Analysis of Network Coding in Cooperative Wireless Multi-Hop Networks

By

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Approval

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Abstract

We investigate the performance of two multi-hop network topologies in which multiple sources have independent information to be transmitted to a far off common destination. Linear Network coding technique is used by relays to transmit the combined information of multiple sources. The first topology contains nodes that are placed in a regular pattern and divide this deterministic topology in two categories of distributed and co-located. These topologies are modeled with a quasi-stationary Markov chain. The relay nodes use decode and forward (DF) mechanism at each hop. We find the transition probability matrix of the Markov chain assuming that all the nodes have same transmit power and the channel is Rayleigh fading. The second random network topology has fixed number of nodes that are randomly placed in a strip-shaped network. The outage probabilitiy of each node is found and the state distribution at each hop is used to analyze the network coverage for a given signal-to-noise ratio (SNR) margin. Theoretical results have been included that match with the simulation results.

Dedication

I dedicate this thesis to my parents, teachers and friends who pray for me and helped me throughout my research phase.

Certificate of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any degree or diploma at National University of Sciences & Technology (NUST) School of Electrical Engineering & Computer Science (SEECS) or at any other educational institute, except where due acknowledgement has been made in the thesis. Any contribution made to the research by others, with whom I have worked at NUST SEECS or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

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I am very thankful to Allah who gave me strength, support and special blessing on me without all this; it cannot be possible for me to complete my research.

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Muhammad Arslan Aslam

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Chapter 1

Introduction

Cooperative transmission (CT) has become an area of prime interest in wireless systems which are affected by multi path fading. By having spatially separated radios transmitting the same message signal through uncorrelated fading channels, CT reaps the advantages of distributed multi-input multiple-output (MIMO) systems such as range extension [1] and energy-efficiency [2]. These networks have many applications in both the wireless sensor networks (WSNs) and the mobile and cellular networks. The cooperative communication gives high reliability, lower error probability with more throughput. It also reduce transmit power with less interference [23]. The range extension and resistance to large scale shadowing is the beauty cooperative communication. For example, physical layer cooperation among wireless nodes or users resembles virtual MIMO spatial diversity or rate multiplexing gains. There are two basic cooperative strategies.

1) Amplify and forward
2) Decode and forward

Amplify and forward (AF) allows relays to amplify the received noisy signal from the source node and then forward it to the destination. It is the simplest relaying strategy with the low implementation cost. It works well when relay node is placed in the middle of the distance between source and destination. While in decode and forward (DF) relay node decode the received signal and re-encode it and forward to the destination. DF outperforms AF when the source relay channel ensures error free detection of the received signal at the relay.

Opportunistic large arrays (OLAs) is one of the most promising techniques at the physical layer incorporating CT and is used in multi-hop networks to deliver data to far off destinations. In basic OLA transmission, the source transmits the message to a cluster of nodes. All the nodes that can decode this message relay the message to another cluster of nodes without any coordination with one another. This process continues in a multi-hop fashion until the message is received at the destination [3]. OLA networks offer a physical layer routing mechanism and they have been shown very promising in large-scale dense WSNs. These networks are suitable for applications such as security, maintenance and control signal.

The network modeling of general OLA algorithms have always been critical owing to the random node locations and random boundaries of each hop. Initial studies used the continuum assumption implying an infinite node density per unit area with fixed transmit power [3]. A finite node density analysis of linear networks was studied in [4] and many variants on 2D random networks and effects of various channel impairments were carried out [5-6]. All these works study a single source with one message traversing the multi-hop network. Multi-packet insertion by same single source was studied in [7-8] for continuum networks. A similar concept of nodes broadcasting/unicasting a message was studied in [9] and optimization on the distance has been carried out to place nodes for better range extension.

In above-mentioned works, a single source excites the whole network. However, due to distributed nature of the nodes (and assuming no MAC protocols) there is always a possibility that simultaneous transmission of more than one sources start. In this case, a relay node at some hop may receive two distinct signals, which may cause interference. To cope with this, physical layer network coding (NC) has been suggested in literature to combine the information. In this thesis, we use NC at a relay node to combine informations of difference sources and investigate the multi-hop propagation of these codewords in opportunistic networks.

1.1 Problem Statement

“The objective of this thesis is to stochastically model the OLA network with the help of physical layer network coding to achieve diversity when more than one sources transmit at the same time”

1.2 Thesis Contribution

We stochastically modelled two network topologies in which there are more than one source that are transmitted at the same time to excite the relay or intermediate nodes between the sources and the common destination. In the first topology, the sources and the nodes are placed deterministically and in second topology nodes are randomly placed in a square region. These topologies are analyzed by finding the outage probability and coverage of the system. We checked our analytical model with the help of Matlab simulation and proved that both analytical and simulation matched with each other.

1.3 Thesis Organization

The rest of the thesis is organized as follows.

1.3.1 Chapter 2

Chapter 2 describes the background and literature review of the thesis. In this chapter, different types of network coding techniques are explained e.g., binary network coding , non-binary network coding, random network coding etc. The use of network coding in wireless network is discussed. And in the end modeling of OLA network and research idea is explained.

1.3.2 Chapter 3

In chapter 3, system arctitecture of deterministic topologies distributed and co-located is discussed. After that encoding and decoding of network codewords is explained along with examples. In the end topologies are statistical modeled with Markov chain.

1.3.3 Chapter 4

In chapter 4, random strip-shaped network topology is discussed and outage probability is calculated to find the coverage of the multi-hop cooperative network topology.

1.3.4 Chapter 5

In Chapter 5, we present our analytical and simulation results. And compare our analytical results with simulation results for matching.

1.3.5 Chapter 6

The thesis conclusion and future work is given in the Chapter 6. In which future research aspects are disscussed.

Chapter 2

Literature Review and Background

2.1 Network Coding

It is a technique in which node can combine the two or more different information that is received by him. Hence the transmitted information by this node is a combination of all received information or packets.

Network coding is first introduced in 2000 in [24]. After that it gets substantial intention from researcher to explore it more. The number of articles on network coding increased significantly in last few years [25].

Network coding techniques have also been used widely to improve throughput, energy efficiency, and robustness of wireless networks. Particularly, Nguyen et al. [26] have proposed an XOR-based network coding for wireless broadcast networks. In this work, the authors have shown that by combining the lost packets together before broadcasting them to all receivers, the source can save a number of retransmissions. Also, Tran et al. [27] have shown that by joining network coding and appropriate channel coding, the network throughput efficiency can be improved up to 3.5 times over the traditional approach ARQ. Furthermore, the network throughput region can be substantially enlarged in prioritized transmission scenarios consisting of an oracle source and multiple wireless users [28]. The benefits of network coding have been further studied in multi-hop wireless networks. The main idea in this research theme is to exploit the natural propagation of wireless signals; thus, by one transmission, several neighbouring nodes of a sender can receive the transmitted data. Wu et al. [31] have shown a wireless data exchange example in which the relay node uses network coding to reduce the number of transmissions. Moreover, Katti et al. [30] have demonstrated a practical network coding system, namely, COPE, in wireless ad hoc networks to improve the network throughput. It has been shown that by allowing the nodes snoop on the medium, learn the neighbour status, and detect coding opportunities, network coding technique can offer several-fold gain in throughput. In addition, the transmission delay using network coding has been investigated [29].

