Performance Evaluation of Bandwidth Request in WiMAX Networks

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Abstract

WiMAX have emerged as a promising wireless broadband access technology with throughputs comparable to wired DSL connections (digital subscriber line). Cellular deployment of WiMAX allows multiple subscriber connections with each base station. Bandwidth allocation to subscribers follows Demand Assignment Multiple Access (DAMA) where a subscriber is allocated the bandwidth based on its request. Performance of WiMAX deployment significantly depends upon the efficient bandwidth request and allocation. In this thesis, we investigate the bandwidth request and grant mechanism of WiMAX using Markov chain based analytical model. We study the impact of different parameters on the grant probability of bandwidth requests. Our analysis shows that grant probability is heavily dependent upon the probability of bandwidth request transmission without collision. We conclude that the grant probability can be improved significantly by reducing the wait time of subscribers after transmitting the bandwidth request.

Certificate of Originality

I hereby declare that this submission is my own work and to the best of my knowledge it contains no materials previously published or written by another person, nor material which to a substantial extent has been accepted for the award of any degree or diploma at NUST SEECS or at any other educational institute, except where due acknowledgment has been made in the thesis. Any contribution made to the research by others, with whom I have worked at NUST SEECS or elsewhere, is explicitly acknowledged in the thesis.

I also declare that the intellectual content of this thesis is the product of my own work, except for the assistance from others in the project's design and conception or in style, presentation and linguistics which has been acknowledged.

Author Name: **Amber Madeeha Zeb**

Signature: ________________

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Chapter 1

Introduction

WiMAX is a broadband wireless access technology with data rates in the order of Mbps and coverage in the range of kilometers for a single base station. Higher data rates and QoS mechanism make WiMAX appropriate for broadband and VoIP applications.

1.1 Physical Frame in WiMAX

Physical Layer of WiMAX uses Orthogonal Frequency Division Multiple Access (OFDMA). A physical layer frame consists of multiple orthogonal carriers, grouped together into sub-channels. Each frame comprises of multiple time slots as shown in Figure 1.1. A time slot and a sub-channel make a unit of allocation and is referred as mini-slot. A frame is logically divided into two parts: Uplink sub-frame and Downlink sub-frame. Uplink sub-frame is used for data transmission from subscribers to the base station while downlink sub-frame is used for data transmission from base station to subscriber stations. Each subscriber is allocated zero or more mini-slots, both in uplink and downlink direction in each frame [1, 2].

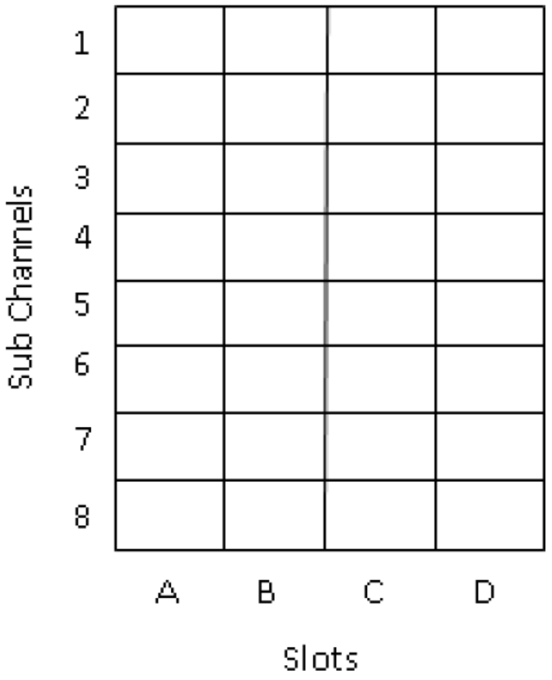


Figure 1.1: BW contention minislots

1.2 Bandwidth Request and Resource Allocation

Base station centrally allocates mini-slots to subscribers – both in uplink as well as downlink direction – using centralized scheduling and resource allocation algorithm. The allocation is communicated to subscribers using DL-MAP and UL-MAP MAC messages on per frame basis. Centralized resource allocation ensures contention free data transmissions for all subscribers. QoS enabled MAC Layer of WiMAX provides four service classes: UGS, rTPS, nrTPS and BE. UGS class guarantees a constant data rate. rTPS and nrTPS are for real-time and non-real time prioritized applications like VoIP and video streaming. BE class behaves like ordinary Internet.

Subscribers having connections belonging to UGS service class are allocated constant number of resources both in uplink and downlink direction, irrespective of their actual data demand. Subscribers with connections belonging to rTPS class are allocated constant resources based on their demand which is known to base station for downlink direction. For uplink direction, polling opportunities are provided to subscribers to establish their demand. For nrTPS and BE service class, the resources in uplink direction are allocated depending upon the demand of subscribers and available resources using Demand Assignment Multiple Access (DAMA) mechanism.

According to DAMA, Whenever a subscriber has data that it wants to transmit to base station (Internet), it should request for uplink resources from base station through a bandwidth request. Upon receiving the bandwidth request, base station allocates appropriate number of mini-slots in the subsequent frames, depending upon the number of resources available. Consequently, the actual data is always transmitted in a contention free manner. However, the transmission of bandwidth requests is not contention free and is based on a combination of Carrier Sense Multiple Access (CSMA) and Code Division Multiple Access (CDMA). All subscribers send their bandwidth requests during a set of four mini-slots within the uplink frame. One out of eight codes can be used to transmit the bandwidth request. The combination of mini-slots and codes results in 32 transmission opportunities (TO) for subscribers in each frame. To avoid collision, subscribers use CSMA in conjunction with binary exponential backoff algorithm.

1.3 Resource Allocation Probability for Bandwidth Request

Bandwidth request and resource allocation mechanism in WiMAX suggests that resource allocation to a given subscriber connection is dependent upon two factors. First, a subscriber should be able to successfully transmit the bandwidth request without collision. If collision free bandwidth request transmission is not achieved, base station will have no means to know about the resource requirement of the subscriber. Second, the resources must be available with base station in the subsequent frames.

Experience from multiple WiMAX operators (names excluded because of NDA agreements) suggests that only limited number of subscribers can be accommodated in a single frame, resulting in starvation for a number of users. This observation suggests that resource allocation probability in WiMAX is low. In this thesis, we investigate the poor performance of bandwidth request mechanism as a possible cause of low resource allocation probability and consequently limited number of users per frame.

1.4 Thesis Contributions

This thesis is based on the hypothesis that, "limited number of WiMAX users accommodated per frame is caused by low resource allocation probability, which in turn is caused by high collision probability of bandwidth requests."

Our contributions can be summarized as follows.

- We have developed analytical model for bandwidth request and allocation mechanism in WiMAX.
- We use the analytical model to investigate the impact of different parameters on grant probability in WiMAX.
- We conclude that for more than 70 users per base station sector, the probability that users will successfully be granted resources is significantly low. Consequently only a limited number of users can be accommodated in the system.

- Wait time for receiving a response from base station, after transmitting a bandwidth request, is a major factor that limits the grant probability. Our analysis shows that this value should be reduced from 20 frames to 5 frames for optimal results.

1.5 Thesis Organization

Rest of the thesis is organized as follows. Chapter 2 reviews the basics of WiMAX and the related work that analyzes the performance of WiMAX. In Chapter 3, we model the bandwidth request and allocation mechanism in WiMAX using a two dimensional Markov chain and derive the expression for bandwidth request grant probability in terms of the parameters affecting the bandwidth request mechanism. In Chapter 4, we evaluate the analytical model and investigate the dependence of grant probability on different parameters. Chapter 5 concludes the thesis.

Chapter 2

Literature Review

In this chapter, we will go through the basics of WiMAX networks that are relevant to this thesis. We will also review the related literature where performance of WiMAX networks have been evaluated.

2.1 Bandwidth Request Mechanism in WiMAX

WiMAX network can operate in point-to-point and point-to-multi-point mode. Focus of this thesis is point-to-multi-point mode. In this mode, multiple subscribers (SS) are connected with a single base station (BS). The entire network infrastructure consists of multiple base stations connected directly or indirectly with Internet, each serving multiple subscribers. The communication from base station to subscriber is known as downlink while the communication from subscribers to base station is known as uplink.

Subscribers can transmit data in uplink direction, only if they are allocated bandwidth in uplink direction. Subscribers generate explicit bandwidth requests for bandwidth allocation. Uplink access and bandwidth allocation is achieved in one of the four ways: (i) Unsolicited bandwidth grants where dedicated slots are reserved for the uplink and downlink of UGS connection for an SS without making any request. Unsolicited grants are useful in the applications that require fixed data rate. (ii) Piggybacked bandwidth request where SS uses data slots allocated to it for transmitting bandwidth request. (iii) Unicast polling sometimes referred as simply polling, where BS grants limited bandwidth to the SS for the purpose of making bandwidth requests. (iv) Contention based bandwidth requests. The fourth category is the focus of this thesis and will be explained in detail in subsequent sections.

An uplink grant is defined as the right of an SS to transmit data in the allocated set of slots (frequency time resource). The BS gives grants after the receipt of a bandwidth request. There are two possible types of bandwidth requests/grants; Incremental and aggregate bandwidth requests. When a BS receives an incremental bandwidth request it adds the quantity of the bandwidth requested into the current perception of the bandwidth need of the connection. When a BS receives an aggregate bandwidth request, it replaces the current bandwidth need of the connection with the requested bandwidth.

