

# **GEOMETRIC ERROR MEASUREMENT AND COMPENSATION IN TWO AXIS CNC MACHINE**



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**MASTER THESIS WORK**

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## **DEDICATION**

*“Dedicated to all those who are part and parcel of my life”.*

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## **ABSTRACT**

CNC machine tools are commonly used for manufacturing these days. CNC machines are meant for consistently producing parts, with high accuracy, with minimum scrap while maintaining high productivity. This big task can be achieved by precise and consistent positional control. Many factors affect the positional accuracy of CNC machines in which geometry of machine, temperature variations and force induced errors are major contributors. Geometric errors of the machine are result of manufacturing imperfections, assembly errors and wear and tear in the machine elements with the passage of time.

To monitor the accuracy of the machines, machine measurement has become an important field of interest for research to ensure the quality assurance of manufacturing processes. Measuring machine errors in suitable manner and utilizing these machine measurements to enhance the accuracy and repeatability of the machines is the focus of this research. Geometrical errors of a two-axis CNC grinding machine has been measured and modeled to achieve better positional accuracy. Laser interferometer and geometrical artifacts are used for machine error measurement. Compensation for positional effect of these geometric errors was calculated based on the model and was compensated through NC program modification. Results were verified by taking measurements on a diagonal with laser interferometer which show a significant reduction in positional error.

In addition to this work effect of software error compensation on accuracy of machined part has been studied on a milling machine. Positional errors of each axis were measured with Laser interferometer and compensated in the machine controller. Parts were machined before and after compensation and measured on CMM to analyze the effect. The CMM results show improvement in positional features of part machined after compensation.

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## LIST OF ABBREVIATIONS

CNC	Computer Numerically Controlled
NC	Numerically Controlled
CMM	Coordinate Measuring Machine
ISO	International Organization for Standardization
ASME	American Society of Mechanical Engineers
ANSI	American National Standard Institute
MIT	Massachusetts Institute of Technology
NURBS	Non-Uniform Rational Basis Spline

# Introduction

## 1.1 Introduction

The dimensional accuracy of the parts produced by a machine tool depends upon the positional accuracy of the machine tool. In present day machining parts with tighter tolerances have increased the need to have more accurate machine.

The recent philosophies of Quality assurance have changed the traditional approach of defect detection through inspection. Instead, focus is on defect prevention, i.e. making the product right at the first time. To implement this philosophy, quality assurance actions must be incorporated into manufacturing systems to actively monitor and correct the error sources of manufacturing processes.

## 1.2 CNC Machine Tools

CNC machine tools are backbone of present day manufacturing. Improved automation is the basic benefit offered by CNC machine tools. The intervention of operator is minimized or sometimes eliminated. Since the machines are capable of running unattended during the machining cycle thus freeing the operator to perform other tasks. The skill level of operator required for machining parts is also reduced as CNC operates under program control. Other benefits of automation are reduced operator fatigue, fewer human errors and predictable machining time required for each work piece.

The other major benefit of CNC technology is production of accurate and consistent work piece. With superior accuracy and repeatability CNC machines are capable of producing identical parts with precision and consistency.

Besides the above two benefits CNC machines are flexible as they are programmable. Programs are saved for different work pieces and recalled for a particular work piece whenever needed.

## 1.3 Accuracy of CNC Machines

The most basic function of a CNC machine is its automatic, precise and consistent positional control. Accurate positioning is achieved by a CNC command which is executed within the control and results in rotating the servo motor a precise number of times. The rotation of servo motor in turn rotates the ball screw. And rotation of ball screw results in motion of axis. The precise numbers of rotations of ball screw are ensured by feedback device installed at the other end. This basic advantage of CNC machines makes them valuable for manufacturing systems, and that's why these machines are widely used worldwide.

## **1.4 CNC Machine Errors**

But the accuracy of the CNC machines get effected by many type of errors including the geometric error of the body structure, the distortion of machine body structure due to temperature variations, the distortion of machine body structure due to cutting forces, backlash of lead screw, servo-control errors, and error motion of spindle axis rotation etc.

Geometric errors arise from manufacturing defects, machine wear and static deflection of machine components. Temperature gradients within the body structure of machine caused by the environment or by the heat generated during the cutting process results in structural deformations and lead screw expansion. Cutting forces also cause deformation in machine structural elements. Deflections or deformations in machine structure although very minor can result in positional errors in CNC machines.

## **1.5 Accuracy Enhancement Through Error Compensation**

Previously accuracy of machine was maintained by increasing mechanical accuracy of machine elements, for example, hand scraping and lapping guide ways. This required substantial effort and cost to manufacture and maintain accurate machines. To overcome these difficulties, computer software error correction is used these days to compensate for hardware imperfections. In software compensation errors are measured and stored in the memory, then in real time machine path is adjusted accordingly. Software compensation has an additional advantage that it can be applied to any existing machine and not requiring any mechanical design changes.

## **1.6 Problem Statement**

There are three main sources of errors in machine tools that determine machine tool accuracy. These are: (1) errors due to geometric inaccuracies; (2) thermally induced errors; and (3) load induced errors. The direct result of these types of errors is in the dimensional and geometric error of the part produced [1].

Geometric errors are one of the major sources of error in CNC machines accuracy and can be further categorized as linear positional errors, straightness errors, angular errors and squareness errors. How these errors can be compensated?

The simplest level of error compensation in CNC machines is compensating for linear positional error or scale error. Most of the CNC machines have provision for compensation of this kind of error now days. But in general full compensation of geometric errors including straightness, angular and squareness errors is commonly not implemented.

## **1.7 Research Aims**

To compensate all the possible geometric errors in a two axis CNC machine to acquire better positional accuracy and to study the effect of software error compensation on part accuracy.

## **1.8 Research Objectives**

The objective of this study is to show in detail the full sequence of operations required to perform geometric error compensation of a CNC machine.

1. To familiarize with machine errors, machine error measuring techniques and equipment and error compensation of CNC machines.

2. To develop a suitable mathematical model for geometric errors of two axis CNC machine.
3. To measure all geometric errors required for geometric error compensation.
4. To compensate the machine for geometric errors.
5. To verify the effectiveness of geometric error compensation.
6. In extension to this work, effect of software error compensation on part accuracy will also be studied.

## **1.9 Methodology**

The objective of this study is to show in detail the full sequence of operations from the measurement of error components to the verification of the positioning accuracy after compensation for geometric errors.

- i. Development of mathematical model for positional error incorporating geometric errors.
- ii. Measurement of geometric errors
- iii. Calculating the resultant positional error using the model and compensating the errors
- iv. verifying the results of compensation

The other part of the study aims to measure the effect of software error compensation on part accuracy;

- i. Machining a test part on CNC milling machine.
- ii. Measuring positional errors of axes and compensating through pitch error compensation.
- iii. Machining another similar test part.
- iv. Measuring both the test parts on CMM.

## **1.10 Thesis Outline**

The thesis contains seven chapters. Chapter two is the literature review in which history of CNC machine compensation is discussed and the existing geometric and thermal error compensation approaches are surveyed.

In Chapter three Machine tool errors are discussed in detail and the basic steps and procedures of error compensation methods are summarized.

In Chapter four machine measurement methods and error measuring techniques are discussed.

Chapter five is meant for modeling the geometric errors of CNC machines.

Chapter six presents the measurement results, error compensation based on the model and verification of results.

Chapter seven is devoted for studying the effect of software error compensation of a CNC machine on part accuracy.

Chapter eight is dedicated to summarize the discussions and conclusions of the study.



# Literature Review

## 2.1 Introduction

In this chapter, an overview of the related researches has been given, that has been performed in the field of CNC machine tools and CMMs. Some of researches related to coordinate measuring machine (CMM) has also been included since CNC machines and CMMs are closely related, though their usage is different.

Machine tool error modeling, error measurement and error compensation has been a subject of interest for many researchers in the past. Review of error modeling and compensation has been presented briefly in this chapter. Literature review of types of machine tool errors and error measurement will be discussed in separate chapters.

## 2.2 Machine Tool Accuracy Enhancement

The history of numerically controlled (NC) machine tools is not very old. Massachusetts Institute of Technology (MIT) first introduced the NC machine tools for the United States Air Force in 1950 [28]. Research started from the first day to improve working performance and accuracy of these machines. The research can be mainly divided into two categories.

One is the design stage and the second one is the operation stage. Machine tools builders mainly focus on the design stages, and the end users focus on the operation stages. End users try to enhance or maintain the accuracy level of machine tools without hardware replacement, which is always an expensive process. End users try to modify the control parameters (for example, the pitch error compensation table, backlash error compensation) to maintain the accuracy level [3]. In many cases, software error compensation has been utilized in research laboratory in the field of CMM and NC machine tools. Software compensation for CMM/NC machine errors is not a replacement for designing major considerations related to errors. This is because for software correction to be effective, two major points have to be considered: (1) it is impossible to get a perfect or a completely general model, and (2) correction can only be achieved for a small error range. The bigger correction comes from the proper machine design [4].

The problem of error in motion is very difficult to be removed completely by design solely. No matter how well a machine is designed, there is a limit to the accuracy that could be achieved through design refinement. Errors like thermal deformation, cutting force deformation, geometric errors etc., cannot be completely accounted for by detailed design. Besides these facts design refinement is costly as tighter tolerances are required to be maintained during manufacturing and assembling machine elements and components. Because of these limitations another approach became popular in 1980's to go for software compensation for reduction of residual errors which was quite easy and cost effective in CNC machines. The basic philosophy behind this concept is that it is very difficult to construct a perfect machine tool because despite the perfect design the accuracy of the machine changes when subjected to thermal loads and excessive cutting forces. It is, however, a much easier

task to monitor or measure the amount of inaccuracy and compensate through changes in the commanded position of the different axes which is applicable only to CNC machine tools. Software compensations are effective only if the machine errors are systematic and repeatable and also measurable [5].

### **2.3 Machine Modeling And Compensation Review**

Compensation for systematic errors has a long history. Some times before 1830, Edward Troughton used a look-up table of previously measured displacement errors to correct the position of the slide way on his linear dividing machine used for manufacturing scales [6]. Software based compensation concepts started in 1970's. The first practical implementation was on Moore N.5 CMM by Prof. R. Hokens [7]. The pioneer work was presented by J. Denavit, et al in form of ideal kinematic model based on homogenous transformation matrices (HTMs) later modified by P. P. Paul, considering a reference frame and laid the analytical foundation of a generalized error model. In 1973, W. J. Love, et al analyzed the volumetric errors by determining the combined effects through trigonometric technique. In 1977, R. Schultschik introduced the close vector chain technique. In the same year R. Hocken developed a matrix translation method and presented a calibration technique [8].

Zhang et al [9] developed an algorithm based on rigid body kinematics and small angle assumption to compensate geometric errors on a Coordinate measuring machine. Donmez et al [10] proposed a general methodology of error modeling and compensation by utilizing homogeneous transformation matrix. The method was implemented in several steps, and a kinematic error modeling method for geometric and thermally induced errors was further proposed. Ferreira [11] proposed an analytical quadratic model for the prediction of geometric errors. Chen et al [12] addressed the non-rigid body effects associated with the volumetric accuracy of a horizontal spindle machine tool. Srivastava et al [13] proposed a volumetric error model based on homogeneous transformation matrix for five axis CNC machine incorporating geometric and thermal errors. Yang et al [14] proposed a polynomial form of the volumetric error model to combine both the geometric and thermal errors.

R.G. Wilhelm et al [15] performed a controlled series of experiments to test the efficacy of performance measurement tests in the prediction of part form errors. Results were shown for flatness, squareness, position, and profile tolerances. He also suggested that standard machine tool performance tests can also be used to predict the "best-case" tolerances that can be achieved for particular part features. Jingxia [16] developed a real time error compensation technique for compensation of geometric, force and thermal errors of CNC machines. He applied the technique to 10 different kinds of machines including turning centers and machining centers; small, medium and large machines; new products and retrofitted machines. Accuracy improvement of the range of 3-10 times was observed. Eung-Suk Lee et al [17] compensated a vertical 3-axis CNC machine for all the 21 geometric errors. Errors were measured using laser interferometer and electronic level and compensated and results were verified with ball-bar measurement.

A.C. Okafor [1] developed an error model for three axis vertical machining center using rigid body kinematics and compensated the machine for geometric/thermal errors through machine controller. K. G. Ahn [18] proposed a volumetric error compensation model considering the backlash error of a vertical machining center. K. K. Tan et al [19] compensated a two-axis precision measuring table for geometric errors based on an error model using radial base function. Barakat et al [4] developed a strategy to compensate for geometric and kinematic

errors in Coordinate measuring machines. G. Chen et al [20] used the diagonal displacement test (with some additional measurements) required by ANSI/ASME standard for the performance evaluation of CNC machining centers to determine the 21 geometric errors, thus minimizing the time required for geometric error assessment. Chana [21] proposed an offline error compensation model by taking into account of geometric and cutting force induced errors in three-axis milling machine. Mahbubur Rahman [3] has modeled the machine tools based on a kinematics chain. He modeled and developed a mathematical model for measuring systems such as laser interferometer, ballbar, the linear comparator and the electronic inclinometer. He also established co-relation among several measurement techniques, which are used today to validate the measurement results.

T.O Ekinici [22] tried to develop a relationship between straightness and angular errors in machines. He developed an algorithm to convert straightness errors to angular errors and then physically measured the angular errors and compared the results. W.T. Lei [23] compensated geometric errors of CNC machine by using approximation function based on NURBS (Non-uniform rational basis spline). L. B. Kong et al [24] developed a kinematic model of two-axis ultra-precision machining system based on the theory of multi-body system. Khan [8] modeled a Five-axis turbine blade grinding machine based on rigid body kinematics and homogeneous transformation matrix. 39 position dependent and position independent systematic geometric errors out of 52 potential errors of five-axis machine tool were considered. C. Zhang et al [30] proposed a decoupling method for geometric errors in CNC machines.

## **2.4 Software Error Compensation**

Error compensation in CNC machines can be implemented in two ways:

- Error compensation through hardware
- Software error compensation

Compensation through hardware means for example minimizing errors through design, manufacturing machine components with tighter tolerances, replacing worn out machine components, adjusting squareness error between two perpendicular axes etc. Obviously this is costly alternative and manufacturing defects cannot be completely eliminated.

In software compensation CNC signal is modified directly or indirectly based on some measurement data. The most commonly used software compensations in CNC machines are pitch error, backlash, servo gain, position loop gain compensation etc. [26]. For software compensation of CNC machine an appropriate mathematical model and measurement of machine tool is required. The software error compensation idea has evolved from coordinate measuring machines to CNC machine tools. In early software compensation works in CNC machines, Donmez [10] compensated a turning center for geometric errors and thermal errors. He developed a mathematical model based on rigid body kinematics and took measurements with laser interferometer. Thermal drift was incorporated in the model based on time history. The compensation signal was fed to the controller through input/output ports. Significant enhancement in accuracy was observed. Chen J. S. et al have compensated for real time time-variant volumetric error [12]. They have expressed geometric and thermal error components in a mathematical model and introduced 11 error components to represent thermal errors, thereby making a total of 32 error components.

Sartori et al [6] explains the limitations of software compensation that only systematic error components can be compensated and they must be significantly greater than random errors. The results of software compensation depend on the thermal conditions of the machine and of objects used for its calibration. This is probably the most important limitation. Z.-Q. Liu [27] used Parametric programming for error compensation on the basis of a simple model of machining system deflections induced by the radial cutting force in CNC turning operations.

## **2.5 Summary**

A lot of work has been done in last three decades regarding modeling of machines and software compensation to enhance the accuracy of CNC machines but still a generalized model could not have been developed therefore research still continues in the area.

# Machine Tool Errors

### 3.1 Purpose

The purpose of this chapter is to get a detail view of sources and types of errors in CNC machine tools.

### 3.2 Introduction

Machine tools are complicated in their architecture and control, and are built from many components and each component contributes motion error to the final tool tip position. Main reasons of these motion errors are geometric errors of the machine tool, cutting process, driving mechanism and environment [26].

Error in machine tools can be understood as any deviation in the relative position of the tool tip and work piece from the theoretically required value. The extent of error in a machine gives a measure of its accuracy. According to ASME, *Error* is difference between the actual response of a machine to a command issued according to the accepted protocol of that machine's operation and the response to that command anticipated by that protocol, and *accuracy* is closeness of the agreement between the result of a measurement and a true value of the measurand [28].

Machine errors can be divided into many categories based on different criteria. Some of the classifications are discussed below,

### 3.3 Systematic and Random Errors

Errors in machine tools can be classified into two main categories which are systematic errors and random errors. Systematic errors are consistent in nature and repetitive, recurring consistently every time the measurement is made, but varying very slowly with passage of time due to degradation of a machine system. Systematic errors are predictable and can be modeled mathematically. In random errors numerical value or sign changes unpredictably. Random errors are difficult to model and to compensate. [8]

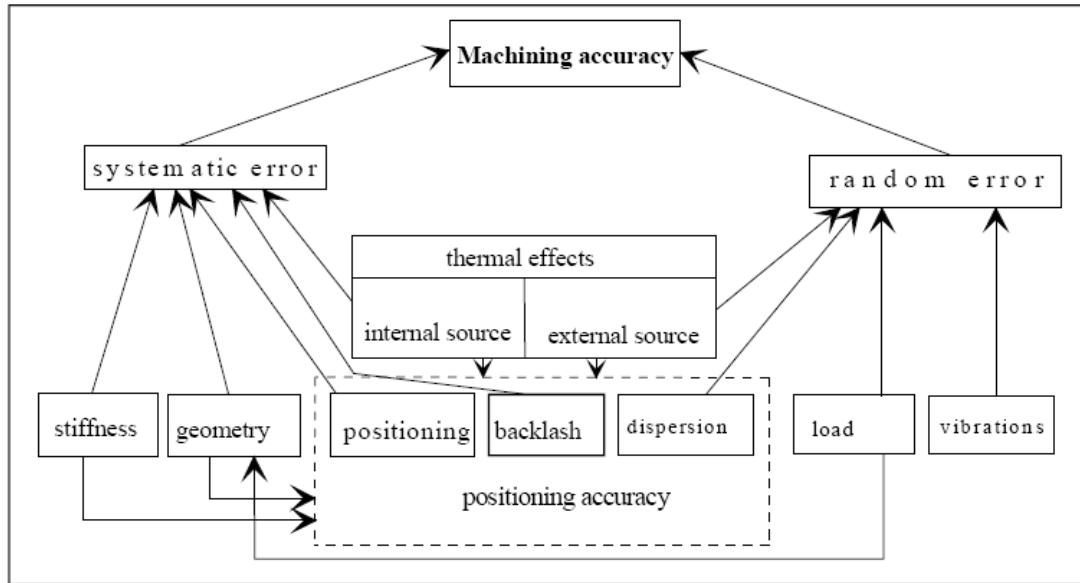


Figure 3.1 Systematic and Random Errors in Machine tools (Anderson 1992)

### 3.4 Quasi-static Errors and Dynamic Errors

In general, there are two basic categories of errors, quasi-static errors and dynamic errors. *Quasi-static* machine tool errors are those positional errors between the tool and the work piece that change slowly in time. These types of errors can be categorized into three classes: *kinematic*, or *geometric*, errors caused by inaccuracies in the machine tool itself or in the relative motion between its components; *thermal* errors caused by temperature variations in either the machine structure or the environment it resides in; and *load-induced* errors caused by the weights of the machine tool components, over-constrained slides, and part weights.

Quasi-static errors are errors in the machine, fixturing, tooling, and work piece that occur relatively slowly. Sources of this type of errors include geometric errors, kinematic errors, thermal errors, cutting force induced errors etc. Dynamic errors are, on the other hand, primarily caused by structural vibration, spindle error motion, controller errors, etc. Quasi-static errors account for about 70 percent of the total error of the machine tool and as such, are a major focus of error compensation research [5].

### 3.5 Types of Machine Tool Errors Based on Error Sources

Machine tool errors can be broadly categorized into following categories based on error sources,

1. Geometric errors due misalignment of machine structure and machine elements
2. Errors caused by thermal gradients/distortions
3. Errors induced by cutting forces including
  - (i) cutting action
  - (ii) gravity loads
  - (iii) accelerating axes
4. Kinematic errors i.e. errors due to velocity, acceleration etc.
5. Errors due to material non-uniformity
6. Errors due to imperfect machine assembly
7. Instrumentation errors
8. Tool wear

9. Spindle errors
10. Fixturing errors and
11. Other sources of errors like servo errors of the machine (following errors and interpolation algorithmic errors) [5].