In earlier literature, network coding [10] is proposed for networks where wireless links are assumed to be noiseless. XOR is the simple example of binary network coding in which intermediate node just XOR the incoming packets and transmit them but to recover information the direct information is also required. This technique is followed by non-binary and linear NC technique. These techniques provide better diversity than simple binary NC. As in these techniques, we can recover the information with the help of codewords only, no direct transmission or information is required. Later on physical layer NC is introduced for multiuser networks. The results show that transmission efficiency is improved in the sense of energy consumption and error probability [12]. To achieve diversity in NC, many code design techniques are proposed e.g., LDPC-based coding [11], and Reed-Solomon codes [13]. Random linear network coding within defined field is proposed in [14]. In random network coding nodes randomly make the codewords of received information by choosing random coefficients to make linear combination.

2.2 Wireless Cooperative Communication using Network Coding

The two hop network is studied in which there are M users that are broadcasting their information using orthogonal channels [13]. The nodes that received user information combine the information and transmit them to the destination. The short fall in their study is that the destination is not faraway and directly receives sources information and also only two-hop network is studied, which can be useful for cellular systems. The outage probability and diversity is calculated at the destination node in the presence of NC [13].

Network coding is proposed for the topology in which multi-user transmitting to the base station using orthogonal channels. The transmission is divided in to two phases broadcast phase and cooperative phase. In broadcast phase all users broadcast their information to basestation as well as other users. Hence in cooperative phase the users that decode the information of other users combine the all informations along with their own information using network coding and transmit to the basestation to achieve diversity

[15]. The technique used by nodes for selecting coefficients to make network codewords is Reed solomon parity check matrix. The limitation is only one hop network topology is considered and inter-user communication is possible. The technique used by nodes for network coding is good for deterministic topologies.

2.3 Modeling of OLA Network

The high density network is modelled using continuum assumption [16]. There is a 2-D strip network in which node or relay density goes to infinity while the power per unit area remains constant. The nodes that decode at the previous level will only participate in the next transmission. If the decoding threshold is below a critical value then continuum model assures that the message will reach the destination irrespective of the distance between source and destination and the transmissions will die out otherwise. The probability of successful decoding decreases with the increase in decoding threshold. Hence the probability of reaching the destination depends up on the node density. The continuum model is applicable only when the nodes density is too high. Therefore, it is not good for the applications which required low node density. The modeling of finite density network is done in [4]. In this paper author model the linear finite network model in which nodes are placed deterministically. The coverage is found using transition matrix of Markov chain. The quasi stationary distribution is found by computing the maximum eigen vector of the transition probability matrix without the killing states. As the distance between source and destination increases, transmissions will eventually die out. The generated transition probability matrix is too large.

This deterministic network topology is only applicable on linear grid node topologies.

2.4 Research Idea

The authors in [17] analyze the impact of intra-flow interference caused by multiple packets transmitted from the same source in OLA network using continuum assumption. The behaviour of cooperative network is analyzed and observes whether transmission reaches its destination or die off. Numerical simulation is used to find the optimal packet insertion rate. In this study the considered model is deterministic and infinite node density is not realistic for practical networks. The multiples flows in this study is from the same source. In [4] multi-hop cooperative network is analyzed with linear network topology and finite density of nodes. The network is modeled with Markov chain to find the outage probability and coverage of the system. Also in this study single source is considered.

The interference analysis in cooperative linear multi-hop network subject to multiple flows is studied in [18]. Packet insertion rate is analyzed subject to interference from other flow of the same source. The closed form expression for the ratio of two hypoexponential random variables is found in this paper. This gives us idea to study the multi-hop cooperative network in which different source excites the network how the information of these sources transmits to the far off destination with the help of relays at the intermediate hops. The generalized stochastic modelling of OLA network is analyzed in [5] with 2D random geometry. The wireless channel is Rayleigh faded. There is only one source node which excites the whole strip-shaped random network. Hence when there are large number of nodes deployed to form a multi-hop wireless network e.g., WSN for sensing purpose. There is the probability that more than one sensor node broadcast their information. Therefor, we consider multiple sources for deterministic as well as random topologies and stochastically model them with Markov chain. We use linear network coding for combining the information of different users to form network coded codeword and avoid interference.

Chapter 3

Deterministic Network Topology

In this chapter, the system model and their parameters are described. Network coding technique to form codewords and decoding of the codewords is explained. The analytical modeling of deterministic distrbuted and co-located topology are done with the help of Markov chain. Quasi stationary distribution is found using transition matrix by excluding killing probabilities. Analytical modeling is done for M sources and N relays for both distributed and co-located topologies.

3.1 Distributed Topology

Consider a network topology in which multiple sources M, where $M \geq 2$, are transmitting their information to a far off destination D. The transmission traverses a multi-hop network where each hop (or level) contains N relay nodes which are at a distance d from the adjacent cluster of relay nodes as shown in Fig. 3.1. To keep a deterministic path loss, the distance between the adjacent nodes of each cluster is kept at w in this paper. This assumption however, can be relaxed by having random node locations and averaging the results over Monte-Carlo trials or by assuming a Poisson point process (PPP), which could be a future direction to this work.

We assume that all the nodes including the sources transmit with a power P_t on orthogonal channels [20]. Hence there is no interference while receiving the information of multiple sources at a relay node. A relay node in the first cluster decodes the information from the all sources independently. The decode-and-forward (DF) technique is used by all relay nodes to broadcast the decoded information to the next cluster of nodes. Only those nodes in a level participate in transmission that decoded the information of all sources, hence the name opportunistic. The DF nodes of the first hop perform suitable processing at the physical layer to combine the information of all sources to make codewords and transmit the network coded data to the next cluster of nodes after applying network coding (NC). The nodes of the second hop and onwards decode the codewords to extract the information of the sources. These nodes are DF nodes if they sucessfully decode the codewords. For example, in Fig. 3.1, the nodes represented by filled circle are DF nodes, whereas the nodes that decode either L data information, where $L < M$, or do not decode any information are represented as hollow circles.

The nodes will relay the network codeword, $C_{l,m}^{(n)}$, with power P_t to the next cluster of nodes. The $C_{l,m}^{(n)}$ is the codeword from node l at level $(n-1)$ to a node m at level n. The received signal at node m at level n is given as

$$Y_{l,m}^{(n)} = h_{l,m} C_{l,m}^{(n-1)} + w_{l,m}, \tag{3.1}$$

where l,m denote the transmitting and receiving nodes, respectively $l, m \in \{1, 2, 3, ...N\}$, while the superscript denotes the receiving level. Here $w_{l,m}$ is the additive white Gaussian noise with double-sided power spectral density $N_o/2$ and $h_{l,m}$ takes into account the flat fading channel and path loss, i.e., $h_{l,m} = \frac{g_{l,m}}{{d_{l,m}}^{\beta}}$, where $\mid g_{l,m} \mid\sim$ Rayleigh distributed and $d_{l,m}$ is the Euclidean distance between nodes l and m. The β is the path loss exponent. These codewords are decoded by the nodes if and only if nodes decode codewords greater than or equal to the number of transmitting sources. The received information from codewords are again coverted into codewords for further transmission. This process continue until the number of nodes to relay information is lesser than number of sources. Therefore, to propagate the information of all sources we need at least M relays i.e., $N \geq M$.