2.1.1 Contention Based Bandwidth Request Mechanism

Contention based bandwidth requests are transmitted by subscribers during a specific set of slots in each uplink subframe. These slots are known as the contention slots. Using contention slots a SS asks BS for bandwidth allocation in UL slots for its data transmission. If the BS receives the request in this contention slot, it responds based on the QoS class and the connection of the SS, network state and the contention algorithm. The response is in the form of bandwidth grant (allocating slots in the uplink), in which the SS can send data. Multiple subscribers can send their bandwidth request in same contention slots. For this reason contention resolution algorithms are applied to select a slot in which a SS should send its request. This mechanism results in a fair distribution of bandwidth without using a dedicated slot for individual SSs. If an SS does not receive a grant within a specified time, also known as timeout interval (or sometimes called as contention based reservation time out), it assumes that a collision has occurred and goes for contention resolution algorithm before requesting again.

2.1.2 Contention Resolution Algorithm

The method of contention resolution in WiMAX standard is based on a truncated binary exponential backoff with the initial backoff window and maximum backoff window values selected by the BS and can be changed if needed. When an SS has to transmit a bandwidth request, it selects a random value between 0 and $W_0 - 1$ ($W_0 = 2^4 = 16$). SS waits for the selected number of slots before transmitting the request. After the request transmission, SS waits for a specific number of frames for the grant of bandwidth from BS. If grant is not received during the specified number of frames, the SS goes to next backoff stage by selecting the random value between 0 and $W_i - 1$ where i is the backoff stange and $W_i = W_0 * 2^i$. Maximum value of backoff stages is 16. If a bandwidth grant is not received after 16 attempts, the SS reports the error to higher layers and reconnection procedure is initiated.

2.2 Related Work

We now discuss the set of literature where bandwidth allocation mechanism similar to the one used in WiMAX has been evaluated.

Heusse et. al. [3] has used DCF access mechanism for data rate adjustment in IEEE 802.11 radio based networks. If a host detects repeated unsuccessful transmissions it lowers its bitrate. If there is at least one host with a lower rate, then 802.11 performs performance anomaly: the throughput of all the hosts is reduced below the lower rate. This behavior is the result of the basic CSMA/CA channel access mechanism in which the long term channel capture probability for all users is equal. Analysis in this paper shows that throughput of 802.11b is much less than the supposed bit rate, and throughput significantly depends upon number of competing hosts. Studies presented in [3] are not much related to our work but this work lays a foundation for basic CSMA/CA and its effect on throughput.

Bianchi [4] has carried out performance analysis of the DCF scheme, in assumption of ideal channel conditions and finite number of terminals. It provides a simple model that accounts for all the exponential backoff protocol details and allows computing the saturation throughput performance of DCF for both standardized access mechanism and also for the combination of the two methods. The model presented in this paper is based on the assumption that the collision probability is constant and independent. This leads to extremely accurate results especially when the number of stations in wireless LAN is fairly large (say greater than 10). This work gives the optimal value of throughput for specified transmission probability, contention window size and number of stations, through a couple of simulations. Our model is based on the same basic assumption and can be considered as the extension of the model presented by Bianchi in [4]. Bianchi proposed the model for 802.11 which cannot be directly mapped onto 802.16 mainly because in 802.11 channel is accessed for each packet transmitted whereas in 802.16 channel is accessed based on some vendor algorithm. Hence further enhancements were required for 802.16 which resulted in our proposed model.

Fallah et al [5] have addressed the problem of bandwidth request contention resolution. Their model is also inspired by the model presented by Bianchi in [4] and is a 2-D markov chain model in 2 variables but the model sates are not all discrete. However, it gives an insight into dependence of grant request onto number of users, number of requests, collision probability and various other parameters. The dependence of throughput onto above mentioned parameters is proved through simulations by varying one of the parameter while keeping rest of the parameters as constant. Gebali et al [6] gives a study of contention resolution strategies using collaborative modulation codes.

Hwang et al [7] has analyzed truncated binary exponential backoff in terms of distributed delay of request packets and loss probability assuming request arrival to be Bernoulli distributed. The work concludes by finding optimal number of transmission opportunities while satisfying quality of service QoS parameters and delay bound and loss bound. Polling schemes are compared on the basis of arrival probability of requests and it is shown numerically that truncated binary exponential backoff performs better than polling when the request arrival probability is small and vice versa. Focus of this work is on finding optimal transmission opportunities with limitations on delay and loss whereas our focus is on finding optimal transmission opportunities with limitations on loss and bandwidth wastage. However work in [7] is helpful in detailed analysis of truncated binary exponential backoff mechanism.

He et al [8] proposed an analytical model for truncated binary exponential backoff mechanism for IEEE 802.16 and modeled bandwidth efficiency and mean delay as functions of the network and scheme parameters such as contention window size and number of slots allocated for bandwidth request and data transmission. This model is an analytical mapping of Bianchi's model in [4] for 802.11 onto 802.16. This analytical model is based on the assumption of constant and independent collision probability and resource request probability. [8] provides assistance while configuring design and scheme parameters for slot allocation in 802.16. Bianchi et al [9] propose an adaptive backoff window protocol for 802.11 to enhance binary exponential backoff by eliminating the dependency on initial backoff window size. It selects the optimal backoff window size depending upon the number of contending stations and overall load on the system. The work shows that the system performance is further enhanced by considering RTS/CTS along with CSMA.

Hoymann [10] has discussed OFDM based transmission mode with MAC physical layer in detail, especially MAC frame structure is elaborated. Interaction of fragmentation and padding of OFDM symbols and its effect on system capacity is evaluated. Different MAC layers with different level of robustness are analyzed and system is optimized while maintaining necessary robustness. Heusse et al [11] shows a tradeoff between throughput enhancement and short term fairness , adaptation to channel conditions, or handling multiple bit rates. Gusak et al [12] showed experimentally that average packet queuing delay is same for weighted round robin and weighted fair queuing scheduling algorithm. Adapting uplink and downlink ratios as network load and channel condition vary, also play a crucial role. This paper proposes an algorithm for adaptive frame partitioning for downlink and uplink channel and investigates its performance. It also studies the performance of a network as a function of changing transmitting environment, maximum segment size and the duration of the 802.16 frame.

Gosh et al [13] has discussed that WiMAX competitiveness in the market place largely depends upon the actual data rates and ranges that are achieved. Based on the extensive studies, this article presents the realistic attainable throughput and performance of expected WiMAX compatible systems based on

802.16d. Ziouva et al [14] evaluate throughput and delay under traffic conditions. They take into account the busy medium conditions and how they affect the use of backoff mechanism. Bianchi did not take into account busy network conditions for invoking backoff procedure so Bianchi [4] results would have less accuracy compared to the results in this paper.

Cicconetti et al [15] verified the effectiveness of rtps, nrtps and BE in managing traffic generated by that and multimedia sources for a point to multipoint mode. Yang et al[16] discussed that since BE traffic does not have any specific delay or bandwidth requirement, high utilization and fair BW sharing are the major concerns of BE scheduling bandwidth allocation without request scheme is evaluated. This paper reveals following significant observations: (i)Performance of the BE traffic in IEEE 802.16 networks is seriously impacted by the request collision rate. (ii) For a given number of request slots and SSs, the collision rate can be effectively reduced. (iii) nrTPS can be implemented as a simple extension of BE class without additional cost or efforts. BE is allowed to use only contention based request, that is, there are several slots shared for BW request and each BE connection contends for sending its request to the BS via the shared slots. Nrtps is basically the same as BE except that it may have additional BW through non-periodic pollings. In this paper, a relationship between the number of request slots and the collision rate is developed and the realized throughput as a result of that relation is evaluated.

Cho et al [17] proposed a bandwidth allocation and admission control policy: An SS sends a connection request in signaling connection; BS sends a response accepting or rejecting the request; If yes, BS notifies BS scheduler and traffic management module. Sudarev et al [18] proposed an analytical model based on discrete time markov chain model to compute essential performance characteristics. DCF is used for contention services. It is observed that wireless network performance strictly depends upon backoff interval selection.

Contention resolution in IEEE 802.16 is similar to the contention resolution in IEEE 802.11. Therefore, a detailed study of CSMA/CA, exponential backoff procedures and their effects on throughput, loss and delay is significantly related to this thesis. However, as we shall later see in this thesis, the effect of wait time after transmission of a bandwidth request is a major factor that affects the performance of bandwidth request mechanism. In IEEE 802.11 MAC, devices do not wait for the the grant unlike WiMAX subscribers. The effect of wait time for WiMAX networks has not been studied in detail. This thesis models and evaluates the effect of all parameters including wait time in detail.

Chapter 3

Analytical Model for Bandwidth Allocation in WiMAX

Having developed a basic understanding of bandwidth allocation mechanisms in WiMAX, we now model the contention based bandwidth request and allocation mechanism as markov chain model. The behavior of a user is modeled to find out grant probability in terms of available resources and probability of no collision. This model will help us study the impact of different parameters on the grant probability.

