Compress and Forward Coding for Half Duplex Gaussian Relay Channels

Defining futures

By
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Dedication

Dedicated to my parents.

For always encouraging me and being there when I needed them the most.

Certificate of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any degree or diploma at NUST SEECS or at any other educational institute, except where due acknowledgement has been made in the thesis. Any contribution made to the research by others, with whom I have worked at NUST SEECS or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

Author Name: **Jaweria Amjad**

Signature: ____________________

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Jaweria Amjad

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Abstract

Cooperative communication has become one of the fastest growing domain of wireless communication and is likely to become one of the mainstream technologies in future due to its efficient use of spectrum. This interest in cooperation is not undeserved since it offers advantages of diversity and multiplexing which make it highly desirable for use in wireless channels.

A three node network known as relay channel forms the basic building block of cooperative networks. Therefore a large body of research specially in the realm of information theory has been dedicated to relay channel. Since its inception, several coding schemes have been proposed to enable cooperation between nodes. They can mainly be classified into Decode and Forward, Compress and Forward and Amplify and Forward.

This monologue presents a multi-level compress and forward coding scheme for a three-node relay network in which all transmissions are constrained to be from an M-ary PAM constellation. The proposed framework employs a uniform scalar quantizer followed by Slepian-Wolf coding at the relay. We first obtain a performance benchmark for the proposed scheme by deriving the corresponding information theoretical achievable rate. A practical coding scheme involving multi-level codes is then discussed. At the source node, we use multi-level low-density parity-check codes for error protection. At the relay node, we propose a multi-level distributed joint source-channel coding scheme that uses irregular repeat-accumulate codes, the rates of which are carefully chosen using the chain rule of entropy. For a block length of 2×10^5 symbols, the proposed scheme operates within 0.56 and 0.63 dB of the theoretical limits at transmission rates of 1.0 and 1.5 bits/sample, respectively.

Chapter 1

Introduction

1.1 Introduction

Multi antenna systems offer increased system capacity, spectral efficiency and range. However wireless networks are comprised of energy-limited devices that can't afford this luxury on the up-link. Therefore, they *cooperate* i-e., different nodes pool their antenna resources to form a "virtual-antenna array". This is possible due to the broadcast nature of wireless channels. Therefore without employing extra resources users can emulate MIMO systems by introducing transmit antenna diversity.

The idea that willingness to share power and computation with other nodes in the network can lead to savings in overall network resources has led to an increased exploration of this domain.

Since cooperation can take place whenever the number of nodes exceed two, a three node network forms the fundamental unit of cooperative communication thus making it the major focus of research in this domain. It was introduced by Van der Meulen [1]. He gave upper and lower bounds on the capacity of relay channel in his preliminary work. His observations led to several improvements (of his results) in coming years by Cover and El Gamal[2]. They gave tightest upper and lower bounds for the relay channel capacity. These bounds haven't been met for any channel with the exception of degraded channel (which is not a realistic channel).

With optimum processing at relay still unknown, several coding schemes have been proposed to achieve the capacity bounds given in [2]. They can be broadly classified into the *Decode and Forward* (DF), *Amplify and Forward*

(AF), and the *Compress and Forward* (CF) categories [2].

In DF, relay decodes the message and forwards it to the destination. It is a well researched protocol since its practical implementation is conceptually straight forward. Several of its practical implementations have been proposed using different error-correcting codes [3]-[11]. Performance of DF is hampered by the fact that channel between the source and relay dictates the ultimate achievable rate. Thus failure in decoding at relay results in poor performance.

On the other hand, in AF and CF, the relay does not attempt to decode. In AF, the signal received at the relay is simply amplified before being transmitted to the destination. On the other hand, in CF, the relay compresses its received block and sends the compressed information to the destination. This protocol uses the principle of Wyner-Ziv(WZ) coding[12] to exploit the correlation between signals received at the relay and the destination. This is possible since both relay and destination receive the corrupted version of the same source signal. Once relay transmits the compressed information, destination can recover the original message by jointly decoding the signals received from relay and direct path. Unlike DF, CF always outperforms direct transmission and it has been shown that WZ based CF relaying exhibits optimal behavior asymptotically, achieving upper bound for receiver cooperation in ad hoc networks[13]. Therefore, it can attain several rate points that are not achievable with DF coding strategy.

1.2 Research Statement

The research statement of my thesis is:

"To propose a Compress and Forward relaying scheme for Half Duplex Gaussian relays using Wyner Ziv Coding at the relay."

1.2.1 Contributions

CF relaying promises tremendous gains in terms of capacity, range and reliability as compared to point to point systems theoretically but it can only be used in practical systems if the achievable gains are comparable to those claimed in theory. In this work, we intend to investigate the feasibility of CF based cooperation by first deriving the information theoretic bounds and then comparing them with the simulated practical results. Therefore the objective of this work is to present

- an information theoretic coding scheme for a compress and forward relaying.
- a practical code design that emulates the above information theoretic analysis.

Keeping in view the above mentioned objectives, we propose a multi-level CF (ML-CF) coding scheme for a half-duplex Gaussian relay channel where all transmissions (from both the source and the relay) are constrained to be from an M-ary PAM constellation. The scheme utilizes uniform scalar quantization (USQ) followed by SW coding for compression of the quantization indices at the relay. We first present the information theoretic achievable rates under the M-ary constellation constraint, which serve as a performance benchmark for our subsequent code designs. With the help of numerical results, we demonstrate that a quantizer with M levels suffices for an M-ary PAM constellation. Since the quantization indices need to be compressed, as well as transmitted over a noisy relay-to-destination link, we propose a multi-level distributed joint source channel coding (ML-DJSCC) strategy, implemented with the help of IRA codes, to provide joint compression and error protection to the quantization indices. The rates of the individual IRA codes are carefully chosen using chain rule of entropy. For transmissions from the source, we employ multi-level LDPC codes to provide error protection. The degree distributions for the IRA and the LDPC codes are optimized using the EXIT chart strategy [14] and the Gaussian assumption [15]. Simulations using optimized codes with a block length of 2×10^5 symbols show a performance gap of only 0.56 and 0.63 dB from the theoretical limits at transmission rates of 1.0 and 1.5 bits/sample (b/s), respectively.

1.3 Organization

We start by briefly discussing the related work in the domain of CF relaying. In the same Chapter we summarize some of the preliminary concepts required for understanding our work. Chapter 3 presents the information theoretic expressions that bound the proposed scheme. Following that, we discuss some of the numerical observations and simplifications of our scheme. In Chapter 4, we discuss practical CF scheme using LDPC and IRA codes. In Chapter 5, we examine the convergence behavior of our scheme and present simulation results that evaluate the gap of practical scheme from the theoretical limits.

Chapter 2

Background

2.1 Related Work

Only a handful distinguished works can be found on the practical implementation of CF. This is because practical implementations of WZ was not proposed until recently[16, 17]. First CF relaying scheme using half-duplex Gaussian relays was proposed in [18, 19]. Another CF coding scheme was presented in [20]. This scheme was significantly sub-optimal since the relay was silent in the second time slot resulting in loss of spectral efficiency. It was followed by another work by the same authors for CF coding[21]. They used scalar quantizer followed by convolutional codes to implement WZ at relay. Polar codes were used to implement CF coding by [22]. Authors propose coding strategies based on Slepian Wolf(SW) coding [23] and WZ at relay. With SW coding, the proposed scheme approaches capacity coinciding with the cut-set bound. Some research works have also focussed on quantize-forward compression strategies without the use of WZ at relay [24, 25].

In [26], a CF relaying scheme for the half-duplex Gaussian relay channel was proposed. Instead of using Gaussian modulation, Binary-Phase Shift Keying(BPSK) was used. Authors gave numerical rate bounds of the proposed scheme using Slepian Wolf Coded Nested Scalar Quantizer(SWCNSQ)[27] to implement WZ coding at relay. They then presented practical code design that performed within a negligible gap from the theoretical limit. The scheme was later extended to fading relay channels under a rateless coded setting in [28].

