passband signal, the generation of which is accomplished in the transmission using a process known as modulation.

1.2 Modulation Techniques

Baseband signals are usually modified to facilitate transmission. This process is known as modulation. One of the parameters of a carrier is varied in proportion to the baseband signal. This results in amplitude modulation (AM), phase modulation (PM) or frequency modulation (FM) depending upon the parameters used. At the receiver the modulated signal is passed through a reverse process in order to construct the original signal.

1.2.1 FSK

When the data is transmitted by varying the carrier frequency it is called Frequency Shifting Keying (FSK). FSK modulation is characterized by the information being contained in the frequency of the carrier. FSK is characterized in Cartesian coordinate space, with each axis representing a frequency tone from the M-ary set of orthogonal tones. The amplitude and phase of the carrier remain constant.

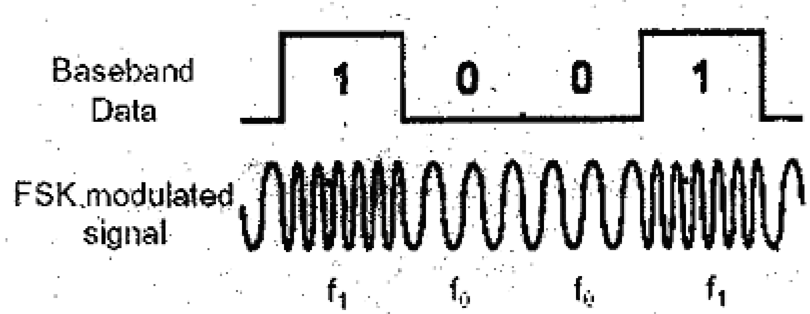


Figure 1.1: Frequency Shift Keying

1.2.2 PSK

In phase shift keying (PSK),it is the phase changes of the carrier that represents the binary 1s and 0s of a digital signal. A binary0 is 180 degree out of phase.

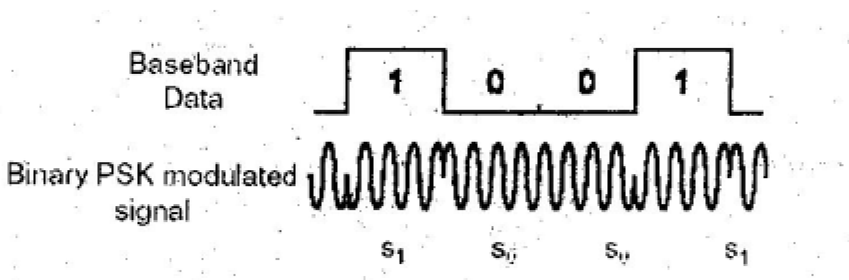


Figure 1.2: Phase Shift Keying

1.2.3 ASK

In amplitude shift keying (PSK),it is the amplitude difference or the presence or absence of a sine wave that represents the binary 1s and 0s of a digital signal. A fixed amplitude carrier represents a binary 1 and its absence is binary 0.

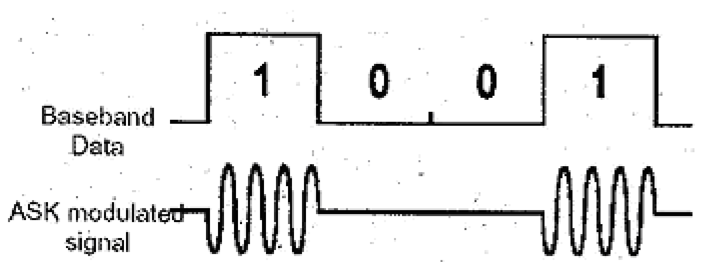


Figure 1.3: Amplitude Shift Keying

CHAPTER 2

OFDM THEORY

2.1 Introduction to OFDM

One way to transmit information in communication systems is to use serial transmission where only one carrier wave utilizes all available bandwidth. However in case of high data rate transmission severe problems with symbol errors in the receiver may occur if the channel has a non-ideal frequency response.

To avoid this problem, data can be transmitted in parallel. Thus, effectively transforming the available frequency channel into a number of sub-channels with a more narrow bandwidth. By this operation the potential problems of non-ideal frequency response can be avoided, due to the almost constant frequency response of the sub-channels that can be expected if the number of sub-channels are chosen sufficiently large. OFDM avoids this problem by sending many low speed transmissions simultaneously. For example, Figure 2.1 shows two ways to transmit the same four pieces of binary data.

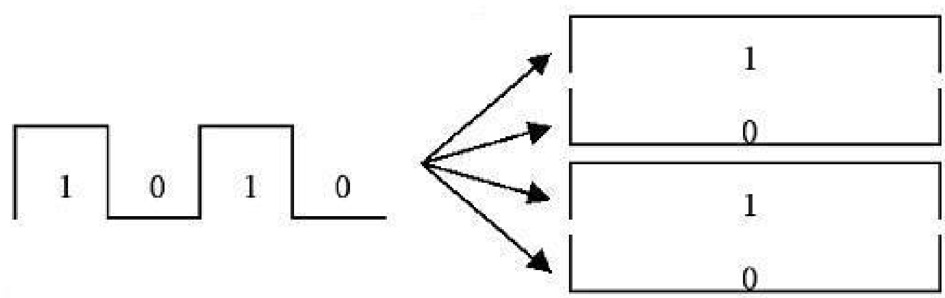


Figure 2.1: Traditional vs OFDM Communication System

Suppose that this transmission takes four seconds. Then, each piece of data in the left picture has a duration of one second. On the other hand, OFDM would send the four pieces simultaneously as shown on the right. In this case, each piece of data has a duration of four seconds.

One spectral efficient technique allowing a large number of sub-channels to co-exist in a given bandwidth is *Orthogonal Frequency Division Multiplexing (OFDM)* is a multicarrier transmission technique used in applications catering to both Wired and Wireless Communications. However, in the wired case, the usage of the term Discrete Multi-Tone is more appropriate. The OFDM technique divides the frequency spectrum available into many closely spaced carriers, which are individually modulated by low-rate data streams. In this sense, OFDM is similar to FDMA (The bandwidth is divided into many channels, so that, in a multi-user environment, each channel is allocated to a user). However, the difference lies in the fact that the carriers chosen in OFDM are much more closely spaced than in FDMA (1kHz in OFDM as opposed to about 30kHz in FDMA), thereby increasing its spectral usage efficiency. The orthogonality between the carriers is what facilitates the close spacing of carriers. The OFDM technique has special features that make it especially robust against so-called intersymbol interference (ISI), which is one of the limiting factors when data is transferred over a wired transmission medium. OFDM is a so-called multicarrier system. The principle of a multicarrier system is to divide the available bandwidth into sub channels, as depicted in figure 2.2, and to transmit the information in parallel on these.

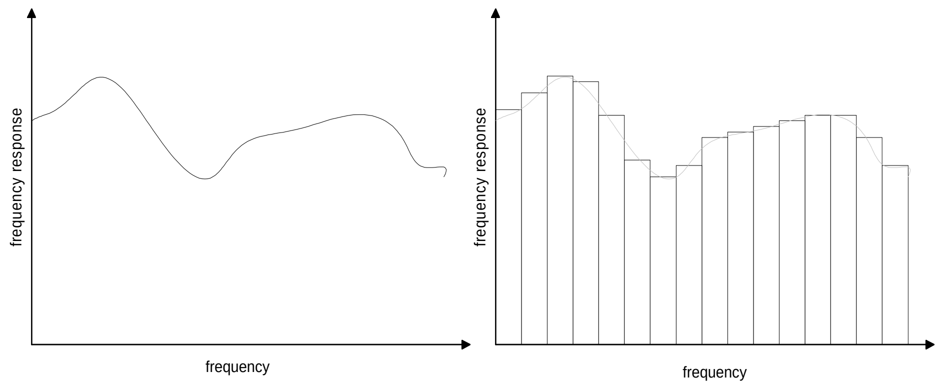


Figure 2.2: Dividing the Bandwidth

2.2 Overview

With the rapid growth of digital communication in recent years, the need for high-speed data transmission has increased. New multicarrier modulation techniques such as OFDM are currently being implemented to keep up with the demand for more communication capacity. Multicarrier communication systems were first conceived and implemented in the 1960s, but it was not until their all-digital implementations with the FFT that their attractive features were unraveled and sparked widespread interest for adoption in various single-user and multiple access (MA) communication standards". The processing power of modern digital signal processors has increased to a point where OFDM has become feasible and economical. OFDM is modulation and multiple-access technique that has been explored for more then 20 years but only recently has it been finding its way into commercial communication system, as the cost of the signal processing it needs has come down.

OFDM, or multitone modulation as it is sometimes called, is presently used in a number of commercial wired and wireless applications. On the wired side, it is used for a variant of digital subscriber line (DSL). For Wireless, OFDM is the basis for several television and radio broadcast applications, including the European digital broadcast television standard, as well as digital radio in North America. OFDM is also used in several fixed wireless systems and wireless local are networks (LAN) products. A system based on OFDM has been developed to deliver mobile broadband data service at data rates comparable to those of wired services such as DSL and cable modems. It was also a candidate for the radio interface in the third generation mobile communication system in Europe.

OFDM enables the creation of a very flexible system architecture that can be used efficiently for a wired range of services, including voice and data. For any mobile system to create a rich user experience, it must provide ubiquitous, fast and user-friendly connectivity. OFDM has several unique properties that make it especially well suited to handle the challenging environmental conditions experienced by mobile wireless data applications.

Before discussing OFDM theory in detail, a brief discussion about orthogonally and discrete Fourier transform is necessary.

2.3 Fourier Transform

The advent of the Fourier Transform eliminated the initial complexity of the OFDM scheme where the harmonically related frequencies generated by Fourier and Inverse Fourier transforms are used to implement OFDM systems. The Fourier transform is used in linear systems analysis, antenna studies, optics, random process modelling, probability theory, quantum physics, and boundary-value problems and has been very successfully applied to restoration of astronomical data.

The Fourier transform, in essence, decomposes or separates a waveform or function into sinusoids of different frequencies which sum to the original waveform. It identifies or distinguishes the different frequency sinusoids and their respective amplitudes.

The Fourier transform of *f(x)* is defined as:

$$F(\omega) = \int_{-\infty}^{\infty} f(x) \cdot e^{-j\omega x} dx \qquad \textbf{(2.1)}$$

and its inverse is denoted by:

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot e^{j\omega x} d\omega \qquad \textbf{(2.2)}$$

However, the digital age forced a change upon the traditional form of the Fourier transform to encompass the discrete values that exist is all digital systems. The modifiedseries was called the **D***iscrete* **F***ourier* **T***ransform* (DFT). The DFT of a discrete-time system, *x(n)* is defined as:

$$\mathrm{X}(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}kn} \qquad 1 \le k \le N \qquad \textbf{(2.3)}$$

and its associated inverse is denoted by:

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} \mathrm{X}(k) \cdot e^{j\frac{2\pi}{N}kn} \qquad 1 \le n \le N \qquad \textbf{(2.4)}$$

However, in OFDM, another form of the DFT is used, called the **F***ast* **F***ourier* **T***ransform* (FFT), which is a DFT algorithm developed in 1965. This "new" transform reduced the number of computations from something on the order of

$$N^2 \text{ To } \frac{N}{2} \cdot \log_2 N. \qquad \textbf{(2.5)}$$

2.4 Orthogonality

In geometry, orthogonal means, "involving right angles" (from Greek *ortho*, meaning *right*, and *gon* meaning *angled*). The term has been extended to general use, meaning the characteristic of being independent (relative to something else). It also can mean: non-redundant, non-overlapping, or irrelevant. Orthogonality is defined for both real and complex valued functions. The functions $\varphi_m(t)$ and $\varphi_n(t)$ are said to be orthogonal with respect to each other over the interval $a < t < b$ if they satisfy the condition:

$$\int_a^b \varphi_m(t)\,\varphi_m^*(t)\,dt = 0, \qquad \text{Where } n \neq m \qquad \textbf{(2.6)}$$

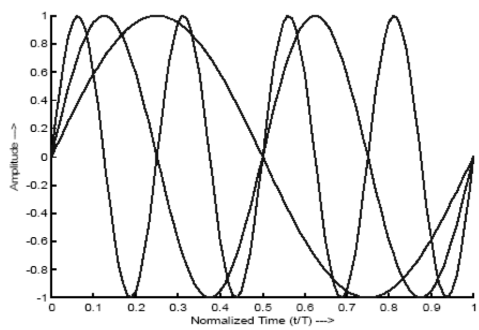


Figure 2.3 : Three Subcarriers within an OFDM symbol

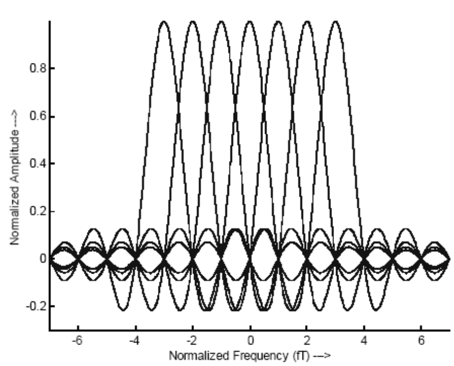


Figure 2.4: Spectra of Individual Sub-Carriers

It is easy to note that

1. The length of the OFDM signal is T.
2. The spacing between the carriers is equal to 1/T.
3. The OFDM symbol-rate is N times the original baud rate.
4. There are N orthogonal sub-carriers in the system.

2.5 OFDM System Model

The purpose of this chapter is basically to discuss in general the different building blocks required for the system to work. A general discrete time OFDM system is shown below. Every block of the OFDM system is discussed one by one.

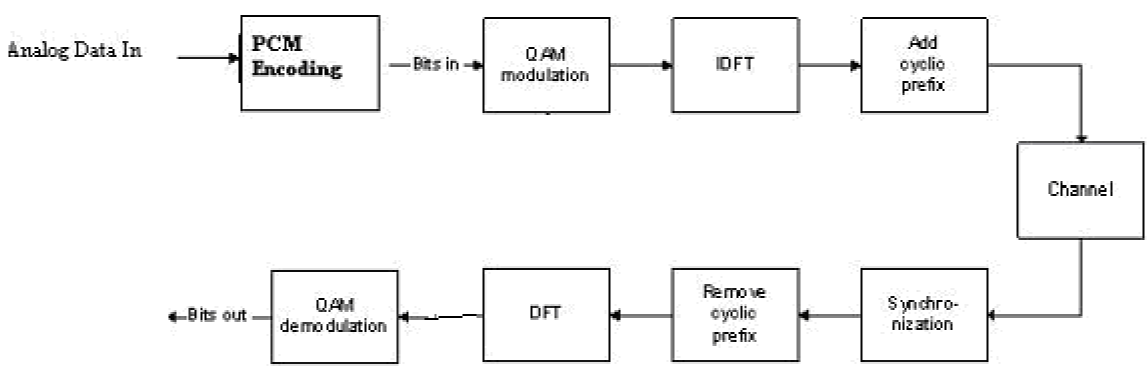


Figure 2.5: OFDM System Model

QPSK

2.5.2 IDFT

Back in the 1960s, the application of OFDM was not very practical. This was because at that point, several banks of oscillators were needed to generate the carrier frequencies necessary for sub-channel transmission. Since this proved to be difficult to accomplish during that time period, the scheme was deemed as not feasible.

But with the arrival of cheap electronics and digital signal processors, the idea of OFDM got a huge boost because DSP's could replace bank of oscillators which were to be used for orthogonal modulation. The expression is

$$\mathrm{X}(k) = \sum_{n=0}^{N-1} x(n) \cdot e^{-j\frac{2\pi}{N}kn} \qquad \textbf{(2.7)}$$

As a property of DFT, the basis functions of discrete Fourier transform are orthogonal; therefore, they can be used as carriers for OFDM.