3.1 Model Notations

Following notations have been used for modeling the contention based bandwidth request mechanism in WiMAX networks. Notations are summarized in Table 3.1. Let the number of users taking part in contention process be represented by n. Number of contention slots per frame that these n users are contending for is represented by x. W_0 is the initial contention window size. Let M represent the maximum number of frames that a subscriber will wait after transmitting a bandwidth request and m be the maximum number of backoff stages (number of

Table 3.1: Table of Notations

Notations	Description
n	No. of active SS
x	No. of contention slots per frame
W_0	Initial or minimum contention window size
W_i	Contention window size after i^{th} retransmission
M	Maximum number of wait frames
m	Maximum number of backoff stages
τ	Probability of transmission of an SS in a slot
q	Probability of grant for a successfully transmitted request
P_r	Probability of resource availability
R_M	No. of resources in M frames
d	Demand of user
P_{tr}	Probability of a given contention slot containing atleast one bandwidth request
P_{nc}	Probability of any given contention slot containing exactly one bandwidth request

retransmission attempts. Let τ be the probability with which a subscriber can transmit a bandwidth request in a particular contention slot and P_{tr} be the probability that there is atleast one bandwidth request transmission in a particular slot. Let P_{nc} be the probability that there is only one bandwidth request in a particular slot and P_r be the probability that there resources are available within M frames after transmission of a particular bandwidth request. Let d be demand of a subscriber. We assume uniform demand for all subscribers, however, non-uniform demand can easily be accommodated into the model. Let R_M be the number of resources (data minislots) available in M frames for allocation using contention based mechanism. Let q be the grant probability subject to the condition that bandwidth request is transmitted successfully.

3.2 Problem Statement

We intend to find out the grant probability q of a bandwidth request as a function of the parameters W_0, m, M, d, n, τ and x.

3.3 Model State Representation

Bandwidth request and grant mechanism of a single subscriber can be modeled using three state variables (i, j, k) where i represents the backoff stage, j represents the value of backoff counter and k represents the number of wait frames elapsed after transmitting a bandwidth request. We use (i, j, k) to represent a state in our Markov chain model.

3.4 Model Explanation

The model is a $2D$ Markovian model in three variables: i, j and k.The values of i range from $(0 - m)$ where m is the maximum number of retransmission attempts. j ranges from $(0-(W_i-1))$ where $W_i = 2^4 * 2^i$ because in standard initial window size is set as 16 for the first backoff stage. So when $i = 0$ the maximum value of $W_0 - 1$ would be 15. W_i depends upon i which is the number of bandwidth request attempts that have failed so far. The maximum value of m can be 12 because WiMAX standard restricts maximum window size to be 65535. k ranges from $(0 - (M - 1))$ where M is the number of frames a request can wait before its timer expires. Usually this timer is the timer $T16$ whose time period is of 100 msec. That makes M to be 20 owing to the fact that the WiMAX frame is 5ms in length.

Figure 3.1 shows the model for Bandwidth request and grant for a single station.We assume that a station is backlogged as far as bandwidth requests are concerned. Bandwidth request is transmitted in states $(i, 0, 0)$ when the backoff counter value reaches 0. After request transmission, the subscriber may receive grant in one of the subsequent $M - 1$ frames. If the grant is received, the subscriber moves to one of the state $(0, j, 0)$ $(j \in 0- > W_0 - 1)$ for next bandwidth request transmission. If grant is not received within $M-1$ frames at any stage i, the subscriber moves to one of the states $(i+1, j, 0)$ $(j \in 0- > W_{i+1}-1)$. In the following, we discuss the state transition equations in detail.

3.4.1 State Transition Equations

Let $\pi_{(i,j,k)}$ be the probability of being in state (i, j, k). The state transitions of Markov model of Figure 3.1 can be represented by five general state transition equations.

Probability of Entering State $(0, j, 0)$

When a bandwidth request is granted in any of the states $(i, 0, k)$, the SS enters in to one of the states $(0, j, 0)$ with the probability of $\frac{q}{W_0}$.

$$\pi_{(i,0,k)}\frac{q}{W_0} = \pi_{(0,j,0)}; i \in (0, m), j \in (0, W_0 - 1), k \in (1, M - 1) \tag{3.1}$$

Note that the SS chooses a random value of wait period before transmitting a bandwidth request, therefore, the probability of entering into any of the states $(0, j, 0)$ is equal.

Probability of Entering States $(i, 0, 1)$

When the SS transmits a bandwidth request in state $(i, 0, 0)$, it goes to state $(i, 0, 1)$ with probability 1.

$$\pi_{(i,0,0)} = 1.(\pi_{(i,0,1)}); i \in (0, m) \tag{3.2}$$

From here on, the station continues to wait for grant and keeps on incrementing its wait frame every time it does not get a grant in a frame.The probability of entering next frame from the current wait frame is given as;

$$\pi_{(i,0,k)}(1-q) = \pi_{(i,0,k+1)}; i \in (0,m), k \in (1, M-2) \tag{3.3}$$

Probabilty of Entering Next Backoff Stage

When a SS does not get a grant at state $(i-1, 0, M-1)$ i.e all M frames have passed and timer has expired, it moves to next backoff stage. The probability of entering into any of random wait states of next backoff stage is given by the equation:

$$\pi_{(i-1,0,M-1)}\frac{(1-q)}{W_i} = \pi_{(i,j,0)}; i \in (1,m), j \in (0, W_i - 1) \tag{3.4}$$

Probability of Entering State $(\mathbf{m}, \mathbf{j}, \mathbf{0})$

For the last backoff stage, when a SS does not get a grant at state $(m-1, 0, M-1)$ it enters into state $(m, j, 0)$ with probability $\frac{(1-q)}{W_m}$. Similarly, if the SS does not get a grant at state $(m, 0, M-1)$, it does not increment its window size any further and enters into the state $(m, j, 0)$. In other words a station enters state $(m, j, 0)$ from two states $(m-1, 0, M-1)$ and $(m, 0, M-1)$. Therefore the probability of entering state $(m, j, 0)$ is

$$\left[\pi_{(M-1,0,m-1)} + \pi_{(m,0,M-1)}\right]\frac{(1-q)}{W_m} = \pi_{(m,j,0)}; , j \in (0, W_m - 1) \tag{3.5}$$

Normalization Equation

The normalization equation is given as ,

$$1 = \sum_{i=0}^{m}\sum_{j=0}^{W_i-1}\sum_{k=0}^{M-1}\pi_{(i,j,k)} \tag{3.6}$$

We split the normalization equation into five parts where all states in each part behave in a similar fashion. These parts can be seen in the Figure 3.1.

$$1 = \sum_{j=0}^{W_0-1} \pi_{(0,j,0)} + \sum_{i=1}^{m-1} \sum_{j=0}^{W_i-1} \pi_{(i,j,0)} + \sum_{j=0}^{W_m-1} \pi_{(m,j,0)} + \sum_{i=0}^{m} \pi_{(i,0,1)} + \sum_{i=0}^{m} \sum_{k=2}^{M-1} \pi_{(i,0,k)} \tag{3.7}$$

We'll solve each part separately.

Solving Part a:

$$\begin{aligned} \sum_{j=0}^{W_0-1} \pi(0,j,0) \\ &= \sum_{j=0}^{W_0-1} \frac{W_0-j}{W_0} \sum_{i=0}^{m} \sum_{k=1}^{M-1} q\pi_{(i,0,k)} \\ &= \sum_{j=0}^{W_0-1} \frac{W_0-j}{W_0} \sum_{i=0}^{m} \sum_{k=1}^{M-1} q\left[(1-q)^{i(M-1)+(k-1)}\pi_{(0,0,0)}\right] \\ &= q\pi_{(0,0,0)} \sum_{j=0}^{W_0-1} \frac{W_0-j}{W_0} \sum_{i=0}^{m} \sum_{k=1}^{M-1} (1-q)^{i(M-1)+(k-1)} \\ &= \frac{W_0+1}{2} q\pi_{(0,0,0)} \sum_{i=0}^{m} \sum_{k=1}^{M-1} (1-q)^{i(M-1)+(k-1)}. \end{aligned} \tag{3.8}$$

Using

$$\sum_{i=0}^{m} \sum_{k=1}^{M-1} (1-q)^{i(M-1)+(k-1)} = \frac{1-(1-q)^{(M-1)(m+1)}}{q}$$

in 3.8 ,we get

$$\sum_{j=0}^{W_0-1} \pi(0,j,0) = \frac{W_0+1}{2} \pi_{(0,0,0)} (1-(1-q)^{(M-1)(m+1)}). \tag{3.9}$$

Solving Part b:

$$\begin{aligned}
\sum_{i=1}^{m-1}\sum_{j=0}^{W_i-1}\pi_{(i,j,0)} \\
&= \sum_{i=1}^{m-1}\sum_{j=0}^{W_i-1}\frac{W_i-j}{W_i}\pi_{(i,j,0)} \\
&= \sum_{i=1}^{m-1}\sum_{j=0}^{W_i-1}\frac{W_i-j}{W_i}(1-q)^{i(M-1)}\pi_{(0,0,0)} \\
&= \pi_{(0,0,0)}\sum_{i=1}^{m-1}(1-q)^{i(M-1)}\frac{W_i+1}{2} \\
&= \frac{1}{2}\pi_{(0,0,0)}\left[\sum_{i=1}^{m-1}W_i*(1-q)^{i(M-1)}\right. \\
&\qquad \left.+\sum_{i=1}^{m-1}(1-q)^{i(M-1)}\right]. \qquad (3.10)
\end{aligned}$$