A CF strategy named Quantize Map and Forward(QMF) was proposed in [29]. In this scheme it is assumed that relay doesn't have any Channel

State Information(CSI) and the quantized indices are mapped on to random Gaussian codewords. Destination decodes the relay and source information jointly without first recovering the quantized indices. Encoding and decoding complexities for such a scheme are polynomial and exponential respectively. A practical implementation of this scheme with simplifications for binary codewords was presented in [30]. Authors used LDPC codes for encoding at source and relay. A certain variation of [29] was proposed in [31]. In this paper authors used vector quantizer for compression instead of symbol-wise quantization. Instead of transmitting once, binning indices are sent multiple times using independent code books.

2.2 Preliminaries

This section discusses some of the preliminary foundations for our proposed setup.

2.2.1 Channel Capacities

Achievable rates for CF and DF with Gaussian channel inputs are presented in [32]. Since capacity expressions for Additive White Gaussian Noise(AWGN) channel and quadratic Gaussian WZ coding exist in closed form expressions, achievable rates were somewhat simpler to calculate.

We on the other hand modulate the source and relay messages to an M-PAM signalling scheme therefore we have an M-PAM input AWGN (MAWGN) channel between each pair of nodes, the capacity of which can only be computed numerically as

$$C(P) = m - \sum_{i=1}^{M} \int f\left(y, \sqrt{P}x_i\right) \log \sum_{j=1}^{M} \frac{f\left(y, \sqrt{P}x_j\right)}{f\left(y, \sqrt{P}x_i\right)} \mathrm{d}y \tag{2.1}$$

where $f(z, \mu)$ is the Gaussian probability density function with unit-variance, mean μ, and evaluated at z. We will also need the capacity of a channel in which there is M-ary PAM interference in addition to the AWGN. If P and S are the signal and interference power, respectively, the capacity of such a channel is given as (assuming that the AWGN is unit-variance)[1]

$$C(P,S) = m - \sum_{i=1}^{M} \int \Gamma_i(y) \log \left(1 + \sum_{\substack{j=1 \\ j \neq i}}^{M} \frac{\Gamma_i(y)}{\Gamma_j(y)} \right) \mathrm{d}y \qquad (2.2)$$

with $\Gamma_i(y) = \sum_{k=1}^{M} \frac{1}{M} f\left(y, \sqrt{P}x_i + \sqrt{S}x_k\right)$, $i = 1, \ldots, M$.

A detailed derivation for (2.1) and (2.2) can be found in Appendix. A and B respectively.

[1]With a slight abuse of notation, we will denote the constrained channel capacity with interference as $C(\cdot,\cdot)$ with two arguments and that without interference as $C(\cdot)$ with a single argument.

Chapter 3

CF for Half Duplex Gaussian Relays

In this chapter we first describe the system and channel model used in our work. It is followed by information theoretic achievable bounds of the proposed scheme.

Before moving on, we summarize the important notations used in this work in Table 3.1.

3.1 System Model

As shown in Fig. 3.1, we consider a three-node relay network with d_{sd}, d_{sr} and d_{rd} being the source-destination, source-relay, and relay-destination link distances, respectively. Throughout this thesis, we assume that the distance d_{sd} is fixed while the other two are variable. The exact value at which d_{sd} is not important since the same channel coefficients can be obtained by scaling all distances appropriately. However, for expositional clarity, we assume that $d_{sd} = 1$ meters (m). The corresponding (real) channels suffer path-loss, with the channel gains given as $c_{sd} = 1$, $c_{sr} = (d_{sd}/d_{sr})^{3/2}$ and $c_{rd} = (d_{sd}/d_{rd})^{3/2}$. We assume the presence of global channel state information, i.e. each node is assumed to be aware of all three channel coefficients. All channels are assumed to have additive white Gaussian noise (AWGN), the variance of which we assume, without loss in generality, to be unity. The transmissions from the source as well as the relay are assumed to be modulated on to an M-ary PAM constellation. Both the source and the relay are assumed to have an *average* power constraint of P_s and P_r, respectively. Owing to the half-duplex nature of the relay node, the total transmission period of N symbols is divided

Table 3.1: Important Notations

Entity	Notation	Description	Remarks
Transmission	α	Half-duplex time sharing constant	$\alpha \in (0,1), \bar{\alpha} = 1-\alpha$
	T, T_1, T_2	Length of complete transmission cycle,BC and MAC mode respectively	$T_1 = \alpha T, T_2 = \bar{\alpha} T, T = T_1 + T_2$
	X_{s1}, X_{s2}	Source transmission in BC and MAC mode respectively	-
Modulation	M	Number of constellation points in PAM	$M = 2^m$
	m	Number of bits in a PAM symbol	
	$\mathcal{X}_M$	PAM signal set	$\mathcal{X}_4 \in \{-3A, -A, A, 3A\}$
	A	Amplitude of the PAM signal set	
Quantization	L	Quantization levels in scalar quantizer	$L = 2^l$
	l	Bits in the Quantizer output	
	q	Quantization step size of scalar quantizer	-
	W	l-bit Compression index(Quantizer output)	$W = W_1, W_1, .., W_l$
Codes	X_{ri}	Multilevel DJSCC codeword for the i^{th} code	$i = \{0, 1, .., b-1\}$
	β_i	Multilevel DJSCC constant	$\beta_i \in (0, \bar{\alpha}), \sum_i \beta_i = \bar{\alpha}, i = \{1, 1, .., l\}$

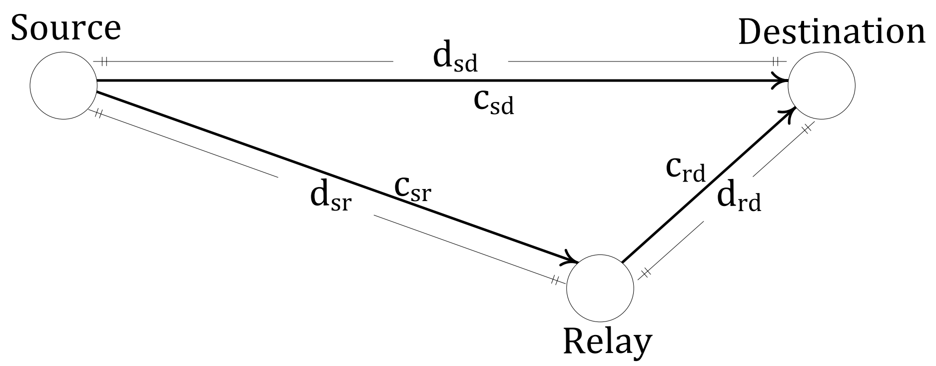

Figure 3.1: The relay channel.

into the relay receive period (denoted as T_1) of length αN symbols, and the relay transmit period (denoted as T_2) of length $\bar{\alpha}N$ symbols, where $\alpha \in [0, 1]$ is the half-duplex time sharing constant and $\bar{\alpha} = 1 - \alpha$. Throughout the rest of the thesis, we will denote sequences with boldface and the associated random variables with italic letters. All logarithms used are to the base 2.