2.6 Raised Cosine Windowing

The sharp phase transitions caused by phase modulation results in very large side-lobes in the PSD and the spectrum falls off rather slowly (according to a *sinc* function). If the number of sub-carries were increased, the spectrum roll-off will be sharper in the beginning, but gets worse at frequencies a little further away from the 3-dB cut-off frequency. To overcome this problem of slow spectrum roll-off, a windowing may be used to reduce the side-lobe level. The most commonly used window is the **Raised Cosine Window** given by:

$$w(t) = \begin{cases} 0.5 + 0.5\cos(\pi + \pi t/(\beta T_r)), & 0 \le t \le \beta T_r \\ 1.0, & \beta T_s \le t \le T_r \\ 0.5 + 0.5\cos((t\text{-}T_r)\pi/(\beta T_r)), & T_s \le \mathrm{t} \le (1+\beta)T_r \end{cases} \qquad \textbf{(2.8)}$$

Here Tr is the symbol interval which is chosen to be shorter than the actual OFDM symbol duration, since the symbols are allowed to partially overlap in the roll-off region of the raised cosine window. Incorporating the windowing effect, the OFDM symbol can now be represented as:

$$y(t) = 2\,\mathrm{Re}\left\{ w(t) \sum_{n=0}^{N-1} d_n \exp(j2\pi \frac{n}{T} t) \right\}, \text{ for } 0 \le t \le T \qquad \textbf{(2.9)}$$

It must be noted that filtering can also be used as a substitute for windowing, for tailoring the spectrum roll-off. But windowing is preferred to filtering because, it can be carefully controlled. With filtering, one must be careful to avoid rippling effects in the roll-off region of the OFDM symbol. Rippling causes distortions in the OFDM symbol, which directly leads to less-delay spread tolerance.

2.7 OFDM Generation

Based on the previous discussions, the method for generating an ODFM symbol is as follows.

1. First, the *N* input complex symbols are padded with zeros to get *Ns* symbols that are used to calculate the IFFT. The output of the IFFT is the basic OFDM symbol.
2. Based on the delay spread of the multi-path channel, a specific guard-time must be chosen (say Tg). A number of samples corresponding to this guard time must be taken from the beginning of the OFDM symbol and appended at the end of the symbol. Likewise, the same number of samples must be taken from the end of the OFDM symbol and must be inserted at the beginning.

3. The OFDM symbol must be multiplied with the raised cosine window to remove the power of the out-of-band sub-carriers.
4. The windowed OFDM symbol is then added to the output of the previous OFDM symbol with a delay of Tr, so that there is an overlap region of *r T* between each symbol.

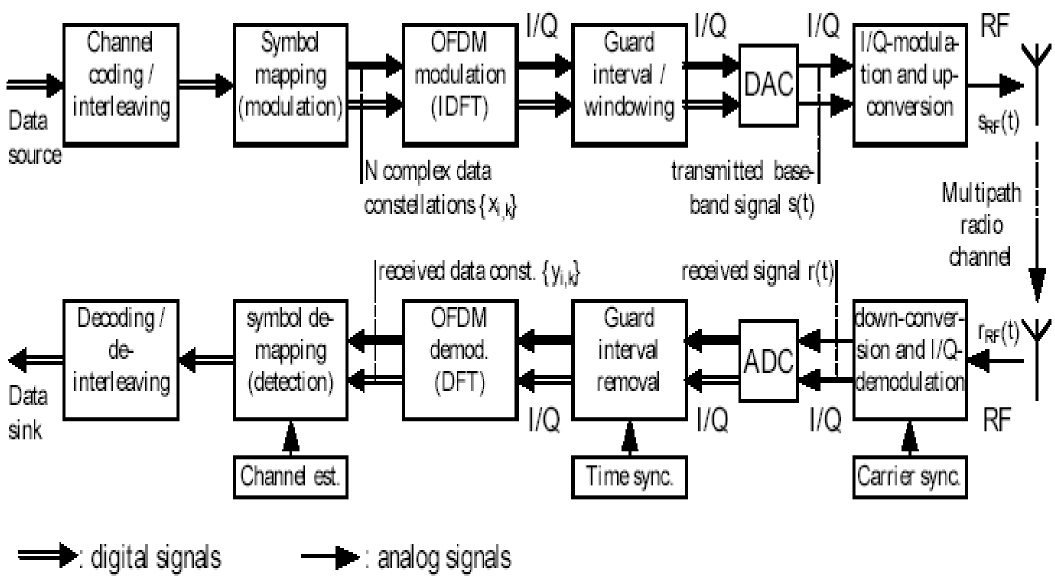


Figure2.6 : OFDM Sytem Block Diagram

2.8. Advantages of OFDM

OFDM possesses some inherent advantages for Wireless Communications. This section glances on few of the most important reasons on why OFDM is becoming more popular in the Wireless Industry today.

2.8.1. Multi-path Delay Spread Tolerance

As discussed earlier, the **increase in the symbol time** of the OFDM symbol by *N* times (*N* being the number of sub-carriers), leads to a corresponding increase in the effectiveness of OFDM against the ISI caused due to multi-path delay spread. Further, using the cyclic extension process and proper design, one can completely eliminate ISI from the system.

2.8.2. Effectiveness against Channel Distortion

In addition to delay variations in the channel, the **lack of amplitude flatness** in the frequency response of the channel also causes ISI in digital communication systems. A typical example would be the twister-pair used in telephone lines. These transmission lines are used to handle voice calls and have a poor frequency response when it comes to high frequency transmission. In systems that use single-carrier transmission, an **equalizer** might be required to mitigate the effect of channel distortion. The complexity of the equalizer depends upon the severity of the channel distortion and there are usually issues such as equalizer non-linearities and error propagation etc that cause additional trouble. In OFDM systems on the other hand, since the bandwidth of each sub-carrier is very small, the amplitude response over this narrow bandwidth will be basically flat (of course, one can safely assume that the phase response will be linear over this narrow bandwidth). Even in the case of extreme amplitude distortion, an equalizer of very simple structure will be enough to correct the distortion in each sub-carrier.

2.8.3. Throughput Maximization (Transmission at Capacity)

The use of sub-carrier modulation improves the flexibility of OFDM to channel fading and distortion makes it possible for the system to transmit at maximum possible capacity using a technique called **channel loading**. Suppose the transmission channel has a fading notch in a certain frequency range corresponding to a certain sub-carrier. If we can detect the presence of this notch by using channel estimation schemes and assuming that the notch doesn't vary fast enough compared to the symbol duration of the OFDM symbol, it can be possible to change (scale down/up) the modulation and coding schemes for this particular sub-carrier (i.e, increase their robustness against noise), so that capacity as a whole is maximized over all the sub-carriers. However, this requires the data from channel-estimation algorithms. In the case of single-carrier systems, nothing can be done against such fading notches. They must somehow survive the distortion using error correction coding or equalizers.

2.8.4. Robustness against Impulse Noise

Impulse noise is usually a burst of interference caused usually caused in channels such as the return path HFC (Hybrid-Fiber-Coaxial), twisted-pair and wireless channels affected by atmospheric phenomena such as lightning etc. It is common for the length of the interference waveform to exceed the symbol duration of a typical digital communication system. For example, in a 10 MBPS system, the symbol duration is 0.1 *s* □, and a impulse noise waveform, lasting for a couple of micro-seconds can cause a burst of errors that cannot be corrected using normal error-correction coding. Usually complicated Reed-Solomon codes in conjunction with huge interleaves are used to correct this problem.OFDM systems are inherently robust against impulse noise, since the symbol duration of an OFDM signal is much larger than that of the corresponding single-carrier system and thus, it is less likely that impulse noise might cause (even single) symbol errors. Thus,complicated error-control coding and interleaving schemes for handling burst-type errors are not really required for OFDM Systems simplifying the transceiver design.

2.8.5. Frequency Diversity.

OFDM is the best place to employ Frequency Diversity. In fact, in a combination of OFDM and CDMA called the MC-CDMA transmission technique, frequency diversity is inherently present in the system. (i.e, it is available for free).

2.9 Channel

It is known that the performance of any wired or wireless system's performance is affected by the medium of propagation, namely the characteristics of the *channel*. In telecommunications in general, a channel is a separate path through which signals can flow. In the ideal situation, a direct line of sight between the transmitter and receiver is desired. But alas, it is not a perfect world; hence it is imperative to understand what goes on in the channel so that the original signal can be reconstructed with the least number of errors. Usually, the received signal is a combination of attenuated, reflected, refracted, and diffracted replicas of the transmitted signal. To top it off, the channel adds noise and if the receiver is in motion, then the Doppler effect has to be taken into consideration.

The system operates over a time-varying frequency selective channel. By placing pilots at certain frequencies, the behavior of the channel can be examined. Figure 3.6 shows an estimated frequency response of the channel when the filters are not connected. As seen in figure 2.7, the frequencies close to half the sampling frequency are strongly attenuated. This attenuation is due to the anti-alias filter in the DSP. Consequently, the subchannels close to half the sampling frequency are not suitable for transmission.

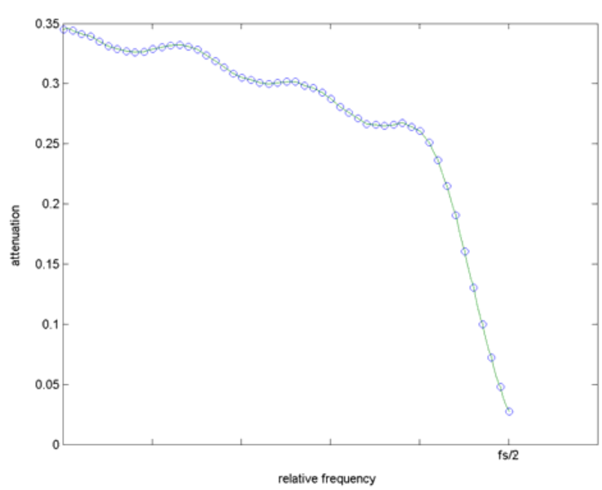


Figure 2.7: Channel Attenuation

To address this problem, subchannel extinguishing is used. That is, zeros are transmitted on the attenuated channels. The obvious disadvantage of this method is that the effective data-rate decreases and some allocated bandwidth is not used.

2.9.1Time-Dispersive Channel

The influence of the time-variant, multipath fading radio channel is expressed by its (lowpass equivalent) impulse response $h(\tau,t)$ plus AWGN $n(t)$:

$$r(t) = h(\tau,t) * s(t) + n(t) = \int_0^{\tau_{max}} h(\tau,t)s(t-\tau)d\tau + n(t) \qquad (2.9)$$

The range of integration in this convolutional integral (* denotes convolution) has been limited to [0, τmax], because the channel impulse response is zero elsewhere. Excess delay $\tau = 0$ of the channel is defined as the delay time at which the first wave arrives at the receiver. Thus, transmit and receive time instants are mathematically defined equal. τmax is the maximum excess delay of the channel. Two assumptions are made to simplify the derivation of the received signal. The channel is considered quasi-static during the transmission of the k-th OFDM symbol, thus $h(\tau,t)$ simplifies to $hk(\tau)$. Furthermore, we define the maximum excess delay $\tau max < Tguard$. Therefore, there is no interference of one OFDM symbol on the effective period of the consecutive one . I.e., inter-symbol-interference (ISI) is suppressed in case of sufficiently accurate time synchronization.

2.9.2 Rayleigh Channel

2.9.3 Dopler Effects

2.9.4 Slow Fading

2.9.5 Fast Fading

CHAPTER 3

SYNCHRONIZATION AND CYCLIC PREFIX

3.1 Inter symbol Interference

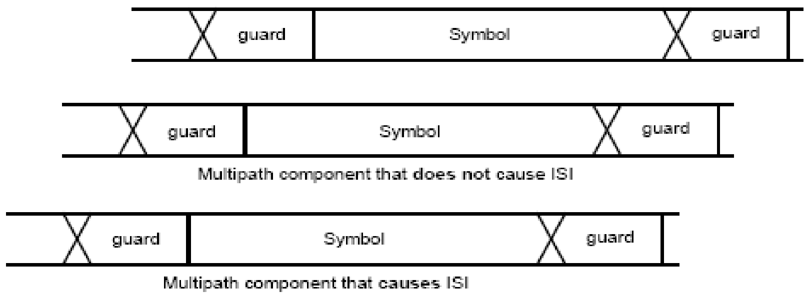
As communication systems evolve, the need for high symbol rates becomes more apparent. However, current multiple access with high symbol rates encounter several multipath problems, which leads to ISI. An *echo* is a copy of the original signal delayed in time. ISI takes place when echoes on different-length propagation paths result in overlapping received symbols. Problems can occur when one OFDM symbol overlaps with the next one. There is no correlation between two consecutive OFDM symbols and therefore interference from one symbol with the other will result in a disturbed signal

In addition, the symbol rate of communications systems is practically limited by the channel's bandwidth. For the higher symbol rates, the effects of ISI must be dealt with seriously. Several channel equalization techniques can be used to suppress the ISI's caused by the channel. However, to do this, the CIR – channel impulse response, must be estimated.

Recently, OFDM has been used to transmit data over a multi-path channel. Instead of trying to cancel the effects of the channel's ISIs, a set of *sub-carriers* can be used to transmit information symbols in parallel *sub-channels* over the channel, where the system's output will be the sum of all the parallel channel's throughputs. This is the basis of how OFDM works. By transmitting in parallel over a set of sub-carriers, the data rate per sub-channel is only a fraction of the data rate of a conventional single carrier system having the same output. Hence, a system can be designed to support high data rates while deferring the need for channel equalizations.

3.2 Guard Time and Cyclic Extension

One of the main advantages of OFDM is its effectiveness against the multi-path delay spread frequently encountered in Mobile communication channels. The reduction of the symbol rate by N times, results in a proportional reduction of the relative multi-path delay spread, **relative** to the symbol time. To completely eliminate even the very small ISI that results, a guard time is introduced for each OFDM symbol. The guard time must be chosen to be larger than the expected delay spread, such that multi-path components from one symbol cannot interfere with the next symbol. If the guard time is left empty, this may lead to **inter-carrier interference (ICI),** since the carriers are no longer orthogonal to each other. To avoid such a cross talk between sub-carriers, the OFDM symbol is **cyclically extended** in the guard time. This ensures that the delayed replicas of the OFDM symbols always have an integer number of cycles within the FFT interval as long as the multi-path delay spread is less than the guard time.



. Figure 3.1 : Guard Time and Cyclic Extension - Effect of Multi path

3.3 Cyclic Prefix

By adding a cyclic prefix (CP), the interchannel interference (ICI) within an OFDM frame can be avoided. The cyclic prefix is simply a copy of the *M* last symbols of the *N* samples placed first, making the signal appear as periodic in the receiver, see figure 3.2. The received signal, *r(k)*, consisting of *y(n)* and the cyclic prefix, is demodulated using the DFT forming *Y(n)*.