Using

$$\sum_{i=1}^{m-1}W_i*(1-q)^{i(M-1)} = 2W_0(1-q)^{M-1}*\frac{1-2^{(m-1)}(1-q)^{(m-1)(M-1)}}{1-2(1-q)^{(M-1)}},$$

and

$$\sum_{i=1}^{m-1}(1-q)^{i(M-1)} = \frac{(1-q)^{(M-1)}-(1-q)^{m(M-1)}}{1-(1-q)^{M-1}}$$

in equation 3.10 will give

$$\begin{aligned}
\sum_{i=1}^{m-1}\sum_{j=0}^{W_i-1}\pi_{(i,j,0)} = \frac{1}{2}\pi_{(0,0,0)}(1-q)^{M-1}\left[\frac{2W_0*1-2^{(m-1)}(1-q)^{(m-1)(M-1)}}{1-2(1-q)^{(M-1)}}\right. \\
\left.+\frac{1-(1-q)^{(m-1)(M-1)}}{1-(1-q)^{(M-1)}}\right]. \\
(3.11)
\end{aligned}$$

Solving Part c:

$$
\begin{aligned}
\sum_{j=0}^{W_m-1} \pi_{(m,j,0)} \\
&= \sum_{j=0}^{W_m-1} \frac{W_m-j}{W_m}(1-q)\left[\pi_{(m-1,0,M-1)}+\pi_{(m,0,M-1)}\right] \\
&= \pi_{(0,0,0)} \sum_{j=0}^{W_m-1} \frac{W_m-j}{W_m}\left[(1-q)^{(M-1)(m-1)}+(1-q)^{(M-1)(m)}\right] \\
&= \pi_{(0,0,0)}\left[(1-q)^{(M-1)(m-1)}+(1-q)^{(M-1)(m)}\right] \\
&\left(\frac{W_m+1}{2}\right). \qquad (3.12)
\end{aligned}
$$

Solving Part d:

$$
\begin{aligned}
\sum_{i=0}^{m} \pi_{(i,0,1)} \\
&= 1*\sum_{i=0}^{m} \pi_{(i,0,0)} \\
&= \sum_{i=0}^{m}(1-q)^{i(M-1)}\pi_{(0,0,0)} \\
&= \pi_{(0,0,0)}\frac{1-(1-q)^{(M-1)(m+1)}}{1-(1-q)^{(M-1)}} \qquad (3.13)
\end{aligned}
$$

Solving Part e:

$$\sum_{i=0}^{m}\sum_{k=2}^{M-1}\pi_{(i,0,k)}$$

$$\begin{aligned}
&= \sum_{i=0}^{m}\sum_{k=2}^{M-1}(1-q)^{i(M-1)+(k-1)}\pi_{(0,0,0)} \\
&= \pi_{(0,0,0)}\sum_{i=0}^{m}\sum_{k=2}^{M-1}(1-q)^{i(M-1)+(k-1)} \\
&= \pi_{(0,0,0)}\sum_{i=0}^{m}(1-q)^{i(M-1)} * \sum_{k=2}^{M-1}(1-q)^{(k-1)} \\
&= \pi_{(0,0,0)}(1-q)\left(\frac{1-(1-q)^{M-2}}{q}\right) \\
&\quad \left(\frac{1-(1-q)^{(m+1)(M-1)}}{1-(1-q)^{M-1}}\right)
\end{aligned} \tag{3.14}$$

Combining 3.9, 3.11, 3.12, 3.13 and 3.14

$$\begin{aligned}
1 = &\left(\frac{W_0+1}{2}\right)\pi_{(0,0,0)}\left(1-(1-q)^{(M-1)(m+1)}\right)+ \\
&\frac{1}{2}\pi_{(0,0,0)}\left(1-q\right)^{M-1}\left[\frac{2W_0*(1-2^{(m-1)}(1-q)^{(M-1)(m-1)})}{1-2(1-q)^{(M-1)}}\right. \\
&\left.+\frac{1-(1-q)^{(M-1)(m-1)}}{1-(1-q)^{M-1}}\right]+\frac{W_m+1}{2}\pi_{(0,0,0)} \\
&\left[(1-q)^{(M-1)(m+1)}+(1-q)^{(M-1)m}\right]+ \\
&\pi_{(0,0,0)}\frac{1-(1-q)^{(M-1)(m+1)}}{1-(1-q)^{(M-1)}}+ \\
&\pi_{(0,0,0)}\left[\frac{(1-q)}{q}\left(1-(1-q)^{M-2}\right)\right]* \\
&\left[\frac{1-(1-q)^{(M-1)(m+1)}}{1-(1-q)^{M-1}}\right].
\end{aligned} \tag{3.15}$$

gives the combined normalization equation.
The probability of being in state $\pi_{(0,0,0)}$ is also equal to the probability of grant subject to the condition that a bandwidth request was transmitted in a slot. Therefore:

$$q\tau = \pi_{(0,0,0)} \tag{3.16}$$

Replacing the value of $\pi_{(0,0,0)}$ from Equation 3.16 in Equation 3.15 and simplifying, we get:

$$\begin{aligned}
1 = \quad & \left(\frac{W_0+1}{2}\right)(q\tau)\left(1-(1-q)^{(M-1)(m+1)}\right)+ \\
& \frac{1}{2}(q\tau)\left(1-q\right)^{M-1}\left[\frac{2W_0 * (1-2^{(m-1)}(1-q)^{(M-1)(m-1)})}{1-2(1-q)^{(M-1)}}\right. \\
& \left.+\frac{1-(1-q)^{(M-1)(m-1)}}{1-(1-q)^{M-1}}\right]+\frac{W_m+1}{2}(q\tau) \\
& \left[(1-q)^{(M-1)(m+1)}+(1-q)^{(M-1)m}\right]+ \\
& (q\tau)\frac{1-(1-q)^{(M-1)(m+1)}}{1-(1-q)^{(M-1)}}+ \\
& \tau\left[(1-q)\left(1-(1-q)^{M-2}\right)\right] * \\
& \left[\frac{1-(1-q)^{(M-1)(m+1)}}{1-(1-q)^{M-1}}\right].
\end{aligned} \tag{3.17}$$

Note that above equation contains only two variables q and τ as a function of parameters W_0, m and M. We need another equation of the two variables in order to solve for any one of these two variable.

3.4.2 Grant Probability

To compute the grant probability in terms of resources available, we observe that the grant probability is the joint probability that resources are available with BS and a bandwidth request is successfully transmitted.

$$\begin{aligned}\text{Grant Probability} = & \quad \text{(Probability of No Collision)} \\ & * \text{(Probability that atleast 1 resource is} \\ & \quad \text{available in M frames)}\end{aligned}$$

$$q = P_{nc} * P_r \tag{3.18}$$

Probability of successful bandwidth request transmission in one slot has been computed by Bianchi [4] in terms of the transmission probability τ. This expression can be extended to x slots as follows:

$$\begin{aligned} P_s = & \quad \frac{n\tau(1-\tau)^{n-1}}{1-(1-\tau)^n} \\ P_{nc} = & \quad (1-(1-P_s)^x), \end{aligned} \tag{3.19}$$

where x is the number of slots. Probability of resources available is given as:

$$P_r = 1 - (n-1)qd \tag{3.20}$$

Thus 3.18 becomes,

$$q = \left(1 - \left(1 - \frac{n\tau(1-\tau)^{n-1}}{1-(1-\tau)^n}\right)^x\right) * (1-(n-1)qd). \tag{3.21}$$

Solving 3.17 and 3.21 simultaneously for q will give us grant probability in terms of the parameters W_0 , m, M, x, d and n. We can now proceed with the evaluation of the contention based bandwidth request mechanism in WiMAX networks.