3.2 CF Relaying and Performance Bounds

The source partitions its message into $m = \log M$ streams and encodes each stream with a separate length-N LDPC code. The individual rates of these LDPC codes are denoted as $R_1, \ldots, R_m$ with the overall transmission rate in b/s given as $R = \sum_{i=1}^{m} R_i$. The LDPC codewords are then modulated to an M-ary PAM constellation to obtain N symbols. The first αN symbols denoted by $\mathbf{X}_{s1}$ are transmitted during T_1 subject to a power constraint $E[X_{s1}^2] \leq P_{s1}$, where X_{s1} is the random variable associated with the i.i.d. sequence $\mathbf{X}_{s1}$. The remaining $\bar{\alpha}N$ symbols are transmitted during T_2 and satisfy the constraint $E[X_{s2}^2] \leq P_{s2}$. Due to the average power constraint at the source, we have $\alpha P_{s1} + \bar{\alpha} P_{s2} \leq P_s$. The αN signal sequences received at the relay and destination during T_1 are given as

$$\mathbf{Y}_r = c_{sr}\mathbf{X}_{s1} + \mathbf{Z}_r \text{ and } \mathbf{Y}_{d1} = c_{sd}\mathbf{X}_{s1} + \mathbf{Z}_{d1},$$

respectively, where $\mathbf{Z}_r$ and $\mathbf{Z}_{d1}$ are i.i.d. zero-mean unit-variance Gaussian noise sequences. The relay quantizes $\mathbf{Y}_r$ using an L-level USQ to obtain a sequence $\mathbf{W}$ of quantization indices. Let $k_0, \ldots, k_L$ be the quantization region boundaries. If q is the quantization step size, we have $k_0 = -\infty$, $k_i = \left(i - \frac{L}{2}\right) q$ for $i = 1, \ldots, L-2$, and $k_L = +\infty$. The quantizer output $W = w \in \{0, \ldots, L-1\}$ if the received signal $Y_r \in \{x : x \in \mathbb{R}, k_w \leq x < k_{w+1}\}$. The quantization indices are SW coded with Y_{d1} as the decoder side-information and provided error protection to form a length $\bar{\alpha}N$ codeword sequence $\mathbf{X}_r$ drawn from an M-ary PAM constellation subject to a power constraint $E[X_r^2] \leq P_r/\bar{\alpha}$. Note that since the relay does not transmit anything during T_1, normalizing the power constraint by $\bar{\alpha}$ makes sure that the average power consumption at the relay is P_r. The codeword $\mathbf{X}_r$ is then transmitted to the destination during T_2, the same time as $\mathbf{X}_{s2}$ is transmitted. The destination receives the superposition of both signals:

$$\mathbf{Y}_{d2} = c_{sd}\mathbf{X}_{s2} + c_{rd}\mathbf{X}_r + \mathbf{Z}_{d2},$$

where $\mathbf{Z}_{d2}$ once again is an i.i.d. zero-mean, unit-variance Gaussian noise sequence.

The destination first attempts to recover the quantization indices $\mathbf{W}$ by treating the transmission $\mathbf{X}_{s2}$ from the source as interference. It can do so if [26, 12]

$$\alpha H\left(W \mid Y_{d1}\right) \leq \bar{\alpha} I\left(Y_{d2}; X_r\right) \tag{3.1}$$

The information term on the right hand side of (3.1) is the constrained capacity of an AWGN channel with an M-ary input and which also sees an M-ary interference and can be computed numerically using (2.2)

After recovering $\mathbf{W}$ (and consequently $\mathbf{X}_r$), the destination cancels the interference caused by $\mathbf{X}_r$ and attempts to recover the source message jointly from $\mathbf{Y}_{d1}$, $\mathbf{Y}_{d2}$ and $\mathbf{W}$. The destination is capable of recovering the source message if the transmission rate satisfies

$$R \;\leq\; \alpha I\left(W, Y_{d1}; X_{s1}\right) + \bar{\alpha} I\left(X_{s2}; Y_{d2} | X_r\right). \tag{3.2}$$

We point out that the information terms in (3.1) as well as (3.2) can only be evaluated numerically using (2.1) and (2.2). It should also be noted that the achievable rate expression in (3.2) corresponds to a particular power allocation P_{s1}, P_{s2}, the quantization parameters L and q, and the half-duplexing parameter α that satisfy the constraint (3.1) and the average power constraint $\alpha P_{s1} + \bar{\alpha} P_{s2} \leq P_s$. This leads to a constrained optimization where one needs to search over these parameters to maximize the achievable rate under the given constraints.

We will now briefly allude to the discussion regarding conditional entropies:

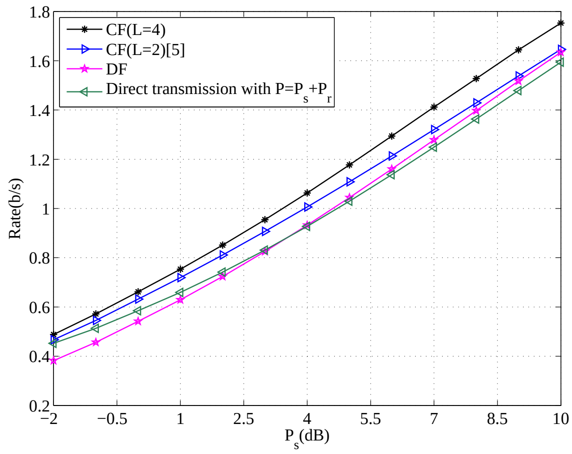

Figure 3.2: CF achievable rates versus the source power P_s for $M = 4$ and their comparison with DF and direct transmission rates. The other parameters are set to P_r = -6 dB, $d_{rd} = 0.2$ m, and $d_{sr} = 0.95$ m.

3.2.1 Conditional Entropies

For computing $H(W \mid X_{s1})$ and $H(W \mid Y_{d1})$, we need $f(y_r \mid y_{d1})$. For N-PAM constellation, it is given by:

$$f(y_r \mid y_{d1}) = \sum_{i=1}^{M} \log \frac{f_g(y_{d1} - x_i)}{\sum_{j=1}^{M} f_g(y_{d1} - x_j)} f_g(y_r - x_i) \tag{3.3}$$

with $x_i \in \mathcal{X}_M$. Using $P_W(w \mid y_{d1})$, SW rate can be computed as:

$$H(W \mid Y_{d1}) = -\int f(y_{d1}) \sum_{w=0}^{L-1} P(w \mid y_{d1}) \log P(w \mid y_{d1}) \,\mathrm{d}y_{d1} \tag{3.4}$$

with $f(y_{d1}) = (1/M) \sum_i f(y_{d1} - x_i)$.

$H(W \mid X_{s1})$ can be computed in analogous fashion.

3.2.2 Numerical Observations

In Fig. 3.2, we present the achievable rates of the CF strategy versus P_s where all transmissions from the source as well as the relay are modulated onto an $M = 4$-ary PAM constellation. In generating the results, we assume that $L = M = 4$, and numerically search over α, q, and the power allocation between P_{s1} and P_{s2} that yield the maximum achievable rate in (3.2) while satisfying the constraint (3.1) (in addition to the constraint $\alpha P_{s1} + \bar{\alpha} P_{s2} \leq P_s$).

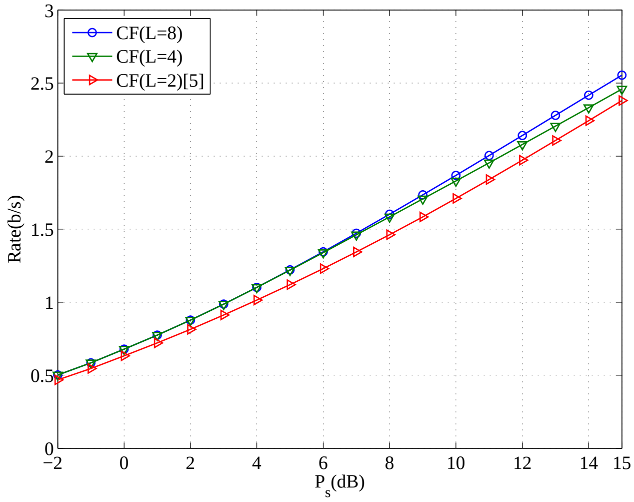

Figure 3.3: A comparison of CF achievable rates for several L. The parameters are set to $M = 8$, P_r = -3 dB, $d_{rd} = 0.15$ m, and $d_{sr} = 0.95$ m.

Some observations that can be made from these numerical results are:

- At an overall transmission rate of 1.0 b/s, the CF strategy outperforms DF by a margin of 1.15 dB, whereas the gain from direct transmission is approximately 1.2 dB. Note that in order to have a fair comparison, we have assumed that for the direct transmission case, the source transmits with a power equal to $P_s + P_r$.