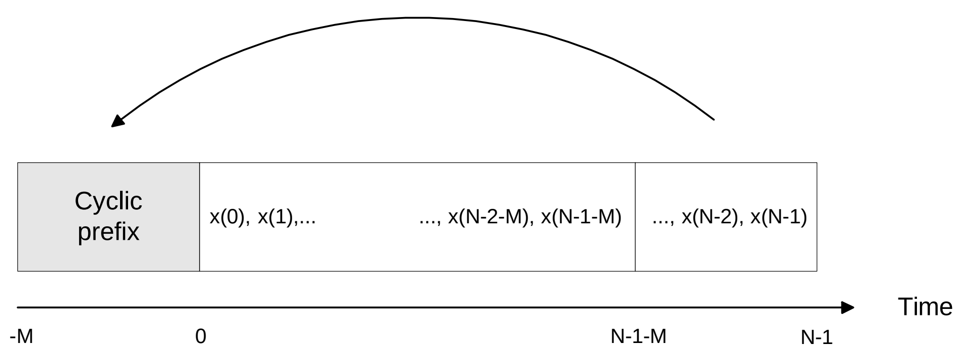


Figure 3.2: The Cyclic Prefix

Another advantage with the cyclic prefix is that it serves as a buffer between consecutive OFDM frames. This is similar to adding guard bits, which means that the problem with ISI also will disappear.

3.4 OFDM Mathematical Model

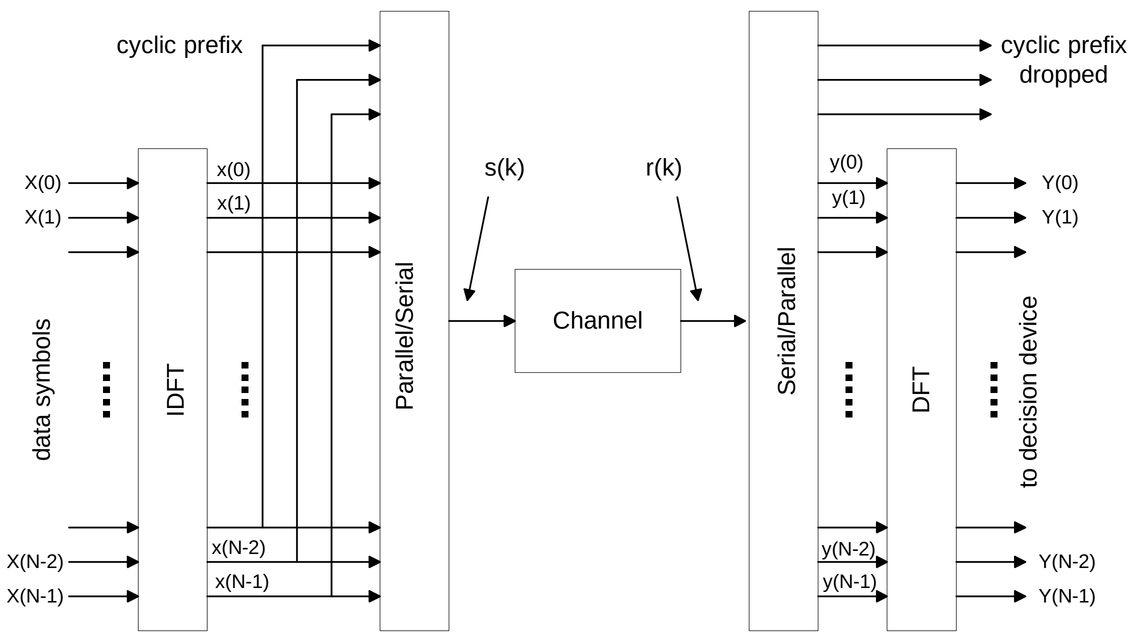


Figure 3.3: A Discrete time OFDM Mathematical Model

Figure 2.7 shows a block diagram of a discrete time OFDM system, where N complex-valued data symbols $X(n)$ modulate N orthogonal carriers using the IDFT forming $x(n)$. The transmitted OFDM signal, $s(k)$, multiplexes N low-rate data streams, each experiencing an almost flat fading channel when transmitted. By adding a cyclic prefix (CP), the interchannel interference (ICI) within an OFDM frame can be avoided. The cyclic prefix is simply a copy of the M last symbols of the N samples placed first, making the signal appear as periodic in the receiver, see figure 2.8. The received signal, $r(k)$, consisting of $y(n)$ and the cyclic prefix, is demodulated using the DFT forming $Y(n)$.

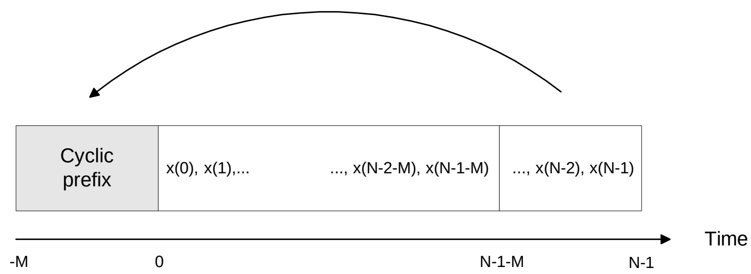


Figure 3.4: The Cyclic Prefix

3.6 Obtaining and Maintaining Orthogonality using the IDFT and a CP

The N complex-valued symbols *X(n), 0≤ n ≤ N-1*, modulate *N* orthogonal carriers using the IDFT:

$$x(k) = \sum_{n=0}^{N-1} X(n) e^{+j2\pi k \frac{n}{N}} \qquad ,0 \le k < N-1 \qquad \textbf{(3.1)}$$

As is well known, the basis functions of the IDFT are orthogonal. By adding a cyclic prefix, the transmitted signal will appear periodic:

$$s(k) = \begin{cases} x(k+N) & ,-M \le k < 0 \\ x(k) = \sum_{n=0}^{N-1} X(n) e^{+j2\pi k \frac{n}{N}} & ,0 \le k < N-1 \end{cases} \qquad \textbf{(3.2)}$$

The received signal can be written as:

$$r(k) = s(k) * h(k) + e(k) \quad ,0 \le k < N-1 \qquad \textbf{(3.3)}$$

where $*$ denotes convolution.

If the cyclic prefix added is longer than the impulse response of the channel, the linear convolution in the channel will, from the receiver's point of view, appear as a circular convolution. This is shown below for any subchannel n, $0 \leq n \leq N-1$.

$$Y(n) = DFT(y(k)) = DFT(IDFT(X(n)) \otimes h(k) + e(k))$$
$$= X(n)DFT(h(k)) + DFT(e(k)) = X(n)H(n) + e'(n) \qquad , 0 \leq k \leq N-1 \quad \textbf{(3.4)}$$

Equation (2.16), where ⊗ denotes circular convolution and e'(n) = DFT(e(k)), shows that there is no interference between the subchannels, i.e. the ICI is zero. Hence, by adding the cyclic prefix, orthogonality is maintained through transmission. Another advantage of using a cyclic prefix is that it acts as a guard space between adjacent OFDM frames, thus making the problem with interframe interference disappear. This holds as long as the cyclic prefix is at least as long as the length of the channel impulse response. The obvious drawback of using a cyclic prefix is that the amount of data that has to be transmitted increases.

3.7 Synchronization in OFDM

The basic idea behind the synchronization is that the first and the last part of each OFDM symbol have a strong correlation due to the addition of cyclic prefix. There are three steps in synchronization.

In order for the data transmission to be able to work correctly the transmitter and the receiver have to be synchronized. This means that the receiver has to know where each frame starts and ends. One way of doing this is to use the cyclic prefix that was added in the transmitter. This will make the frame appear cyclic which can be used in the receiver. The fact that there are great correlation between the first and the last parts of the frame is used in the receiver. This is explained in the figure below.

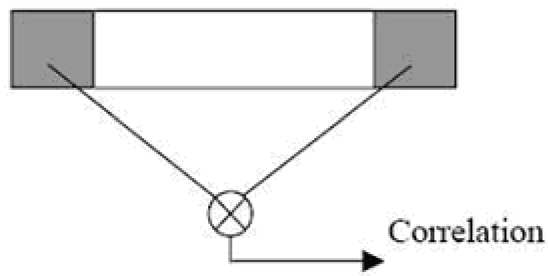


Figure 3.5: The Correlation in OFDM

Assume that an interval of T consecutive samples of the OFDM signal r is observed. It can also be assumed that these samples contain one entire OFDM frame. Since the channel delay is unknown to the receiver a Maximum Likelihood (ML) estimate of the delay Θ is obtained as:

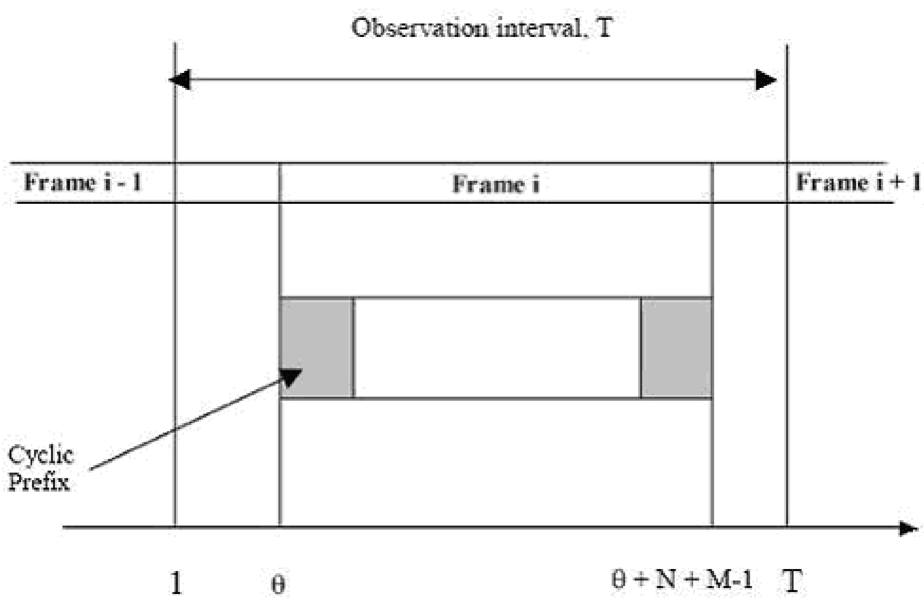


Figure 3.6: The Synchronization in OFDM Frame

$$\hat{\theta} = \arg_{\theta} \max\{|\gamma(\theta)| - \rho\phi(\theta)\} \qquad 1 \le \theta < T - N - M + 1$$

where

$$\gamma(\theta) = \sum_{k=\theta}^{\theta+M-1} r(k) r^*(k+N)$$

$$\phi(\theta) = \frac{1}{2} \sum_{k=\theta}^{\theta+M-1} |r(k)|^2 + |r(k+N)|^2$$

$$\rho = \frac{SNR}{SNR+1}$$

(3.5)

3.7.1 Energy Detection for the Very First Frame

The synchronization process starts with the receiver searching for the first frame. The receiver continuously checks for the received stream of data. As soon as the received data value crosses a certain threshold, it is an indication that the energy is detected and the index is noted.

3.7.2 Observation Interval

As soon as the very first frame is detected, an observation vector is generated around the detected frame. The length of this observation interval is

Length = 2^(length of cyclic prefix) + no. of channels + (length of cyclic prefix)

3.7.3 Maximum Likelihood Estimation

Once the observation interval has been created, the task is now to find the starting point of the frame using the maximum likelihood estimation.

$$\hat{\theta}_{ML} = \arg\max_{\theta}\{|\gamma(\theta)| - \rho\Phi(\theta)\} \qquad ,1 \le \theta < T - N - M + 1 \qquad \textbf{(3.6)}$$

where

$$\gamma(\theta) = \sum_{k=\theta}^{\theta+M-1} r(k) r^*(k+N) \qquad \textbf{(3.7)}$$

$$\Phi(\theta) = \frac{1}{2}\sum_{k=\theta}^{\theta+M-1} |r(k)|^2 + |r(k+N)|^2 \qquad \textbf{(3.8)}$$

After this, the point where the next frame should start is calculated. The important thing here to note is that once the first frame is detected and synchronized correctly, the starting point of all other frames is easy calculated.

3.8 Requirements for a Practical OFDM System

The complete system requires additional blocks.Our system is designed for wireless multipath channels so it is *'Packet'* or *'Burst'* oriented.The following diagram shows the steps needed for synchronization

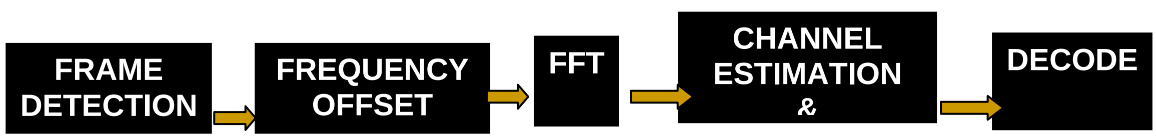


Figure 3.7 OFDM system

3.8.1 Frame Detection/Timing Synchronization

Accurate detection of the start of frame is required to detect the start of OFDM symbols.Only in that case the carriers are accurately mapped. An error in timing in conventional OFDM with Guard Interval induces a phase shift defined by:

$$\varphi_i = 2\pi f_i \tau$$

However in our **BEOFDM** system, the consequence is ISI in the absence of GI.Thus a ***ROBUST*** synchronization scheme is required.

3.9 Correlation using special Training Symbol

Our timing algorithm consists of correlating a known training symbol with the input at the receiver.The peaks in correlation indicate the start of frame.