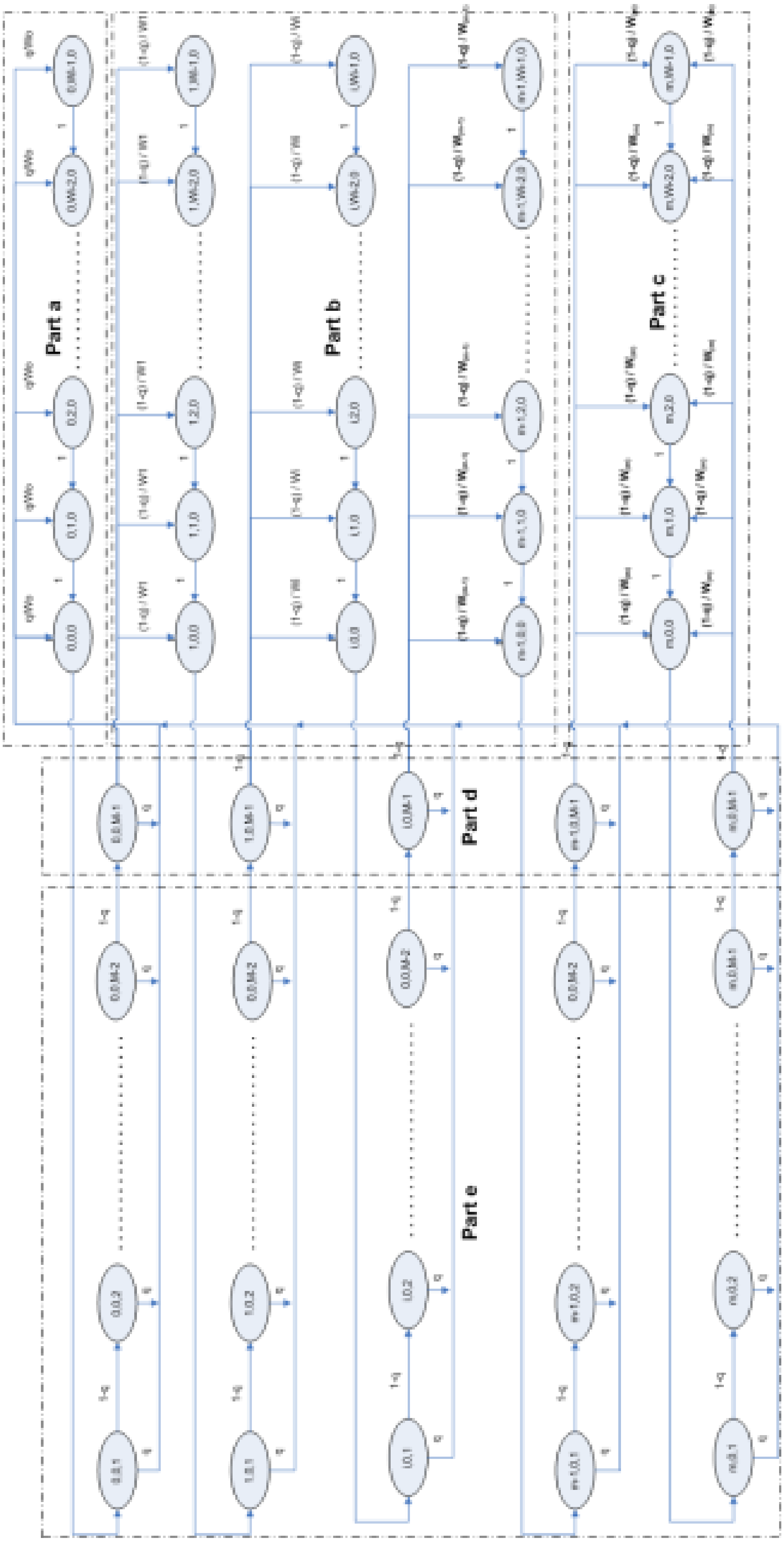


Figure 3.1: Markov Model for the BW request contention resolution

Chapter 4

Performance Evaluation

We now evaluate the performance of contention based bandwidth allocation mechanism in WiMAX networks using the equations derived from analytical model. We start observing the value of grant probability q for varying number of users and the default values for parameters W_0, m, M and x. Subsequently we study the impact of individual parameters on grant probability. Our analysis seeks the answers to following questions.

1. Which factor, probability of no collision or probability of available resources is dominant in grant probability?
2. What is the effect of varying values of different parameters on grant probability?
3. What is the number of users that a system (WiMAX Sector) can support with a reasonable grant probability?

4.1 Grant Probability with Default Parameters

In this section, we use the values $M = 20; m = 12; x = 32;$ and $W_0 = 16$, which are the default values as specified by IEEE 802.16 standard. We calculated grant probability for varying user demand and varying number of users as shown in Figure 4.1(a).

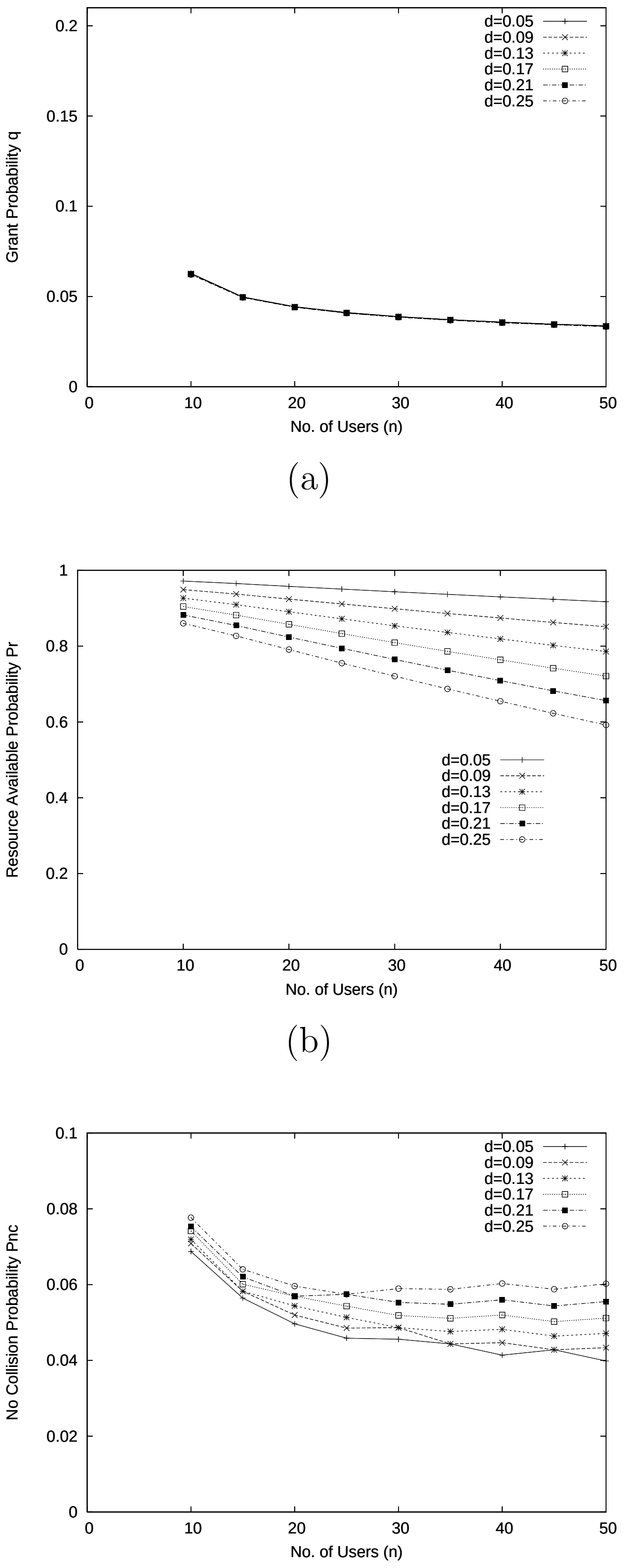
Figure 4.1(a) shows grant probability against varying number of users and different user demands. Observe that varying user demand is not affecting the grant probability. However, increasing the number of users reduces the grant probability from 0.063 for $n = 10$ to 0.033 for $n = 50$. Note that varying user demand should affect the resource availability probability. The invariant behavior of grant probability with reference to user demand in Figure 4.1(a) suggests that resource availability probability is not dominating factor in grant probability because of very small probability of successful request transmission. Figures 4.1(b) and 4.1(c) further confirm our observation.

Figure 4.1(b) shows resource availability probability against varying number of users and different user demand. We observe that resource availability probability is significantly high for all values of n and d (minimum value is 0.6). On the other hand, in Figure 4.1(c), probability of no collision is significantly low for all values of n and d, explaining the low grant probability. For all the values of d, it is observed that the grant probability is mainly dependent on the probability of no collision rather than on probability of available resources. For $d = 0.05$ and $n = 10$; P_{nc} is 0.068 and P_r is 0.97 which results in $q = 0.0629$. Similarly, for $d = 0.05$ and $n = 50$; P_{nc} is 0.0398 and P_r is 0.917 resulting in $q = 0.0338$.

Now we try to find out the parameters that are affecting grant probability.

4.2 Grant Probability with Variable $\mathbf{W_0}$

We use the values of $d = 0.05$, $M = 20$, $m = 12$, and number of slots = 32 and observe the variation in grant probability for variable values of W_0. Figures 4.2(a) and 4.2(b) show the



(a)

(b)

(c)

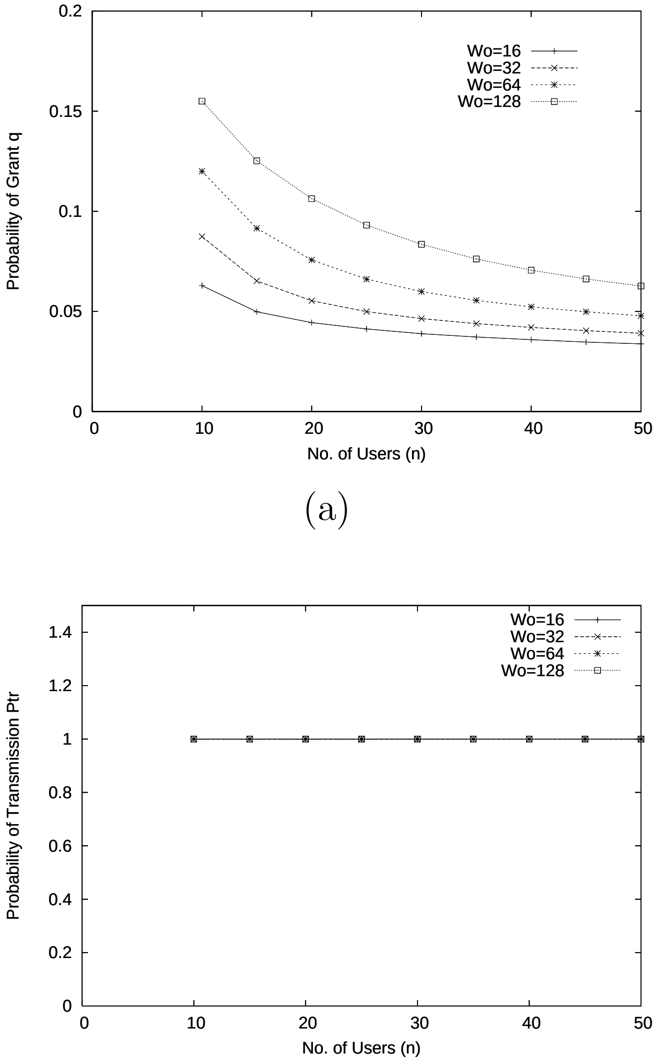
Figure 4.1: Basic Performance

resulting graphs for q and P_{tr} respectively.