- It was shown in [26] that a binary quantizer $(L = 2, q = \infty)$ sufficed for the case when transmission from the source and relay were modulated on to a BPSK constellation $(M = 2)$, i.e. no significant gains were observed with $L > 2$. Results in Fig. 3.2 however indicate that this does not hold true for higher order constellations. For example, for $M = 4$, we observe that in order to achieve a transmission rate of 1.5 b/s, employing a binary quantizer at the relay requires 0.89 dB more transmission power than the case when an $L = 4$ quantizer was used. At the same time, our numerical results (not shown in the figure) seem to indicate that going beyond $L = 4$ does not yield any noticeable gains.

- A similar observation is made for the case of $M = 8$. As shown in Fig. 3.3, achieving a transmission rate of 2.0 b/s with a binary quantizer requires 1.2 dB more power at the source than the case with $L = 4$; the $L = 4$ quantizer on the other hand requires 0.36 dB more power than the case with $L = 8$. We've also found numerically that going beyond $L = 8$ in this case does not give any further noticeable gains.

Chapter 4

Practical CF Relaying System

Global channel state information (CSI) is assumed which is used to compute optimal value of α and source and relay powers.

Instead of operating over $GF(q)$, we use binary component codes to channel encode information at source and relay. Whereas in past, researchers focussed over minimizing Euclidean distance and maximizing asymptotic gains, recently research in the domain of coded modulation has proved that and schemes like multilevel coding (MLC) [33] and bit-interleaved coded modulation (BICM) [34] can achieve capacity while providing both power and bandwidth efficiency. We will now describe our CF coding scheme in detail.

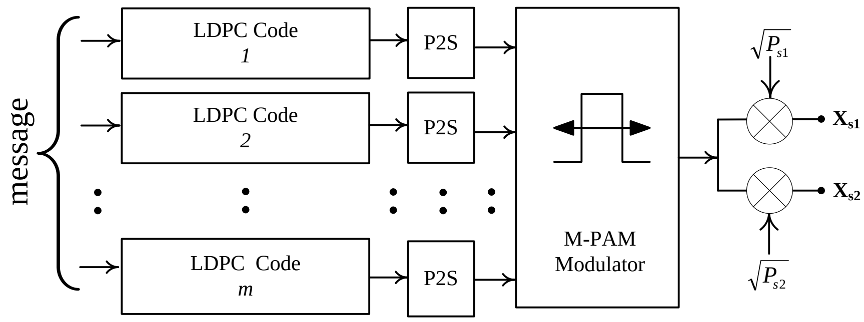

Figure 4.1: Encoding at the Source node using m LDPC codes. The P2S blocks indicate parallel to serial conversion.

4.1 Message Encoding

For a given relay power P_r, as well as the relay position, we use the information-theoretic analysis presented in Section 3.2 to evaluate the optimum parameters α, q, P_{s1} and P_{s2} that are required to achieve a target transmission rate of R b/s. A block diagram of multi-level message encoder at the Source node is shown in Fig. 4.1. The message to be transmitted to the destination is partitioned into m bit-streams. Each bit-stream is encoded with a length-N LDPC code of rate R_i, $i = 1, \ldots, m$ so that the length of the i^{th} message bit-stream is NR_i. The individual code rates satisfy $\sum_{i=1}^{m} R_i = R$. The resulting codewords are serially fed m bits at a time into a unit energy M-PAM modulator, i.e. the k^{th} bit of all codewords forms the k^{th} symbol of the PAM sequence, $k = 1, \ldots, N$. The first αN symbols of this PAM sequence are scaled by $\sqrt{P_{s1}}$ to form the sequence $\mathbf{X}_{s1}$ that satisfies the average power constraint of P_{s1}, and which is transmitted to the relay and the destination during T_1. The remaining $\bar{\alpha}N$ symbols are scaled by $\sqrt{P}_{s2}$ to form the sequence $\mathbf{X}_{s2}$ that satisfies an average power constraint of P_{s2} and which is transmitted during T_2. The individual rates R_i of the LDPC codes can be easily evaluated by representing the PAM sequences $\mathbf{X}_{s1}$ and $\mathbf{X}_{s2}$ as a combination of m bit-levels, plugging this representation into (3.2) and applying the chain rule of mutual information, i.e. the rate of each level is chosen based on the assumption that we have perfect knowledge of the codewords from the previous levels and no knowledge of those from the subsequent levels.

4.2 Multilevel Distributed Joint Source Channel Coding

As mentioned earlier, the relay quantizes the received sequence $\mathbf{Y}_r$ using an L-level quantizer, the quantization step-size of which is chosen to maximize the overall transmission rate in (3.2). The quantizer outputs a sequence $\mathbf{W}$ consisting of αN quantization indices, each of length $l = \log L$ bits. The sequence $\mathbf{W}$ now needs to be compressed using SW coding with $\mathbf{Y}_{d1}$ as the decoder side-information. In addition, channel coding is also required to protect its transmission against noise on the relay to destination link. Instead of providing separate SW and channel coding, we resort to Distributed Joint Source Channel Coding (DJSCC) [26] in which SW coding and error protection is implemented in a joint manner. The challenge however is that for $M > 2$, each quantization index is composed of $l > 1$ bits as opposed to the BPSK case in [26] where the quantization indices were one-bit each. Taking this into account, we propose to use multiple binary IRA codes to implement multi-level DJSCC, the details of which are given below.

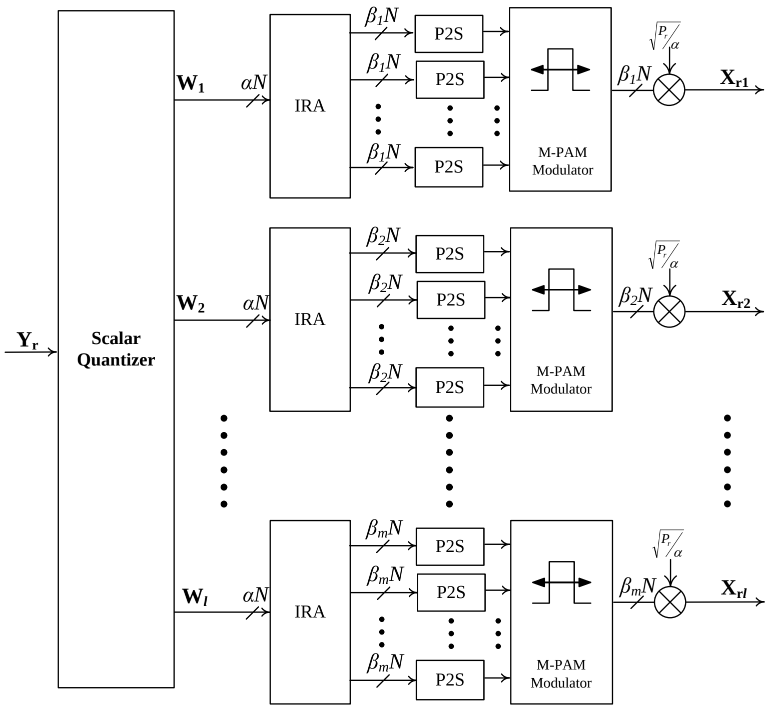

Figure 4.2: Multilevel DJSCC encoding at the relay using l IRA codes.