Thus, errors can broadly be grouped into three major classes namely geometric and kinematic errors, temperature induced errors or thermal errors and cutting force induced errors. Figure 3.2 shows this classification used by Ramesh et al [5].

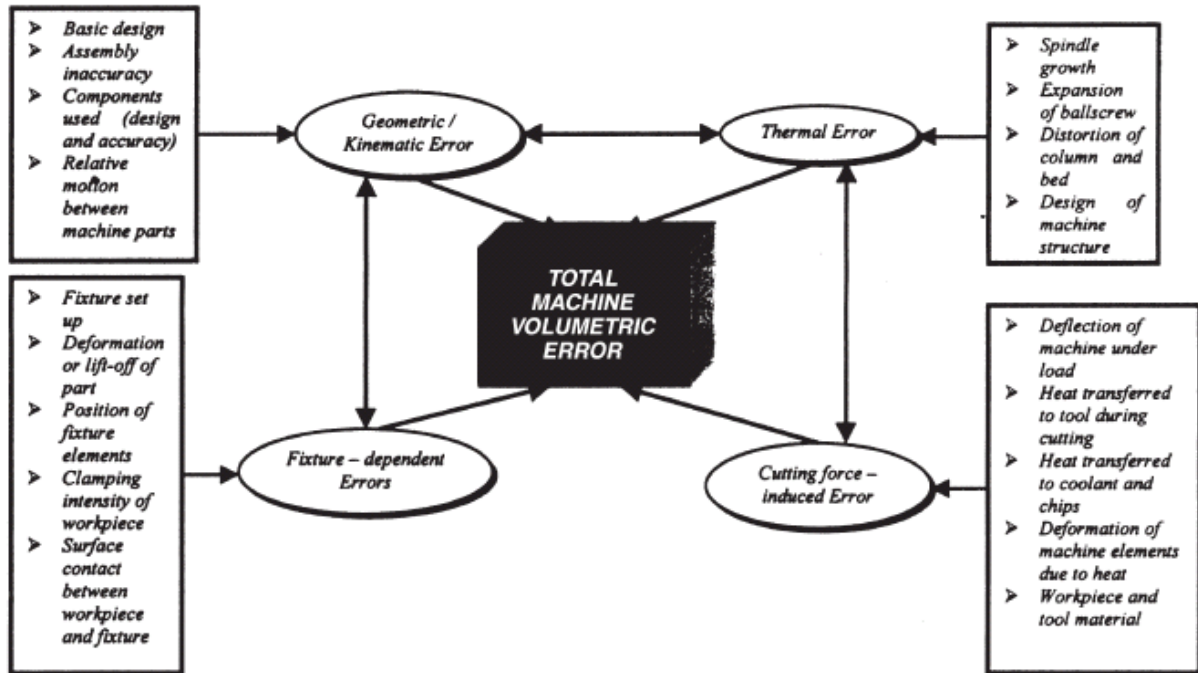


Figure 3.2 Overview of the error budget in a machine tool and the factors affecting it [5]

### 3.5.1 Geometric and Kinematic Errors

Geometric errors arise from misalignment of machine elements. These misalignments are result of manufacturing and assembly imperfections. Thermal distortions and component wear are another factor which can cause or change geometric errors. As a result of geometric errors, squareness and parallelism of moving elements change which in turn result in position errors between tool tip and work piece or imperfect geometry of part produce [1].

Geometric errors can be systematic or can exhibit a random behavior. Because many sources systemic as well as random are involved to produce them.

Kinematic errors related to the relative motion errors of moving machine elements. Acceleration/ deceleration or velocity errors are the examples. These errors become prominent when axes are moving simultaneously. Such errors occur during the execution of linear, circular or other types of interpolation algorithms and are more pronounced during actual machining. Kinematic and geometric errors are inter-related. [5]

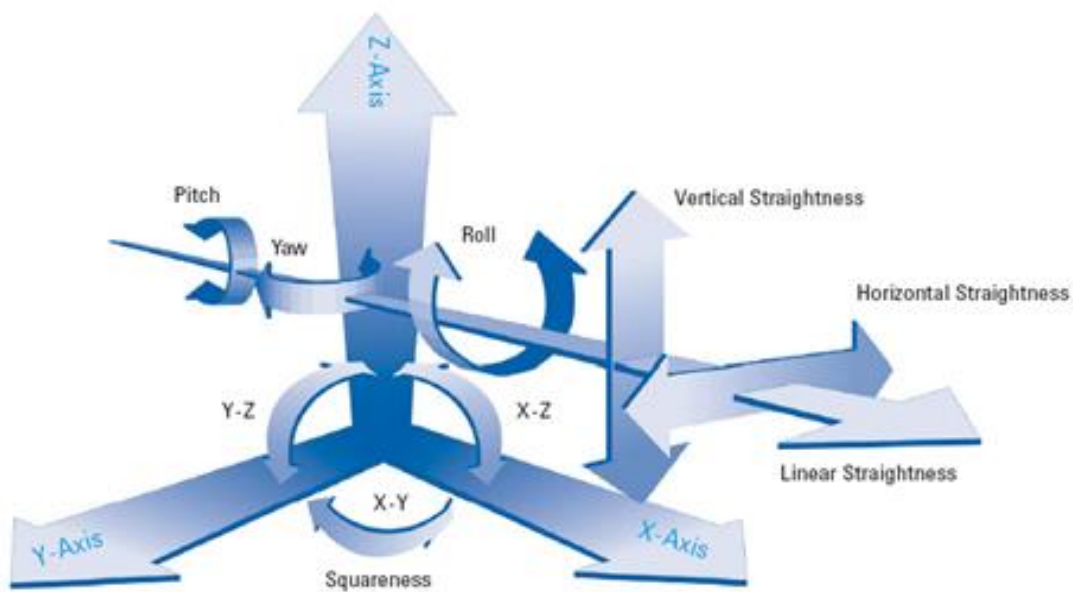
### 3.5.2 Geometric Errors for 3-Axis Machine Tool

All machine tools are composed of moving carriages, tables, or other elements whose purpose is to change the position of the work piece with respect to the cutting tool. Each element of a machine can be considered as rigid body with six degrees of freedom, five of which are constrained and it is driven to move along remaining one degree of freedom and its motion is measured precisely. A typical linear carriage moving in the X direction is shown in

Figure 3.4 below. One can measure six error terms associated with its motion. These measurements are the linear displacement error (positioning accuracy and repeatability) along the intended direction of travel (X); two straightness errors, in this case the Y straightness of X and the Z straightness of X; and three angular errors, rotations about the X, Y, and Z-axes, called roll, pitch and yaw, respectively. Therefore, for a three-axis machine, there are six error terms per axis to measure plus three error terms to define the squareness of the axes with respect to each other, yielding 21 different error terms [28].

**Table 3.1 21 geometric error components of a 3 axes machine**

Linear positioning errors (scale error)	03
Straightness errors	06
Angular errors	09
Squareness errors	03
<b>Total</b>	<b>21</b>



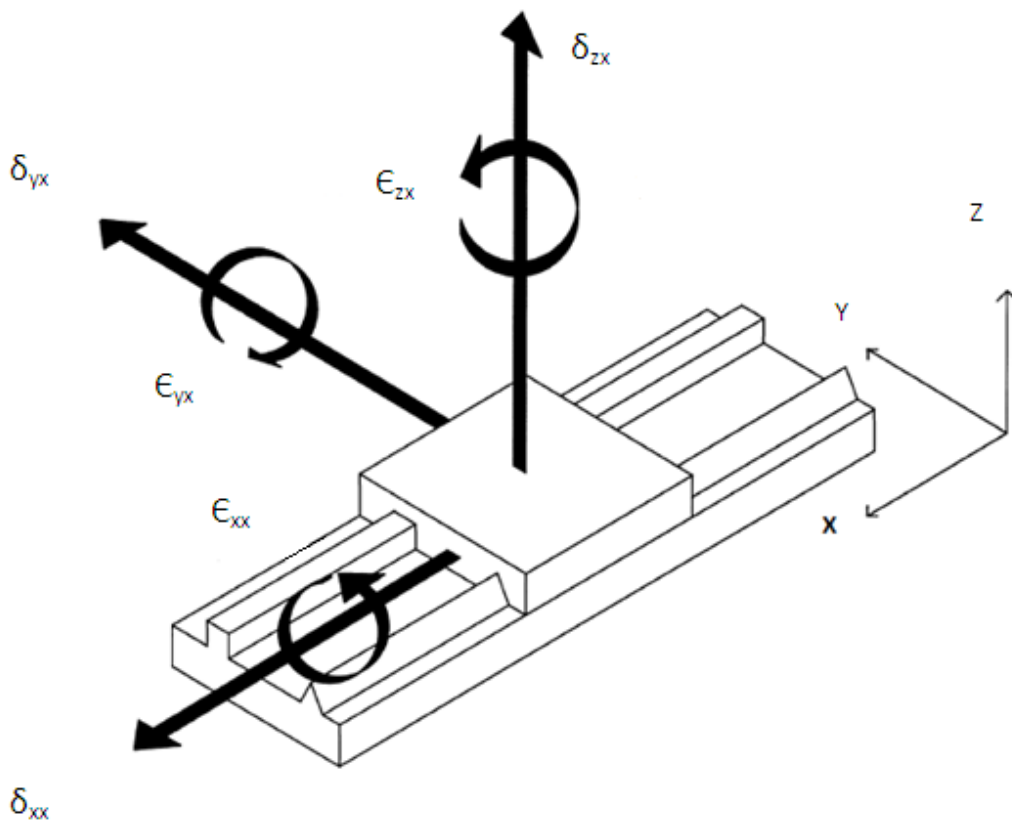
**Figure 3.3 21 geometric error components of a 3 axes machine**

Linear machine tool slides or carriages, like all rigid bodies, have six degrees of freedom of motion, three of which are translational, and three of which are rotational. Typically, five of these degrees of freedom are constrained. Errors of motion in the unconstrained sixth degree of freedom are typically referred to as *scale errors*, since the motion is usually measured using some kind of scale, or *lead-screw errors*, since that is usually the means of providing motion to the carriage. Undesired linear motions in directions orthogonal to the intended motion direction are referred to as *straightness errors*, usually either a *horizontal straightness error* or a *vertical straightness error*, depending on the orientation of the carriage. Rotations of the carriage are referred to as *angular errors* and are usually designated as *roll*, *pitch*, and *yaw*, analogous to the definitions used in aircraft [29].



A schematic of six error components of the three-axis machine's X-axis carriage system is given in Figure 3.4. A summary of the symbols used is as follows:

X	Direction of motion	
$\delta_{xx}$	Translational error along X- axis	(Linear positioning error)
$\delta_{yx}$	Translational error along Y- axis	(Straightness of X along Y-axis)
$\delta_{zx}$	Translational error along Z- axis	(Straightness of X along Z-axis)
$\epsilon_{xx}$	Rotational error about X- axis	(Roll)
$\epsilon_{yx}$	Rotational error about Y- axis	(Yaw)
$\epsilon_{zx}$	Rotational error about Z- axis	(Pitch)
$S_{xy}$	Squareness error between X-axis and Y-axis	Perpendicularity error



**Figure 3.4 Schematic of six degrees of freedom error motion of a machine tool carriage system.**

$\epsilon$  represents angular error motions. The first subscripted letter represents the axis, the carriage rotates about, and the second subscripted letter represents the intended direction of motion; positive rotation is defined by the 'right-hand rule';  $\delta$  represents the translational error motions. The first subscript represents the axis, the carriage translate along while moving the intended direction (the second subscript).  $S$  represents the mutual squareness of two subscripted axis.

A brief description of each geometric error is given below;

### 3.5.3 Linear Positioning Error (Scale Error)



Linear positioning error is the deviation from commanded position of an axis. Linear positioning error of an axis can be combined effect of all other geometric errors and scale error.

### 3.5.4 Straightness Error



Slide straightness error is the deviation from straight line movement. The carriage is exhibiting a lateral translation from a straight line while moving in intended axis direction. This erroneous movement is called straightness error.

Straightness error represents bending or overall misalignment in the guide ways of a machine. This could be the result of wear in these guide ways, an accident which may have damaged them, or poor machine foundations that are causing the axis to bow. Straightness error will have a direct effect on the positioning and contouring accuracy of a machine [30].

### 3.5.5 Angular Error



Angular error also called rotational error is the deviation of an objects angular orientation from a nominal angle or displacement. It is further categorized as Pitch, Yaw and Roll errors.

#### Pitch

Angular motion of a carriage, designed for linear motion, about an axis perpendicular to the motion direction and perpendicular to the yaw axis [28].

#### Yaw

Angular motion of a carriage, designed for linear motion, about a specified axis perpendicular to the motion direction. In the case of a carriage with horizontal motion, the specified axis shall be vertical unless explicitly specified. For a carriage that does not have horizontal motion, the axis must be explicitly specified [28].

#### Roll

Angular motion or rotation of a carriage, designed for a linear motion, about the linear motion axis [28].

Pitch and yaw errors are among the major contributors to positional inaccuracies of the machine tools. Pitch errors are caused by deflections in guideways due to gravity or some other bowing effects. Yaw errors are result of slackness in guideways in horizontal plane [30].

### 3.5.6 Squareness (Perpendicularity) Error



For two mutually perpendicular linear axes, the angular deviation from 90 degrees is called squareness error or only squareness. It is measured between the best-fit lines drawn through two sets of straightness data derived from two orthogonal axes in a specified work zone [28].

Squareness error can be a result of wear in machine guideways, an accident, poor machine foundation or misalignment of home position sensors. Squareness error results in positional inaccuracy and poor contouring ability of the machine [30].

### 3.5.7 Thermal Errors

One of the principal causes for position inaccuracies in machine tools is thermal deformation. Changes in the environmental temperature affect the machine structure, machine scales and work piece as well. Due to expansion or contraction due to thermal effects the dimensions of machine elements change thus changing overall geometric error of the system. About 40-70% of the total dimensional and shape errors of workpiece are attributed to thermal factors [31]. The environmental temperature effects can be minimized by air conditioning the machine surroundings yet other sources like heat generated from cutting operation are still there. Thermal errors usually have a complex nonlinear behavior which makes them difficult to handle.

Following sources of thermal variations are identified by Ramesh et al [31]:

- (i) heat generated from the cutting process
- (ii) heat generated by moving machine elements due to friction
- (iii) heating or cooling provided by the cooling systems
- (iv) heating or cooling influence of the surroundings
- (v) the effect of people
- (vi) Thermal memory from any previous environment.

Heat generated by moving machine elements is most critical among the above given sources of thermal variations. Continuous operation of the machine for machining a part results in thermal expansion of the machine frame due to friction. The distortion in the machine frame may result in direct position error or may induce a squareness error among axes. If rotary encoders are used with ballscrews to provide linear positioning then thermal monitoring of the ballscrew becomes critical because in this case ballscrew becomes a part of position measuring loop and effects the position of the carriage directly. Thermal expansion of spindle due to rotating motor and excessive loads of cutting operation is called spindle growth. Thus thermal changes can distort machine structure, expand ballscrew and result in spindle growth, changing the tool tip position with respect to the work as a consequence.

Another heat generation source which cannot be avoided completely is heat generation at contact point of tool tip and work piece during cutting operation. By flushing the coolant the temperatures can be controlled to some extent. This heat source can results in expansion of tool or work piece.

Some other internal heat sources in a machine tool are given below

- a) Bearings
- b) Gear and hydraulic oil
- c) Drives and clutches
- d) Pumps and motors

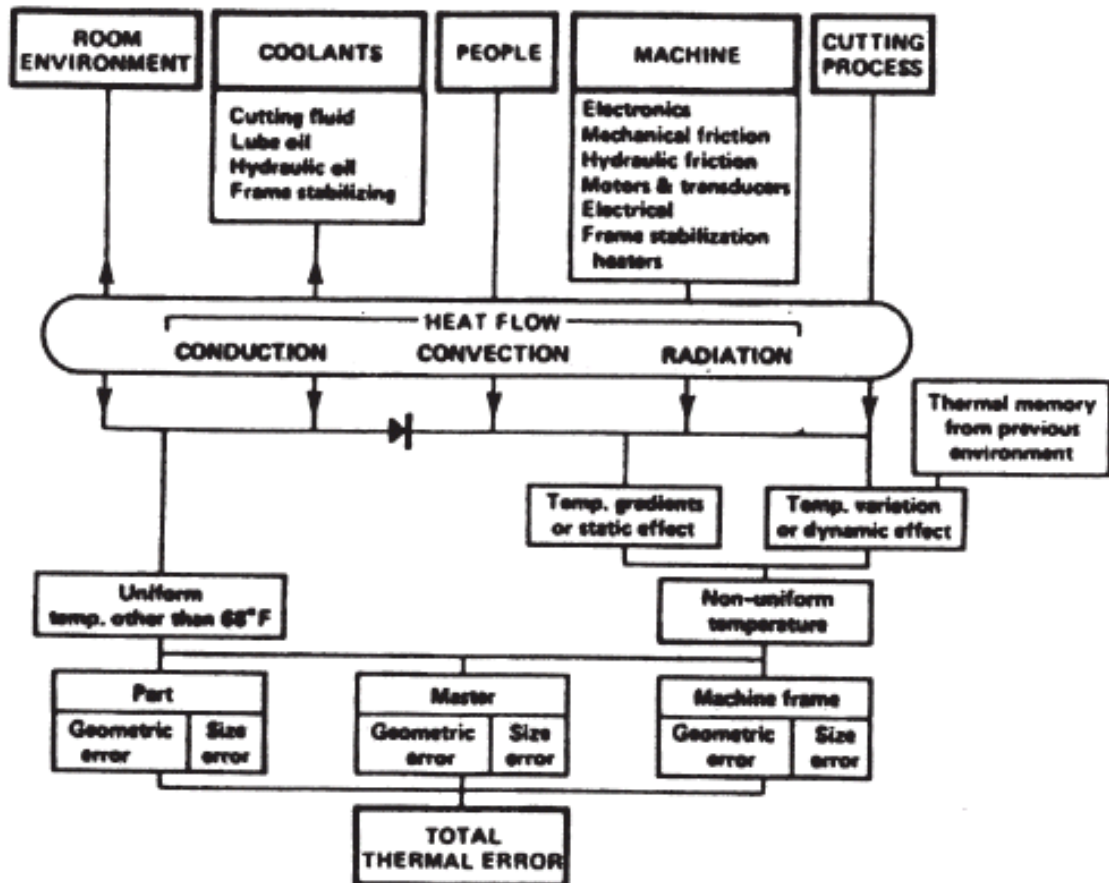


Figure 3.5 Thermal Effects Diagram [36]

Figure 3.5 discusses heat sources, their transmission and their effect on machine and part. The ultimate result of thermal errors is geometry and size errors in part produced. Figure 3.6 shows deformation in machine structure due to internal and external heat sources [31].