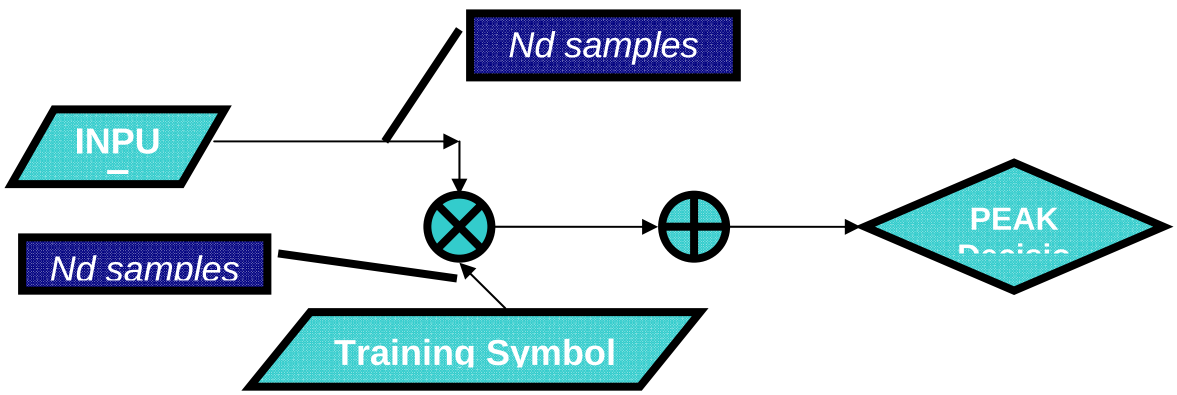


Figure 3.8 correlation using special training symbol

3.9.1 Effects of incorrect frame detection on Conventional OFDM with Guard Interval/CP

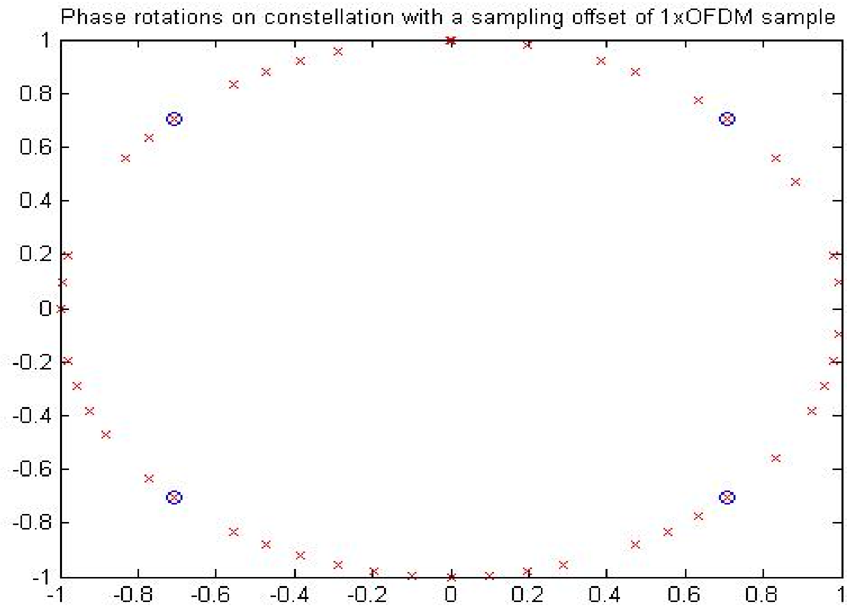


Figure 3.9

3.9.2 Effects of incorrect frame detection on Bandwidth Efficient OFDM without GI/CP

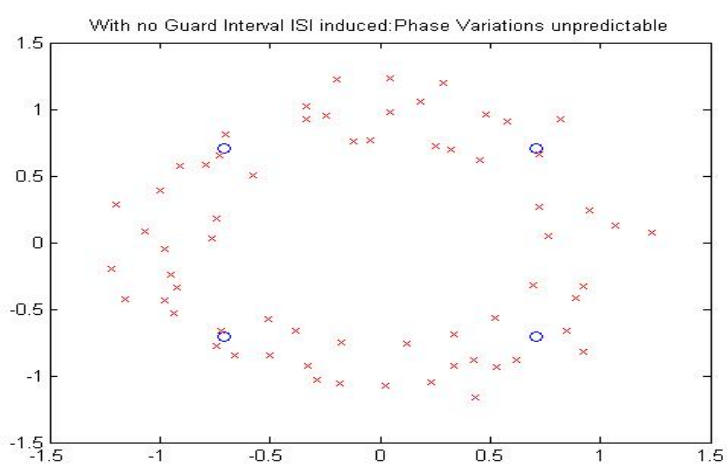


Figure 3.10

3.10 Frequency Synchronisation

Orthogonal Frequency Division Multiplexing (OFDM)has gained considerable interest in recent years . One of the biggest problem of OFDM is the synchronization.To ensure ISI free detection, precise timing information (regarding where the symbol boundary lies) is needed so that an uncorrupted portion of the received OFDM symbol can be sampled for FFT. To ensure subcarrier orthogonality and hence ICI-free detection, the transmitter receiver carrier frequency offset must be estimated and compensated.

The ISI-free received signal samples are expressed as

$$y(l) \quad = \quad s(l)e^{j2\pi(\Delta f_c T)\frac{l}{N}} + z(l) \qquad \textbf{(3.9)}$$

where l is the (time-domain) sample index; $y(l)$ is the received signal sample; N is the total number of subcarriers; $z(l)$ is the noise sample; and the desired signal sample
$s(l)$ is expressed as

$$s(l) \quad = \quad \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} U(k)C(k)e^{j2\pi k\frac{l}{N}} \qquad \textbf{(3.10)}$$

where k is subcarrier index, $U(k)$ is the data modulated on the subcarrier, and $C(k)$ is subcarrier frequency response.
Correlating samples at a distance T (i.e. N samples) apart, we have

$$J = \sum_{l=0}^{N-1} y(l)y^*(l+N) \qquad \textbf{(3.11)}$$

and the fraction part of the frequency offset is estimated as

$$\hat{\rho} = \frac{1}{2\pi} \arg\left[J^*\right] \qquad \textbf{(3.12)}$$

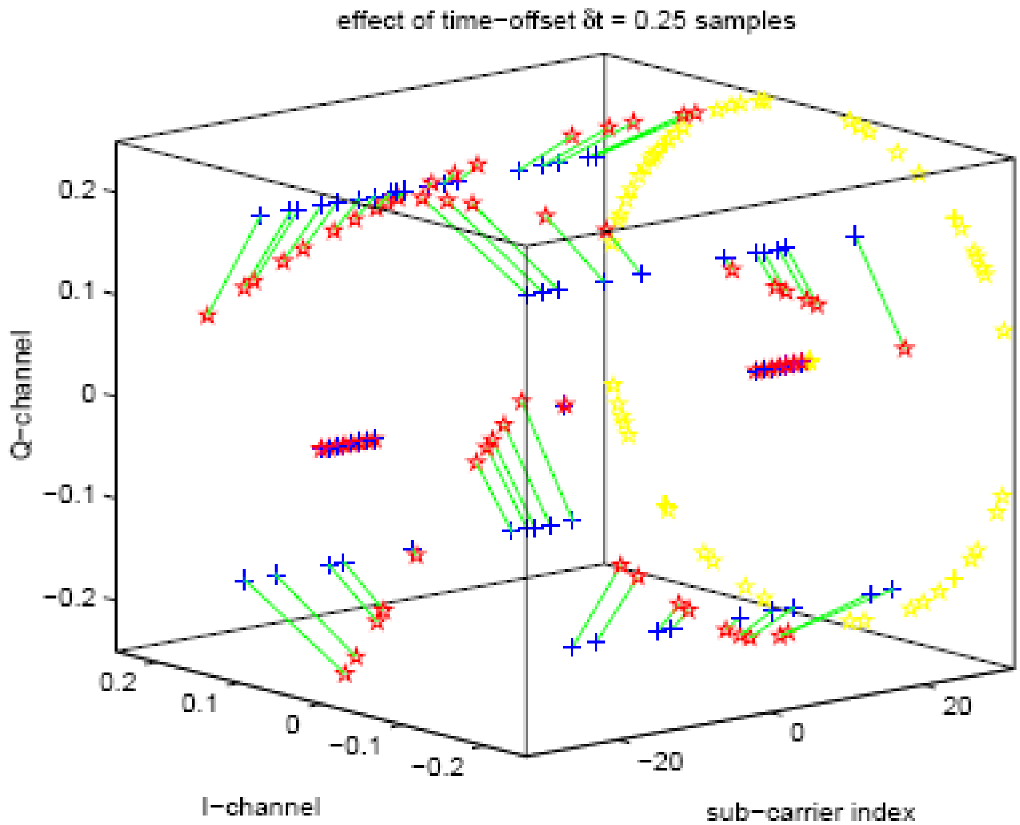


Figure 3.11: Visualization of the influence of an FFT timing offset on the demodulated signal constellations. A linearly increasing phase rotation is observed with increased frequency distance to the center frequency. '+' indicate QPSK constellations without the influence of a timing-offset; '.' depict the rotated data symbols.

3.10.1 Carrier Synchronization Error

Frequency offsets are typically introduced by a (small) frequency mismatch in the local oscillators of the transmitter and the receiver. Doppler shifts can be neglected in indoor environments. The impact of a frequency error can be seen as an error in the frequency instants, where the received signal is sampled during demodulation by the FFT. Figure 4-8 depicts this two-fold effect. The amplitude of the desired sub-carrier is reduced ('+') and inter-carrier-interference ICI arises from the adjacent sub-carriers ('{'). Mathematically, a carrier offset can be accounted for by a frequency shift δf and a phase offset θ in the lowpass equivalent received signal

$$r'(t) = r(t)e^{j(2\pi\delta f t+\theta)} .$$

(3.13)

With eq. (4-9) we obtain

$$y_{i,k} = \frac{1}{T_{FFT}} \int_{t=kT}^{kT+T_{FFT}} r(t)e^{j(2\pi\delta f t+\theta)} e^{-j2\pi i(t-kT)/T_{FFT}}\, dt =$$

$$e^{j2\pi\theta} \frac{1}{T_{FFT}} \int_{t=kT}^{kT+T_{FFT}} \left[\int_{\tau=0}^{\tau_{max}} h(\tau)s(t-\tau)d\tau + n(t) \right] e^{j2\pi\delta f t} e^{-j2\pi i(t-kT)/T_{FFT}}\, dt$$

(3.14)

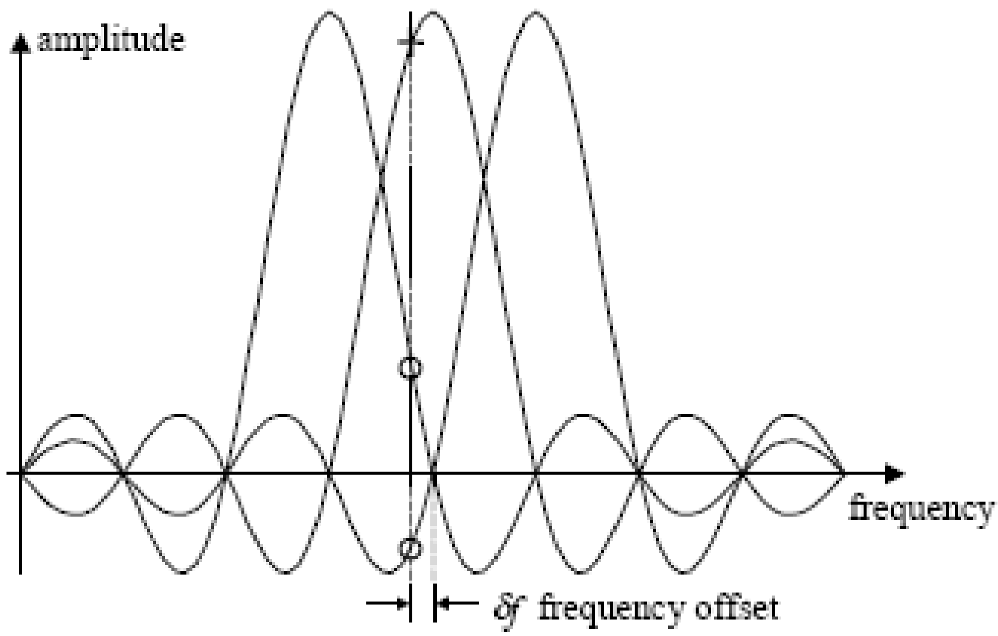


Figure 3.12: Inter-carrier-interference (ICI) arises in case of a carrier synchronization error.The figure illustrates the spectra of three individual sub-carriers. These spectra are superimposed in the OFDM signal spectrum.

Repeating the derivation leading to eq. (4-13), the received constellation points become

$$y_{i,k} = e^{j(\theta+2\pi\delta f kT)} \sum_{i'=-N/2}^{N/2-1} x_{i',k} h_{i',k} \frac{1}{T_{FFT}} \int_{u=0}^{T_{FFT}} e^{-j2\pi\left(\frac{i-i'}{T_{FFT}}-\delta f\right)u}\, du + n_{i,k} .$$

(3.15)

Due to the frequency error, the integral is not equal zero for $i \neq i'$, neither it is one for $i = i'$, as in the idealized case above. I.e., the orthogonality between sub-carriers has been partly lost. The evaluation of this expression yields two terms. The first term (for $i = i'$)

accounts for equal phase rotation and attenuation of all sub-carriers, the second one (for $i \neq i'$) describes the ICI.

$$y_{i,k} = e^{j(\theta + 2\pi\delta f kT)} x_{i,k} h_{i,k} \frac{1}{T_{FFT}} \int_{u=0}^{T_{FFT}} e^{j2\pi\delta f u} du +$$

$$e^{j(\theta + 2\pi\delta f kT)} \sum_{\substack{i'=-N/2 \\ i' \neq i}}^{N/2-1} x_{i',k} h_{i',k} \frac{1}{T_{FFT}} \int_{u=0}^{T_{FFT}} e^{-j2\pi\left(\frac{i-i'}{T_{FFT}} - \delta f\right)u} du + n_{i,k}$$

(3.16)

These expressions are valid for a frequency-offset $\delta f < 0.5$ SC. For larger offsets, the transmitted data symbols $x_{i,k}$ would get shifted by one or more positions in the frequency-direction. I.e., the data symbol of the i-th transmitted SC would appear at the ($i + \delta fi$)-th SC at the receiver, where $\delta fi = \mathrm{round}(\delta f/F)$ is the integer part of the frequency-error in sub-carriers. The ICI term can be seen as an additional noise term and can thus be represented as a degradation of SNR. The amount of degradation has been evaluated by Pollet *et al.* [10] for AWGN channels and by Moose [11] for dispersive fading channels (see also [7]). Frequency-offsets up to 2 % of the sub-carrier spacing F are negligible, according to their results. Even 5–10 % can be tolerated in many situations.

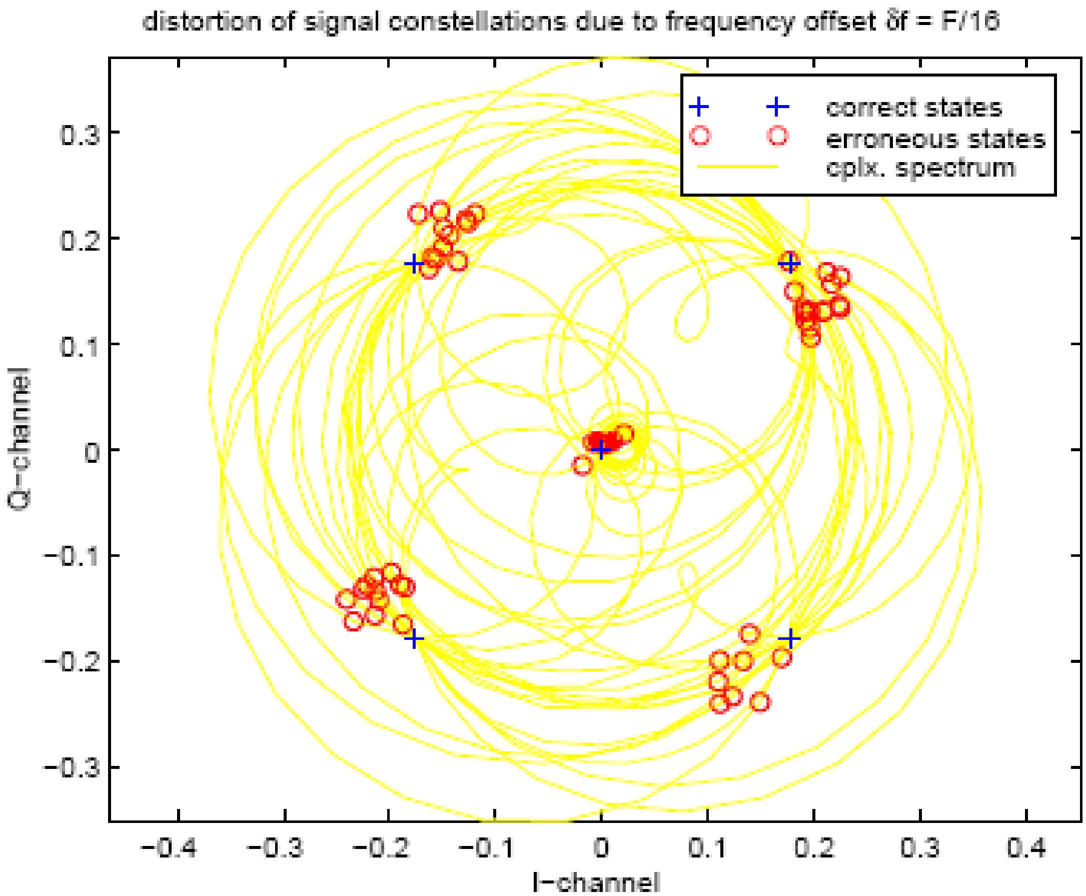


Figure 3.13: Phase rotation due to carrier offset of 1/16 of the sub-carrier spacing. The received signal constellations distorted by ICI are shown.