Encoding: We split the quantization index sequence $\mathbf{W}$ into l bit-plane sequences $\mathbf{W}_1, \ldots, \mathbf{W}_l$ each being of length αN. For example, $\mathbf{W}_1$ could correspond to a sequence comprising of the least-significant bits of the original quantization sequence, whereas $\mathbf{W}_l$ could correspond to the most-significant bits. One possibility could have been to encode each one of the l quantization bit-plane with m IRA codes, the parity bits of which are then mapped to an M-PAM constellation (similar to Fig. 4.1). However, this approach requires the use of $l \times m$ IRA codes, which becomes prohibitive when both l and m are large. Instead, we use a single IRA code for each bit-plane as shown in Fig. 4.2. Bit-plane i, $i = 1, \ldots, l$ is encoded with a code that has $m\beta_i N$ parity bits (the appropriate choice of β_i's would be discussed later). These parity bits are then mapped m bits at a time to a length-$\beta_i N$ symbol sequence $\mathbf{X}_{ri}$ (with an average power $P_r/\bar{\alpha}$. These symbol sequences are transmitted to the destination one after the other, starting with $\mathbf{X}_{r1}$ and ending with $\mathbf{X}_{rl}$. Since the total number of transmissions from the relay are $\bar{\alpha}N$ symbols, we have the constraint $\sum_{i=1}^{l} \beta_i = \bar{\alpha}$.

Decoding: Note that the systematic bits of the IRA code are not transmitted over the physical channel. However since $\mathbf{Y}_{d1}$ at the destination is

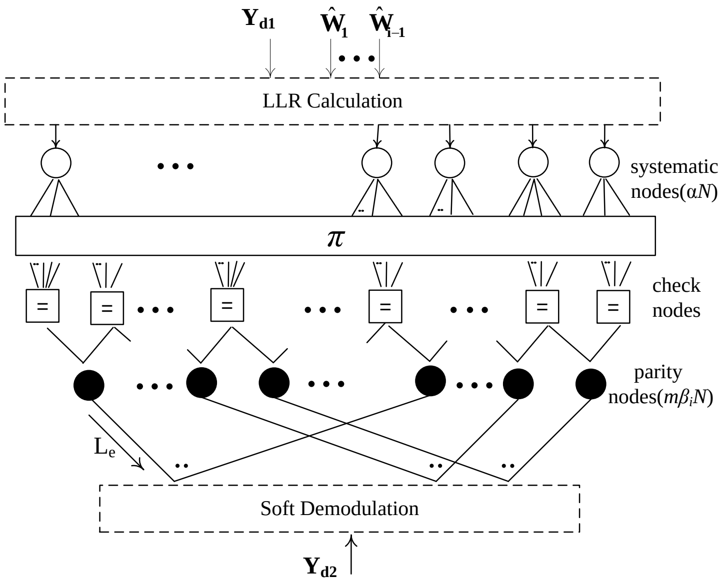

Figure 4.3: DJSCC decoder for the $i-th$ quantization bit plane.

correlated with $\mathbf{W}$, one can think of the systematic bits as being transmitted over a virtual correlation channel with $\mathbf{Y}_{d1}$ as the output. The decoding of the bit-planes is done in stages starting with $\mathbf{W}_1$ and ending with $\mathbf{W}_l$. Thus when attempting to decode $\mathbf{W}_i$, the calculation of log-likelihood ratios (LLRs) corresponding to the systematic bits not only use $\mathbf{Y}_{d1}$ but also $\hat{\mathbf{W}}_1, \ldots, \hat{\mathbf{W}}_{i-1}$, the decoded versions of the respective quantization index bit-planes from the previous stages, as shown in Fig. 4.3. This decoding strategy follows directly from the chain-rule of entropy, using which the information theoretic constraint (3.1) necessary for recovery of the quantization index sequence $\mathbf{W}$ can be rewritten as

$$\alpha \sum_{i=1}^{l} H\left(W_i | Y_{d1}, W_1, \ldots, W_{i-1}\right) \leq \sum_{i=1}^{l} \beta_i I\left(X_{ri}; Y_{d2}\right) \tag{4.1}$$

For each bit-plane i, $i = 1, \ldots, l$, we choose β_i to satisfy the individual constraint

$$\alpha H\left(W_i | Y_{d1}, W_1, \ldots, W_{i-1}\right) \leq \beta_i I\left(X_{ri}; Y_{d2}\right). \tag{4.2}$$

This makes sure that the overall conditional entropy of $\mathbf{W}$ given $\mathbf{Y}_{d1}$ satisfies (4.1). Thus if the codes used are capacity achieving, the proposed methodology should guarantee that the quantization index sequence $\mathbf{W}$ is recoverable.

Noting that multiple parity-bits of an IRA code are mapped to the same modulated symbol, we employ an iterative soft-demodulation strategy [34].

The iterative strategy for recovering the quantization indices is summarized below:

1. **Repeat** for all bit-planes $i = 1, \dots, l$.
2. Initialize extrinsic LLRs (from parity nodes to the soft demodulator) $\mathbf{L}_e = \mathbf{0}$.
3. Use $\mathbf{Y}_{d1}$ and $\hat{\mathbf{W}}_1, \dots, \hat{\mathbf{W}}_{i-1}$ to calculate the a-priori LLRs for the systematic nodes.
4. **While** (stopping criterion not met)[1]
5. (Soft demodulation) Use $\mathbf{Y}_{d2}$ and $\mathbf{L}_e$ to calculate the a-priori LLRs for the parity nodes.
6. Run one iteration of belief propagation (BP) algorithm on the IRA decoding graph ($\mathbf{L}_e$ is updated).
7. **end while**
8. Obtain $\hat{\mathbf{W}}_i$ by hard-thresholding the a-posteriori LLRs from the systematic nodes.

After decoding all bit-planes, the DJSCC decoder passes the estimates $\hat{\mathbf{W}}_1, \dots, \hat{\mathbf{W}}_l$ to the message decoder.

4.3 Message Decoding

Destination uses MSD on the m LDPC decoding graphs to recover the corresponding codewords, and hence the original message sequence. We have two types of variable nodes at each stage of the LDPC decoder. The first type correspond to the symbols received during T_1. The a-priori LLRs for these type of nodes are calculated using Y_{d1}, $\hat{\mathbf{W}}_1, \dots, \hat{\mathbf{W}}_l$, and the decoded codewords corresponding to the previous stages. The second type of variable nodes are those which correspond to T_2. For these nodes only $\mathbf{Y}_{d2}$ and the decoded codewords corresponding to the previous stages are used to evaluate the a-priori LLRs.

[1] In our simulations, we stop when the maximum number of iterations have exceeded or the correct codeword has been decoded.

Chapter 5

Convergence Analysis

5.1 Code Design

For optimizing the degree distributions of the LDPC and IRA codes, we first use the information-theoretic analysis of Section 3.2 to evaluate (for given relay position and power P_r) the optimum parameters α, q, P_{s1} and P_{s2} that are required to achieve a target transmission rate of R b/s. The analysis also yields the target rates R_i, $i = 1, \ldots, m$ for the individual LDPC codes, as well as β_i, $i = 1, \ldots, l$, that govern the target rates of the individual IRA codes.

The degree distributions for the multi-level LDPC and IRA codes are designed using the EXIT chart strategy [35] with Gaussian approximation [15]. The approach we use is a direct consequence of chain rule of mutual information/entropy, i.e., we assume perfect knowledge of prior bit-planes, and no information about subsequent ones. This simplifies the design process in the sense that the individual codes can be designed one by one, in a serial fashion.

5.1.1 IRA

In the following, we briefly explain how degree distributions of a single level of IRA codes are optimized; the procedure has to be repeated for all l levels.