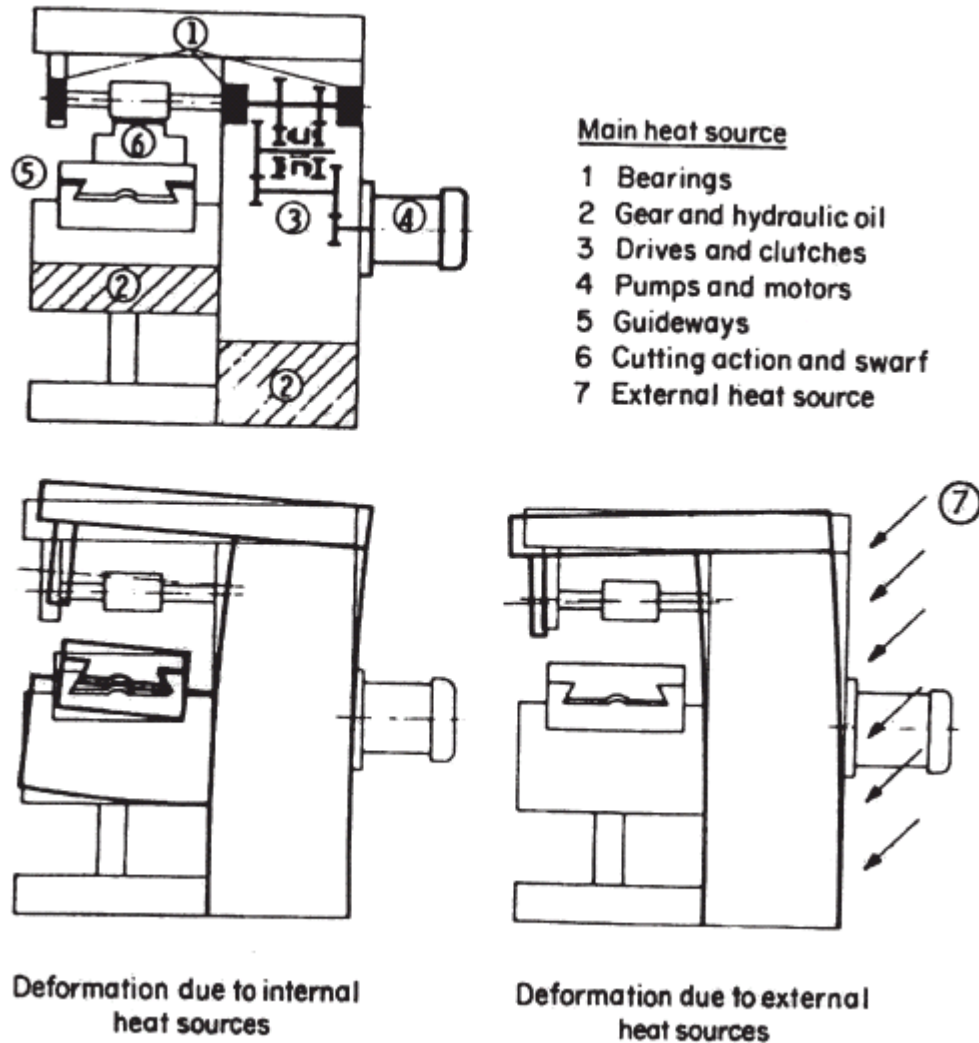


Figure 3.6 Examples of thermally induced displacements on a milling machine [M. Week 1984]

### 3.5.8 Cutting-force Induced Errors

The cutting action generates forces which are exerted on tool tip and work piece and ultimately on machine elements like bed, column guideways etc. The stiffness of these machine elements is responsible for errors caused by cutting forces. As a result of cutting force the position of the tool tip with respect to work piece varies because of the distortion of the various machine elements [5].

Thus machine accuracy is dependent upon stiffness of the machine structure and the forces generated during cutting process. If machine structure has high stiffness, it would have better accuracy. On the other hand for a machine with a given stiffness a heavy cut would generally produce more inaccurate components than a lighter cut.

Cutting force errors are interrelated to thermal and geometric errors. With thermal effects, stiffness of machine elements changes and under forces deformation occurs, distorting the geometry of the machine and as a result inducing position errors. Spindle growth is an example where both thermal effect and cutting forces induce error.

### **3.5.9 Other Errors**

In other errors which contribute to the inaccuracy of machined parts tool wear and fixture errors can be considered. Tool wear compensations are applied in machine controllers to compensate for tool wear. Fixtures are used for clamping and orientation of work pieces using locating elements and clamping devices. Fixture errors arise from fixture setup and geometric inaccuracies of fixture elements [5].

In other sources of errors in machines ball screw errors, guide ways errors, bearings, spindle misalignment, vibrations, controller induced errors, servo control errors etc. can be mentioned.

### **3.6 Summary**

Machine tool errors can be systematic and random errors. Systematic errors are predictable whereas random errors are difficult to predict. Geometric errors dominate in systematic errors and the major origins are ball screws, guide ways, bearings etc. Thermal errors are another major source and can complicate the geometric errors. Force induced errors are another source of errors. These errors are interrelated and form a complex situation for handling. These errors can be minimized through feedback control by modeling the errors and implementing a compensation plan.

# Machine Measurement

### 4.1 Purpose

The purpose of this chapter is to discuss the tools and techniques used in measurement of machines.

### 4.2 Introduction

The basic standard as a guideline for measuring the errors in machine tools are documented in International Organization of Standardization (ISO) 230 series [32, 33, 34, and 35] and American Society of Mechanical Engineers (ASME) B5.54 [28] and some methodologies, techniques and information about instrumentation are already available and are in practice. These basic standards can be applied to any machine tool but there are some specific standards for the calibration of machines including the tolerance for deviations of errors from the measured values.

Usually following measuring equipment are used for CNC machine performance measurements,

- Optical instruments (Laser Interferometers, Auto collimators)
- Linear Comparator (Precision scales, displacement indicators)
- Circular test equipment (Ball bar, Grid encoders)
- Angle measuring devices (Inclinometers, Electronic levels, etc.)
- Artifact measurements (Step gauge, Ball plates, test mandrel, Square block, straight edge)

### 4.3 Geometric Error Measurement

Table 4.1 gives typical devices used for different geometric error measurements.

**Table 4.1 References and devices used to measure geometric errors in machines [6].**

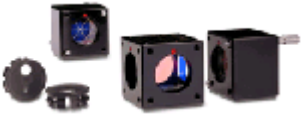




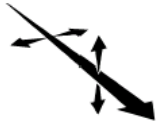



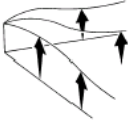


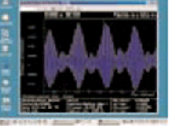

Type of geometrical error	References and devices
Positioning error	<ul style="list-style-type: none"> <li>• Laser interferometers</li> <li>• Set of gauge blocks / end bars</li> <li>• Step gauges</li> <li>• Ball arrays</li> </ul>
Straightness error	Straightness references: <ul style="list-style-type: none"> <li>• Taut wire</li> <li>• mechanical and optical straight edge</li> <li>• wing reflector</li> <li>• Laser beam</li> </ul> Displacement indicator: <ul style="list-style-type: none"> <li>• capacitance gage</li> <li>• electronic gage and LVDT</li> <li>• plane mirror laser interferometer</li> <li>• photodiode</li> </ul>
Angular errors ( Pitch, Yaw)  Angular Error (Roll)	Autocollimator Angular laser interferometer Mechanical level Electronic level Straightness measuring devices separated at certain distance
Squareness	Mechanical square with collimator Diagonal measurements
Parallelism	Collimator with optical square Laser interferometer with optical square

### 4.3.1 Laser Measuring System

Almost all geometric measurements can be made with laser interferometers designed for machine tool measurements for example Renishaw laser measuring system ML10 has been used in this study. It's based on ML10 (HeNe laser) laser source and EC10 environmental compensation unit which provides for wavelength and material thermal expansion compensation (Environmental compensation is only required for linear displacement measurements). With different set of optics the system is capable of measuring various machine errors; Table 4.2 gives measuring options available. Figure 4.1 shows a typical setup for linear measurements.



**Table 4.2 Measurement options of Renishaw laser measuring system [30]**

Linear positioning accuracy and repeatability of an axis		
Angular pitch and yaw of an axis		
Straightness of an axis		
Squareness between axes		
Flatness of a surface		
Rotary axis/table angular positioning		
Dynamic characteristics of a machine		

### 4.3.2 Displacement (Positioning Error) Measurement

Linear positioning error of an axis can be measured with Laser interferometer, Precision scales, artifacts like gauge blocks, step gauges etc. using displacement indicators.

ISO 230-2 [33] and ASME B5.54 [28] recommend laser interferometer for positional error measurement in CNC machines. For positional accuracy and repeatability measurements machine is programmed to move the axis under test and to position it at a series of target positions. At a target position, the machine will remain at rest long enough for the actual position reached to be measured and recorded. . Fig 4.1 shows setup of laser interferometer for linear positioning error measurement. The measurement setup is designed to measure the relative displacements between the component that holds the tool and the component that holds the workpiece in the direction of motion of the axis under test [28].

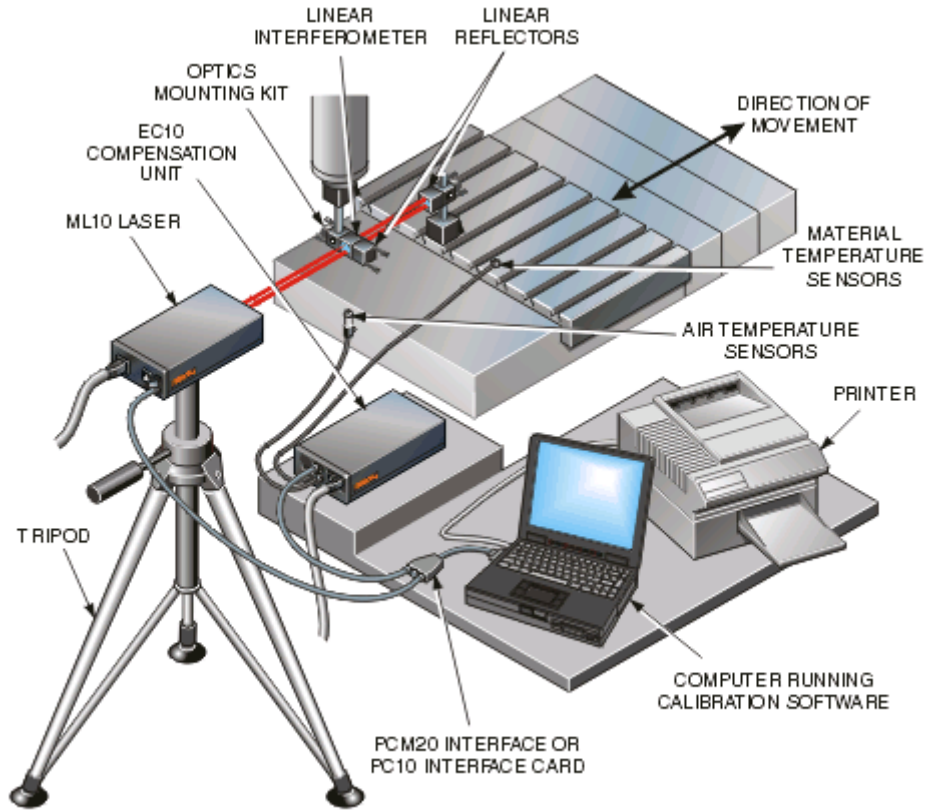


Figure 4.1 Linear positioning error measurement setup with Laser interferometer [30]

### 4.3.3 Straightness Measurement

The laser system measures the machine straightness accuracy and repeatability by moving the machine to a number of target positions and measuring the straightness deviations. These measurements must be repeated for the two measurement planes, e.g. the vertical measurement plane and the horizontal measurement plane when measuring the straightness of a horizontal axis [30].

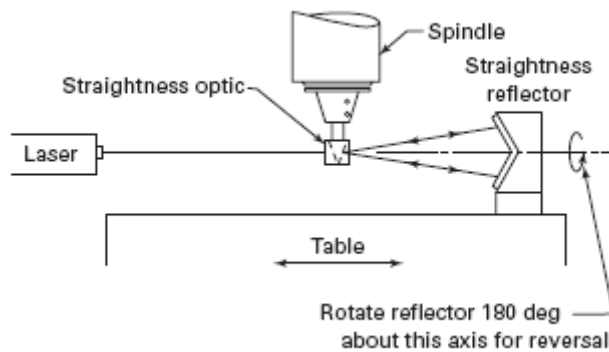
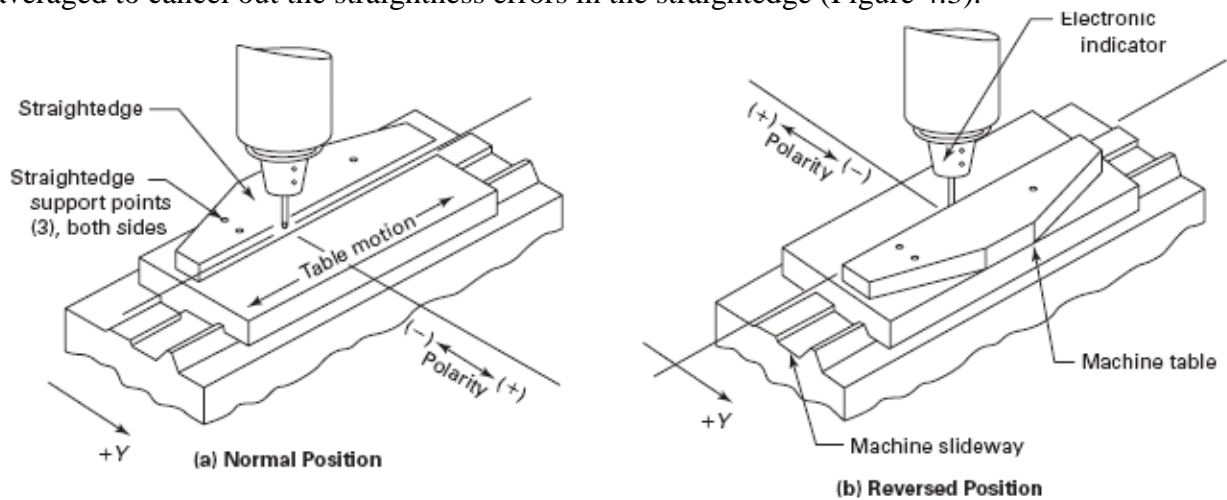


Figure 4.2 Straightness measurement using laser interferometer

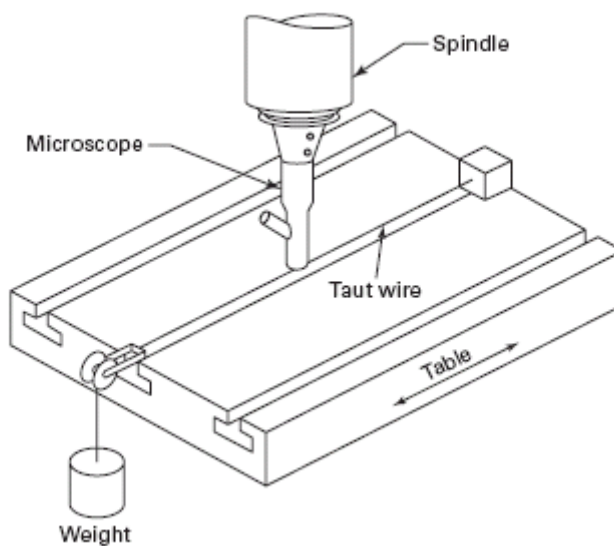
Straightness can also be measured using a straightness and displacement indicator using reversal technique for horizontal axis. First measurement is taken along the straight edge and

then it is rotated 180° and measurement taken again. Two sets of readings are suitably averaged to cancel out the straightness errors in the straightedge (Figure 4.3).



**Figure 4.3 Setup for Measuring Straightness Using an Electronic Indicator and a Mechanical Straightedge [28]**

Taut wires (Figure 4.4) are often used for measuring the horizontal straightness on large machines. The wire is stretched along the axis direction and measurements of wire position are made with a sensor (proximity sensor or microscope) mounted in the machine spindle [28].



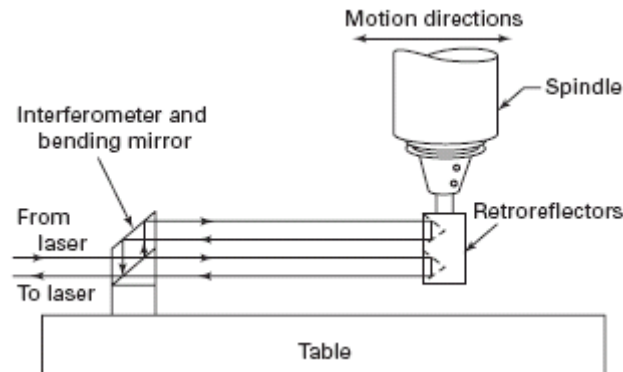
**Figure 4.4 Test setup for measuring straightness using taut wire [28]**

Straightness can also be measured using a precision level; the instrument is fixed to the moving component. The component is moved incrementally and the level readings are recorded after each move [32].

#### 4.3.4 Angular Measurements

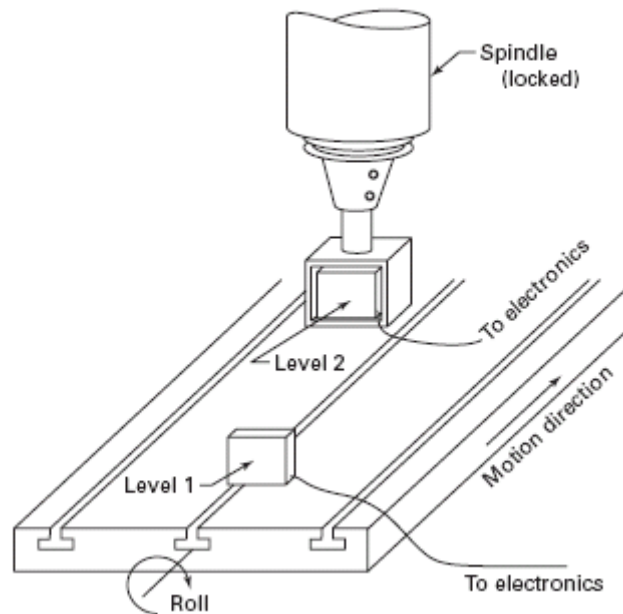
The common methods of angular error measurement use (1) laser angular interferometers or autocollimators to measure pitch and yaw of an axis (rotations about axes orthogonal to the direction of axis motion), and (2) differential levels to measure the roll (rotation about the axis of linear motion) of horizontal axes. Differential straightness measurements are used to

assess the roll of vertical axes. Differential levels may also be used for measuring the pitch of horizontal axes and the yaw and pitch of vertical axes [28].



**Figure 4.5 An Angular Interferometer Setup to Measure Pitch on a Machine Where the Spindle Moves Relative to the Table [28]**

Differential levels can be used for measuring roll and pitch of horizontal axes and yaw and pitch of vertical axes, as shown in Figure 4.6. One level is placed in the part position and a second measuring level in a fixture attached to the machine spindle. The readout device is set to display the difference between the two readings of the levels.



**Figure 4.6 Typical Setup Showing Differential Levels to Measure the Roll of a Horizontal Axis [28]**

### 4.3.5 Squareness Measurement

Squareness is measured with a variety of instruments as described below. The procedure is similar in all these cases. Two nominally square reference lines are established in the middle of the work zone where possible, each nominally parallel to one of the axes whose squareness to each other is to be measured. For each axis in turn, the machine is traversed along its motion axis and the lateral motion between the nominal tool point and the reference surface is

measured in a direction orthogonal to the traverse direction using an indicator in the tool position [28].

**Mechanical Square:** The Square is placed in such a way that its reference surfaces are nominally aligned with the two axes whose squareness is to be measured. A mechanical indicator is used to trace the reference surfaces of the square along the axes of travel.

**Optical Square and Straightness Interferometer:** With laser interferometer squareness measurements are carried out by making straightness measurements along each of the two nominally orthogonal axes of interest, using a common reference. A common reference is required so that the two sets of straightness measurements can be compared and the out of squareness of the two axes calculated. The common reference is normally the optical alignment of the straightness reflector, which is neither moved (relative to the table), nor adjusted, between the two straightness measurements. An optical square is used for at least one of the straightness measurements to allow the laser beam to be aligned along each axis without touching the straightness reflector [30].

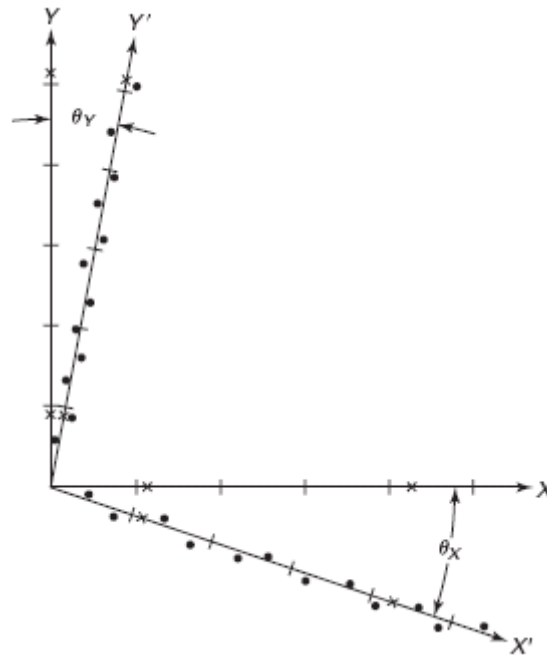


Figure 4.7 Conceptual Diagram Showing the Angles Obtained in a Squareness Measurement [28]

#### 4.3.6 Parallelism Measurement

It is often necessary to measure parallelism between linear axes where there are more than the traditional three linear axes. For example, parallelism may be checked between the Z and W axes on a machining center, where the W is an extending spindle. This test is performed by using any straightness measurement device [28].