It is observed from Figure 4.2(a) that for W_0 =16 and $n = 10$, q is 0.0629 which increases to 0.155 when W_0 is increased to 128. This shows that for $n = 10$, increasing W_0 from 16 to 128 is making q better by a factor of 2. For $n = 50$, q is 0.0338 which increases to 0.0627 when W_0 is increased to 128. This shows that for $n = 50$, increasing W_o from 16 to 128 is making q better again by a factor of 2. Note however, that increasing initial contention window size means the user has to wait for a longer time before attempting transmission, resulting in reduced number of bandwidth request opportunities. We confirm this by looking at the transmission probability P_{tr} for $W_0 = 16$, 32 , 64 and 128 in 4.2(b).

4.3 Grant Probability with variable M

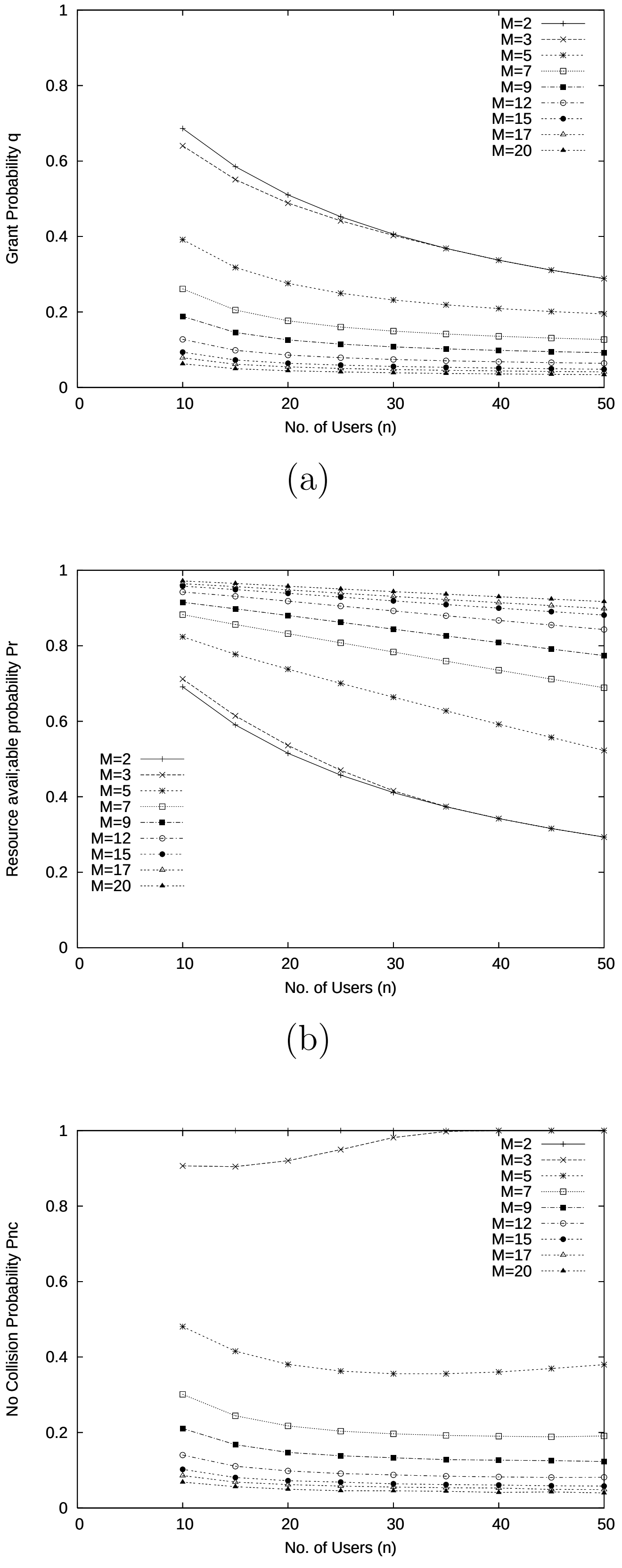
We use the values of $d = 0.05$, $m = 12$, $W_0 = 16$ and $x = 32$, and try to find out the affect of varying M on grant probability Vs number of users. Figure 4.3(a), 4.3(c), 4.3(b) show the results for grant probability, probability of no collision and probability of resources available. For $n = 10$ and $M = 20$, q is 0.0629; whereas, as we reduce M, q becomes better and at $M = 2$, q becomes 0.686 for $n = 10$. This is a ten times improvement which is a significant affect. For $n = 50$, q is 0.288 for $M = 2$, and q reduces to 0.0338 for $M = 20$. However, the value of $M = 2$ means that the user waits for 2 frames to get grant before assuming that the request is lost due to collision. Whereas, very likely it is possible that the request was received by the BS but was not granted due to absence of available resources in the next 2 frames.Therefore, to stabilizes the no collision probability and hence for realistic results we recommend $M = 5$.



(a)

(b)

Figure 4.2: Grant probability for variable W_0



(a)

(b)

(c)

Figure 4.3: Grant Probability with variable M

4.4 Grant Probability with variable x

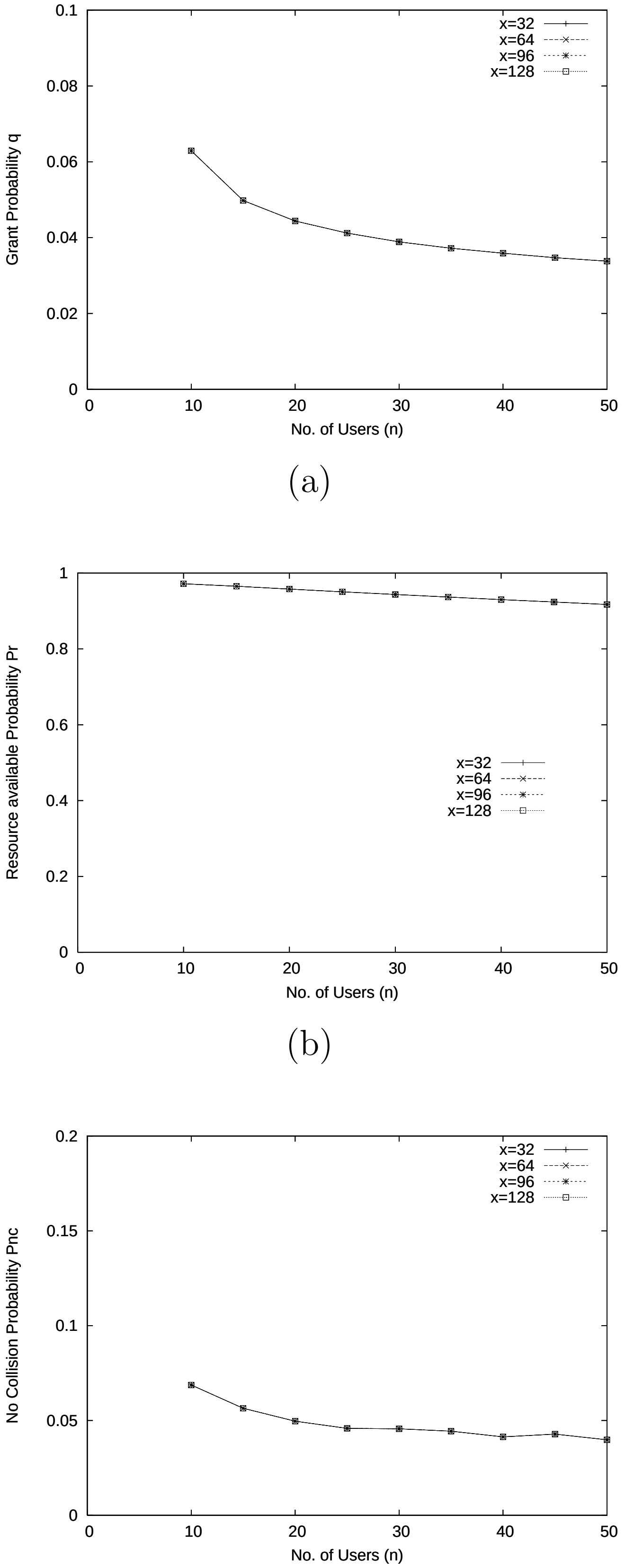
We use values of $d = 0.05$, $m = 12$, $W_0 = 16$ and $M = 20$ and try to find out the affect of varying x on grant probability Vs number of users. Figure 4.4(a), 4.4(c), 4.4(b) grant probability, no collision probability and resource available probability respectively. For $n = 10$ and $x = 32$, q is 0.0629; whereas, as we increase x, q becomes better and at $x = 128$; q becomes 0.0714. For $n = 50$, q is 0.0338 for $x = 32$, and q increases to 0.0353 for $x = 128$. This is an improvement but a negligible one. Hence we conclude that number of slots is not playing any significant role so we need not alter its value for improving grant probability.