Each IRA code has two types of variable nodes, the systematic and the parity nodes. The parity nodes are divided into m groups; corresponding bits from each group map to the same M-ary symbol. As shown in Fig. 4.3 each parity node is connected to two consecutive check nodes and vice-versa, with as many parity nodes as the check nodes. Thus the check nodes can also be divided into m types. We fix the check nodes within group i, $i = 1, \ldots, m$,

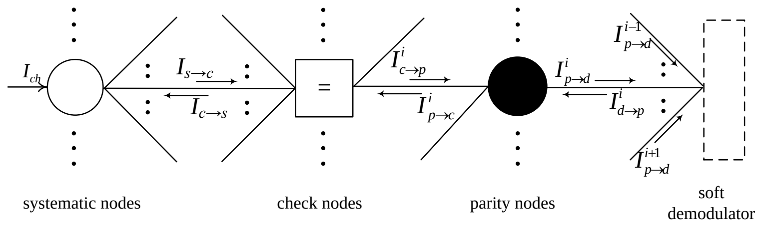

Figure 5.1: Information flow for IRA codes.

to have a regular degree of d_i, and then design the systematic node degree distribution $\lambda(x) = \sum_{d=1}^{D} \lambda_d^j x^{d-1}$, where D is the maximum systematic node degree. Since each group contains equal number of check nodes, the overall check node degree distribution is given as $\rho(x) = \sum_{i=1}^{m} \rho_i x^{d_i - 1}$ with $\rho_i = d_i \left(\sum_{j=1}^{m} d_j\right)^{-1}$. Let $I_{s\to c}$ be the a-priori information from the systematic nodes to the check nodes as shown in Fig. 5.1. If $I^i_{p\to c}$ is the information flow from the parity to check nodes in group i, the information from check to parity nodes can be evaluated using the approximate check to bit-node duality[14] as

$$I^i_{c\to p} \approx 1 - J\Bigg(d_i J^{-1}\left(1 - I_{s\to c}\right) + J^{-1}\left(1 - I_{p\to c}\right)\Bigg) \tag{5.1}$$

where $J(\mu)$ is the information that a log-likelihood ratio drawn from a Gaussian distribution of mean μ and variance 2μ conveys about the bit it represents. The information from the parity nodes in group i to the soft demodulator is given as $I^i_{p\to d} = J\left(2J^{-1}\left(I^i_{c\to p}\right)\right)$. For informations $I^j_{p\to d}$, $j = 1, \ldots, m$, and given channel conditions, we evaluate the EXIT function of the soft demodulator using Monte-Carlo simulations [36] to obtain $I^i_{d\to p}$ and consequently

$$I^i_{p\to c} = J\Bigg(J^{-1}\left(I^i_{c\to p}\right) + J^{-1}\left(I^i_{d\to p}\right)\Bigg) \tag{5.2}$$

for all $i = 1, \ldots, m$. This completes one iteration of decoding from the check- to parity- to soft demodulator back to parity- to check-node. In order to simplify the optimization process, we assume that the iterations on this side of the decoding graph (right hand side of the check node in Fig. 5.1) continue until a fixed point is reached. In other words, for a given $I_{s\to c}$, we initially assume $I_{p\to c}$ to be zero, and then continue the iterations specified by (5.1) and (5.2) until the point where all $I^1_{p\to c}, \ldots, I^m_{p\to c}$ converge to fixed values. If $\tilde{I}^i_{p\to c}$ denotes the fixed point, the check-nodes to systematic node information is given as

$$I_{c\to s} \approx 1 - \sum_{i=1}^{m} J\Bigg((d_i - 1) J^{-1} \left(1 - I_{s\to c}\right) + 2J^{-1}\left(1 - \tilde{I}^{i}_{p\to c}\right)\Bigg) \tag{5.3}$$

For convergence of the IRA code at the j-th level, we need to satisfy the constraint

$$\sum_{d=1}^{D} \lambda_d^j J\Bigg((d-1) J^{-1}\left(I_{c\to s}\right) + J^{-1}\left(I_{ch}\right) \Bigg) > I_{s\to c} \tag{5.4}$$

for all $I_{s\to c} \in [0, 1)$, where I_{ch} is the information on the systematic nodes obtained from the (virtual correlation) channel. When designing the i-th level IRA code, we have $I_{ch} = I\left(W_i; Y_{d1} | W_1, \ldots, W_{i-1}\right)$. Note that $I_{c\to s}$ in (5.4) is in fact a function of $I_{s\to c}$, but we omit that dependence for notational convenience. In addition to (5.4), we have the trivial constraints $\sum_{i=1}^{D} \lambda_i = 1$, and $\lambda_i > 0$ for all $i = 1, \ldots, D$. Under these constraint, the IRA code rate needs to be maximized which is equivalent to maximizing $\sum_{d=1}^{D} \frac{\lambda_d^j}{d}$. By discretizing $I_{s\to c}$ on the interval $[0, 1)$, the optimization can be easily solved using linear programming.

5.1.2 LDPC

For each level of multilevel LDPC codes, we have two group of bit nodes; Type-1 nodes correspond to the information received in T_1 and Type-2 to information received in T_2. Both these groups have different SNR characteristics and therefore we design their degree distributions separately. We use a set of constraints that transforms the design process into a linear optimization problem[26].

Let $\Theta_i^{1(2)}$ denote the fraction of degree i variable nodes of type-1(2). Noting that the fraction of type-1 nodes is α and that of type-2 is $\bar{\alpha}$, following constraints can be devised[26].

$$\sum_{i=2}^{V_{max}} \Lambda_i^1 = \alpha; \qquad \sum_{i=2}^{V_{max}} \Lambda_i^2 = \bar{\alpha} \tag{5.5}$$

With V_{max} being the maximum variable node degree. Also if $\eta_i^{1(2)}$ denote the fraction of edges connected to type-1(2) variable nodes of degree i. We then have[26]:

$$
\begin{aligned}
\eta_i^{1(2)} &= \frac{\text{Edges connected to degree } i \text{ nodes of type-1(2)}}{\text{Total edges}} \\
&= \frac{\Lambda_i^{1(2)} i}{\sum_{j=2}^{V_{max}} \left(\Lambda_j^1 + \Lambda_j^2\right) j}
\end{aligned}
\tag{5.6}
$$

Since

$$\sum_{i=2}^{V_{max}} \left(\Lambda_i^1 + \Lambda_i^2\right)$$

Summing over all the degrees and both types of variable nodes gives[26]:

$$\sum_{i=2}^{V_{max}} \frac{\eta_i^1 + \eta_i^2}{i} = \frac{1}{\sum_{i=2}^{V_{max}} \left(\Lambda_i^1 + \Lambda_i^2\right)} \tag{5.7}$$

Summing (5.7) over i and using (**??**), (5.6) can be transformed into a constraint into a constraint in terms of degree distributions $\eta_i^{1(2)}$

$$\sum_{i=2}^{V_{max}} \bar{\alpha}\frac{\eta_i^1}{i} - \alpha\frac{\eta_i^2}{i} = 0 \tag{5.8}$$

And we already know that degree distributions should add up to one:

$$\eta_i^1 + \eta_i^2 = 1 \tag{5.9}$$

It can be easily verified that both the sub-channels are symmetric which allows us to use all-zero codeword assumption. Also since all the messages are assumed to be Gaussian, we can use Gaussian assumption. Let $I_{ac} \in [0, 1]$ be the apriori information to the check nodes. We can use bit-check node duality to evaluate the average extrinsic information from check to variable nodes using:

$$
\begin{aligned}
I_{ec}(I_{ac}) &= \sum_{j=2}^{D_c} \rho_j (1 - J((j-1)J^{-1}(1 - I_{ac}))) \\
&\approx 1 - \sum_{j=2}^{D_c} \rho_j J((j-1)J^{-1}(1 - I_{ac}))
\end{aligned}
\tag{5.10}
$$

Using I_{ec}, the extrinsic information from bit to check nodes i s given by:

$$I_{ev}(I_{ac}) = \sum_{k=1}^{2} \sum_{\eta_i^k} J(J^{-1}(I_{ch}^k) + (i-1)J^{-1}(I_{ec}(I_{ac}))) \tag{5.11}$$

with $I_{ch}^{1(2)}$ being the channel information on sub-channel-1(2).

For a zero error probability, (5.8),(5.9) along with the constraint given below have to be satisfied:

$$I_{ev}(I_{ac}) > I_{ac}, \qquad \forall I_{ac} \in [0,1) \tag{5.12}$$

Since all the constraints are linear in nature; the variable node degree distribution can be optimized easily using linear programming.

5.2 Simulation Results

In this Section, we present simulation results for a 4-PAM CF relaying system with transmission rates of 1.0 and 1.5 b/s. The setup we consider corresponds to $d_{rd} = 0.2$ m, $d_{sr} = 0.95$ m and $P_r = -6$ dB. We list the optimized information theoretic parameters required to achieve the target transmission rates in Table 5.1.