CHAPTER 4

BANDWIDTH EFFICIENT OFDM

In an OFDM system for wireless communications, a guard interval longer than channel delay has to use to avoid interchannel interference (ICI) and intersymbol interference (ISI), and so the multiplexed subchannels can be decoupled by the discrete Fourier transform (DFT). However, this ICI- and ISI-free OFDM system reduces the efficiency of bandwidth usage and prevents the use of sufficient statistics in channel equalization. In this paper, we consider the OFDM system that achieves one hundred percent efficiency of bandwidth usage and uses sufficient statistic. A one-tap decision feedback equalizer (DFE) is proposed. It is demonstrated that though occupying narrower bandwidth, the one-tap DFE based, ICI and ISI OFDM system achieves lower symbol error rate and higher mutual information between input of decision device and transmitted symbols.

4.1. Introduction

In an OFDM system , the entire channel is divided into many narrow subchannels, which are utilized in parallel transmission, thereby increasing the symbol duration and reducing ISI. Therefore, OFDM is an effective technique for combating multipath fading and for high-bit-rate transmission over mobile wireless channels. In addition, an OFDM system can achieve adaptive allocation of transmission load in different subchannels to achieve optimum transmission rate. Besides, because of the longer duration of symbols, the OFDM system can alleviate the effect of impulse noise. When an OFDM system is designed such that there is neither ICI nor ISI, the computationally efficient FFT can be applied to decouple subchannels and the channel equalization is accomplished simply by a complex scalar for each subchannel [5].

There are two major drawbacks for ICI- and ISI-free OFDM systems. First, a guard interval longer than channel delay has to use in each OFDM symbol period, thus resulting a considerable loss in the efficiency of bandwidth utilization. Second, FFT based demodulation methods, although computationally simple, do not use sufficient statistics for channel equalization, which degrades performance.

Although there are quite a few studies in channel equalization for OFDM systems in literature , most of them are for the DFT-based OFDM systems that employ a guard interval or a cyclic prefix. They either equalize each subchannel independently or equalize an equivalent baseband channel to decrease the effective channel delay.

We consider the OFDM systems without a guard interval and with one hundred percent efficiency of bandwidth usage. A simple one-tap DFE is proposed for channel equalization. It is shown that this one-tap DFE based OFDM system performs better in both SER and the mutual information between the input of decision device and transmitted symbols.

4.2. Conventional OFDM system

4.2.1. OFDM signal model

Consider an OFDM system that transmits N symbols $s_n(i)$ in the ith OFDM symbol period through N subchannels of subcarrier spacing 1 / (NT_f). The transmitted baseband OFDM signal is expressed as

$$x(t) = \sqrt{\frac{T_s}{NT_f}} \sum_{i=-\infty}^{\infty} \sum_{n=0}^{N-1} s_n(i) v(t - iNT) e^{j \frac{2\pi \left(n - \frac{N-1}{2}\right)(t - iNT)}{NT_f}} \quad \textbf{(4.1)}$$

where T is the data symbol period, NT is the OFDM symbol period, 1 / T_f is approximately the total bandwidth of the OFDM signal, and $v(t)$ is the OFDM symbol pulse shaping filter. We consider the following baseband radio channel model,

$y(t) = \sum_{j=1}^{d} \alpha_j(t) x(t - \tau_j) + n(t)$ where d is the number of paths, $\alpha_j(t)$ is the fading coefficient of the jth path with delay τ_j, and $n(t)$ is additive white Gaussian noise. Assume that $\tau_1 = 0$, $\tau_j > 0$ for $j > 1$, and $\tau_{\max} = \max \tau_j < NT$. Within the OFDM symbol period [0, NT), totally N_s samples are taken with sampling period T_s where the first sample is taken at time c such that $0 \le c \le NT - N_sT_s$. In the lth OFDM symbol period, the kth sample is taken at time $t = lNT + c + kT_s$ for $k = 0, 1, \ldots, N_s - 1$. Consider the samples in the tth OFDM symbol period (from now on, t denotes the index of OFDM symbols) and omit frequency shift. Assume that timing is perfect and c is properly chosen so that OFDM symbol $t + 1$ is not involved, then only OFDM symbols $i = t - 1$ and t are involved in the data $y_k(t) = y(tNT + c + kT_s)$ for $k = 0, 1, \ldots, N_s - 1$. If denote $\mathbf{y}(t) = (y_0(t), \ldots, y_{Ns-1}(t))^{\mathrm{T}}$, $\mathbf{s}(t) = (s_0(t), \ldots, s_{N-1}(t))^{\mathrm{T}}$, and $\mathbf{n}(t) = (n_0(t), \ldots, n_{Ns-1}(t))^{\mathrm{T}}$, then $\mathbf{y}(t) = \mathbf{H}^{(0)}(t)\mathbf{s}(t) + \mathbf{H}^{(1)}(t)\mathbf{s}(t-1) + \mathbf{n}(t)$ where $\mathbf{H}^{(m)}(t) = \mathbf{G}^{(m)}(t) \circ \tilde{\mathbf{F}} = (\mathbf{C}^{(m)}(t)\mathbf{E}^{(m)}) \circ \tilde{\mathbf{F}}$ for m = 0 and 1. Operation $\circ$ denotes the Hadamard product. The (k, j)th element of $\mathbf{C}^{(m)}(t) \in C^{Ns \times d}$ is $c_{k,j}^{(m)}(t) = \alpha_j(tNT + c + kT_s)\, v(mNT + c + kT_s - \tau_j)$, the ($j$, n)th element of $\mathbf{E}^{(m)} \in C^{d \times N}$ is $\tilde{f}_{k,n} = \exp[j2\pi(mNT + c - \tau_j)\, n / (NT_f)]$, and the ($k$, n)th element of $\tilde{\mathbf{F}} \in C^{Ns \times N}$ is $\tilde{f}_{k,n} = \exp[j2\pi nk / (NT_f / T_s)] / \sqrt{NT_f / T_s}$. We assume that $E[s_i(t_1)s_j^*(t_2)] = \delta(t_1 - t_2)\delta(i - j)$, $E[s_i(t_1)n_j^*(t_2)] = 0$ for $\forall i, j, t_1, t_2$, and $E[\mathbf{n}(t_1)\mathbf{n}^*(t_2)] = \sigma^2\delta(t_1 - t_2)\mathbf{I}$. Fig. 1 shows the received OFDM signal waveform.

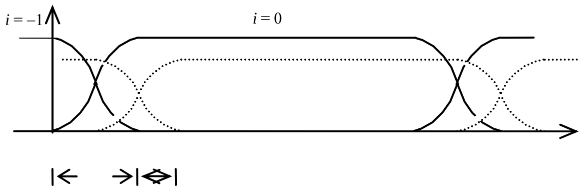


Fig. 4.1. Received OFDM signal waveform

4.2.2 Conventional OFDM Symbol Structure

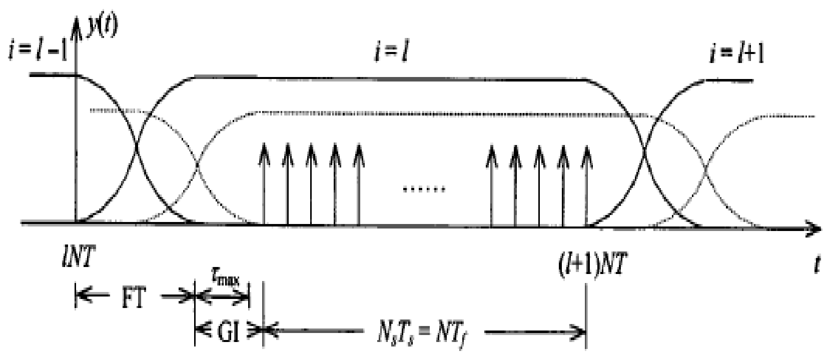


Fig 4.2 **Conventional OFDM Symbol Structure**

- The signal in FT and GI is discarded (t0=FT+GI)
- Samples are ICI and IBI free
- Efficiency of data usage < 100%

4.3 B.W. Efficient OFDM

- B.W Efficient OFDM achieves high efficiency of bandwidth usage, allows ISI and ICI to exist, and uses sufficient statistics in symbol demodulation.
- A one-tap decision feedback equalizer (DFE) is implemented for equalization of the bandwidth-efficient OFDM system

4.3.1 B.W. Efficient OFDM Symbol Structure

- The Signal in each entire OFDM period is sampled (t0=0)

• Guard Interval = 0

• Efficiency of data usage = 100%

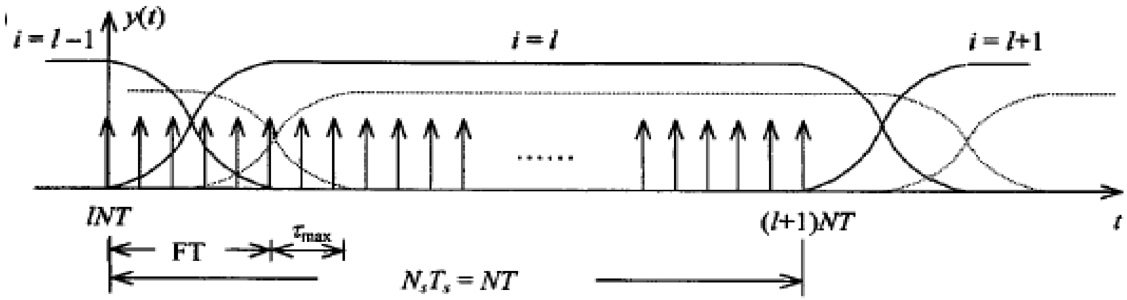


Figure 4.3 B.W. Efficient OFDM Symbol Structure

4.3.2. Efficiency of bandwidth usage

We are concerned with the choice of parameters: T, T_f, N, T_s, N_s, and c. Parameters T, T_f, and N are transmitter parameters that determine the transmitted continuous-time OFDM signal. Assume that T and N are fixed. Then T_f determines the dimension of the transmitted continuous-time OFDM signal as well as the efficiency of bandwidth usage that is defined by

$$\eta \equiv \frac{1}{\text{Bandwidth} \times \text{Symbol period}} = \frac{T_f}{T} . \qquad \textbf{(4.2)}$$

In terms of the 2WT principle [10], the transmitted continuous-time OFDM signal $x(t)$ has dimension NT / T_f in one OFDM symbol period. Hence, to provide sufficient freedom for reliable transmission, we must have $T / T_f \geq 1$ or $\eta \leq 1$. It is desired that an OFDM system has $\eta = 1$.

Parameters N_s, T_s, and c are receiver parameters that determine how to use the received continuous-time OFDM signal. Note that to obtain sufficient dimension for symbol detection we must have $N_s \geq N$ and that to satisfy the Nyquist sampling rate we have $T_s \leq T_f$. An interesting case of $\tilde{\mathbf{F}}$ is that its columns are orthogonal.

Lemma 1: When $T_s \leq T_f$, the necessary and sufficient condition for the columns of $\tilde{\mathbf{F}}$ to be mutually orthogonal is $N_s = KNT_f / T_s$ where K is a natural integer.

The systems with different choice of T_f, N_s, T_s, and c have different advantages. An OFDM system free of both ICI and ISI is obtained if the following conditions hold: (i) $\alpha_j(tNT + c + kT_s) = \alpha_j(tNT + c)$ for $k = 0, \ldots, N_s - 1$, (ii) $N_sT_s \le NT - \mathrm{FT} + \tau_{\max}$, and (iii) the columns of $\tilde{\mathbf{F}}$ are orthogonal. In this system,

$$\mathbf{y}(t) = \tilde{\mathbf{F}}\mathrm{diag}(g_0^{(0)}, \ldots, g_{N-1}^{(0)})\mathbf{s}(t) + \mathbf{n}(t) \quad \textbf{(4.3)}$$

Its subchannels can be decoupled by $\tilde{\mathbf{F}}^*$ and the channel equalization is accomplished by a complex scalar for each subchannel. In the conventional, ICI- and ISI-free OFDM system, $\tilde{\mathbf{F}}$ is designed to be an inverse DFT matrix.

It is easy to show that the efficiency of bandwidth usage of this ICI- and ISI-free OFDM system is

$$\eta \le \frac{1}{K}\left(1 - \frac{\mathrm{FT} + \tau_{\max}}{NT}\right) < 1 \quad \textbf{(4.4)}$$

4.3.3 Efficiency of data usage

To measure how efficiently the received data are used, we define the efficiency of data usage as

$$\gamma \equiv \frac{\text{Length of sampling interval}}{\text{OFDM symbol period}} = \frac{N_sT_s}{NT} \quad \textbf{(4.5)}$$

Note that if $\gamma = 1$ and the received signal is sampled at the sampling rate higher than or equal to the Nyquist rate, the samples are sufficient statistics. A higher efficiency of data usage usually implies a better performance in channel equalization.

Due to condition (ii), the efficiency of data usage for the ICI- and ISI-free OFDM system is

$$\gamma \le 1 - \frac{\mathrm{FT} + \tau_{\max}}{NT} \quad \textbf{(4.6)}$$

4.4. OFDM system with η = 100%

4.4.1. A one-tap decision-feedback equalizer

In this paper, we consider the OFDM system that achieves one hundred percent of efficiency of bandwidth usage. The ICI and ISI are allowed in the received data. Because only one previous OFDM symbol is involved in the data, we propose a one-tap DFE for channel equalization. As shown in Fig. 2, its forward and backward matrices are

$$\mathbf{F}_0(t) = \mathbf{V}_s \Lambda_s \left(\Lambda_s^2 + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{U}_s^* \qquad \textbf{(4.7)}$$

and

$$\mathbf{B}_1(t) = \mathbf{H}^{(1)}(t)\ , \qquad \textbf{(4.8)}$$

respectively. Λ_s is the diagonal matrix whose diagonal elements are non-zero singular values of $\mathbf{H}^{(0)}(t)$, and $\mathbf{V}_s$ and $\mathbf{U}_s$ are corresponding row and column vectors of signal space. $\mathbf{B}_1(t)$ tries to eliminate $\mathbf{s}(t-1)$ by using previous estimate while $\mathbf{F}_0(t)$ achieves the MMSE estimation.

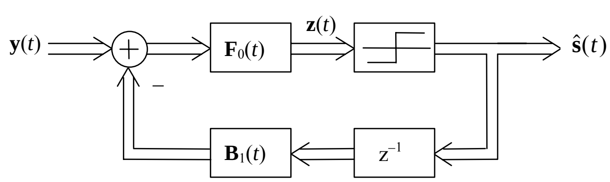


Figure. 4.4. One-tap DFE

4.5 Performance comparison

In the comparison, the configuration of the WAND system [3] is considered. The carrier frequency is 5.1 GHz. Symbol period is T = 0.075 μs. If user's velocity is three meters per second, the Doppler frequency times OFDM symbol period is only 6.12×10^{-5}. Hence, in the indoor application at which the WAND aims, the Doppler effect is negligible. We assume that α_j's are time-invariant in each data burst and are randomly generated. There are N = 16 subchannels and d = 4 rays with delays equal to 0, 0.02, 0.05, and 0.1 μs, respectively. Symbols are modulated by 8PSK in each subchannel and are uncoded. The OFDM pulse shaping filter is the time-domain raised cosine function $v(t) = \sqrt{W}$ for $0 \le |t-\frac{1+\beta}{2W}| \le \frac{1-\beta}{2W}$ and $\sqrt{\frac{W}{2}\left(1-\sin\frac{\pi W(t-1/2W)}{\beta}\right)}$ for $\frac{1-\beta}{2W} \le |t-\frac{1+\beta}{2W}| \le \frac{1+\beta}{2W}$ where β = 0.2 and W = 5/6 MHz. In this case, the filter time is FT = 0.24 μs. The SNR is defined as

$$\mathrm{SNR} = 10\log_{10}[P_s/(NT/T_s)\sigma^2] \tag{14}$$

where P_s is the energy of received continuous-time OFDM signal in one OFDM symbol period $[0,NT)$. Two systems are compared.