3.2 Network Coding Scheme

We use the network coding coefficient matrix [19] to design the network codes (linear finite field codes) for our system. The network coding coefficient matrix T is a $N \times M$ matrix given as

$$T = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,M} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N,1} & a_{N,2} & \cdots & a_{N,M} \end{pmatrix},$$

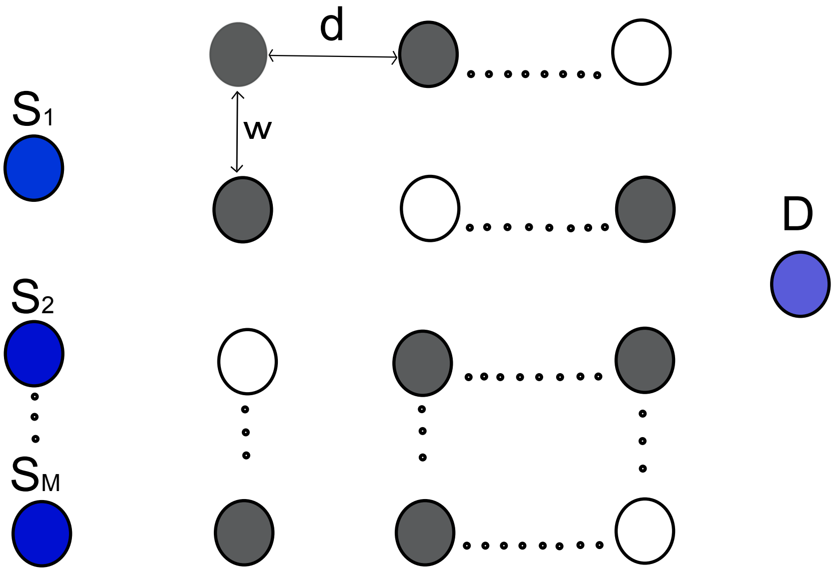

Figure 3.1: Realization of deterministic network topology for M Sources

where each row makes a codeword for one relay, which is a linear combination of source informations I_i $\forall i \in \{1, ..., M\}$. The coefficient $a_{l,m}$ is selected within a defined finite field (Galois field). To acheive diversity in this system, the selected codewords should be linearly independent from one another. Hence to make linearly independent codewords, the coefficient matrix is maximum distance separable (MDS). The Reed-Solomon (RS) codes are commonly used as a variant of MDS codes [13], which are used in this study.

3.3 Decoding of codewords

Let M sources transmit their respective information I_1,I_2,...,I_M, to the first hop nodes. The outage probability of a node m at the first hop for any of the M sources is given as

$$P^{(1)}_{o_{l,m}} = \mathbb{P}\left\{P^{(1)}_{r_{l,m}} < \tau\right\}, l \in \{S_1, S_2, ..., S_M\}, \tag{3.2}$$

where $P^{(1)}_{r_{l,m}}$ is the received power from node l to node m at first hop and τ is the modulation dependent threshold. Assuming $\mid h_{l,m} \mid^2 \sim$ Exp $(d^{\beta}_{l,m})$, (3.2) can be written as

$$P^{(1)}_{o_{l,m}} = 1 - \exp(-d^{\beta}_{l,m}\tau). \tag{3.3}$$

The nodes at level 1 can be considered as DF nodes if they decode all M sources information I_1,I_2,...,I_M, i.e., $P^{(1)}_{r_{l,m}} \geq \tau$, where $l \in \{S_1, S_2, ..., S_M\}$ and m denotes the receiving node index at level 1. In other words,

$$\mathbb{P}\{\textit{node m is DF in level } 1\} = \prod_{l=1}^{M} \mathbb{P}\{P^{(1)}_{r_{l,m}} \geq \tau\},$$

These DF nodes form linearly independent network codes using rows of matrix T. These codewords are linearly independent from one another so that only M relayed signals are sufficient to recover M sources information. For example, in case of two sources $I_1 + I_2$ and $2I_1 + 3I_2$ are linearly independent; one can recover both sources information using a simple Gaussian elimination method. This condition can also be checked by evaluting the rank of network coding coefficient matrix T, which should be equal or greater than the number of sources to perfectly decode the signals for M sources.

Denoting $\xi^{(n-1)}$ to be a set of indices of DF nodes at previous hop or level $n-1$, the probability of outage at current hop or level n ($n \geq 2$) for a node m can be calculated as

$$P^{(n)}_{o_{(m|\xi^{(n-1)})}} = \prod_{l\in\xi^{(n-1)}} P^{(n)}_{o_{l,m}} + \sum_{z=1}^{M-1}\left(\sum_{v=1}^{\chi}\left(\prod_{a=1}^{z} P^{(n)}_{o_{\varphi,m}} \prod_{b=z}^{M-z}(1 - P^{(n)}_{o_{\vartheta,m}})\right)\right), \tag{3.4}$$

where $\chi \doteq \mid \xi^{(n-1)} \mid$ is the cardinality of set $\xi^{(n-1)}$. For successful transmission $\mid \xi^{(n-1)} \mid \geq M$, $\varphi \doteq \left(\xi^{(n-1)}(v+a-1)\right)_{\chi}$ and $\vartheta \doteq \left(\xi^{(n-1)}(b+v)\right)_{\chi}$ denotes the transmitting node from level $(n-1)$ and $(.)_{\chi}$ shows modulo-χ operator. The $P^{(n)}_{o_{l,m}}$ is the outage probability of the codeword from node l at level $(n-1)$ to node m at level n given in (3.3). These probabilities are different because of different path loss and will be same if we consider all nodes at each hop to be co-located, which is described in next section.