Table 5.1: Optimized parameters for 4-PAM CF relaying with $d_{sr} = 0.95, d_{rd} = 0.2$ m and $P_r = -6$ dB. All powers are specified in dB and rates in b/s.

Rate	1.0	1.5
α	0.57	0.63
β_0	0.26	0.28
β_1	0.17	0.09
P_s	4.06	8.38
P_{s1}	4.63	8.65
P_{s2}	2.61	7.25
q	1.55	2.44
R_1	0.63	0.37
R_2	0.83	0.67

Note that if the above information theoretic parameters are used to design the LDPC and IRA codes, the practical coding losses would imply that the

Table 5.2: Degree Distributions for $R = 1(b/s)$ using 4-PAM CF Relaying

-	IRA_0		IRA_1		$LDPC_0$			$LDPC_1$		
x	ρ_0	λ_0	ρ_1	λ_1	ρ_0	η_0^1	η_0^2	ρ_1	η_1^1	η_1^2
2		0.4021		0.1825		0.1031	0.0151			0.1023
3	0.42587	0.0997		0.0623			0.1690		0.1257	0.045
4	0.57143	0.1795							0.0244	
5				0.2185						
6		0.0884				0.0045			0.0892	
7							0.0219			
8						0.0969		0.35		
10			0.47619	0.0986		0.1215		0.3	0.0913	
11			0.52381							
12									0.0716	
14					0.1			0.35		
15					0.1					
16					0.1	0.0747				
17					0.2				0.1185	
18					0.5					
22						0.1177				
30		0.2303								
90				0.4407						
120						0.2758				0.3320

rates of the optimized codes are less than those required. Therefore we keep α, P_{s2}, β_0, and β_1 fixed at the theoretical values and increase P_{s1} gradually until codes of required rates are achieved. The power P_{s2} is not increased from its theoretical minimum so as not to increase the interference that $\mathbf{X}_{s2}$ causes while decoding $\mathbf{W}$ [26].

We simulate the optimized degree distributions at a finite block-length of $N = 2 \times 10^5$ symbols. In this case too we fix all parameters except P_{s1} to the information theoretic values and gradually increase this power until the desired bit-error rate (BER) of 10^{-5} is achieved. Simulation results indicate that the overall power P_s required to achieve the target BER at a transmission rate of 1.0 b/s is only 0.56 dB more than the theoretical limit. On the other hand, at a transmission rate of 1.5 b/s, the proposed ML-CF scheme suffers a loss of only 0.63 dB from the theoretical bound.

Check and Variable node profile for both IRA and LDPC codes have been given in Table 5.2 for an overall rate of 1.0 b/s.

The BER performance for $R = 1(b/s)$ using optimized degree distributions is shown in Fig. 5.2. For finite block lengths the gap to theoretical capacity for a $BER = 10^{-6}$ is $0.56dB$. For an over rate of $1.5(b/s)$ the

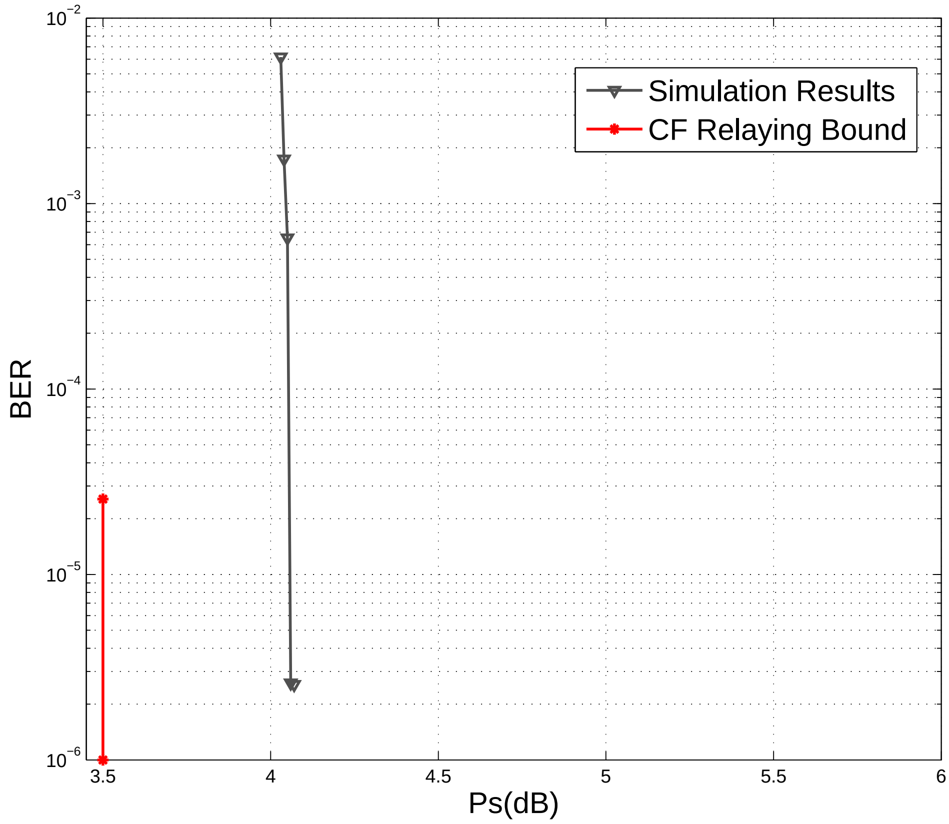

Figure 5.2: BER Vs. average source power P_s for an overall rate $R_{CF} = 1(b/s)$. Channel parameters are given Table 2.

simplified CF scheme suffers a loss of $0.63dB$.

Chapter 6

Conclusion & Furure Work

6.1 Conclusion

In this thesis, we have presented an Compress and Forward strategy for a half-duplex Gaussian relay network where the transmissions from the source and the relay are drawn from an M-ary PAM constellation. The compression of the signal received at the relay is achieved by quantizing it before applying SW compression.

Numerical evaluation of information theoretic analysis indicates that it is sufficient to consider an M-level quantizer, i.e. one does not gain much by going beyond M levels. At the same time, one suffers a significant degradation in performance by considering less than M levels.

A coding scheme using LDPC and IRA codes was also presented. Multi-level LDPC codes were used to encode the source message, whereas multiple IRA codes were used to implement multi-level DJSCC of the quantization indices. Simulation of the proposed methodology indicates performance close to the theoretical bound.

6.2 Future Work

There can be many interesting directions to which the current work can be extended.

Currently we are focussing on optimizing the quantizer design. Based on the preliminary results, we believe that a quantizer design can be presented that eliminates the need of optimization over quantization parameters. If solved, this would be a huge leap in the field of CF coding strategies where optimizer quantizer design is one of the key challenges.

Also the exploration of the proposed CF coding scheme for other channel codes and conditions would be interesting.