The laser system can be used to measure linear and rotational parallelism. Linear parallelism measurements are made to determine the misalignment between two nominally co-axial axes. Rotational parallelism measurements are made to determine the misalignment between a

rotational axis and a nominally co-axial linear axis, for example, the out of parallelism of a lathe's carriage axis relative to the spindle's axis of rotation [30].

#### 4.4 Diagonal Displacement Test

Diagonal displacement tests are used to determine displacement accuracy of the machine along body or face diagonals. To obtain an estimate of the volumetric positioning capability of the machine, one has to combine the results of these tests with those of linear displacement tests. [28].

In diagonal displacement test measurements are taken along body or face diagonals. Laser interferometer with specific optics is used to measure these diagonals.

*Body Diagonals:* space diagonal of a rectangular prism within the working volume of the machine tool [35], Figure 4.8.

*Face diagonals:* diagonal in a face plane of a rectangular prism within the working volume of a machine tool [35] Figure 4.9.

G Chen et al [20] showed that with diagonal displacement measurements by adding some additional measurements as required by ISO 230-6 [35], all 21 geometric parameters can be determined.

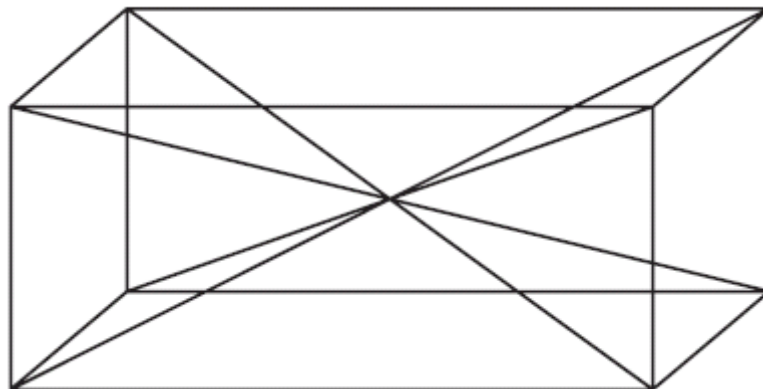


Figure 4.8 Four Body Diagonals of a Rectangular Prism [28]

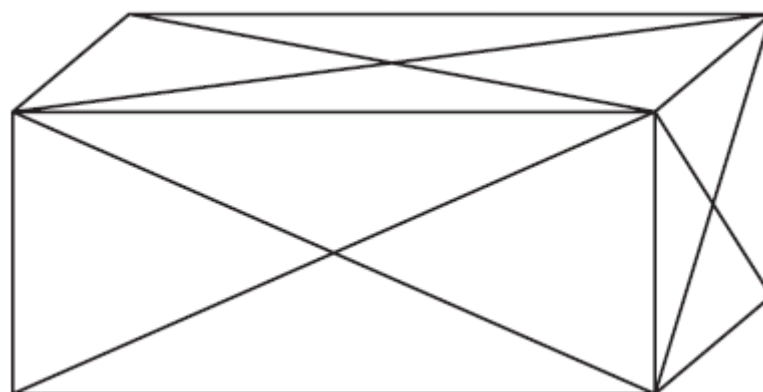


Figure 4.9 Face Diagonals of a Rectangular Prism [28]

#### 4.5 Circular Tests

Circular tests provide one of the best checks for contouring performance evaluation. When contouring a circle, a machine is running with multiple axes along a circular path and each

axis goes through sinusoidal acceleration, velocity, and position changes. Circular tests can be performed using telescoping ball bars, grid encoders, or precision disks [28].

In circular test CNC machine is programmed to move in circle and small deviations are measured using a transducer and data is captured by software. The resultant data is plotted to reveal the performance of the machine. Ideally if there is no error the plot is a perfect circle. But the errors distort the shape of circle for example circle may be elliptical or spikes at reversal movement points. These different kinds of patterns of circles can be used to diagnose problems and inaccuracies in machine.

Circular test is a handy diagnostic tool and can reveal geometric errors (straightness, squareness), ball screw errors (backlash) or errors in servo drives (servo mismatch).

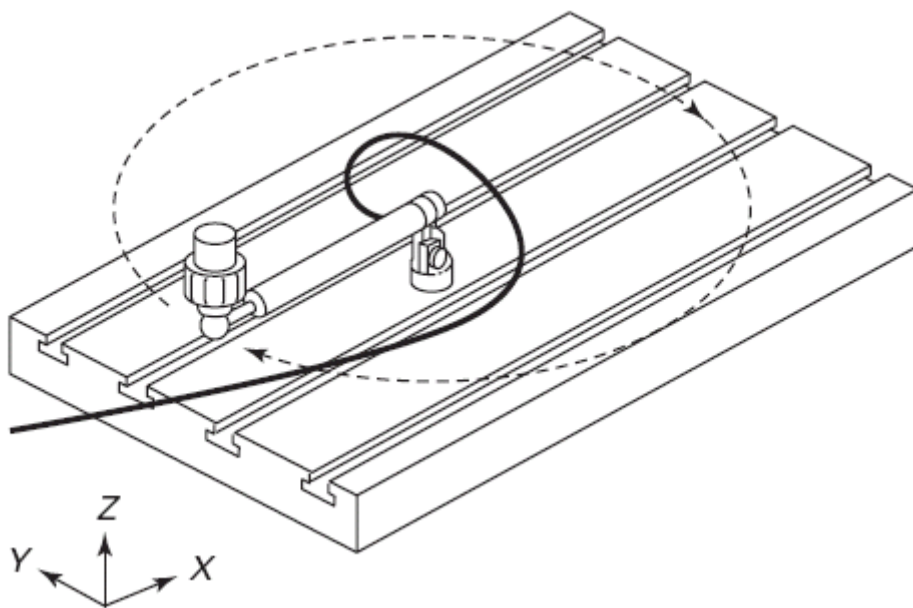


Figure 4.10 Typical setup for 360° circular test [28]

## 4.6 Other Tests

Variety of other tests are defined by standards as given in [28, 32, 33] for machine performance measurements which include spindle alignment test, alignment of machine tables and other elements, rotary axis test, part machining tests, dynamic tests, test for thermal effects, force and vibrations etc.

### 4.6.1 Part Machining Tests

Machining test parts have an important role in the acceptance testing of machining centers. The machining accuracy is checked under a limited set of machining conditions, to arrive at standard means of machine comparison. Since such tests necessarily include factors that may not have any direct relation to the machine tool, such as materials variation, tooling variations, thermal effects of cutting, and fixturing variations, careful attention must be paid to the minimization of these effects for accurate comparison of different machining centers [28].

## 4.7 New Developments in The Field of Machine Measurement

### 4.7.1 Laser Tracer

For highest precision applications the LaserTRACER was developed in cooperation with the German Physikalisch-Technische Bundesanstalt (PTB) and the National Physical Laboratory (NPL) in the UK. This is a self-tracking laser interferometer in which laser beam tracks automatically a reflector and determines the distances with highest accuracy in all directions. Figure 4.11 shows a typical setup of laser tracer on a CNC machine.

The time needed for a measurement has reduced considerably. The laser tracer method does not require precise adjustment or positioning of the instruments and it require less number of measurements as compared to conventional laser instruments. After a few steps all systematic geometric deviations of Cartesian and rotary axes can be calculated.

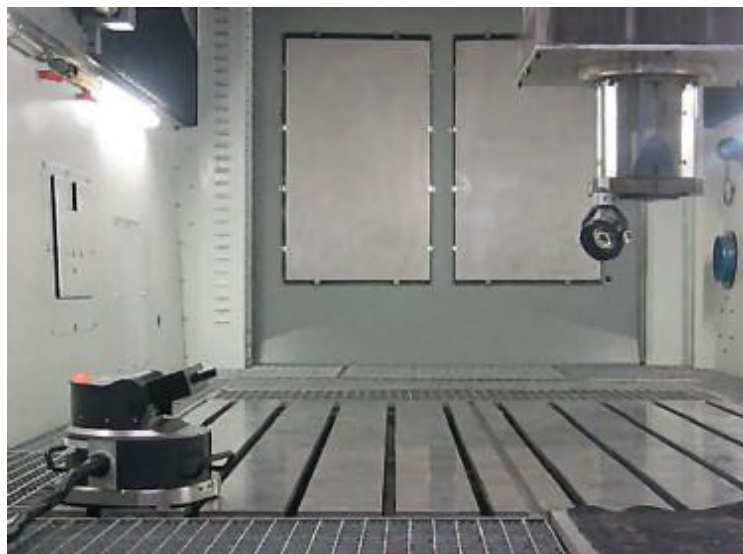
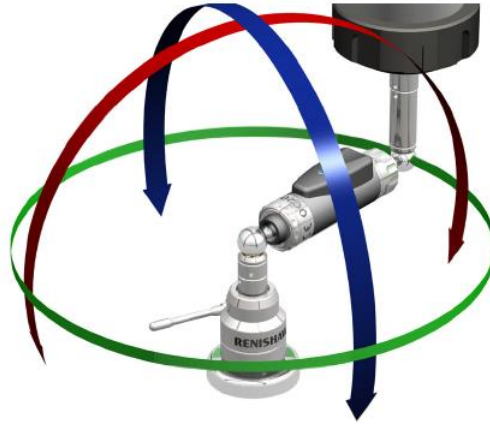


Figure 4.11 Laser Tracer [Etalon]

### 4.7.2 Ball-bar With Volumetric Testing

Ball-bar is a circular test device but Renishaw has upgraded its ball-bar ability in QC20W. It performs testing in three perpendicular planes through a single reference point. It is used as quick check for the performance of positional and geometric accuracy of machines in limited volume. In Figure 4.12 measuring paths for a 3D ballbar are shown.





**Figure 4.12 3D Ball Bar [Renishaw]**

#### **4.8 Summary**

Many methods are described in standards like ISO 230 series [32-35] and ASME B5.54 [28] for performance evaluation and testing of CNC machine tools. Measuring methods and equipment related to geometric error measurement have been discussed in this chapter. New developments in this field are intended to minimize time required for testing.

## Geometric Error Modeling

### 5.1 Purpose

The purpose of this chapter is to develop a mathematical model of two axes CNC machine based on which geometric error compensation will done later on.

### 5.2 Introduction

Errors present in the machines due to the mechanical imperfections (either due to manufacturing defects or produced because of wear of moving elements) and misalignment of machine tool elements are called geometric error, which include, linear positional errors or scale errors, straightness errors, angular errors and squareness errors. For a three axis machine a total of 21 errors exist. Based on the number of axis, criticality, and modeling approach certain errors can be neglected to minimize measurement effort.

### 5.3 Geometric Error Modeling of Three Axis Machine Tool

A Schematic of six error components of X-axis is given in Figure 5.1 Details of symbols and errors is given as under

X	Direction of motion	
$\delta_{xx}$	Translational error along X- axis	(Linear positioning error)
$\delta_{yx}$	Translational error along Y- axis	(Straightness of X along Y-axis)
$\delta_{zx}$	Translational error along Z- axis	(Straightness of X along Z-axis)
$\epsilon_{xx}$	Rotational error about X- axis	(Roll)
$\epsilon_{yx}$	Rotational error about Y- axis	(Yaw)
$\epsilon_{zx}$	Rotational error about Z- axis	(Pitch)
$S_{xy}$	Squareness error between X-axis and Y-axis	

These are the geometric errors which can be present in an axis for slide motion. Similarly symbols can be assumed for other two axes.

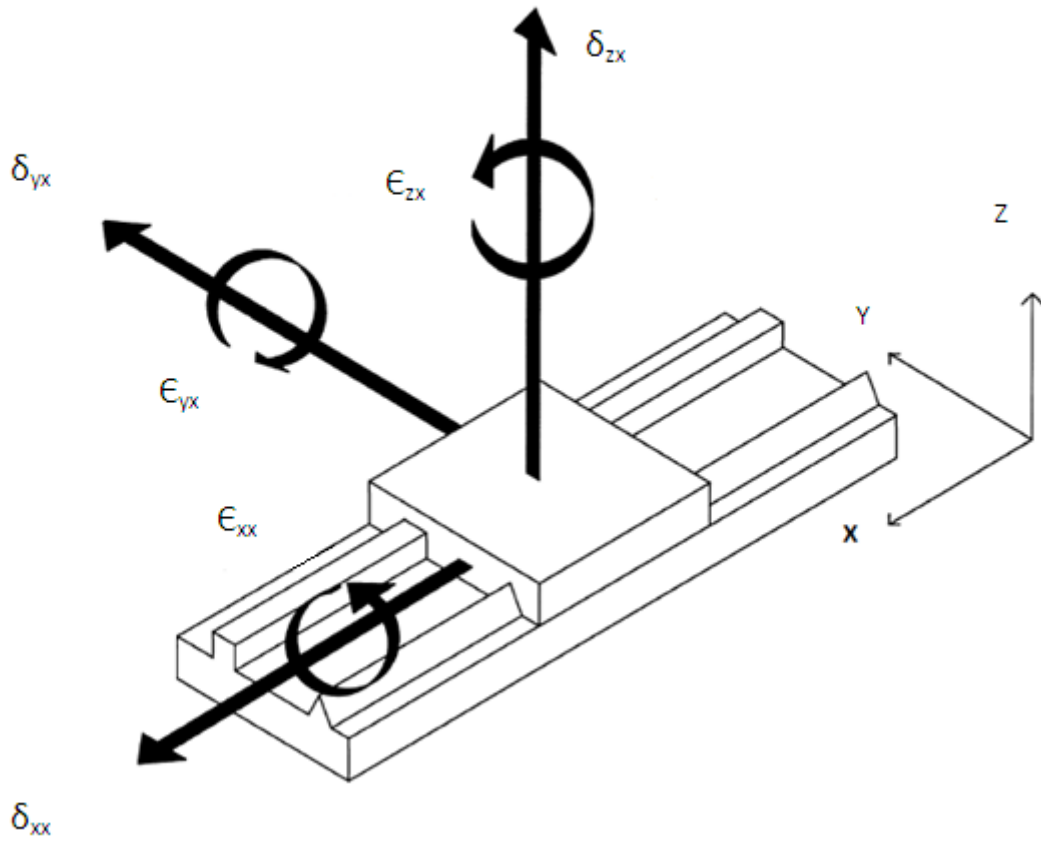


Figure 5.1 Schematic of six degrees of freedom of error motion of a machine tool carriage in an axis

### 5.3.1 Homogeneous Transformation Matrix

Let's assume that the slide moves along X-axis from position O to Position O1 (Figure 5.2). To perform the transformation from O1 to O following steps are required [24]

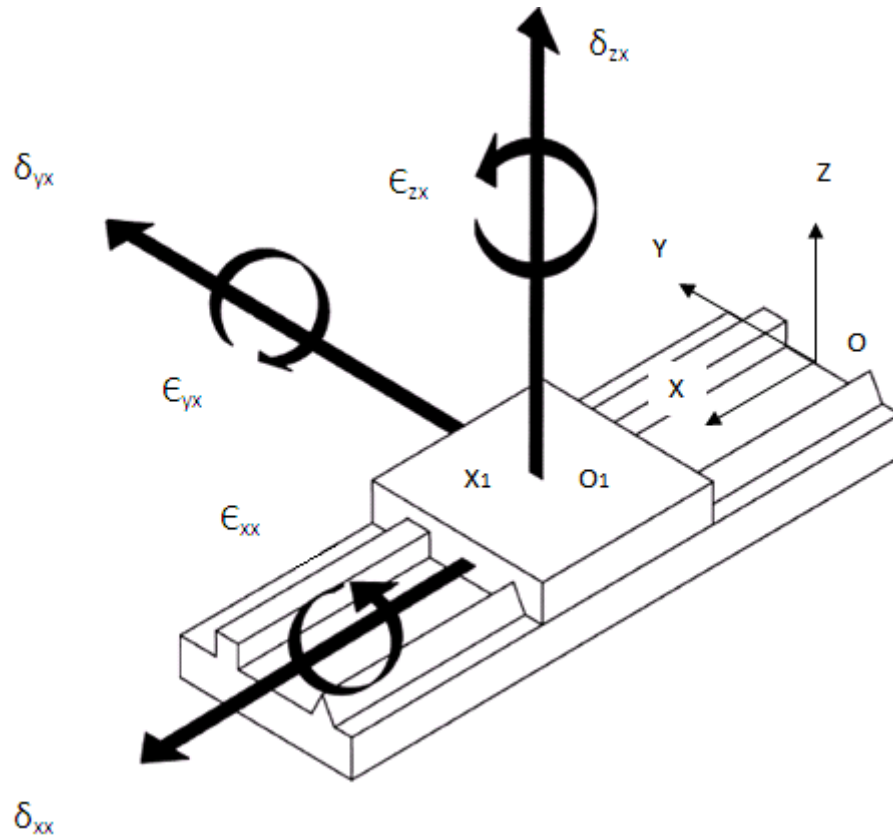


Figure 5.2 Schematic of translation of machine tool carriage in X- axis from O to O1

Its angular motions cause angles  $\alpha$  (about X-axis)  $\beta$  (about Y-axis) and  $\gamma$  (about Z-axis) then

$$T_o^{O1}(\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \alpha & -\sin \alpha & 0 \\ 0 & \sin \alpha & \cos \alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_o^{O1}(\beta) = \begin{bmatrix} \cos \beta & 0 & \sin \beta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin \beta & 0 & \cos \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_o^{O1}(\gamma) = \begin{bmatrix} \cos \gamma & -\sin \gamma & 0 & 0 \\ \sin \gamma & \cos \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_o^{O1}(R) = T_o^{O1}(\alpha)T_o^{O1}(\beta)T_o^{O1}(\gamma)$$

$$T_o^{O1}(R) = \begin{bmatrix} \cos \beta \cos \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma & 0 \\ \cos \beta \sin \gamma & \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & \cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma & 0 \\ -\sin \beta & \sin \alpha \cos \beta & \cos \alpha \cos \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

Transformation matrices by translation of slide distances  $\delta_x, \delta_y, \delta_z$  along x, y, and z axis.

$$T_o^{O1}(x) = \begin{bmatrix} 1 & 0 & 0 & \delta x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_o^{O1}(y) = \begin{bmatrix} 1 & 0 & 0 & \delta y \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_o^{O1}(z) = \begin{bmatrix} 1 & 0 & 0 & \delta z \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_o^{O1}(T) = T_o^{O1}(x)T_o^{O1}(y)T_o^{O1}(z)$$

$$T_o^{O1}(T) = \begin{bmatrix} 1 & 0 & 0 & \delta x \\ 0 & 1 & 0 & \delta y \\ 0 & 0 & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

The integrated transformation matrix can be derived from equations (1) and (2)

$$T_o^{O1} = T_o^{O1}(R)T_o^{O1}(T)$$

$$T_o^{O1} = \begin{bmatrix} \cos \beta \cos \gamma & \sin \alpha \sin \beta \cos \gamma - \cos \alpha \sin \gamma & \cos \alpha \sin \beta \cos \gamma + \sin \alpha \sin \gamma & \delta x \\ \cos \beta \sin \gamma & \sin \alpha \sin \beta \sin \gamma + \cos \alpha \cos \gamma & \cos \alpha \sin \beta \sin \gamma - \sin \alpha \cos \gamma & \delta y \\ -\sin \beta & \sin \alpha \cos \beta & \cos \alpha \cos \beta & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

Now the actual rotations as defined above are  $E_{xx}, E_{yx}, E_{zx}$  abbreviated as  $E_x, E_y, E_z$ . the matrix becomes as

$$T_o^{O1} = \begin{bmatrix} \cos \varepsilon_y \cos \varepsilon_z & \sin \varepsilon_x \sin \varepsilon_y \cos \varepsilon_z - \cos \varepsilon_x \sin \varepsilon_z & \cos \varepsilon_x \sin \varepsilon_y \cos \varepsilon_z + \sin \varepsilon_x \sin \varepsilon_z & \delta x \\ \cos \varepsilon_y \sin \varepsilon_z & \sin \varepsilon_x \sin \varepsilon_y \sin \varepsilon_z + \cos \varepsilon_x \cos \varepsilon_z & \cos \varepsilon_x \sin \varepsilon_y \sin \varepsilon_z - \sin \varepsilon_x \cos \varepsilon_z & \delta y \\ -\sin \varepsilon_y & \sin \varepsilon_x \cos \varepsilon_y & \cos \varepsilon_x \cos \varepsilon_y & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Since the angles are very small ignoring the higher order derivations the matrix can be expressed as

$$T_o^{O1} = \begin{bmatrix} 1 & -\varepsilon_z & \varepsilon_y & \delta x \\ \varepsilon_z & 1 & -\varepsilon_x & \delta y \\ -\varepsilon_y & \varepsilon_x & 1 & \delta z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

#### 5.4 Geometric Error Modeling of Two Axis Grinding Machine

Now in our case of two axes grinding machine, axis configuration is as given in Figure 5.3 X-axis moves over Z-axis and axis directions are given in upper left corner. Y-axis is not present but for the sake of modeling it is assumed to be positive upwards as shown in figure.