3.4 Markov chain Modeling

At any time instant, a relay node at level n will transmit the codeword to the next hop with a condition that the node had already decoded the M sources information using codewords perfectly, or it will not participate, if it had not decode at least M codewords. The state of system at nth level can represented as $\mathbb{X}(n) = [\mathbb{I}_1(n), \mathbb{I}_2(n), \mathbb{I}_3(n), ..., \mathbb{I}_N(n)]$, where $\mathbb{I}_m(n)$ is the binary random variable for the mth node at level n and can be represented as

$$\mathbb{I}_m(n) = \begin{cases} 0 & \textit{node m does not decode data} \\ \\ 1 & \textit{node m decodes data} \end{cases} \tag{3.5}$$

Therefore, each node of a particular hop can be represented by either 0 or 1 depending whether it has successfully decoded the M codewords or not. For N nodes in a cluster, there are 2^N states within the whole system. We use discrete-time finite-state Markov chain to model the system and assume that the statistical conditions of the channel are same for all the hops. Moreover, there is a probability that the Markov chain can go into an absorbing state and the process of transmission is stopped. This happens when all the nodes of a hop can't decode the information. Hence we define two matrices to represent the Markov chain. One is full transition probability matrix $\mathbf{P}'$ for all absorbing and irreducible states of the system and the other matrix $\mathbf{P}$ will model transient states only. The rows of the matrix $\mathbf{P}'$ sum to 1. The matrix $\mathbf{P}$ is a submatrix of $\mathbf{P}'$ and due to the killing probabilities, the row sum is always less than unity. These killing probabilities can be express as

$$\kappa_i = 1 - \sum_{j \in \mathcal{B}} \mathbf{P}_{ij}, \;\; i \in \mathcal{B} \tag{3.6}$$

where $\mathbf{P}$ is a square irreducable and non negative matrix and $\mathcal{B}$ denotes the irreducible state space of the system. According to Perron-Frobenius theorem [21], there exists an exclusive and maximum eigenvalue, ρ, such that the related eigenvector is unique and contains only positive entries [21]. The theory of Markov chain says that if $\mathbf{u}$ is the left eigenvector of matrix $\mathbf{P}$, then the distribution $\mathbf{u} = u_i, i \in \mathcal{B}$ is called ρ-invariant distribution, where ρ is corresponding eigenvalue of matrix $\mathbf{P}$, i.e., $\mathbf{uP} = \rho\mathbf{u}$.

As the probability of going into absorbing state is greater than zero, therefore eventually killing of process is occurred. However the probability for the system to be in state j at level n is given as

$$\mathbb{P}\{\mathbb{X}(n) = j\} = \rho^n u_j, \;\; j \in \mathcal{B}, \;\; n \geq 0. \tag{3.7}$$

Hence ρ can be used to characterize the coverage of the network using the properties of transition matrix.

3.4.1 Transition probability matrix

To compute the entries of transition probability matrix $\mathbf{P}$, we have to find probability of success/outage at certain level n. Let i and j be the states of system such that $i, j \in \mathcal{B}$. For instance, if N=4, then i = [1101] at level 1 from Fig. 3.1. A sucessful hop occurs when a minimum of M nodes are in state 1, i.e., $\sum_{m=1}^{N} \mathbb{I}_m(n) \geq M$, where the probability of a node m to go in state 1 is given by $1 - P^{(n)}_{o_{(m|\xi^{(n-1)})}}$ and $P^{(n)}_{o_{(m|\xi^{(n-1)})}}$ is calculated using (3.4). The probability of going from any transient state i to any transient state j depends upon the position of nodes, which decode the informations. The one-step transition probability from state i to j can be found by using

$$P_{ij} = \prod_{m \in \xi^{(n)}} (1 - P^{(n)}_{o_{(m|\xi^{(n-1)})}}) \times \prod_{m \in \bar{\xi}^{(n)}} (P^{(n)}_{o_{(m|\bar{\xi}^{(n-1)})}}) \tag{3.8}$$

where $\xi^{(n)}$ and $\bar{\xi}^{(n)}$ are the set of indices of nodes which are 1 and 0, respectively, in state j at level n. For example, if there are four nodes at each hop, $N = 4$ then there are eleven transient states for M=2, i.e., at a certain level n, state of system is [1 0 1 1] then depending upon previous state i, the one-step transition probability is given as

$$P_{ij} = \prod_{m \in \{1,3,4\}} P_{s_{(m|\xi^{(n-1)})}} . P_{o_{(2|\xi^{(n-1)})}}, \tag{3.9}$$

where the probability of success is $P_{s_{(m|\xi^{(n-1)})}} = 1 - P_{o_{(m|\xi^{(n-1)})}}$, $\xi^{(n)} \in \{1, 3, 4\}$ and $\bar{\xi}^{(n)} \in \{2\}$. When all the entries of the transition matrix are calculated, the Perron eigenvalue gives us the information about one-hop success probability, which is further used to find the coverage of the network.

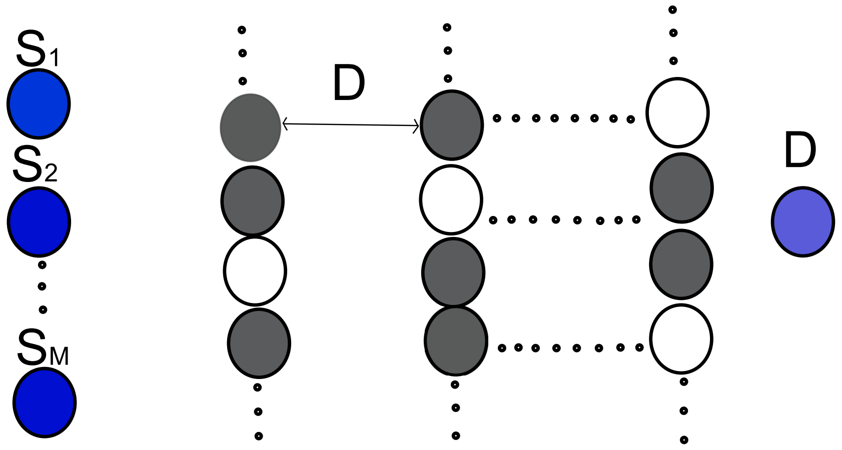

Figure 3.2: Realization of co-located network topology for M Sources

3.5 Co-located Topology

We consider another network topology in which the distance w between the adjacent nodes of a cluster is negligible hence the N relay nodes form a co-located group. The only distance left is the inter-group distance D therefore one co-located group is D distance apart from another co-located group as shown in Fig. 3.2. As previously, those nodes that decode the message, make network codewords and transmit these codewords to the next level using orthogonal channels. The probability of outage of node m at current level n can be calculated as

for χ=M,

$$P^{(n)}_{o_{m,\xi^{(n-1)}}} = \sum_{\alpha=1}^{\chi} {}^{\chi}\mathcal{C}_{\alpha}\, P_o^{\alpha}(1-P_o)^{\chi-\alpha}. \tag{10a}$$

and for $\chi > M$,

$$P^{(n)}_{o_{m,\xi^{(n-1)}}} = \sum_{\alpha=\chi-(M-1)}^{\chi} {}^{\chi}\mathcal{C}_{\alpha}\, P_o^{\alpha}(1-P_o)^{\chi-\alpha}. \tag{10b}$$

where ${}^{\chi}\mathcal{C}_{\alpha} = \frac{\chi!}{\alpha!(\chi-\alpha)!}$ denotes the 'combination' operator and $P_o = 1 - exp(-D^{\beta}\tau)$, is the outage probability of any node in a group. This probability of outage is same for all nodes because of the same path loss.

The rest of the procedure remains the same. Eq. (10) is used alongwith other parameters to formulate the transition matrix, whose properties provide us information about system performance, which is explained in next section.