Appendix A

Capacity of AWGNC

Capacity of a channel is given by:

$$\begin{aligned} C &= H(y) - H(y \mid x) \\ &= -\int f(y) \log f(y)\, \mathrm{d}y + \int f(y,x) \log f(y \mid x)\, \mathrm{d}y \end{aligned} \tag{A.1}$$

For M-PAM Signalling, the channel transmission is of the form $Y = X + Z$. Here X is the equiprobable M-PAM transmissions from the source node. Let x_i denote any instance of input power levels, with $i = 1, 1, ..m, l = \log M$

$$\begin{aligned} C &= -\int \sum_i (p(x_i) f(y \mid x_i)) \log f(y)\, \mathrm{d}y + \int \sum_i (f(y, x_i) \log f(y \mid x_i))\, \mathrm{d}y \\ &= \int \sum_i (p(x_i) f(y \mid x_i)) \log \left(\frac{f(y \mid x_i)}{f(y)} \right) \mathrm{d}y \end{aligned} \tag{A.2}$$

$$C = \frac{1}{M} \sum_i \left(\int f(y \mid x_i) \log \left(\frac{f(y \mid x_i)}{f(y)} \right) \mathrm{d}y \right) \tag{A.3}$$

For an AWGN channel

$$f(y \mid x_i) = \frac{1}{\sqrt{2\pi}} \exp \left(\frac{-(y - x_i)^2}{2} \right)$$

Substituting in (A.3) :

$$C^{MAWGNC} = \sum_i \frac{1}{M} \int \frac{\exp\left(\frac{-(y-x_i)^2}{2}\right)}{\sqrt{2\pi}} \log\left(\frac{\frac{\exp\left(\frac{-(y-x_i)^2}{2}\right)}{\sqrt{2\pi}}}{f(y)}\right) \mathrm{d}y \tag{A.4}$$

$$\begin{aligned} C &= \sum_i \frac{1}{M} \int \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-(y-x_i)^2}{2}\right) \log\left(\frac{\frac{1}{\sqrt{2\pi}} \exp\left(\frac{-(y-x_i)^2}{2}\right)}{\frac{1}{M\sqrt{2\pi}} \sum_j e^{\frac{-(y-x_j)^2}{2}}}\right) \mathrm{d}y \\ &= -\sum_i \frac{1}{M} \int \frac{1}{\sqrt{(2\pi}} \exp\left(\frac{-(y-x_k)^2}{2}\right) \log\left(\frac{\sum_j e^{\frac{-(y-x_j)^2}{2}}}{M \exp\left(\frac{-(y-x_i)^2}{2}\right)}\right) \mathrm{d}y \end{aligned}$$

$$\begin{aligned} C &= \log(M) \int \sum_i p(x_i) \frac{1}{\sqrt{2\pi}} e^{\frac{-(y-x_i)^2}{2}} \mathrm{d}y \\ &\quad - \sum_i \frac{1}{M} \int \frac{e^{\frac{-(y-x_i)^2}{2}}}{\sqrt{2\pi}} \log\left(\frac{e^{-\frac{(y-x_i)^2}{2}}}{\sum_j e^{-\frac{(y-x_j)^2}{2}}}\right) \mathrm{d}y \\ &= \log(M) \int f(y)\, \mathrm{d}y \\ &\quad - \sum_i \frac{1}{M} \int \frac{\exp\left(\frac{-(y-x_i)^2}{2}\right)}{\sqrt{2\pi}} \log\left(\frac{\exp\left(-\frac{(y-x_i)^2}{2}\right)}{\sum_j \exp\left(-\frac{(y-x_j)^2}{2}\right)}\right) \mathrm{d}y \\ C &= \log(M) - \sum_i \frac{1}{M} \int \frac{e^{\frac{-(y-x_i)^2}{2}}}{\sqrt{2\pi}} \log\left(\frac{e^{-\frac{(y-x_i)^2}{2}}}{\sum_j e^{-\frac{(y-x_j)^2}{2}}}\right) \mathrm{d}y \end{aligned}$$

As all the symbols are equiprobable and we are integrating over the same gaussian with shifted means:

$$C = m - \int \frac{e^{\frac{-(y-x_i)^2}{2}}}{\sqrt{2\pi}} \log \frac{e^{-\frac{(y-x_i)^2}{2}}}{\sum_j e^{-\frac{(y-x_j)^2}{2}}} \mathrm{d}y \tag{A.5}$$

Appendix B

Capacity of MMGNC

In this appendix we derive the capacity expression for M-PAM Mixture Gaussian Noise Channel(MMGNC).The channel transmission for a MMGNC is of the form $Y = X + S + Z$. Here X and S are the equiprobable M-PAM transmissions from the source node and the interferer node. Let x_i and s_j denote any instance of input power levels, with $i, j = 0, 1, ..l-1, l = \log_2 M$

$$\begin{aligned} C^{MMGNC} = \quad & -\int \sum_i \sum_j (p(x) f(s \mid x) f(y \mid s, x)) \log f(y) \, \mathrm{d}y \\ & + \int \sum_i \sum_i (f(y, x, s) \log \big(\sum_s f(y \mid s, x) p(s) \big) \, \mathrm{d}y \end{aligned} \tag{B.1}$$

Simplifying

$$\begin{aligned} C^{MMGNC} &= \sum_i \int \sum_j f(y \mid x_i, s_j) f(s_j, x_i) \log \left(\frac{\sum_k f(y \mid x_i, s_k) p(s_k)}{f(y)} \right) \mathrm{d}y \\ &= \sum_i \int \sum_j f(y \mid x_i, s_j) p(x_i) p(s_j) \log \left(\frac{\sum_k f(y \mid x_i, s_k) p(s_k)}{f(y)} \right) \mathrm{d}y \end{aligned}$$

For Additive Mixture Gaussian Noise Channel:

$$f(y \mid x_i, s_j) = \frac{1}{\sqrt{(2\pi)}} \exp \left(\frac{-(y - x_i - s_j)^2}{2} \right) \tag{B.2}$$

Substituting (B.2)

$$C^{MMGNC} = -\sum_i \int \sum_j \frac{e^{\left(\frac{-(y-x_i-s_j)^2}{2}\right)}}{M^2\sqrt{2\pi}} \log\left(\frac{f(y)}{\sum_k \frac{e^{\frac{-(y-x_i-s_k)^2}{2}}}{M\sqrt{2\pi}}}\,\mathrm{d}y\right) \tag{B.3}$$

Replacing $f(y)$ with $\sum_i \sum_j \frac{e^{\frac{-(y-x_i-x_j)^2}{2}}}{M^2\sqrt{2\pi}}$ in (B.3), we get

$$C^{MMGNC} = m - \sum_i \int \Gamma_i(y) \log\left(1 + \sum_{\substack{j \\ j\neq i}} \frac{\Gamma_j(y)}{\Gamma_i(y)}\right) \mathrm{d}y \tag{B.4}$$

with

$$\Gamma_i(y) = \sum_k \frac{1}{M\sqrt{2\pi}} e^{\left(\frac{-(y-x_j-s_k)^2}{2}\right)}$$
$$\Gamma_j(y) = \sum_k \frac{1}{M\sqrt{2\pi}} e^{\left(\frac{-(y-x_i-s_k)^2}{2}\right)}$$

Appendix C

Condition Pdf $f(y_r \mid y_d)$

$$
\begin{aligned}
f(y_r \mid y_d) &= \frac{f(y_r, y_d)}{f(y_d)} \\
&= \frac{\sum_s f(y_r, y_d \mid s)p(s)}{\sum_s f(y_d \mid s)p(s)} \\
&= \frac{\sum_s f(y_r \mid y_d, s)f(y_d \mid s)p(s)}{\sum_s f(y_d \mid s)p(s)} \\
&= \frac{\sum_s f(y_r \mid s)f(y_d \mid s)p(s)}{\sum_s f(y_d \mid s)p(s)} \\
&= \frac{\sum_s f(y_r \mid s)f(y_d \mid s)}{\sum_s f(y_d \mid s)} \\
&= \frac{\sum_i f(y_r \mid s_i)f(y_d \mid s_i)}{\sum_k f(y_d \mid s_i)} \\
&= \sum_i \frac{\frac{1}{\sqrt{(2\pi\sigma^2)}} \exp\left(\frac{-(y-s_i)^2}{2\sigma^2}\right)}{\sum_j \frac{1}{\sqrt{(2\pi\sigma^2)}} \exp\left(\frac{-(y-s_j)^2}{2\sigma^2}\right))} f(y_r \mid s_i) \\
&= \sum_i \frac{\exp\left(\frac{-(s_i^2-2ys_i)}{2\sigma^2}\right)}{\sum_j \exp\left(\frac{-(2s_j^2-2ys_j)}{2\sigma^2}\right))} f(y_r \mid s_i)
\end{aligned}
$$

with

$$
f(y_r \mid s_i) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(y_r-s_i)^2}{2}}
$$

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