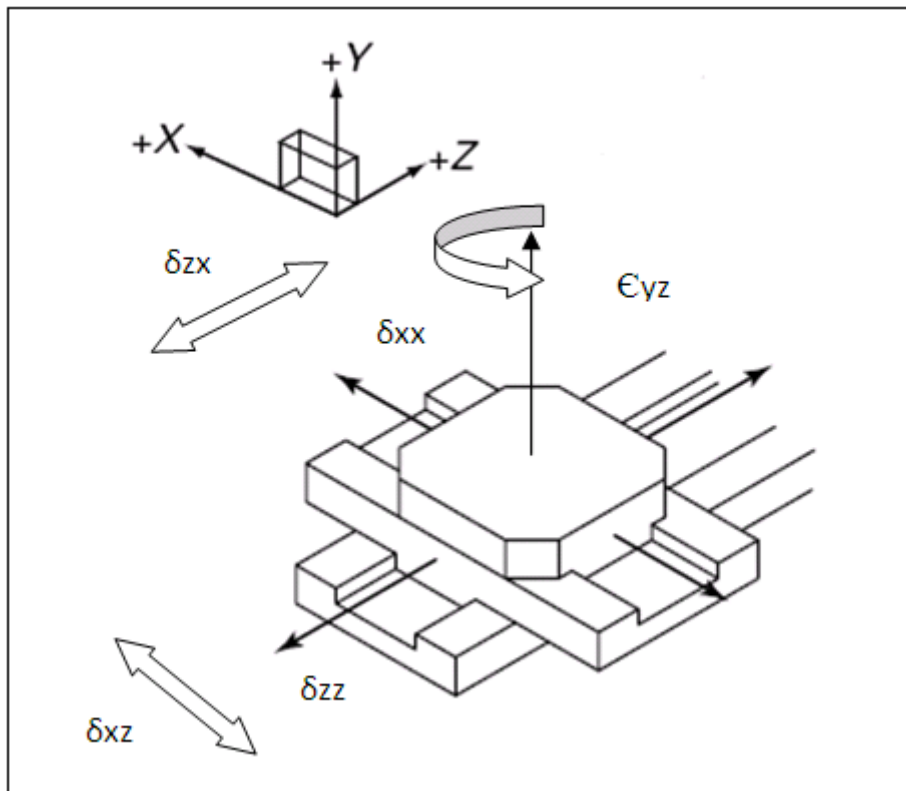
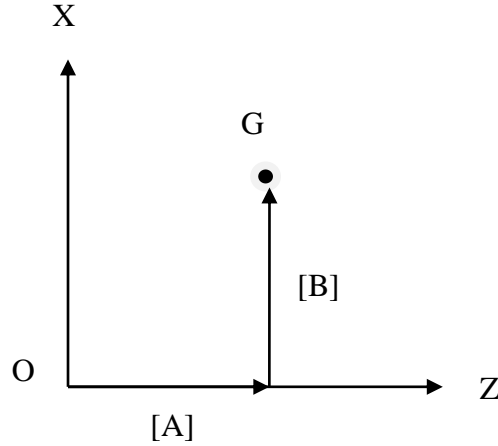


Figure 5.3 Machine axis configuration for two axis machine under test



**Figure 5.4 Schematic of translation in two-axes machine**

Now consider that a two axis machine moves to a position  $G(z,x)$  from the origin  $O(0,0)$  as shown in figure 5.4 through the arrowed path. The desired coordinates of  $G$ , if there is no error, can be represented by  $K. G. Ahn [18]$ .

$$[G] = [A] [B] \quad (6)$$

Where  $[A]$  and  $[B]$  are the kinematic transformation matrices in the  $z$  and  $x$  directions respectively.

Now if we consider the actual position of  $G$  since the transformation matrices contain translational and angular errors equation (6) can be re-written as

$$G_{\text{actual}} = [G \Delta G] = [A \Delta A] [B \Delta B] \quad (7)$$

Where

$$[A] = \begin{bmatrix} 1 & 0 & 0 & z \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

As transformation for a three axis machine is given by equation (5) which results in over all 21 error components for all three axis but for two axis machine these reduce to a total of six errors components, including two linear errors, two straightness errors, one angular error and the squareness error between the  $Z$  and  $X$  axis [19]. Since roll error are usually neglected being small and pitch error and vertical straightness are irrelevant while considering a two axis horizontal machine. Therefore the error matrix reduce to

$$[\Delta A] = \begin{bmatrix} 1 & -\varepsilon_{yz} & 0 & \delta_{zz} \\ \varepsilon_{yz} & 1 & 0 & \delta_{xz} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & x \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[\Delta B] = \begin{bmatrix} 1 & -\varepsilon_{yx} & 0 & \delta_{zx} - S_{zx}x \\ \varepsilon_{yx} & 1 & 0 & \delta_{xx} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Where z, x are the nominal positions and  $\delta_{xx}$ ,  $\delta_{zz}$  are the respective positional errors  $\delta_{xz}$ ,  $\delta_{zx}$ , are the straightness errors, where the first subscript represents the error direction and the second refers to the moving direction, and  $\varepsilon_{yx}$   $\varepsilon_{yz}$  are the angular (yaw) errors, where the first subscript represents the axis of rotation and the second refers to the moving direction, and  $S_{zx}$  is the squareness error between the two axis.

After expanding the terms in equation (7),  $[\Delta G]$  can be re-written as:

$$[\Delta G] = \begin{bmatrix} 1 & a_{12} & 0 & Pz \\ a_{21} & 1 & 0 & Px \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By neglecting the higher order terms and solving by transformation matrices defined in equation (7)

$$Pz = \delta_{zz} + \delta_{zx} - S_{zx}.x - \varepsilon_{yz}.\delta_{xx}$$

$$Px = \delta_{xx} + \delta_{xz} + \varepsilon_{yz}.\delta_{zx} - \varepsilon_{yz}S_{zx}.x$$

Thus from above two equations, Pz and Px are the combined positional errors caused by the constituent geometric errors.

## 5.5 Summary

Modeling machine tool errors is a complex process but this complexity can be reduced by using small angle assumption. In this case scale errors, mutual straightness errors of both the axis, yaw error of Z-axis and squareness error are the constituent geometric errors which needs to be measured to find resultant Pz and Px.



## Experimental Work

### 6.1 Purpose

The purpose of this chapter is to document the experimental work performed regarding geometric error measurement, geometric error compensation and verification of results.

### 6.2 Introduction

A two axis cylindrical grinding machine (STUDER S-36) was selected for the experiments. A mathematical model for the two axis machine has been developed in previous chapter. The geometric error information contained in the final equation was required to be measured to perform the geometric error compensation. Several methods are available for measuring these geometric errors of machine tool axes as discussed in Chapter 4. Most of these errors can be measured with laser interferometer system by using different set of optics. Laser interferometer provides accuracies in the order of submicron, and can be set up for automated data capturing. The disadvantage of these systems is the time required to set up the laser and optics for a particular measurement.

### 6.3 Measuring Strategy

Based on the mathematical model following geometric errors need to be measured

$$P_Z = \delta_{zz} + \delta_{zx} - S_{zx} \cdot X - \varepsilon_{yz} \cdot \delta_{xx}$$

$$P_X = \delta_{xx} + \delta_{xz} + \varepsilon_{yz} \delta_{zx} - \varepsilon_{yz} S_{zx} \cdot X$$

1. Positional errors were measured along Z-axis ( $\delta_{zz}$ ) and X-axis ( $\delta_{xx}$ ).
2. Straightness errors were measured i.e. straightness of X in Z ( $\delta_{zx}$ ) direction and straightness of Z in X direction ( $\delta_{xz}$ ).
3. Yaw error of Z axis was measured.
4. Mutual Squareness error of Z and X axes ( $S_{zx}$ ) was measured.

#### 6.3.1 Linear Measurements ( $\delta_{xx}$ , $\delta_{zz}$ )

Laser interferometer was aligned along an axis; machine was programmed to move along that axis while other axis movements were restricted. Laser measurements were taken on intervals for which machine was programmed and data was captured during dwell periods. Figure 6.1 shows the schematic for target position in laser measurements. Similarly measurements are taken on all axes.



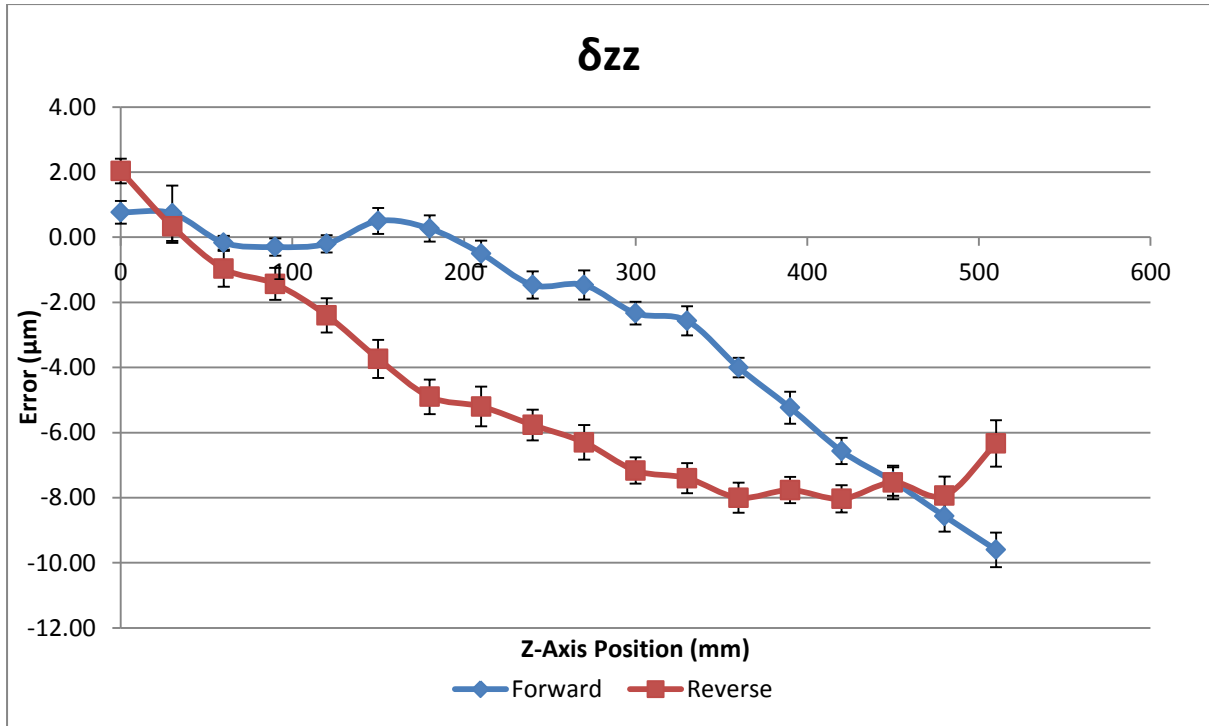


Figure 6.3 Z-Axis Linear Position Errors, Studer S-36

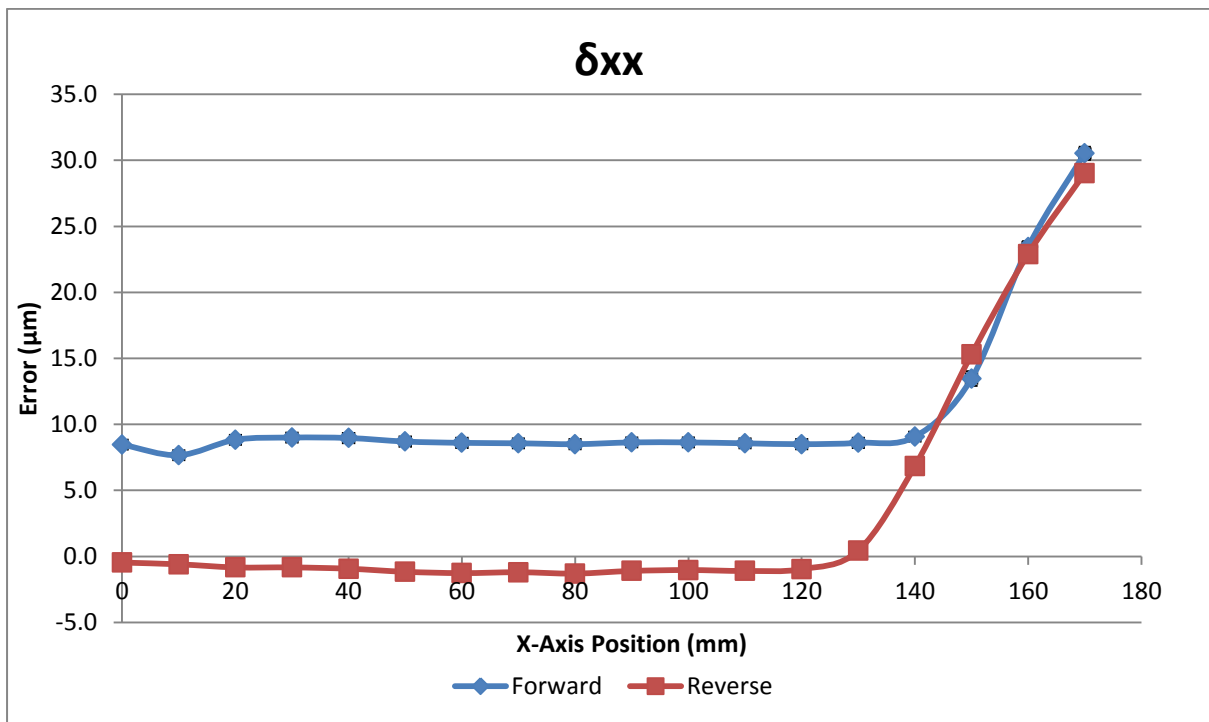


Figure 6.4 X-Axis Linear Position Errors, Studer S-36

Z-axis position error ( $\delta_{zz}$ ) ranges within 12  $\mu\text{m}$  and have poor bidirectional repeatability (Figure 6.3). X-axis position error ( $\delta_{xx}$ ) in figure 6.4 has poor repeatability up to 140mm target position and beyond 140mm a steep up rise to 30  $\mu\text{m}$  at 160mm. This may be because of geometric error like straightness.

### 6.3.2 Straightness Measurement ( $\delta_{zx}$ , $\delta_{xz}$ )

Straightness error measurements were taken on same intervals and travel as for linear measurements given in Figure 6.2 for both axes with laser interferometer. Three bidirectional runs were taken to ensure repeatability. Sign conventions used are as given in Figure 6.5. Straightness of Z along X is nominal but straightness of X along Z axis has a large value (Figure 6.7).

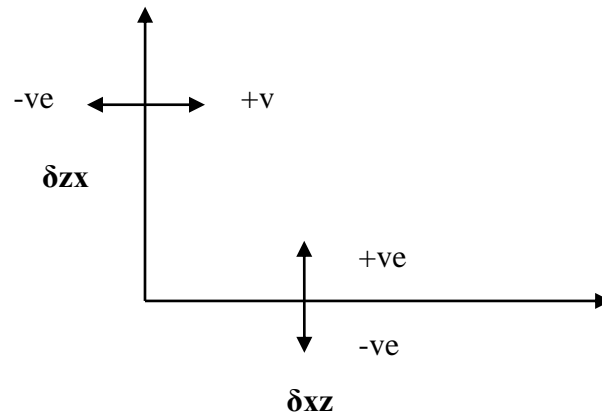


Figure 6.5 Straightness measurement sign conventions



Figure 6.6 Z-axis Straightness errors  $\delta_{xz}$ , Studer S-36

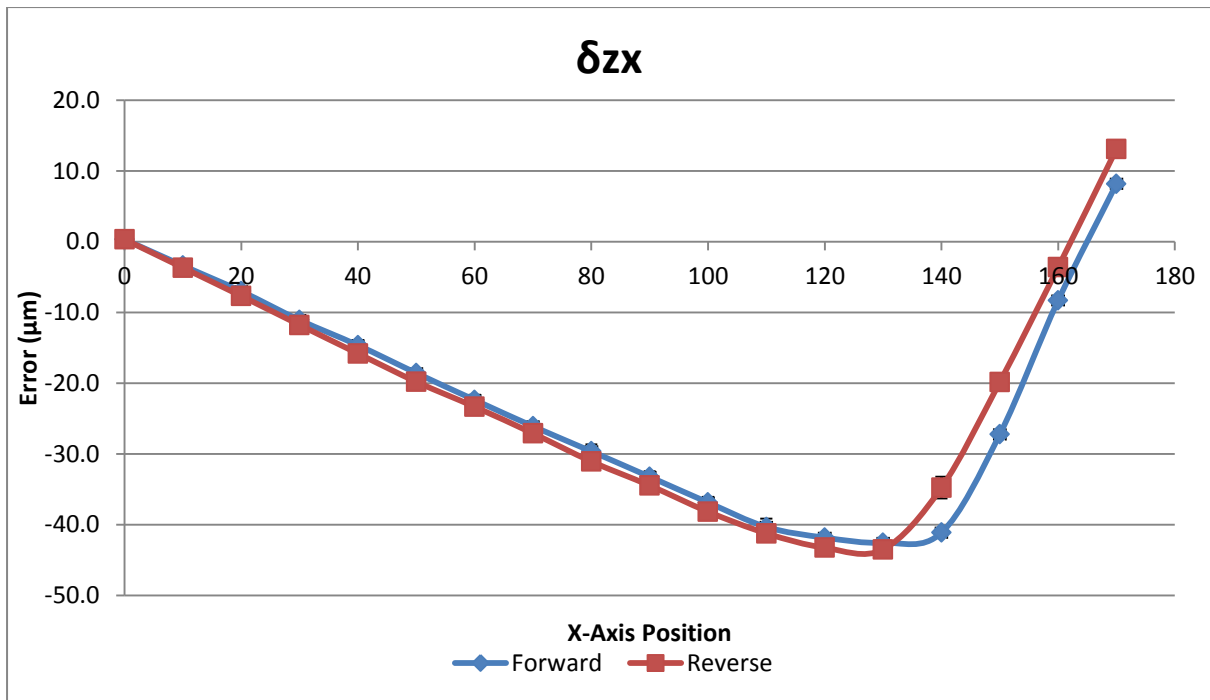


Figure 6.7 X-Axis Straightness Errors  $\delta_{zx}$ , Studer S-36

### 6.3.3 Yaw Error of Z Axis ( $\epsilon_{yz}$ )

Angular error (Yaw) measurements were taken on same intervals and travel as for linear measurements given in Figure 6.2 for Z axis with laser interferometer. Three bidirectional runs were taken to ensure repeatability. Sign convention was determined by right hand rule about auxiliary axis as shown in Figure 6.8.