Chapter 4

Modeling of Random Topology

In OLA network nodes are distributed randomly in a large area. Hence to make our model close to OLA we model the strip-shaped random network of area L. We also consider the nodes to participate in transmission which, decode the partial information of sources. There are total four states in the network by considering two sources. Therefore, we found the state distribution at each hop to find the coverage of the network and analytically model them.

4.1 System Model

In the previous chapter, we model the scenario in which the nodes are placed deterministically. Now in this topology, we assume that the sources are uniformly distributed in a region of area $L \times L$. Similarly, at each hop there are N relay nodes, which are randomly placed in regions of area $L \times L$ each. The network is extended in the horizontal direction with each $L \times L$ regions that are contigious as shown in Fig. 4.1. There are fixed number of nodes at each hop but randomly placed, making a binomial point process (BPP) at each hop. The distance between nodes of the adjacent hop is random. The transmission strategy remains the same, i.e., the nodes which decode at level $(n-1)$ will participate in transmission to the nodes of level n. Decoding depends on the channel condition and location of node in a region. The nodes that receive both sources information will make network codewords and transmit them to the next level. We assume that all the nodes that decode both sources' information use random network coding and the codewords made by the nodes are independent. However, in this topology, we also assume that the nodes which decode either I_1 or I_2 can also forward their decoded information to the next hop.

It can be observed that the receiving nodes at the first hop can thus be categorized in three different groups; one having the information of source 1, second with the information of source 2 and third group contains the nodes that receive both informations. For example, in Fig. 4.1., the nodes represented by hollow circles decode nothing, filled circle nodes decode both informations, horizontally-hashed nodes decode source 1 information and vertically-hashed nodes decode source 2 information, respectively. Therefore, there are four possible states for a node at a receiving level as follows

- State 0= if node decodes nothing,
- State 1= if node decodes only I_1,
- State 2= if node decodes I_1 and I_2,
- State 3= if node decodes only I_2.

If we model this network with a Markov chain, there are 4^N states in

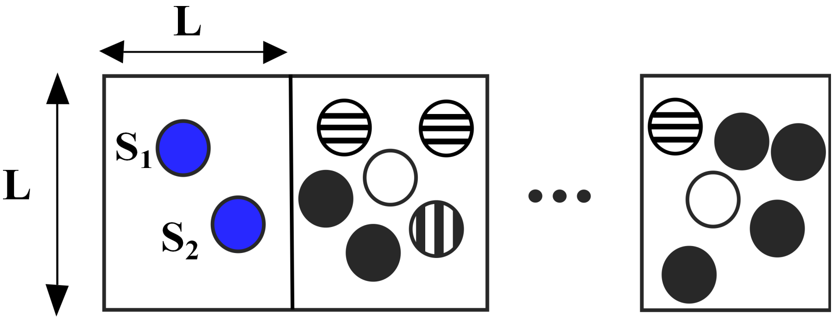

Figure 4.1: Random topology with 6 nodes at each hop

the system e.g., if N=6 then total states will be 4096. As the number of nodes in a hop increases, the problem of state-space explosion occurs, which implies that the states of the system become enormously large. Therefore, we calculate the probability of state distribution at each hop to analyze our model. In other words, we are interested in finding the percentage of the nodes in a level that are in 0, 1, 2,or 3 state, respectively.

4.2 State Distribution Probability

The nodes at the first hop receive two signals from both sources S_1 and S_2. As the informations are different, there is a SISO link between the sources and each of the nodes at the first hop. The power received at a j^{th} node in a SISO link is given as

$$P^{(1)}_{r_{s_i,j}} = \frac{P_t \mu_{i,j}}{d^{\beta}_{i,j}}, \tag{4.1}$$

where the $s_i \in (S_1, S_2)$ and $\mu_{i,j}$ is a random variable (RV) that denotes the channel gain between nodes i and j, which is exponentially distributed with unit mean. The sources and the nodes of the first hop are randomly

deployed so the Euclidean distance between them becomes random. From [22], the distribution of the squared distance between two randomly placed node in a contigious $L \times L$ regions is given by Weibull distribution. The outage probability of a SISO link for this model in the presence of Rayleigh fading and arbitrary path loss exponent β is calculated as [5]
for $c \geq 1 (\beta \leq 3.1612)$

$$P_{o(SISO)} = \frac{\tau\chi}{P_t} \sum_{n=0}^{\infty} \frac{1}{(n+1)!} \Gamma\left(\frac{1+c+n}{c}\right) \left(-\frac{\tau\chi}{P_t}\right)^n . \tag{4.2}$$

where for Weibull distribution shape parameter, $k = 1.5806$ and variable scale parameter $\lambda = \frac{4L^2}{3\Gamma(1.6327)}$, $c = \frac{2k}{\beta}$ and $\chi = \lambda^{\frac{\beta}{2}}$.

As stated earlier, the nodes at the first hop can be divided in three groups ignoring the state 0 nodes. These groups are G_1, G_2, and G_3 representing nodes in State 1, 2 and 3, repectively. Using Eq. (4.2), the success probability of a node to go in any state $State1$, $State2$ or $State3$ can be found. Also a node j is in outage (state 0) if it does not decode any source information. Hence the probability of state distribution for node j can be given by a 4×1 vector $\mathbf{V}_j$, where

$$\mathbf{V}_j = [P^2_{o(SISO)}, \quad P_{s(SISO)} \times P_{o(SISO)}, \quad P^2_{s(SISO)}, \quad P_{o(SISO)} \times P_{s(SISO)}]^T .$$

In the above definition, $P_{s(SISO)} = 1 - P_{o(SISO)}$, whereas the entries of vector $\mathbf{V}_j$ are the probabilities of node j to go in state 0, 1, 2 and 3, respectively.

From second hop and onwards, the three groups of nodes (in state 1, 2, and 3) transmit their informations. It can be observed that as time proceeds, there will be more nodes which could have decoded both the information. This is because there is a greater possibility that the receiving node receives information by two different groups; either it is by the nodes which have information of a single source or by the nodes which already have information from both sources. In any case, the receiving nodes will have two different informations, i.e., from both the sources. Hence more number of nodes will have both informations. As the hop count increases, the number of nodes in state 2 increases until it reaches a maximum value.

The power received for node j at a level n $(n \geq 2)$ from either G_1 or G_3 is given as

$$P_{r_{i,j}}^{(n)} = \sum_{\xi_k^{(n-1)}} \frac{P_t \mu_{i,j}}{d_{i,j}^{\beta}} \tag{4.3}$$

where $k \in \{1, 3\}$ and $\xi_k^{(n-1)}$ denotes the set of indices of state 1 and 3, respectively.