System I: This is the same system as in the WAND. Bandwidth is $1 / T_f$ = 20 MHz, sampling period is $T_s = T_f$, c = 0.40 μs, and number of samples per OFDM symbol is $N_s = N = 16$. The efficiency of bandwidth usage is η = 67%. The filter time is FT = 0.24 μs and the guard interval GI = 0.16 μs is used. Hence, the efficiency of data usage is γ = 67%. The other 33% data, though received, are discarded. The sampled data are ICI- and ISI-free. A DFT is used to decouple subchannels.

System II: $T_f = T$, $T_s = T_f / 2$, c = 0 μs, and $N_s = 2N$. Then η = 100%. Samples are taken from entire duration of an OFDM symbol period, and therefore the efficiency of data usage is γ = 100%. The signal is oversampled and there are ICI and ISI in sampled data. The proposed one-tap DFE is used in channel equalization.

4.6 SER in Simulation

In simulation, each data burst contains 100 OFDM symbols. 500 bursts are tested at each SNR value. As shown in Fig. 3, although the one-tap DFE based OFDM system uses less bandwidth and the ICI and ISI exist, it performs better than the ICI- and ISI-free, DFT-based OFDM system by 3 dB in SNR. The reason is due mainly to the fact that the ICI and ISI system uses sufficient statistics.

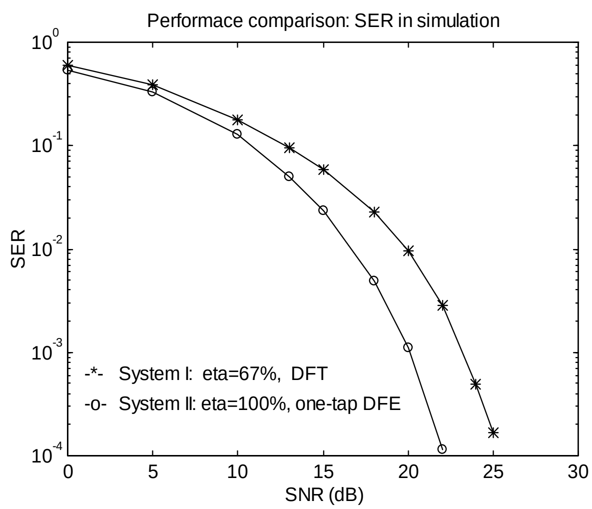


Fig. 4.5. SER in simulation

Fig. 4. Mutual information

The comparison of the two systems is listed in Table 4.1.

Table 4.1. Comparison of two OFDM systems

	Conventional system	Proposed system
Bandwidth efficiency	67%	100%
Data use efficiency	67%	100%

Interference	ICI- and ISI-free	ICI and ISI
Equalization	DFT	One-tap DFE
Computational efficiency	High	Low
SER	Higher	Lower (3dB)
Mutual information	Lower	Higher (1.5 dB)

4.7 Conclusions

In OFDM systems, a guard interval is necessary to obtain ICI- and ISI-free data. However, the price paid is the efficiency of bandwidth usage and the performance is reduced due to the use of insufficient statistics in channel equalization. Though the bandwidth is much narrower, the proposed one-tap DFE based, ICI and ISI system performs better than the ICI- and ISI-free, DFT based system in both symbol error rate and mutual information. This justifies that the equalization of this bandwidth-efficient OFDM system is worth to explore in semi-blind and blind cases.

CHAPTER 5

Estimation And Equalization Alogorithm

5.1 Residual ISI Cancellation (RISC) for OFDM

An iterative technique is developed for orthogonal frequency division multiplexing (OFDM) systems to mitigate the residual intersymbol interference (ISI) that exceeds the length of the guard interval. The technique, called residual ISI cancellation (RISIC), uses a combination of tail cancellation and cyclic restoration and is shown to offer large performance improvements. The effects of imperfect channel estimation are also considered. The RISIC algorithm is applied to a typical terrestrial high-definition television (HDTV) broadcasting system that uses a concatenated coding scheme for error control. Results show that the RISIC algorithm can effectively mitigate residual ISI on static or slowly fading ISI channels.

We assume a data rate of 20 Mb/s where the data are independent bits. A 16-quadrature amplitude modulation (QAM) constellation is used, which requires about 5 MHz of bandwidth. As in [8], an inverse fast Fourier transform/fast Fourier transform (IFFT/FFT) pair is used as a modulator/demodulator.

For the th block, the -point IFFT output sequence is

$$x_{i,k} = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} X_{i,n} \exp\left\{j\frac{2\pi nk}{N}\right\}, \qquad 0 \le k \le N-1 \tag{5.1}$$

where $\{X_{i,n}\}_{n=0}^{N-1}$ is the transmitted symbol sequence and N is the block size. If the guard interval is a cyclic extension of the IFFT output sequence [2], then the IFFT output sequence with the addition of the guard interval is

$$x_k^g = x_{(k+N-G)_N}, \qquad 0 \le k \le N+G-1 \tag{5.2}$$

where G is the length of the guard interval, and $(k)_N$ is the residue of k modulo N. The samples $\{x_k^g\}_{k=0}^{N+G-1}$ are passed through a digital-to-analog (D/A) converter with rate $1/T_s$ where T_s is the sample period of the OFDM signal with the guard interval. We will consider both static and fading ISI channels. For both channels, a maximum delay spread of MT_s is assumed. Throughout the paper, perfect carrier and timing synchronization are assumed. In the absence of noise, if $G \ge M$ the received samples for the th block after removal of the guard interval are

$$r_{i,k} = \sum_{m=0}^{M} h_{m,k} x_{i,(k-m)_N}, \qquad 0 \le k \le N-1 \tag{5.3}$$

where $h_{m,k}$ is the channel impulse response at lag and instant k.

Two static ISI channels are considered, as shown in Table I. Channel 1 has subchannels with a moderate null, whereas Channel 2 has several subchannels with a severe null(Fig.1). For fading channels, we choose the six-tap typical urban (TU) and hilly terrain (HT) power delay profiles from the COST- 207 studies [9]. The tap gains are zero-mean complex Gaussian random processes that are generated using Jakes' method [10] corresponding to a two-dimensional (2-D) isotropic scattering environment.

5.2. PERFORMANCE OF OFDM ON ISI CHANNELS

5.2.1. Static ISI Channel

If $M > G$, then the received samples in (3) can be modified as the sum of two components, viz.

$$r_{i,k} = r_{i|i-1,k} + r_{i|i,k}$$

where $r_{i|i-1,k}$ is the received sample component with contributions only from block $i-1$ and $r_{i|i,k}$ is the received sample component with contributions only from block ***i***

5.2.2. RISIC

If the channel changes little during the block duration and the guard interval is sufficiently large, i.e. $G \geq M$, then the channel output is

$$\tilde{r}_{i,k} = \sum_{m=0}^{M} h_m x_{i,(k-m)_N}, \qquad 0 \leq k \leq N-1 \qquad \textbf{(5.4)}$$

where $\tilde{r}_{i,k}$ represents the desired channel output that is free of ISI. To achieve the desired channel output $\tilde{r}_{i,k}$ in the presence of residual ISI two steps must be taken. The first is to remove the residual ISI from the received signal, and the second is to use reconstruction to restore cyclicity and avoid ICI. These two procedures are called tail cancellation and cyclic reconstruction, respectively . The procedure can
be described by

$$\tilde{r}_{i,k} = r_{i,k} - r_{i-1,k} + \sum_{m=G+1}^{M} h_m x_{i,(k-m)_N} \cdot (1 - u(k - m + G)). \quad \textbf{(5.5)}$$

The residual ISI is removed from the received signal by subtracting the second term in (29). Cyclicity is restored by the last term in (29).The feasibility of implementing the tail cancellation and cyclic reconstruction procedures depends on the availability of the transmitted signal $\{x_k\}$ at the receiver. Echo cancellers have exact knowledge of the transmitted symbols and, therefore, the above procedures have been successfully implemented [6]. The vast majority of communication applications that require mitigation of ISI, however, do not enjoythis luxury. We now describe the method for reducing the effect of ISI, using the aforementioned procedures, when the transmitted symbols are not available to the receiver a priori.

5.2.2.1. RISIC Algorithm

We assume that the channel impulse response is constant over a block period, i.e $h_{m,k} = h_m, 0 \leq k \leq N-1$., The RISIC algorithm proceeds as follows.

1) An estimate of the channel impulse response, $\hat{h}_m$ is obtained from a training sequence and updated in a decision-directed mode. Channel estimation will be treated in more detail in Section V-C.

2) Decisions on the transmitted symbols $\{\hat{X}_{i-1,n}\}_{n=0}^{N-1}$ from block are obtained for use in tail cancellation. Since the decisions are affected by residual ISI, some may be erroneous. These symbols are converted back to the time domain using IFFT giving $\{\hat{x}_{i-1,k}\}_{k=0}^{N-1}$.

3) For the block of index we perform tail cancellation by calculating the residual ISI and subtract it from $r_{i,k}$ i.e.,

$$\tilde{r}_{i,k}^{(0)} = r_{i,k} - \sum_{m=G+1}^{\hat{M}} \hat{h}_m \hat{x}_{i-1,(k-m+G)_N} \cdot (1 - u(k - m + G)), \qquad 0 \leq k \leq N-1 \quad \textbf{(5.6)}$$

where $\hat{M}$ is the estimate of the maximum channel impulse response length.

4) The $\{\bar{r}_{i,k}^{(0)}\}_{k=0}^{N-1}$ obtained in Step 3 are converted to the frequency domain using FFT and decisions are made.Afterwards, the decisions are converted back to the time domain to give $\{\hat{x}_{i,k}^{(0)}\}_{k=0}^{N-1}$.

5) Next, we perform cyclic reconstruction by forming

$$\bar{r}_{i,k}^{(I)} = \bar{r}_{i,k}^{(0)} + \sum_{m=G+1}^{\hat{M}} \hat{h}_m \hat{x}_{i,(k-m)_N}^{(I-1)} \cdot (1 - u(k-m+G)), \qquad 0 \le k \le N-1 \tag{5.7}$$

where I represents an iteration number with an initial value of $I = 1$.

6) The $\{\bar{r}_{i,k}^{(I)}\}_{k=0}^{N-1}$ are converted to the frequency domain

and decisions are made yielding $\{\hat{X}_{i,n}^{(I)}\}_{n=0}^{N-1}$. This completes the th iteration in the RISIC algorithm.

7) To continue iterations, convert the $\{\hat{X}_{i,n}^{(I)}\}_{n=0}^{N-1}$ to $\{\hat{x}_{i,k}^{(I)}\}_{k=0}^{N-1}$ and repeat Steps 5–7 with

8) The end of the RISIC algorithm for block i.

C. Channel Estimation

All the remarkable improvements that RISIC has shown so far are achieved under the premise of perfect channel

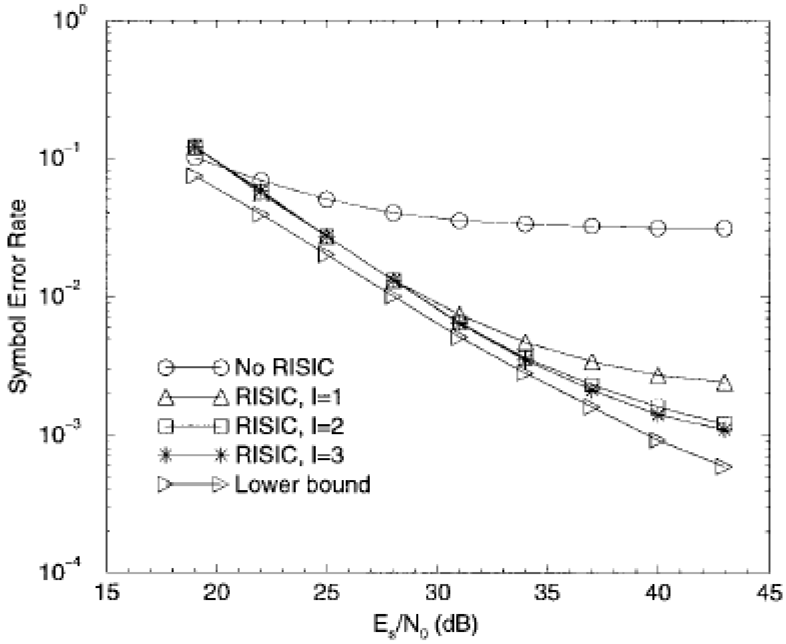


Fig.5.1. Performance of the RISIC technique on a fading ISI channel: G = 0; N = 1024; fDNTs = 0:005:

information. For a strictly static ISI channel, the channel estimation process needs to be carried out only once. In fact, almost perfect channel estimates can be obtained using a training sequence [6]. Time-varying channels present the real challenge for channel estimation.

For fading ISI channels, we propose a channel estimation technique that can provide accurate estimates, even in the presence of residual ISI. A special OFDM training block is used where a chirp sequence of $N/2$ symbols are used [6]

$$C_n = \exp\left(j\frac{2\pi}{N}n^2\right), \qquad 0 \le n \le \frac{N}{2} - 1. \tag{5.8}$$

The chirp sequence provides a desirable peak-to-average power ratio of one and weighs each subchannel equally in estimating the channel, since the FFT of a chirp sequence is also chirp sequence. However, we modify the training sequence to increase resilience to residual ISI, by inserting zeros in every odd subchannel. The training block symbols are

$$D_n = \begin{cases} \sqrt{2}C_{n/2}, & n = 0, 2, \cdots, N-2 \\ 0, & n = 1, 3, \cdots, N-1 \end{cases} \tag{5.9}$$

where $\sqrt{2}$ is a normalization factor. The N-point IFFT of D_n is

$$d_k = c_{(k)_{N/2}}, \qquad 0 \le k \le N-1. \tag{5.10}$$

From (35), the first half of the time domain training sequence $\{d_k\}$ is identical to the second half. This is a valuable property for long channel impulse responses because the first half of $\{d_k\}$ can be used just like a guard interval, while $\{d_k\}$ still possesses a peak-to-average power ratio of one. The following is the channel estimation procedure using the proposed training block.

1) After removal of the guard interval, the received samples are rearranged as

$$\bar{r}_{ts,k} = r_{ts,k+(N/2)}, \qquad 0 \le k \le \frac{N}{2} - 1 \tag{5.11}$$

where the subscript indicates that the received samples are for a training block.

2) $N/2$ channel estimates are calculated by

$$\bar{\eta}_n = \frac{Z_{ts,n}}{C_n}, \qquad 0 \le n \le \frac{N}{2} - 1 \tag{5.12}$$

where $\{Z_{ts,n}\}_{n=0}^{(N/2)-1}$ is the $N/2$-point FFT.

3) The estimates $\bar{\eta}_n$ are converted to $\bar{h}_m$ by

$$\bar{h}_m = \frac{1}{\sqrt{\frac{N}{2}}} \mathrm{IFFT}\,\{\bar{\boldsymbol{\eta}}\}(m) \tag{5.13}$$

where an $N/2$-point IFFT is used. Then, $\{\bar{h}_m\}_{m=0}^{(N/2)-1}$ is passed through a rectangular window to zero $\bar{h}_m, m > \hat{M}$ and the result is $\{\hat{h}_m\}_{m=0}^{(N/2)-1}$ which is used for RISIC.