4.5 Grant Probability with variable m

We use the values of $d = 0.05$, $W_0 = 16$, $x = 32$ and $M = 20$ and try to find out the affect of varying m on grant probability Vs number of users. Figure 4.5(a), 4.5(c), 4.5(b) show the results for grant probability, probability of no collision and probability of resources available respectively. For $n = 10$ and $m = 12$, q is 0.0629 whereas, as we decrease m, q becomes better and at $m = 3$; q become 0.209. For $n = 50$, q is 0.1607 for $m = 3$, and q reduces to 0.0338 for $m = 12$. This shows that by reducing m, q becomes better by a factor of 4.7. This is a significant improvement. Hence, we conclude that backoff stage is playing a significant role so we need to alter its value for improving grant probability.

4.6 Grant Probability with variable n

Now we intend to analyze the effect of increasing number of active users n on the grant probability. For this we use the values of the parameters as $M = 20$, $m = 12$, $x = 32$, $W_0 = 16$,



(a)

(b)

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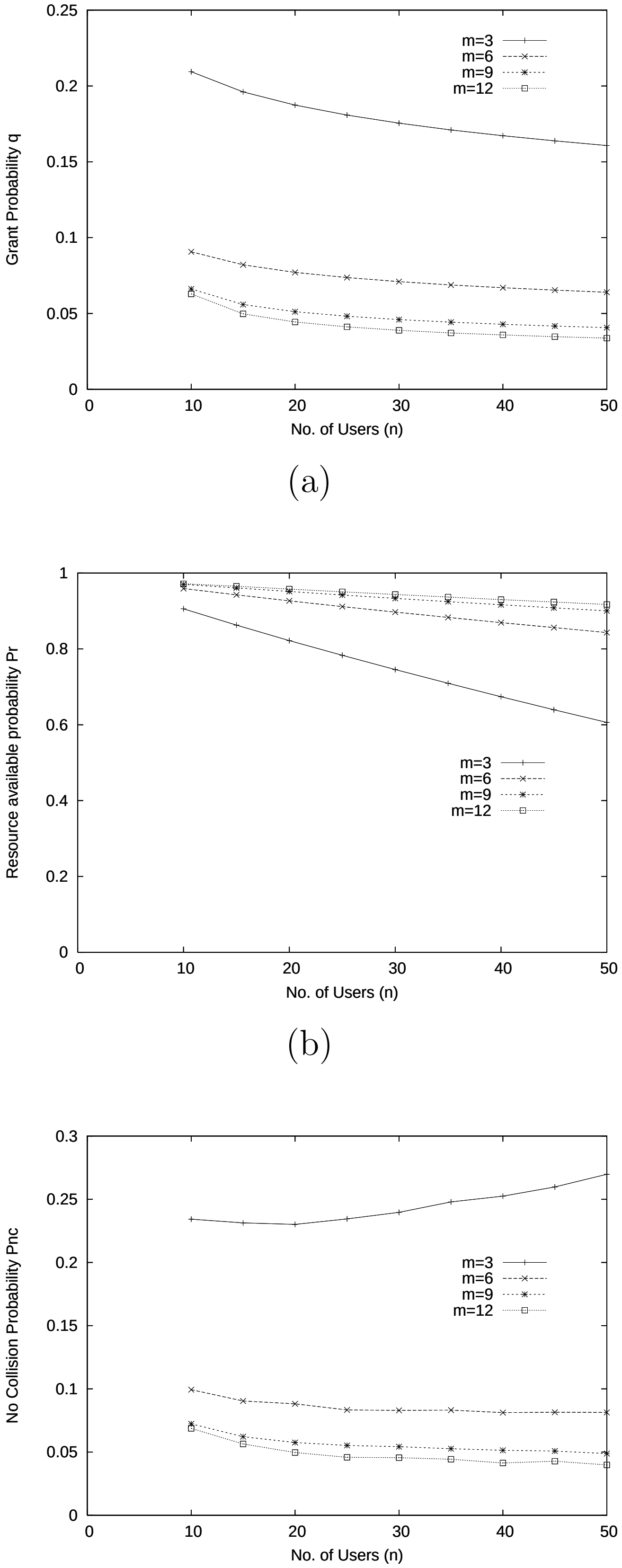
Figure 4.4: Grant Probability with variable x



(a)

(b)

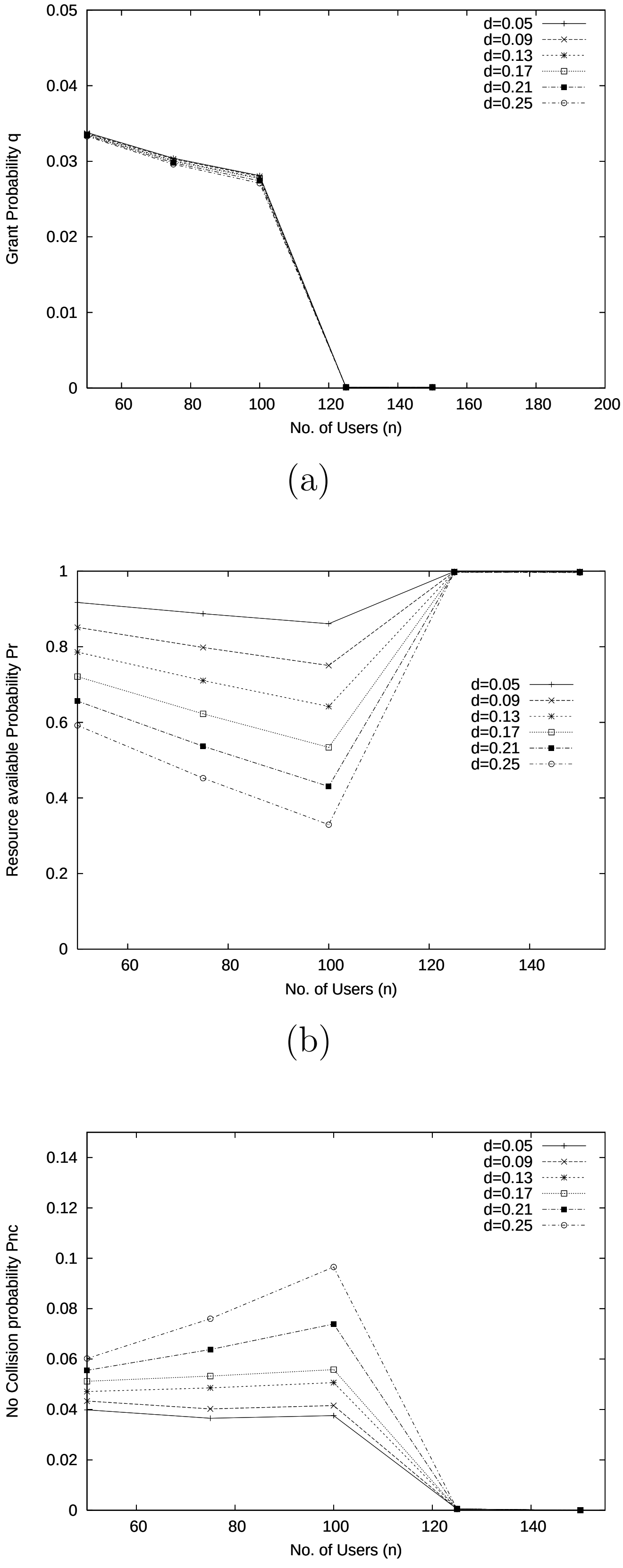
(c)

Figure 4.5: Grant Probability with variable m

and we plotted d against number of users from 50 to 150. Figure 4.6(a), 4.6(c) and 4.6(b) show the results. It is observed that as the number of users increase, q reduces significantly and at $n = 100$ probability of no collision becomes 0 and hence the grant probability drops down to zero. As a collision is unavoidable at users around 100 and above, no resources are allocated and hence the resource available probability reaches 1.

4.7 Grant Probability with variable m and large n

We now try to analyze the effect of m on number of users n when they vary from 1 to 100. For $M = 20$, $x = 32$, $W_0 = 16$ and $d = 0.05$; we varied n from 1 to 100 and found q, P_r and P_{nc}.Figure 4.7(a), 4.7(c),4.7(b) shows the results for q , P_{nc}, P_r respectively. When $m = 3$, q has a dip from 0.995 at $n = 1$ to $q = 0.2378$ at $n = 5$. It then reduces slowly until $n = 67$, however, it drops significantly as it reaches $n = 68$ where q drops from 0.158 to 0.001. This tells that with the given set of parameters, n up to 67 are accommodated with a grant probability. However, as n is increased further grant probability becomes zero which eventually results in waste of resources. When $m = 12$; q drops from 0.995 at $n = 1$ to 0.1183 at $n = 5$ and continues reducing slowly to 0.06 at $n = 9$ to 0.028 at $n = 100$. Therefore we conclude that as n increases, m should be chosen 12. However, for values of $n < 68$, $m = 3$ gives better results of grant probability.