Maximal ratio combining (MRC) is used at a receiving node, therefore, the SNR at the receiver will be the sum of the individual SNRs of signals received from the relaying nodes. MRC will be used for the node, which has either transmitted I_1 or I_2. This is a MISO link as multiple inputs are transmitting towards a single receiver e.g., nodes that decode I_1 at level $(n-1)$ will transmit same information to a node at level n. This creates a virtual MISO system. The outage probability of MISO system is calculated as [5]

$$P_{o(MISO)} = \chi^{\sigma} \sum_{a_1=0}^{\infty} \dots \sum_{a_\sigma=0}^{\infty} \frac{(\tau/P_t)^{a_1+a_2+\dots+a_\sigma+\sigma}}{(a_1+a_2+\dots+a_\sigma+\sigma)!} \prod_{i=1}^{\sigma} \Gamma\left(\frac{1+c+a_i}{c}\right)(-\chi)^{a_i} \tag{4.4}$$

where σ is the number of transmitter in the previous level i.e., $\sigma \in G_1$ and G_3. From second hop and onwards, we calculate the state distribution of all the states of the system using (4.2) and (4.4). We use (4.2) for the nodes in state 2 because these nodes make linearly independent combinations.

The outage probability of SISO link can be used to calculate the outage probability of state 2 nodes beacause these nodes are transmitting different information in the form of network codewords. To understand, consider an example that at level $(n-1)$ two of the nodes are in state 1, two in state 2 and a single node in state 3. Node j that is at level n can be in one out of four states depending upon its decoding. The probability of being in state 0, i.e. it has not decoded any information and nothing to forward, is given as

$$\begin{aligned}\mathbb{P}\left\{node\ j\ in\ outage\right\} = &\left(P_{o(MISO)} \times P^{k}_{o(SISO)}\right) \\ &+ \left(P_{o(MISO)} \times P_{o(SISO)} \times \binom{2}{1} P_{o(SISO)} \times P_{s(SISO)}\right)\end{aligned}$$

Probability of the node j being in state 1 is;

$$\mathbb{P}\left\{node\ j\ in\ state\ 1\right\} = \left(P_{s(MISO)} \times P^{k}_{o(SISO)}\right)$$

where $k \in$ number of nodes that make SISO link with node j at the next level i.e., in the given example, k=3. Similarly we can calculate state 2 and state 3 distributions.

To find number of nodes of each state in a single hop, we multiply the probability of state distribution with the total number of nodes at each hop. As we discussed that the number of nodes in state 2 increases when we come close to the destination. Therefore, the coverage depends on the number of nodes that are in state 2. The system is in coverage untill the number of nodes in state 2 is greater than or equal to two.

Chapter 5

Results and Analysis

In this chapter, we analyze the numerical results generated through simulations. Fig. 5.1 shows the relation between the probability of a certain state of the system versus the number of hops. In this result, the distance w is kept at 2, while the inter-hop distance d is 1. Also the path loss exponent is taken to be 2 while $M = 2$ and the number of nodes at each hop is four, i.e., $N = 4$. The SNR margin, γ, which is normalized SNR, i.e., $\gamma = P_t/\tau N_o$ is taken to be 10 dB. It can be seen that analytical results obtained by the use of Markov chain method almost approach the simulation results. Analytically, the transition matrix is computed and the left eigenvector, corresponding to Perron eigenvalue, gives the state distribution probabilities. It can be seen that as the number of hops increases, the probability of achieving a certain transient state decreases. This is due to the âĂIJkilling probabilitiesâĂİ as discussed earlier in the previous sections. The state of the system in which all nodes are DF nodes can cover more hops as compared to other states of the system. To avoid repetition, only 3 states out of 11 transient states are shown in the graph.

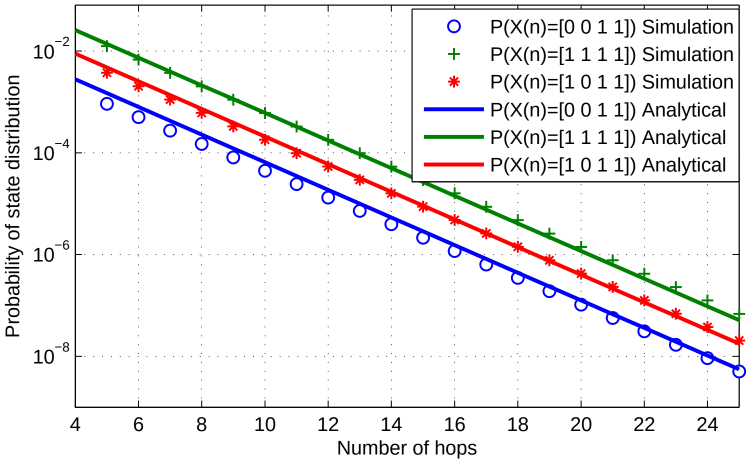

Figure 5.1: Probability of states distribution; P_t=1, w=2, d=1, $\beta = 2, \tau = 0.1, M = 2, N = 4$.

Fig. 5.2 shows the relationship between the SNR margin and the Perron eigenvalue, ρ. The eigenvalue represents the one-hop success probability in our network. The one-hop success probability implies that at least M nodes are in success for onward transmission of data. As the system has more SNR margin to offer, the eigenvalue approaches to 1. It can also be seen from the figure that as N increases, the eigenvalue converges to 1 with a lesser SNR margin. This is due to the fact that as N increases, the diversity gain is increased and thus the one-hop success probability is increased. The one-hop success probability is increased for N=6 and after that there is no considerable increase in the offered diversity gain because of increased path loss.

From the network designer point of view, a certain quality of service (QoS), η, is required to be maintained in the system. In other words, if we

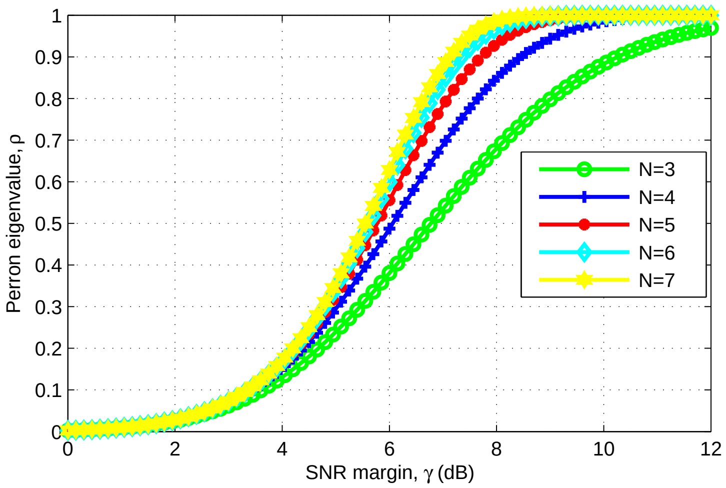

Figure 5.2: Perron eigenvalue for different values of N; M=2, P_t=1, $\beta = 2, d = 1, w = 1$.

are interested in calculating the probability of delivering the message to n^{th} hop and require this probability to be greater than η, then $\rho^n \geq \eta$ where n is the number of hop. Fig. 5.3 shows the relation between QoS and normalized distance, where we define the normalized distance as $n \times d$, where d is distance between two adjacent hops. We show results for different values of N i.e., 5, 6, and 7 and QoS η. We can see that by increasing N, we can deliver our message far away with a particular QoS. Hence if we want to deliver message with QoS ($\eta = 0.9$) and our destination is far away then we need to increase N or we can say that we may require more transmitting nodes at each hop.