4) $\{\hat{h}_m\}_{m=0}^{(N/2)-1}$ is padded by zero sequence of length $N/2$ and then converted to $\{\hat{\eta}_n\}_{n=0}^{N-1}$ by

$$\hat{\eta}_n = \sqrt{N}\,\mathrm{FFT}\,\{\hat{\boldsymbol{h}}\}(n), \qquad 0 \le n \le N-1 \tag{5.14}$$

where, M this time, an -point FFT is used. The channel estimates for all subchannels are now available for data demodulation.

When is not known, the window size must be large enough to include the entire channel impulse response yet as small as possible to minimize the effect of noise. The impact of $\hat{M} > M$ is considered in Section VI.

On static ISI channels, the channel estimation is performed only once, at the beginning of each simulation run, and the channel estimates are averaged over four blocks. On fading ISI channels, the channel estimation is performed by periodically sending one training block out of every 20 blocks transmitted, while updating the channel estimate in the remaining blocks in a decision-directed mode [12]. In the decision directed mode, the decisions made after the final iteration of the RISIC algorithm are used to update the channel estimates. Decision errors do not degrade the accuracy of the estimates excessively, because the rectangular window can smooth out the aberrations caused by the decision errors.

Fig. 8 shows the performance of the RISIC technique with channel estimation on static and very slowly fading ISI channels. For a static ISI channel, the SER obtained is virtually identical to that obtained with perfect channel estimation. For a slowly varying fading ISI channel $(f_D N T_s = 0.001)$ the RISIC technique works well with the channel estimation, especially at high SNR. However, for faster fading $(f_D N T_s = 0.005)$ the degradation is severe, as shown in Fig. 9. This suggests that the proposed RISIC technique with channel estimation is well suited for static and slowly time-varying ISI channels.

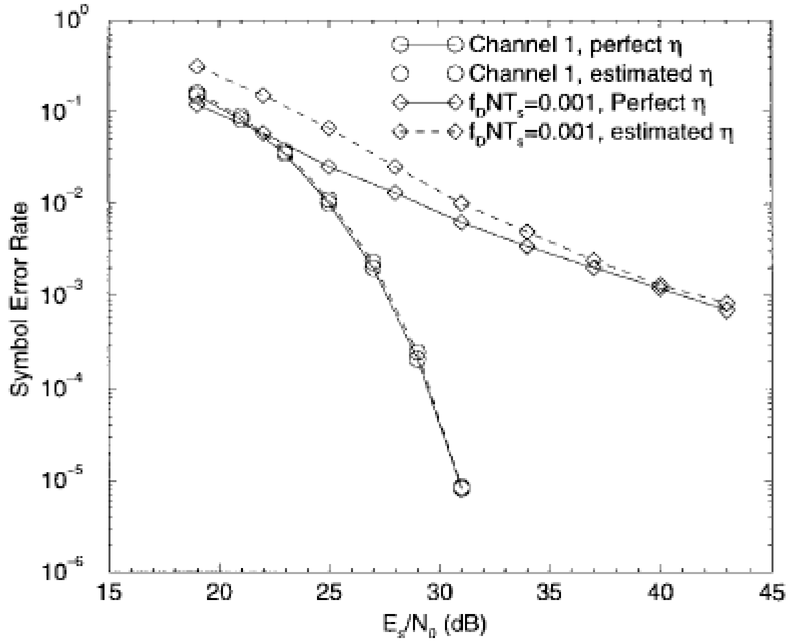


Fig. 5.2. Effect of imperfect channel estimation with the RISIC technique for static and slowly fading ISI channels with N = 128 and N = 1024; respectively: G = 0; I = 3:

5.3. TRANSLATION TO PASSBAND AT TRANSMITTER

In our modem, carrier frequency is 2 Khz. In discrete domain it comes out to be pi/2. This complex carrier is generated by using look up table. Carrier takes four values of look up table periodically.

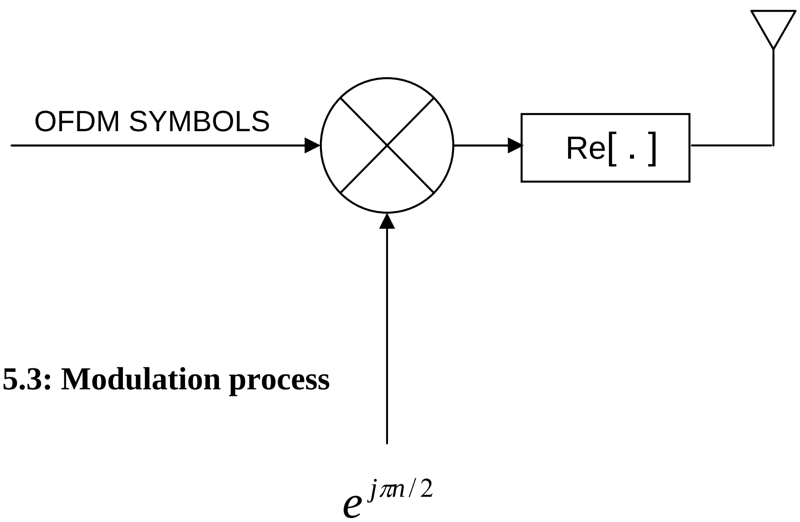


Figure 5.3: Modulation process

5.4. DOWN CONVERSION AT RECEIVER

The procedure of down conversion is shown in figure below.

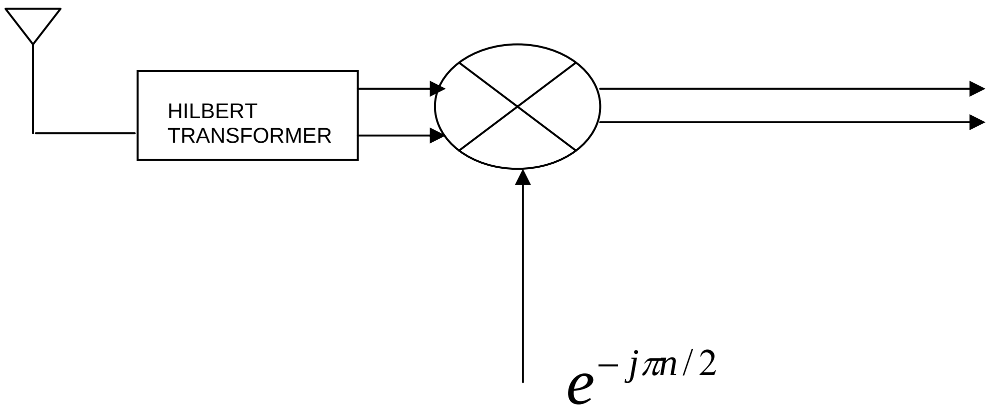


Figure 5.4 : Complex signal recovery

The down conversion requires Hilbert transformer. Hilbert transform recovers quadrature part from inphase Hilbert transformer is just an all pass phase shifter. Its magnitude and phase responses are shown [14].

$$h(t) = \frac{1}{\pi t} \tag{5.15}$$

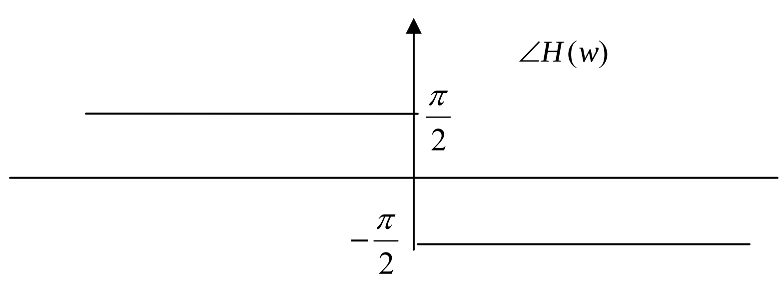


Figure 5.5 : Phase response

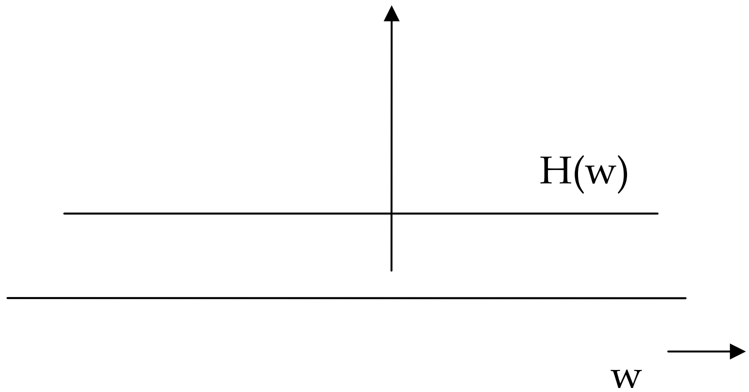


Figure 5.6 : Magnitude response

CHAPTER 6

PERFORMANCE ANALYSIS

6.1 System Parameters:

The system parameters for the Bandwidth Efficient OFDM have been analyzed in great detail. They are explained below:

6.1.1 Number Of Carriers: (N)

The main parameter in an OFDM system is the number of carriers or the FFT size. Generally larger number of carriers for a given bandwidth will give better performance over frequency selective channels. However increasing the number of carriers increases the complexity of calculations. Another drawback is sensitivity to frequency offsets at the transmitter or receiver oscillators. The more the carriers, the less the bandwidth per carrier and thus the more the system is prone to frequency errors.

The number of Carriers also determines the length of the OFDM symbol in time domain. As is obvious, the larger length symbols perform better. Normally the FFT size is decided by the max delay spread of the channel. It is chosen so that the length of the symbol is larger by 2 to 3 times than the maximum delay spread of the channel.

6.1.2 Guard Interval: (GI)

The second fundamental system parameter is the guard interval. Usually chosen just greater than the max delay spread of the channel, in our system, the guard interval is zero. In conventional OFDM systems, longer guard intervals improve performance.

6.1.3 Guard Interval of the Pilot Symbol: (GI_pilot)

As in our system the pilot symbol is used for channel estimation, so it should be free of ISI. To ensure this a guard interval greater than the maximum delay spread of the channel is inserted before the pilot. This is also utilized for frequency synchronization. There is a certain value for different channels which maximizes the performance.

6.1.4 Frame Length: (FL)

In our BE OFDM system, one of the fundamental parameters is the frame length. This is important as the Channel Estimation and Frequency Offset estimation is only done once for each frame. Larger frame length increases effective utilization of bandwidth as the pilot symbol at the start does not contain any useful data. In slow fading channels, the frame length can be large but for fast fading channels, the frame length must be kept small to keep the BER in control.

6.1.5 Sampling Rate: (Fs)

The system has been tested at different sampling rates, from 8Khz to 48Khz in ideal environments using the sound card of a PC. The bandwidth being 4Khz and 24Khz respectively. However hardware limits the use of higher sampling rates. Sampling rates for GSM and 802.16 are in the range of 100Khz and 2Mhz which cannot be used in our test setup. However the effects on such high sampling rates will be analyzed.

6.1.6 Channel Coding:

We have used a Convolutional coder operating at a rate of ½ with a Viterbi decoder to minimize error for speech transmission.

6.1.7 Modulation Scheme:

All work has been done with QPSK as the modulation scheme. However it can easily be extended to M-PSK or QAM.

6.2 System Test Environment:

For demonstration of our system, we have made simulated a practical communication environment by connecting two computers through their sound cards over wireless and wired mediums. We have developed a GUI in matlab which transmits/receives. We have transmitted wave files to demonstrate the effectiveness of the system.

Also we have demonstrated that our system works over wireless by transmitting and receiving over DSB-WC (AM). The system works for……………………..

6.3 Performance over frequency selective channels

For this purpose, we have considered 3 different frequency selective channels.

A GUI was developed in MATLAB to demonstrate the effects on the BER etc. of the different channels.

As was mentioned earlier the channel is modeled as a tap delay line given by the equation:

$$y(t) = \sum_{j=1}^{d} \alpha_j(t) x(t - \tau_j) + n(t) \qquad \textbf{(6.1)}$$

The different channels have different values of α and τ_j, thus a different frequency response. The first is moderately frequency selective but the second and third have a sharp spectral nulls which degrade performance. This degradation can be overcome by using channel codin and/or larger number of carriers.

The maximum delay spread is assumed to be 8 samples in the simulation. This is a safe assumption and can be proven as follows. If the sampling rate considered to be 1MHz then time for one sample Ts=1us. The maximum delay spread in indoor environments is approximately 500ns=0.5us which is covered within one sample. However for worst case scenarios in outdoor environments, the delay spread may reach much further. So an assumption of 6 samples provides for 6us delay.

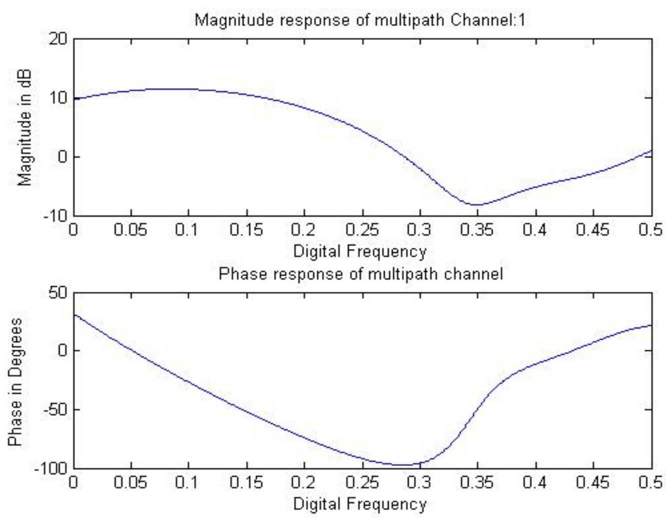
TABLE 6.1

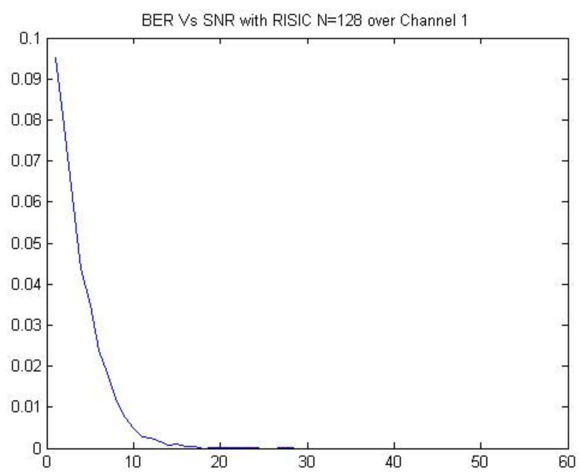
Channel No	Tap No 0α	Tap No 1α	Tap No 2α	Tap No 3α	Tap No 4α	Tap No 5α	Tap No 6α
1.	1.165-.6965i	0	.6268+1.6961i	.751+.591i	0	0	0
2.	1.165-.6965i	0	0	.6268+1.6961i	.751+.591i	.3516+1.7971i	0
3.	0.165-.6965i	0	.6268+0.6961i	0	1.51+.591i	0	.516+1.7971i

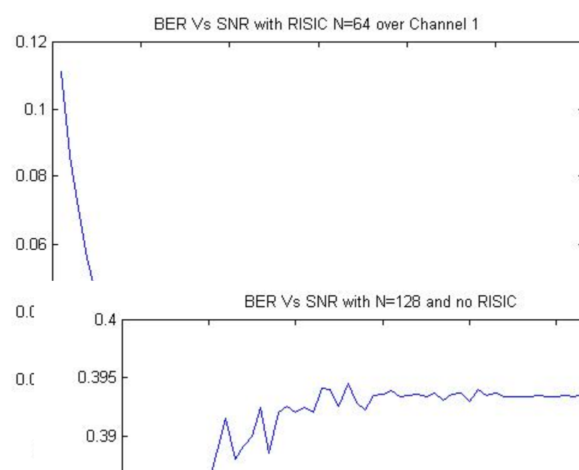
Simulation Parameters:

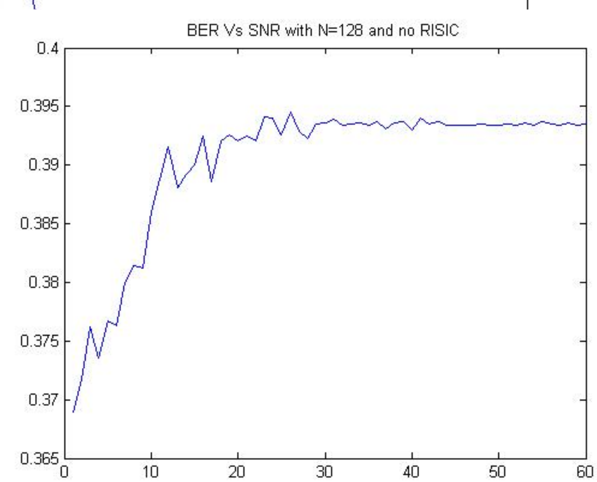
- No of Carriers (N): Variable.
- Frame Length(FL): FL=10 OFDM symbols per frame.
- Guard Interval (G): G=0.
- Guard Interval for Pilot Symbol (GI_pilot): 20 samples.
- Channel Coding: No channel Coding is done for the analysis.
- Modulation Scheme: QPSK was used.