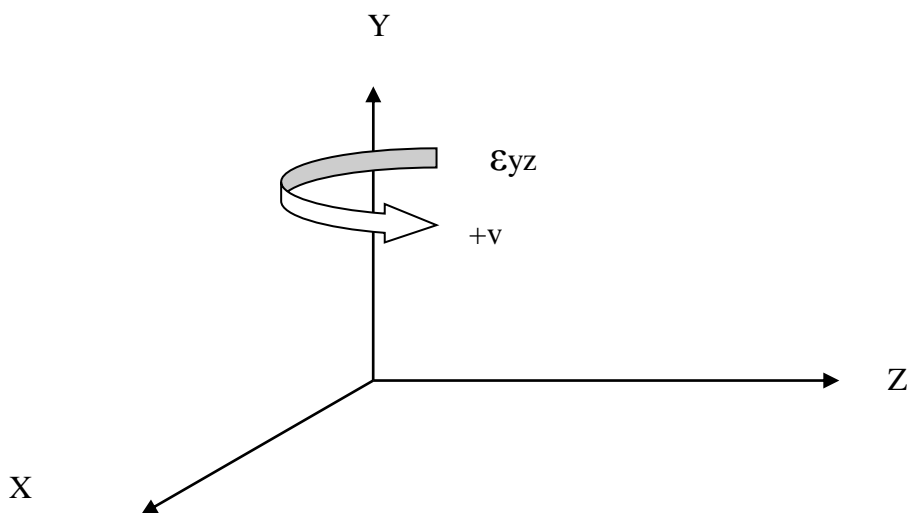


Figure 6.8-Angular measurement sign convention

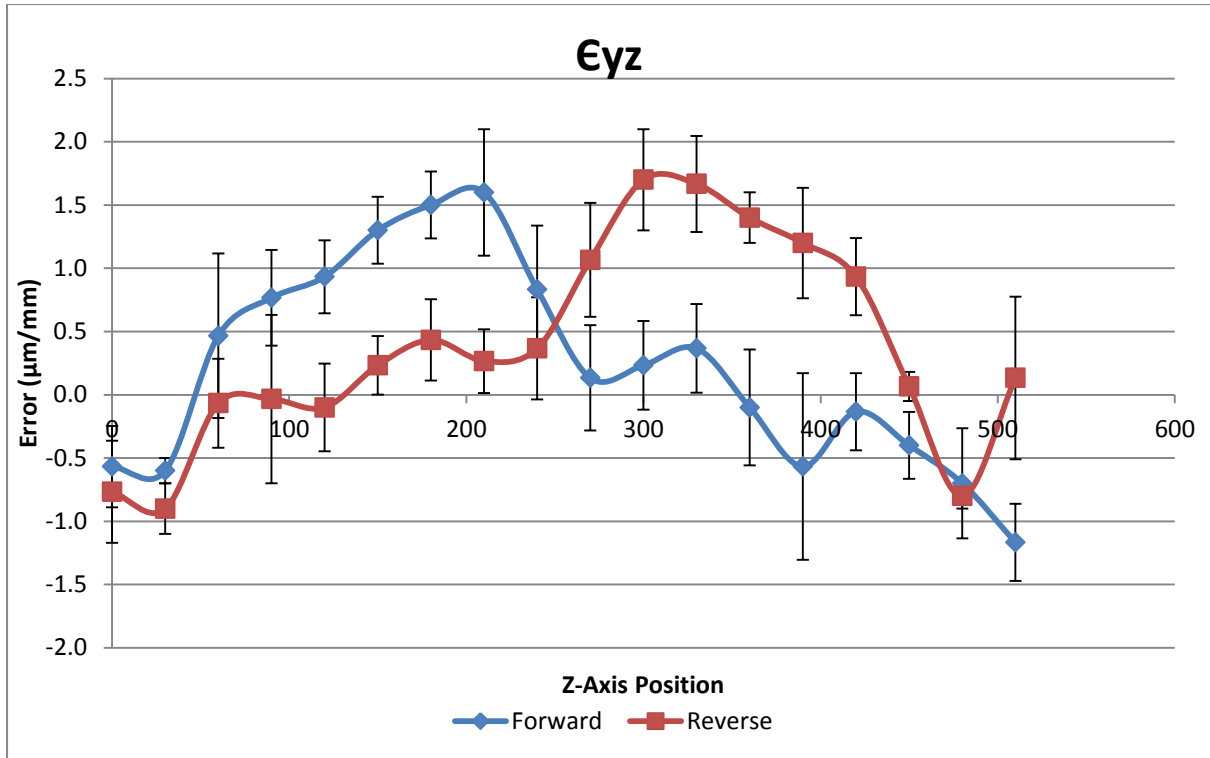


Figure 6.9 Z-Axis Yaw error (Eyz), Studer S-36

### 6.3.4 Squareness Error Measurement (Sxz)

Squareness was determined by the means of a mechanical square and indicator. Square was aligned with one axis (Z-axis) and deviations were measured on the other axis (X-axis). Positive squareness will indicate greater than 90 degree, and the negative squareness will indicate less than 90 deg. Measured squareness error was -0.065mm at 170mm.

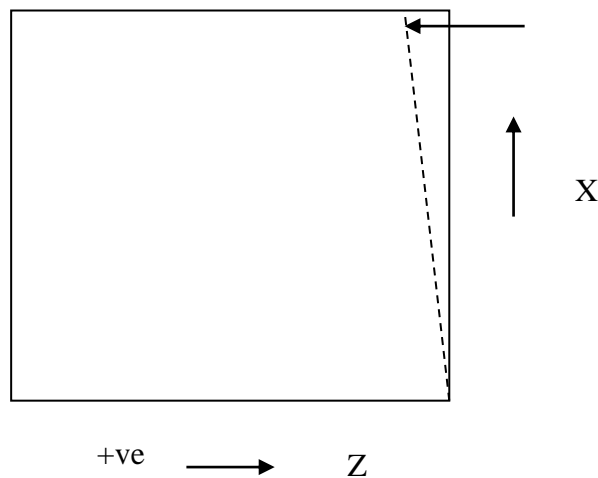


Figure 6.10 Squareness measurement Schematic

### 6.3.5 Measured Data Summary

Table 6.1 and 6.2 summarize the measurements made on machine. F stands for Forward, R for Reverse and Avg. for average. This data will be further used in calculating the geometric error.

**Table 6.1 Measured Data (Z-axis)-All Values in mm**

Target Position	$\delta_{zz}$			$\delta_{xz}$			$\epsilon_{yz}$		
	F	R	Avg.	F	R	Avg.	F	R	Avg.
0	0.0007	0.0020	0.0014	0.0000	-0.0005	-0.0003	0.0030	0.0023	0.0027
30	0.0007	0.0003	0.0005	0.0017	-0.0021	-0.0002	0.0007	0.0002	0.0005
60	-0.0001	-0.0009	-0.0005	-0.0002	-0.0030	-0.0016	0.0000	-0.0011	-0.0006
90	-0.0003	-0.0014	-0.0009	-0.0001	-0.0025	-0.0013	-0.0016	-0.0037	-0.0027
120	-0.0002	-0.0024	-0.0013	-0.0006	-0.0022	-0.0014	-0.0021	-0.0023	-0.0022
150	0.0005	-0.0037	-0.0016	0.0000	-0.0017	-0.0009	-0.0021	-0.0030	-0.0026
180	0.0002	-0.0049	-0.0024	0.0008	-0.0010	-0.0001	-0.0028	-0.0037	-0.0033
210	-0.0005	-0.0052	-0.0029	0.0001	-0.0012	-0.0006	-0.0044	-0.0044	-0.0044
240	-0.0014	-0.0057	-0.0036	0.0008	-0.0011	-0.0002	-0.0037	-0.0056	-0.0047
270	-0.0014	-0.0063	-0.0039	0.0008	-0.0011	-0.0002	-0.0049	-0.0056	-0.0053
300	-0.0023	-0.0071	-0.0047	0.0011	-0.0005	0.0003	-0.0050	-0.0064	-0.0057
330	-0.0025	-0.0074	-0.0050	0.0017	0.0000	0.0009	-0.0050	-0.0072	-0.0061
360	-0.0040	-0.0080	-0.0060	0.0023	0.0005	0.0014	-0.0063	-0.0072	-0.0068
390	-0.0052	-0.0077	-0.0065	0.0024	0.0007	0.0016	-0.0056	-0.0079	-0.0068
420	-0.0065	-0.0080	-0.0073	0.0029	0.0011	0.0020	-0.0056	-0.0073	-0.0065
450	-0.0075	-0.0075	-0.0075	0.0033	0.0014	0.0024	-0.0061	-0.0077	-0.0069
480	-0.0085	-0.0079	-0.0082	0.0034	0.0018	0.0026	-0.0072	-0.0085	-0.0079
510	-0.0096	-0.0063	-0.0080	0.0035	0.0021	0.0028	-0.0075	-0.0087	-0.0081

**Table 6.2 Measured Data (X-axis)-All Values in mm**

Target Position	$\delta_{xx}$			$\delta_{zx}$		
	F	R	Avg.	F	R	Avg.
0	0.0084	-0.0004	0.0040	0.0000	0.0000	0.0000
10	0.0076	-0.0006	0.0035	-0.0033	-0.0039	-0.0036
20	0.0088	-0.0008	0.004	-0.007	-0.0078	-0.0074
30	0.0090	-0.0008	0.0041	-0.0110	-0.0120	-0.0115
40	0.0089	-0.0009	0.0040	-0.0149	-0.0157	-0.0153
50	0.0087	-0.0011	0.0038	-0.0185	-0.0195	-0.0190
60	0.0086	-0.0012	0.0037	-0.0224	-0.0234	-0.0229
70	0.0085	-0.0012	0.0037	-0.0261	-0.0272	-0.0266
80	0.0085	-0.0013	0.0036	-0.0297	-0.0311	-0.0304
90	0.0086	-0.0011	0.0038	-0.0334	-0.0315	-0.0324
100	0.0086	-0.0010	0.0038	-0.0368	-0.0382	-0.0375
110	0.0085	-0.0011	0.0037	-0.0401	-0.0412	-0.0406
120	0.0085	-0.0009	0.0038	-0.0420	-0.0437	-0.0428
130	0.0086	0.0004	0.0045	-0.0429	-0.0437	-0.0433
140	0.0090	0.0068	0.0079	-0.0414	-0.0348	-0.0381
150	0.0134	0.0153	0.0144	-0.0271	-0.0196	-0.0233
160	0.0234	0.0229	0.0232	-0.0083	-0.0037	-0.0060
170	0.0305	0.0290	0.0298	0.0083	0.0138	0.0110
<b>Squareness error (Szx)</b>			<b>-0.065mm at 170mm</b>			

## 6.4 Verification Strategy

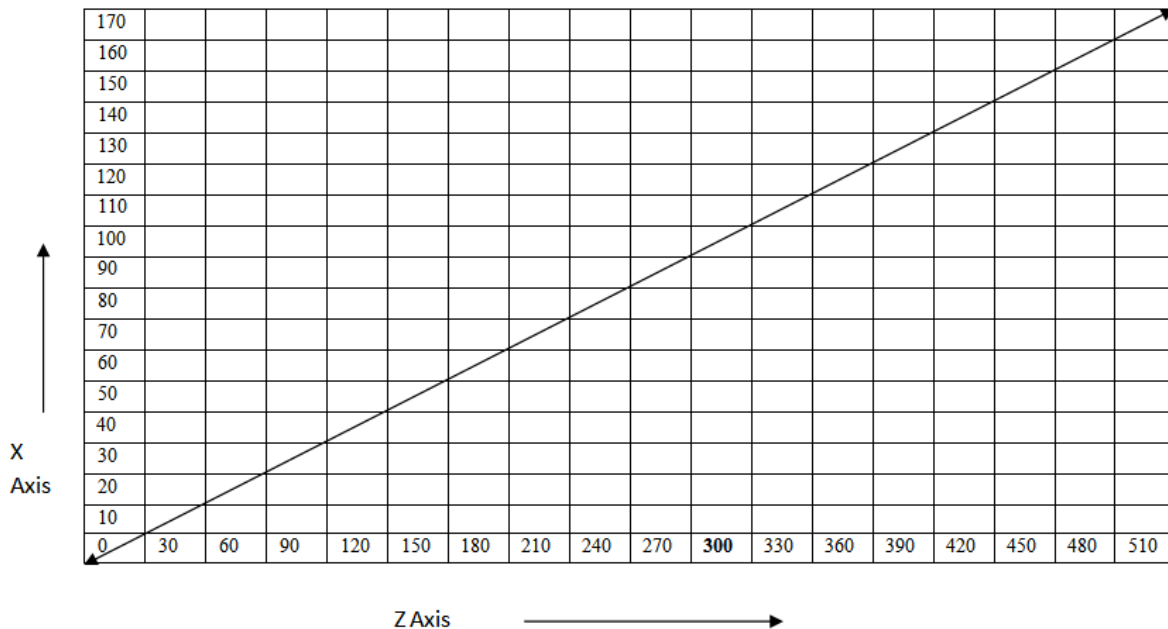
To test the mathematical model experimentally a diagonal path was chosen as shown in Figure 6.11 as used by K. K. Tan [19]. For which compensation will be calculated using mathematical model and measured data on points shown in Figure 6.11.

Diagonal measurements with laser interferometer for verification/validation of model will be performed

1. Without any compensation
2. Compensation with modified program calculated on positional error only
3. Compensation with modified program calculated on geometrical errors using mathematical model

$$Pz = \delta_{zz} + \delta_{zx} - S_{zx}.X - \epsilon_{yz}.\delta_{xx}$$

$$Px = \delta_{xx} + \delta_{xz} + \epsilon_{yz}.\delta_{zx} - \epsilon_{yz}S_{zx}.X$$



**Figure 6.11 Diagonal measurement for Verification**

### 6.4.1 Diagonal Measurement Setup

Laser was aligned in diagonal with the help of turning mirror. Figure 6.12 shows the setup for diagonal measurement on Studer S-36 CNC grinding machine. A diagonal part program on following target positions was made for forward and reverse runs. For X-axis diametral target points were given. Reversal movements were given on the end points to observe any backlash. Dwell time after every traverse was given to let the laser software capture data.

Z	0	30	60	90	120	150	180	210	240	270	300	330	360	390	420	450	480	510
X	0	20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340



Typical Part Program for diagonal measurement on above given targets is given below;

```
G01 X0 Z0 F1000;  
G04 X5;  
X-2 Z -1;  
G04 X5;  
X0 Z0;  
G04 X5;  
X20 Z30;  
G04 X5;  
X40 Z 60;  
G04 X5;  
-  
-  
X340 Z 510;  
G04 X5;  
X342 Z 512;  
G04 X5;  
X340 Z 510;  
G04 X5;  
X320 Z480  
-  
-  
X0 Z0;  
M30;
```



Figure 6.12 Laser setup for Diagonal measurement, Studer S-36

Figure 6.13 show the results of diagonal measurement. It is apparent that error has increased a lot while both axes move simultaneously. This is the effect of geometric error, straightness error of X-axis and squareness error in this case.

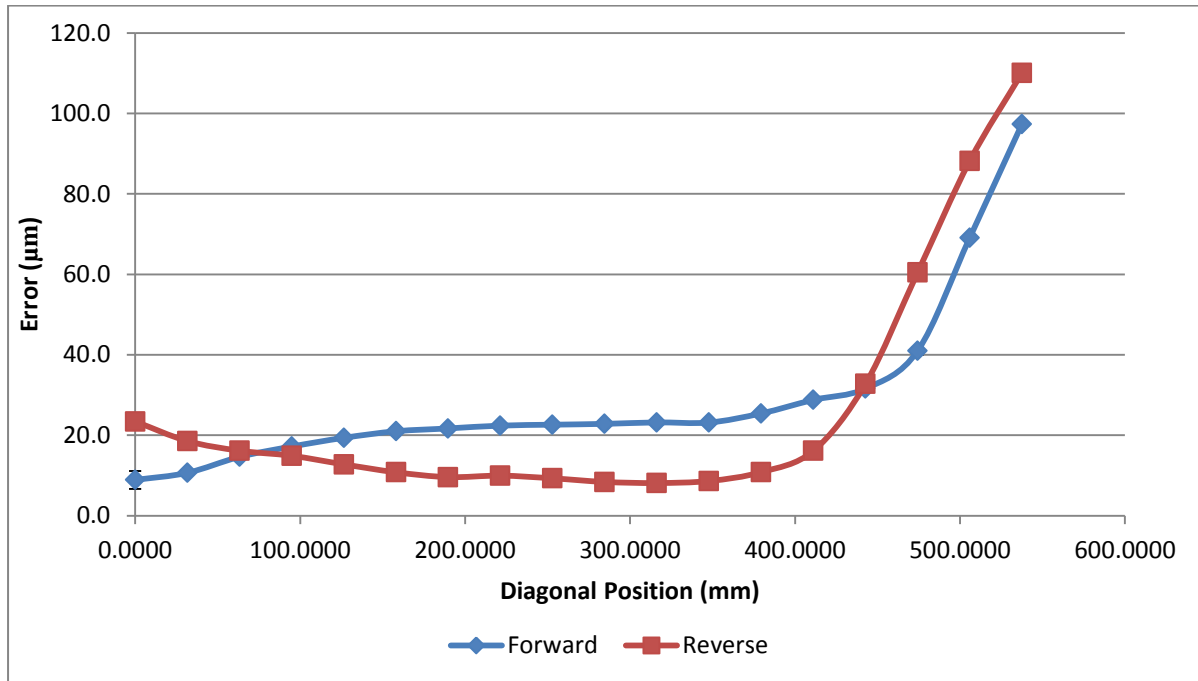


Figure 6.13 Positional error on diagonal without compensation, Studer S-36

### 6.4.2 Compensation Calculation

Step by step procedure for calculation of compensation is as follow;

1. First the combined error was calculated based on the model for both Z and X axis for a given path.
2. The error was subtracted from nominal target positions to get compensated values.  

$$\text{Position (z,x)modified} = \text{Position (z,x)nominal} - \text{Error (Pz, Px)}$$
3. Calculations were made separately for both forward and reverse runs.
4. The program was modified based on compensated values as given Table 6.3 below.
5. Typical programs are also given for diagonal and compensated diagonal positions.
6. For diametral values of X-axis compensation is also multiplied by 2.
7. Typical Part Program for diagonal measurement with compensated values is given below.

```
G01 X-0.0016 Z-0.0007 F1000;
G04 X5;
X-2 Z -1;
G04 X5;
XX-0.0016 Z-0.0007;
G04 X5;
X19.9814 Z29.9988;
G04 X5;
X39.9828 Z 59.9995;
G04 X5;
```

-

x339.9322 z 509.9363;  
G04 X5;  
X342 Z 511;  
G04 X5;  
x339.9380 z 509.9275;  
G04 X5;  
x319.9508 z479.9504;  
-  
-  
x0.0018 z-0.0020;  
M30;

**Table 6.3 Compensated values for diagonal part program with both geometric and positional compensations**

Geometric Compensation				Positional Compensation			
Forward		Reverse		Forward		Reverse	
Z	X	Z	X	Z	X	Z	X
-0.0007	-0.0168	-0.0020	0.0018	-0.0007	-0.0168	-0.0020	0.0008
29.9988	19.9814	29.9998	20.0054	29.9993	19.9848	29.9997	20.0012
59.9995	39.9828	60.0011	40.0076	60.0001	39.9824	60.0009	40.0016
89.9998	59.9822	90.0019	60.0066	90.0003	59.9820	90.0014	60.0016
119.9998	79.9834	120.0028	80.0062	120.0002	79.9822	120.0024	80.0018
149.9989	99.9826	150.0041	100.0056	149.9995	99.9826	150.0037	100.0022
179.9992	119.9812	180.0054	120.0044	179.9998	119.9828	180.0049	120.0024
209.9998	139.9828	210.0056	140.0048	210.0005	139.9830	210.0052	140.0024
240.0005	159.9814	240.0062	160.0048	240.0014	159.9830	240.0057	160.0026
270.0004	179.9812	270.0033	180.0044	270.0014	179.9828	270.0063	180.0022
300.0009	199.9806	300.0071	200.0030	300.0023	199.9828	300.0071	200.0020
330.0005	219.9796	330.0065	220.0022	330.0025	219.9830	330.0074	220.0022
360.0001	239.9784	360.0058	240.0008	360.0040	239.9830	360.0080	240.0018
389.9983	259.9780	390.0016	259.9978	390.0052	259.9828	390.0077	259.9992
419.9943	279.9762	419.9893	279.9842	420.0065	279.9820	420.0080	279.9864
449.9772	299.9667	449.9697	299.9667	450.0075	299.9732	450.0075	299.9694
479.9556	319.9465	479.9504	319.9508	480.0085	319.9532	480.0079	319.9542
509.9363	339.9322	509.9275	339.9380	510.0096	339.9390	510.0063	339.9420

### 6.4.3 Verification Results

Figures 6.14 and 6.15 (below) show the diagonal measurements results for verification of positional and geometric error compensations. With applying positional error compensation only (Figure 6.14) very little effect on error has been observed in diagonal measurement involving both the axis simultaneously. Whereas with geometric error compensation, (Figure 6.15), error has reduced considerably. Which means machine geometry plays important role in determining machine errors especially when axis are moving simultaneously. Therefore only positional error compensation is not enough for machines.