The one-hop success probability of co-located network topology is shown in Fig. 5.4. In this plot, the one-hop success is increased significantly when the number of nodes in a co-located group increases as compared to the previous plot of the ρ in Fig. 5.2. In co-located topology, the path loss

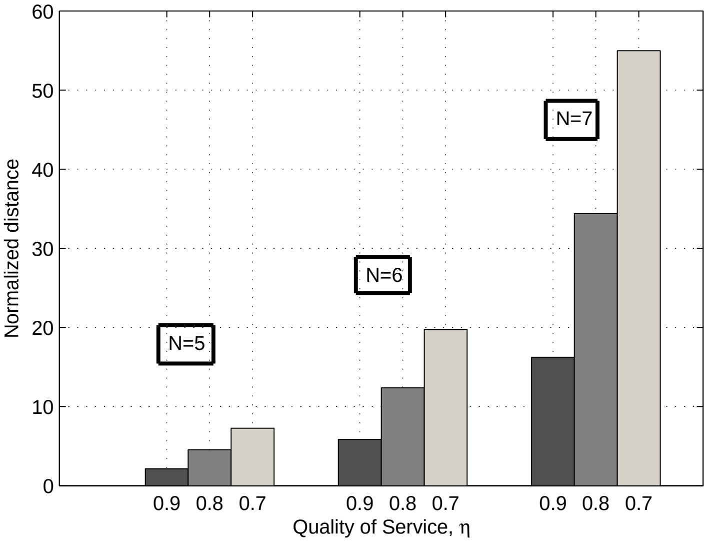

Figure 5.3: Normalized distance for different value of N; γ=6 dB, P_t=1, M=2, $\beta = 2, d = 1, w = 1$.

is compensated after a specific SNR margin, however, this is not the case with distributed topology. Hence the co-located topology provides us better coverage than the distributed topology.

Fig. 5.5 shows the Perron eigenvalue difference of co-located and distributed topologies, where ρ_D and ρ_d are the one-hop success probability of co-located and distributed topology, repectively. The difference is high for the lesser SNR-margin and as γ increases the difference becomes small because the effect of path loss diminish with very high transmit power in both topologies. At lower SNR-margin, the co-located network topology performs better than the distributed topology.

Fig. 5.6 represents the number of relays per hop, N, required to acheive a certain QoS, $\eta \geq 0.8$, versus SNR-margin. This shows that, as number of

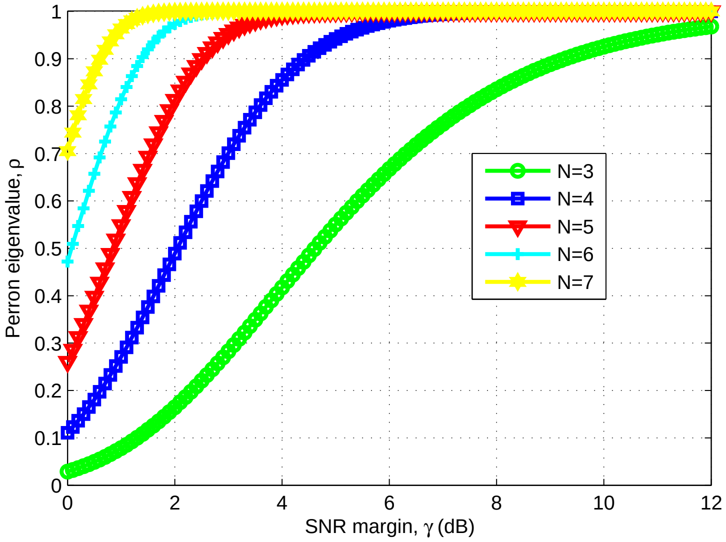

Figure 5.4: Perron eigenvalue for co-located network topology; M=3, P_t=1, $\beta = 2, D = 1$.

M increases, we require more relays nodes per hop to maintain the QoS at a fixed SNR-margin. The required N can be small if SNR margin becomes high. Hence if the information of about the number of sources is known we can design the network by finding the number of relay nodes N required to maintain the QoS. For instance, if SNR margin=5dB, M=5, then we need at least eight nodes (N=8) in each hop to acheive success probability greater than 80%. However, for the same number of sources if more SNR margin is available, e.g., 10dB, then the required nodes per hop becomes 5.

We now discuss the random topology in which the sources and the relay nodes are randomly placed. Fig. 5.7 shows the analytical and simulation matching of the state distribution of this topology. We have shown the state 2 and state 0 distribution of the analytical and simulation models. As we can

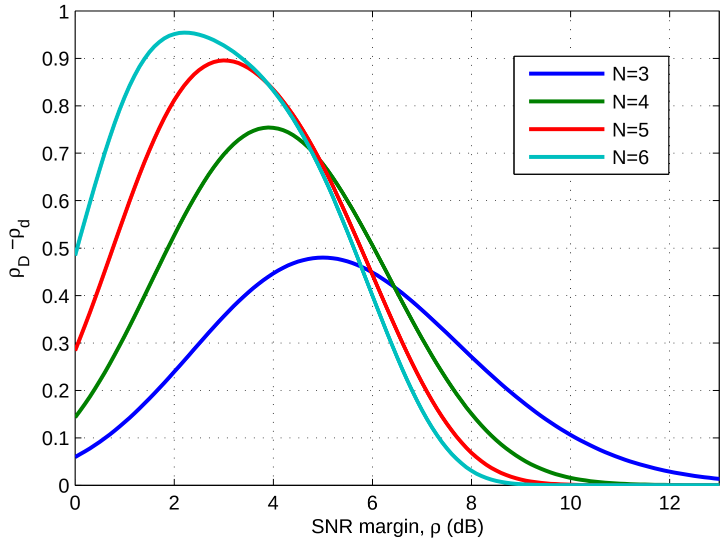

Figure 5.5: Perron eigenvalue difference between co-located and distributed network topology; M=2, P_t=1, $\beta = 2, D = 1, d = 1, w = 1$.

seen that these state distribution probabilities match for both cases. This plot also depicts the relationship of state distribution to the number of hops. The system parameters are; $P_t = 1$, $\beta = 2$ and γ=16 dB. It can be see that as the number of hops increases, the distribution of state 2 increases or in other words, the number of nodes in state 2 increases. Since the total number of nodes at each hop is constant, this implies that the number of nodes in state 1 and 3 decreases with time.