6.4.1 Channel 1:









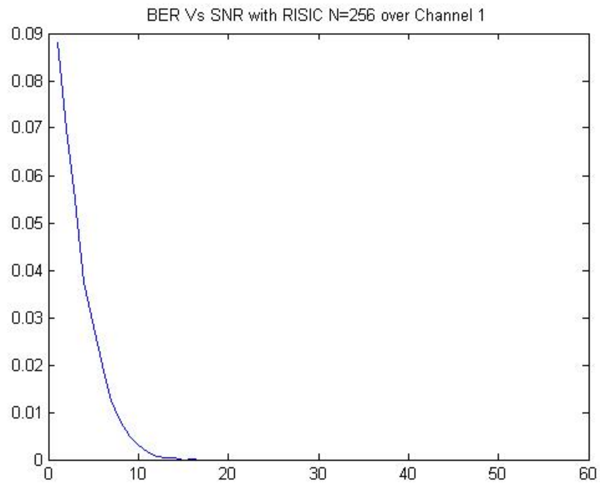
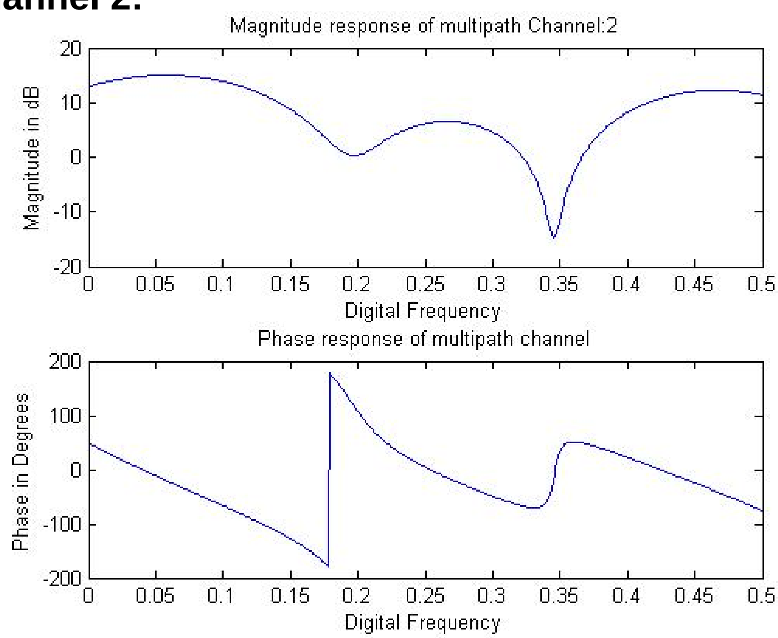
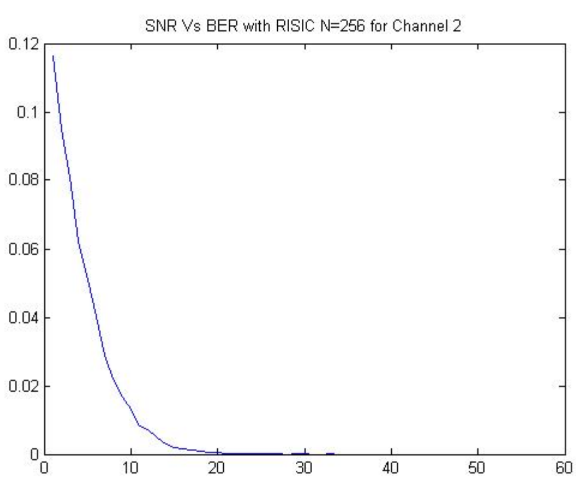
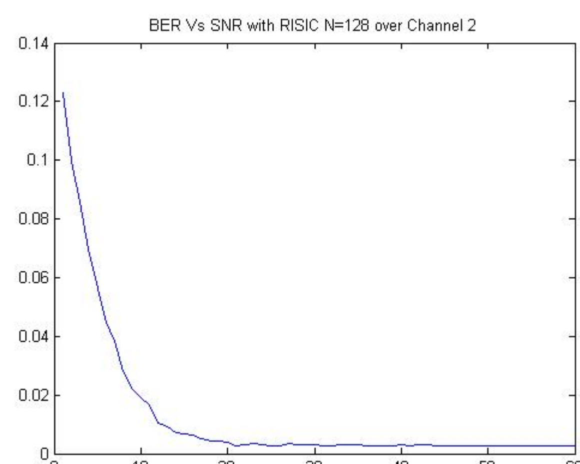


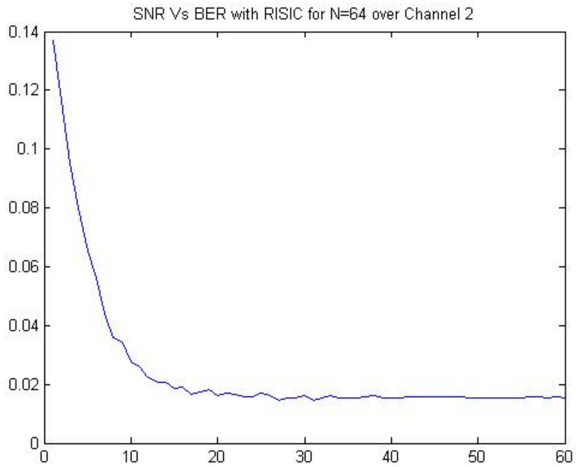
FIGURE 6.1

6.4.2 Channel 2:









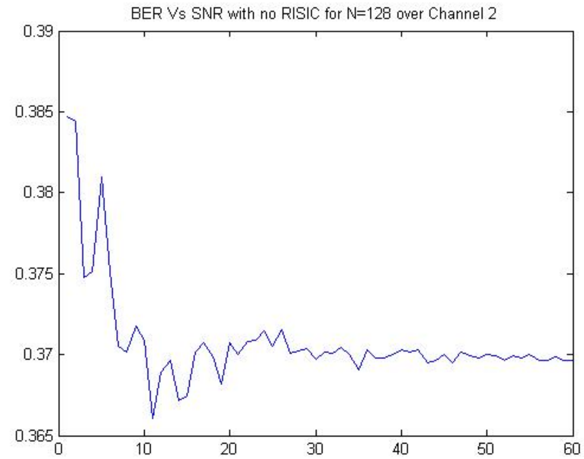
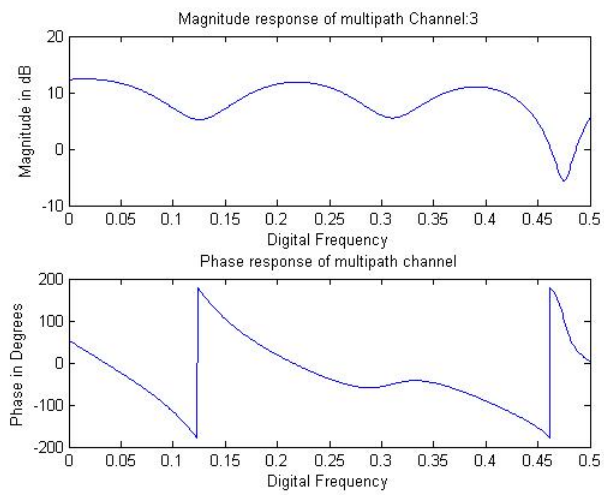
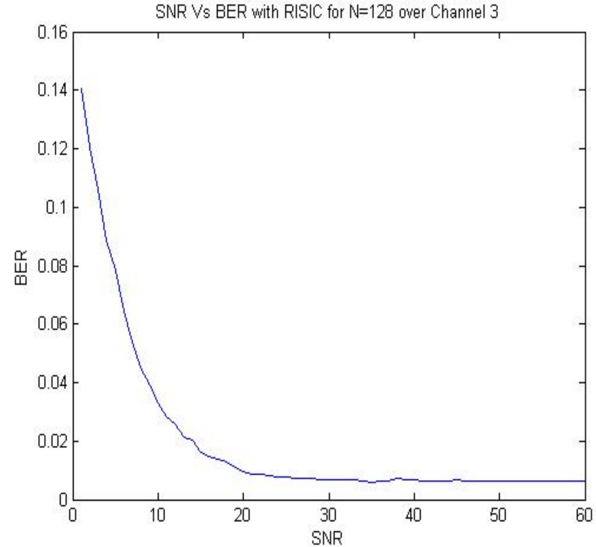


FIGURE 6.2

6.4.3 Channel 3:





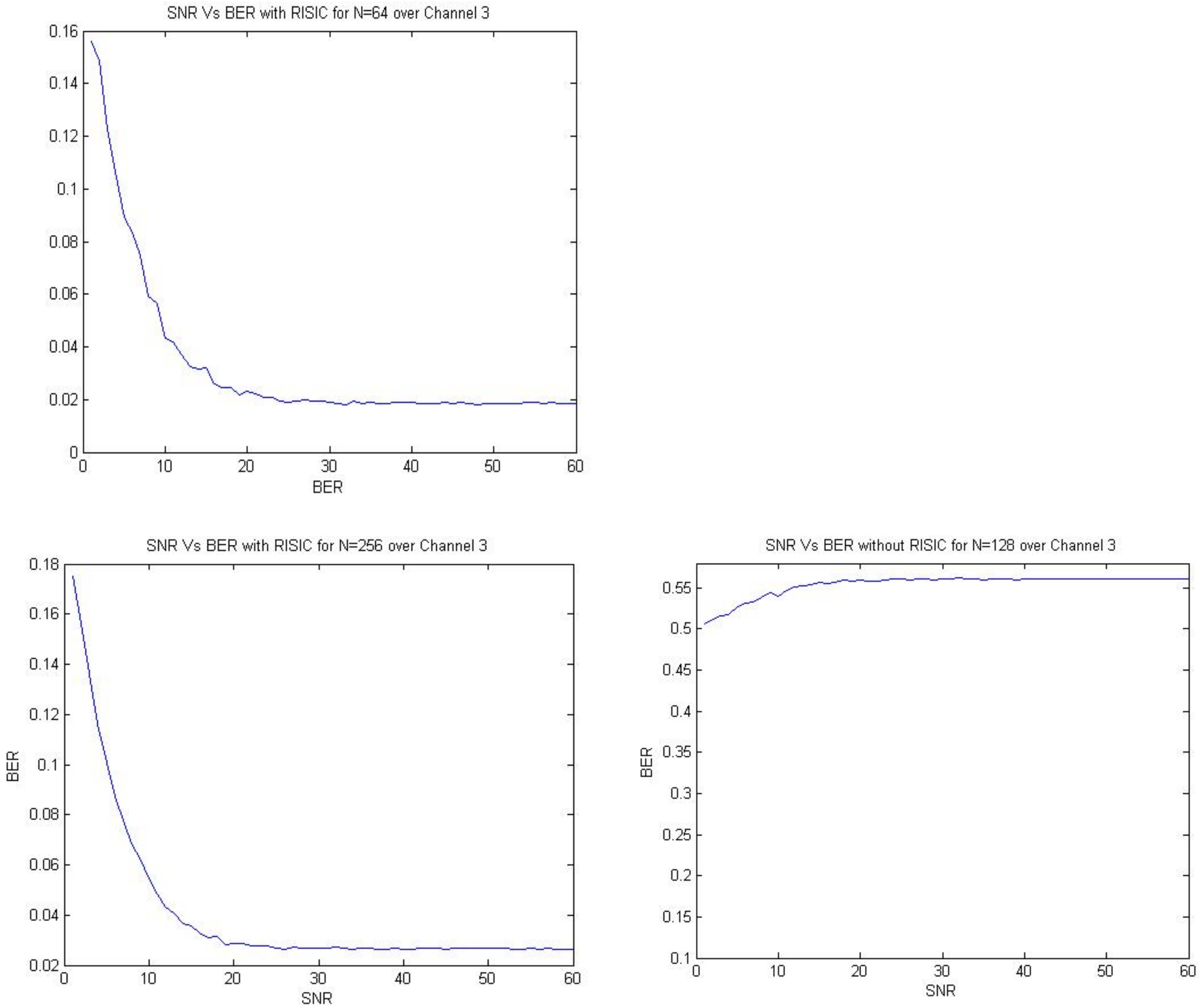

SNR Vs BER with RISIC for N=64 over Channel 3
BER
SNR Vs BER with RISIC for N=256 over Channel 3
BER
SNR
SNR Vs BER without RISIC for N=128 over Channel 3
BER
SNR

FIGURE 6.3

6.5 Performance over mobile channels:

Doppler Shift Table:

Relative Velocity	Doppler Shift @ 900Mhz	Doppler Shift @ 2.4Ghz
5Km/hr	4.02 Hz	10 Hz
10Km/hr	8.34 Hz	22.24 Hz
60Km/hr	50 Hz	133 Hz
100Km/hr	83.34 Hz	222.24 Hz

Done for Fs=100KHz

APPENDEXIS

APPENDIX A Complete Matlab Source Code

The OFDM Modem Simulation code for the Important steps is present here. This code is implemented in the *MATLAB Version 6.5.0.180913a (R13)* is present here.

APPENDIX B List of Abbreviations

ADSL	**Asymmetric Digital Subscriber Line**
AWGN	**Additive White Gaussian Noise**
CP	**Cyclic Prefix**
BER	**Bit Error Rate**
DAB	**Digital Audio Broadcast**
DFT	**Discrete Fourier Transform**
DMA	**Direct Memory Access**
DSP	**Digital Signal Processor**

DVB	**Digital Video Broadcast**
FFT	**Fast Fourier Transform**
ICI	**Interchannel Interference**
IDFT	**Inverse Discrete Fourier Transform**
ISI	**Intersymbol Interference**
MAP	**Maximum a Posteriori**
ML	**Maximum Likelihood**
MMSE	**Minimum Mean Square Error**
OFDM	**Orthogonal Frequency Division Multiplexing**
PAM	**Pulse Amplitude Modulation**
QAM	**Quadrature Amplitude Modulation**
SNR	**Signal to Noise Ratio**
TDD	**Time Division Duplex**

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