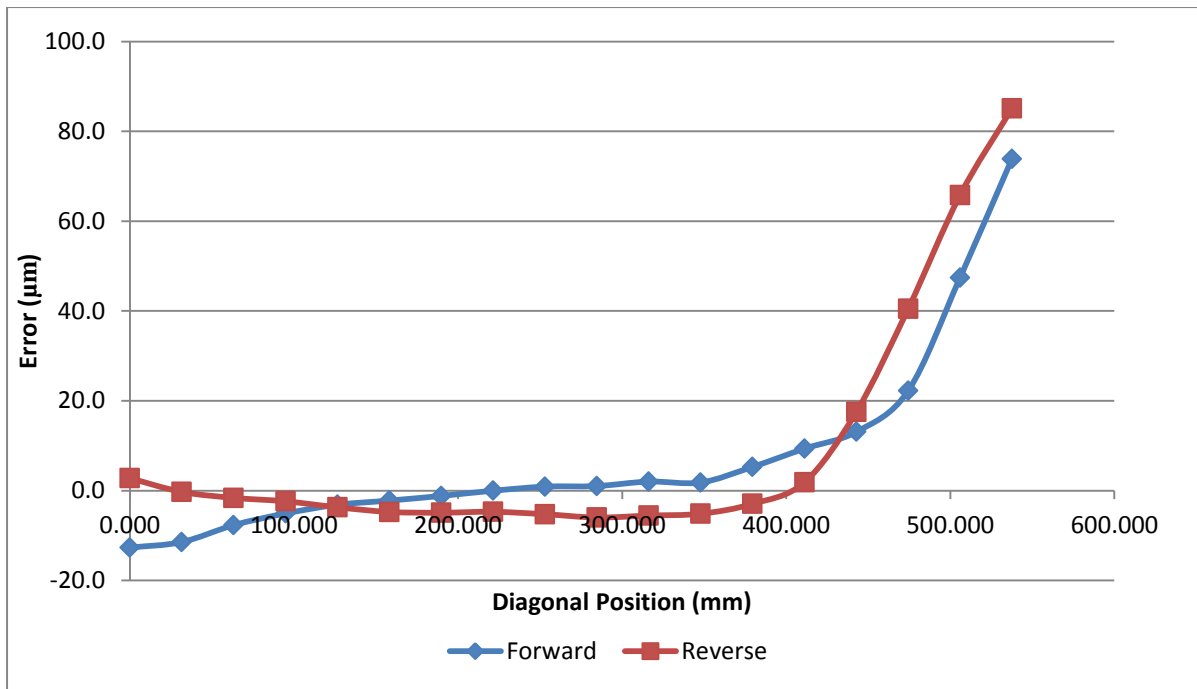


Figure 6.14 Positional error on diagonal positional compensation, Studer S-36

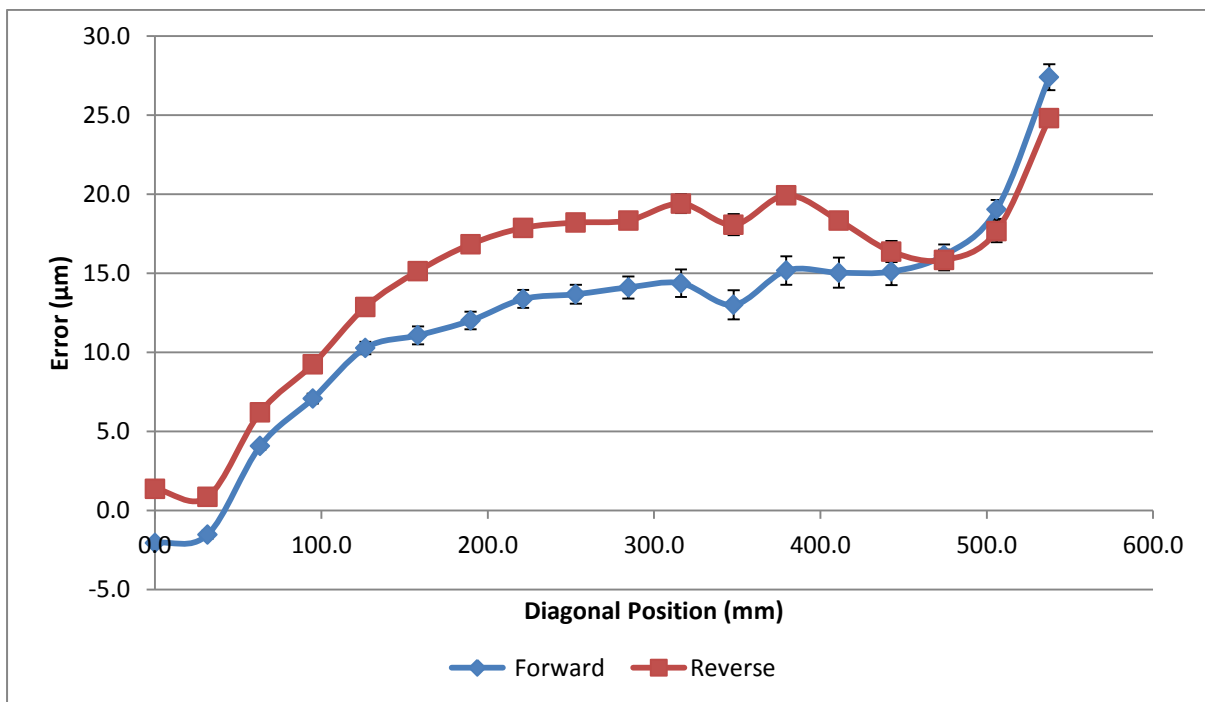
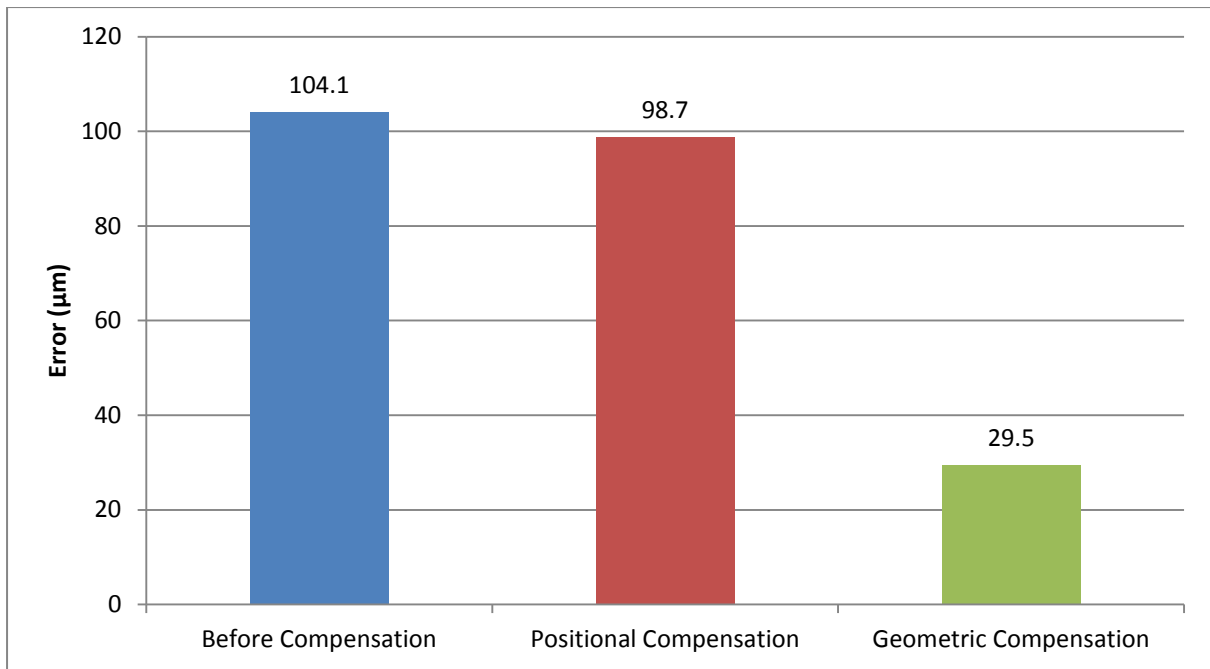


Figure 6.15 Positional error on diagonal with Geometric compensation, Studer S-36

Figure 6.16 shows the comparison of error measured on diagonal without compensation, with positional error compensation and with geometric error compensation. Error is calculated according to ISO 230-2 [33].

$$E = \max.[x_i \uparrow; x_i \downarrow] - \min.[x_i \uparrow; x_i \downarrow] \quad [33]$$

Error is the difference of maximum point and minimum point on the graph.



**Figure 6.16 Positional error comparison**

## 6.5 Discussions

Diagonal measurement of machine involving two axes simultaneously has provided an opportunity to visualize that how geometric error can affect the positional accuracy and in turn part accuracy while involving two axis even though it has smarter positional errors in individual axis.

Machine under study have large straightness error in X-axis along Z-axis ( $\delta_{zx}$ ) and squareness error  $S_{zx}$ . A large positional error was observed on diagonal. This is because of the straightness error of X-axis and squareness error. This shows how geometric errors can affect the positional error while axes are moved simultaneously. After compensation position error was reduced to 29.5 µm, a considerable reduction in this case.

## 6.6 Summary

Linear, straightness, angular errors were measured in individual axis and squareness of both the axis and then these errors were combined based on the mathematical model to compensate for geometric errors. The results were later verified by programming the machine in diagonal movement and aligning the laser interferometer along this diagonal. Compensations were calculated on this path using mathematical model and measurements were taken without compensation, with positional error compensation and with geometric error compensations. Results show a considerable decrease of error while considering geometric errors in compensation strategies.

# Effect of Software Error Compensation on Part Accuracy

## 7.1 Purpose

The purpose of the experiment is to check the effect of positional error compensation and error reduction through software error compensation on part accuracy.

## 7.2 Introduction

Software compensation techniques are popular in CNC machines because of the convenience and cost effectiveness. The most common software compensation is positional error compensation or sometimes called pitch error compensation. In positional error compensation of CNC machines, positional error of each individual axis is measured with laser interferometer and compensated in the pitch error compensation file of the machine controller. This measurement is performed in “No Load” conditions. Therefore it’s important to monitor the effect of this type of calibration on actual part because during machining forces are acting on tool as well as on part through other elements of machine. Therefore this test was done to measure this effect.

## 7.3 Test Requirements

ASME B5.4-2005 defines detailed requirements for machining test parts in [28]. Some of which are given as below;

- The test part shall be placed as close as possible to the position where operations will be performed on the machine in production.
- Depending on the material chosen for test parts, appropriate process parameters and tooling shall be selected.
- The tooling used for machining test parts in this Standard shall be standard tooling, thoroughly inspected to ensure tool accuracy.
- Test part material, tools, process parameters (feeds, speeds, depths of cut), and the part program shall be thoroughly documented.
- The machine should be warmed up by moving the machine axes for a minimum of 1 hr before starting this test.
- After machining the part should be measured with instrument having 1/10 of specified measurement accuracy and measurement should be performed at 20°C.

## 7.4 Experimental Procedure

1. Figure 7.1 shows the step by step procedure adopted to perform the experiment.
2. A part was machined on MV1060 machining center according to the part drawing given in Figure 7.2 by aligning features (holes and pocket dimensions) with X and Y axes of the machine.

3. Positional error compensation of machine was performed and machine was being compensated for positional errors in all axes.
4. After calibration the same part was being machined on new blank of same material with same location, fixturing, cutting feed, speeds, depth of cut and part program.

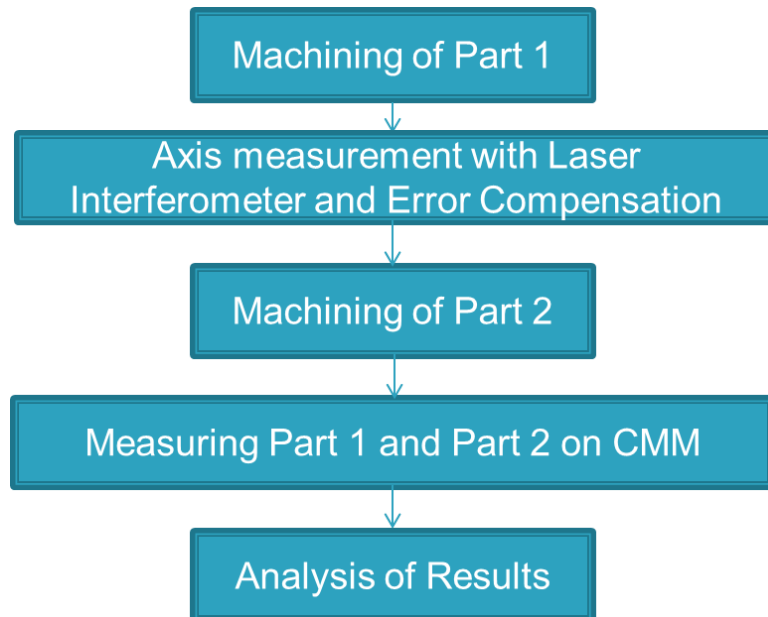


Figure 7.1 Procedure for Experiment

5. After that both the parts were measured in same environment on a Coordinate Measuring Machine (CMM).
6. The difference between the two parts measured results was analyzed.

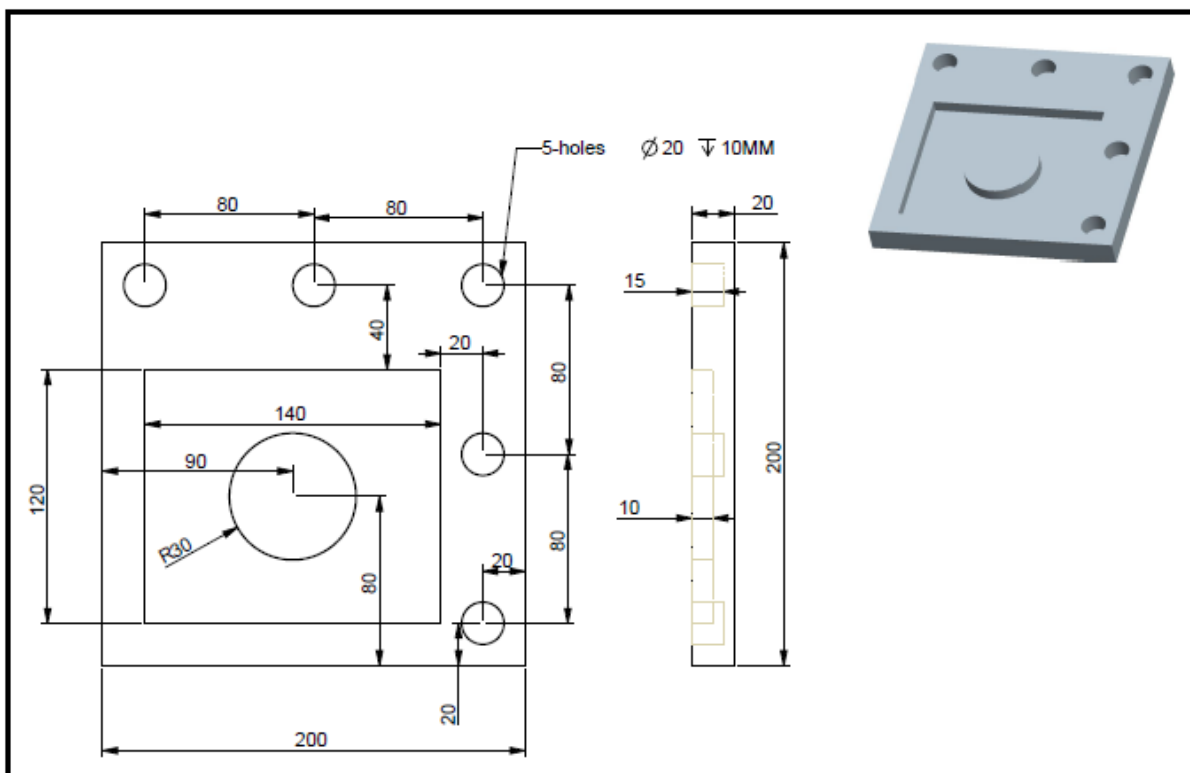


Figure 7.2 Drawing of the part having positional features in both X and Y axis of Machine

## 7.5 Positional Error Compensation of Machining Centre MV-1060

In Positional error compensation positional error of each axis is measured using laser interferometer and then compensated in Pitch error compensation file of machine controller. This process not only assesses machine's positional accuracy but also helps to remove positional error to some extent through controller compensation. This test is also a performance criterion for CNC machines set by ISO 230-2 [33] and American Society of Mechanical Engineers (ASME) B5.54 [28].

Positional error compensation of Machining Centre MV-1060 was performed using laser interferometer ML10. Figure 7.3 shows the setup for laser measurement of MV1060 at SMME NUST.



Figure 7.3 Laser Calibration of Machining Centre MV-1060 at SMME NUST



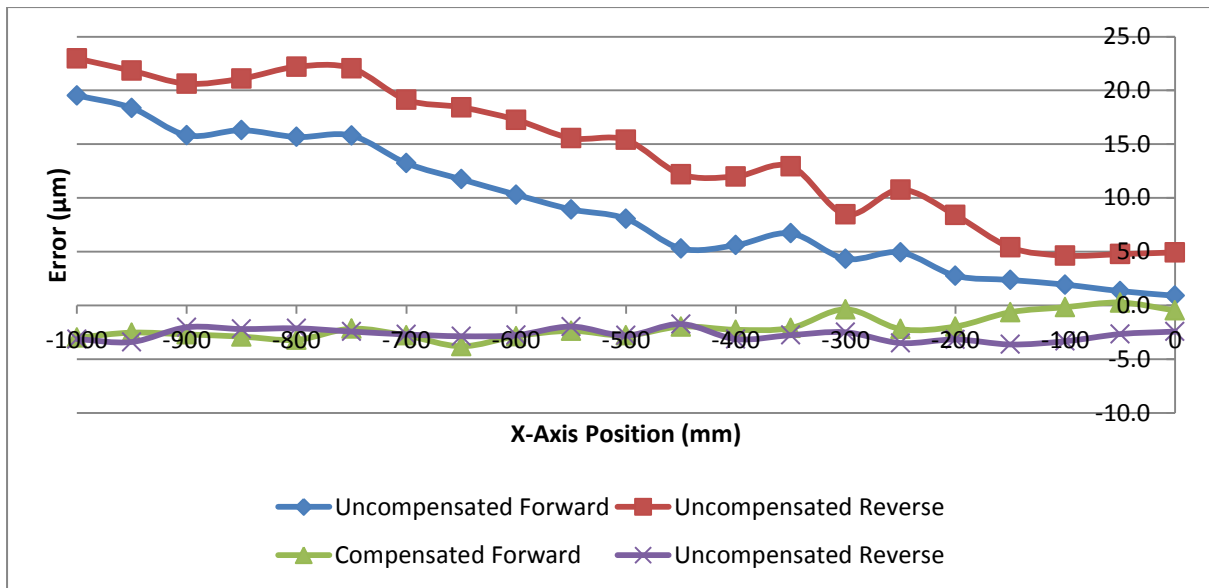


Figure 7.4 X-axis positional error before and after compensation

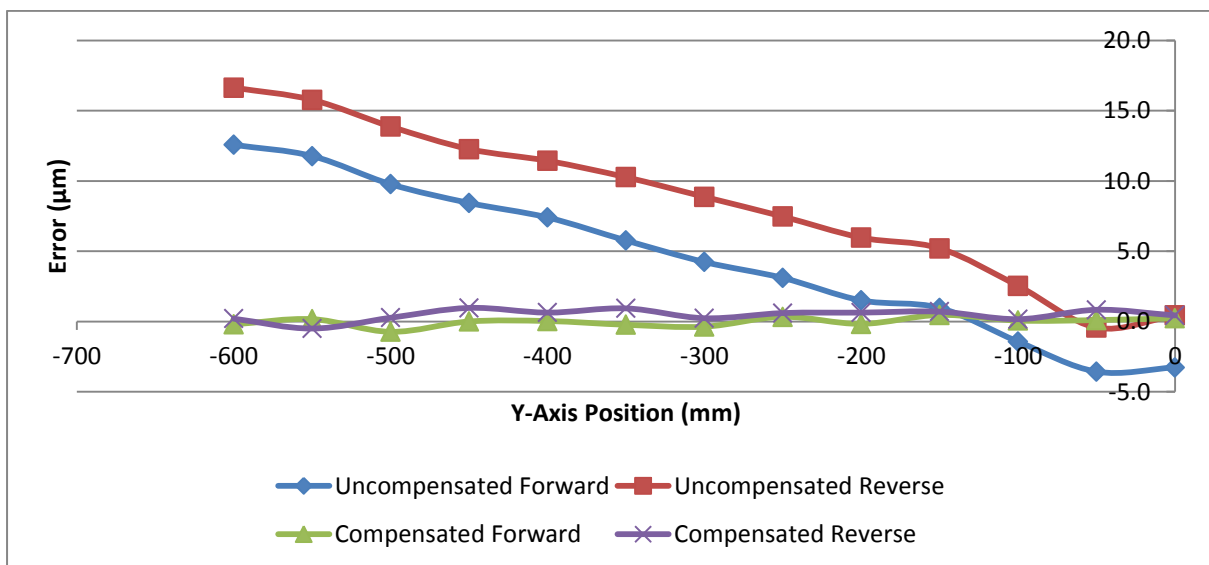
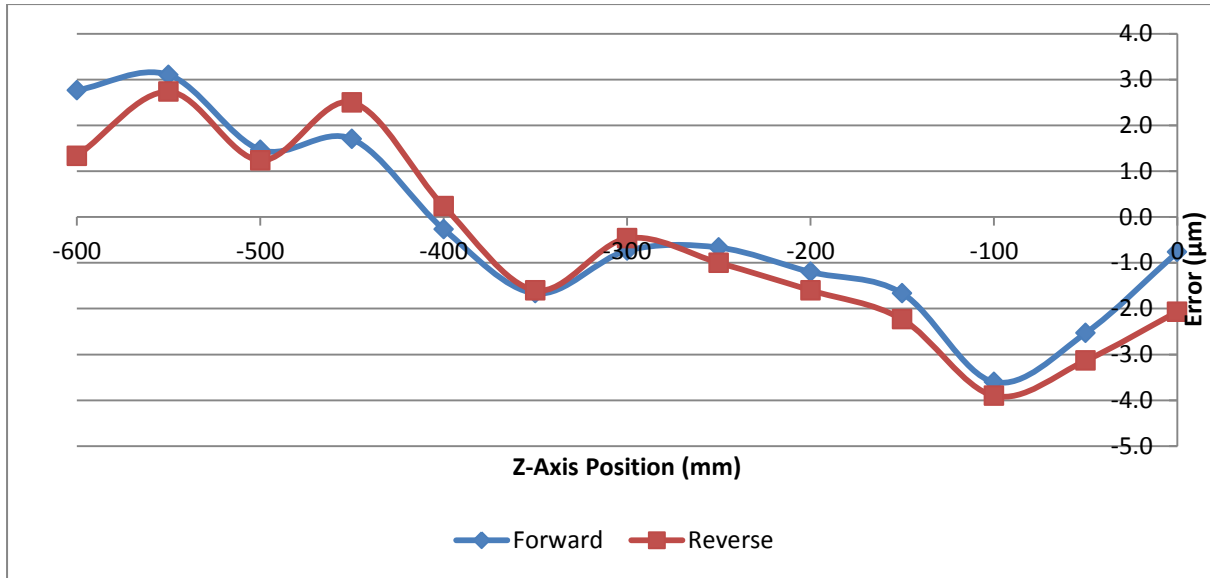