Fig. 5.8 shows the relation of the number of nodes in state 2 versus the SNR margin. As we know that the coverage depends on the number of nodes in state 2 because the number of nodes in state 1 and 3 approach to zero as we move closer to the destination. Hence we require minimum of two nodes that are in state 2 to recover both sources' informations. Therefore, we can

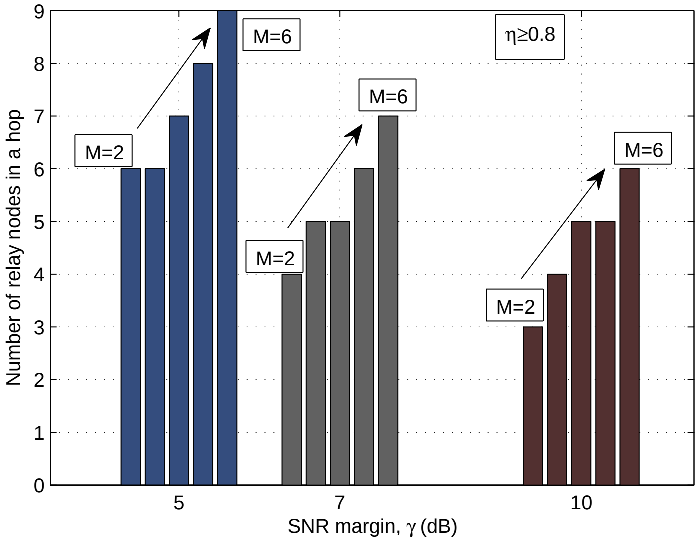

Figure 5.6: Number of N required for different values of M at certain QoS ; P_t=1, $\beta = 2, d = 1, w = 1$.

find the coverage of the system using this result. As we increase the SNR margin, the number of nodes in state 2 increases because more nodes can decode both sources information. This results is taken at the fifth hop of the system with path loss exponent β is 2 and L is taken to be 3.

In Fig. 5.9, we compare the coverage of the network for different number of nodes in each hop. In order to survive, a hop should contain more than two nodes which are in state 2. Therefore, the hop-count with at least two nodes is the coverage of the network. From Fig. 8, it can be seen that the number of state 2 nodes increases until a certain hop and then this number decreases because of the killing probabilities. Hence, for N=8, seven hops are in coverage, while for N=10 and 12, thirteen and twenty three hops, respectively, are in coverage. Therefore, the role of diversity is seen in this

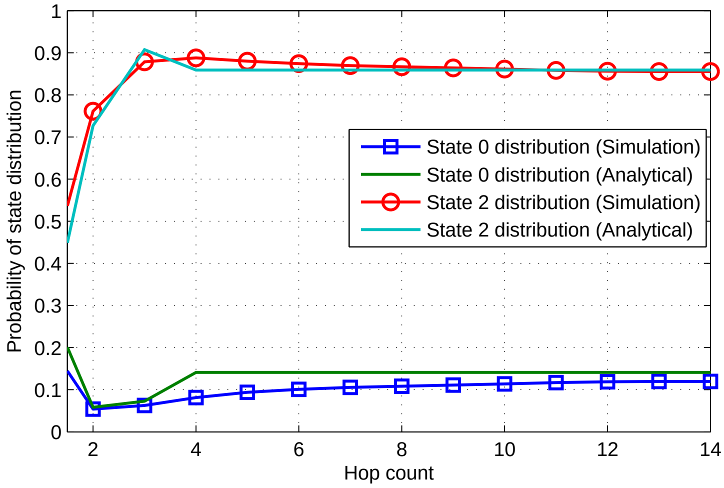

Figure 5.7: States distribution for random topology;L=3,N=8

figure i.e., providing more coverage at the expence of deploying more nodes per hop.

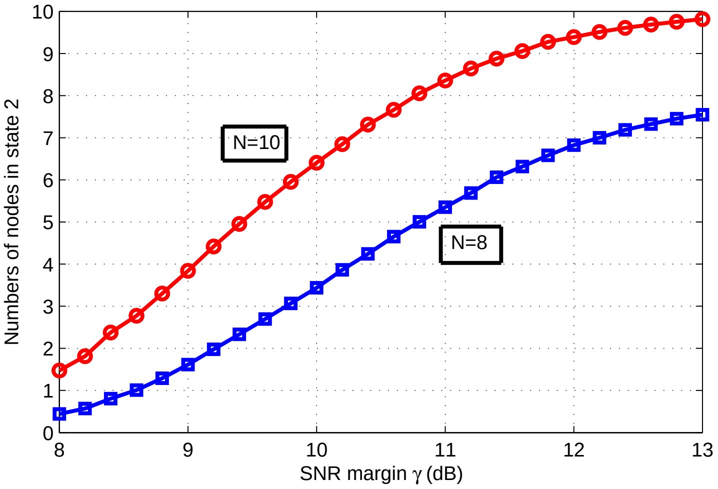

Figure 5.8: Number of nodes in state 2 at 5^{th} hop;P_t=1

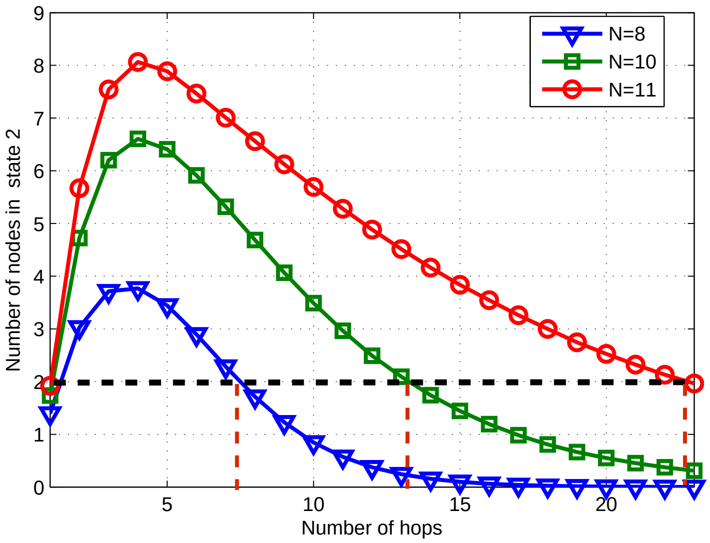

Figure 5.9: Coverage of system with different N;P_t=1,$\beta = 2$,L= 3, $\gamma = 10dB$

Chapter 6

Conclusions and Future Work

6.1 Conclusions

In this thesis, two topologies deterministic and random were analyzed to model the multi-hop wireless cooperative network. In deterministic topology, we have investigated the distributed multi-source multi-hop network topology and modeled the topology with Markov chain and analyzed this topology by calculating the outage probabilities in the presence of network coding. Using these probabilities, we formulated the state transition matrix and determined the coverage and reliable delivering of message to the destination for different M. We also formulated the state transition matrix for the M source co-located topology.

In random topology, we calculated the state distribution probabilities at each hop. In this topology, we also considered the nodes which decoded only one source information and cannot make network codewords. Using state distribution probabilities, we calculate the coverage of the strip-shaped network.

6.2 Future Work

A potential future work is to model a network where a subset of nodes from one hop can decode partial information from all sources. The source nodes (more than two) are distribution in a region and number of nodes at each hop is random. You can say that nodes are distributed according to Poisson point process instead of Binomial point process. The source nodes can broadcast their information any time in the network and the placement of source nodes are also random (any where in the network). Hence information is propageted in network from any side. The design of an efficient NC will be an interesting problem to investigate in these topologies. And try to develop a technique to achieve maximum diversity using linear or physical layer network coding. The outage of the network can be caluculated by considering the deterministic and random network coding.

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