(a)

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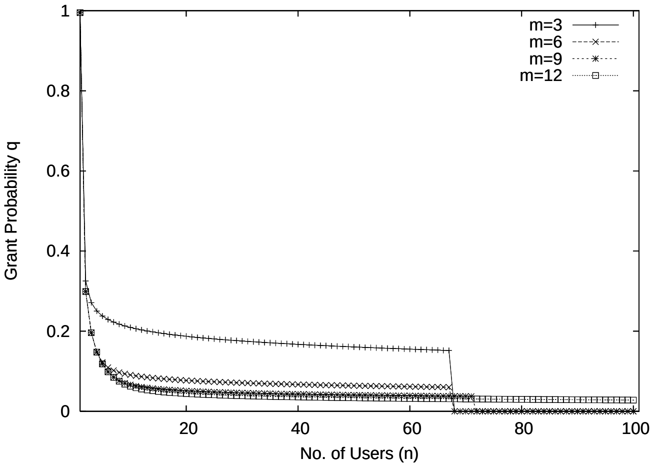
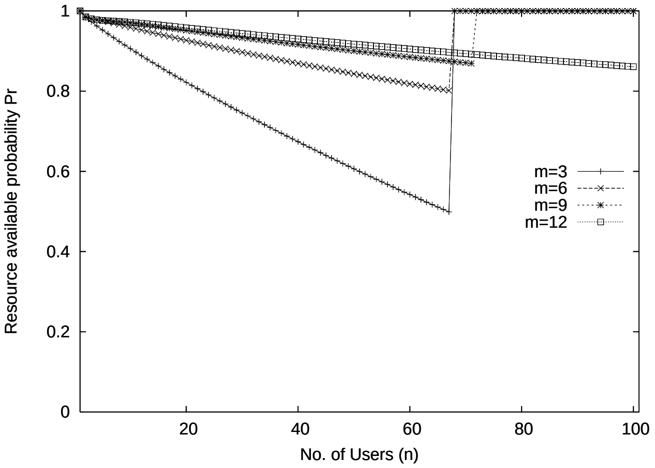
(c)

Figure 4.6: Grant Probability with variable n



(a)



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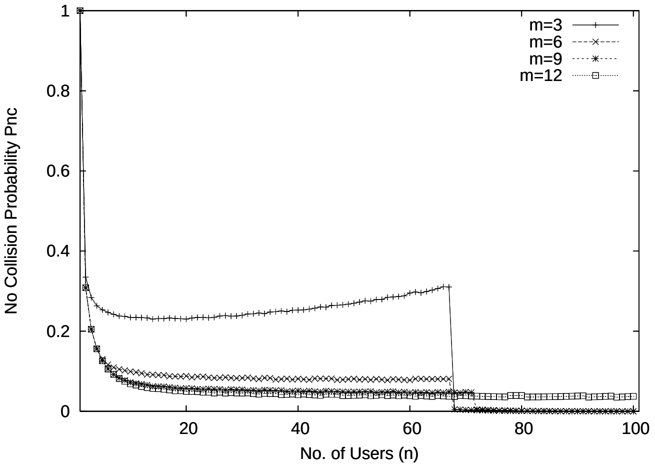


(c)

Figure 4.7: Grant Probability with variable m and large n

4.8 Grant Probability with variable m and $\mathbf{M} = 5$

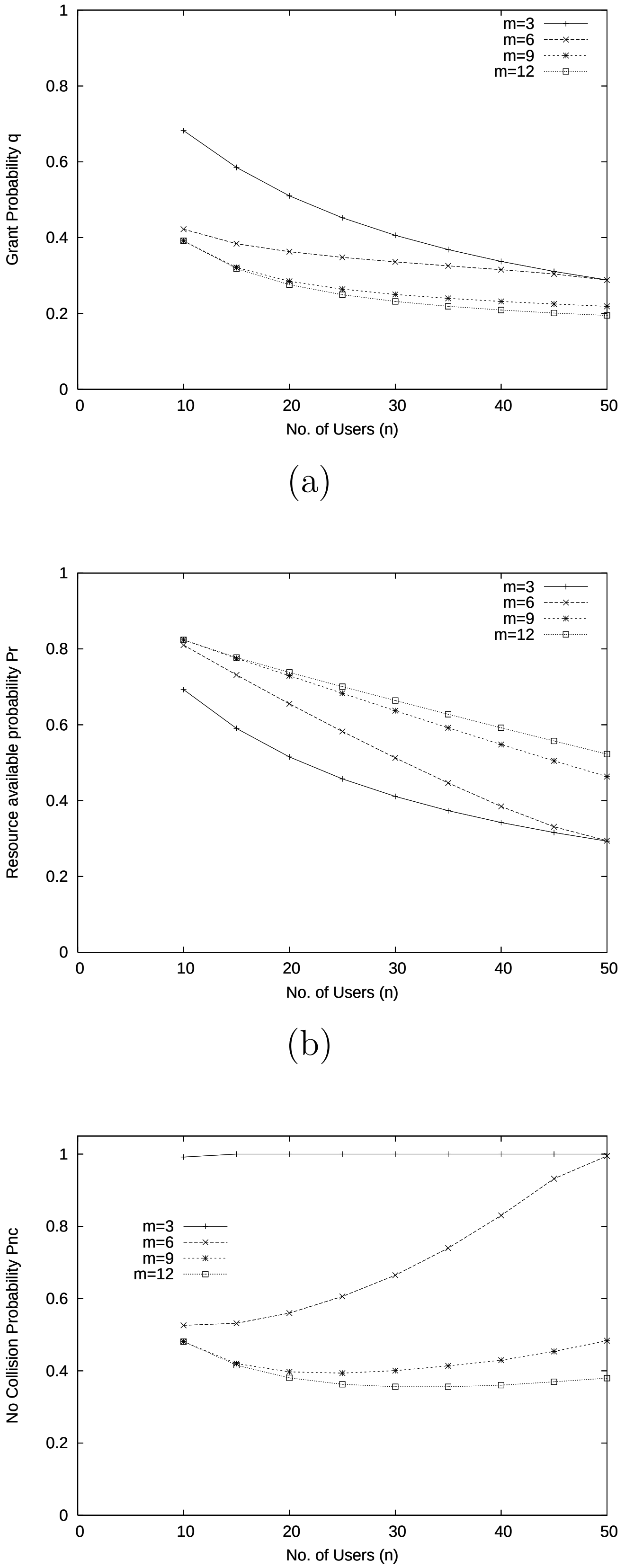
Now we try to analyze the effect of variation of both M and m on grant probability. The users n are kept 10 to 50, $d = 0.05$, $x = 32$, $W_0 = 16$ and $M = 5$. We then calculated q for $m = 3, 6, 9$ and 12. The results for q, P_{nc} and $P_r$are shown in figure 4.8(a), 4.8(c) and 4.8(b) respectively.

We are trying to observe the combined effect of reducing m and M. It is observed that as M is kept 5 for varying m, there is a significant improvement in q as compared to $M = 20$ in figure 4.5. Also for small values of m, grant probability is significantly better as obvious from figure 4.8(a).

q improved from 0.209 for $m = 12$ and $n = 10$ to 0.682 at $m = 3$. This is an improvement by a factor of 3.7. However, as n is made large, small value of m adversely affects q because of repeated retransmissions and lesser value for backoff counter in wait state, hence resulting in high probability of collision. This can be avoided by increasing m as the number of users increase.

4.9 Conclusion from Results

From a number of simulations we verified that collision of BW requests in UL contention slots is the reason for the lesser number of users accommodated than the theoretical limits. Due to collisions, BW request do not reach the BS and as a result SS do not get a grant and resources in the UL frame are wasted. It is important to note here that the DL frame has more resources than the UL frame; however, resources in DL frame are allocated when collision free requests are received in the UL contention slots. When resources in UL frame are wasted i.e contention slots go empty due to collision among the requests,



(a)

(b)

(c)

Figure 4.8: Grant Probability with variable m and $M = 5$

the DL frame resources are also wasted because of no allocation. This analysis is for common user and not for corporate users. Corporate users are treated separately. We found out that m should be kept small for small number of users, and it should be increased as the number of active users increases. Reducing frames to wait M before getting a grant improves grant probability almost ten times. Also, with a given set of parameters, maximum number of users that can be accommodated with a reasonable grant probability is also found.

Chapter 5

Conclusion

In this thesis we have presented a simple analytical model for contention resolution in BW requests of WiMAX networks as a markov model in three variables. Performance evaluation is carried out to find out the effect of varying various parameters on grant probability. The model assumes a finite number of users and ideal channel conditions. Using the proposed model we have evaluated the effect of various parameters involved on the grant probability. We have shown that the grant probability mainly depends upon the system parameters, mainly on number of backoff stages, frames to wait, initial backoff window size and the number of active users in the BW request mechanism. When m is reduced from 12 to 3, grant probability q improves by a factor of 4.7. Similarly, reducing frames to wait M from 20 to 2, q is improved by a factor of 10. However, we found that $M = 5$ gives practically realistic results and improvements. Increasing W_0 from 16 to 128 improves grant probability q by a factor of 2. However, varying contention slots x has negligibly small effect on grant probability. It is also found that with the given set of parameters,$M = 20$, $x = 32$, $W_0 = 16$, $d = 0.05$ and varying m; a maximum of 67 users can be accommodated with a grant probability. At $n = 68$ and above, probability of no collision decreases to 0, resulting in zero grant probability. Similarly, maximum number of users accomodated with any given set of parameters can be found.

The analytical model developed in this thesis can be used to develop the optimization schemes to maximize the grant probability of the system. It can also be used to maximize the efficiency of the contention process. Adjustment of the system parameters is dynamic in 802.16, therefore we can adjust the system parameters to maintain the optimal performance of the system under varying conditions. Adaptive algorithms can also be devised to maintain the optimal performance under varying load.

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