Figure 7.5 Y-axis positional error before and after compensation

The existing compensation was zeroed and measurements were made. Based on these results new compensation was calculated and values were fed to the controller. Table 7.2 shows the parameters for compensations for X and Y axes. X-axis was measured again and accuracy improvement is apparent from Figure 7.4.

Similarly results were measured for Y-Axis and Z-axis, Figures 7.5 and 7.6 show the results. Y axis was compensated and error was reduced as shown in Figure 7.5. Z-axis was not compensated.



**Figure 7.6 Z-axis positional error**

Table 7.1 shows the summary of the measured results with respect to the compensations used. Since Z-axis was good in accuracy (approximately within 10µm), no compensation was performed for Z-axis.

**Table 7.1 Summary of measured results**

Axis	Accuracy (µm)		
	Existing Comp.	Zero Comp.	New Comp.
X-Axis	30.17	42.7	9.14
Y-Axis	24.02	22.26	3.8
Z-Axis	10.8	---	---

Table 7.2 shows the new compensation parameter values which were uploaded on machine controller after being calculated from laser measurements of each axis.

**Table 7.2 Parameters for Compensation (MV-1060)**

Parameter No.	Value		
	<b>X</b>	<b>Y</b>	<b>Z</b>
3620	100	200	300
3621	1	102	202
3622	101	201	301
3623	1	1	1
3624	20000	20000	20000
3625	0	0	0
1851	0	2	1
Pitch error Parameters			
X-Axis		Y-Axis	
Parameter No.	Value	Parameter No.	Value
51	0	171	0
52	0	172	0
53	0	173	1
54	0	174	1

**Table 7.3 Parameters for Compensation (MV-1060) (Continues)**

X-Axis		Y-Axis	
Parameter No.	Value	Parameter No.	Value
55	0	175	0
56	1	176	2
57	1	177	2
58	0	178	1
59	1	179	0
60	1	180	0
61	0	181	-1
62	0	182	-1
63	1	183	0
64	1	184	1
65	1	185	1
66	0	186	1
67	1	187	1
68	0	188	1
69	1	189	1
70	1	190	0
71	1	191	0
72	1	192	1
73	1	193	0
74	1	194	0
75	0	195	0
76	1	196	1
77	2	197	0
78	1	198	0
79	0	199	0
80	1	200	-1
81	1	201	0
82	0		
83	1		
84	2		
85	2		
86	0		
87	0		
88	0		
89	1		
90	0		
91	2		
92	1		
93	1		
94	0		
95	1		
96	1		
97	1		
98	1		
99	1		
100	1		
101	0		

## 7.6 Test Part Machining

Two test parts of mild steel (MS) were machined i.e. one before compensation and the other after compensation.

### 7.6.1 Machining Details

Part Drawing	Figure 7.2
Part material:	MS
Blank Dimension:	200x200x20 mm
Cutting feed	500
Spindle speed	1300rpm
Depth of Cut	1mm
Tool Diameter	12mm
Tool material	High speed steel
Job location (from machine reference)	X= -980mm      Y= -380mm
Coolant Used	Yes

### 7.6.2 Part Measurement Results

Figure 7.7 shows the inspection drawing for the parts. Symbols shown in Figure 7.7 were measured on CMM where X1, X2, Y1, Y2 are center to center distances and D, C are diameter and circularity of extrusion and X<sub>L</sub> and Y<sub>L</sub> are pocketing sizes machined in X and Y directions respectively.

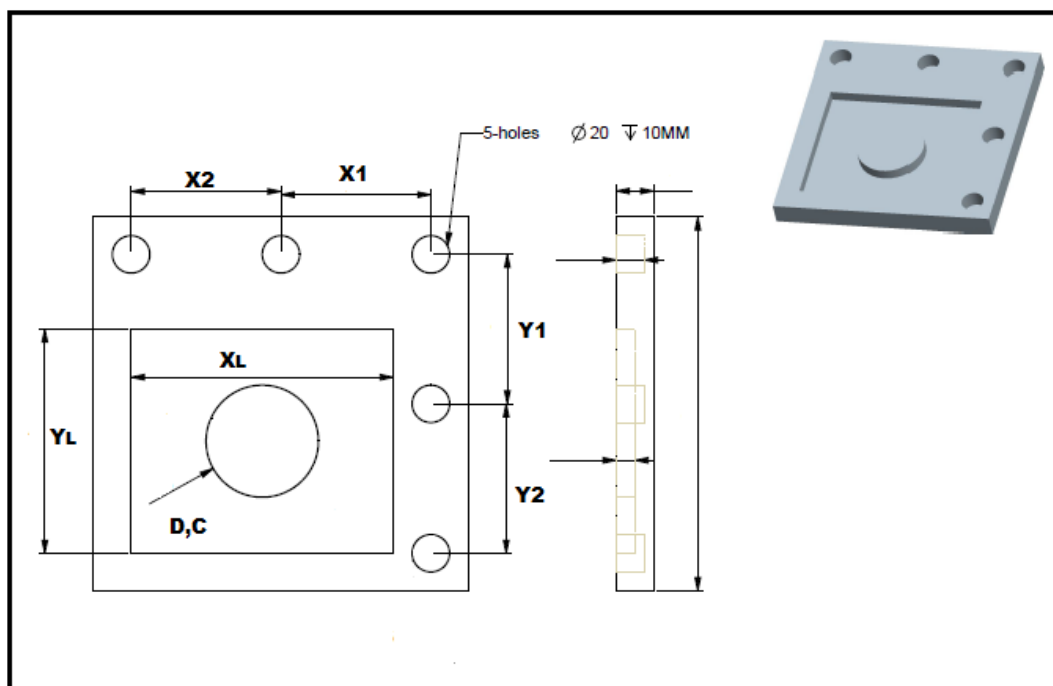


Figure 7.7 Parts Inspection Drawing

Parts were properly cleaned and a soaking time of about eight hours was given in measurement lab to achieve temperature stabilization.

Measuring Equipment      Coordinate measuring machine (CMM)

Accuracy       $(3+4L/1000) \mu\text{m}$ , L in mm

Environmental Conditions      Temperature  $20 \pm 1^\circ\text{C}$ , Humidity <60%

Parts were probed three times on CMM on automatic mode. Table 7.3 shows the measured results. Mean errors of measured results have been plotted on bar graphs for different features and are shown in Figures 7.8 and 7.9.

**Table 7.3 Parts measured results**

	Part 1 Before Compensation (mm)					Part 2 After Compensation (mm)				
	Measured Readings			Mean	Mean Deviation	Measured Readings			Mean	Mean Deviation
	1	2	3			1	2	3		
X1	79.983	79.975	79.983	79.980	-0.020	79.989	79.992	79.991	79.991	-0.009
X2	79.998	79.995	79.997	79.997	-0.003	80.000	80.001	80.002	80.001	0.001
Y1	79.992	79.994	79.991	79.992	-0.008	80.001	80.003	80.000	80.001	0.001
Y2	79.993	79.984	79.988	79.988	-0.012	79.991	79.991	79.989	79.990	-0.010
XL	140.105	140.099	140.101	140.102	0.102	140.085	140.09	140.086	140.087	0.087
YL	120.119	120.114	120.119	120.117	0.117	120.089	120.094	120.094	120.092	0.092
D	59.883	59.874	59.876	59.878	-0.122	59.903	59.907	59.9	59.903	-0.097
C	0.007	0.006	0.008	0.007	0.007	0.003	0.001	0.004	0.003	0.003

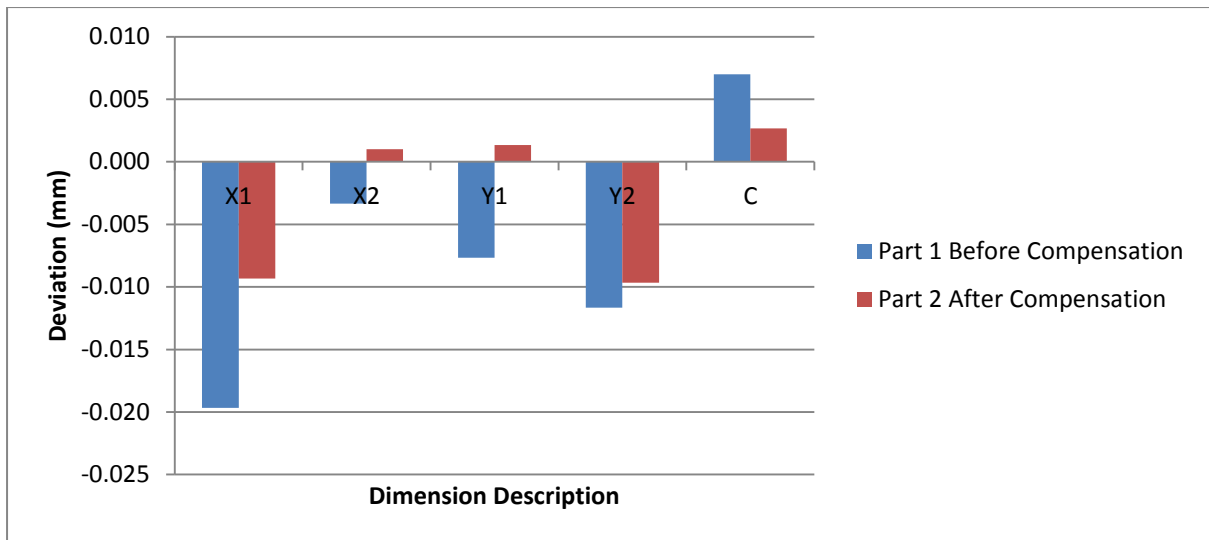


Figure 7.8 Hole position and Circularity Deviation

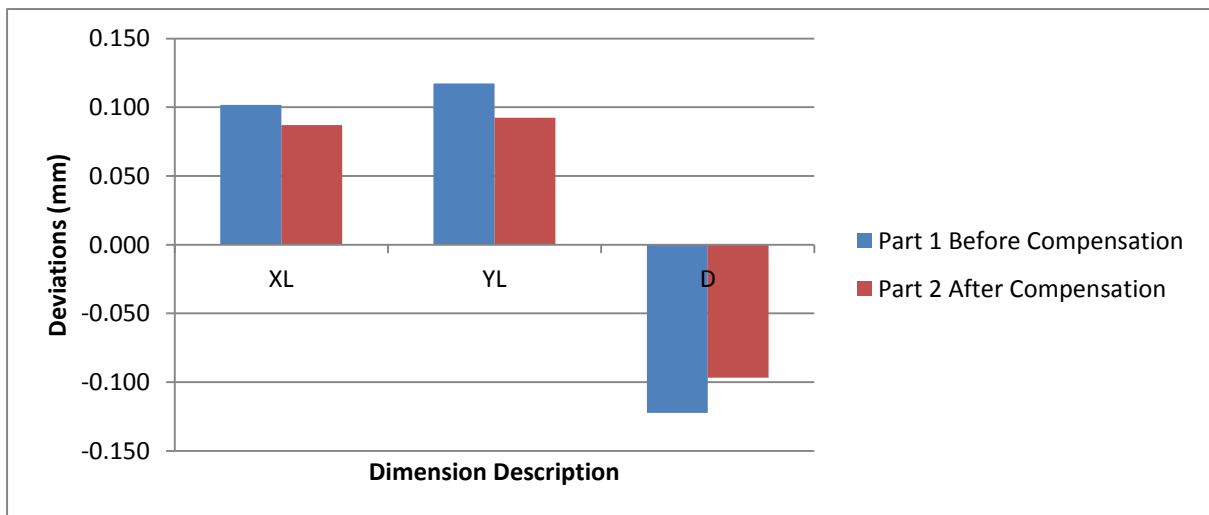


Figure 7.9 Pocket Size and Extrusion diameter Deviation

## 7.7 Discussions

Referring to Table 7.3 and Figures 7.8 and 7.9, here follows the discussion regarding measured deviations of part features.

### Hole Position Deviations (Figure 7.8)

Center to center distances X1, X2, Y1, Y2 between the consecutive holes, as explained in Figure 7.7 were measured on a calibrated Coordinate Measuring Machine (CMM) of accuracy  $(3+4L/1000) \mu\text{m}$ . The lab temperature was maintained within  $20 \pm 1^\circ\text{C}$  and humidity less than 60% and both parts were soaked in this environment for eight hours.

The deviations of Part 2 machined after compensation are less than that of Part 1. This means positional compensation through software has definitely a positive effect on part accuracy.

### **Pocket Size and Extrusion Diameter Deviations (Figure 7.9)**

Pocket sizes  $X_L$ ,  $Y_L$  and extrusion diameter  $D$  have larger deviations in both the parts in the range of 0.1mm which is not comparable to the capability of a CNC machine. The reason lies with tool radius compensation which was not applied because no precise tool measuring equipment was available. Although the high speed steel tool in this case was measured with digital Vernier Caliper and tool diameter was considered to be exactly 12mm whereas results claim that tool was a little larger in size.

The results of Part 2 are better than Part 1 which means accuracy of pocket sizes also improved with compensation but there may be another uncertainty involved in this situation that is tool wear. Tool wear could not affect the center to center distances of holes but it must affect the pocket sizes or extruded diameter. Since tool diameter was not monitored as discussed above, therefore improvement in  $X_L$ ,  $Y_L$  and  $D$  cannot be certainly associated with compensation it may also be due to tool wear or both in this case.

### **Circularity Deviations (Figure 7.8)**

Circularity results are also better in Part2 which shows that while X and Y axis are moving simultaneously the software compensation do effect the accuracy of the parts..

## **7.8 Summary**

The experiment was performed to monitor the effect of software compensation on part accuracy. Positional error compensation through controller was performed on machining center and effect was measured by machining parts before and after compensation. Results show accuracy improvement in machined part after compensation.

# Discussion on Results and Conclusions

## 8.1 Purpose

Purpose of this chapter is to summarize and conclude the experimentation work performed in this research work.

## 8.2 Geometric Error Compensation Results

Geometric error compensation on the diagonal path shows a considerable reduction in position error (Figure 6.16). About 70% of the error was removed with geometric error compensation as compared to the uncompensated diagonal path error. In the initial measured error on the diagonal (Figure 6.13), a steep slope can be observed after 400mm nominal position which is result of X-axis straightness ( $\delta_{zx}$ ). The poor straightness of X-axis is distorting machine geometry in this section and resulting in poor position accuracy in this section of diagonal path. Since this straightness was incorporated in the mathematical model its effect was minimized after geometric error compensation (Figure 6.15) whereas in position error compensation the problem remains as it is (Figure 6.14). This verifies the effectiveness of mathematical model developed for the two axis CNC grinding machine.

The results show that the model is appropriate but still about 30% of diagonal position error is remaining. For this remainder error there may be many reasons which are discussed in this discussion;

1. Machine tools are complex and other errors like thermal errors, force errors etc. make effect alongside geometric errors as discussed in Chapter 3-Machine Tool Errors.
2. Error measurement on limited area-assumption that error will be uniform throughout the length spans of axis.
3. Temperature effects are not considered in the modeling.
4. Mass, velocity and acceleration effects have not been considered.
5. Assumptions are made during mathematical modeling. Also geometric errors in the plane were considered only.
6. Measurements cannot be made at same time. Environment varies during different measurements. Although ML10 has environmental compensation unit but temperature variations in machine elements can cause erroneous results.
7. Machine repeatability is another source of uncertainty in measurement results.
8. Variation in warm up times for CNC machine can result in measurements error.



### **8.3 Software Error Compensation Effect on Part Accuracy**

As discussed in Chapter 7 the deviations of Part 2 machined after compensation are less than that of Part 1 machined before compensation. After compensation part errors were reduced hence proving a positive effect on part accuracy. XL, YL and D have larger deviations which indicate that tool was little oversized. Circularity of the circular feature also shows improvement with compensation. The difference in pocket sizes and extrusion diameter may be because of tool wear. Which shows that tool monitoring with precise measuring equipment is mandatory for precise machining.

### **8.4 Conclusions Geometric Error Compensation**

Two axis grinding machine was successfully modeled for geometric errors using homogeneous transformation matrix with small angle assumption. The geometric errors derived from error model were successfully compensated through part program modification. The compensation was performed on a diagonal, involving both axes simultaneously. About 70% improvement in the positional accuracy on diagonal was observed. The results show considerable improvement in accuracy but an error of 29.5  $\mu\text{m}$  still exists. This remaining error can be attributed to many variables which are discussed in section 8.2. The diagonal measurement has also revealed that machines with geometry problems show large position errors when axis are moved simultaneously although individual axis position errors are small. It was observed that the machine has competitively larger straightness and squareness errors.

### **8.5 Conclusions Software Error Compensation Effect on Part Accuracy**

Improvement in the accuracy of test part machined after positional error compensation depicts the effectiveness of software compensation applied in CNC machines. Although the compensation is based on measurements performed on no load conditions yet their effect was observed on part machining. But to achieve conclusive results tight machining conditions and part measurement conditions are required to be maintained. Tool radius and tool wear compensations are necessary for precise pocketing/slotting, which were not applied.

### **8.6 Future Work**

As future work other major sources of errors e.g. thermal errors, force errors can be incorporated in the mathematical model to further enhance the accuracy of the machine.

Software can be developed to make calculations easy for any given path within the machine axis limits.

Instead of limited area assumption for Laser measurements an appropriate function can be assumed to estimate intermediate values reasonably.

In future work for part accuracy experiments can be performed in better way by freezing other variables by which better results can be achieved.

Also effect of geometric errors can be measured on machined parts by measuring geometric errors and then machining features on test parts.

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